Latest results from EPOS

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1 **Glauber** and **Gribov-Regge** approach

Following two lectures given at the **ISAPP 2018**
see https://indico.cern.ch/event/719824/timetable/

for more details in lectures given at the **37th Joliot-Curie School**
https://ejc2018.sciencesconf.org/data/pages/joliot.20.pdf

2 Some **recent results** concerning **EPOS3-EPOS LHC** fusion

1 Glauber and Gribov-Regge approach

Glauber approach

Nucleus-nucleus collision A + B (including p+A)**:**

- Sequence of independent binary nucleon-nucleon collisions
- Nucleons travel on straight-line trajectories
- The inelastic nucleon-nucleon cross-section σ_{NN} is independent of the number od NN collisions

Monte Carlo version: Two nucleons collide if their transverse distance is less than $\sqrt{\sigma_{NN}/\pi}$.

Analytical formulas from geometry

Probability of one interaction (T_A, T_B normalized to 1)

$$P = \underbrace{\int T_A(b')T_B(b'-b)d^2b'}_{\text{thickness function } T_{AB}(b)} \times \sigma_{NN}$$

Having AB possible pairs: probability of n interactions :

$$P_n = \binom{AB}{n} P^n (1-P)^{AB-n}$$

Probability of at least one interaction (given b):

$$\sum_{n=1}^{AB} P_n = 1 - P_0 = 1 - (1-P)^{AB}$$

And finally the AB cross section (called optical limit):

$$\sigma^{AB} = \int \left\{1 - (1 - P)^{AB}\right\} d^2b.$$

Theoretical justification?

... based on relativistic quantum mechanical scattering theory, compatible with QCD

=> Gribov-Regge approach

Gribov-Regge approach and cut diagrams

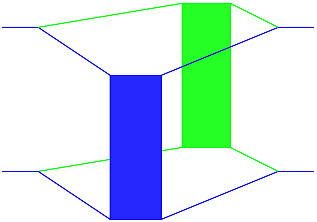
Be T the elastic (pp,pA,AA) scattering T-matrix =>

$$2s\,\sigma_{\mathrm{tot}} = \frac{1}{\mathrm{i}}\mathrm{disc}\,T$$

Basic assumption :
Multiple "Pomerons"

$$iT = \sum_k \frac{1}{k!}\left\{iT_{\mathrm{Pom}} \times \ldots \times iT_{\mathrm{Pom}}\right\}$$

(QCD hidden in T_{Pom})

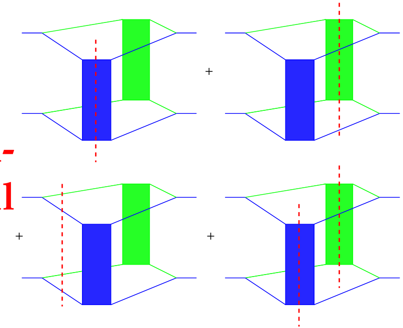


Evaluate

$$\frac{1}{i}\mathrm{disc}\left\{iT_{\mathrm{Pom}} \times \ldots \times iT_{\mathrm{Pom}}\right\}$$

using "cutting rules" :

A "cut" multi-Pomeron diagram amounts to the sum of all possible cuts



Summing uncut contributions
=> MuScatt weights, n-fold integrals (n up to 10^7)
=> Sampled via Markov chains

Gribov Regge for A+B scattering

In the GR framework, defining

$$\int dT_{AB} := \int \prod_{i=1}^{A} d^2b_i^A \, T_A(b_i^A) \prod_{j=1}^{B} d^2b_j^B \, T_B(b_j^B),$$

we obtain (neglecting energy sharing):

$$\frac{d\sigma^{AB}}{d^2b} = \int dT_{AB} \underbrace{\sum_{m_1} \ldots \sum_{m_{AB}}}_{\sum m_i \neq 0} \prod_{k=1}^{AB} \frac{W(b_k)^{m_k}}{m_k!} e^{-W(b_k)})$$

Relaxing the condition $\sum m_i \neq 0$ gives unity. Interpretation: $f = 1 - e^{-W(b_k)}$ = probability of an interaction in pp.

We get (in **GR** without energy sharing)

$$\frac{\sigma^{AB}}{d^2b} = 1 - \left\{ \int dT_{AB} \prod_{k=1}^{AB} (1 - f) \right\},$$

which corresponds to "Glauber Monte Carlo".

Similar but not equal to the "Glauber optical limit":

$$\frac{\sigma^{AB}}{d^2b} = 1 - \{1 - T_{AB}(b)\,\sigma_{NN}\}^{AB}$$

But – to be consistent –

one has to cut Pomerons!

2 Recent results

Current activities: Unification of

EPOS LHC (normal LHC observables, public)

& EPOS3 (collectivity, semi-public)

to have one "General Purpose Model",

for soft and hard physics, being publicly available

- □ Selfconsistent implementation of HF
- □ Saturation and factorization
- □ Low energies
- □ Statistical hadronization

Statistical hadronization

- EPOS 3:
  - Hydrodynamic expansion of core
  - Statistical decay of fluid
    (Grand canonical, big systems)

- EPOS LHC:
  - Effective flow, droplet decay
    (like resonance decay, small systems, small hadron list)

- "Unification": Microcanonical decay (small and big)

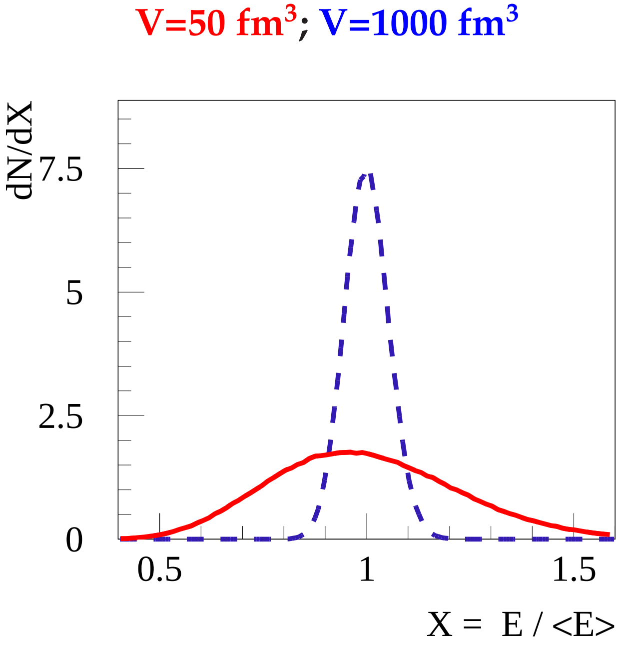
2.1 Microcanonical hadronization of plasma droplets

- **No need to match dynamical part of hydro evolution**

- **Energy and flavor conservation for small systems**

- **Needed to “unify” EPOSLHC and EPOS3**

Grand canonical decay, T = 130 MeV



Microcanonic decay

of given volume in its CMS into n hadrons

$$dP = C_{\mathrm{vol}}\, C_{\mathrm{deg}}\, C_{\mathrm{ident}} \times \delta(E - \Sigma E_i)\, \delta(\Sigma \vec{p}_i) \prod_A \delta_{Q_A, \Sigma q_{A\,i}} \prod_{i=1}^{n} d^3 p_i$$

$$C_{\mathrm{vol}} = \frac{V^n}{(2\pi\hbar)^{3n}}\,, \quad C_{\mathrm{deg}} = \prod_{i=1}^{n} g_i\,, \quad C_{\mathrm{ident}} = \prod_{\alpha \in \mathcal{S}} \frac{1}{n_\alpha !}$$

(n_α is the number of particles of species α, $\mathcal{S}$ is the set of particle species)

Different from decay rate of a massive particle (using LIPS), where asymptotic states are defined over an infinitely large volume

(see Becattini et al, EPJC35:243-258,2004). **But $E_i = \sqrt{p_i^2 + m_i^2}$**

Microcanonical decay

$$dP \propto d\Phi_{\mathrm{NRPS}} = \delta(M - \Sigma E_i)\, \delta(\Sigma \vec{p}_i)\, \prod_{i=1}^{n} d^3 p_i$$

- □ Hagedorn 1958 methods to compute Φ_{NRPS}
- □ Lorentz invariant phase space (LIPS) (James 1968)
- □ Hagedorn methods used for decaying QGP droplets (Werner, Aichelin, 1994, Becattini 2003)
- □ 2012 (Bignamini,Becattini,Piccinini) compute Φ_{NRPS} via the Lorentz invariant phase space (LIPS)

- **Hagedorn integral method can be made very efficient at large n (new),** but it is VERY time consuming at small n

- **LIPS method very fast for small n,** gets time consuming at large n

- **around $n \approx 30 - 40$ both methods work (=> checks)**

Hagedorn integral method

The phase-space integral:

$$\phi_{\mathrm{NRPS}}(M, m_1, \ldots, m_n)$$
$$= (4\pi)^n \int \prod_{i=1}^{n} p_i^2\, \delta(E - \sum_{i=1}^{n} E_i)\, W(p_1, \ldots, p_n) \prod_{i=1}^{n} dp_i,$$

with the "**random walk function**" W (angular integral)

$$W(p_1, \ldots, p_n) := \frac{1}{(4\pi)^n} \int \delta(\sum_{i=1}^{n} p_i \times \vec{u}_i) \prod_{i=1}^{n} d\Omega_i$$

New: Very efficient procedures to compute W for large *n* with high precision

We obtain (Werner, Aichelin 94)

$$\phi(M, m_1, \ldots, m_n) = \int_0^1 dr_1 \ldots \int_0^1 dr_{n-1} \psi(r_1, ..., r_{n-1})$$

$$\psi = \frac{(4\pi)^n \, T^{n-1}}{(n-1)!} \prod_{i=1}^{n} p_i \, E_i \, W(p_1, \ldots, p_n),$$

with $z_i = r_i^{1/i}$, $x_i = z_i x_{i+1}$, $s_i = x_i T$, $t_i = s_i - s_{i-1}$,
$E_i = t_i + m_i$, $T = M - \sum_{i=1}^{n} m_i$

Suitable for MC provided *W* is known

Sampling hadron configurations $K = \{h_1, \ldots, h_n; \vec{p}_1, ...\vec{p}_n\}$ via Markov chains

We construct sequences of random configurations

$$K_1\,,\ K_2\,,\ K_3, ...K_t\,, ...$$

such that $f_t(K_t)$ converges towards $f(K)$ for $t \to \infty$

with f = microcanonical probability distribution

The law changes step by step ($f_t \to f_{t+1}$) :

$$f_{t+1}(K) = \sum_{K'} f_t(K')\, p(K' \to K)\,.$$

with $p(K \to K')$ of the form

$$w(K \to K') \times \min\left(1,\, \frac{f(K')}{f(K)}\, \frac{w(K' \to K)}{w(K \to K')}\right)$$

(which guarantees convergence)

2.2 Grand canonical limit

For very large M we should recover the "grand canonical limit" for single particle spectra:

$$f_k = \frac{g_k V}{(2\pi\hbar)^3} \exp\left(-\frac{E_k}{T}\right),$$

The average energy is

$$\bar{E} = \frac{g_k V}{(2\pi\hbar)^3} \sum_k \int_0^\infty E_k \exp\left(-\frac{E_k}{T}\right) 4\pi p^2 dp$$

Changing variables via $E_k dE_k = p dp$, and using $K_1(z) = z \int_1^\infty \exp(-zx)\sqrt{x^2-1}dx$, and $3\,K_2(z) = z^2 \int_1^\infty \exp(-zx)\sqrt{x^2-1}^3 dx$,

=>

$$\bar{E} = \frac{4\pi g_k V}{(2\pi\hbar)^3} m^2 T \left(3TK_2(\frac{m}{T}) + mK_1(\frac{m}{T}) \right).$$

The microcanonical decay of an object of mass M and volume V should converge (for $M \to \infty$) to the GC single particle spectra

with T obtained from $M = \bar{E}$.

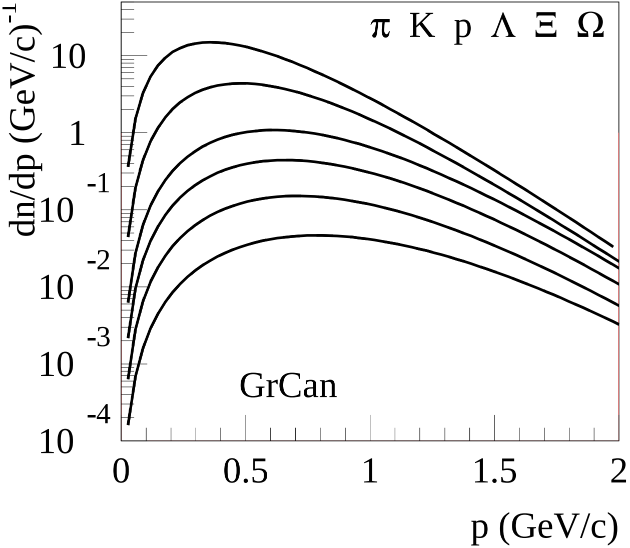
2.3 Comparing GC et MiC decay

We consider a complete (?) set of hadrons ($\approx$ 400, PDG list)

We check the effect of

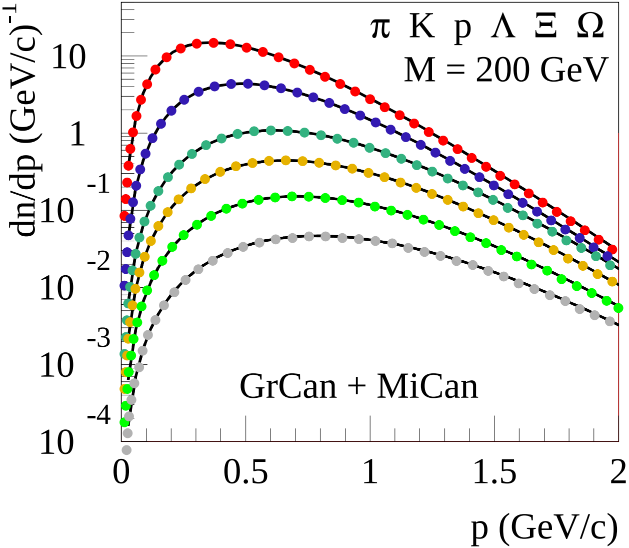
- □ **energy conservation**
- □ **flavor conservation**

GC decay, E/V= 0.333 GeV/fm^3 T=164 MeV



V=150 fm^3

GC+MiC decay, E/V= 0.333 GeV/fm^3 M=200 GeV

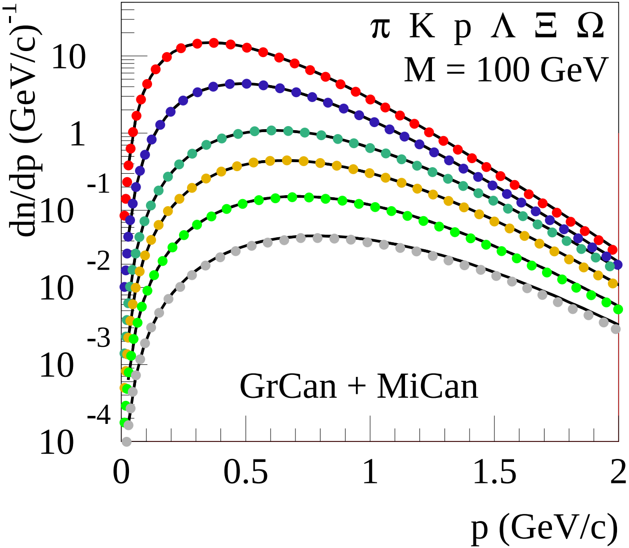


V=600 fm^3

$\times \frac{1}{4}$

good test for
Metropolis proposal

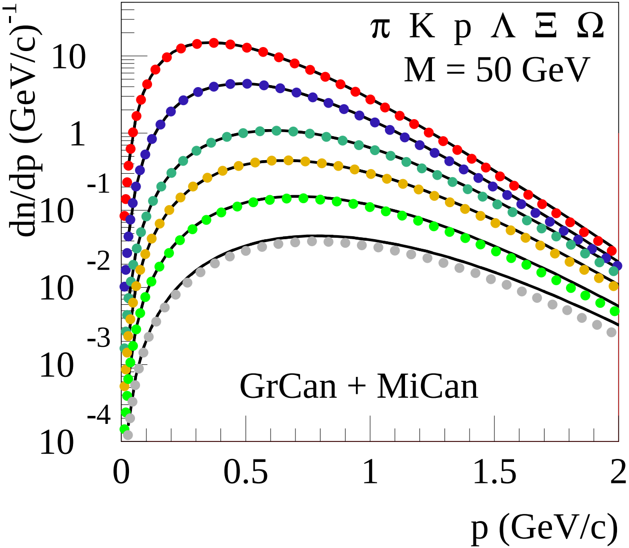
GC+MiC decay, E/V= 0.333 GeV/fm^3 M=100 GeV



V=300 fm^3

$\times \frac{1}{2}$

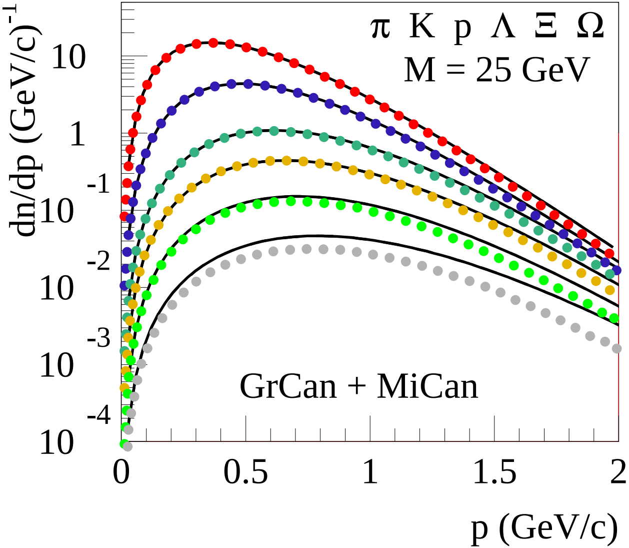
GC+MiC decay, E/V= 0.333 GeV/fm^3 M=50 GeV



V=150 fm^3

×1

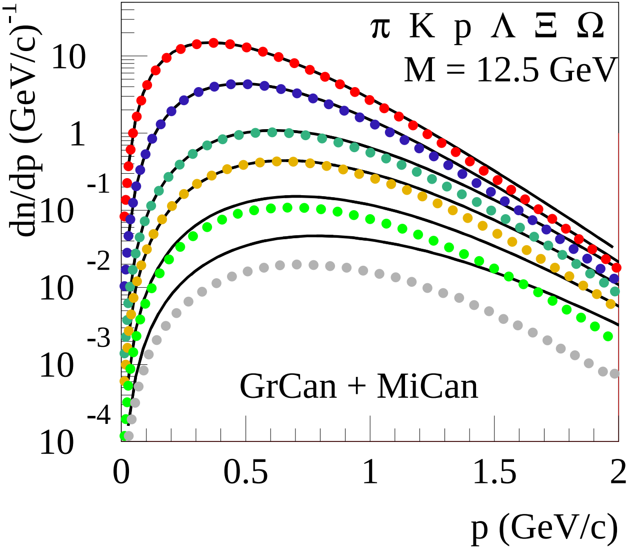
GC+MiC decay, E/V= 0.333 GeV/fm^3 M=25 GeV



V=75 fm^3

×2

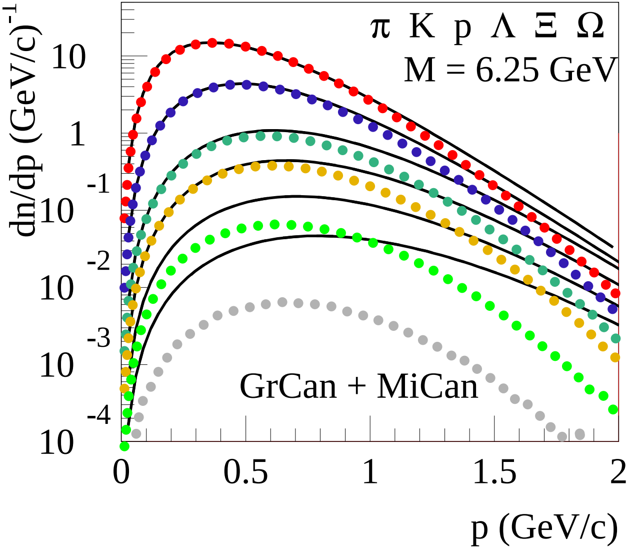
GC+MiC decay, E/V= 0.333 GeV/fm^3 M=12.5 GeV



V=37.5 fm^3

×4

GC+MiC decay, E/V= 0.333 GeV/fm³ M=6.25 GeV



V=18.75 fm³

×8

3 Summary

- **New microcanonical hadronization procedure:**
  - **Very efficient, possible for "big" systems**
  - **Works for "complete" hadron set (PDG)**
  - **Coincides with GC results for big systems**
  - **Unique procedure, for big and small systems**
  - **Todo : Incorporate flow**
- **Todo: Public unified EPOS**