35th Winter Workshop on Nuclear Dynamics, Beaver Creek, January 6-12, 2019

Quantum entanglement and and eigenstate thermalization in QCD

Dmitri Kharzeev

Based on:

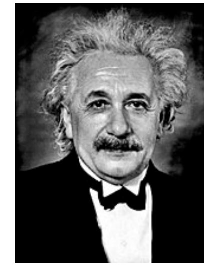
DK, E. Levin, Phys Rev D 95 (2017) 114008

O.K. Baker, DK, Phys Rev D 98 (2018) 054007

Z. Tu, DK, T. Ullrich, to appear

DK, to appear

Quantum entanglement



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

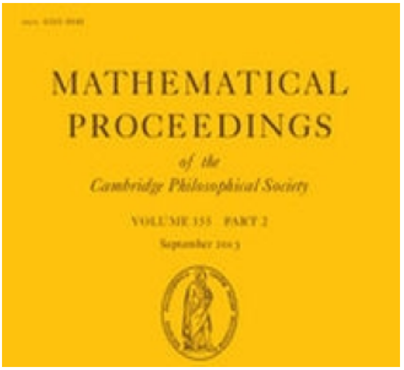
OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*
(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS



BY E. SCHRÖDINGER

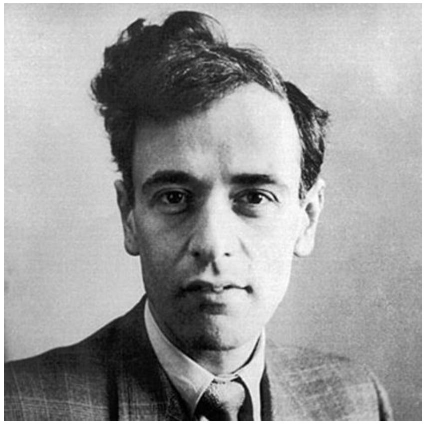
[Communicated by Mr M. BORN]

[*Received* 14 August, *read* 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one privilege: to restrict the experiments to one only of the two systems. After re-establishing one representative by observation, the other one can be inferred simultaneously. In what follows the whole of this procedure will be called *the disentanglement*. Its sinister importance is due to its being involved in every measuring process and therefore forming the basis of the quantum theory of

Describing entanglement: the density matrix

(1927)



L.D. Landau,
1908-1968

EPR state (2 qubits):

$$\frac{|0\rangle_A|0\rangle_B \pm |1\rangle_A|1\rangle_B}{\sqrt{2}}$$

J. von Neumann,
1903-1957

The corresponding density matrix:

$$\rho_{\mathrm{AB}}\left(\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\right) = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}\,\frac{\langle 00| \pm \langle 11|}{\sqrt{2}} = \frac{|00\rangle\langle 00| \pm |00\rangle\langle 11| \pm |11\rangle\langle 00| + |11\rangle\langle 11|}{2}$$

If the state of B is unknown, A is described by the reduced density matrix:

$$\begin{aligned}\rho_A &= \mathrm{tr}_B(\rho_{AB}) = \frac{|0\rangle\langle 0|\ \langle 0|0\rangle \pm \cancel{|0\rangle\langle 1|\ \langle 0|1\rangle} \pm \cancel{|1\rangle\langle 0|\ \langle 1|0\rangle} + |1\rangle\langle 1|\ \langle 1|1\rangle}{2} \\ &= \frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{\mathbf{I}}{2}.\end{aligned}$$

Mixed state! $\rho_A \neq \rho_A^2$

Das Dämpfungsproblem in der Wellenmechanik.

Von **L. Landau** in Leningrad.

(Eingegangen am 27. Juli 1927.)

Es wird eine Formel für die wellenmechanische Behandlung der Dämpfung aufgestellt. Mit ihrer Hilfe werden einige diesbezügliche Fragen untersucht; auch die Kohärenzerscheinungen finden ihre Aufklärung. Ein Ausdruck für spontane Emission wird ermittelt, und die Intensitätsfrage der Spektrallinien auf diese Weise gelöst.

§ 1. Gekoppelte Systeme in der Wellenmechanik. In der Wellenmechanik kann ein System nicht eindeutig definiert werden; wir haben es immer mit einer Wahrscheinlichkeitsgesamtheit zu tun (statistische Auffassung)[1]. Ist das System mit einem anderen gekoppelt, so tritt in seinem Verhalten eine doppelte Unbestimmtheit auf.

Der Zustand des ersten Systems sei charakterisiert durch die Größen a_n in

$$\psi = \sum a_n \psi_n; \tag{1}$$

für das zweite gelte

$$\psi' = \sum b_r \psi'_r. \tag{2}$$

Die Schrödingersche Funktion für beide Systeme zusammen ist dann:

$$\Psi = \psi\psi' = \sum_n \sum_r a_n b_r \psi_n \psi'_r = \sum_n \sum_r c_{nr} \psi_n \psi'_r, \tag{3a}$$

wo

$$c_{nr} = a_n b_r. \tag{3b}$$

Tritt eine Kopplung auf, so wird c_{nr} Funktion der Zeit und kann nicht mehr der Gleichung (3b) gemäß zerlegt werden. Es können also a_n und b_r hier nicht mehr einzeln angewandt werden.

Quantifying entanglement: von Neumann entropy

$$\rho = \sum_n p_n |n\rangle\langle n|$$

Entanglement entropy:

$$S = -\mathrm{tr}\rho \ln \rho = -\mathrm{p_n} \ln \mathrm{p_n}$$

Pure states:

$$S = 0$$

e.g.

$$p_0 = 1, \; p_{n \neq 0} = 0$$

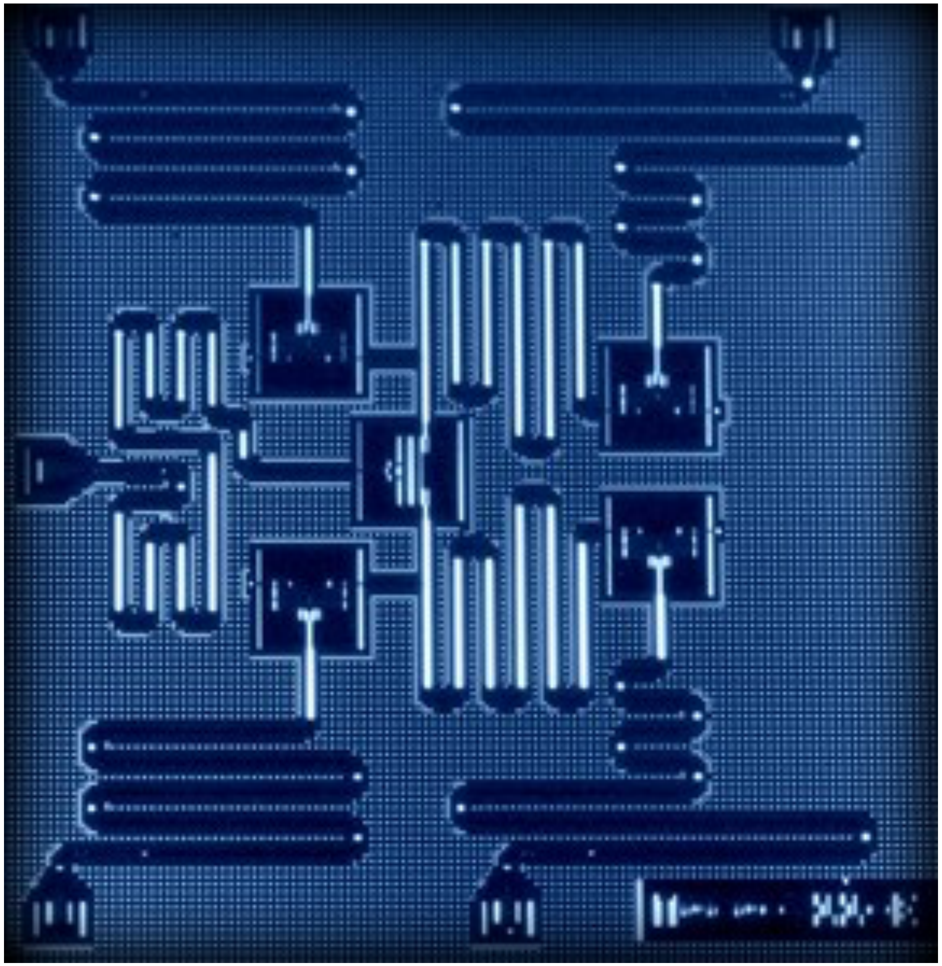
Mixed states:

$$S \neq 0$$

e.g. for EPR $\rho_A = \frac{\mathbf{I}}{2}$

$$p_0 = p_1 = \frac{1}{2} \rightarrow S = \ln 2$$

Entanglement at work: quantum computing



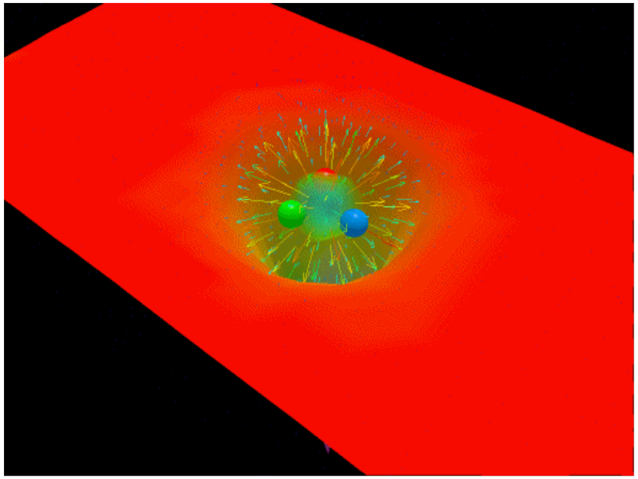
IBM five qubit processor   credit: IBM-Q

Entanglement is real.

Image credit: Evolving Science, 2018

What are the implications for the structure of the proton and the nuclei?

Color confinement is the ultimate realization of entanglement: not only the quarks are entangled, they cannot even exist as separate particles.



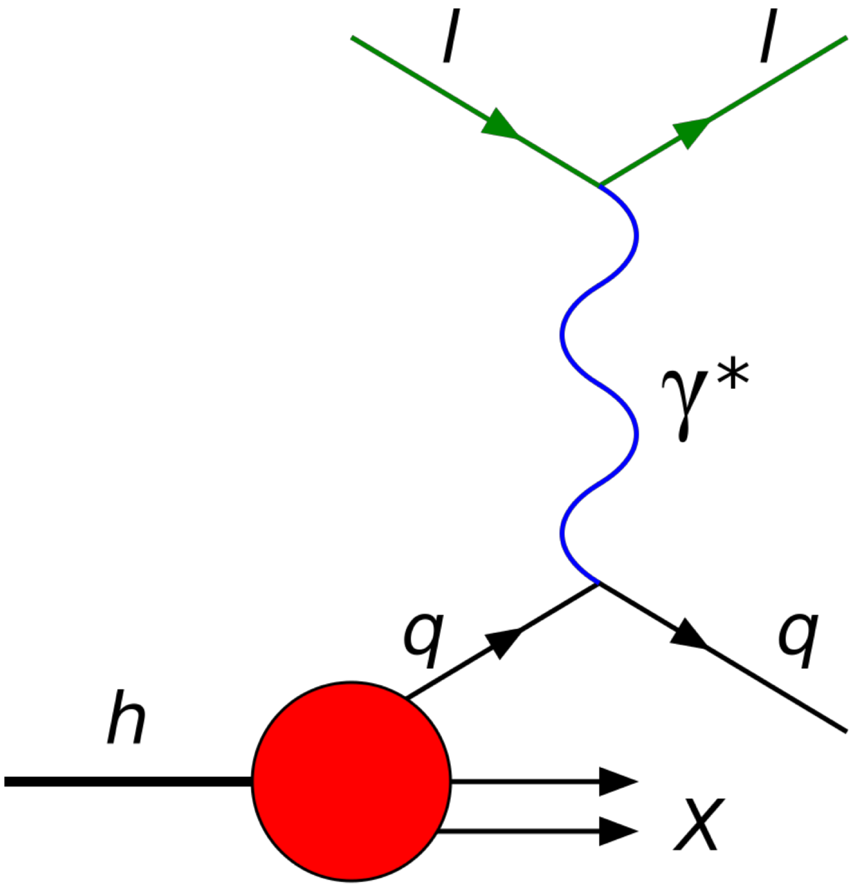
How to reconcile this with the parton model that seemingly ignores the entanglement and deals with individual partons rather than the entire proton, and with probabilities rather than amplitudes?

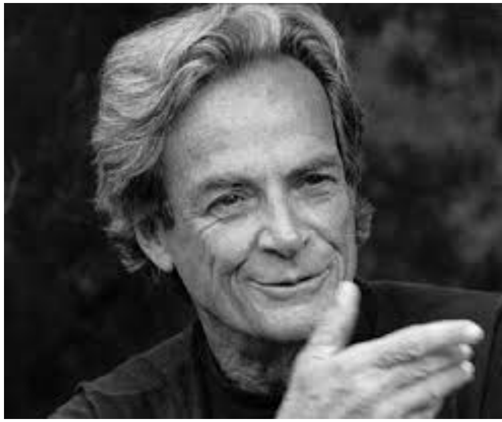
Quantum entanglement

“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences ... .”

Erwin Schrödinger, 1952

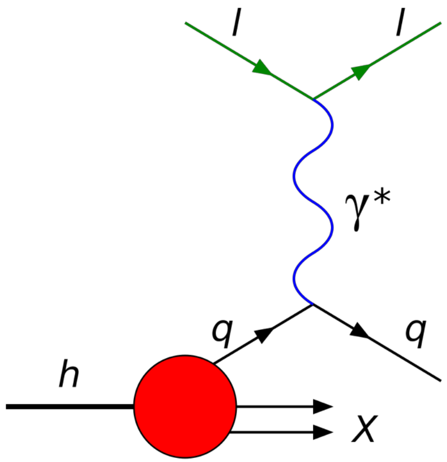
The parton model: 50 years of success





In almost fifty years that have ensued after the birth of the parton model, it has become an indispensable building block of high energy physics

The parton model: basic assumptions



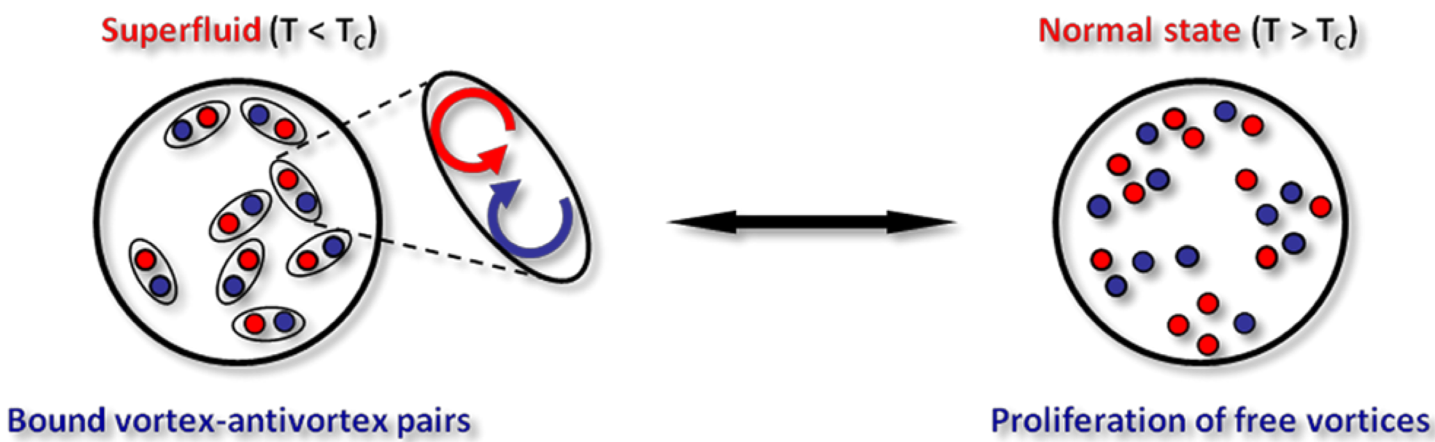
In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?

The puzzle of the parton model

In quantum mechanics, the proton is a pure state with zero entropy. Yet, a collection of free partons does possess entropy… Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems: BKT phase transition (Nobel prize 2016)

The quantum mechanics of partons and entanglement

Our proposal: the key to solving this apparent paradox is entanglement.

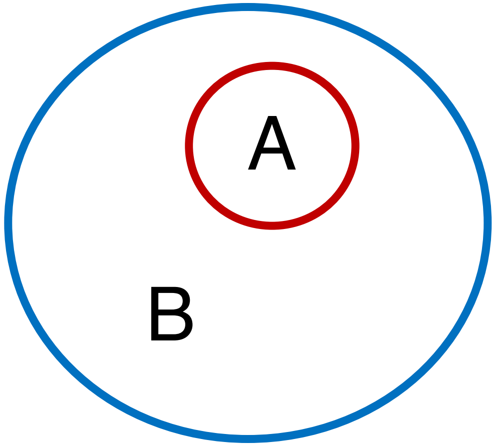
DK, E. Levin, arXiv:1702.03489

DIS probes only a part of the proton's wave function (region A). We sum over all hadronic final states; in quantum mechanics, this corresponds to accessing the density matrix of a <u>mixed state</u>

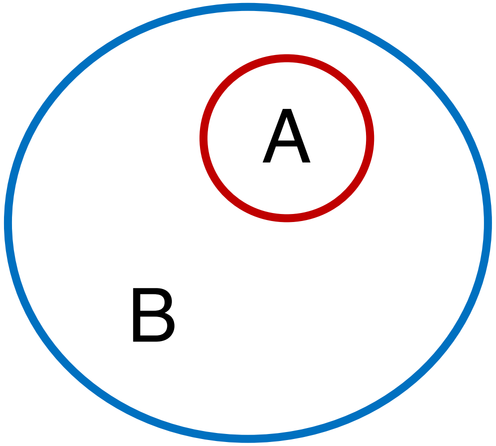
$$\hat{\rho}_A = \mathrm{tr}_{\mathrm{B}}\hat{\rho}$$

with a non-zero <u>entanglement entropy</u>

$$S_A = -\mathrm{tr}\left[\hat{\rho}_{\mathrm{A}} \ln \hat{\rho}_{\mathrm{A}}\right]$$



The quantum mechanics of partons and entanglement



The proton is described by a vector

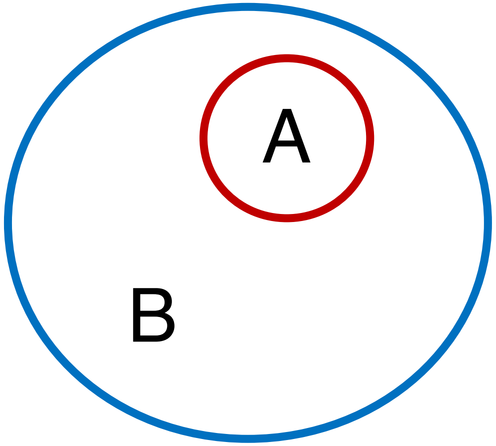
$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} \; |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$

If $|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$ only one term

contributes, then the state is separable (not our case!).
Otherwise, the state is **entangled**.

The quantum mechanics of partons and entanglement



The Schmidt decomposition theorem:

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

There exist the orthonormal states $|\Psi_n^A\rangle$ and $|\Psi_n^B\rangle$ for which the pure state can be represented as a single sum with real and positive coefficients α_n

If only one term (Schmidt rank one), then the state is separable. Otherwise, it is **entangled**; but no interference between different n's.

The quantum mechanics of partons and entanglement

We assume that the Schmidt basis $|\Psi_n^A\rangle|\Psi_n^B\rangle$ corresponds to the states with different numbers of partons n; since it represents a relativistic quantum field theory (QCD), the Schmidt rank is in general infinite.

The density matrix is now given by

$$\rho_A = \mathrm{tr_B}\ \rho_{\mathrm{AB}} = \sum_{\mathrm{n}} \alpha_{\mathrm{n}}^2\ |\Psi_{\mathrm{n}}^{\mathrm{A}}\rangle\langle\Psi_{\mathrm{n}}^{\mathrm{A}}|$$

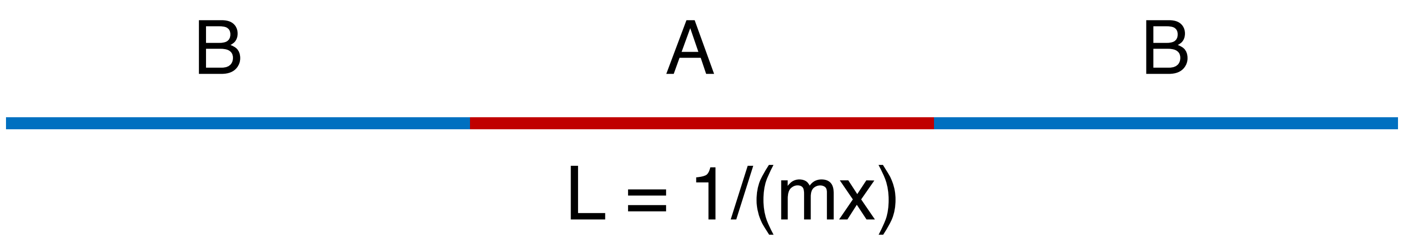
where $\alpha_n^2 \equiv p_n$ is the probability of a state with n partons

The entanglement entropy

The entanglement entropy is now given by
the von Neumann's formula

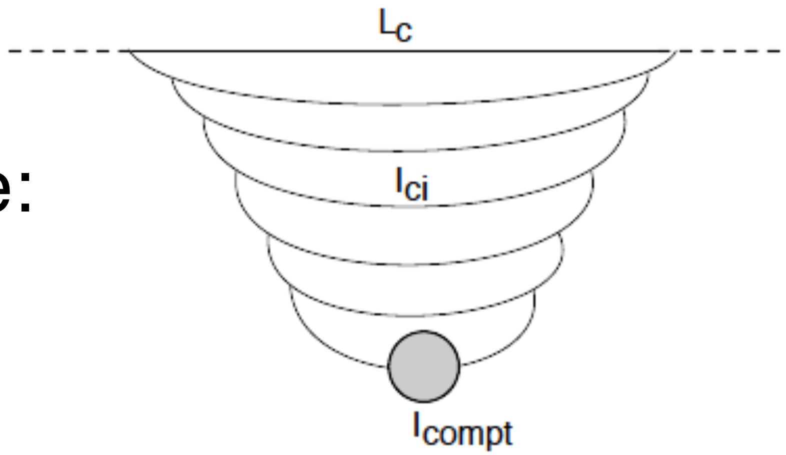
$$S = -\sum_n p_n \ln p_n$$

and can be evaluated by using the QCD evolution equations.
Let us start with a (1+1) case, followed by (3+1).



The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) \;+\; (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta\, n\, P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method

(A.H.Mueller ‘94; E.Levin, M.Lublinsky ‘04):

$$Z(Y,u) = \sum_n P_n(Y) u^n.$$

Solution:

$$P_n(Y) = e^{-\Delta Y}(1 - e^{-\Delta Y})^{n-1}.$$

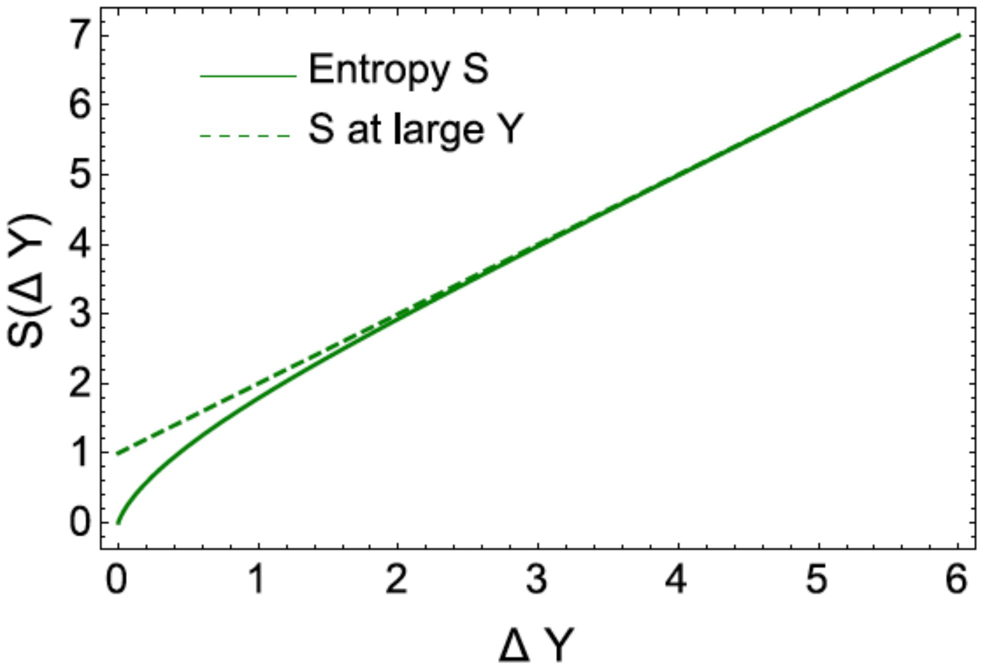
The resulting von Neumann entropy is

$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln\left(\frac{1}{1 - e^{-\Delta Y}}\right)$$

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \to \Delta Y$$



This “asymptotic” regime starts rather early, at

$$\Delta Y \simeq 2$$

The entanglement entropy from QCD evolution

At large ΔY (x ~ 10^{-3}) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\Delta}$$

becomes very simple:

$$\boxed{S = \ln[xG(x)]}$$

The entanglement entropy from QCD evolution

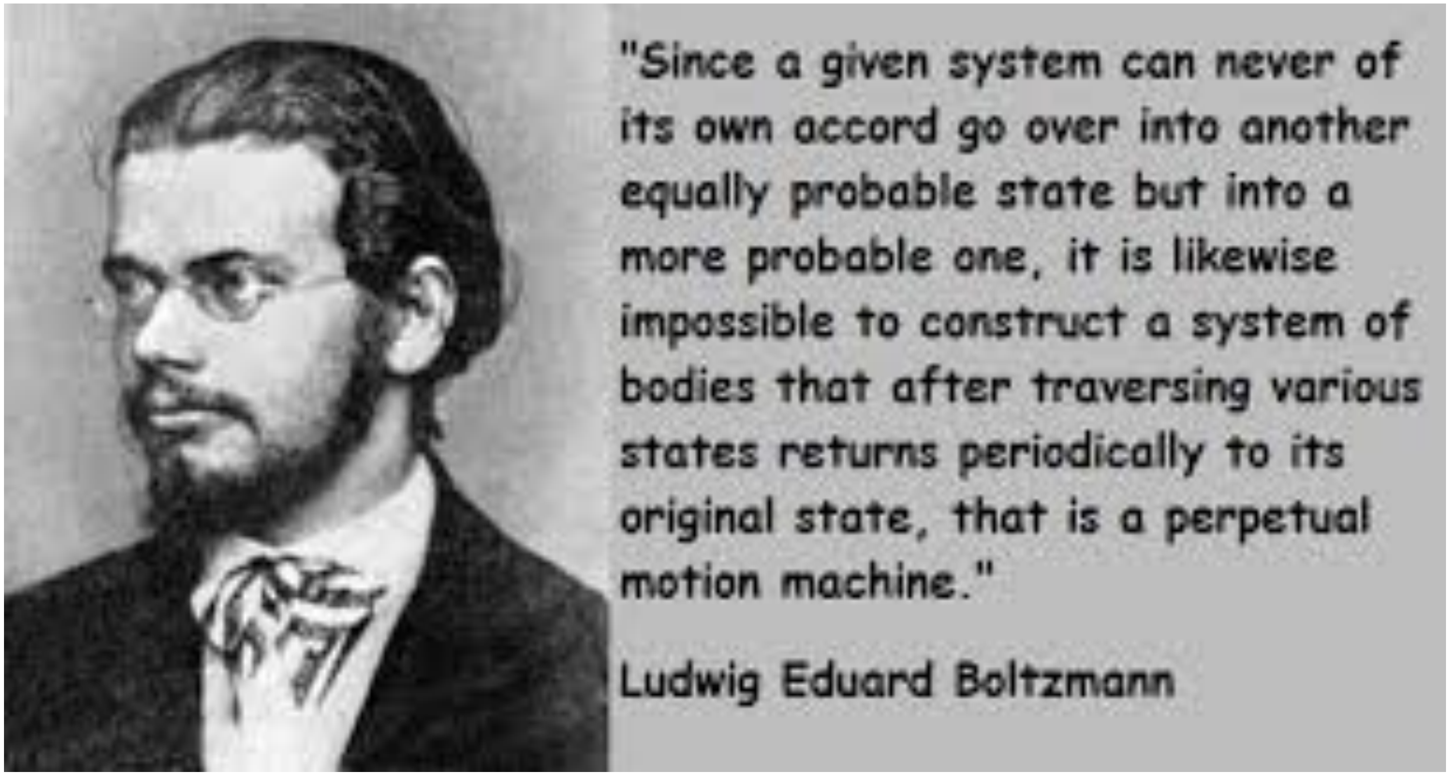
The (3+1) case is cumbersome, but the result is the same, with $\Delta = \bar{\alpha}_s \ln(r^2 Q_s^2)$

What is the physics behind this relation?

$$S = \ln[xG(x)]$$

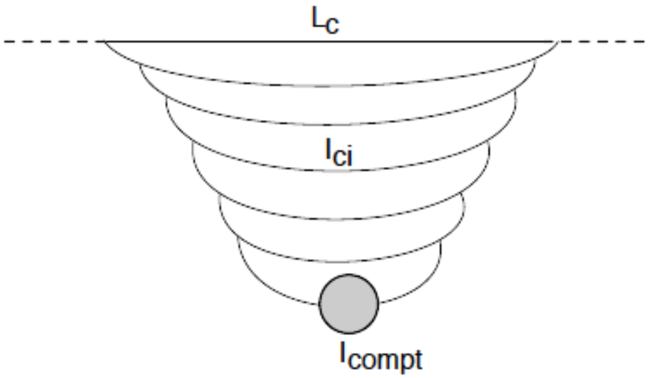
It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

L. Boltzmann:



the system is driven to the most probable state with the largest entropy

Possible relation to CFT



The small x formula

$$S = \ln[xG(x)]$$

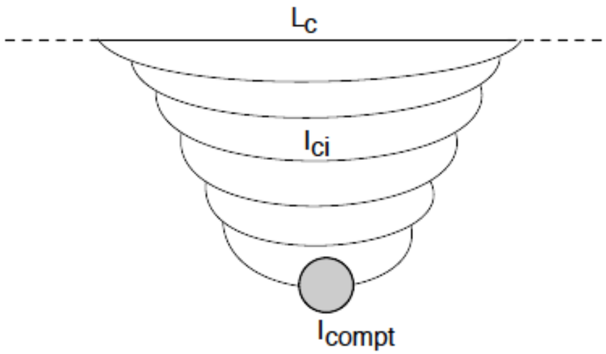
yields

$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

where

$$L = (mx)^{-1}$$

is the longitudinal length probed in DIS (in the target rest frame), and $\epsilon \equiv 1/m$ is the proton's Compton wavelength



Possible relation to CFT

This formula

$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

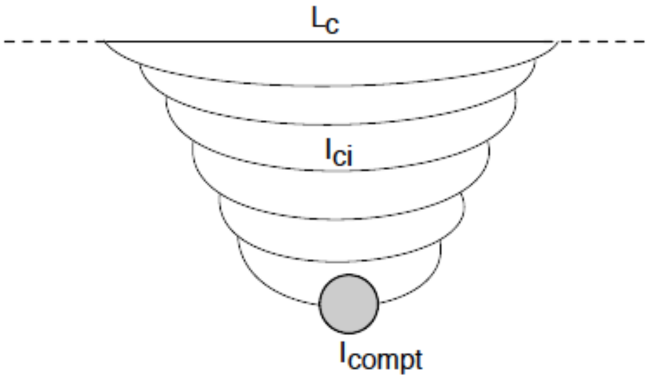
looks very similar to the well-known (1+1) CFT result:

(Holzhey, Larsen, Wilczek '94; Vidal, Latorre, Rico, Kitaev '03, Korepin '04, Calabrese, Cardy '05)

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

where c is the central charge of the CFT, and ε is the resolution scale

Possible relation to CFT



If this is not a mere coincidence:
the central charge $c \leq 1$:

$$c = 1 - \frac{6}{m(m+1)}, \quad m = 3, 4, ..., \infty$$

c=1 corresponds to free bosonic theory.
This means that $\Delta \leq 1/3$;
a bound on the growth of the structure function!

$$\boxed{xG(x) \leq \text{const}\ \frac{1}{x^{1/3}}}$$

The Second Law for entanglement entropy?

If the Second Law applies to entanglement entropy (EE) (a number of indications, e.g. from black hole physics), then the entropy of hadronic final state in DIS has to be equal or larger than the EE of the initial state measured through the structure function:

$$S_{\mathrm{hadrons}} \geq S_{EE}(x)$$

Indications from holography that the entropy does not increase much at strong coupling; this leads to

$$S_{\mathrm{hadrons}} \simeq S_{EE}(x)$$ **parton liberation, LPHD** ?

Fluctuations in hadron multiplicity

What is the relation between the parton and hadron multiplicity distributions?

Let us assume they are the same ("EbyE parton-hadron duality"); then the hadron multiplicity distribution should be given by

$$P_n(Y) \;=\; e^{-\Delta Y}(1 - \, e^{-\Delta Y})^{n-1}.$$

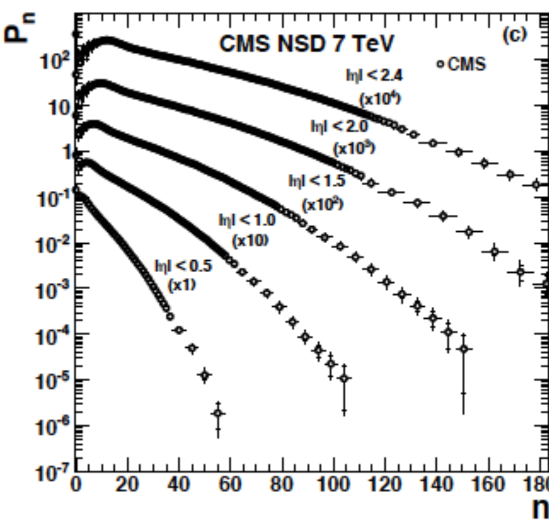
Consider moments

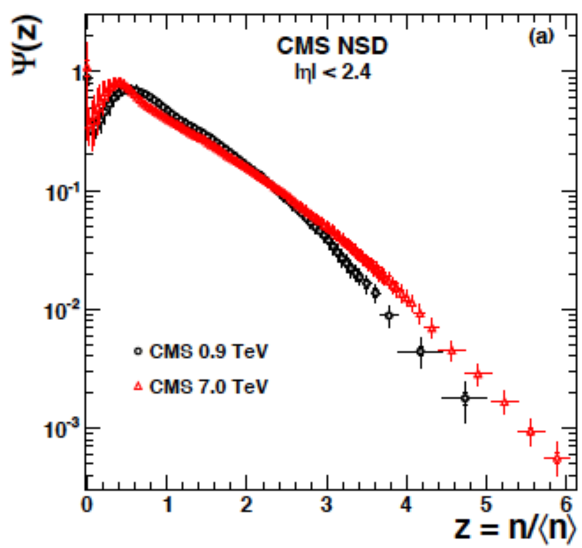
$$C_q = < n^q > / < n >^q$$

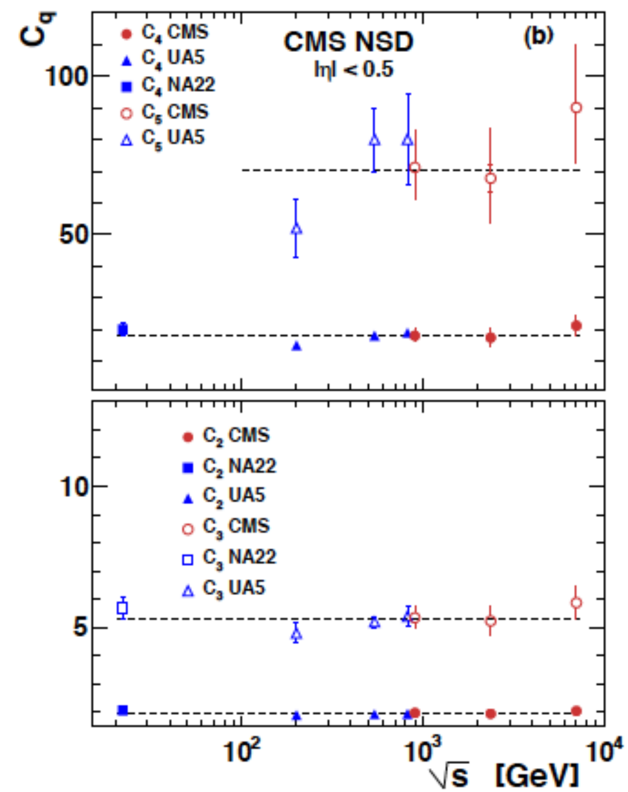
Fluctuations in hadron multiplicity

The CMS Coll., arXiv:1011.5531, JHEP01(2011)079

Charged particle multiplicities in pp interactions at $\sqrt{s} = 0.9, 2.36$, and 7 TeV







KNO scaling violated!

Fluctuations in hadron multiplicity

The moments can be easily computed by using the generating function

$$C_q = \left(u \frac{d}{du} \right)^q Z(Y,u) \Bigg|_{u=1}$$

We get

$$C_2 = 2 - 1/\bar{n}; \quad C_3 = \frac{6(\bar{n}-1)\bar{n}+1}{\bar{n}^2};$$

$$C_4 = \frac{(12\bar{n}(\bar{n}-1)+1)(2\bar{n}-1)}{\bar{n}^3}; \quad C_5 = \frac{(\bar{n}-1)(120\bar{n}^2(\bar{n}-1)+30\bar{n})+1}{\bar{n}^4}.$$

Fluctuations in hadron multiplicity

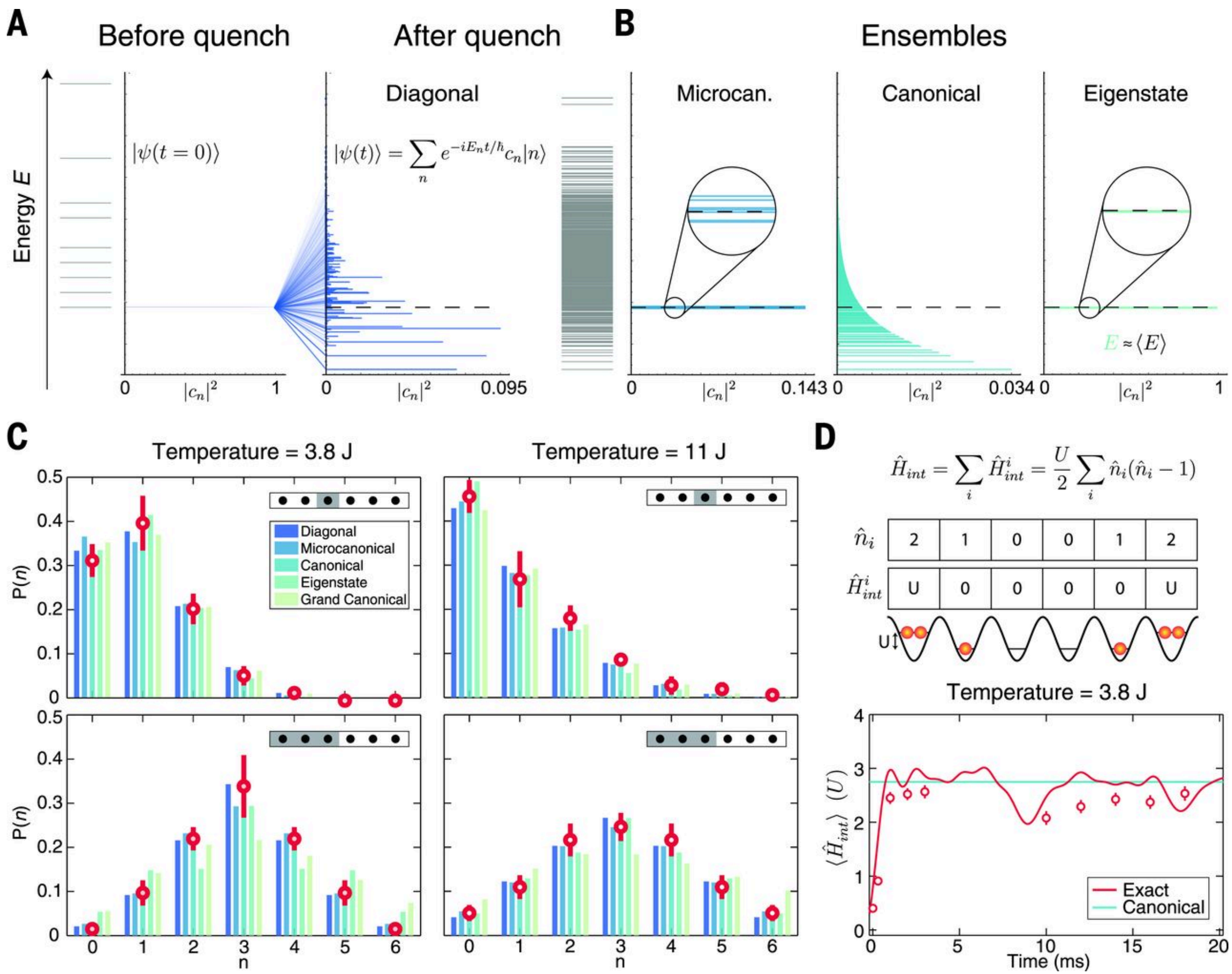
Numerically, for $\bar{n} = 5.8 \pm 0.1$ at |η|<0.5, E_{cm}=7 TeV
we get:

theory	exp (CMS)	theory, high energy limit
$C_2 = 1.83$	$C_2 = 2.0$+-0.05	$C_2 = 2.0$
$C_3 = 5.0$	$C_3 = 5.9$+-0.6	$C_3 = 6.0$
$C_4 = 18.2$	$C_4 = 21$+-2	$C_4 = 24.0$
$C_5 = 83$	$C_5 = 90$+-19	$C_5 = 120$

It appears that the multiplicity distributions of final state hadrons
are very similar to the parton multiplicity distributions –
this suggests that the entropy is close to the entanglement entropy

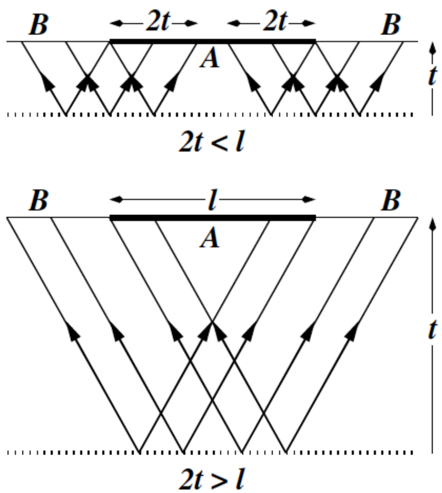
Quantum thermalization through entanglement in an isolated many-body system

A. M. Kaufman et al, Science 353 (2016) 794



Quantum thermalization through entanglement in high energy collisions?

Quenches in entangled systems described by CFT yield thermalization with an effective temperature proportional to the UV cutoff, $1/\tau_0$



$$S_A(t) \simeq \frac{c}{3}\ln\tau_0 + \begin{cases} \dfrac{\pi c t}{6\tau_0} & t < \ell/2\,, \\ \dfrac{\pi c \ell}{12\tau_0} & t > \ell/2\,, \end{cases}$$

P. Calabrese, J. Cardy, arxiv:1603.02889

In high energy collisions, the UV cutoff is given by the "hardness" Q of the collision. Is the effective temperature deduced from the transverse momentum spectra proportional to Q?

Use the LHC data to check this!

O.K. Baker, DK, PRD'18

EIC study: Z.Tu, DK, T.Ullrich, to appear

R. Bellwied, arxiv:1807.04589

Summary

1. Entanglement entropy (EE) provides a viable solution to the apparent contradiction between the parton model and quantum mechanics.
2. The use of EE allows to generalize the notion of parton distribution to any value of the coupling, and possibly to elucidate the role of topology.
3. If the connection to CFT is confirmed, it will put a bound on the growth of parton distributions at small x.
4. Entanglement may provide a new mechanism for thermalization in high-energy collisions.