Wayne State University
College of Liberal Arts & Sciences
Department of Physics and Astronomy

General Balance Functions (and other correlation functions)

Winter Workshop Nuclear Dynamics,
Beaver Creek, CO

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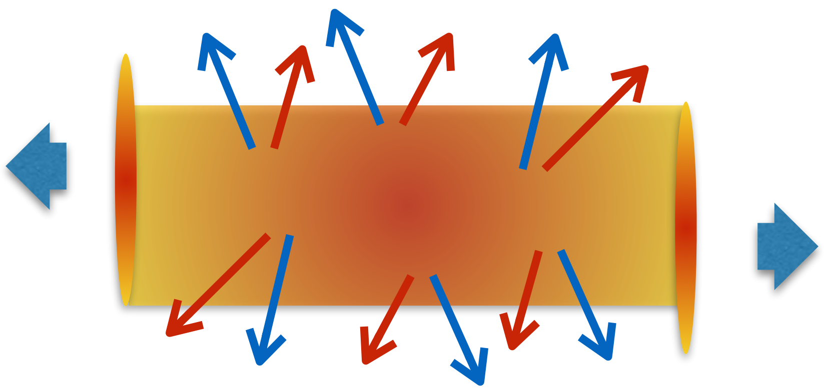
Outline
- LS,US,CI, CD Correlation Functions
- General Balance Function (GBF)
- Shear Viscosity w/ pT correlations

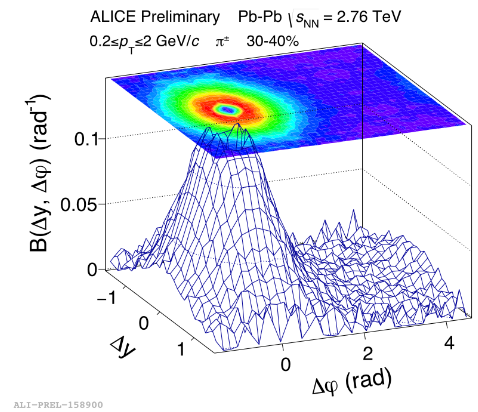
Six Reasons to Measure General Balance Functions

- Understand/Probe
  - **Two-stage charge production model**
  - **Collision dynamics, e.g., radial flow**
  - **Hadro-chemistry** - Charge/Strangeness/Baryon/Resonance production
- Background/Support for other studies
  - Search for CME/CMW effects:
    - BF -> Better system expansion models
    - More reliable calculations of charge conservation backgrounds in CME searches.
  - Search for DCC production
    - Differential correlators - neutral and charge kaons
  - Studies of (higher-moments) net charge/baryon fluctuations.

Two-wave Production

Bass, Danielewicz, Pratt PRL. 85, 2689 (2000)
Pratt, Cheng PRC 68, 014907 (2003)
Bozek PLB 609 (2005) 247–251
Kapusta, Plumberg PRC 97, 014906 (2018)

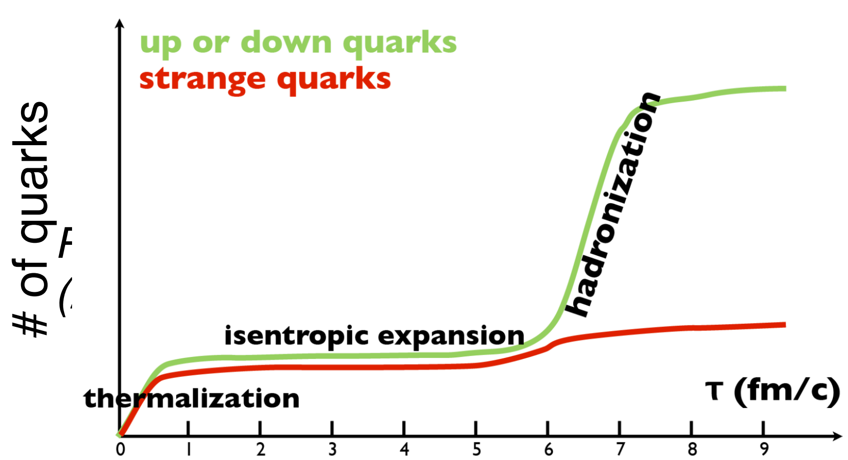


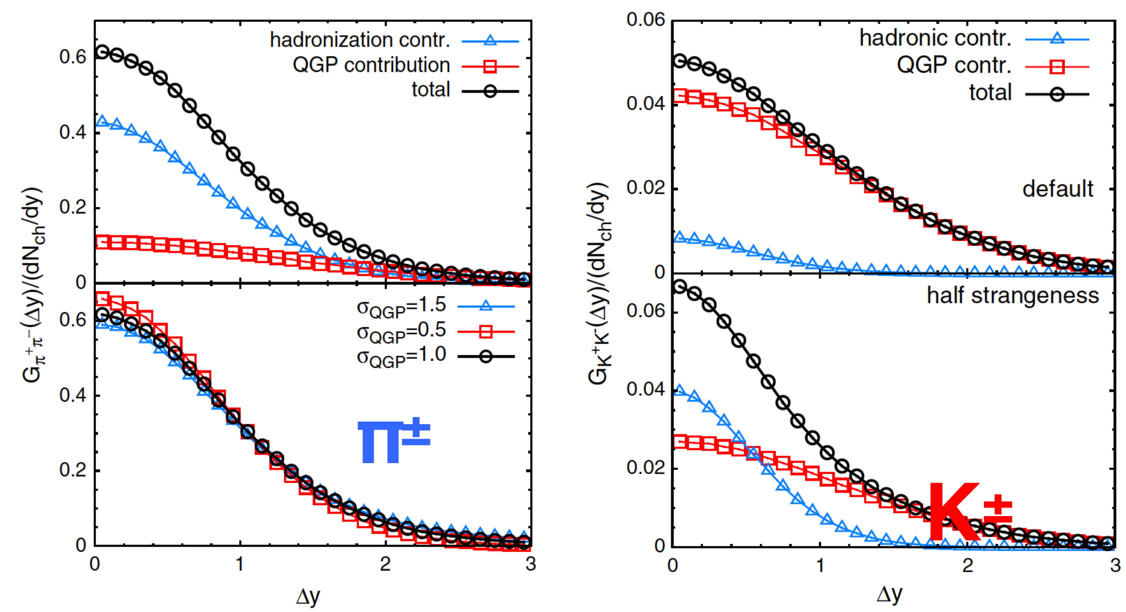


$$B_{\alpha,\beta}(\Delta\eta,\Delta\phi)=\frac{dN_{\beta}}{d\eta}R_{\alpha,\beta}^{(CD)}(\Delta\eta,\Delta\phi)$$

Correlations: Coordinate Space ⟶ **Correlations:Momentum**

Quark Production vs. Time





Two-wave quark production model:
- $\pi^{\pm}$: predominantly produced at late stage
- $K^{\pm}$: predominantly produced at early stage

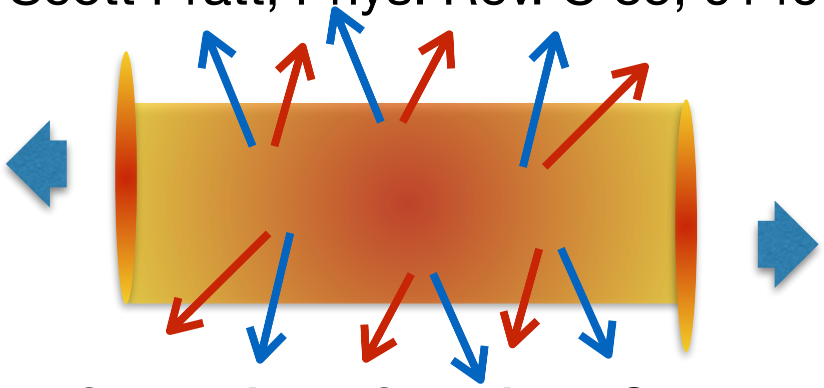
- Strong late stage contribution
- Hadronization part narrower
- Narrows with centrality

- Weak late stage contribution
- Weak centrality dependence.

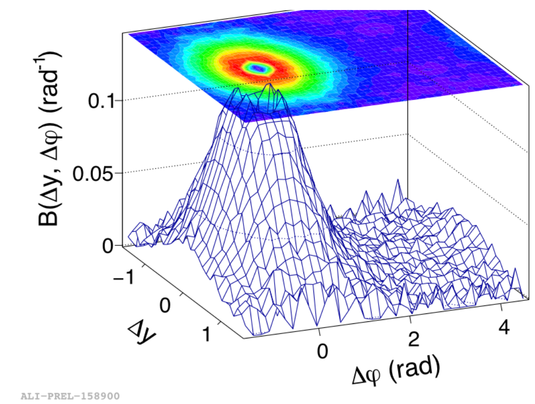
Investigate if BFs for $\pi^{\pm}$, $K^{\pm}$, …, evolve differently with centrality at LHC & RHIC (BES)

Hadro-chemistry

Scott Pratt, Phys. Rev. C 85, 014904



Longitudinal & Radial Expansion
Hadronization



Correlations: Coordinate Space **Correlations:Momentum**

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a, b: net charge, strangeness, baryon **α, β : particle species (pion, kaon, etc)**

$$g_{ab}(\eta_1,\eta_2) = g_{ab}^{(\mathrm{QGP})}(\eta_1,\eta_2) + g_{ab}^{(\mathrm{had})}(\eta_1,\eta_2),$$

$$g_{ab}^{(\mathrm{had})}(\eta_1,\eta_2) = -\left[\chi_{ab}^{(\mathrm{had})}(\eta_1) - \chi_{ab}^{(\mathrm{QGP})}(\eta_1)\right]\delta(\eta_1-\eta_2),$$

$$\chi_{ab}^{(\mathrm{had})}(\eta) = \sum_{\alpha\in\mathrm{had}} q_{\alpha,a}q_{\alpha,b}\langle n_\alpha(\eta)\rangle$$

$$\chi_{ab}^{(\mathrm{QGP})}(\eta) = \sum_{\alpha\in\mathrm{QGP}} q_{\alpha,a}q_{\alpha,b}\langle n_\alpha(\eta)\rangle,$$

$$g_{ab}(\eta_1,\eta_2) = \sum_{\alpha\beta}\langle n_\alpha(\eta_1)\rangle q_{\alpha,a}q_{\beta,b}\langle n_\beta(\eta_2)\rangle \exp\left\{\sum_{cd} q_{\alpha,c}\mu_{cd}(\eta_1,\eta_2)q_{\beta,d}\right\}$$

$$= \sum_{cd}\chi_{ac}(\eta_1)\mu_{cd}(\eta_1,\eta_2)\chi_{db}(\eta_2),$$

$$\mu_{ab}(\eta_1,\eta_2) = \sum_{cd}\chi_{ac}^{(-1)}(\eta_1)g_{cd}(\eta_1,\eta_2)\chi_{db}^{(-1)}(\eta_2).$$

A tool for studying the chemical evolution of the quark-gluon plasma

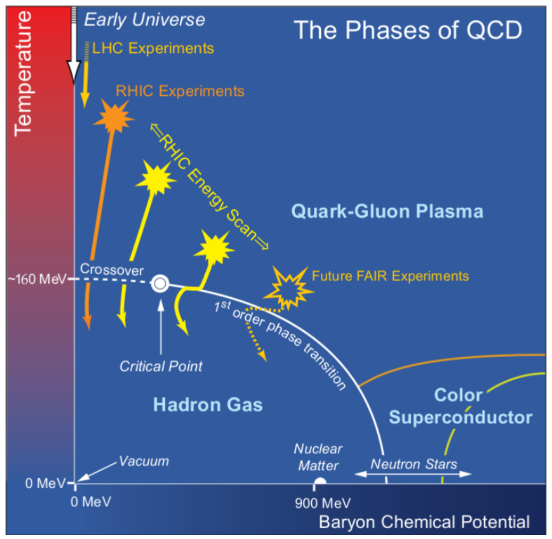
$$g_{\alpha\beta}(\eta_1,\eta_2) = \left\langle[n_\alpha(\eta_1) - n_{\bar\alpha}(\eta_1)]\,[n_\beta(\eta_2) - n_{\bar\beta}(\eta_2)]\right\rangle$$

$$\simeq 4\langle n_\alpha(\eta_1)\rangle q_{\alpha,a}\mu_{ab}(\eta_1,\eta_2)q_{\beta,b}\langle n_\beta(\eta_2)\rangle.$$

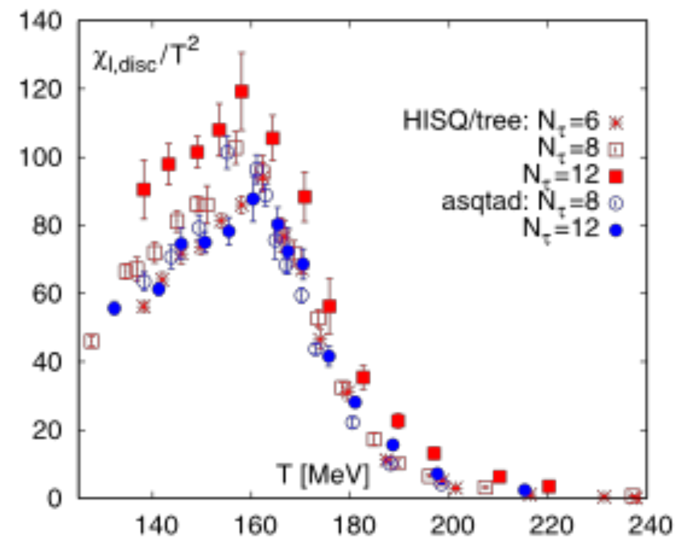
→

$$B_{\alpha,\beta}(\Delta\eta,\Delta\phi) = \frac{dN_\beta}{d\eta}R_{\alpha,\beta}^{(CD)}(\Delta\eta,\Delta\phi)$$

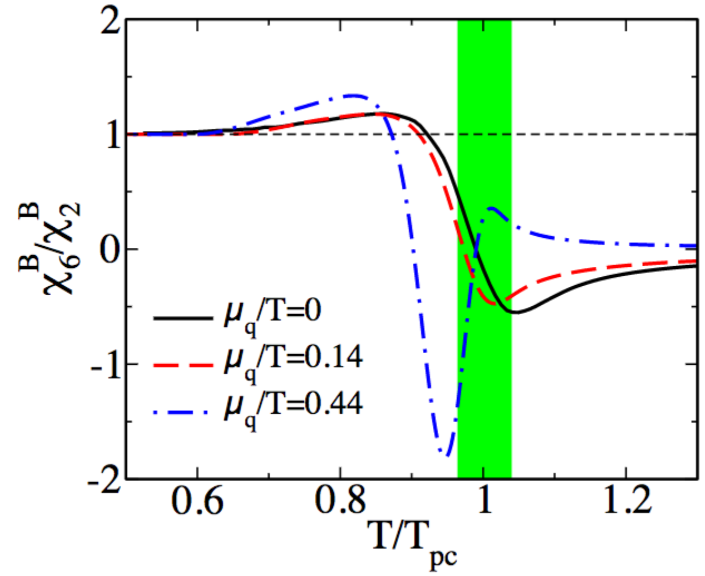
Support for net-charge (baryon) fluctuation studies



A. Bazavov et al. PRD 85 (2012) 054503



B. Friman, et al. EPJC 71 (2011) 1694,



- E-By-E fluctuations of net charge/baryons/strangeness probe properties (phase structure) of QCD matter.
- LHC: Test lattice QCD predictions at $\mu_B = 0$; If close to 2nd-order phase transition for vanishing quark masses → signs of criticality?
- RHIC/BES: Search for critical point.
- Measure susceptibilities

$$\chi_n^B = \frac{\partial^n \left(P/T^4\right)}{\partial\left(\mu_B/T\right)^n}$$

$$\langle \Delta N_B \rangle = VT^3\chi_1^B$$

$$\left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^2 \right\rangle = VT^3\chi_2^B = \sigma^2$$

$$\left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^3 \right\rangle / \sigma^3 = \frac{VT^3\chi_3^B}{\left(VT^3\chi_2^B\right)^{3/2}} = S$$

$$\left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^4 \right\rangle / \sigma^4 - 3 = \frac{VT^3\chi_4^B}{\left(VT^3\chi_2^B\right)^2} = k$$

Caveats: GCE expectations must be "corrected" for various effects:

- **Charge Conservation**
- **V_x =?= V_p correspondance**
- **Energy-momentum conservation**
- **Quantum number conservation**
- **Finite system size and lifespan**
- **Stopping/Fluctuations**

Definitions

Densities:

$$\rho_1(\vec{p}_1) \equiv \rho_1(\phi_1, \eta_1, p_{T,1})$$

$$\rho_2(\vec{p}_1, \vec{p}_2) \equiv \rho_2(\phi_1, \eta_1, p_{T,1}, \phi_2, \eta_2, p_{T,2})$$

2-Cumulant:

$$C_2(\eta_1, \eta_2) \equiv \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

Normalized Cumulants:

$$R_2(\Delta\eta, \Delta\phi) \equiv \frac{\rho_2(\Delta\eta, \Delta\phi)}{\rho_1(\eta_1, \phi_1) \otimes \rho_1(\eta_2, \phi_2)} - 1$$

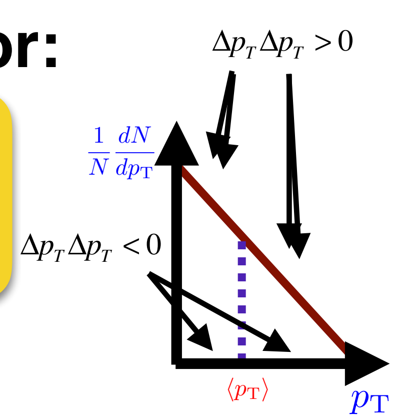
Transverse Momentum Correlator (1):

M. Sharma & C.P., PRC 79, 024905 (2009)

$$\langle \Delta p_T \Delta p_T \rangle(\Delta\eta, \Delta\phi) \equiv \frac{\int \rho_2(\vec{p}_1, \vec{p}_2) \Delta p_{T,1} \Delta p_{T,2} \, dp_{T,1} \, dp_{T,2}}{\rho_2(\Delta\eta, \Delta\phi)}$$

Dimensionless pT Correlator:

$$P_2(\Delta\eta, \Delta\phi) = \frac{\langle \Delta p_T \Delta p_T \rangle(\Delta\eta, \Delta\phi)}{\langle p_T \rangle^2}$$



Transverse Momentum Correlator (2):

S. Gavin Phys.Rev.Lett. 97 (2006) 162302
M. Sharma & C.P. et al (STAR), PLB704, 467 (2011)

$$G_2(\Delta\eta, \Delta\phi) \equiv \frac{\int \rho_2(\vec{p}_1, \vec{p}_2) p_{T,1} p_{T,2} \, dp_{T,1} \, dp_{T,2}}{\rho_1(\eta_1, \phi_1) \otimes \rho_1(\eta_2, \phi_2)} - \langle p_{T,1} \rangle \langle p_{T,2} \rangle$$

Charged particle pair combinations:

- LS : Like-sign pairs $\quad O^{(LS)} = \frac{1}{2}\left(O^{(++)} + O^{(--)}\right)$
- US : Unlike-sign pairs $\quad O^{(US)} = \frac{1}{2}\left(O^{(+-)} + O^{(-+)}\right)$
- CI : Charge Independent $\quad O^{(CI)} = \frac{1}{2}\left(O^{(LS)} + O^{(US)}\right)$
- CD: Charge Dependent $\quad O^{(CD)} = \frac{1}{2}\left(O^{(US)} - O^{(LS)}\right)$

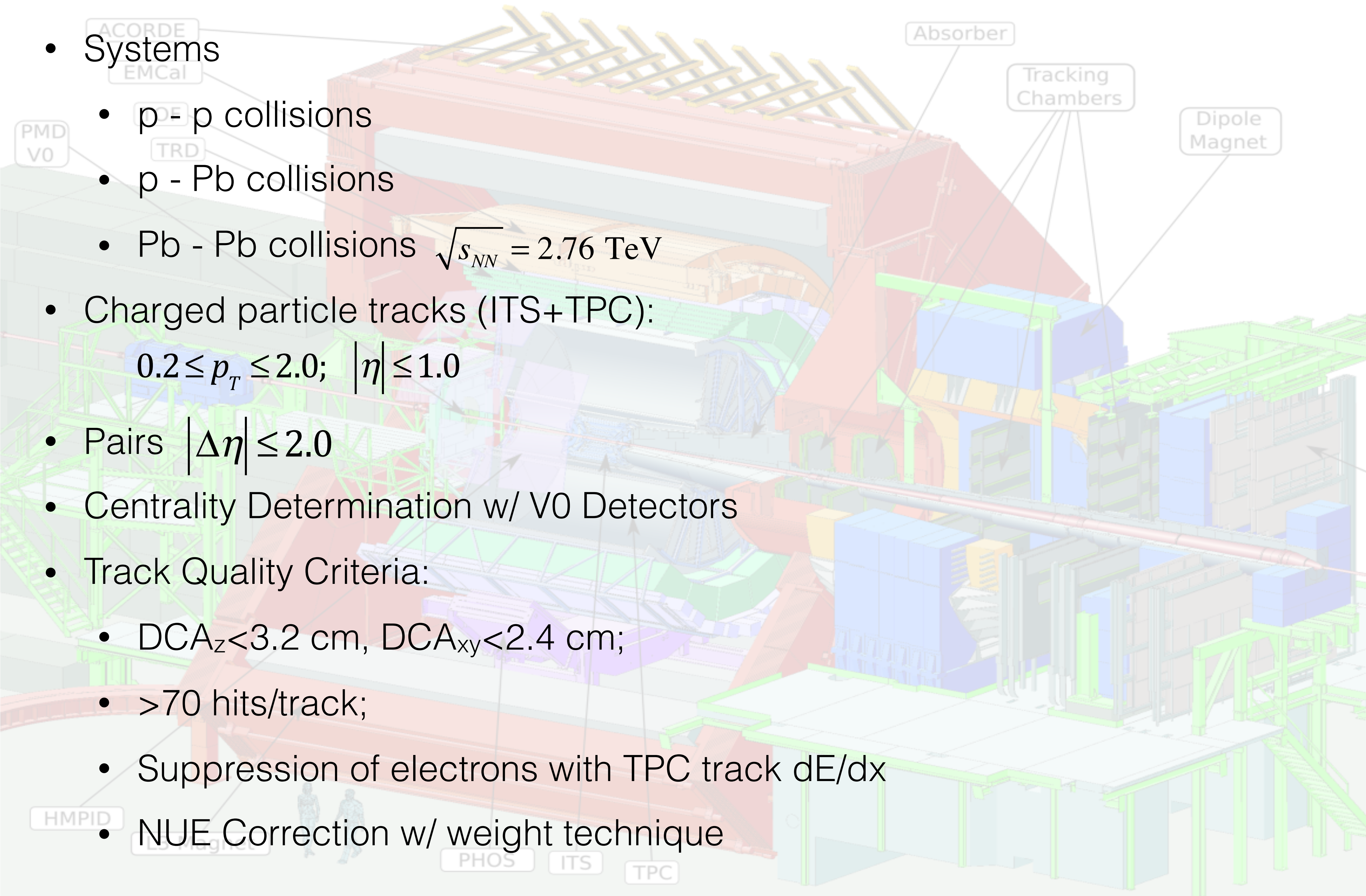
Balance Functions (BF):

$$B(\Delta\eta, \Delta\phi) \equiv \frac{dN}{d\eta} R_2^{(CD)}(\Delta\eta, \Delta\phi)$$

Note:

$$B(Y|Y) = \frac{\langle N \rangle}{4}\{2R_{+-} - R_{++} - R_{--}\} = -\frac{\langle N \rangle}{4}\nu_{dyn}.$$

Measurements by ALICE



- Systems
  - p - p collisions
  - p - Pb collisions
  - Pb - Pb collisions $\sqrt{s_{NN}} = 2.76$ TeV
- Charged particle tracks (ITS+TPC):

  $0.2 \le p_T \le 2.0; \quad |\eta| \le 1.0$
- Pairs $|\Delta\eta| \le 2.0$
- Centrality Determination w/ V0 Detectors
- Track Quality Criteria:
  - DCA_z<3.2 cm, DCA_{xy}<2.4 cm;
  - >70 hits/track;
  - Suppression of electrons with TPC track dE/dx
  - NUE Correction w/ weight technique

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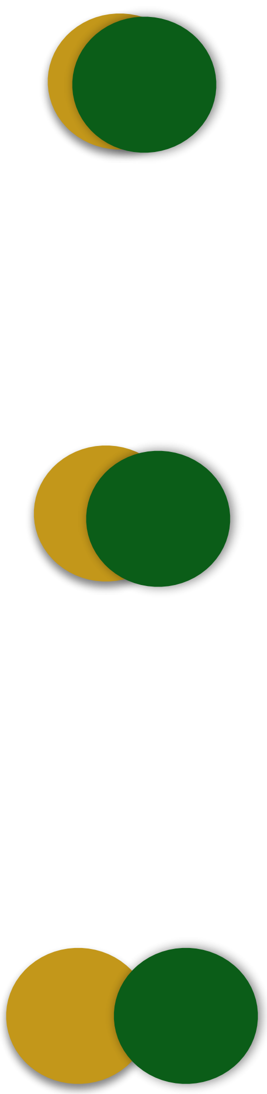
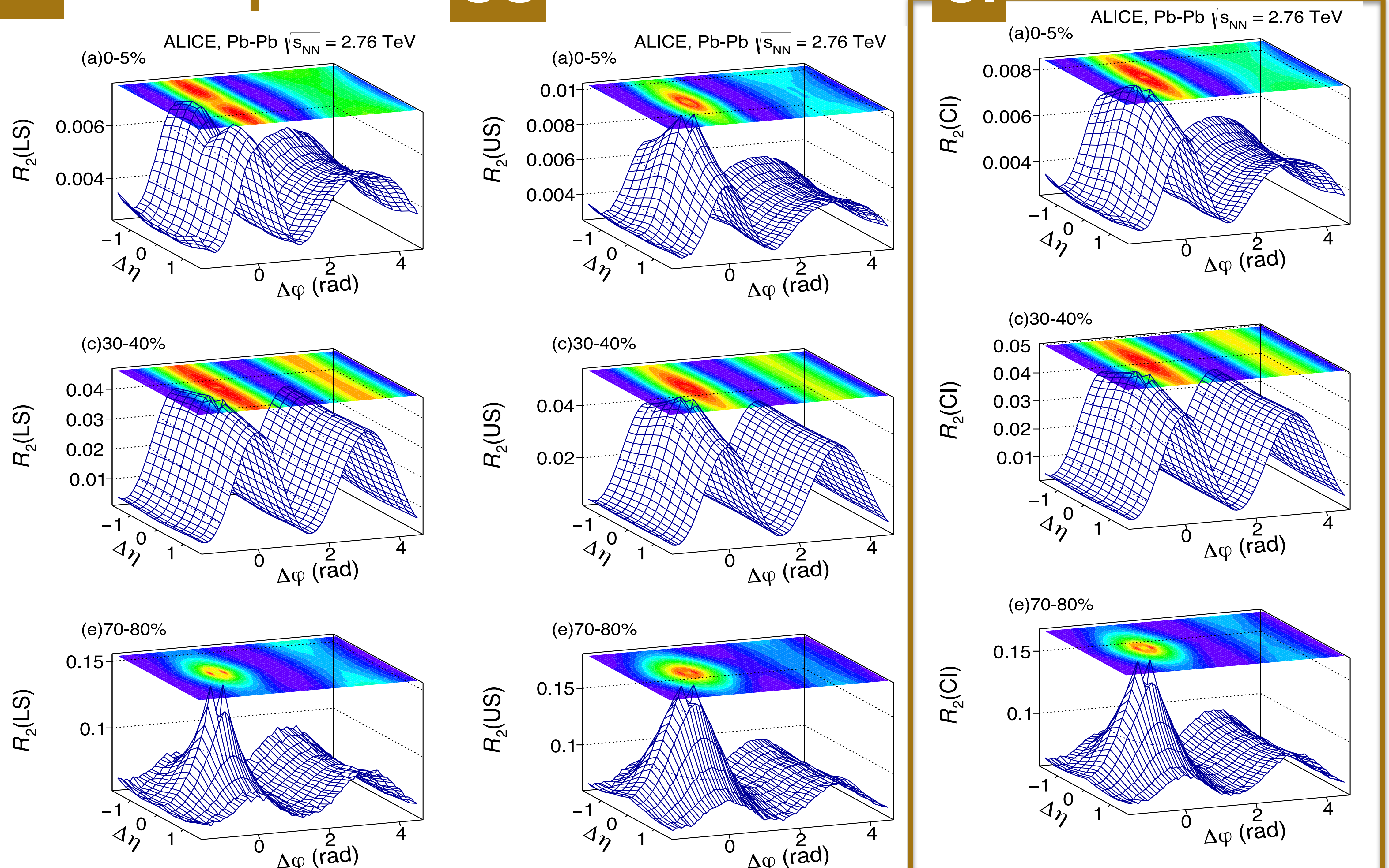
P. Pujahari, et al., arXiv:1805.04422, Submitted to PRC.

$R_2^{(CI)}$ in Pb — Pb @ 2.76 TeV

LS + US → CI

0.2 < pT < 2.0 GeV/c



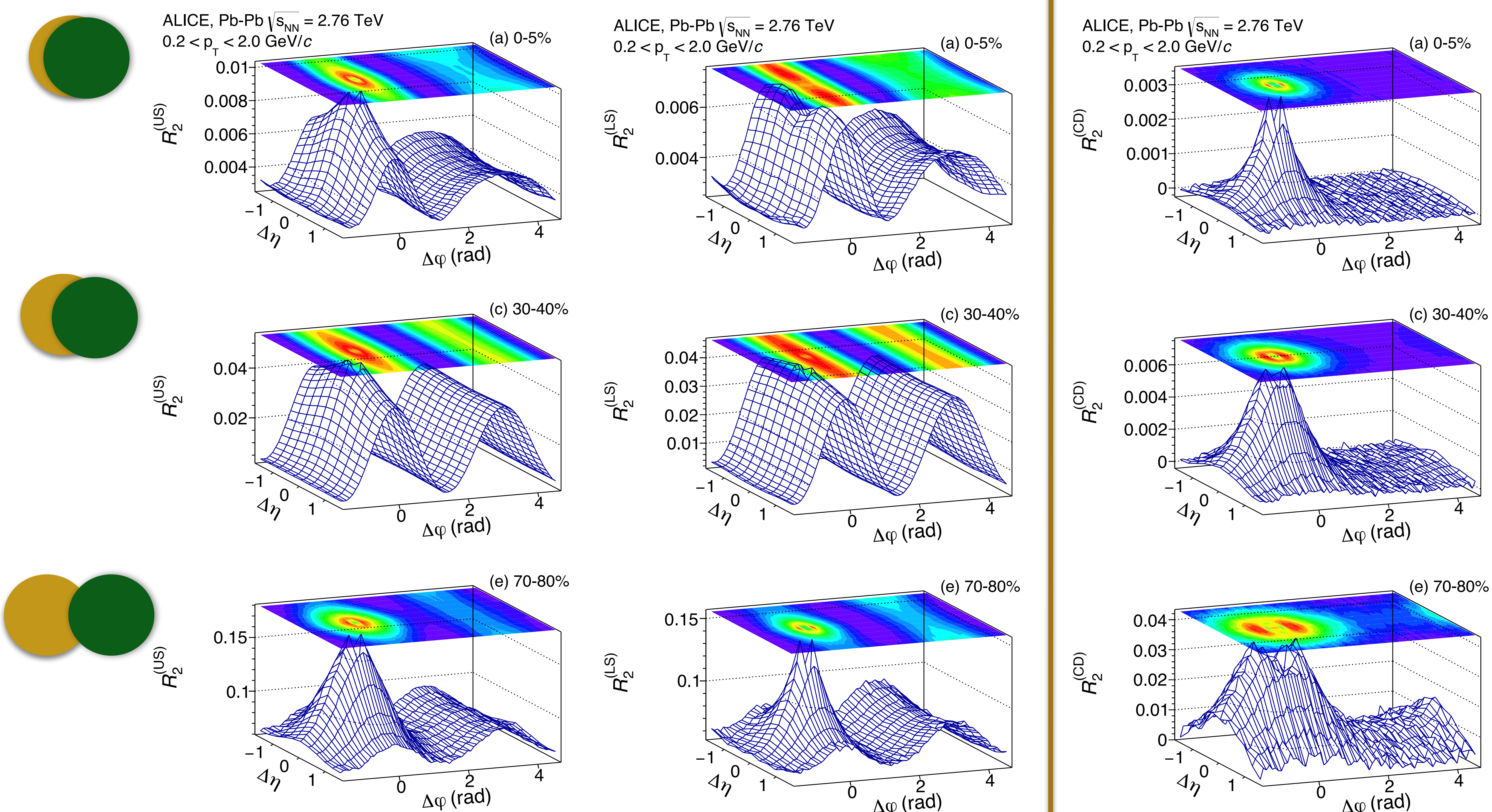


P. Pujahari, et al., arXiv:1805.04422, Submitted to PRC.

$R_2^{(CD)}$ in Pb — Pb @ 2.76 TeV

US — **LS** **CD**

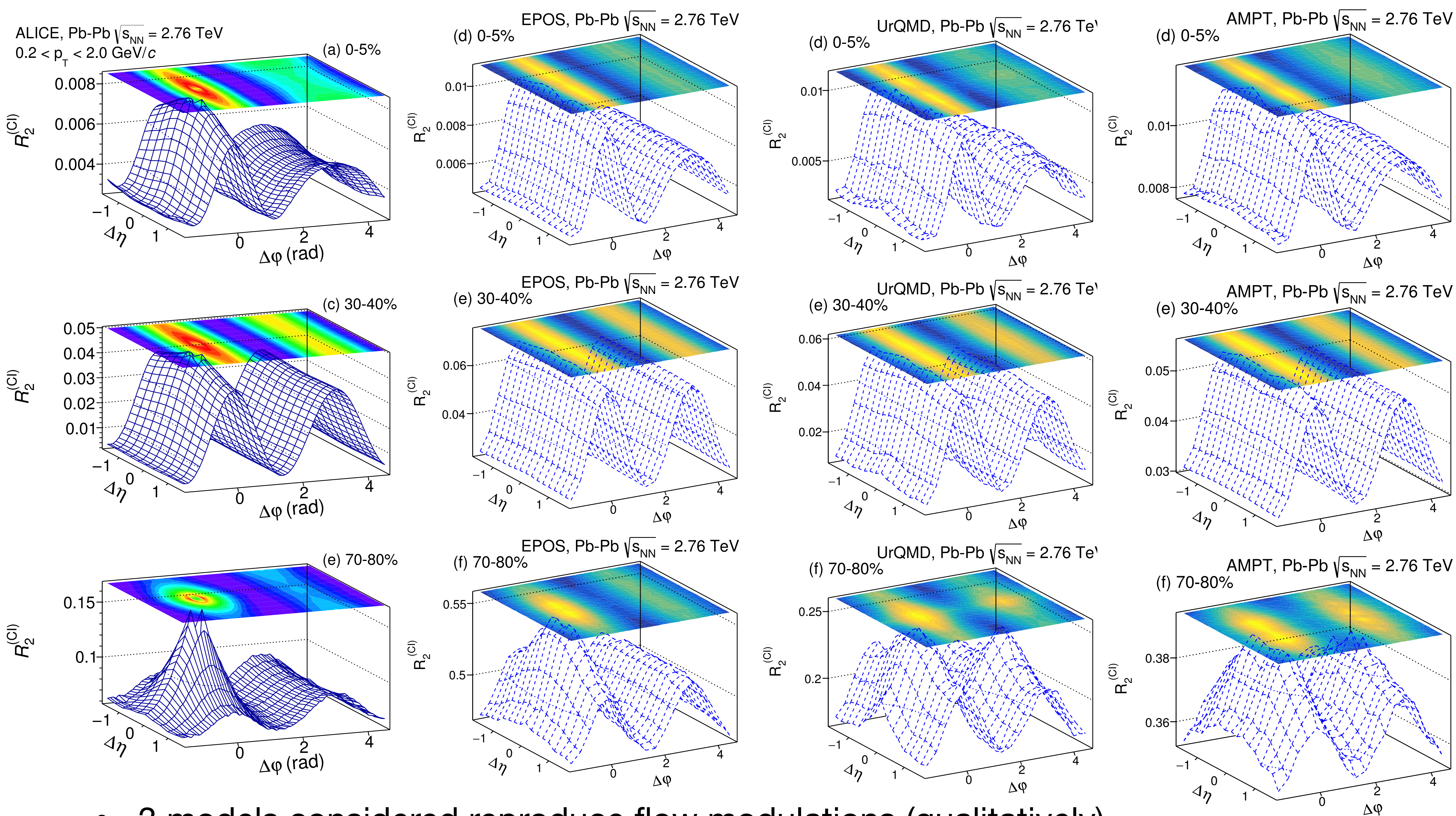
$$O^{(CD)} = \frac{1}{2}\left(O^{(US)} - O^{(LS)}\right)$$



In Preparation, S. Basu et al.

R_2^{CI} — Comparison w/ Models

ALICE DATA | **EPOS 3.0 (320k events)** | **UrQMD (225k events)** | **AMPT (200k events)**

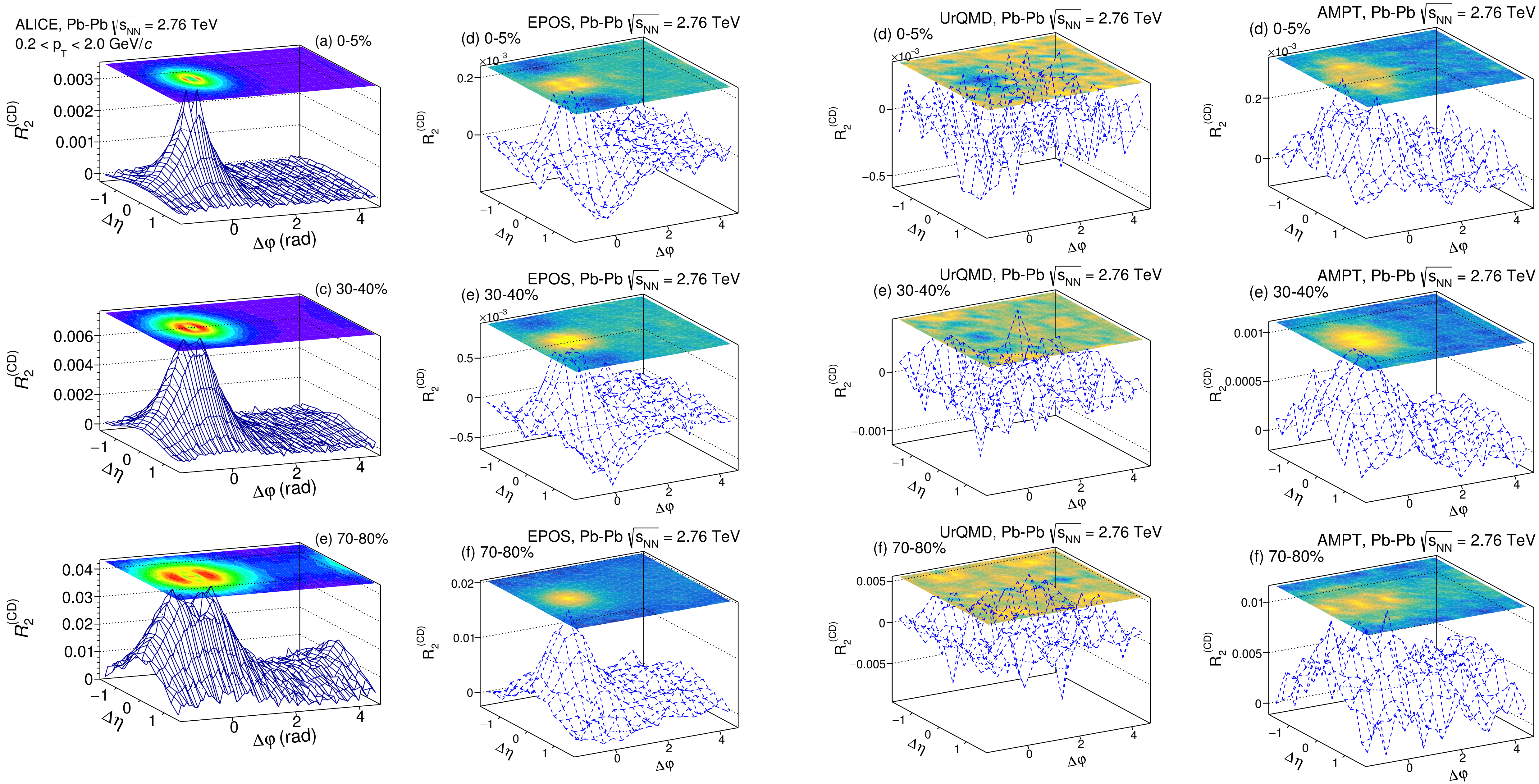


- 3 models considered reproduce flow modulations (qualitatively)
- Near-side/Away-side shapes challenge models.
- EPOS qualitatively best for this observable.

In Preparation, S. Basu et al.

R_2^{CD} — Comparison w/ Models

ALICE DATA | **EPOS 3.0** | **UrQMD** | **AMPT**



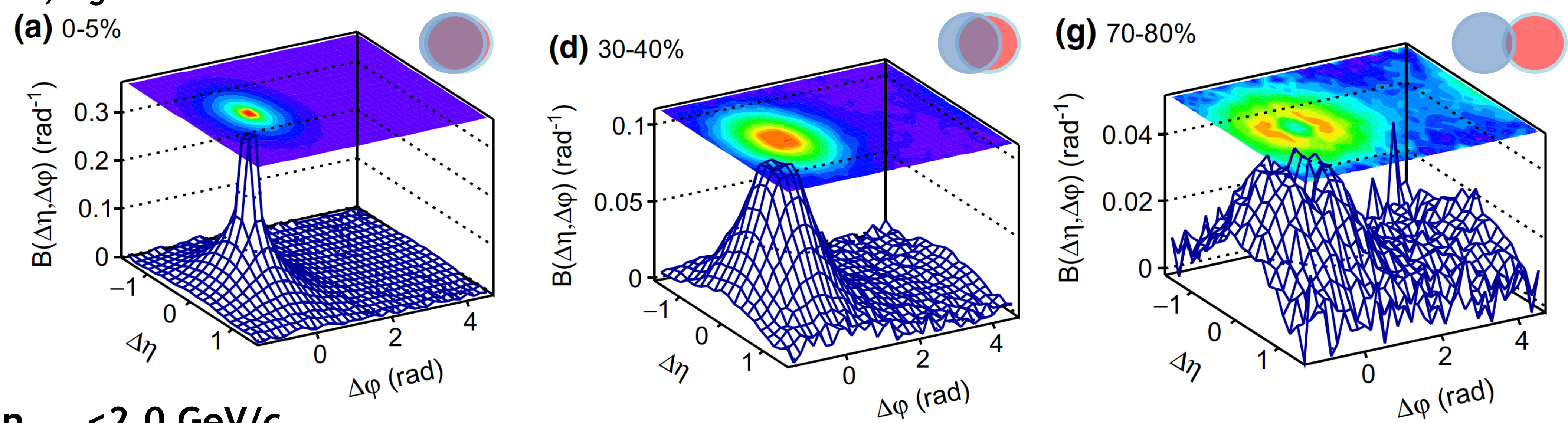
- **EPOS: Qualitative Agreement, Insufficient correlation strength (corona/core?)**
- **UrQMD: No agreement**
- **AMPT: Qualitative Agreement, Correlation strength incorrect, no centrality evolution**

Lesson: Need to account for charge conservation!

Pb-Pb, p-Pb & pp Collisions

Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV

0.2<$p_{T,assoc}$<$p_{T,trig}$<2.0 GeV/c

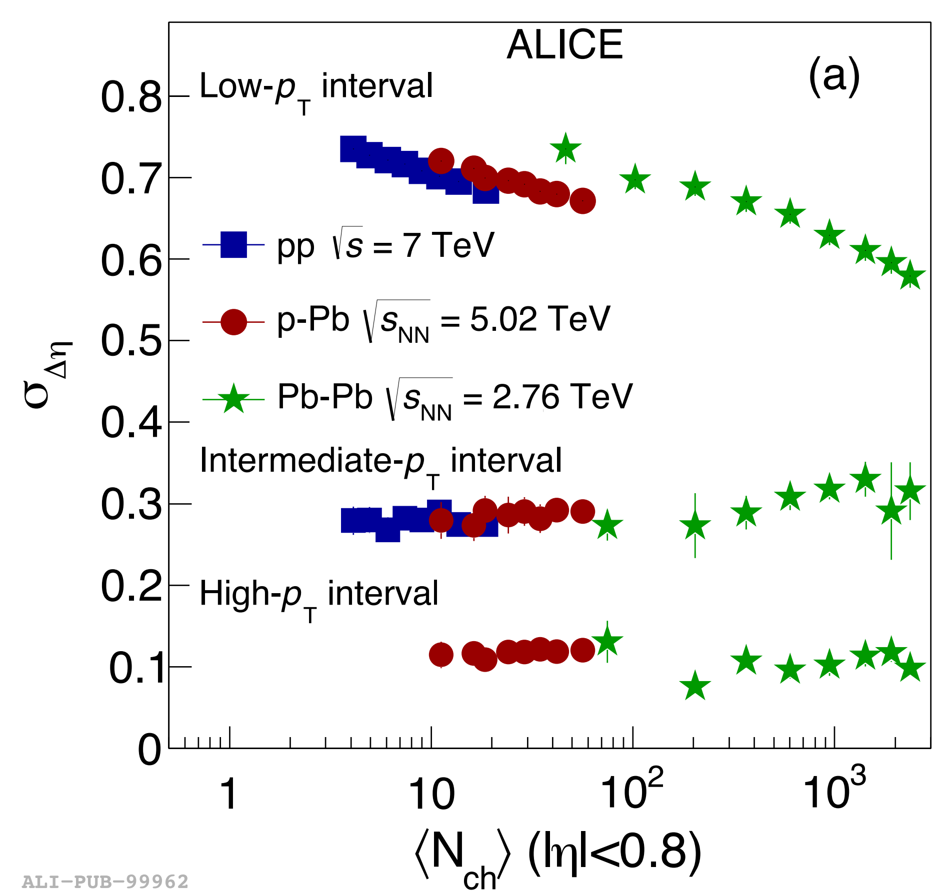


◇ **0.2<$p_{T,assoc}$<$p_{T,trig}$<2.0 GeV/c**

- Pb-Pb: narrowing towards central collisions
  - -> delayed hadronization (longer system lifetime)
  - -> radial flow
- p-Pb, pp: narrowing towards large multiplicity collisions

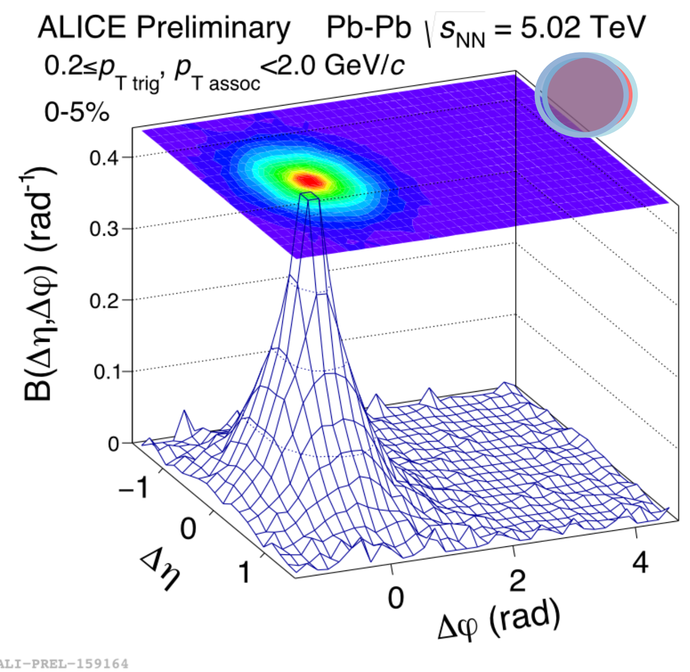
◇ **2.0<$p_{T,assoc}$<3.0<$p_{T,trig}$<4.0 GeV/c & 3.0<$p_{T,assoc}$<8.0<$p_{T,trig}$<15.0 GeV/c**

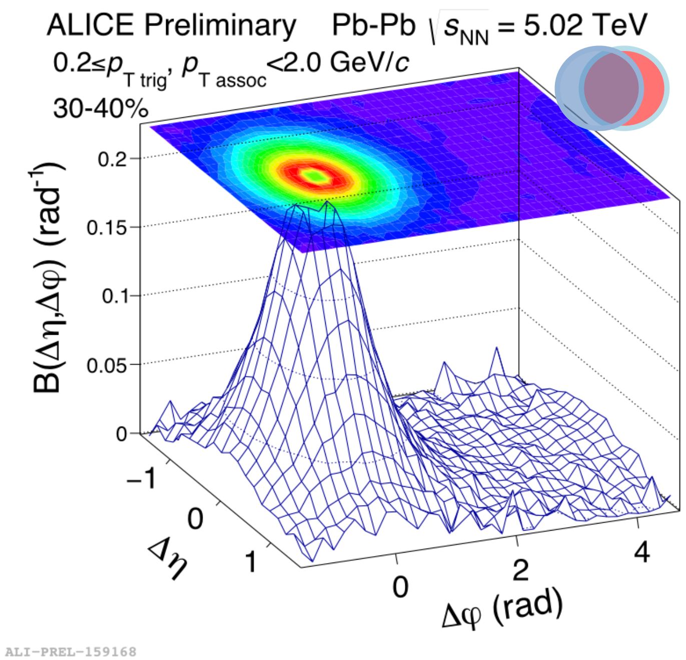
- Pb-Pb, p-Pb, pp: no multiplicity dependence
- may indicate different quark production mechanisms (interplay of bulk & jets)

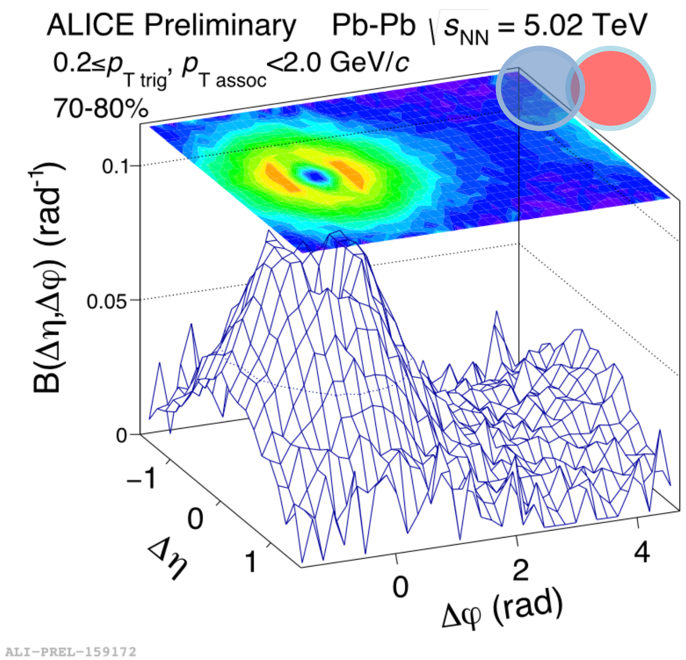


Pb-Pb @ 5.02 TeV

0.2<$p_{T\ trig}$, $p_{T\ assoc}$<2.0 GeV/c



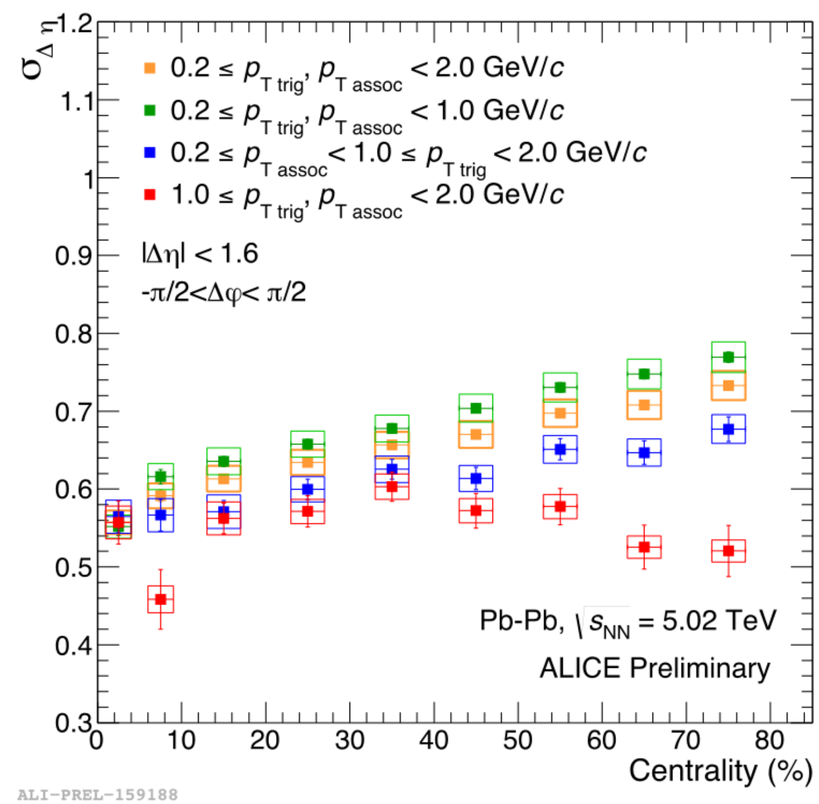


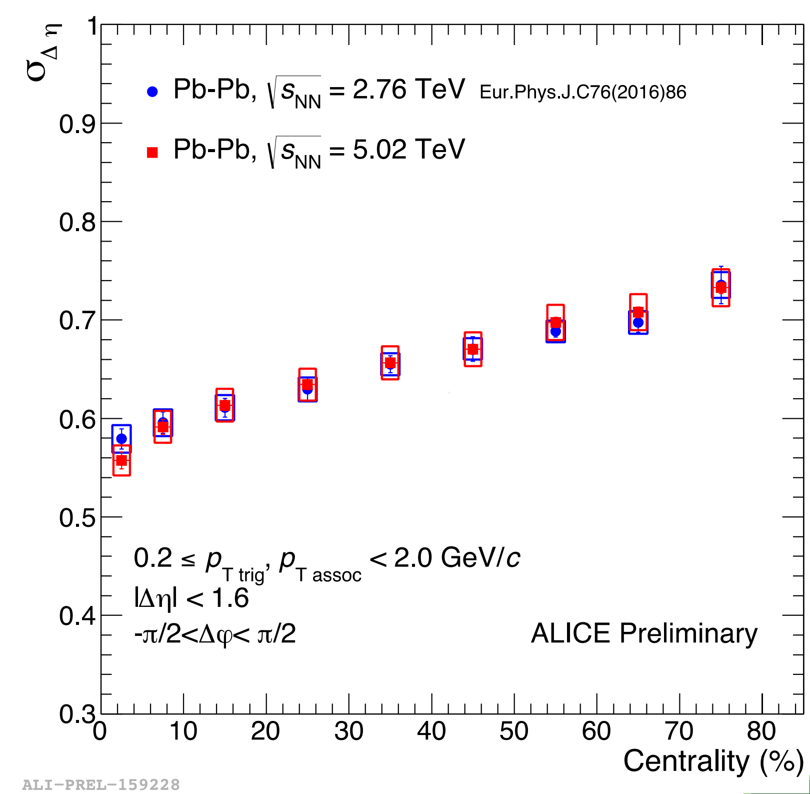


Near-side Peak Widths

pT dependent BF:
centrality dependence changes with pT range
no narrowing for 1.0<pT trig, pT assoc<2.0 GeV/c

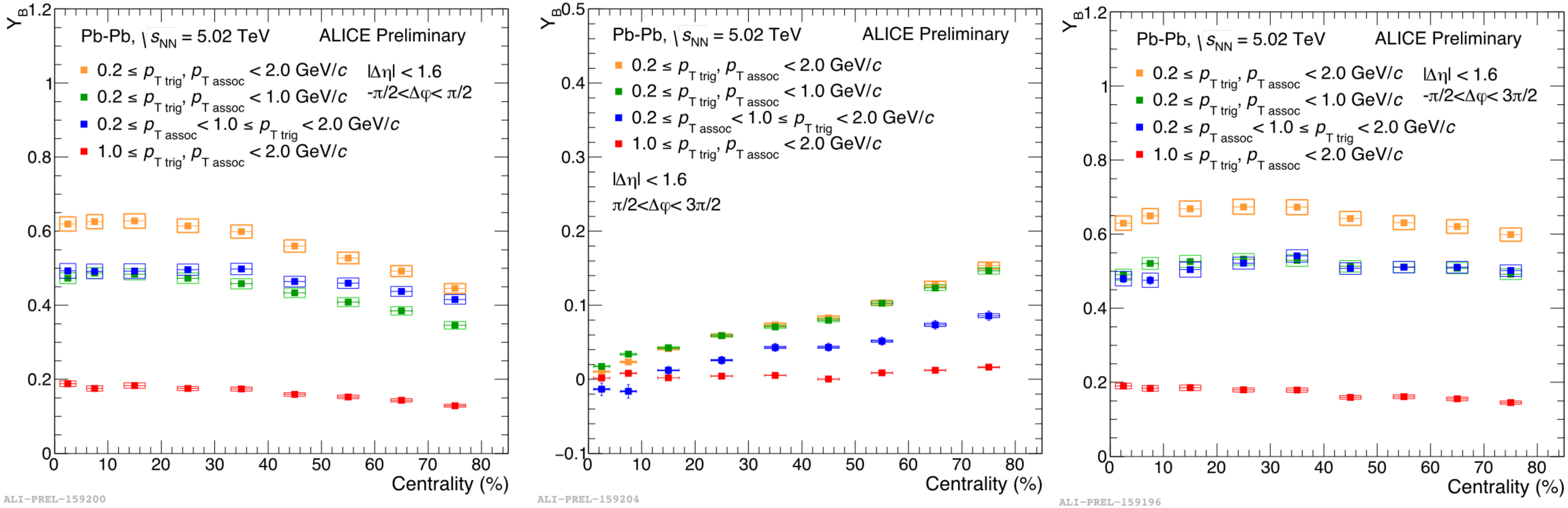
No difference between 2.76 & 5.02 TeV





Associated Charge Yield — Pb-Pb @ 5.02 TeV

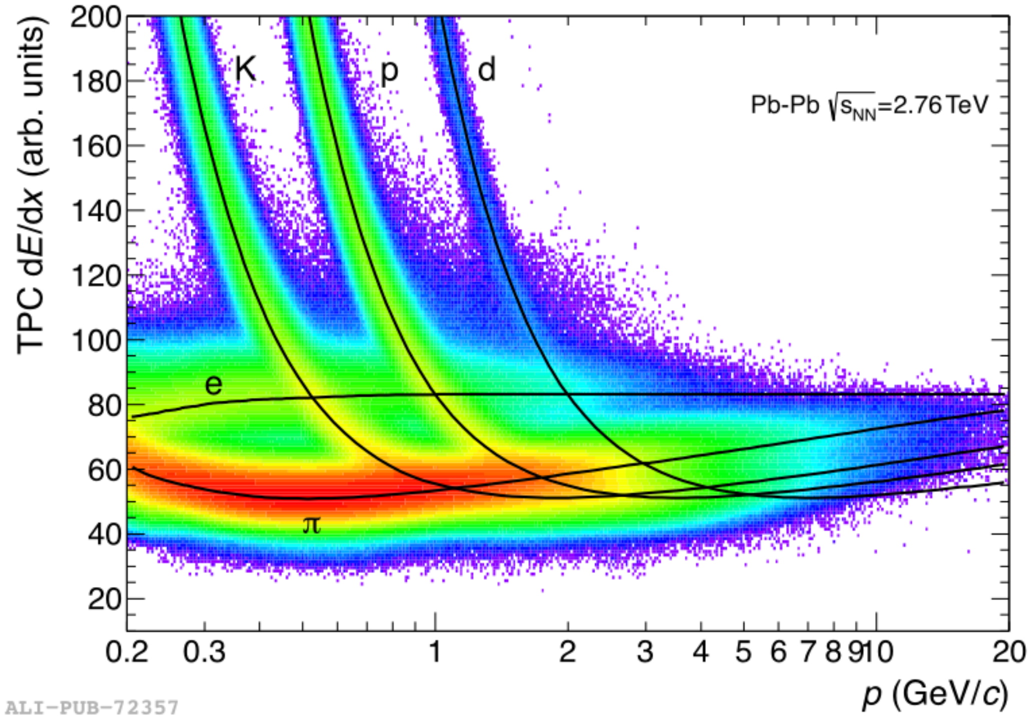
Near-side Away-side Total

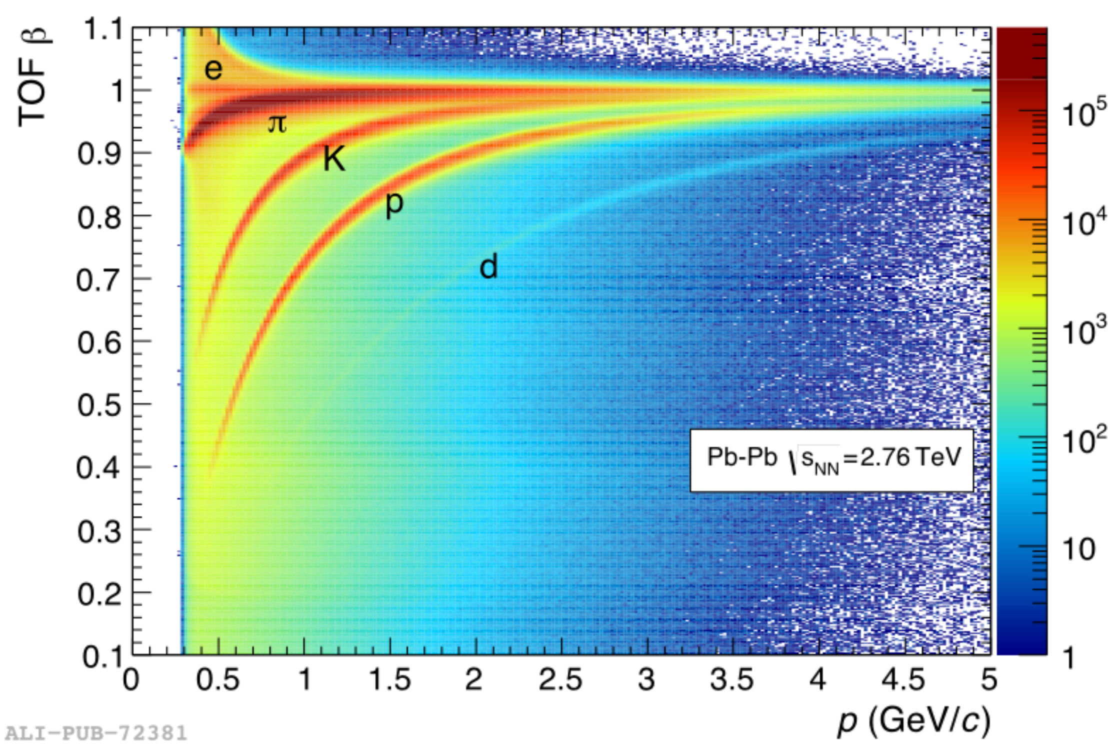


Balancing Charge Yield (integral of BF):
Amount of balancing charge within experimental acceptance
First measurement of balancing charge yields
pT dependent BF: centrality dependence changes with pT range

Charged pion and kaon identification in ALICE

ALICE Int.J.Mod.Phys. A29 (2014) 1430044



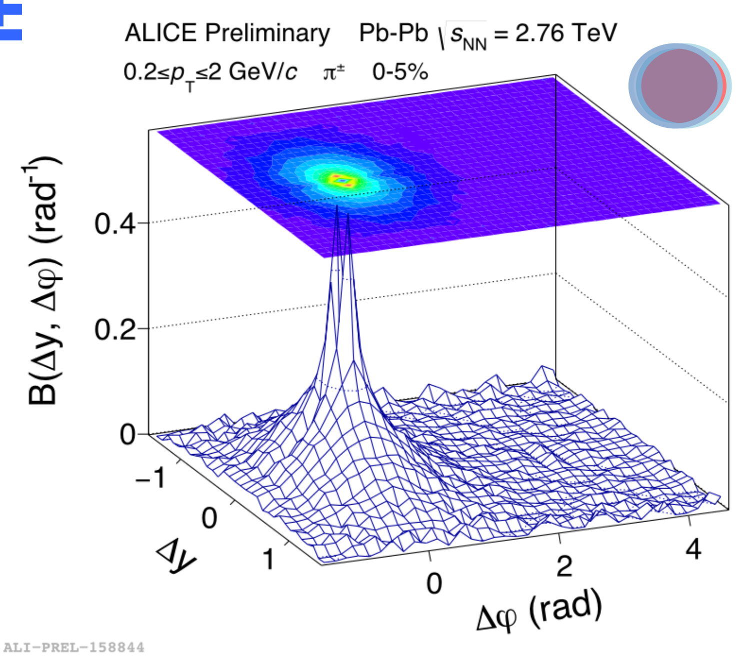


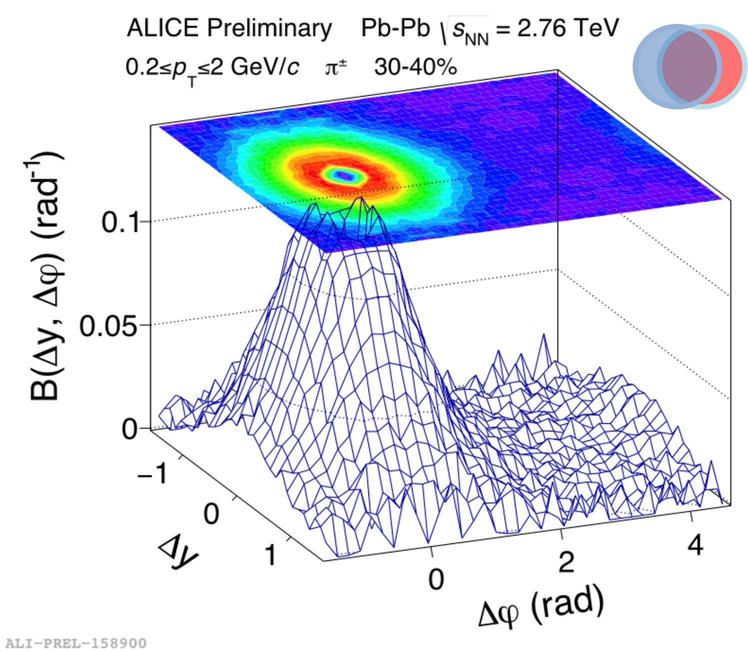
Cuts and Purity

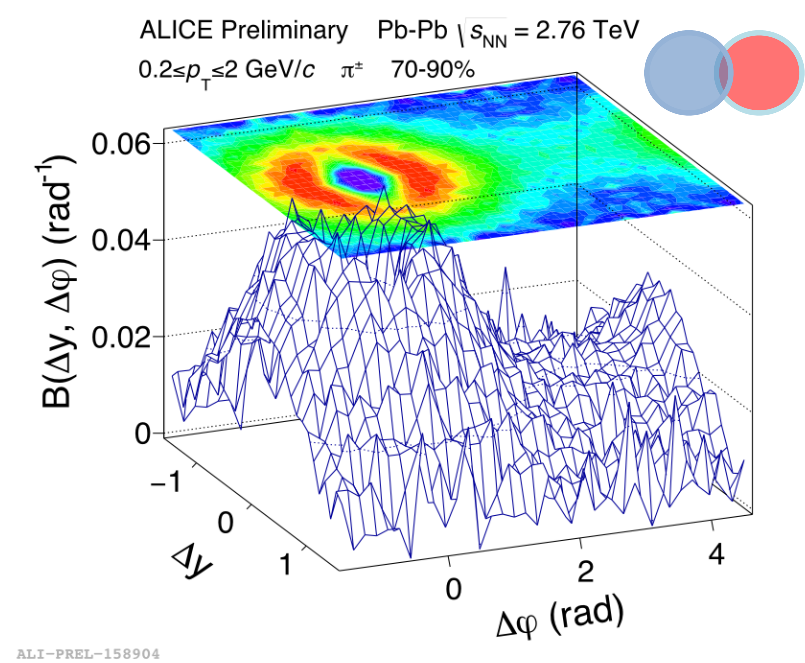
	$\pi^{\pm}$	$K^{\pm}$
TPC *0.2<p_T, p<0.8 GeV*	$n\sigma_{\pi}<2$, $n\sigma_{K,p}>2$, $n\sigma_{e}>1$	$n\sigma_{K}<2$ $n\sigma_{\pi,p}>3$ $n\sigma_{e}>1$
TOF *0.8<p, p_T<2.0 GeV*	$n\sigma_{\pi}<2$, $n\sigma_{K,p}>2$	
TPC + TOF *0.8<p, p_T<2.0 GeV*		$n\sigma_{K}<2$ $n\sigma_{\pi,p}>3$
Purity	>96%	>96%

Pion, Kaon BF: Pb — Pb @ 2.76 TeV

π±

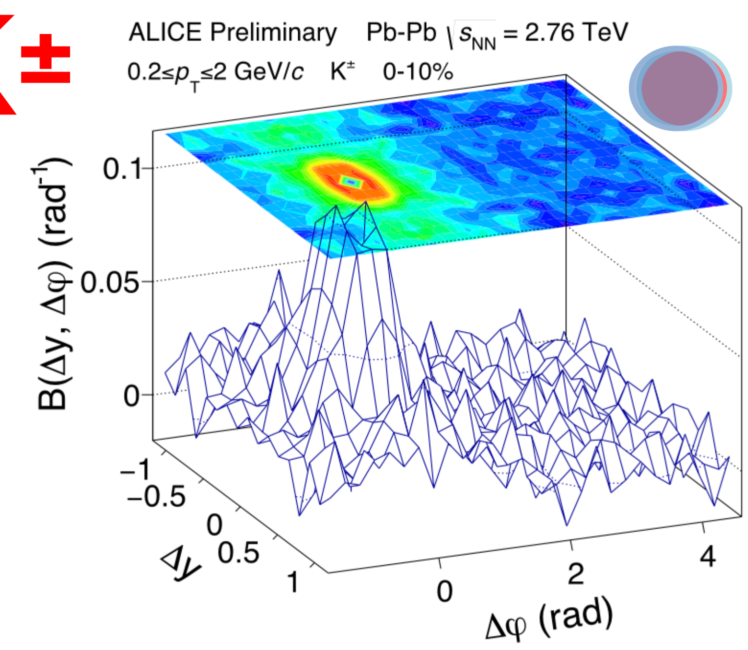


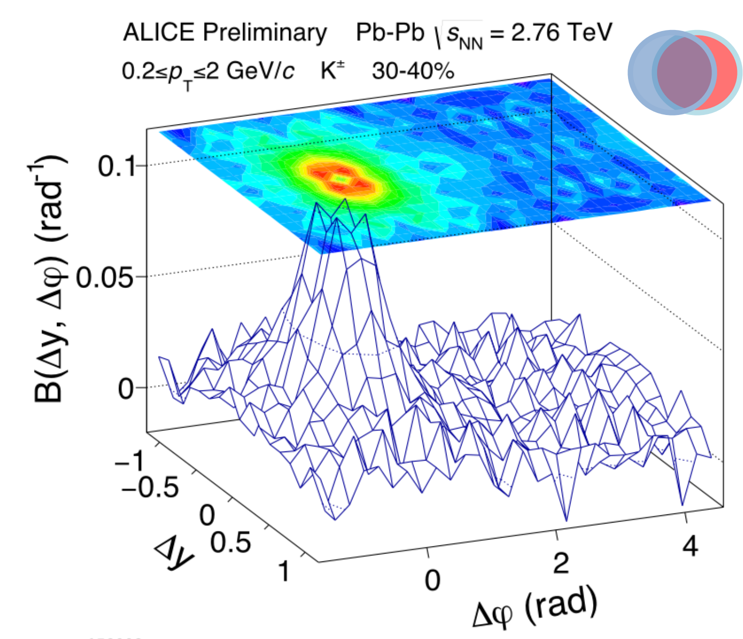


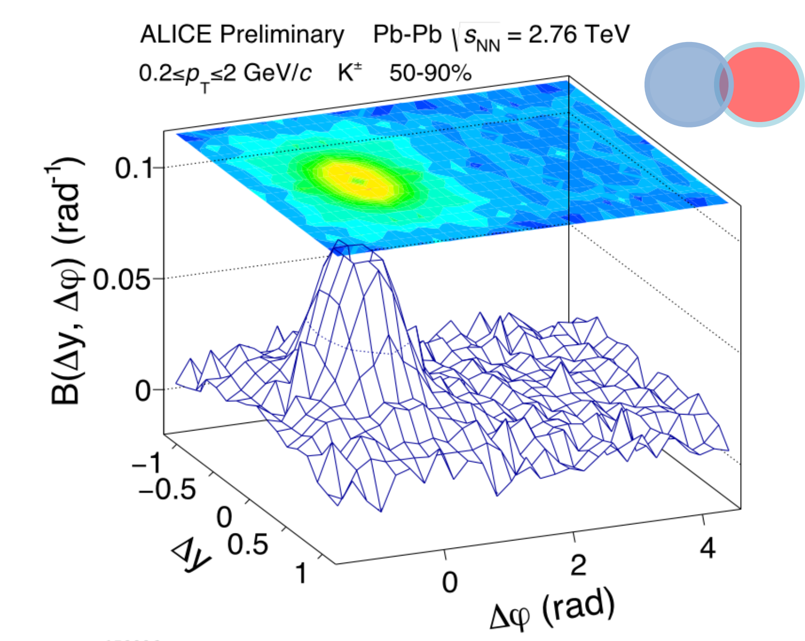


π± : Considerable shape dependence on collision centrality

K±



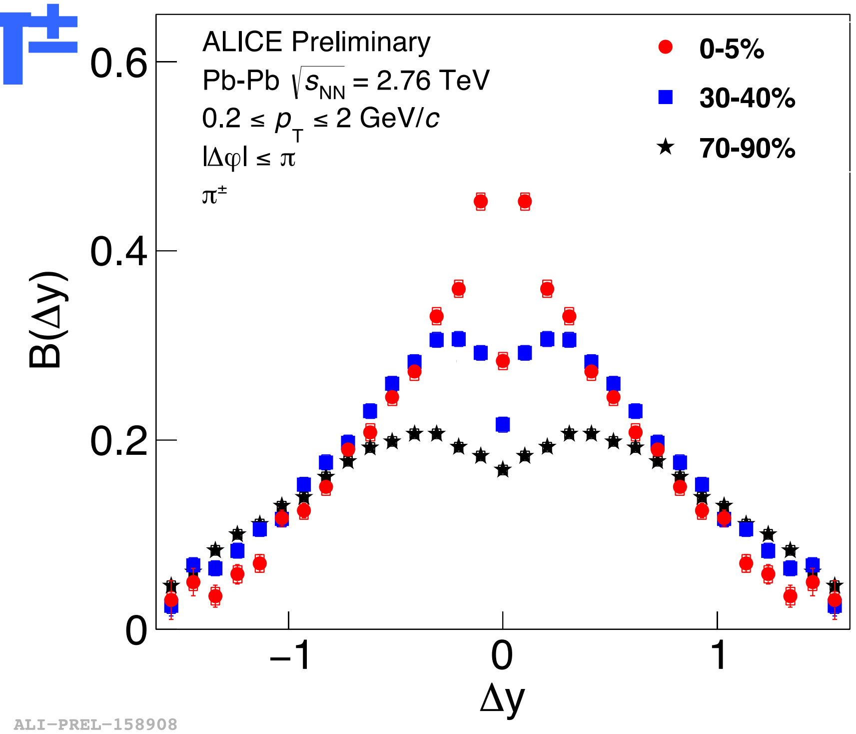


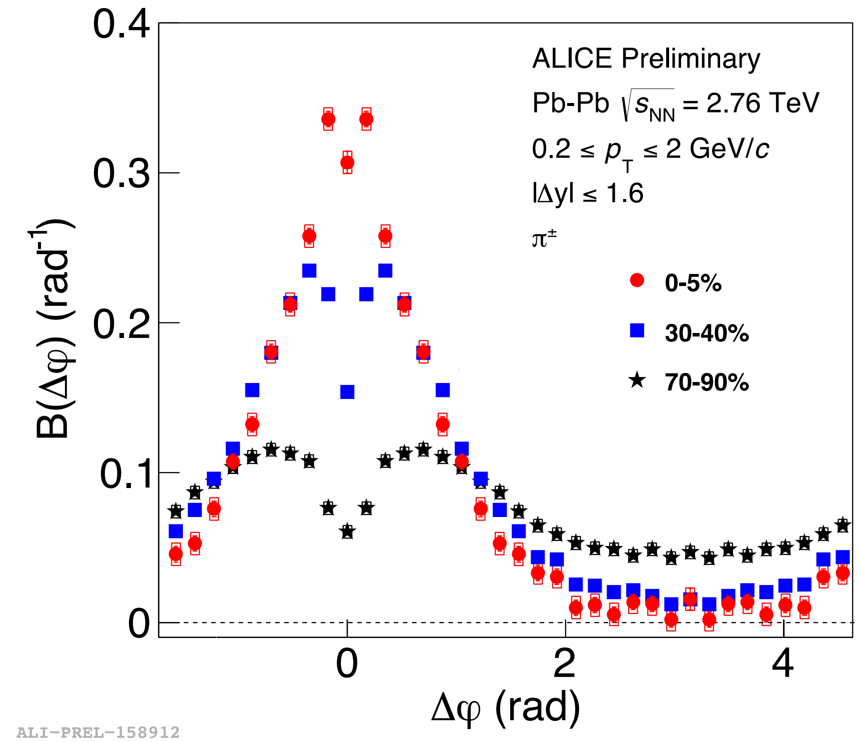


K± : Modest shape dependence on collision centrality

Pion, Kaon— Projections

π±

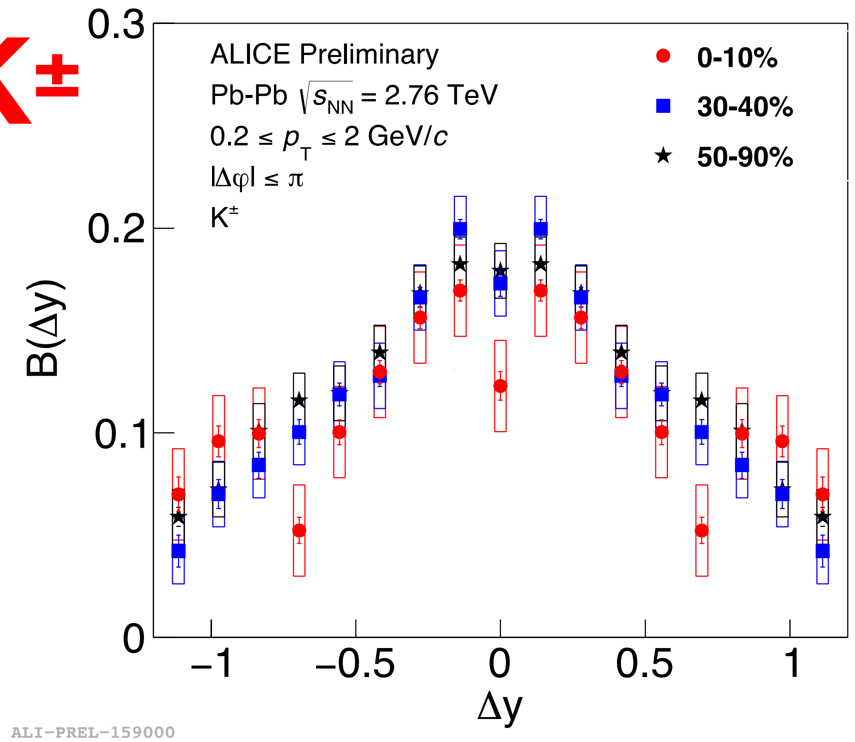


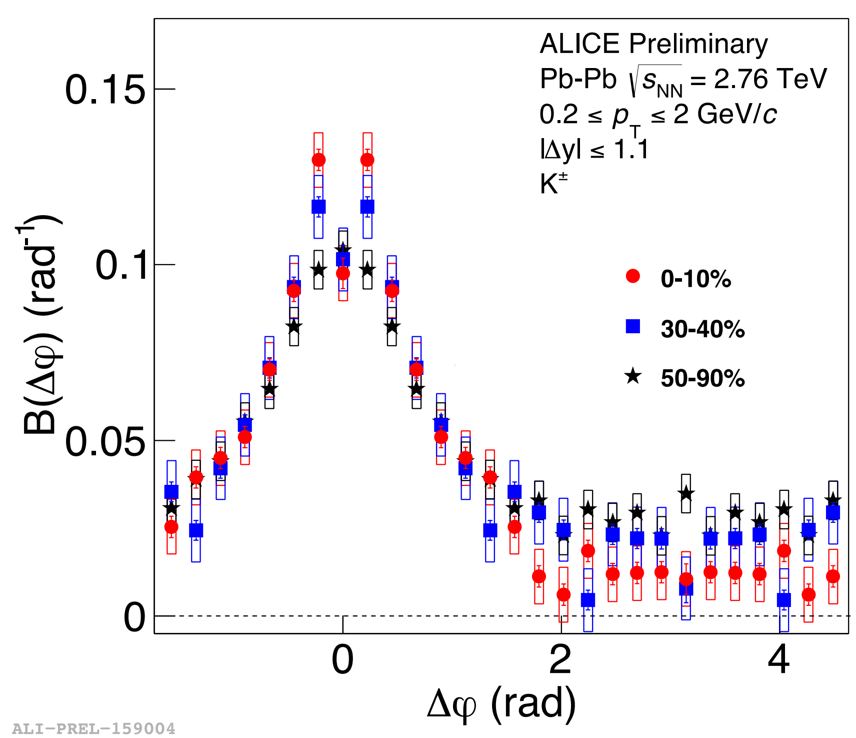


- Efficiency corrected
- Absolute normalization
- Can be integrated meaningfully

π± :
Considerable shape dependence on collision centrality

K±

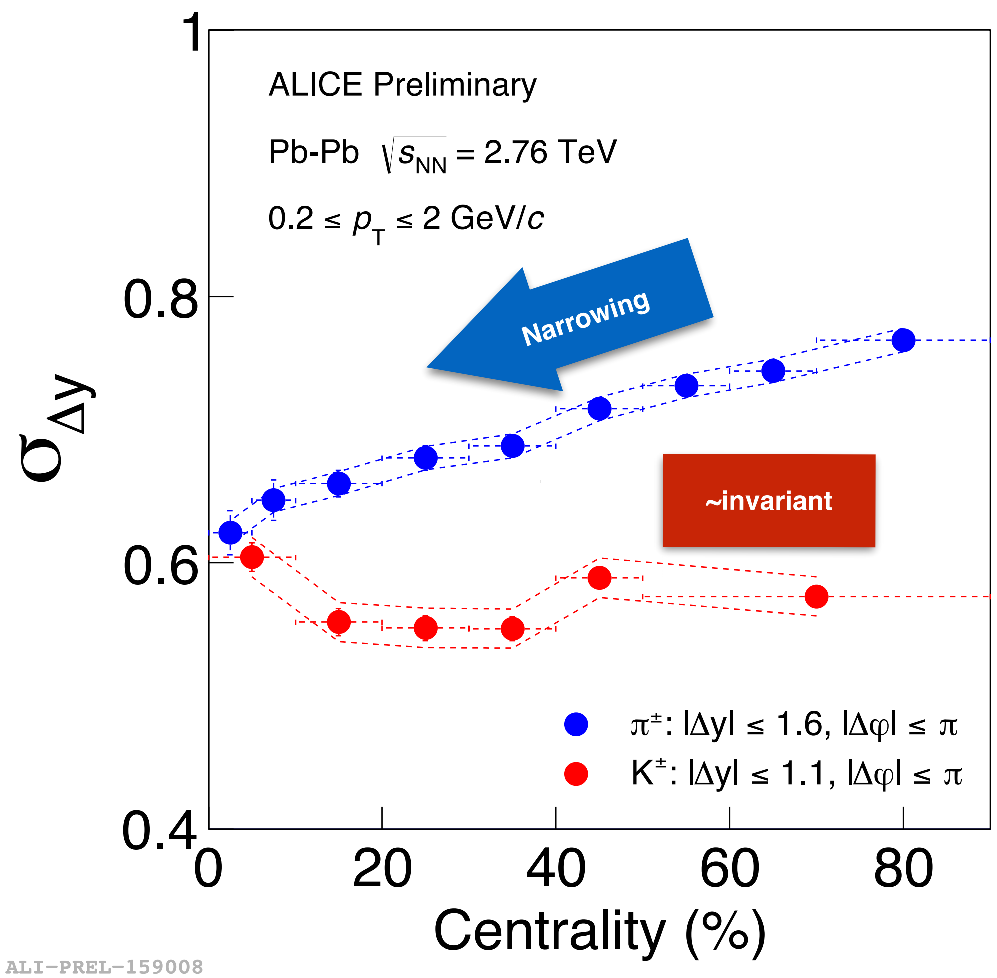


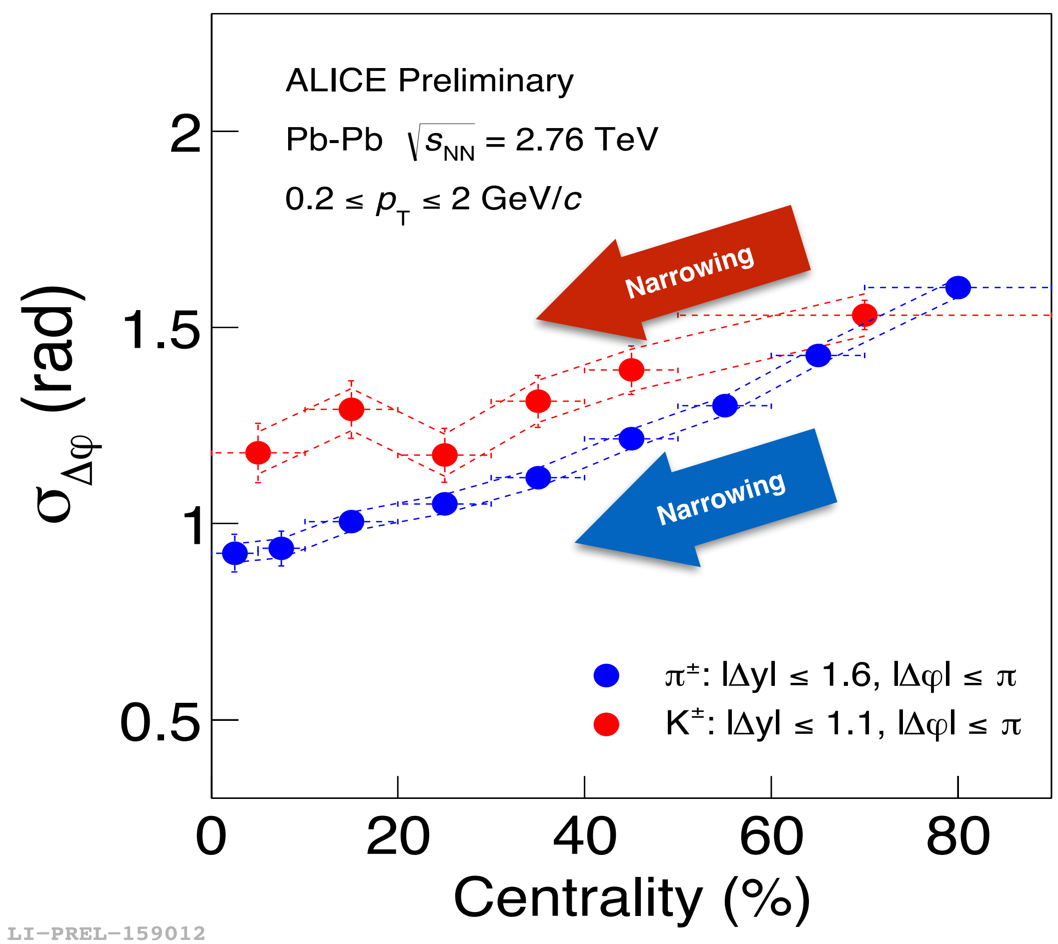


K± :
Modest shape dependence on collision centrality

Pion, Kaon Balance Functions — Pb-Pb

BF Widths

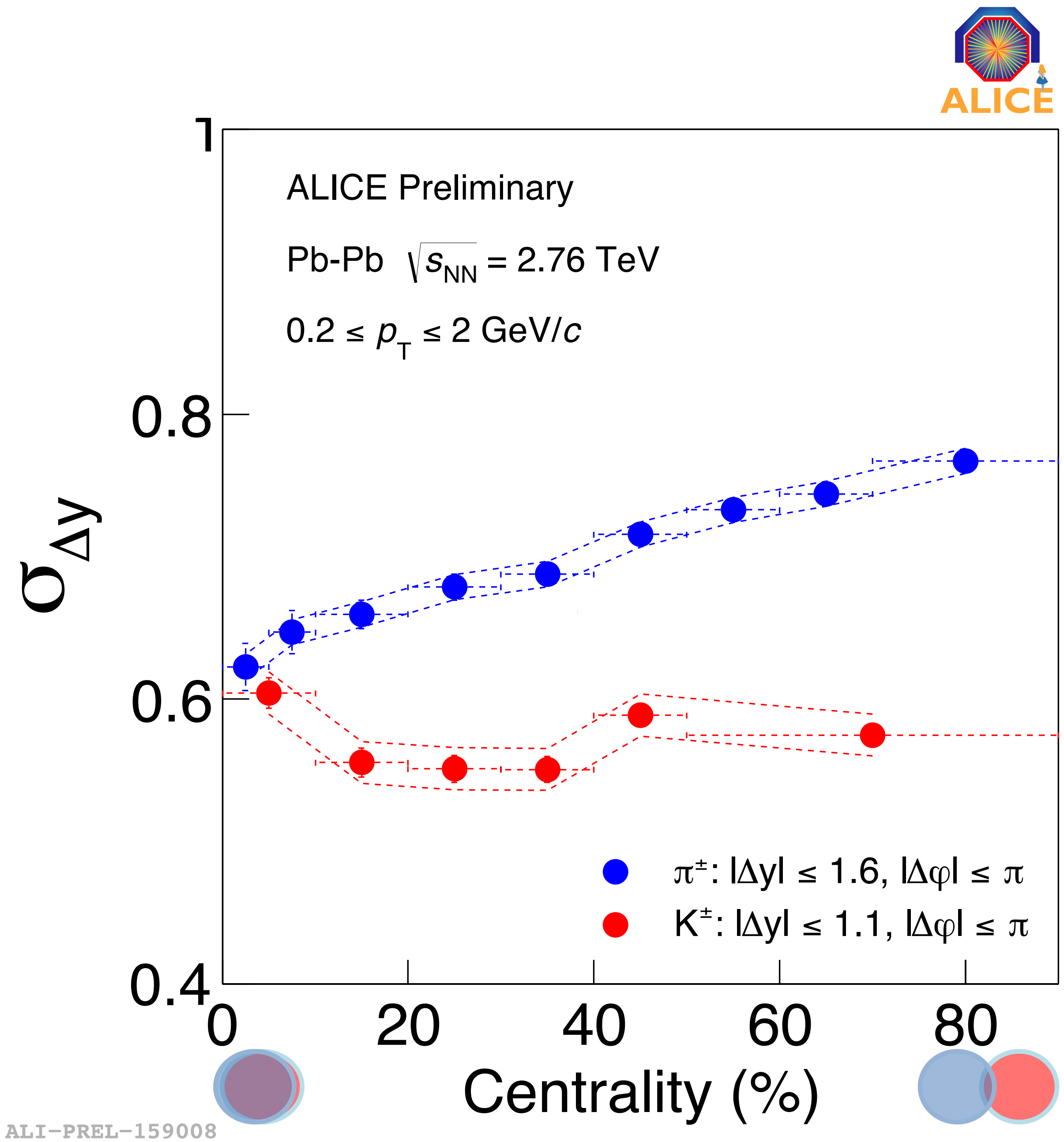


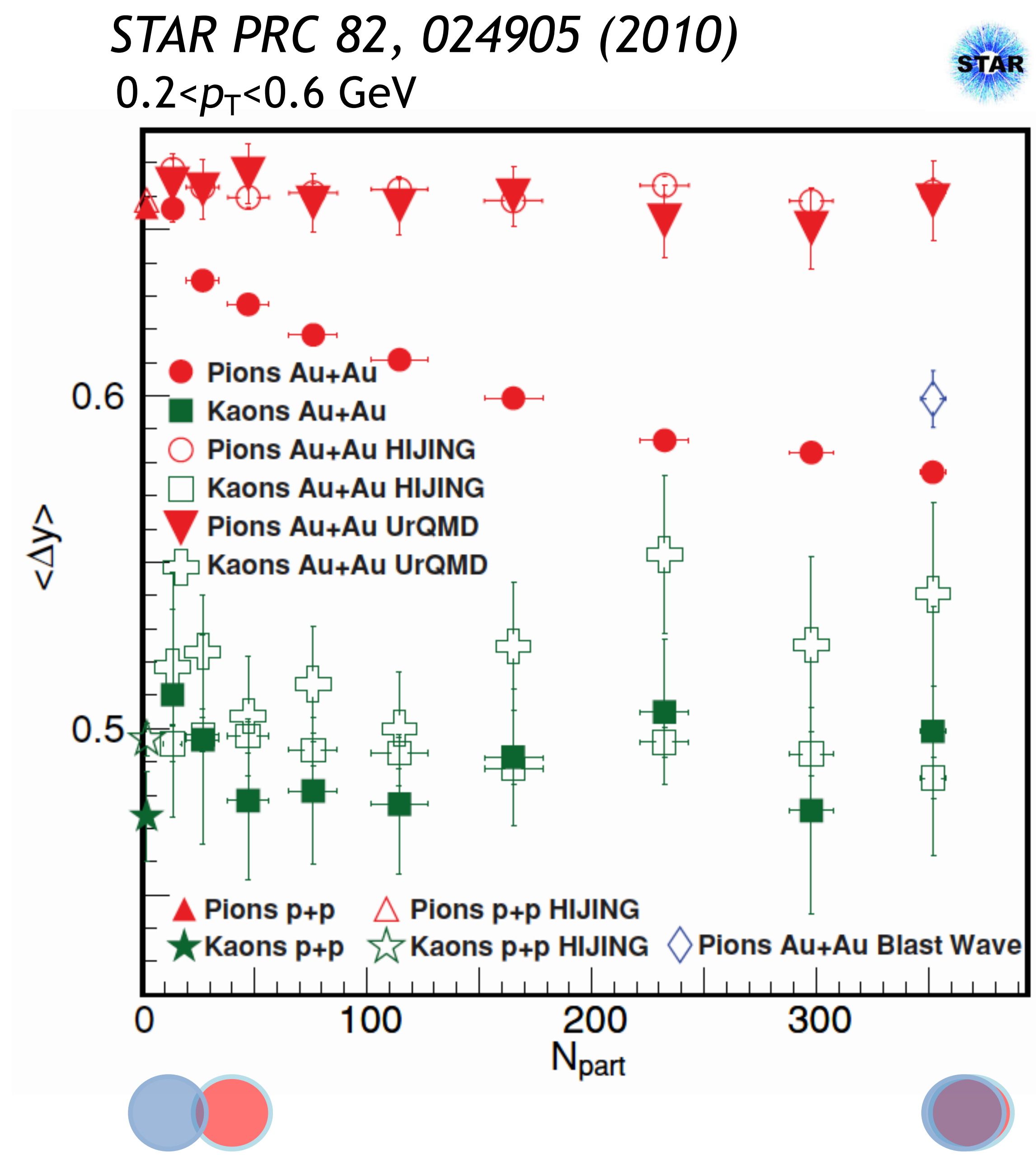


- **Longitudinal Widths**
  - **Pions: Narrowing vs. centrality**
  - **Kaons: ~Invariant vs. centrality**
- **Azimuthal Widths**
  - **Pions: Narrowing vs. centrality**
  - **Kaons: Narrowing vs. centrality**

- **Signature expected (Pratt et al.) for**
  - **Strong radial flow**
  - **Delayed hadronization (pions)**
  - **Two stage charge productionity**

Comparison With STAR Results



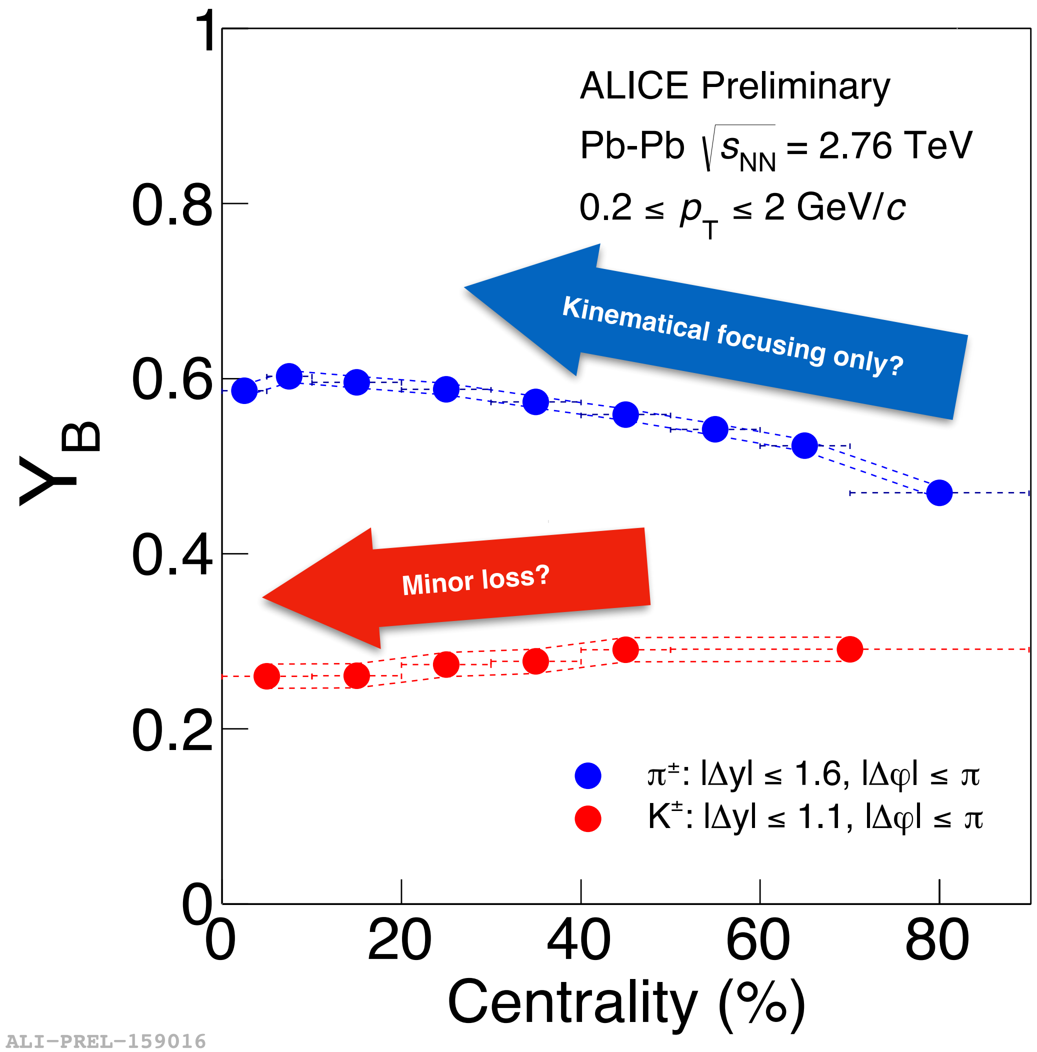


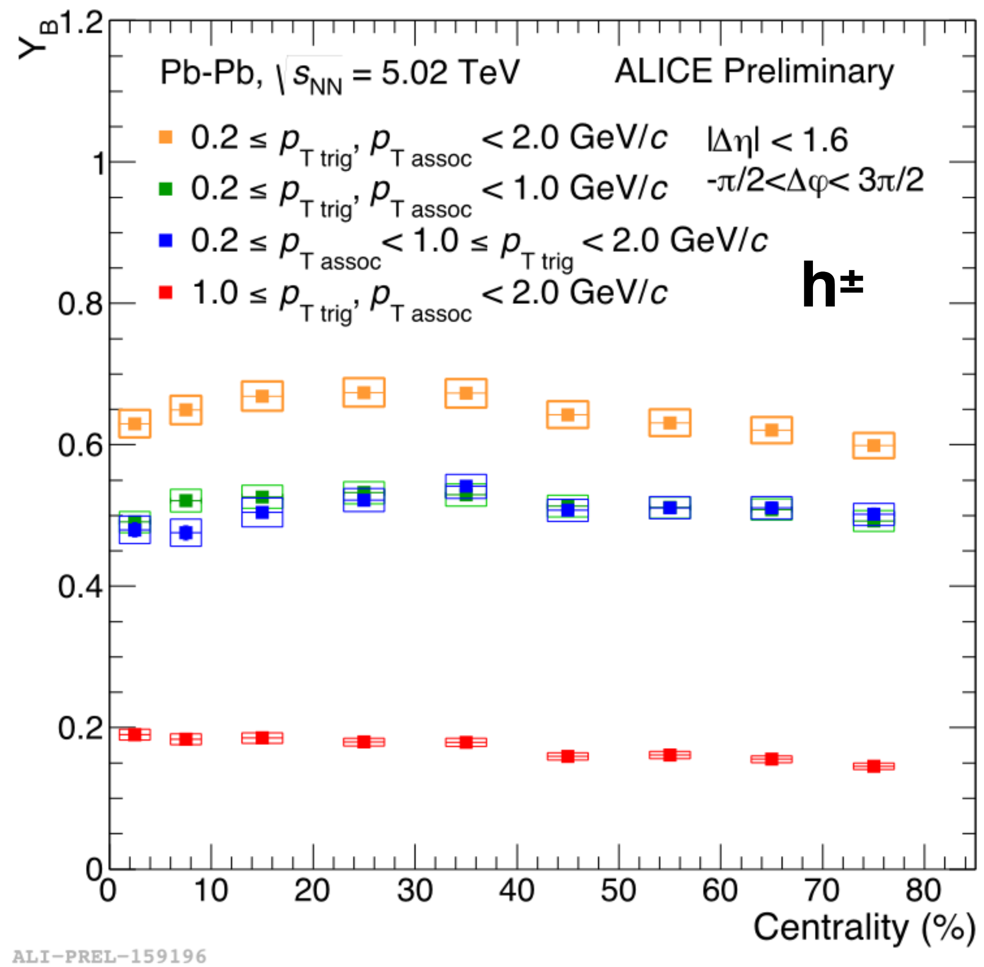
- ◇ $\pi^{\pm}$ width narrowing towards central collisions
- ◇ $K^{\pm}$ no change of width with centrality
- ◇ consistent with two-wave model

- ◇ similar trends and magnitudes measured by STAR
- ◇ In Au-Au @ 200 GeV

Hadron, Pion, Kaon Balance Functions — Pb-Pb

BF Yields





- **Yield vs. centrality is sensitive to …**
  - **Hadro-chemistry, What particles accompany a pion? A pion? A kaon? etc.**
    - **Resonances, string fragmentation/melting, etc**
  - **System expansion dynamics**
    - **Use to constrain BW models or Hydrodynamic models.**

R_2^{CD}, Balance functions (BF), General Balance Function (GBF)

Analysis/Publication Status/Opportunities

	$h^{\pm}$	$\pi^{\pm}$	$K^{\pm}$	p(p)	Λ	?
$h^{\pm}$	Pb-Pb 2.76 TeV[1,2] p-p 7 TeV[2] p-Pb 5.02[2] Pb-Pb 5.02 TeV[3] **Xe+Xe**					
$\pi^{\pm}$		Pb-Pb 2.76 TeV[3] p-p 7 TeV[3] p-Pb 5.02[3] **Xe-Xe** **Pb-Pb (2018)**	Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	**Pb-Pb (2018)**	
$K^{\pm}$		Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	Pb-Pb 2.76 TeV[3] Pb-Pb 5.02 TeV[5] **Xe+Xe**	Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	**Pb-Pb (2018)**	
$p^{\pm}$		Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	Pb-Pb 2.76 TeV[4] **Pb-Pb (2018)**	Pb-Pb 2.76 TeV[4] Pb-Pb 5.02 TeV[5] **Pb-Pb (2018)**	Pb-Pb 5.02 TeV[6] **Pb-Pb (2018)**	
Λ		**Pb-Pb (2018)**	**Pb-Pb (2018)**	Pb-Pb 5.02 TeV[6] **Pb-Pb (2018)**	Pb-Pb 5.02 TeV[6] **Pb-Pb (2018)**	

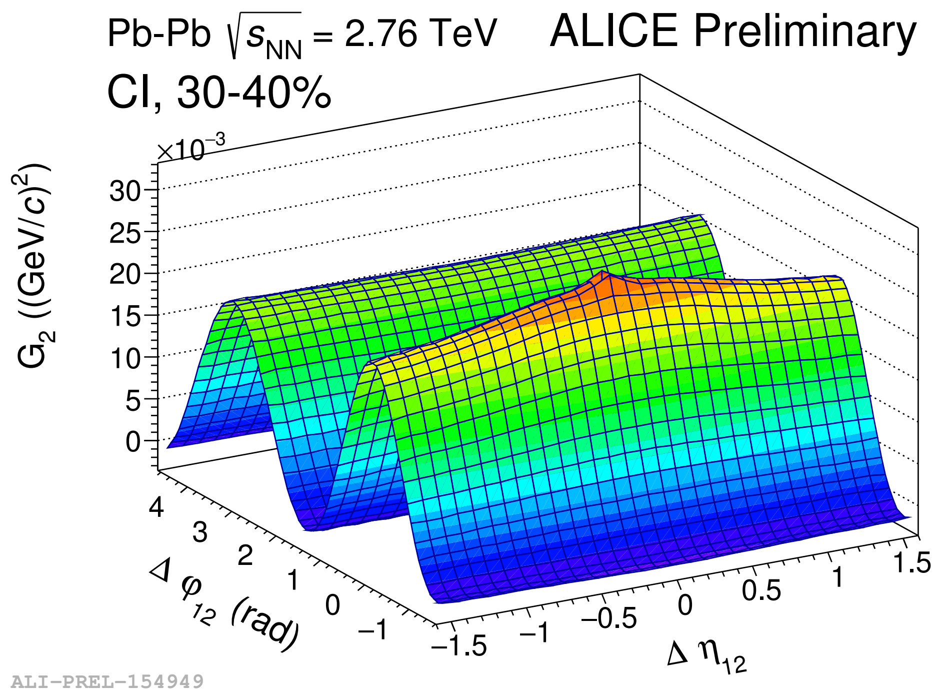
- Currently: PID Cuts based on dE/dx, TOF
- Near future: **Diff. Identity Method**
  - Expanded kinematic range,
  - Better statistics
  - ***OPPORTUNITY FOR "NEW" STUDENTS/ Post-Docs***

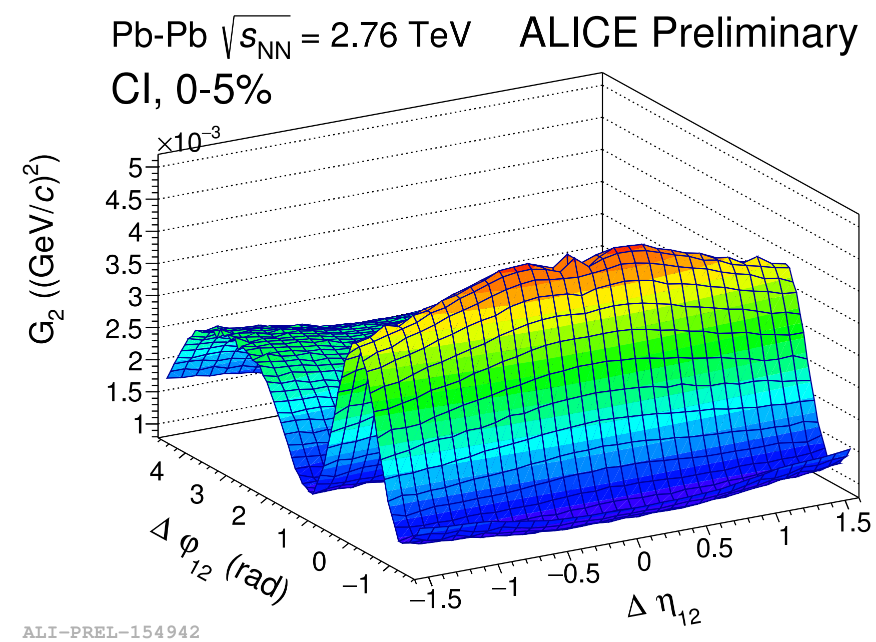
(1) ALICE, PLB 723, 267 (2013)
(2) ALICE, Eur. Phys. J. C76, 86 (2016), 1509.07255
(3) J. Pan, D. Caffarri, QM18
(4) **J. Pan (PhD Thesis) - paper in 2019**.
(5) D. Caffarri - paper in 20192020.
(6) S. Basu - paper in 2019/2020.

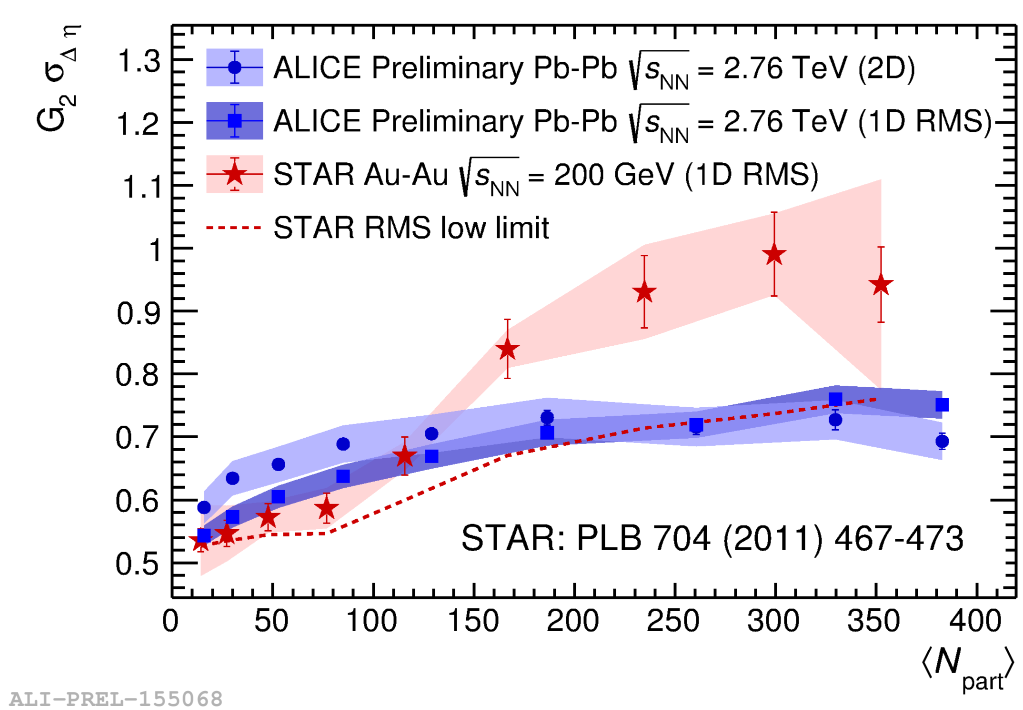
Momentum Correlator G_2

$$G_2(\Delta\eta,\Delta\phi) \equiv \frac{\int \rho_2(\vec{p}_1,\vec{p}_2) p_{T,1} p_{T,2} \, dp_{T,1} \, dp_{T,2}}{\rho_1(\eta_1,\phi_1) \otimes \rho_1(\eta_2,\phi_2)} - \langle p_{T,1} \rangle \langle p_{T,2} \rangle$$

S. Gavin Phys.Rev.Lett. 97 (2006) 162302
M. Sharma & C.P. et al (STAR), PLB704, 467 (2011)







- Broadening of G2 w/ centrality
- Different than STAR's
- Implications on viscosity at LHC ?

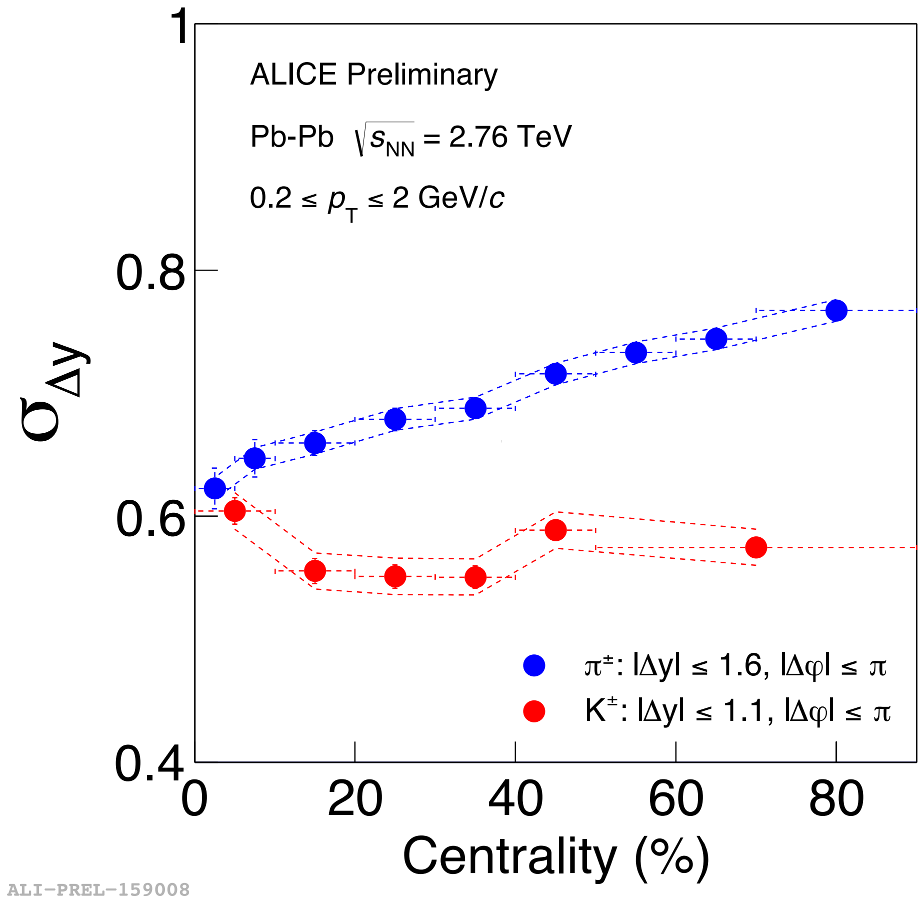
Summary

- **Charged hadrons BF (Pb-Pb @ 5.02 TeV):**
  - p_T dependent BF
  - *narrowing towards central collisions similar to Pb-Pb @ 2.76 TeV for p_T<2.0 GeV/c*
  - Balancing Charge Yield consistent with narrowing towards central collisions

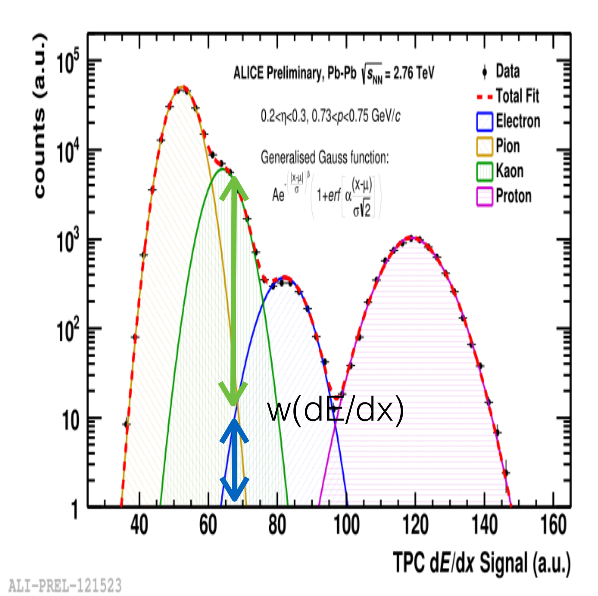
System	$\sqrt{s_{NN}}$ (TeV)	h±	π±	K±
Pb-Pb	2.76	✔	✔	✔
Pb-Pb	5.02	✔		
p-Pb	5.02	✔	✔	
pp	7	✔	✔	

✔ - published
✔ - new results

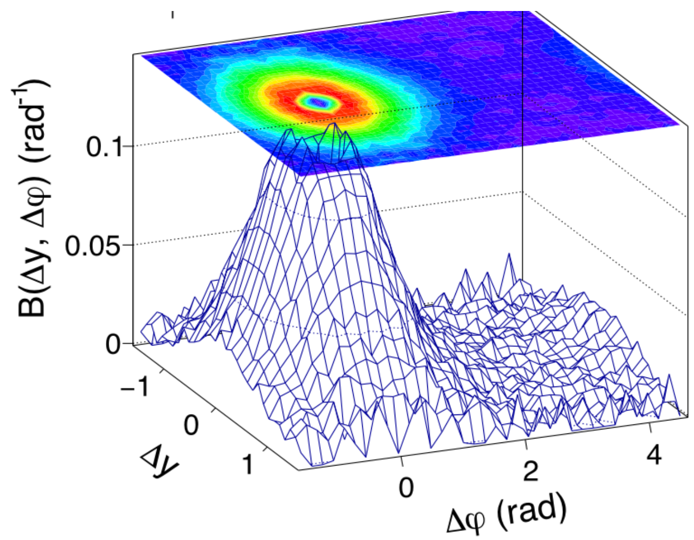
- **Charged pions & kaons BF (Pb-Pb @ 2.76 TeV):**
  - B(Δy): π± narrowing towards central collisions similar to h±
  - K± no centrality dependence
  - Similar trends and magnitude to STAR results for Au-Au @ 200 GeV
  - Consistent with two-wave production model
  - B(Δφ): both π± and K± narrowing towards central collisions
  - Strong radial flow - Tune models
- From Model Comparisons
  - Models need to account properly for charge conservation.



Identity Method for π, K, p, Λ identification



PID Matrix		
π-π	π-K	π-p
K-π	K-K	K-p
p-π	p-K	P-p



Measurements of moments of particle multiplicities w/ IM

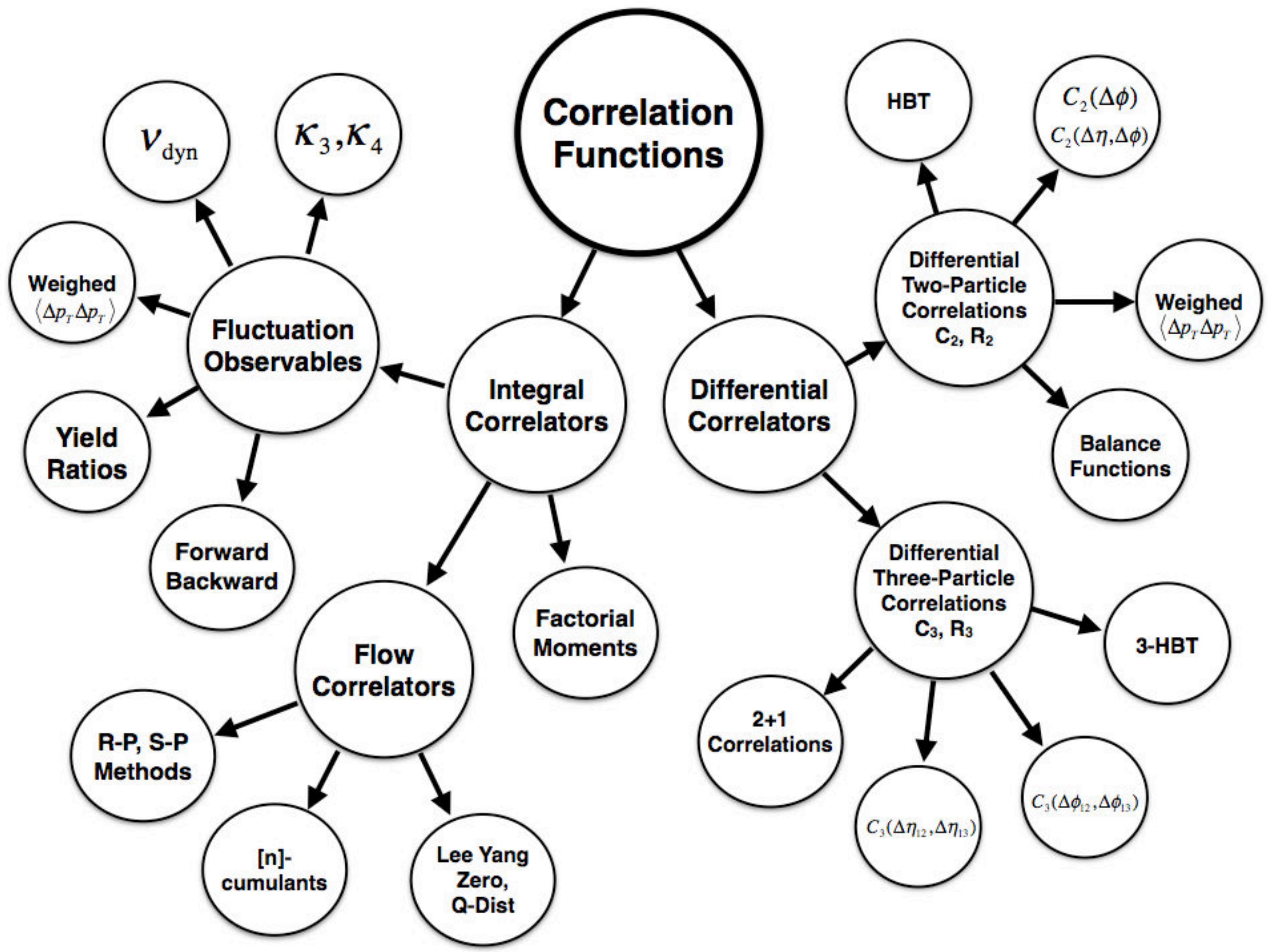
- **Two particle species**: M. Gazdzicki, et al., PRC83 (2011) 054907; M. Gazdzicki, EPJC 8, 131 (1999), nucl-th/9712050.
- **Arbitrary number of species**: M. I. Gorenstein, PRC84, 024902 (2011).
- **Measurements of higher moments**: A. Rustamov and M. I. Gorenstein, PRC86, 044906 (2012).
- Measurements of moments **in the presence of transverse momentum-dependent efficiency losses**: C. A. Pruneau, PRC 96, 054902 (2017)
- **Differential CFs w/ efficiency losses**: C. Pruneau, A. Ohlson, arXiv:1806.02264v1, Accepted PRC
- Nu-Dyn: πK, πp, Kp, ALICE (Mesut Arsland), submitted to EPJC, arXiv: 1712.07929

- **π, K, p identification**:
  - Compute probability (weight) of measured dE/dx, corresponds to π, K, p, fill histograms for each species (statistical identification),
  - Calculate sum of weights (W) instead of multiplicity
  - Calculate moments of W distribution, invert response matrix to determine moments <N> and <N(N-1)>
  - **Account for misidentification/impurity (and efficiency) without lowering efficiency** by imposing strict selection cuts.

- **Essentially an unfolding method**
- **Applicable to integral and differential correlation functions**
- **Concept applicable to primary/secondary track unfolding also…**

Last: a shameless plug…

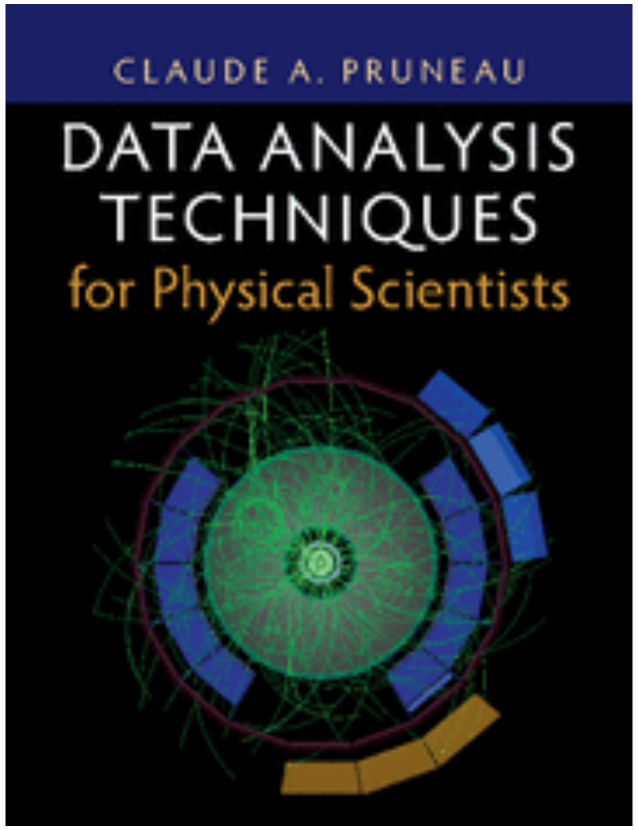
- Correlation observables are all inter-connected …
- Measure/emphasize different aspects of the physics we seek to understand.



~730 pages, ~90$, a very good value…

For basic intro, see:

www.cambridge.org/9781108416788



Topics	Chapters
Classical Statistics	5
Bayesian Statistics	1
Data Reconstruction/ Analysis Methods	2
Correlation Functions	2
Data Correction/Unfolding	1
Basic Monte Carlo Techniques	2