Dim-8 operators and W+h production

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based on 1808.00442 with C. Hays (Oxford), V. Sanz & J. Setford (U. Sussex)

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Why go to dim > 6 in SMEFT?

Generic amplitude: $$A = A_{SM} + A_6 \frac{E^2}{\Lambda^2} + A_8 \frac{E^4}{\Lambda^4} + \cdots$$

$$|A|^2 = |A_{SM}|^2 + \frac{2E^2}{\Lambda^2} Re(A^*_{SM} A_6) +$$

$$\frac{E^4}{\Lambda^4} |A_6|^2 + \frac{2E^4}{\Lambda^4} Re(A^*_{SM} A_8) \cdots$$

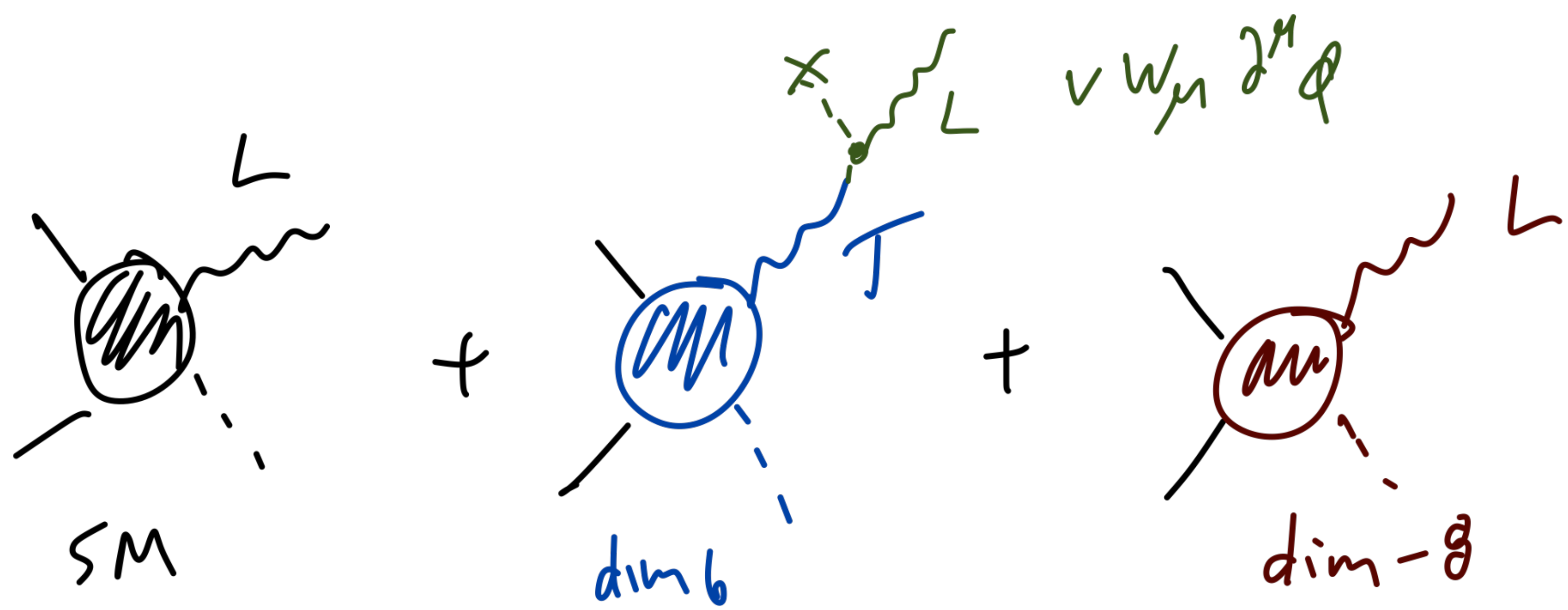
same order in $1/\Lambda$

- $1/\Lambda^4$ important to understand uncertainty on $1/\Lambda^2$
- If you want to include (dim-6)^2, need to include dim-8 to be consistent

Why go to dim > 6 in SMEFT?

While (dim-6)^2 , dim-8 have same $1/\Lambda$, they may have
different energy dependence

e.g. if helicity of process mismatches with dim-6 structure
but not dim-8



[1607.05236 Azatov et al]

Why go to dim > 6?

In that case:

$$|A|^2 \supset \frac{v^2 \hat{s}}{\Lambda^4}|A_6|^2 + 2\frac{\hat{s}^2}{\Lambda^4}Re(A_{SM}^* A_8) + \cdots$$

Dim-8 piece enhanced relative to (dim-6)^2 at large energy

(similar situation if dim-6 violates a symmetry (i.e. CP))

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(similar situation if dim-6 violates a symmetry (i.e. CP))

But: dim > 6 gets complicated because there are more fields = number ways to contract indices grows rapidly. Also IBP and EOM redundancies

ex.) $D_\mu H D^\mu H H^{\dagger 2}, \quad D_\mu H^\dagger D^\mu H^\dagger H^2, \quad D_\mu H D^\mu H^\dagger (H^\dagger H)$

not independent

Powerful new tool

of operators and their field content can be generated automatically via **Hilbert Series**

[Lehman, AM ’15, Henning et al ’15, ’17]

given symmetry group **G**, fields $\boldsymbol{\phi_i}$, $\boldsymbol{\psi_i}$, $\boldsymbol{X_i^{L,R}}$ → Hilbert Series → # and form of **all** invariant (Lorentz & gauge) operators,

- extends to all orders
- includes all IBP, EOM redundancies
- works for all sorts of EFT (SMEFT, nonlinear reps, non-relativistic QFT)

[Kobach, Pal ’17, ’18]

Example output: SMEFT dim-8

$$(e_c^\dagger\, e_c)(L^\dagger\, L)W^L$$
$$(e_c^\dagger\, L^\dagger)(d_c\, Q)W^L$$
$$3\,(L^\dagger\, L)(Q^\dagger\, Q)W^L$$
$$(d_c^\dagger)^2\, d_c^2\, G^L$$
$$4\,(d_c^\dagger\, d_c)(u_c^\dagger\, u_c)\, G^L$$
$$2\,(Q^\dagger\, Q)(L^\dagger\, L)\, G^L$$
$$(d_c\, Q)(e_c^\dagger\, L^\dagger)\, G^L$$

$$\ldots$$

$$3\,D\,(d_c^\dagger\, d_c)(Q\, H\, d_c)$$
$$6\,D\,(Q^\dagger\, Q)(Q\, H\, d_c)$$
$$6\,D\,(L^\dagger\, L)(Q\, H^\dagger\, u_c)$$
$$3\,D\,(u_c^\dagger\, u_c)(Q\, H^\dagger\, u_c)$$

$$\ldots$$

# of operators and field content automatic, but no information about how to attach indices or how to place derivatives

Systematic algorithm for how to add indices & select minimal sets developed in 1808.00442

Finding the form of operators:

$\mathcal{O}(D^0)$ terms are straight forward group theory + Bose/Fermi statistics, trickiness enters with derivatives

$$3D(d_c^\dagger\, d_c^2 H Q) = \,?$$

Problem only gets worse as you add derivatives

ex.)

$$2\,D^3(d_c^\dagger\, d_c\, H^\dagger H) \quad \left\{ \begin{array}{r} d_c^\dagger (D d_c)(D^2 H^\dagger) H, \\ d_c^\dagger (D d_c) H^\dagger\, D^2 H, \\ d_c^\dagger (D d_c)(D H^\dagger)^2 H, \\ d_c^\dagger (D d_c)(H^\dagger)(D H)^2, \\ d_c^\dagger (D d_c)(D H^\dagger)(D H) H^\dagger H \\ \dots \end{array} \right.$$

14 terms

Finding the form of operators:

To proceed: manually reconstruct IBP relationships

Suppose we're interested in operators with **n** fields, **m** derivatives

$$\mathcal{O}^i(D^m\phi^n)$$

Finding the form of operators:

To proceed: manually reconstruct IBP relationships

Suppose we're interested in operators with **n** fields, **m** derivatives

$\mathcal{O}^i(D^m\phi^n)$ i.e. $D^2(\phi^2(\phi^*)^2)$

1.) Look first for operators: $\mathcal{O}^j_\mu(D^{m-1}\phi^n)$ $(D_\mu\phi^*)\phi^*\phi^2\ldots$

one fewer derivative, four-vector representation of Lorentz, invariant under all other symmetries

(#, form already present in Hilbert series)

(see talk by David for related approach)

Finding the form of operators:

2.) apply the last derivative:

$$D^\mu\Big[\mathcal{O}^j_\mu(D^{m-1}\phi^n)\Big] = \sum_i c_{ji}\,\mathcal{O}^i(D^m\phi^n)$$

relation between operators $\mathcal{O}^i(D^m\phi^n)$

ex.) $$D^\mu(\phi^2\,\phi^*\,D_\mu\phi^*) = 2\,\phi\phi^*(D_\mu\phi)(D^\mu\phi^*) + \phi^2\,(D_\mu\phi^*)^2$$

total derivative $$= \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} \phi\phi^*(D_\mu\phi)(D^\mu\phi^*) \\ \phi^2(D_\mu\phi^*)^2 \end{pmatrix}$$

build up IBP matrix c_{ji}

$$\therefore \quad \underset{\text{Including IBP}}{\#\ \mathcal{O}^i(D^m\phi^n)} = \#\ \mathcal{O}^i(D^m\phi^n) - \cancel{\#\ \mathcal{O}^j_\mu(D^{m-1}\phi^n)}$$

$$- \text{rank}(c^{ij})$$

As $\mathcal{O}^j_\mu(D^{m-1}\phi^n)$ operators may not be independent:

- row-reduced c_{ij} tells us complete IBP relations among $\mathcal{O}^i(D^m\phi^n)$

ex.) For $2\,D^3(d_c^\dagger\, d_c\, H^\dagger H)$ (14 terms, 14 conditions from $O(D^2)$)

$$\left.\begin{array}{l}(d^\dagger\bar\sigma^\mu D_\nu d_c)(D^2_{\{\mu\nu\}}H^\dagger H)\\(d^\dagger\bar\sigma^\mu D_\nu d_c)(H^\dagger D^2_{\{\mu\nu\}}H)\end{array}\right\}$$ are sufficient to generate all operators of this type (not unique choice)

$$\left.\begin{array}{l}(d_c^\dagger\bar\sigma^\mu d_c)(D^2_{\{\mu\nu\}}H^\dagger D_\nu H)\\(d_c^\dagger\bar\sigma^\mu d_c)(D_\nu H^\dagger D^2_{\{\mu\nu\}}H)\end{array}\right\}$$ are not sufficient

$$\therefore \quad \underset{\text{Including IBP}}{\#\, \mathcal{O}^i(D^m\phi^n)} \;=\; \#\, \mathcal{O}^i(D^m\phi^n) \;-\; \#\, \cancel{\mathcal{O}^j_\mu(D^{m-1}\phi^n)}$$

$$-\,\text{rank}(c^{ij})$$

As $\mathcal{O}^j_\mu(D^{m-1}\phi^n)$ operators may not be independent:

-

$\mathcal{O}_{2D1}$	$(H^\dagger H)^2(D_\mu H^\dagger\, D_\mu H)$	$\mathcal{O}_{2D14}$	$i\,\epsilon_{IJK}\,(D_\mu H^\dagger\tau^I D_\nu H)(W^J_{\mu\rho}\widetilde{W}^K_{\rho\nu} + \widetilde{W}^J_{\mu\rho}W^K_{\rho\nu})$
$\mathcal{O}_{2D2}$	$\delta_{IJ}\,(H^\dagger H)(H^\dagger\tau^I H)(D_\mu H^\dagger\tau^J D_\mu H)$	$\mathcal{O}_{2D15}$	$\delta_{IJ}\,(D_\mu H^\dagger\,\tau^I D_\mu H)B_{\rho\sigma}W^J_{\rho\sigma}$
$\mathcal{O}_{2D3}$	$(D_\mu H^\dagger\, D_\nu H)B_{\mu\rho}B_{\rho\nu}$	$\mathcal{O}_{2D16}$	$\delta_{IJ}\,(D_\mu H^\dagger\,\tau^I D_\mu H)B_{\rho\sigma}\widetilde{W}^J_{\rho\sigma}$
$\mathcal{O}_{2D4}$	$(D_\mu H^\dagger D_\mu H)B_{\rho\sigma}B_{\rho\sigma}$	$\mathcal{O}_{2D17}$	$i\,\delta_{IJ}\,(D_\mu H^\dagger\tau^I D_\nu H)(B_{\mu\rho}W^J_{\rho\nu} - B_{\nu\rho}W^J_{\rho\mu})$
$\mathcal{O}_{2D5}$	$(D_\mu H^\dagger D_\mu H)B_{\rho\sigma}\widetilde{B}_{\rho\sigma}$	$\mathcal{O}_{2D18}$	$\delta_{IJ}\,(D_\mu H^\dagger\tau^I D_\nu H)(B_{\mu\rho}W^J_{\rho\nu} + B_{\nu\rho}W^J_{\rho\mu})$
$\mathcal{O}_{2D6}$	$\delta_{AB}\,(D_\mu H^\dagger D_\nu H)G^A_{\mu\rho}G^B_{\rho\nu}$	$\mathcal{O}_{2D19}$	$\delta_{IJ}\,(D_\mu H^\dagger\tau^I D_\nu H)(B_{\rho[\mu}\widetilde{W}^J_{\nu]\rho} - \widetilde{B}_{\rho[\mu}W^J_{\nu]\rho})$
$\mathcal{O}_{2D7}$	$\delta_{AB}\,(D_\mu H^\dagger D_\mu H)G^A_{\rho\sigma}G^B_{\rho\sigma}$	$\mathcal{O}_{2D20}$	$i\,\delta_{IJ}\,(D_\mu H^\dagger\tau^I D_\nu H)(B_{\rho\{\mu}\widetilde{W}^J_{\nu\}\rho} + \widetilde{B}_{\rho\{\mu}W^J_{\nu\}\rho})$
$\mathcal{O}_{2D8}$	$\delta_{AB}\,(D_\mu H^\dagger D_\mu H)G^A_{\rho\sigma}\widetilde{G}^B_{\rho\sigma}$	$\mathcal{O}_{2D21}$	$(H^\dagger H)(D_\mu H^\dagger\, D_\nu H)B_{\mu\nu}$
$\mathcal{O}_{2D9}$	$\delta_{IJ}\,(D_\mu H^\dagger D_\nu H)W^I_{\mu\rho}W^J_{\rho\nu}$	$\mathcal{O}_{2D22}$	$(H^\dagger H)(D_\mu H^\dagger\, D_\nu H)\widetilde{B}_{\mu\nu}$
$\mathcal{O}_{2D10}$	$\delta_{IJ}\,(D_\mu H^\dagger D_\mu H)W^I_{\rho\sigma}W^J_{\rho\sigma}$	$\mathcal{O}_{2D23}$	$\delta_{IJ}\,(H^\dagger H)(D_\mu H^\dagger\tau^I D_\nu H)W^J_{\mu\nu}$
$\mathcal{O}_{2D11}$	$\delta_{IJ}\,(D_\mu H^\dagger D_\mu H)W^I_{\rho\sigma}\widetilde{W}^J_{\rho\sigma}$	$\mathcal{O}_{2D24}$	$\delta_{IJ}\,(H^\dagger H)(D_\mu H^\dagger\tau^I D_\nu H)\widetilde{W}^J_{\mu\nu}$
$\mathcal{O}_{2D12}$	$i\,\epsilon_{IJK}\,(D_\mu H^\dagger\tau^I D_\nu H)W^J_{\mu\rho}W^K_{\rho\nu}$	$\mathcal{O}_{2D25}$	$i\,\epsilon_{IJK}\,(H^\dagger\tau^I H)(D_\mu H^\dagger\tau^J D_\nu H)W^K_{\mu\nu}$
$\mathcal{O}_{2D13}$	$\epsilon_{IJK}\,(D_\mu H^\dagger\tau^I D_\nu H)(W^J_{\mu\rho}\widetilde{W}^K_{\rho\nu} - \widetilde{W}^J_{\mu\rho}W^K_{\rho\nu})$	$\mathcal{O}_{2D26}$	$i\,\epsilon_{IJK}\,(H^\dagger\tau^I H)(D_\mu H^\dagger\tau^J D_\nu H)\widetilde{W}^K_{\mu\nu}$

All boson operators with higgses translated, in FeynRules file

First application: $pp \to W h$

Why?

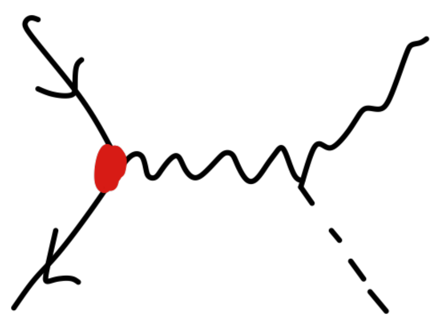
- Few operators contribute: we only care about dim-8 terms that can interfere w/ SM contributions — limits us to LH quarks, CP even: ~15 boson ops, ~14 fermionic, implemented in FeynRules/UFO

- Potential for large effects: most dim-6 terms generate W_T, not W_L, $\therefore$ $pp \to W_L\, h$ will have

$$(\text{dim-6})^2 \sim \frac{v^2\,\hat{s}}{\Lambda^4} \qquad\qquad (\text{dim-8}) \sim \frac{\hat{s}^2}{\Lambda^4}$$

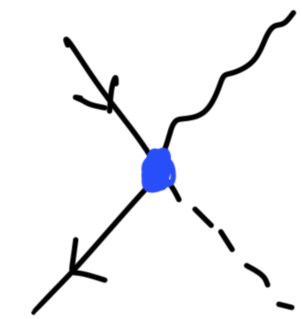
* $N_f = 1$ flavor universal higher dim effects, Warsaw basis

First application: $pp \to Wh$



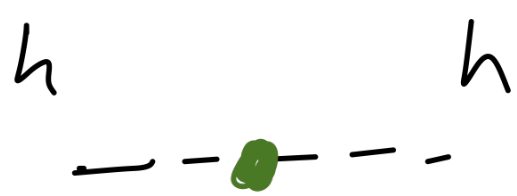
$\bar{f}fV$ modifications

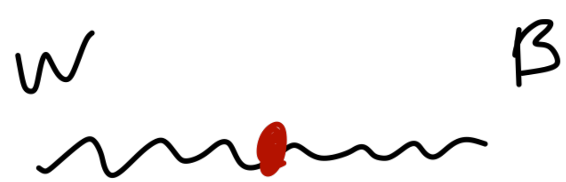
hVV modifications



contact terms

Field strengths, EW inputs modified

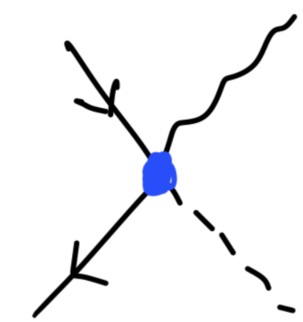




First application: $pp \to W h$

New at dim-8:

operators that affect contact terms but **not** $\bar{f}$fV



ex.) $(Q^\dagger \bar{\sigma}^\mu D_\nu Q)(D^2_{\{\mu\nu\}} H^\dagger) H \supset v\, d_L^\dagger \bar{\sigma}^\mu u_L\, W^-_\nu\, \partial^2_{\{\mu\nu\}} h + \cdots$

(compare to $(Q^\dagger \bar{\sigma}^\mu Q)(H^\dagger \overleftrightarrow{D}_\mu H)$ at dim-6)

important as they have strong momentum dependence,
involve W_L

First application: $pp \to W h$

To estimate impact of dim-8 terms:

- Turn on only one dim-6 operator at a time:

  First example: $c_{HW}\, H^\dagger H\, W_{\mu\nu} W^{\mu\nu}$, $c_{HW} = \dfrac{1}{\Lambda_6^2}$

- Turn on all dim-8, equal magnitude coefficients: $c_8^{(i)} = \dfrac{1}{\Lambda_8^4}$

First application: $pp \to W\,h$

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- Naively: $\Lambda_6 = \Lambda_8$, but we'll float these as a way to explore how big dim-8 could potentially be

- Explore different signs among $c^{(i)}_8$ as there can be cancellations among terms. Results shown for a representative example choice of signs

First application: $pp \to Wh$

Demand EFT consistency conditions:

- (dim-6) x SM > (dim-8) x SM

$1/\Lambda^2$ effects $\qquad\qquad$ $1/\Lambda^4$ effects

- (dim-8) x SM > (dim-8)^2

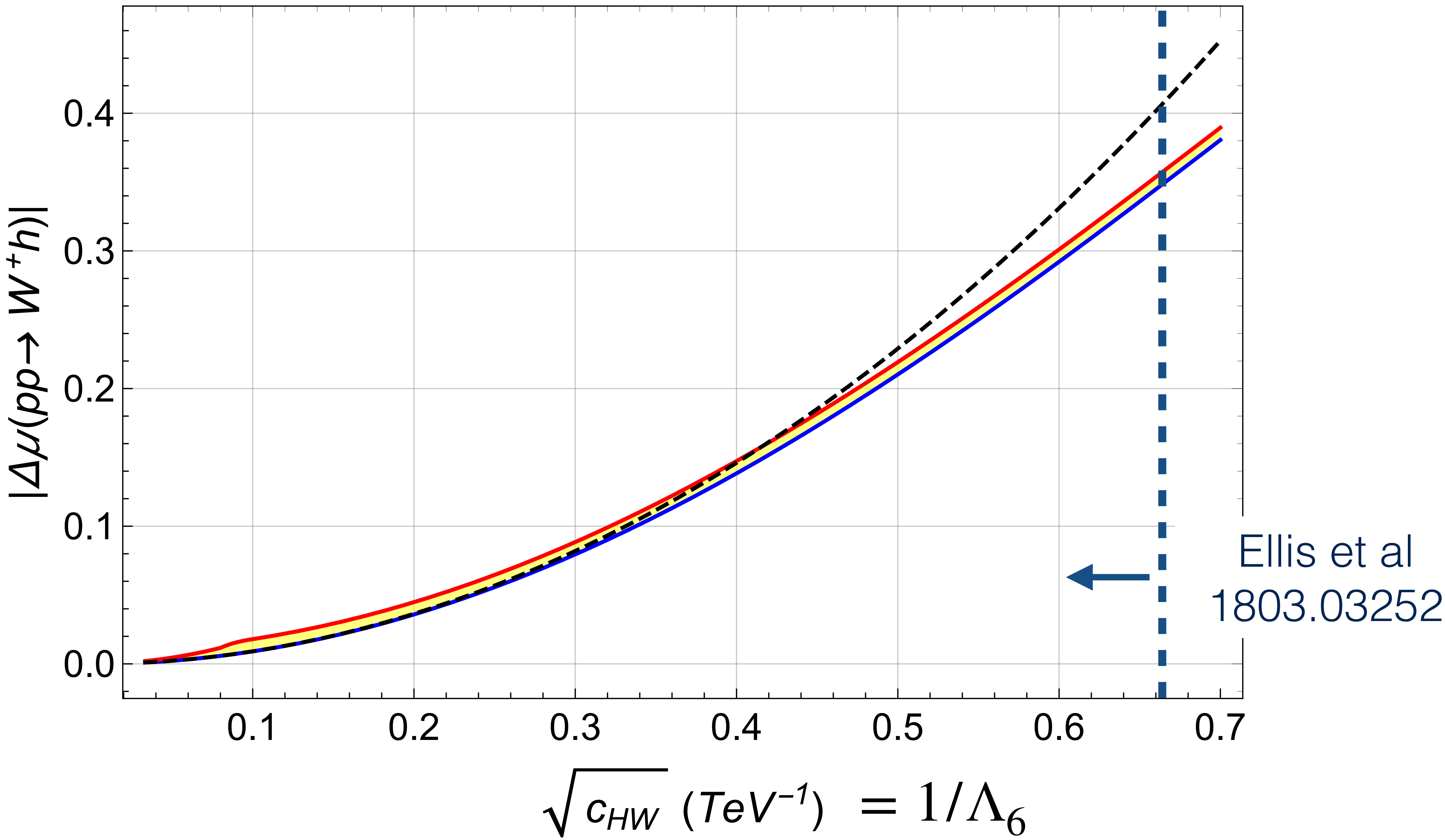
$1/\Lambda^4$ $\qquad\qquad$ $1/\Lambda^8$

Imposed at the level of cross section

Calculate:

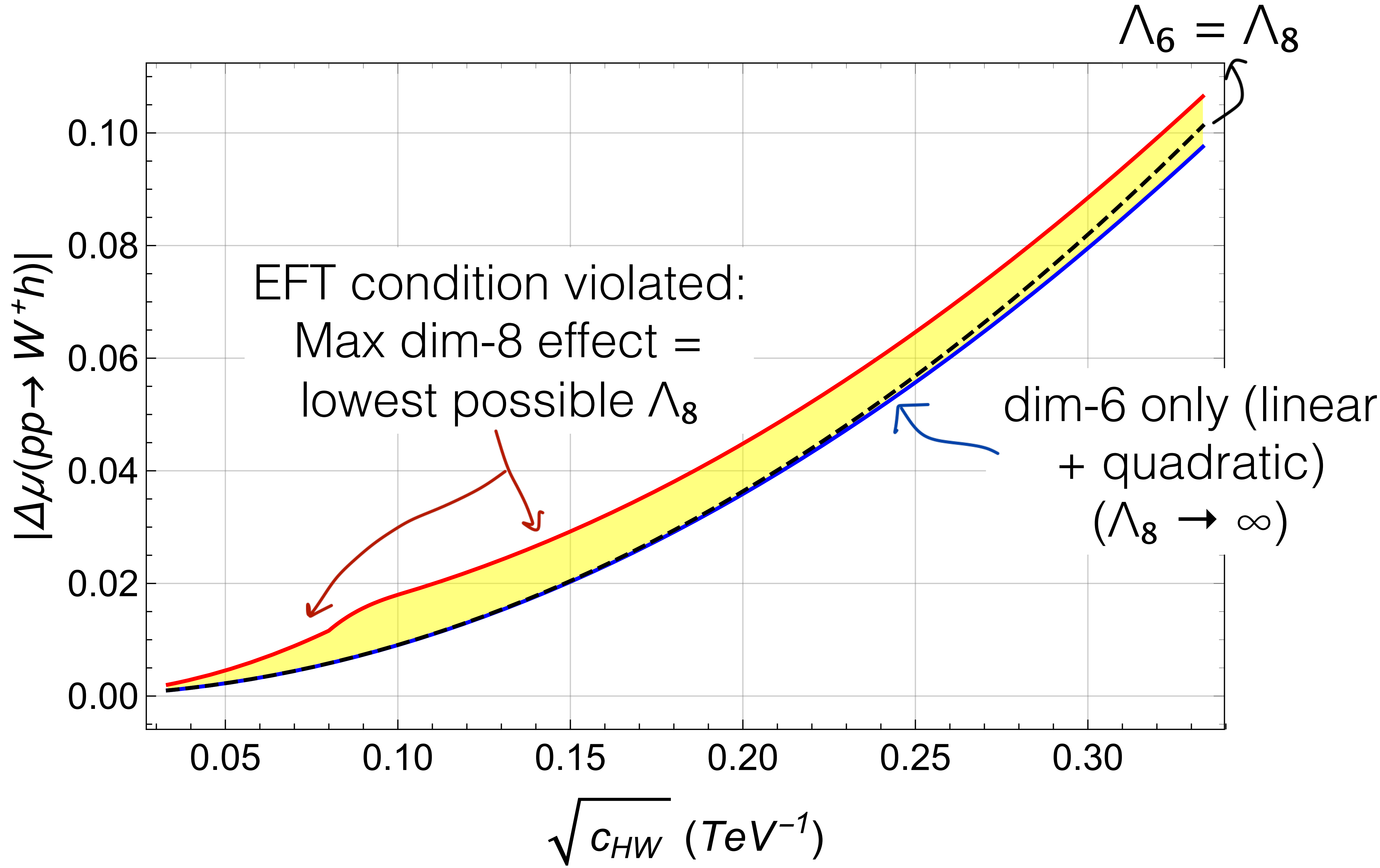
$$|\sigma(pp \to W^+h)_{SMEFT} - \sigma(pp \to W^+h)_{SM}|/\sigma_{SM}$$

Application pp → W H in SMEFT

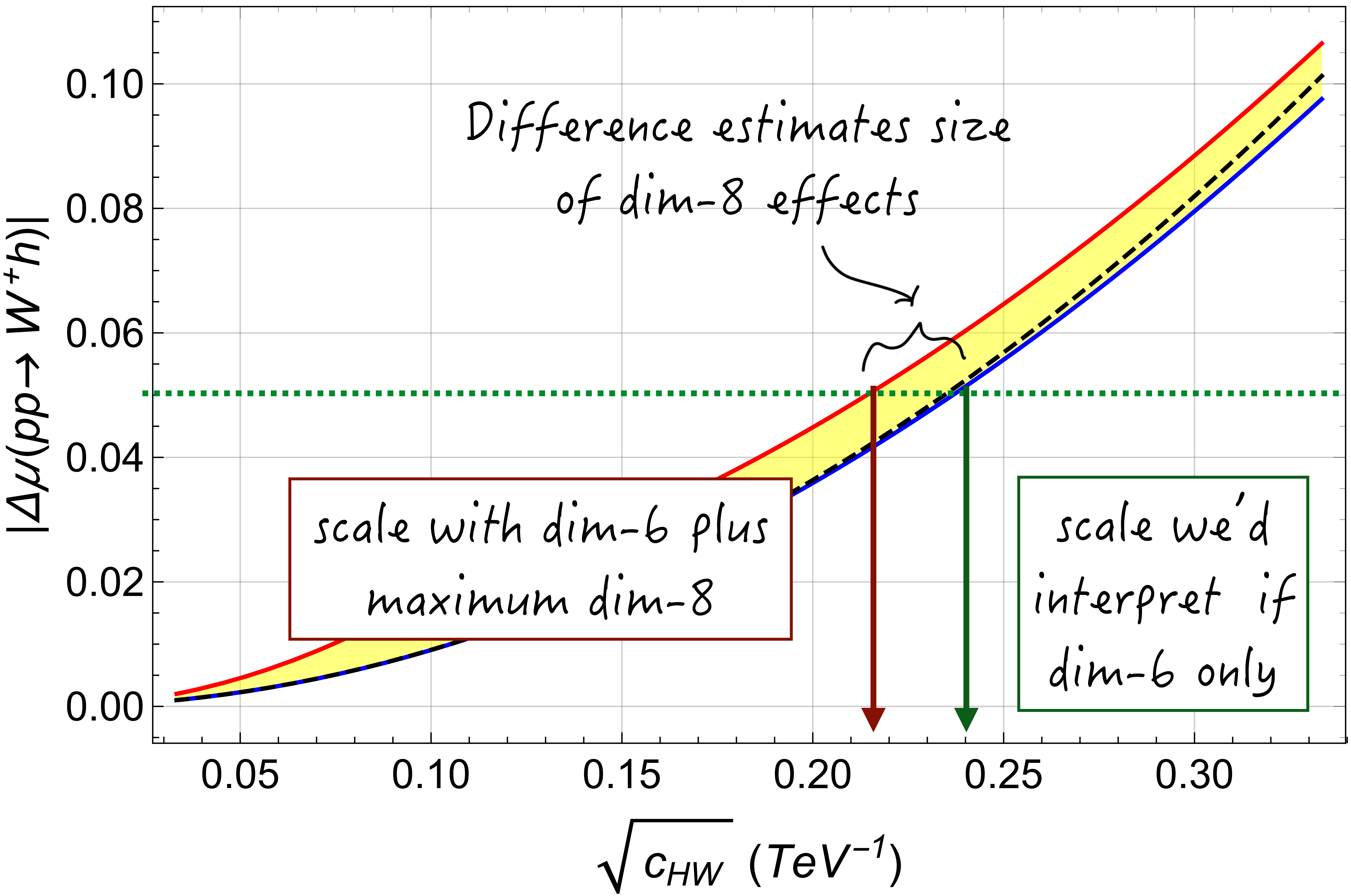


Application pp → W H in SMEFT

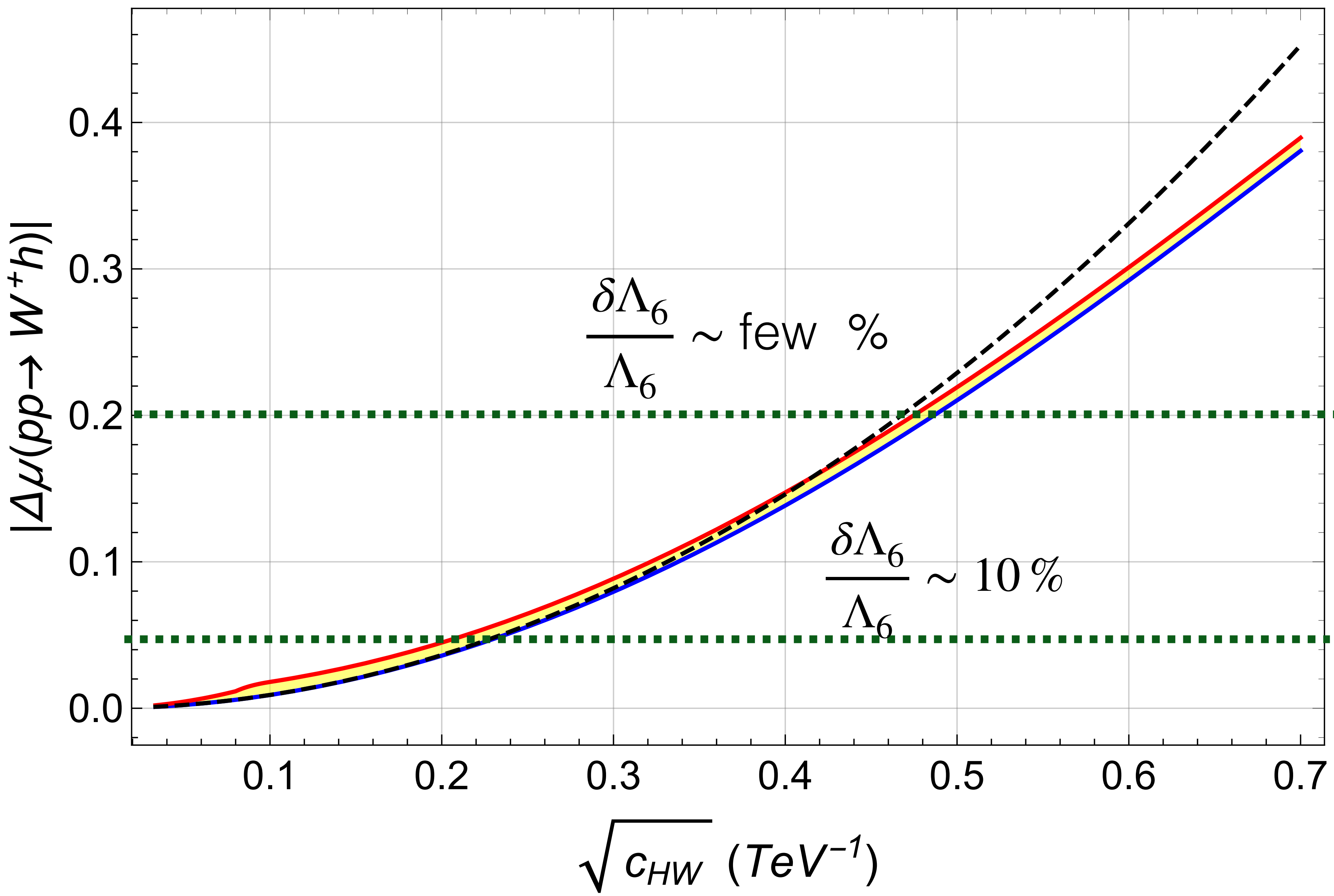
Zoom in



Application pp → W H in SMEFT



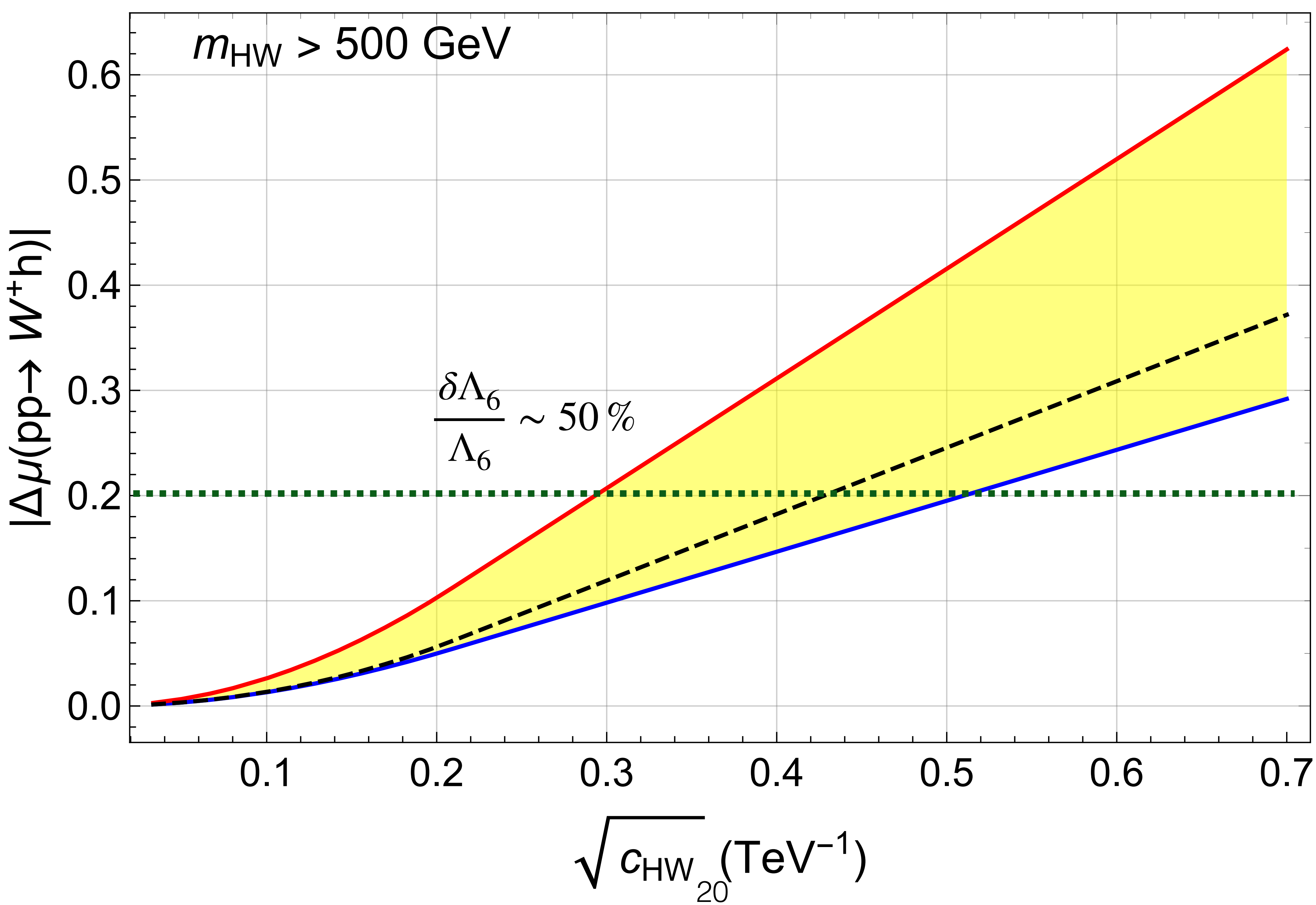
Application pp → W H in SMEFT



as $\hat{s} \sim (m_W + m_H)^2 \sim v^2$, difference in energy dependence of EFT pieces is minimal → dim-8 effects small

Application pp → W H in SMEFT

Same idea, but looking at m_{HW} > 500 GeV: larger $\hat{s}$ means energy differences in EFT terms more prominent



Conclusions & future directions

number and field content of higher dim operators in EFTs: solved

Translation to pheno form for dim-8 done for all bosonic operators with Higgses, subset of fermionic operators (N_f = 1), in Feynrules .fr format (in 1808.00442 source).

Recipe established to construct any others (SMEFT & beyond) based on constructing matrix of IBP relations

Sample application: dim-8 SMEFT effects in pp → W+h

- estimate dim-8 effect at O(few %) for current $\Delta\mu$(pp → hW), growing to O(10%) by $\Delta\mu \sim 0.1$

- Larger effects, up to $(\delta\Lambda_6/\Lambda_6) \sim 50\%$ for boosted/high-$\hat{s}$ regions

Conclusions & future directions

- Improve/optimize FeynRules implementation
- Expanding the set of processes: $pp \to W\gamma$, hh already in progress. Automatization (talk by David) will open up many more
- We only turned on one dim-6 operator, turning on multiple allows the possibility of more intricate/complex effects
- Dim-8 effects largest for specific polarizations. Worth studying how to enhance those populations in analyses

[1712.01310 Franseschini et al,1804.08688 Liu, Wang]

Lots of interesting directions to explore!

EXTRA

Hilbert series:

$$\mathcal{H}_{SM} = \boxed{\int d\mu_{Lorentz}\, d\mu_{gauge}}\, \boxed{\frac{1}{P}}\, \boxed{PE\Big[\sum_{\phi} \frac{\phi}{\mathcal{D}^{d_\phi}} \chi_\phi\Big] PEF\Big[\sum_{\psi} \frac{\psi}{\mathcal{D}^{d_\psi}} \chi_\psi\Big]}$$

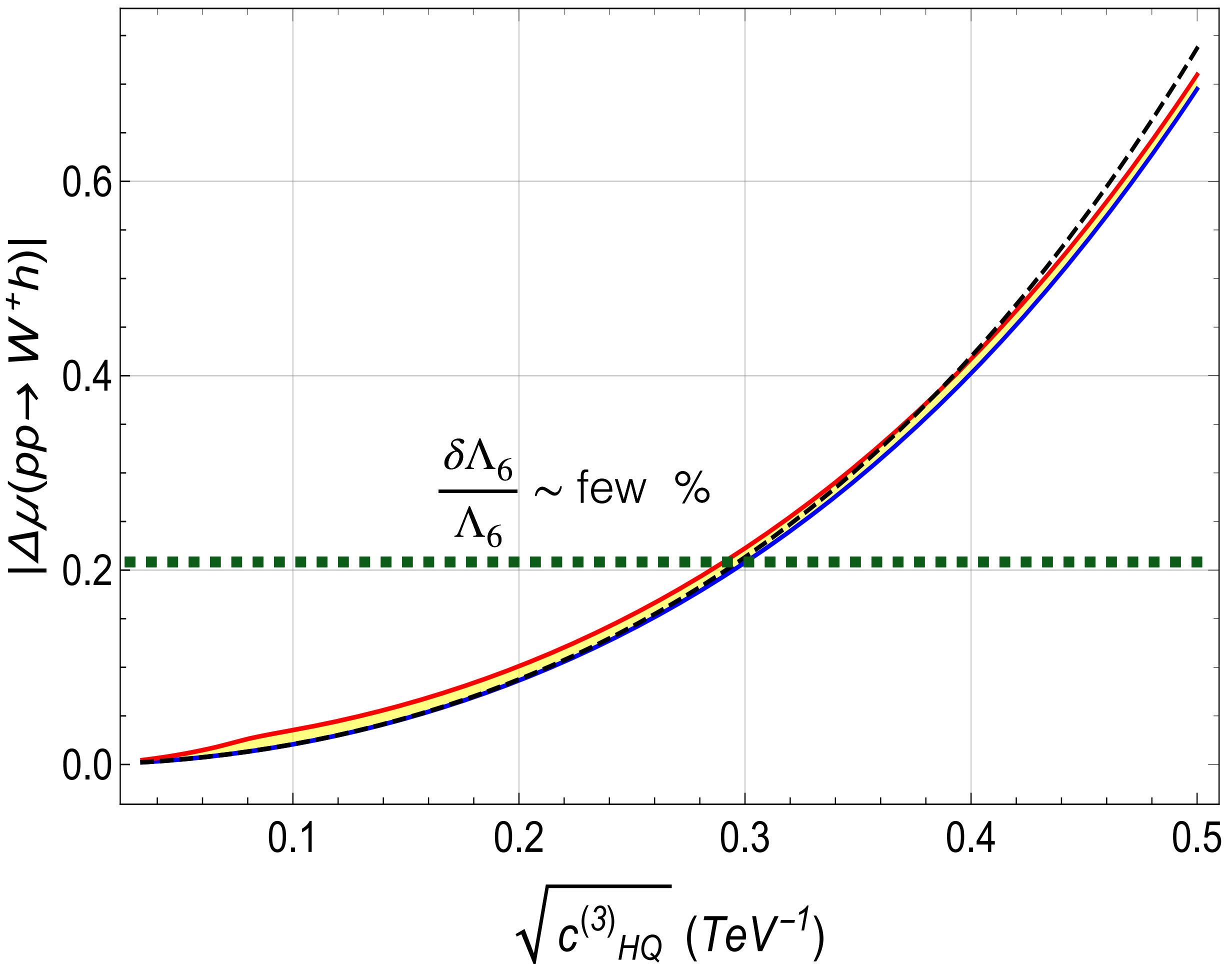
projects out invariants from polynomial (relies on character orthonormality)

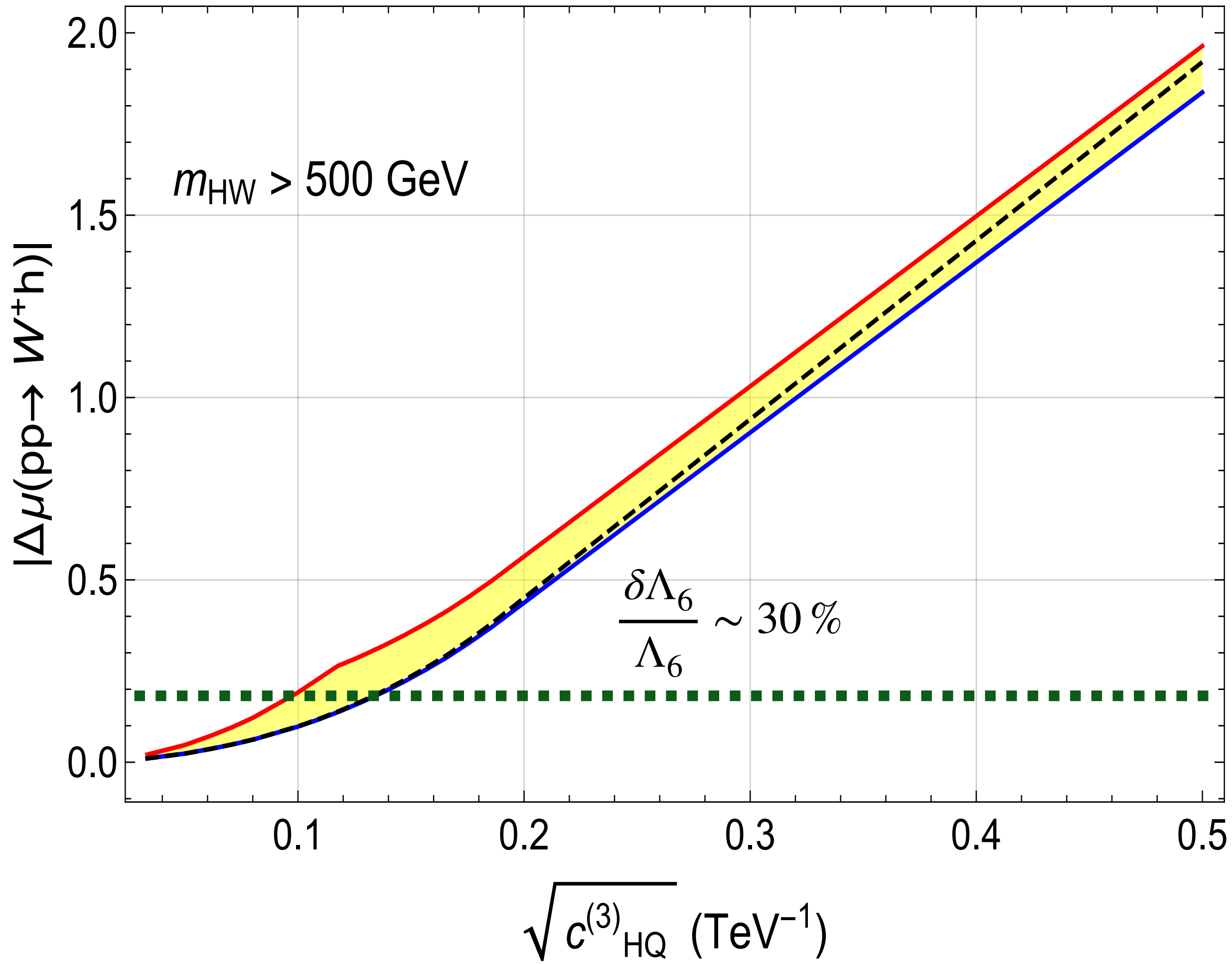
generating function — generates all possible polynomials of fields
(ϕ^2, $\phi\,\psi$, $\psi^2\,\phi$,etc.)
and derivatives

removes IBP redundancies

Using other dim-6 operators

$$c_{Hq}^{(3)}\,(Q^\dagger \bar{\sigma}^\mu Q)(H^\dagger \overleftrightarrow{D}_\mu H)$$

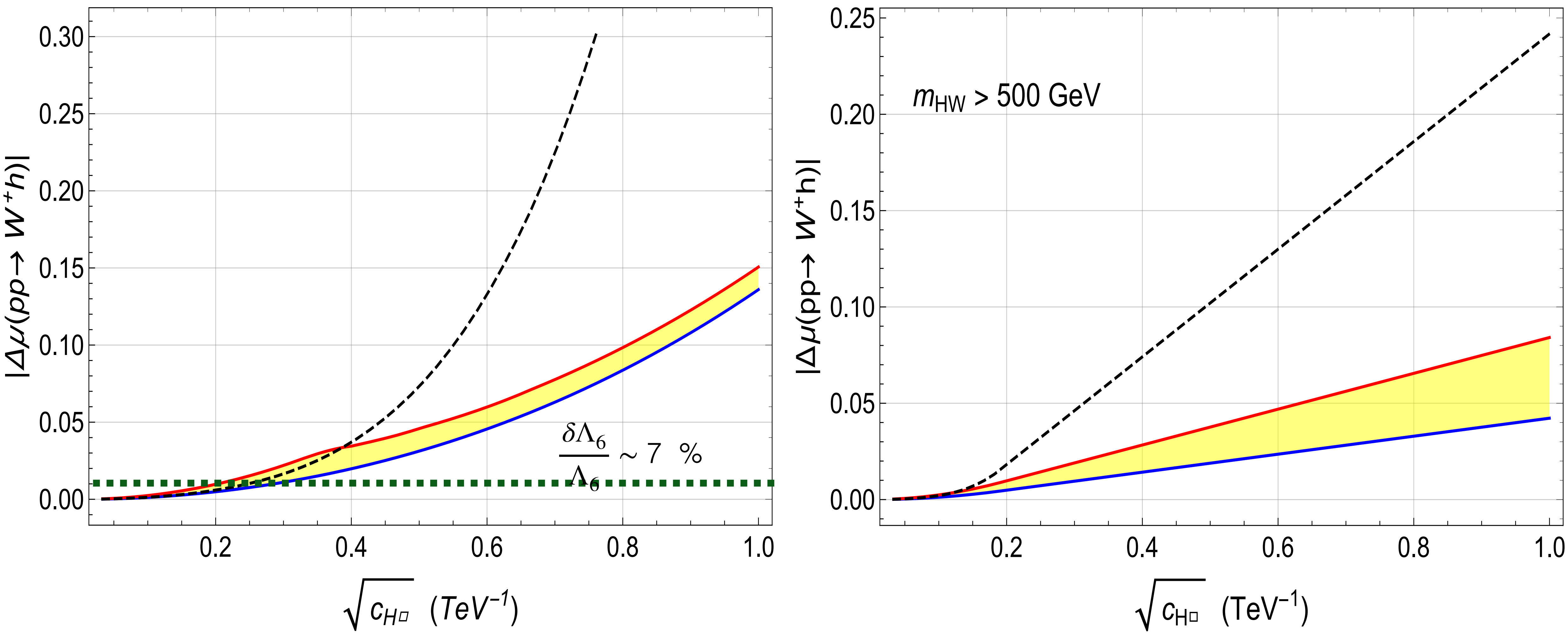




Does contribute to pp → W_L h ∴ larger impact on σ, reduced effect from dimension-8

Using other dim-6 operators

$$c_{H\Box}(H^\dagger H)\Box(H^\dagger H)$$



Only enters via normalization of Higgs field