The LUX approach to the photon PDF

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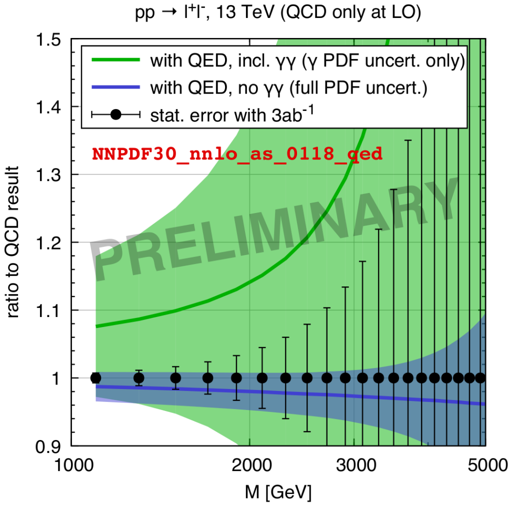
EW WG, CERN, 15-11-2018

Outline

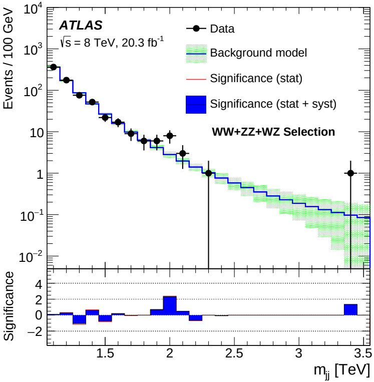
- The need for the photon PDF
- The LUX breakthrough
- The LUX Master Equation
- The original LUX PDF set
- Structure functions data
- The elastic contribution and the elastic data
- Uncertainties
- Conclusions

The need for a photon PDF

Around 2015-2016 people realized that uncertainties in the photon content of the proton where becoming relevant for the production of high mass objects



Atlas Boosted Jets analysis (end 2015)



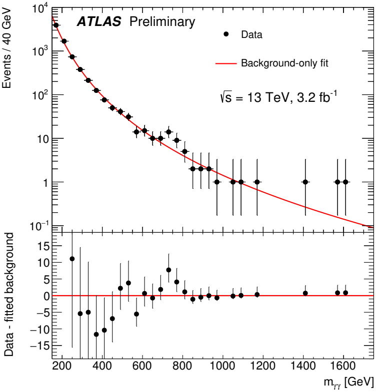
Excess at 2 TeV.

Not confirmed in 13 TeV run.

The worry was that at very high scales gluon and quarks soften due to AP evolution.

Photons mostly stay the same;

The Infamous 750



Towards the end of 2015, data on photon pair production at the LHC were hinting to the presence of a resonance at 750 GeV. No other channels were showing a similar peak.

Disappeared in summer 2016 ...

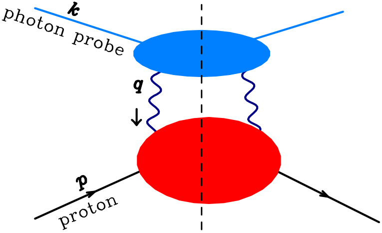
If something decays abundantly into two photons (no other channels were observed) it may also be produced in two photon collisions. Constraints on its couplings depend upon the photon parton density (Franceschini etal, Dec. 2015).

The LUX breakthrough

The photon PDF seems to be difficult to measure, since there is no process in which it enters in a dominant way.

In the work of Manohar,Salam,Zanderighi,PN,2016 it was shown that it is possible to strongly constrain the photon PDF's using already available data.

The key observation is that the process of a hypotetical probe interacting with a proton leads to the exchange of a spacelike photon. Whether we see it as coming from the probe and reaching the proton or viceversa is only a matter of convenience.



In fact, the cross section can be written as the product of a probe tensor and an EM tensor

$$\sigma = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{e^4_{\mathrm{phys}}(q^2)}{q^4} \times \langle k | \tilde{J}_p{}^{\mu}(-q) J_p^{\nu}(0) | k \rangle \times \langle p | \tilde{J}_{\mu}(q) J_{\nu}(0) | p \rangle$$

Kinematics constraints:

$$Q^2 = -q^2 > 0, \qquad 0 < x_{\mathrm{bj}} = Q^2/(2p \cdot q) \leq 1.$$

- Same kinematic restrictions as in DIS.
- Hadronic tensor: same as electroproduction
  $\frac{1}{4\pi} \langle p | \tilde{J}_{\mu}(q) J_{\nu}(0) | p \rangle = -g_{\mu\nu} F_1(Q^2, x_{\mathrm{bj}}) + \frac{p^{\mu} p^{\nu}}{p \cdot q} F_2(Q^2, x_{\mathrm{bj}}) + \dots$
  (Notice: full F_1 and F_2, not only inelastic)
- Photon induced process can be given in terms of F_1, F_2
- Hence: the photon PDF must be calculable in terms of F_1, F_2.

We proceeded as follows:

- Take a BSM interaction of the form $\frac{e}{\Lambda}\bar{l}[\gamma^{\mu},\gamma^{\nu}]LF_{\mu\nu} + \mathrm{cc}$, l massless, L massive with mass M, both neutral. With this choice there are no QED corrections to the probe process. All higher order QED effects are lumped into the physical electromagnetic coupling and in the hadronic tensor.
- Compute the cross section in terms of F_1 and F_2
- Compute the cross section with the Parton Model formula
- Extract f_γ by identifying the two cross sections.

We got a formula for f_γ in the $\overline{\mathrm{MS}}$ scheme at NLO, in term of the electroproduction structure functions F_1 and F_2:

$$x f_{\gamma/p}(x,\mu^2) = \frac{1}{2\pi}\int_x^1 \frac{dz}{z}\Bigg\{ - \overbrace{\alpha(\mu^2) z^2 F_2\left(\frac{x}{z},\mu^2\right)}^{\overline{\rm MS}\text{ correction}}$$

$$+ \underbrace{\int_{\frac{x^2 m_p^2}{1-z}}^{\overbrace{\frac{\mu^2}{1-z}}^{\overline{\rm MS}}} \frac{dQ^2}{Q^2}}_{\mathcal{O}(\log\frac{\mu^2}{m_p^2})} \frac{\alpha^2(Q^2)}{\alpha(\mu^2)}\left[\left((1+(1-z)^2)+\frac{2x^2 m_p^2}{Q^2}\right)F_2(x/z,Q^2) - z^2 F_L\left(\frac{x}{z},Q^2\right)\right]\Bigg\}.$$

- $f_\gamma \approx \alpha \log\frac{\mu^2}{m_p^2} \approx \alpha/\alpha_s$ ($\alpha_s(\mu^2) \approx 1/\log\frac{\mu^2}{\Lambda^2}$) relative to $f_{u/d}$.
- $Q^2 \approx m_p^2$ region formally of order α, i.e. NLO (as $\overline{\rm MS}$ term).
- Straightforward to improve at NNLO in α_s (Master Equation is exact, compute the parton model process at NNLO)
- Also accurate at $(\alpha/\alpha_s)^2$, provided that $\alpha(Q^2)$ and F_2 include leading log electromagnetic evolution.
- Valid at all μ's: MUST match evolution accuracy with one extra α_s. Agrees with De FLorian, Sborlini, Rodrigo $\alpha\alpha_s$ splitting functions, `arXiv:1512.00612`.

Use:

Ideal use:

- Get $F_{2/L}$ at low Q^2 from available data.
- PDF global fit, including EM evolution, with the photon density constrained by the previous equation, $F_{2/L}$ taken from data at low Q^2 and computed from the PDF's at high Q^2

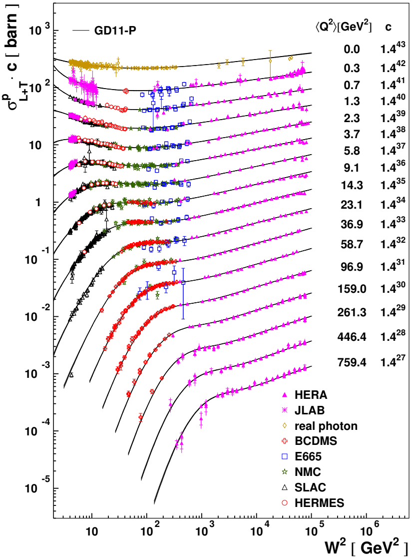
Much can be done without performing a dedicated global fit. However, if we aim at NLO accuracy:

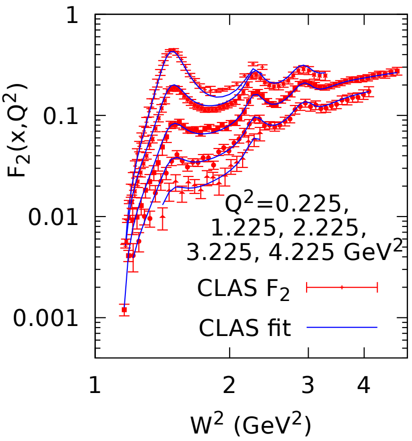
- Low Q^2 region cannot be neglected.
- $(\alpha/\alpha_s)^2$ terms arising from the evolution of QED coupling cannot be neglected $(\alpha(m_\mu^2))/\alpha(M_Z^2) \approx 0.94)$
- $(\alpha/\alpha_s)^2$ terms arising from the QED evolution of the quarks are small, just do something minimal to account for them.

The LUX PDF set

- Start from a standard set (e.g. `PDF4LHC15_nnlo_100`);
- Compute the photon PDF at $\mu = 100\,\mathrm{GeV}$, with the low Q^2 component determined from A1, CLAS and Hermes GD11-P fits, and the high Q^2 part determined from the input PDF with standard NNLO coefficient functions.
- Evolve down to 10 GeV, including QED evolution only for splitting processes that affect the photon: $P_{\gamma q}$, $P_{\gamma g}$, $P_{\gamma\gamma}$ (with $\alpha\alpha_s$ terms included).
- Fix the momentum sum rule by rescaling the gluon (a factor of 0.99299 is needed).
- Evolve up including full QED evolution (with $\alpha\alpha_s$ terms included).

This procedure is such that the structure functions at a scale of 10 GeV, where they are strongly data constrained, remain consistent with the new pdf set, while the $(\alpha/\alpha_s)^2$ due to photon radiation are included in the quark distributions at high scale.





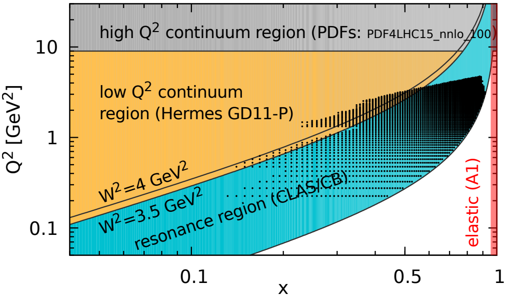
Fitted data from $Q^2 = 0.225$ to 4.725 in steps of $0.05\,\mathrm{GeV}^2$.

Hermes fit: we are interested in the region $Q^2 < 10\,\mathrm{GeV}^2$.

Continuum data region: $4\ \mathrm{GeV}^2 < W^2 \lesssim 10^5\mathrm{GeV}^2$ $(x \to 10^{-4})$.

Inelastic Data coverage

- Low Q^2 continuum essentially covered by data.
- F_2 and F_L must vanish as Q^2 and Q^4 at constant W (by analiticity of $W^{\mu\nu}$).



Also:

$$F_2(x,Q^2) = \frac{1}{4\pi^2\alpha}\frac{Q^2(1-x)}{1+\frac{4x^2m_p^2}{Q^2}}(\sigma_T(x,Q^2)+\sigma_L(x,Q^2)) \underset{Q^2\to 0}{\Longrightarrow} \frac{Q^2\sigma_{\gamma p}(W)}{4\pi^2\alpha^2}.$$

At small Q^2, $\sigma_T \Longrightarrow \sigma_{\gamma p}(W)$, becoming a function of W only (the CM energy in photoproduction), and σ_L vanishes. Photoproduction data included in Hermes and Christy-Bosted parametrizations.

Elastic Contribution

F_2 and F_L receive an elastic contribution that we must include:

$$\begin{aligned} F_2^{\mathrm{el}} &= \frac{G_E^2(Q^2) + G_M^2(Q^2)\tau}{1+\tau}\,\delta(1-x), \\ F_L^{\mathrm{el}} &= \frac{G_E^2(Q^2)}{\tau}\,\delta(1-x), \end{aligned}$$

with $\tau = Q^2/(4m_p^2)$. In the dipole approximation

$$G_E(Q^2) = \frac{1}{(1+Q^2/m_{\mathrm{dip}}^2)^2}, \quad G_M(Q^2) = \mu_p G_E(Q^2), \quad \begin{array}{l} m_{\mathrm{dip}}^2 = 0.71\,\mathrm{GeV}^2 \\ \mu_p = 2.793 \end{array}$$

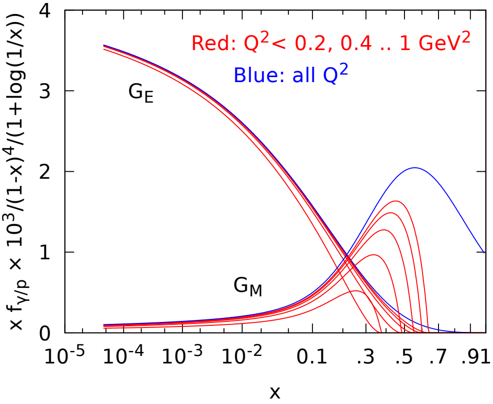
so that the elastic contribution falls rapidly with Q^2.

The elastic contribution to f_γ is

$$x f_\gamma^{\mathrm{el}}(x,\mu^2) = \frac{1}{2\pi}\int_{\frac{x^2 m_p^2}{1-x}}^{\frac{\mu^2}{1-x}} \frac{\mathrm{d}Q^2}{Q^2}\,\frac{\alpha^2(Q^2)}{\alpha(\mu^2)}\left\{\left(1-\frac{x^2 m_p^2}{Q^2(1-x)}\right)\frac{2(1-x)G_E^2(Q^2)}{1+\tau}\right.$$
$$\left. + \left(2-2x+x^2+\frac{2x^2 m_p^2}{Q^2}\right)\frac{G_M^2(Q^2)\tau}{1+\tau}\right\}.$$

Dipole approximation,
($\mu \to \infty$ in figure.)

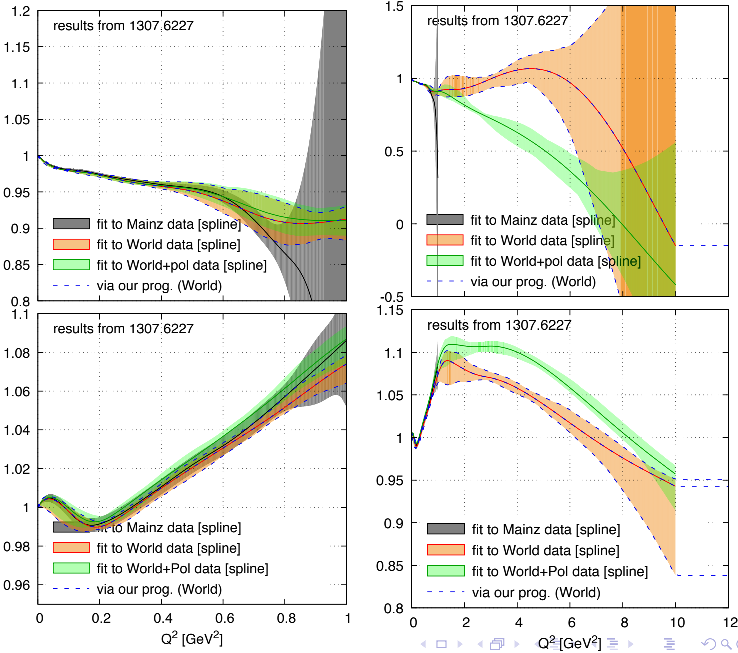
- Mostly G_E at small x.
- Mostly G_M at large x.
- Mostly from $Q^2 < 1 GeV$.



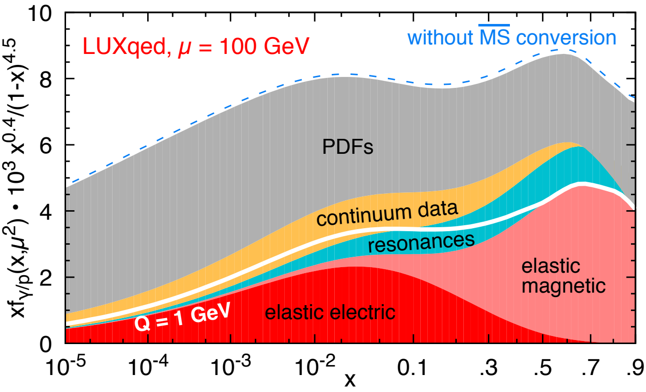
Elastic Data, A1 experiment and World data

$G_E / G_E^{\mathrm{dipole}}$

$G_M / G_M^{\mathrm{dipole}}$

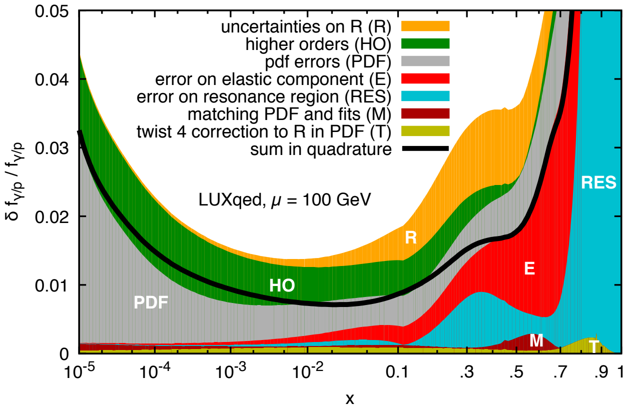


Contributions to f_γ:

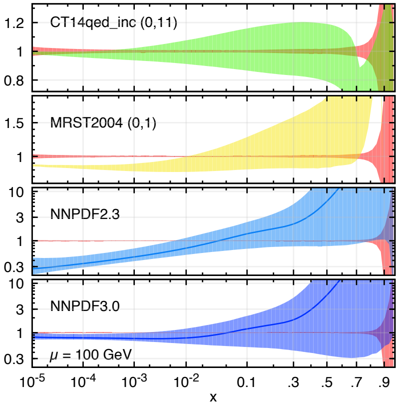


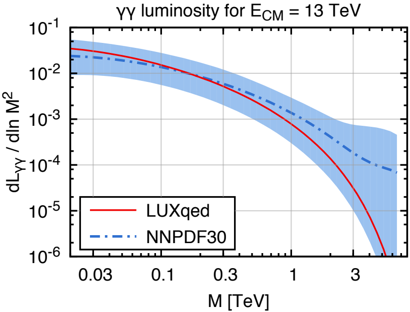
- $Q^2 > 9\, GeV^2$, computed from standard PDF sets
- Important elastic component. Magnetic prevails for $x > 0.2$.
- Continuum and resonance contributions not negligible
- Very important contribution from $Q^2 < 1\, GeV^2$.

Uncertainties



- At small x, higher order effects and PDF's dominate the error.
- At large x, elastic and resonant region dominant.
- Total uncertainty at the percent level.
  Further improvements possible!





The LUX method achieves by far better precision than other methods.

Approaches that use some lepton scattering information (in particular `CT14qed_inc`) achieve better precision than “totally agnostic” approaches (NNPDF) (note different y axis in panel).

- In the original LUX paper, a set was produced by adding the LUX photon to an existing PDF set, typically PDF4LHC. The procedure adopted needed to evolve the PDF backward to a reference scale. Because of a limitation of the PDF4LHC set (being the average of sets obtained with different evolution codes), this backward evolution could not be carried out below 10 GeV (and hence the limitation of the set.)
- A more sensible procedure is to include the LUX equation at the stage when the PDF fit is carried out, with the input on the structure functions at low energy taken from fits to leptoproduction data, and the structure functions at high energy computed self-consistently using the fitted parton densities (as done, for example, in NNPDF3.1luxQED.)

The impact

- The LUX approach has provided a method to strongly constrain the photon PDF. This can lead indirectly to some improvements in the accuracy of PDF fits.
- The photon contribution to high energy processes has ceased to be a main source of theoretical uncertainty. It is now known very precisely.
- The photon PDF is only indirectly related to the distribution of ISR photons in hadronic reactions. These have to be computed much in the same way as soft gluon radiation. If there are contributing photon initiated subprocesses the photon PDF will also enter there.

The PDF tells you the probability to find a parton in a hadron inclusively (i.e. no matter what happens to the rest of the hadron). This is not quite the same as asking the amount of radiation accompanying a given reaction.

- Photon PDF can be extracted with great precision from available knowledge of proton structure function and form factors.
- The needed low Q^2 data is available thanks to extensive low and intermediate energy Nuclear Physics studies.
- Our study aimed at NLO precision including terms suppressed by one power of α_s or by a power of α/α_s relative to the leading term. This leads to precisions at the percent level.
- The study of structure functions and form factors at low energy is still ongoing in the Nuclear Physics Community (further progress will come).
- Formulae for NNLO f_γ have been also worked outManohar,Salam,Zanderighi,PN,2017.

Uncertainties included in LUX

Added members with variations in photon PDF calculation:

- 0-100: original PDF members (`PDF4LHC15_nnlo_100`)
- 101: Replace CLAS parametrization of resonance region with Christy-Bosted one. (Becomes particularly crazy al large x).
- 102: rescale R in low Q^2 region by 1.5.
- 103: rescale R in high-Q^2 region with a higher-twist component.
- 104: Use 'World" elastic fit from A1: no polarization data, no fit to Two Photon Exchange effects.
- 105: Use lower edge of elastic fit error band.
- 106: Start using PDF's from $Q^2 = 5$ rather than $9\,\mathrm{GeV}^2$.
- 107: Upper limit of integration in f_γ formula changed to μ^2 instead of $\mu^2/(1-z)$, with suitable correction of $\overline{\mathrm{MS}}$ term.

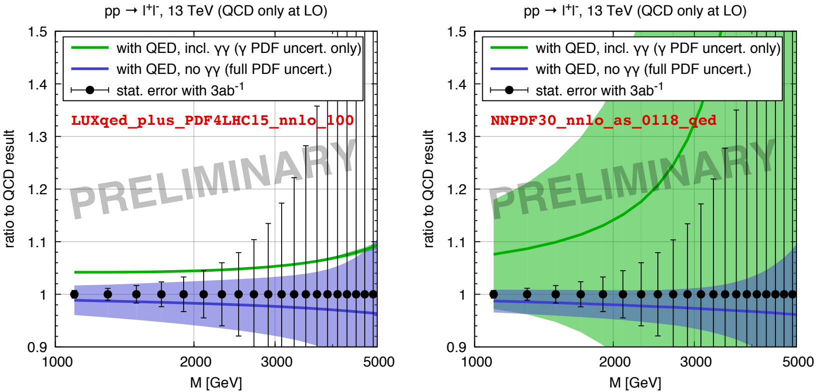
All errors are taken as symmetric.

APPLICATION TO HIGGS PHYSICS

$pp \to H W^+ (\to l^+\nu) + X$ at 13 TeV	
non-photon induced contributions	91.2 ± 1.8 fb
photon-induced contribs (NNPDF23)	$6.0^{+4.4}_{-2.9}$ fb
photon-induced contribs (LUXqed)	4.4 ± 0.1 fb

non-photon numbers from LHCHXSWG (YR4)

di-lepton spectrum



LUXQED photon has few % effect on di-lepton spectrum and negligible uncertainties

RESOURCES

- `LUXqed_plus_PDF4LHC15_nnlo_100` set available from LHAPDF
- Additional plots and validation info available from http://cern.ch/luxqed
- Preliminary version of HOPPET DGLAP evolution code with QED (order α and $\alpha\alpha_s$) corrections available from hepforge:

```
svn checkout http://hoppet.hepforge.org/svn/branches/qed hoppet-qed
```

(look at `tests/with-lhapdf/test_qed_evol_lhapdf.f90` for an example; interface may change, documentation missing)

EXTRA SLIDES

Going NNLO

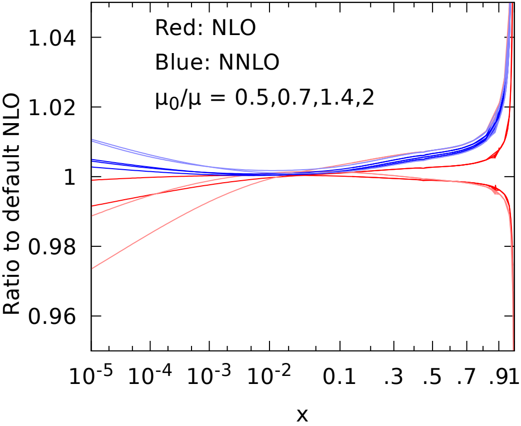
Going to one extra order in α_s is not difficult. We need to compute our "probe" process in the parton model, at NNLO, subtracting the collinear singularities in the $\overline{MS}$ scheme.

The *d*-dimensional NLO and NNLO corrections to the probe process are obtained by

- writing our master formula in $d = 4 - 2\epsilon$ dimension, and replacing the $W^{\mu\nu}$ tensor with the partonic $w_i^{\mu\nu}$ tensor.
- Compute $w_i^{\mu\nu}$ along the lines of the Altarelli-Ellis-Martinelli calculation of NLO corrections to DIS of 1979, keeping however one extra power of ϵ in its expansion.

Going NNLO

PRELIMINARY



Reduction of the uncertainty band visible from $x \approx 0.3$. The crossing point in scale dependence makes it difficult to appreciate what happens for smaller values of x.

Going NNLO

Extending the EM uncertainty from α^2/α_s to full α^2 is more problematic. Corrections of order α to the physical EM coupling $e_{\mathrm{phys}}(q^2)$ cannot be computed from first principles because of the hadronic contributions, so also the EM running coupling should be determined using data driven methods.
The question of the accuracy of the structure functions and form factors would also require a critical analysis of the inclusion of EM corrections in DIS.

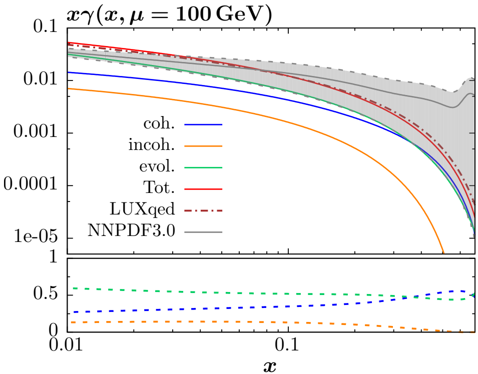
Previous work

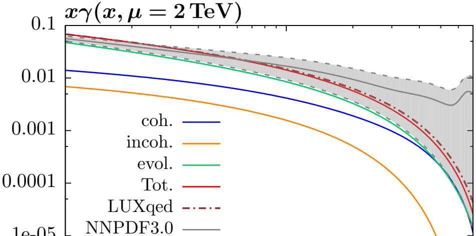
There is a vast literature touching this topic.

- Elastic component: Budnev etal, 1975; Gluck, Pisano and Reya, 2002; Martin and Ryskin, 2014; Harland-Lang, Khoze and Ryskin, 2016; `CTEQ14qed_inc`
- *ep* scattering connection: Mukherjee and Pisano, 2003; Łuszczak, Schäfer, and Szczurek, 2015.

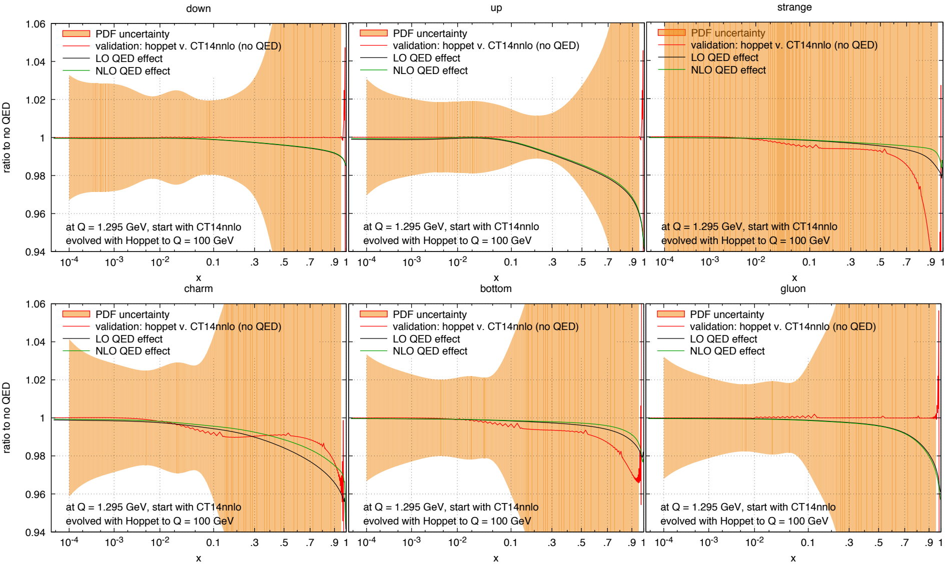
In the work of Mukherjee and Pisano, a formula similar to our master equation appears, except for the inclusion of the $\overline{\mathrm{MS}}$ correction, and for different integration limits.
A similar formula appears also in Łuszczak, Schäfer, and Szczurek, except that, due to their small x approximation, their result does not obey the correct evolution equations. They also make use of data driven parametrizations of structure functions.

Comparison with Harland-Lang, Khoze and Ryskin, 1607.04635v3 (October 10, 2016).

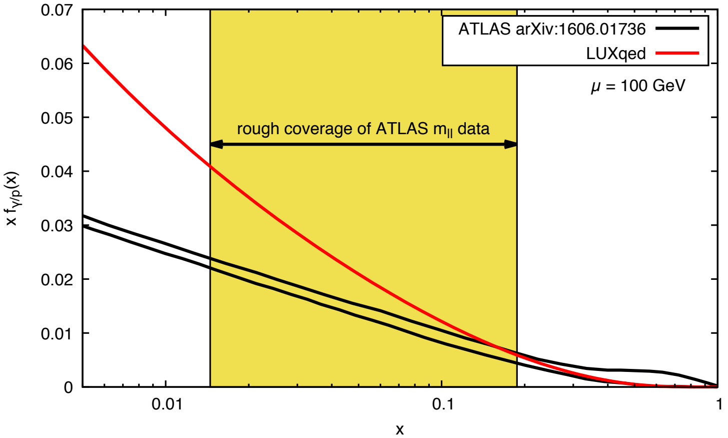




Impact of QED evolution



ratio of ATLAS photon (1606.01736) to LUXqed



ATLAS result based on reweighting of NNPDF23 with high-mass ($M_{ll} > 116$ GeV) data