Theory Correlations Between W and Z p_T Spectra.

Frank Tackmann

Deutsches Elektronen-Synchrotron

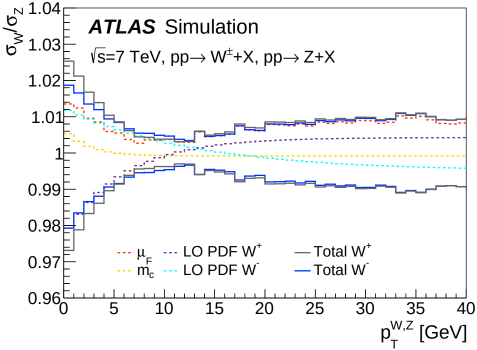
EWWG Workshop
CERN, November 14, 2018

Disclaimer:
Ongoing work, all results are preliminary ...

Extrapolating from Z to W

Small $p_T^W < 40\,\mathrm{GeV}$ is the relevant region for m_W

- $\simeq 2\%$ uncertainties in p_T^W translate into $\simeq 10\,\mathrm{MeV}$ uncertainty in m_W
- Direct calculation of W p_T spectrum will not reach $\lesssim 1\%$ anytime soon



⇒ We need to extrapolate from precisely measured Z p_T spectrum to get precise prediction for W

- Regardless how precisely $\mathrm{d}\sigma(W)/\mathrm{d}p_T$ can be calculated directly, we always want to exploit Z data to combine all available information to maximize precision

Basic Strategy.

$$\underbrace{\frac{\mathrm{d}\sigma(W)}{\mathrm{d}p_T}}_{\text{needed}} = \underbrace{\left[\frac{\mathrm{d}\sigma(Z)}{\mathrm{d}p_T}\right]_{\text{measured}}}_{\text{measure precisely}} \times \underbrace{\left[\frac{\mathrm{d}\sigma(W)/\mathrm{d}p_T}{\mathrm{d}\sigma(Z)/\mathrm{d}p_T}\right]_{\text{theory}}}_{\substack{\text{calculate precisely} \\ \text{theory uncertainties cancel}}}$$

- Ratio is just a proxy
  - More generally: Combine various control measurements, fit to all control and signal processes
  - Tuning Pythia on Z data and use it to predict W is one example of this

Basic Strategy.

$$\underbrace{\frac{\mathrm{d}\sigma(W)}{\mathrm{d}p_T}}_{\text{needed}} = \underbrace{\left[\frac{\mathrm{d}\sigma(Z)}{\mathrm{d}p_T}\right]_{\text{measured}}}_{\text{measure precisely}} \times \underbrace{\left[\frac{\mathrm{d}\sigma(W)/\mathrm{d}p_T}{\mathrm{d}\sigma(Z)/\mathrm{d}p_T}\right]_{\text{theory}}}_{\substack{\text{calculate precisely} \\ \text{theory uncertainties cancel}}}$$

- Ratio is just a proxy
  - More generally: Combine various control measurements, fit to all control and signal processes
  - Tuning Pythia on Z data and use it to predict W is one example of this
- Crucial Caveat: Cancellation fundamentally relies on theory correlations
  - Take 10% theory uncertainty on σ^{signal} and σ^{control}
    - $\rightarrow$ 99.5% correlation yields 1% uncertainty on their ratio
    - $\rightarrow$ 98.0% correlation yields 2% uncertainty on their ratio – 2$\times$ larger!
- In Addition: Must account for all non-cancelling subleading effects
  - Another talk for another day ...

Theory Correlations.

Theory correlations are also necessary for most interpretations

- Correlations across differential spectrum
- Correlations between different signal processes, different E_{cm}, ...

Correlations only come from common sources of uncertainties

- Straightforward for parametric uncertainties (PDFs, ...)

Theory Correlations.

Theory correlations are also necessary for most interpretations

- Correlations across differential spectrum
- Correlations between different signal processes, different E_{cm}, ...

Correlations only come from common sources of uncertainties

- Straightforward for parametric uncertainties (PDFs, ...)

What to do about perturbative theory uncertainties?

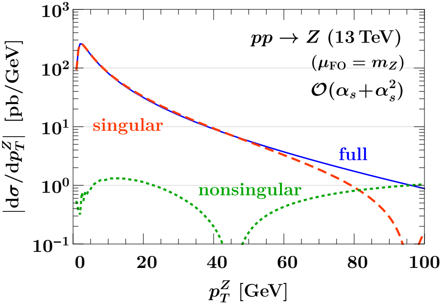
- ✗ Often we don't even really know what our uncertainties mean ...
- ✗ The Issue: Scale variations are inherently ill-suited for this
  - ✗ QCD scales are not physical parameters, they simply specify a particular perturbative scheme
  - ✗ They do not have an uncertainty that can be propagated
  - ✗ They are not the underlying source of uncertainty
  - ✗ Trying to decide how to correlate or decorrelate scale variations is really just a bandaid, but not treating the real problem

Small-p_T Region.

Define scaling variable $\tau \equiv p_T^2/m_V^2$ and expand in powers of τ

$$
\begin{aligned}
\frac{d\sigma}{d\tau} = \delta(\tau) &+ \alpha_s \Big[\frac{\ln\tau}{\tau} + \frac{1}{\tau} + \delta(\tau) + f_1^{\rm nons}(\tau)\Big] \\
&+ \alpha_s^2 \Big[\frac{\ln^3\tau}{\tau} + \frac{\ln^2\tau}{\tau} + \frac{\ln\tau}{\tau} + \frac{1}{\tau} + \delta(\tau) + f_2^{\rm nons}(\tau)\Big] \\
&+ \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \ddots + \ldots\Big] \\
= &\qquad d\sigma^{(0)}/d\tau \qquad + \mathcal{O}(\tau)/\tau
\end{aligned}
$$

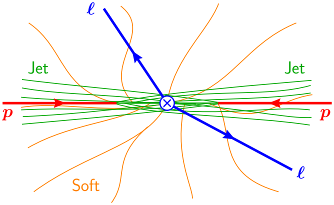
- For small $\tau \ll 1$ (i.e. $p_T^2/m_V^2 \ll 1$)
  - ✓ Logarithmic terms completely dominate perturbative series
  - ✓ Their all-order structure is actually simpler and more universal
  - ✓ Holds the key for a rigorous, quantitative treatment of theory correlations



Factorization and Resummation at Small p_T

Leading-power p_T spectrum factorizes
into hard, collinear, and soft contributions

$$\frac{d\sigma}{d\vec{p}_T} = \sigma_0\, H(Q,\mu) \int d^2\vec{k}_a\, d^2\vec{k}_b\, d^2\vec{k}_s$$
$$\times\, B_a(\vec{k}_a, Qe^{Y}, \mu, \nu)\, B_b(\vec{k}_b, Qe^{-Y}, \mu, \nu)$$
$$\times\, S(\vec{k}_s, \mu, \nu)\, \delta(\vec{p}_T - \vec{k}_a - \vec{k}_b - \vec{k}_s)$$



- Each function is a renormalized object with an associated RGE
  - Structure depends on recoil variable but is universal for all color-singlet processes

⇒ Perturbative series is determined to all orders by a coupled system of differential equations
  - Their solution leads to resummed predictions
  - Each resummation order (only) requires as ingredients anomalous dimensions and boundary conditions entering the RG solution

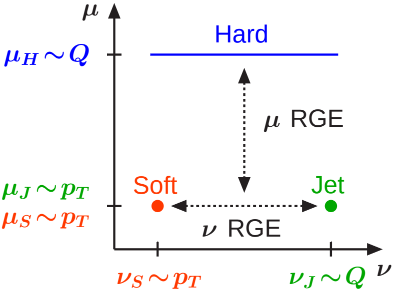
Example: Coupled RGE System for p_T

In virtuality scale μ

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\mu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_B(\mu,\nu)\, B(\vec{p}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\mu} = \gamma_S(\mu,\nu)\, S(\vec{p}_T,\mu,\nu)$$



and rapidity scale ν

$$\nu \frac{\mathrm{d}B(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = -\frac{1}{2}\int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, B(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\nu \frac{\mathrm{d}S(\vec{p}_T,\mu,\nu)}{\mathrm{d}\nu} = \int \mathrm{d}^2\vec{k}_T\, \gamma_\nu(\vec{k}_T,\mu)\, S(\vec{p}_T - \vec{k}_T,\mu,\nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\gamma_\nu(\vec{k}_T,\mu) = \nu \frac{\mathrm{d}}{\mathrm{d}\nu}\gamma_S(\mu,\nu)\delta(\vec{k}_T) = -4\Gamma_{\mathrm{cusp}}[\alpha_s(\mu)]\delta(\vec{k}_T)$$

plus evolution equations for α_s and PDFs

Example: Multiplicative RGE.

All-order RGE and its solution

$$\mu \frac{\mathrm{d}H(Q,\mu)}{\mathrm{d}\mu} = \gamma_H(Q,\mu)\, H(Q,\mu)$$

$$\Rightarrow \qquad H(Q,\mu) = H(Q) \times \exp\Big[\int_Q^\mu \frac{\mathrm{d}\mu'}{\mu'} \gamma_H(Q,\mu')\Big]$$

Necessary ingredients

- Boundary condition

$$H(Q) = 1 + \alpha_s(Q)\, h_1 + \alpha_s^2(Q)\, h_2 + \cdots$$

- *Anomalous dimension*

$$\gamma_H(Q,\mu) = \alpha_s(\mu)\big[\Gamma_0 + \alpha_s(\mu)\,\Gamma_1 + \cdots\big] \ln\frac{Q}{\mu} + \alpha_s(\mu)\big[\gamma_0 + \alpha_s(\mu)\,\gamma_1 + \cdots\big]$$

⇒ Each resummation order determined by a few (universal) parameters

Theory Correlations at Small p_T

Perturbative series at small recoil is determined to all orders by a coupled system of differential equations (RGEs)

→ Each resummation order only depends on a few semi-universal parameters

→ Unknown parameters at higher orders are the actual sources of perturbative theory uncertainty

	boundary conditions			anomalous dimensions			
order	h_n	s_n	b_n	γ_n^h	γ_n^s	Γ_n	β_n
LL	h_0	s_0	b_0	–	–	Γ_0	β_0
NLL′	h_1	s_1	b_1	γ_0^h	γ_0^s	Γ_1	β_1
NNLL′	h_2	s_2	b_2	γ_1^h	γ_1^s	Γ_2	β_2
N^{3}LL′	h_3	s_3	b_3	γ_2^h	γ_2^s	Γ_3	β_3
N^{4}LL′	h_4	s_4	b_3	γ_3^h	γ_3^s	Γ_4	β_4

- Basic Idea: Treat them as theory nuisance parameters (TNPs)
  - ✓ Vary them independently to estimate the theory uncertainties
  - ✓ The extent to which they are common and universal correctly encodes the theory correlations between different processes and kinematic regions
- Price to Pay: Calculation becomes quite a bit more complex

Two Levels.

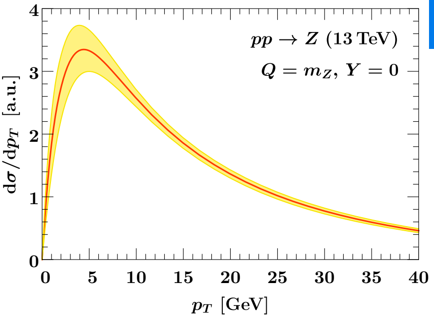
Level 1: most conservative

- Use the highest known order as the TNP
- Vary within some factor of its known value

Level 2: maximal precision

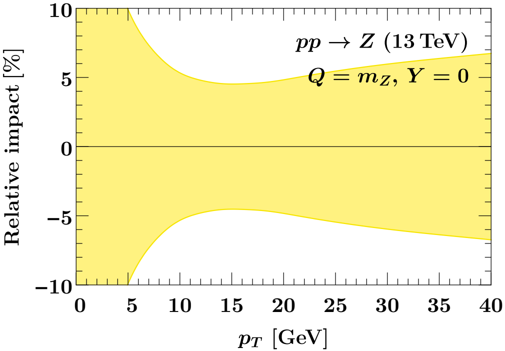
- Work at the next still unknown order
- Vary the TNPs within a reasonable expected range
  - This requires some theory prejudice, but this provides much more control than scale variations
  - Since there are several independent TNPs, more robust against unintentional/accidental underestimate of any one TNP
- TNPs can in principle be constrained by data

Start with Z Production.



Z p_T spectrum at the peak $Q = m_Z$

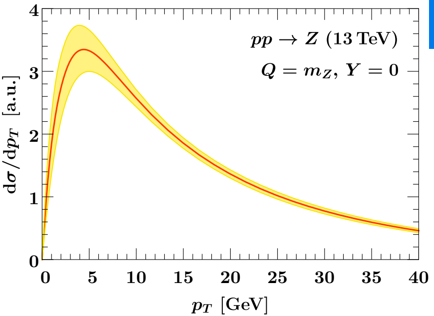
- Level 1 TNPs at NNLL′
  - Two-loop parameters treated as "unknown" nuisance parameters
  - Vary within $\pm 2\times$ their true value



Start with Z Production.

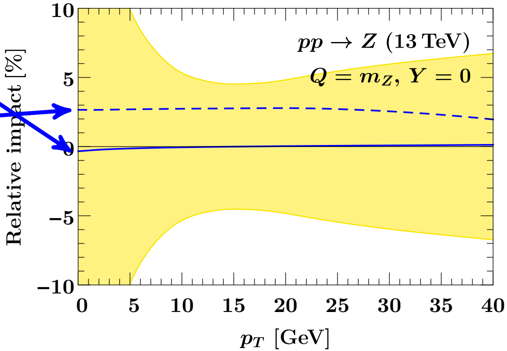
$Z\ p_T$ spectrum at the peak $Q = m_Z$

- Level 1 TNPs at NNLL′
  - Two-loop parameters treated as "unknown" nuisance parameters
  - Vary within $\pm 2\times$ their true value

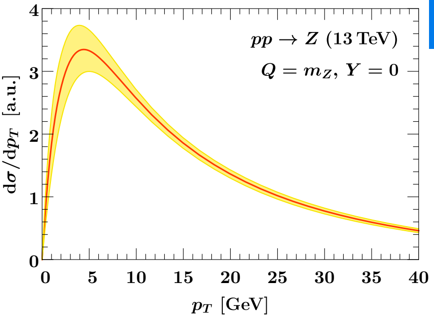


Relative impact of different TNPs and what they (only) depend on

- γ_1^h: color channel (gg vs. $q\bar{q}$)
- h_2: hard process



Start with Z Production.

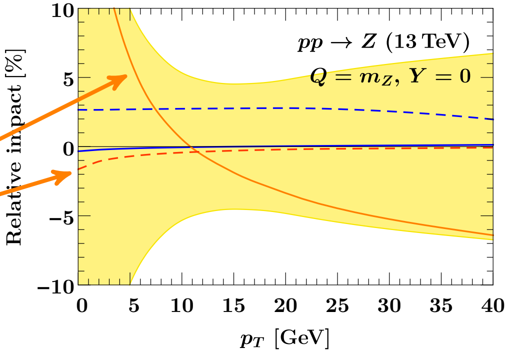


Z p_T spectrum at the peak $Q = m_Z$

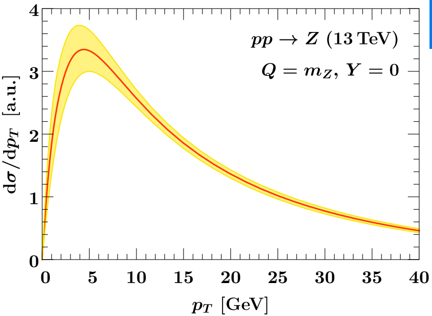
- Level 1 TNPs at NNLL′
  - Two-loop parameters treated as "unknown" nuisance parameters
  - Vary within $\pm 2\times$ their true value

Relative impact of different TNPs and what they (only) depend on

- γ_1^h: color channel (gg vs. $q\bar{q}$)
- h_2: hard process
- γ_1^ν: type of recoil variable
- s_2: type of recoil variable



Start with Z Production.

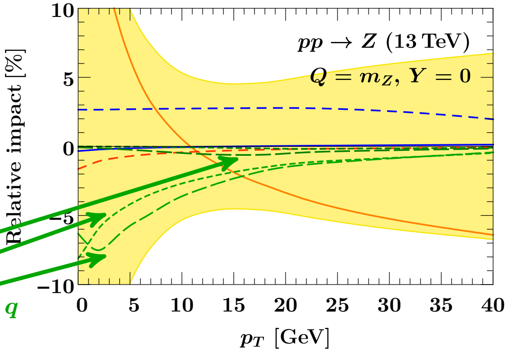


Z p_T spectrum at the peak $Q = m_Z$

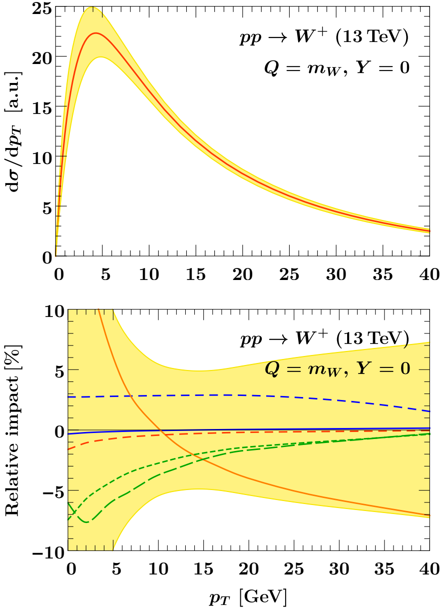
- Level 1 TNPs at NNLL′
  - Two-loop parameters treated as "unknown" nuisance parameters
  - Vary within $\pm 2\times$ their true value

Relative impact of different TNPs and what they (only) depend on

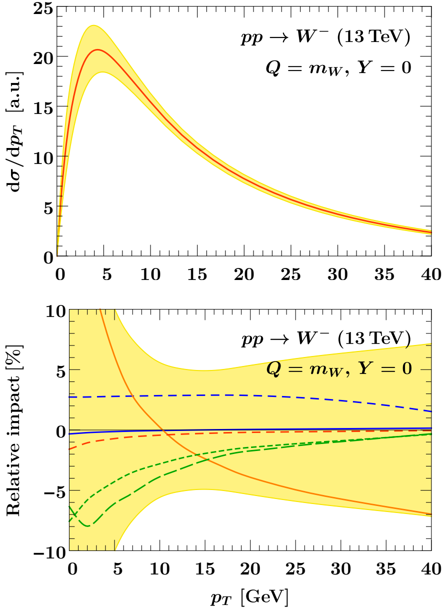
- γ_1^h: color channel (gg vs. $q\bar{q}$)
- h_2: hard process
- γ_1^ν: type of recoil variable
- s_2: type of recoil variable
- b_2: type of recoil variable channel: $g \to b$, $q \to q$, $g \to q$



W^+ and W^-.

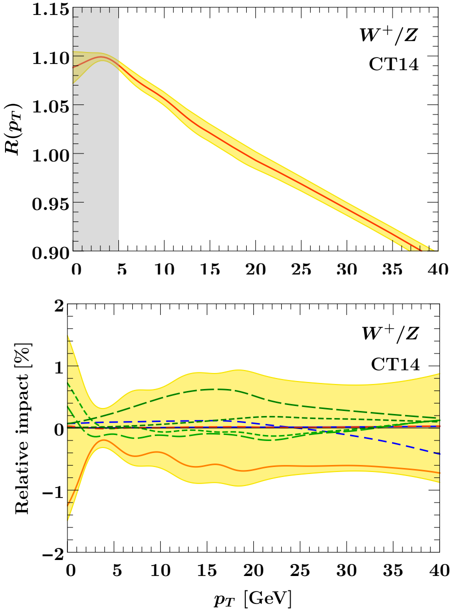


$pp \to W^+$ (13 TeV)
$Q = m_W,\ Y = 0$
$d\sigma/dp_T$ [a.u.]
$pp \to W^+$ (13 TeV)
$Q = m_W,\ Y = 0$
Relative impact [%]
p_T [GeV]

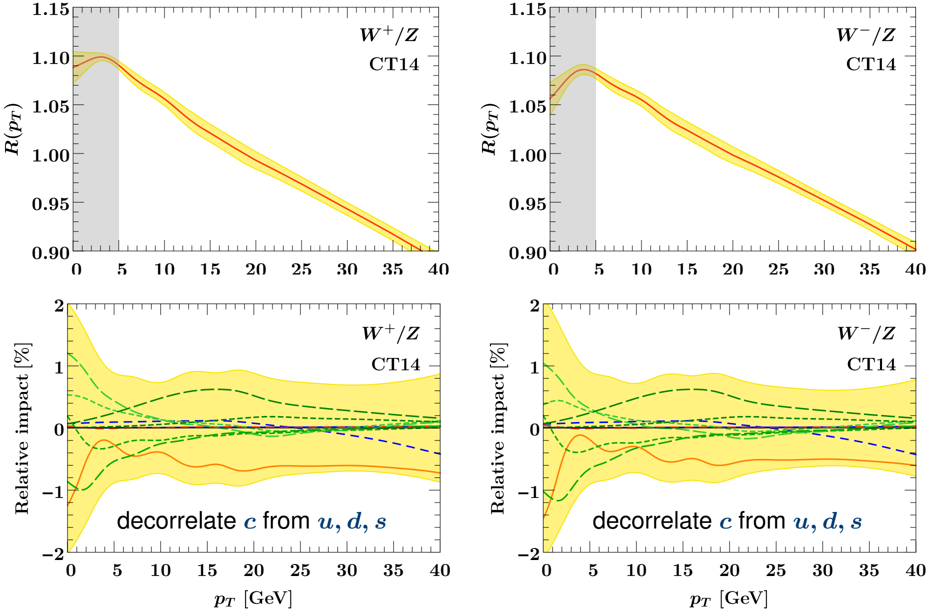


$pp \to W^-$ (13 TeV)
$Q = m_W,\ Y = 0$
$d\sigma/dp_T$ [a.u.]
$pp \to W^-$ (13 TeV)
$Q = m_W,\ Y = 0$
Relative impact [%]
p_T [GeV]

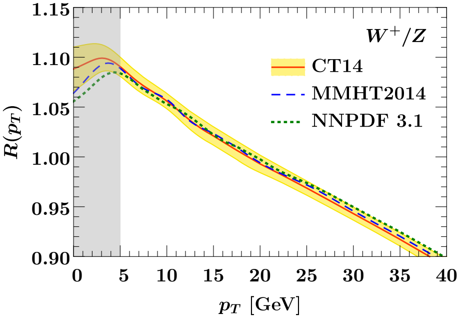
$W^{\pm}/Z$.

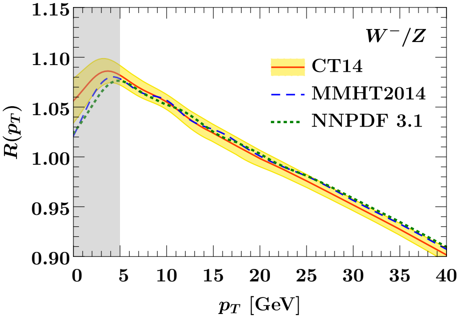


$W^{\pm}/Z$.

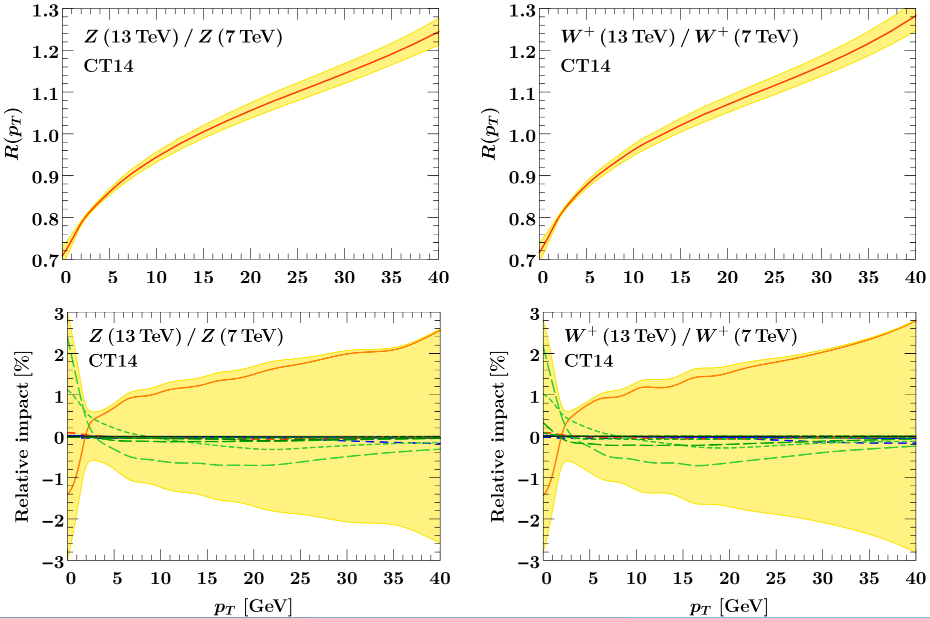

W^+/Z
CT14
$R(p_T)$
W^-/Z
CT14
$R(p_T)$
W^+/Z
CT14
Relative impact [%]
decorrelate c from u, d, s
p_T [GeV]
W^-/Z
CT14
Relative impact [%]
decorrelate c from u, d, s
p_T [GeV]

PDF Dependence.





Different E_{cm}.



Summary.

Benefits of Theory Nuisance Parameters

- Encode true correlations between different
  - p_T values, Q values, E_{cm}, partonic channels, hard processes
  - Different recoil variables ($\vec{p}_T$, ϕ^*, p_T^{jet}, $\mathcal{T}_0$, ...)

  $\Rightarrow$ Predictions differing in any of these can be properly correlated

- Can be propagated straightforwardly (like any other nuisance parameters)
  - Including Monte Carlo, neutral networks, ...

- Can use partial orders and maximally exploit all available information
  - Uncertainties explicitly account for new structures (partonic channels) appearing at higher order
  - Reduced uncertainties because more perturbative information is used

- Can in principle be constrained by data
  - Consistent to use precise control measurements to constrain the total uncertainties on final predictions
  - This also requires some care to not overconstrain them