

Precise predictions for Drell-Yan transverse observables with RadISH/NNLOJET

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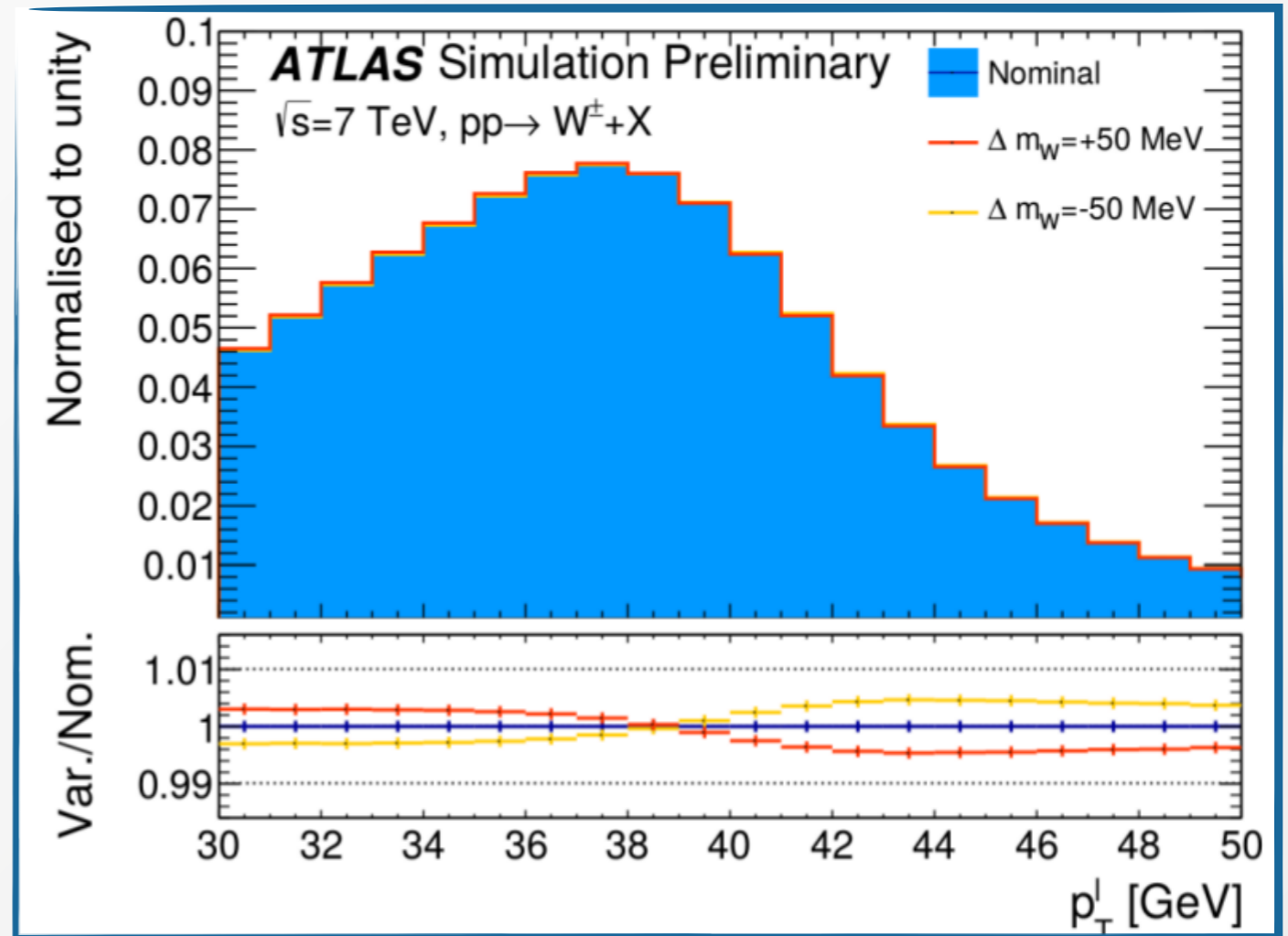
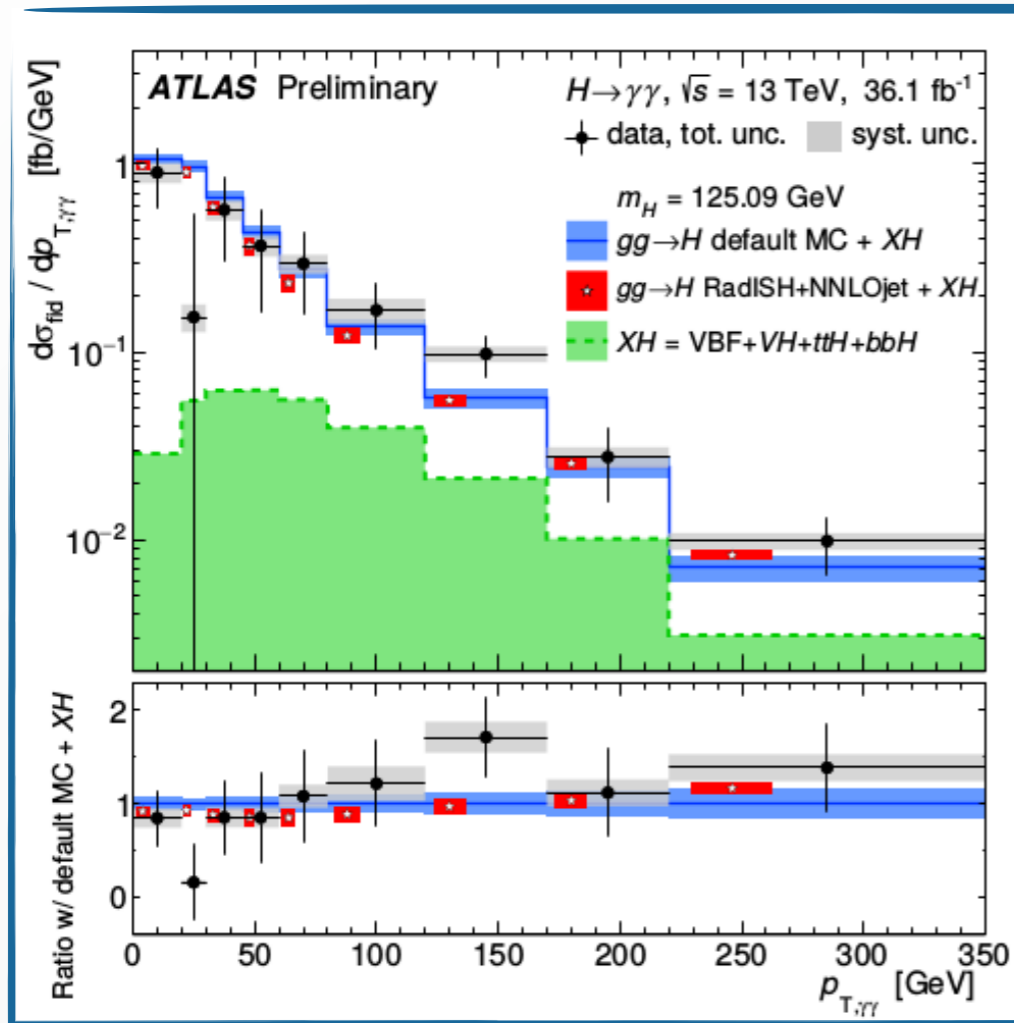


Based on past and ongoing work with

*W. Bizon, X. Chen, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover,
P.F. Monni, A. Huss, E. Re, and P. Torrielli*

[Bizon, Chen et al. 1805.05916 + ongoing work]

Transverse observables in colour-singlet production



Higgs transverse momentum:
 relatively easy to measure;
 sensitivity to BSM physics (e.g.
 Higgs couplings)

DY Transverse observables (p_t, φ^*):
 measured with extremely high precision
 Implications for PDFs, α_s extraction,
 W mass extraction: modeling of p_t^W, p_t^Z
 crucial

Transverse observables in colour-singlet production

Clean experimental and theoretical environment for precision physics

Parameterized as

$$V(k) = \left(\frac{k_t}{M} \right)^a f(\phi)$$

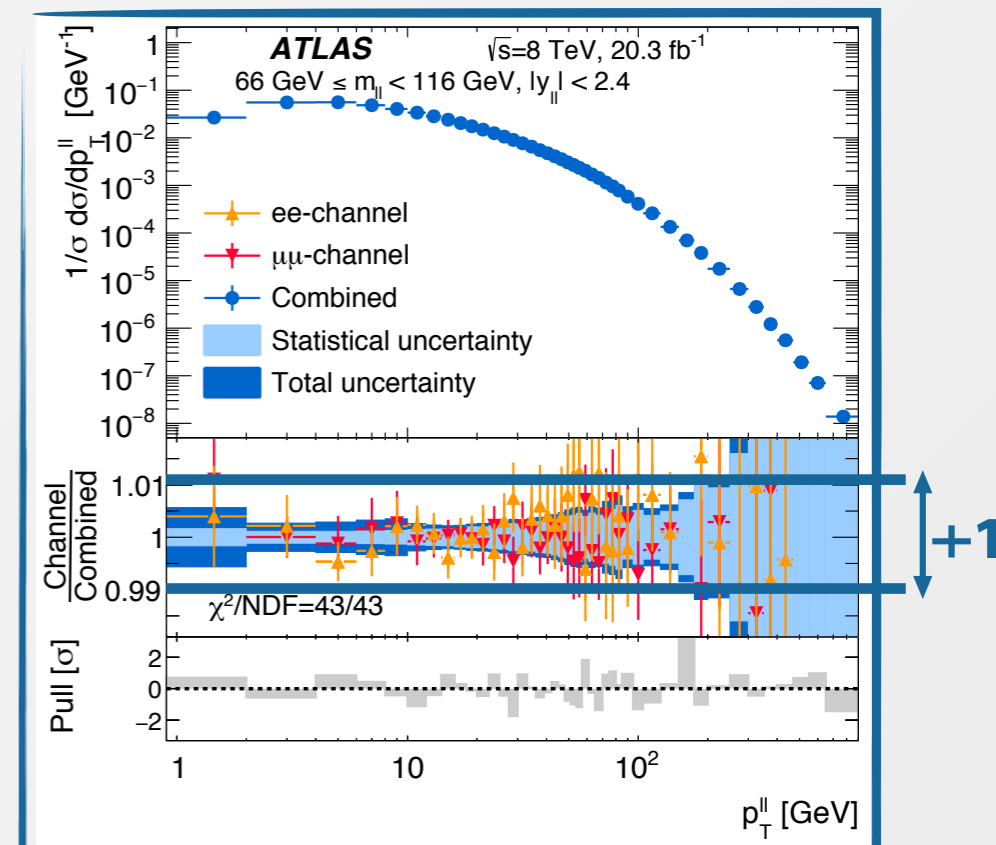
for a **single soft** QCD emission k **collinear** to incoming leg. Independent of the rapidity of radiation. $V \rightarrow 0$ for soft/collinear radiation.

Inclusive observables (e.g. transverse momentum p_t) probe directly the kinematics of the colour singlet

$$V(k_1, \dots, k_n) = V(k_1 + \dots + k_n)$$

- negligible or no sensitivity to multi-parton interactions
- reduced sensitivity to non-perturbative effects
- measured extremely precisely at experiments

Very accurate theoretical predictions needed



Precision physics at the LHC: theory

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda_{\text{QCD}}^p / Q^p)$$

Input parameters:

strong coupling

PDFs

α_s

f

few percent
uncertainty;
improvable

Non-perturbative effects

percent effect;
not yet under
control

Precision physics at the LHC: theory

$$\sigma(s, Q^2) = \sum_{a,b} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda_{\text{QCD}}^p / Q^p)$$



$$\hat{\sigma} = \hat{\sigma}_0 (1 + \alpha_s C_1 + \alpha_s^2 C_2 + \alpha_s^3 C_3 + \dots)$$

LO NLO NNLO N³LO

$\alpha_s \sim 0.1$

$\delta \sim 10\text{-}20\%$

NLO

$\delta \sim 1\text{-}5\%$

NNLO (or even **N³LO**)

All-order resummation

Cumulative cross section

$$\Sigma(v) = \int_0^v dV \frac{d\sigma}{dV} \sim \sigma_0 [1 + \alpha_s \# + \alpha_s^2 \# + \dots]$$

Fixed-order prediction: reliable for **inclusive enough** observables and in regions not marred by **soft/collinear radiation** ($v \rightarrow 0$)

Real and virtual contributions can become **highly unbalanced** in processes where the real radiation is strongly constrained by kinematics

Large logarithms appear at **all order** as a left-over of the real-virtual cancellation of IRC divergences

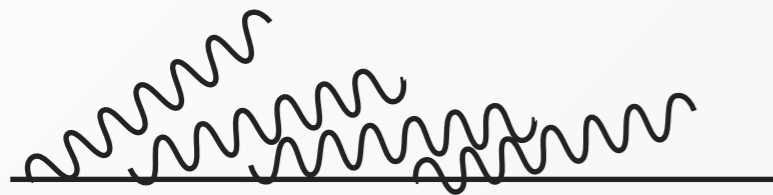
$$\ln \Sigma(v) = \sum_n \left\{ \underbrace{\mathcal{O}(\alpha_s^n L^{n+1})}_{\mathbf{LL}} + \underbrace{\mathcal{O}(\alpha_s^n L^n)}_{\mathbf{NLL}} + \underbrace{\mathcal{O}(\alpha_s^n L^{n-1})}_{\mathbf{NNLL}} + \dots \right\} \quad \begin{array}{l} L = \ln 1/v \\ v = p_t/M \text{ in the transverse} \\ \text{momentum case} \end{array}$$

**Fixed order predictions no longer reliable:
all-order resummation of the perturbative series**

Case study: transverse momentum p_t

Resummation of transverse momentum is particularly delicate because p_t is a **vectorial quantity**

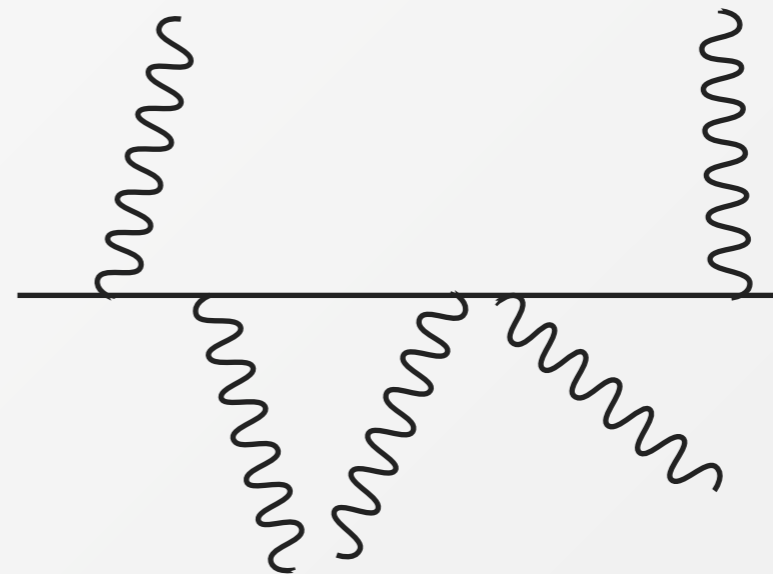
Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression



$$\sum_{i=1}^n \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

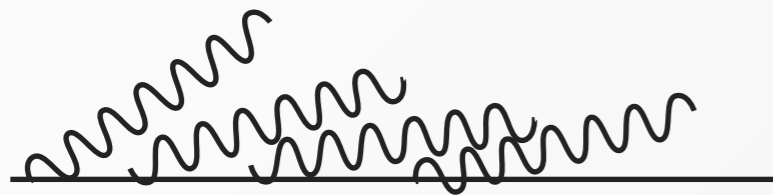
$p_t \sim 0$ far from the Sudakov limit

Power suppression

Case study: transverse momentum p_t

Resummation of transverse momentum is particularly delicate because it is a **vectorial quantity**

Two concurring mechanisms leading to a system with small p_t



$$p_t^2 \sim k_{t,i}^2 \ll M^2$$

cross section naturally suppressed as there is no phase space left for gluon emission (Sudakov limit)

Exponential suppression

Dominant at small p_t



$$\sum_{i=1}^n \vec{k}_{t,i} \simeq 0$$

Large kinematic cancellations

$p_t \sim 0$ far from the Sudakov limit

Power suppression

Resummation in direct and in conjugate space

Phase-space constraints do not usually factorize in **direct space**

Resummation usually performed in impact-parameter (b) space where the two competing mechanisms are handled through a **Fourier transform**. **Transverse-momentum conservation** is respected

$$\delta\left(\vec{p}_t - \sum_{i=1}^n \vec{k}_{t,i}\right) = \int d^2b \frac{1}{4\pi^2} e^{i\vec{b}\cdot\vec{p}_t} \prod_{i=1}^n e^{-i\vec{b}\cdot\vec{k}_{t,i}}$$

[Parisi, Petronzio '78; Collins, Soper, Sterman '85]

Resummation in direct space: not possible to find a closed analytic expression in direct space which is both

- a) free of logarithmically subleading corrections
- b) free of singularities at finite p_t values

[Frixione, Nason, Ridolfi '98]

A naive logarithmic counting at small p_t is not sensible, as one loses the **correct power-suppressed scaling** if only logarithms are retained

Resummation in direct space now possible

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

[Ebert, Tackmann '16] see also [Kang, Lee, Vaidya '17]

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

Translate the resummability of the observable into properties of the observable in the presence of multiple radiation: **recursive infrared and collinear (rIRC) safety**

[Banfi, Salam, Zanderighi '01, '03, '04]

Existence of a **resolution scale** q_0 , **independent of the observable**, such that emissions below q_0 (**unresolved**) do not contribute significantly to the observable's value.

Starting point: all-order cumulative cross section

$$\Sigma(v) = \int d\Phi_B \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^n [dk_i] |\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2 \Theta(v - V(\{\Phi_B\}, k_1, \dots, k_n))$$

single-particle phase space

matrix element for n real emissions

all-order form factor (virtuals)

$v = p_t/M$

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

1. Establish a **logarithmic counting** for the squared matrix element $|\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2$

Decompose the squared amplitude in terms of **n -particle correlated blocks**, denoted by $|\tilde{\mathcal{M}}(k_1, \dots, k_n)|^2$ ($|\tilde{\mathcal{M}}(k_1)|^2 = |\mathcal{M}(k_1)|^2$)

$$\sum_{n=0}^{\infty} |\mathcal{M}(\Phi_B, k_1, \dots, k_n)|^2 = |\mathcal{M}_B(\Phi_B)|^2$$

*expression valid for inclusive observables

$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^n \left(\overset{\text{LL}}{|\mathcal{M}(k_i)|^2} + \int [dk_a][dk_b] \overset{\text{NLL}}{|\tilde{\mathcal{M}}(k_a, k_b)|^2} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} - \vec{k}_{ti}) \delta(Y_{ab} - Y_i) \right. \right.$$

$$\left. \left. + \int [dk_a][dk_b][dk_c] \overset{\text{NNLL}}{|\tilde{\mathcal{M}}(k_a, k_b, k_c)|^2} \delta^{(2)}(\vec{k}_{ta} + \vec{k}_{tb} + \vec{k}_{tc} - \vec{k}_{ti}) \delta(Y_{abc} - Y_i) + \dots \right) \right\}$$

$$\equiv |\mathcal{M}_B(\Phi_B)|^2 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n |\mathcal{M}(k_i)|_{\text{inc}}^2$$

Upon integration over the phase space, the expansion can be put in a **one to one correspondence** with the logarithmic structure

Systematic recipe to include terms up to the desired logarithmic accuracy

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

2. Exploit rIRC safety to single out the IRC singularities of the real matrix element and achieve the cancellation of the exponentiated divergences of virtual origin

Introduce a slicing parameter $\epsilon \ll 1$ such that all inclusive blocks with $k_{t,i} < \epsilon k_{t,1}$, with $k_{t,1}$ hardest emission, can be neglected in the computation of the observable

$$\Sigma(v) = \int d\Phi_B |\mathcal{M}_B(\Phi_B)|^2 \mathcal{V}(\Phi_B) \quad \text{unresolved emissions}$$

$$\times \int [dk_1] |\mathcal{M}(k_1)|_{\text{inc}}^2 \left(\sum_{l=0}^{\infty} \frac{1}{l!} \int \prod_{i=2}^{l+1} [dk_j] |\mathcal{M}(k_j)|_{\text{inc}}^2 \Theta(\epsilon V(k_1) - V(k_j)) \right)$$

$$\times \left(\sum_{m=0}^{\infty} \frac{1}{m!} \int \prod_{i=2}^{m+1} [dk_i] |\mathcal{M}(k_i)|_{\text{inc}}^2 \Theta(V(k_i) - \epsilon V(k_1)) \Theta(v - V(\Phi_B, k_1, \dots, k_{m+1})) \right) \quad \text{resolved emissions}$$

Unresolved emission doesn't contribute to the evaluation of the observable: it can be exponentiated directly and employed to cancel the virtual divergences, giving rise to a Sudakov radiator

$$\mathcal{V}(\Phi_B) \exp \left\{ \int [dk] |\mathcal{M}(k)|_{\text{inc}}^2 \Theta(\epsilon V(k_1) - V(k)) \right\} \simeq e^{-R(\epsilon V(k_1))}$$

Transverse observable resummation with RadISH

[Monni, Re, Torrielli '16, Bizon, Monni, Re, LR, Torielli '17]

Final result at NLL

$$\begin{aligned} \frac{d\Sigma(v)}{d\Phi_B} &= \int \frac{dk_{t,1}}{k_{t,1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(k_{t,1})} e^{R'(k_{t,1})} \mathcal{L}_{\text{NLL}}(k_{t,1}) R'(k_{t,1}) \\ &\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i k_{t,1}) \Theta(v - V(\Phi_B, k_1, \dots, k_{n+1})) \end{aligned}$$

Parton luminosity at NLL reads

$$\mathcal{L}_{\text{NLL}}(k_{t,1}) = \sum_c \frac{d|M_B|_{c\bar{c}}^2}{d\Phi_B} f_c(x_1, k_{t,1}^2) f_{\bar{c}}(x_2, k_{t,1}^2)$$

At higher logarithmic accuracy, it includes **coefficient functions** and **hard-virtual** corrections

All ingredients to perform resummation at **N³LL accuracy** are now available

[Catani *et al.* '11, '12][Gehrmann *et al.* '14][Li, Zhu '16][Moch *et al.* '18]

Fixed-order predictions now available at **NNLO**

[A. Gehrmann-De Ridder *et al.* '15, 16, '17][Boughezal *et al.* '15, 16]

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma_{\text{matched}}^{\text{mult}}(v) \sim \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}} *$$

- allows to include constant terms from NNLO (if N³LO total xs available)
- physical suppression at small v cures potential instabilities

*actual scheme slightly more involved

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \rightarrow \frac{1}{p} \ln \left(1 + \left(\frac{Q}{k_{t1}} \right)^p \right)$$

Q : **perturbative resummation scale**
used to probe the size of subleading logarithmic corrections

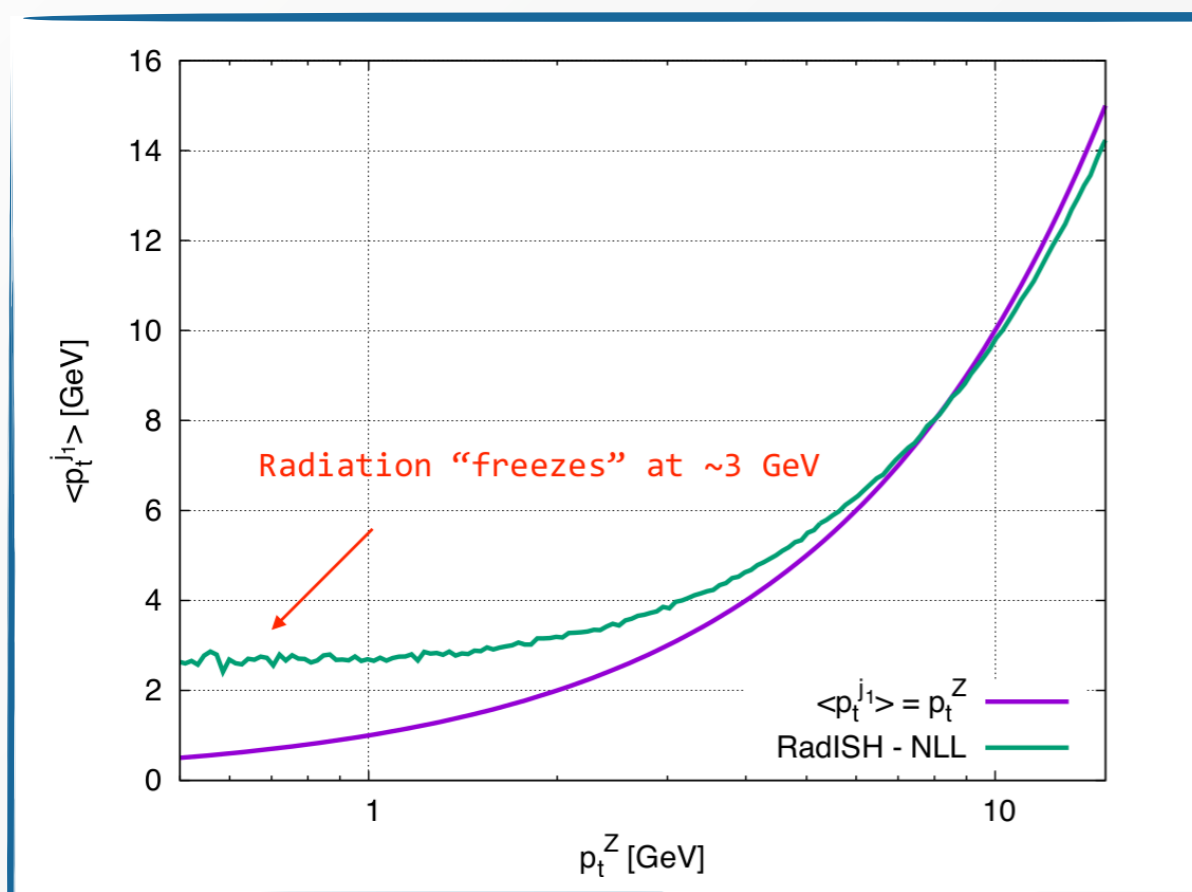
p : arbitrary matching parameter

The Landau pole and the small- p_t limit

Running coupling $\alpha_s(k_{t1}^2)$ and Sudakov radiator hit Landau pole at

$$\alpha_s(\mu_R^2)\beta_0 \ln Q/k_{t1} = \frac{1}{2} \quad k_{t1} \sim 0.01 \text{ GeV}, \quad \mu_R = Q = m_Z$$

Only real cutoff in the calculation: emission probability is set to zero below this scale and parton densities are frozen.



At small p_t the large azimuthal cancellations dominate over the Sudakov suppression: the cutoff is never an issue in practice

$$\frac{d^2\Sigma(v)}{dp_t d\Phi_B} \simeq 2\sigma^{(0)}(\Phi_B)p_t \left(\frac{\Lambda_{\text{QCD}}^2}{M^2} \right)^{\frac{16}{25} \ln \frac{41}{16}}$$

No NP parameters included in the following

Theoretical predictions for Z observables

[Bizon, Chen et al. 1805.05916]

Results obtain using the fiducial cuts of the 8 TeV ATLAS data measurement

[ATLAS 1512.02192]

$$p_t^{\ell^\pm} > 20 \text{ GeV}, \quad |\eta^{\ell^\pm}| < 2.4, \quad |Y_{\ell\ell}| < 2.4, \quad 46 \text{ GeV} < M_{\ell\ell} < 150 \text{ GeV}$$

using NNPDF3.0 with $\alpha_s(M_Z)=0.118$ and setting the central scales to

$$\mu_R = \mu_F = M_T = \sqrt{M_{\ell\ell} + p_T^2}, \quad Q = \frac{M_{\ell\ell}}{2}$$

5 flavour (massless) scheme: no HQ effects and no PDF thresholds

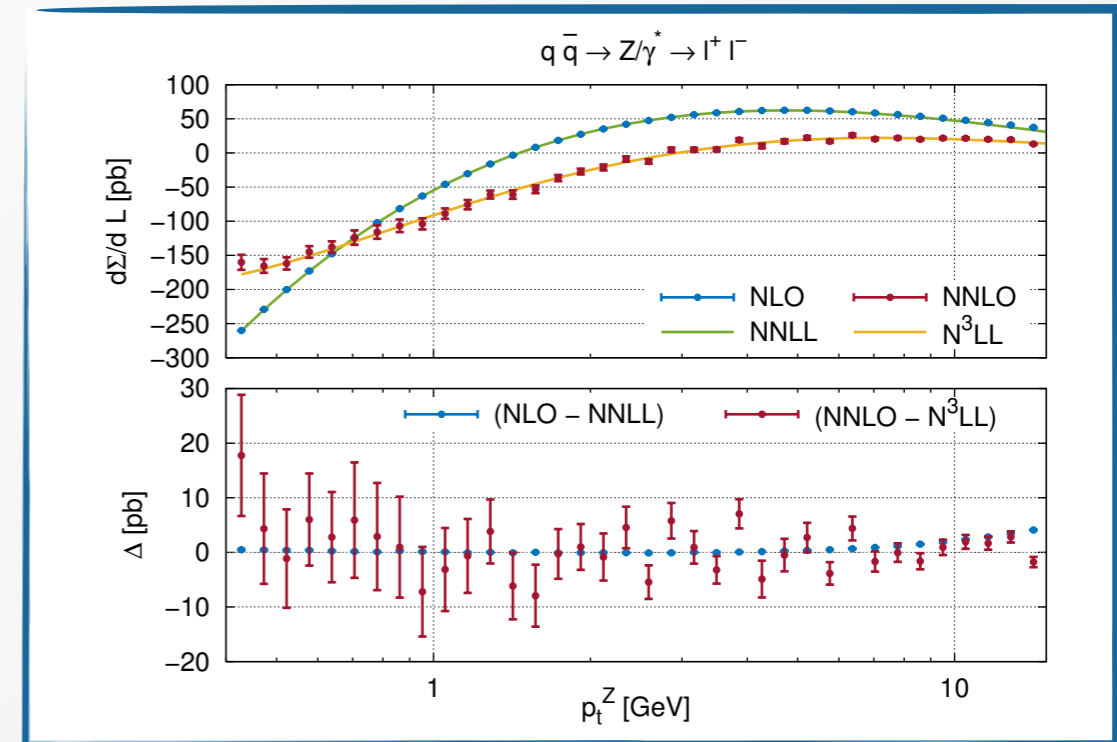
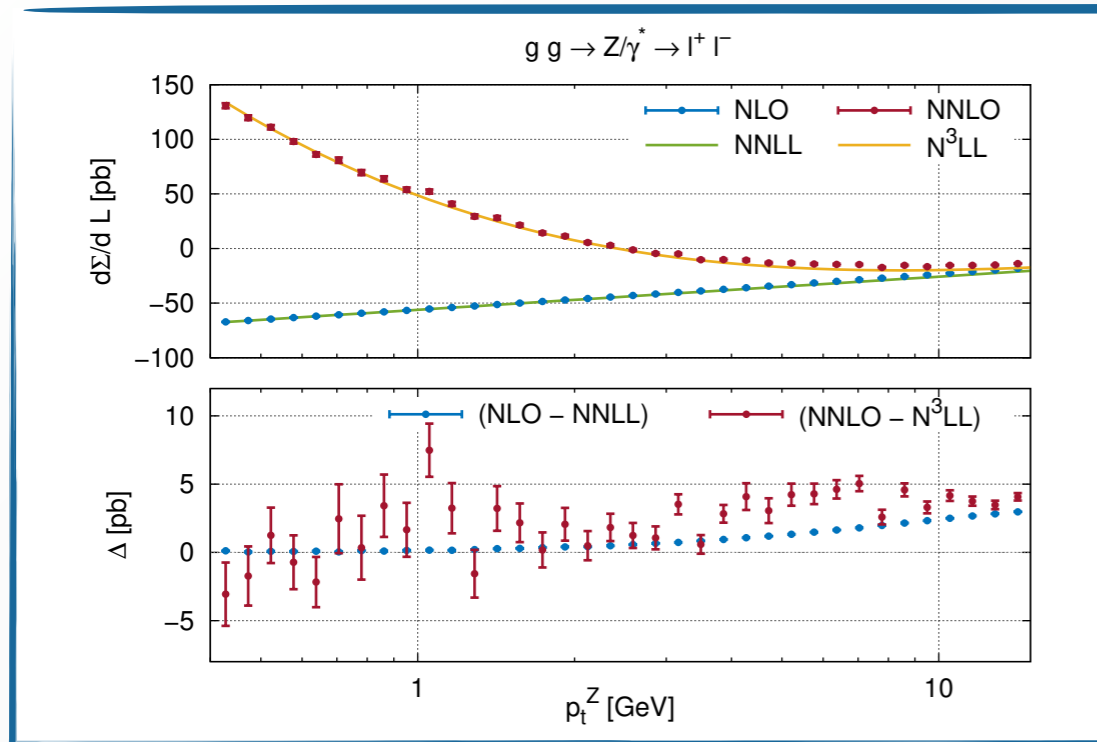
Scale uncertainties estimated by varying **renormalization** and **factorization** scale by a factor of two around their central value (**7 point variation**) and varying the **resummation** scale by a factor of 2 around its central value for factorization and renormalization scales set to their central value: **9 point envelope**

Matching parameter p set to 4 as a default

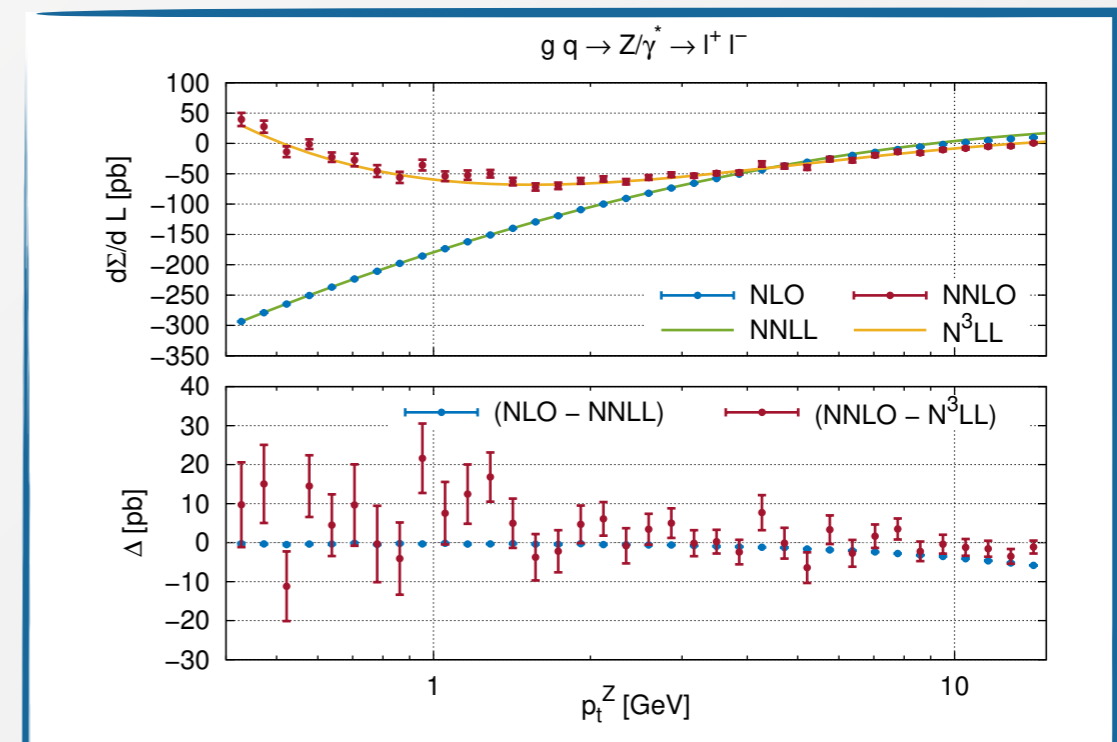
No NP parameters included in the following

Fixed order vs. resummation

[Bizon, Chen et al. 1805.05916]

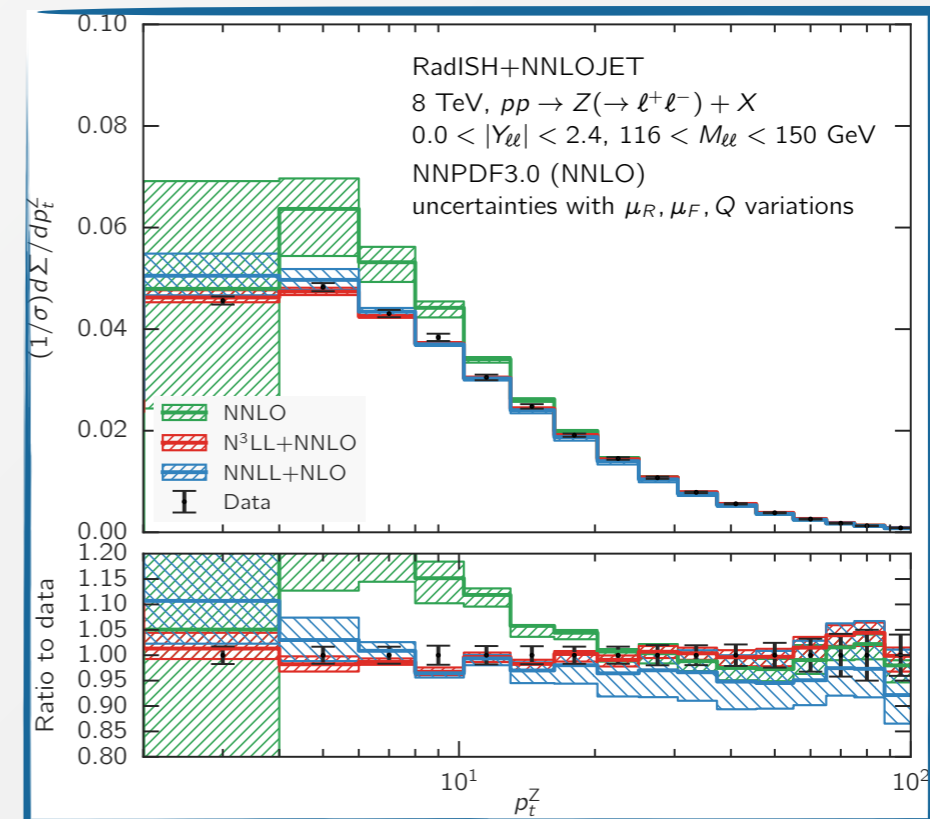
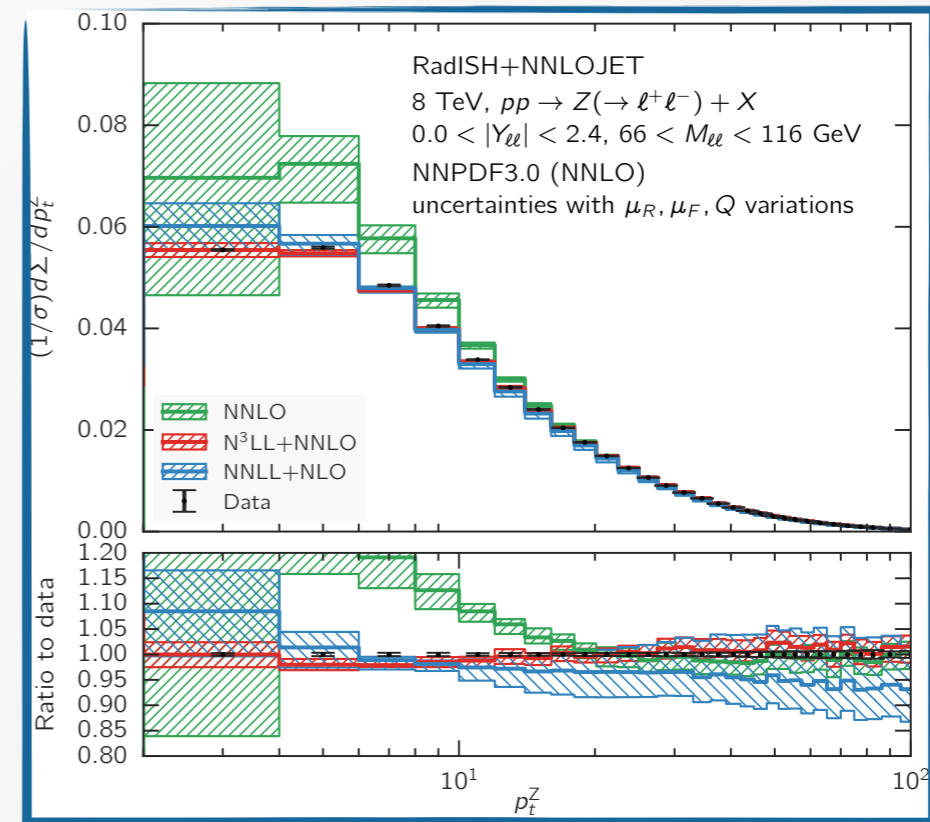
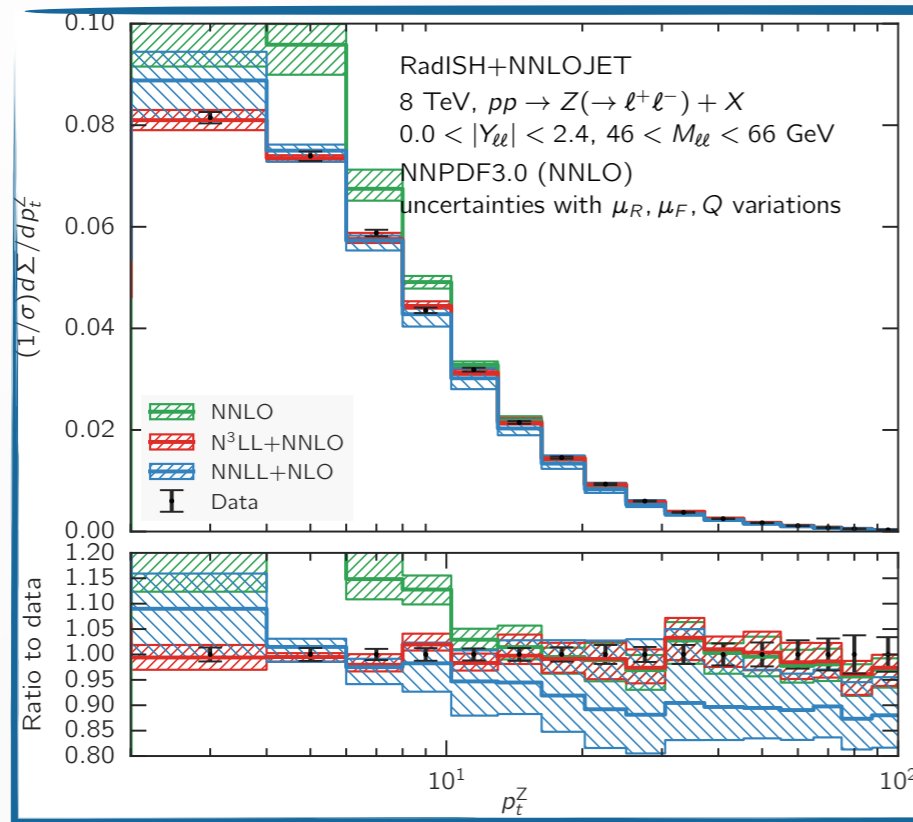


- Very good agreement with the fixed order at small p_t
- Very strong validation of both calculations
- Fixed an implementational error in the fixed order computation



Results for the p_t distribution

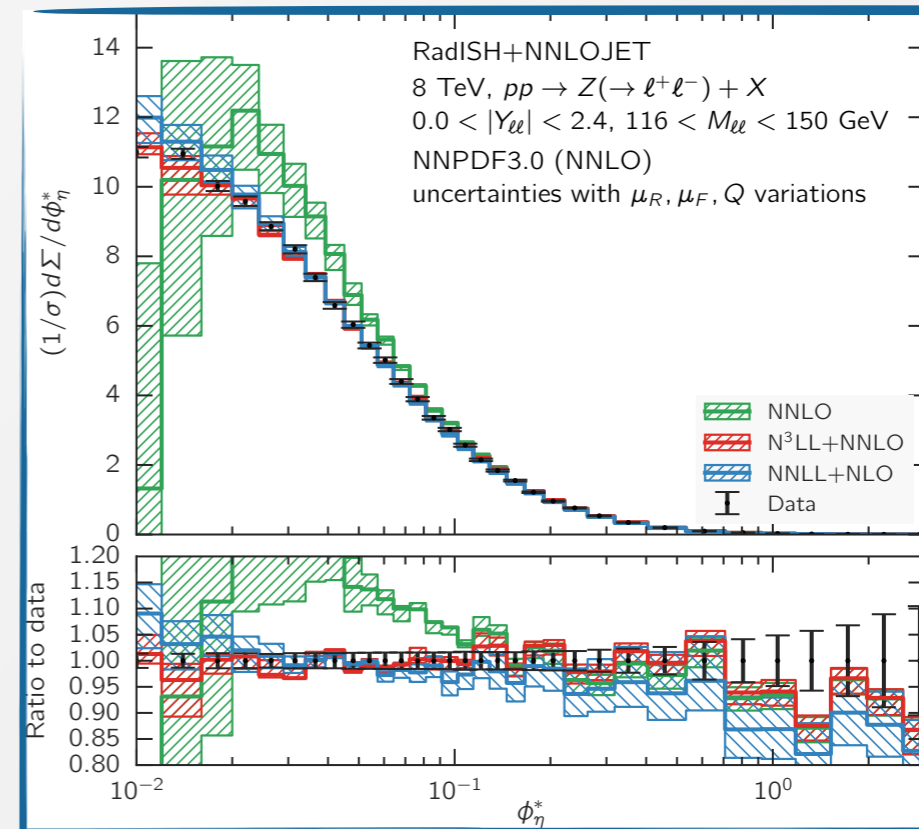
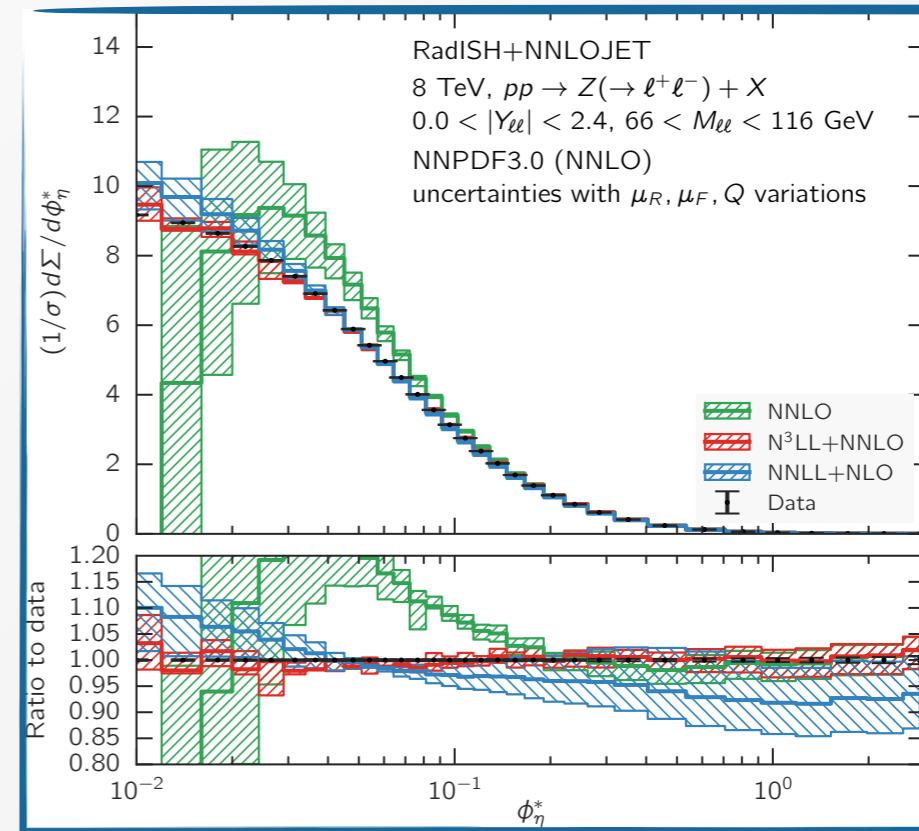
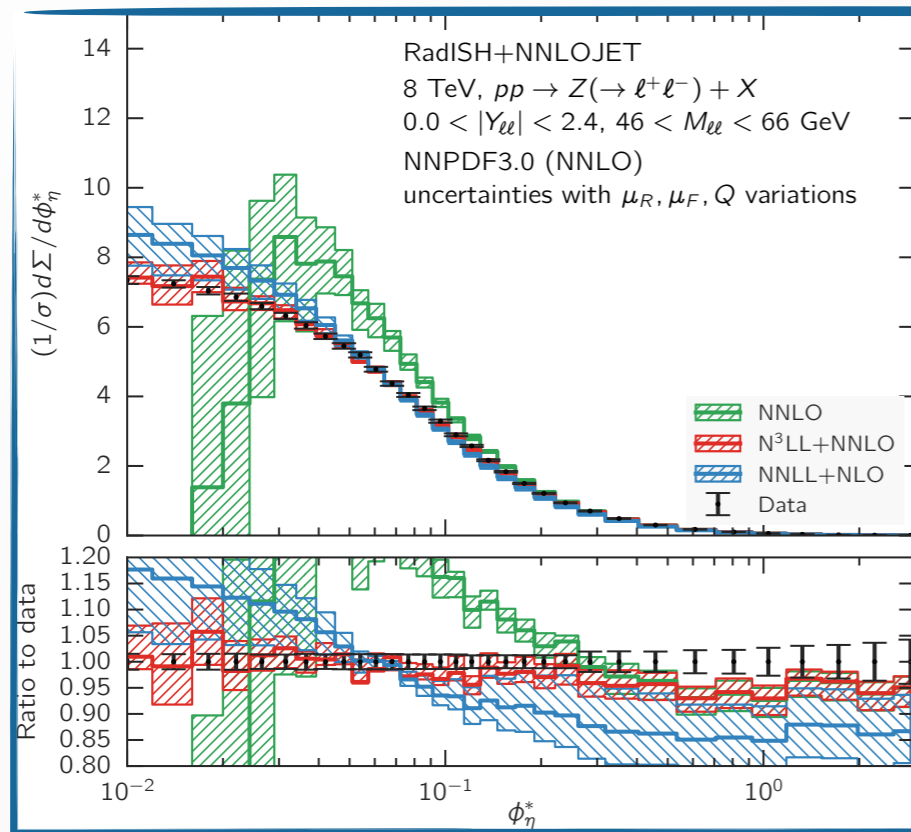
[data from ATLAS 1512.02192]
[Bizon, Chen et al. 1805.05916]



- Very good agreement with data in all fiducial regions
- Uncertainties at the few percent level, still larger than the experimental errors

Results for the ϕ^* distribution

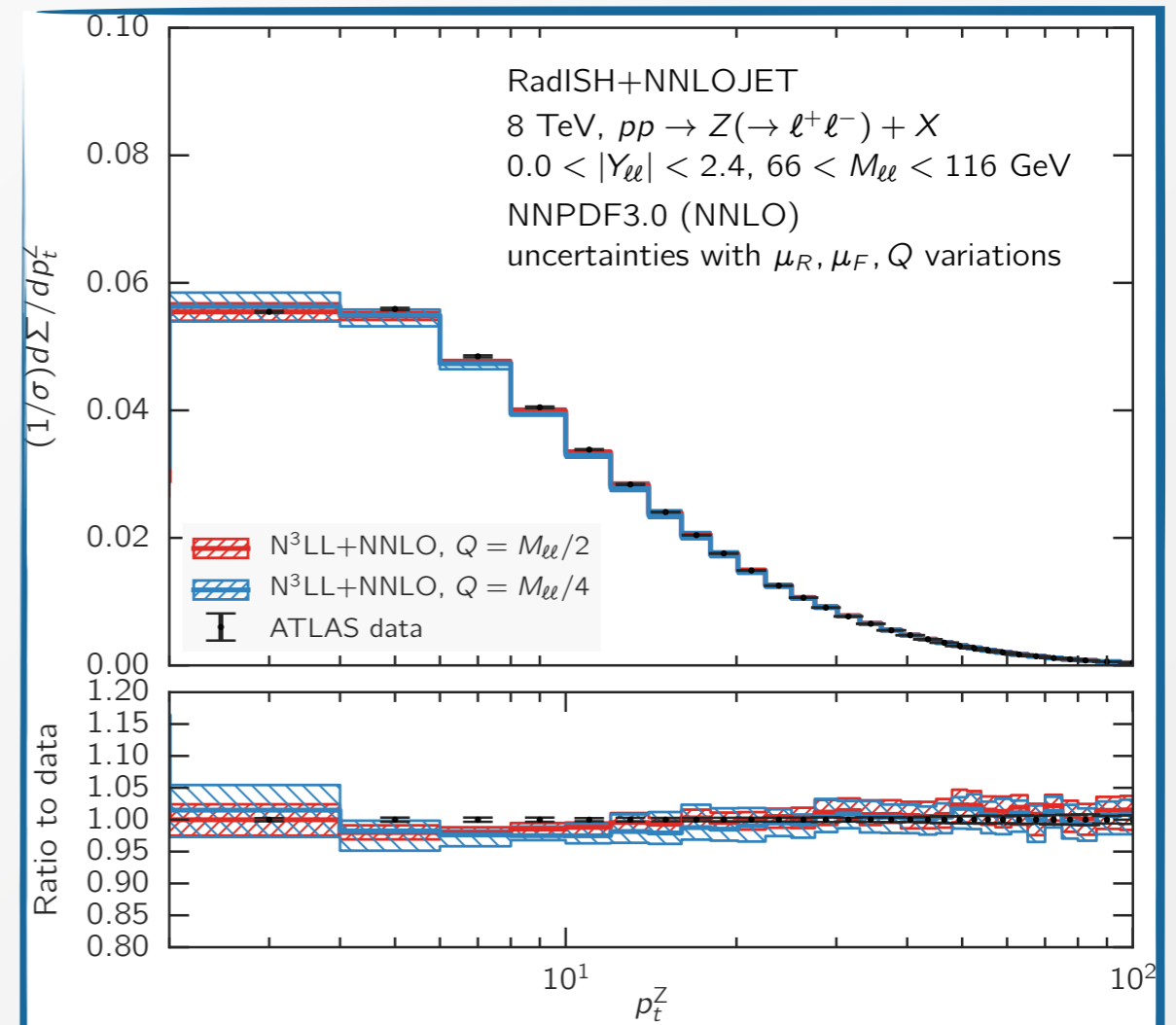
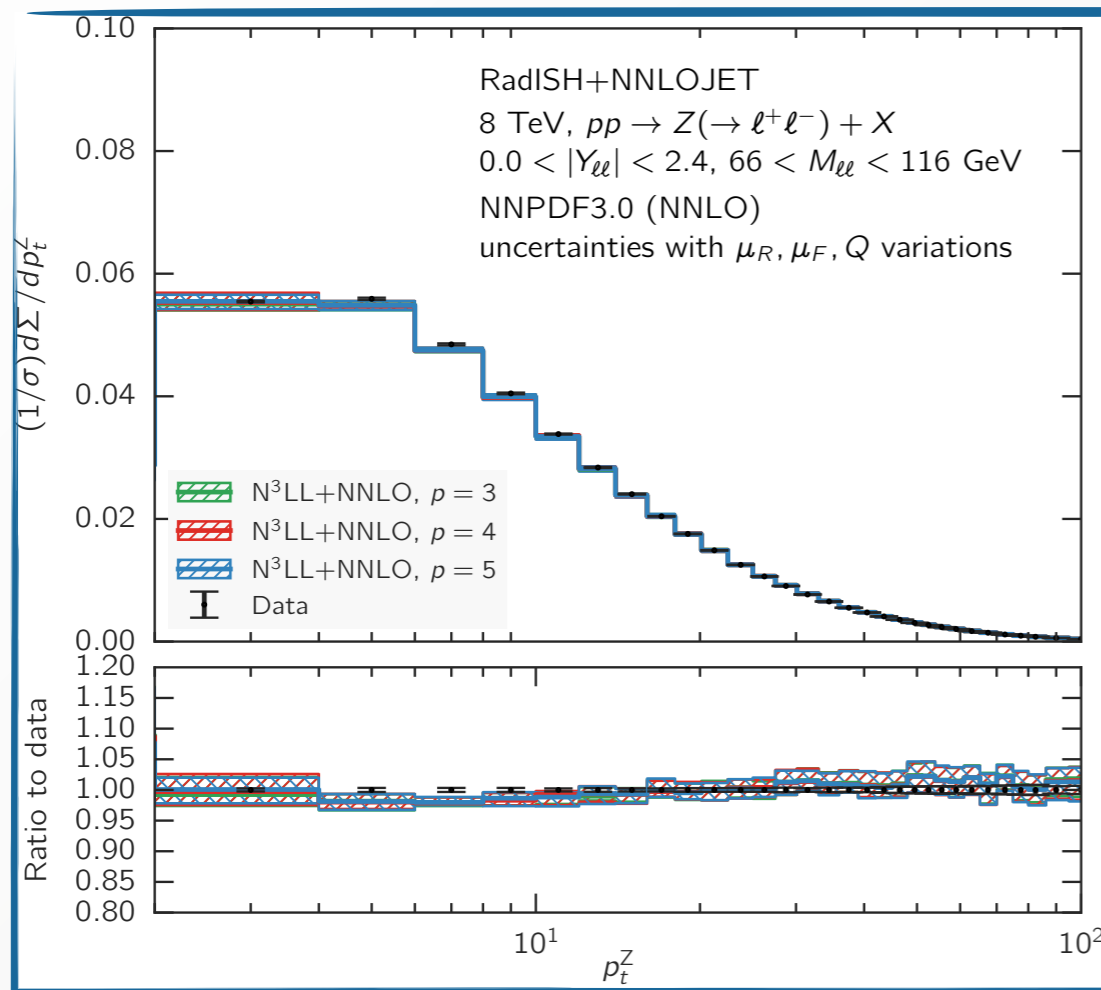
[data from ATLAS 1512.02192]
[Bizon, Chen et al. 1805.05916]



- Similar conclusion for ϕ^* observable

Resummation and matching uncertainties

[data from ATLAS 1512.02192]
[Bizon, Chen et al. 1805.05916]

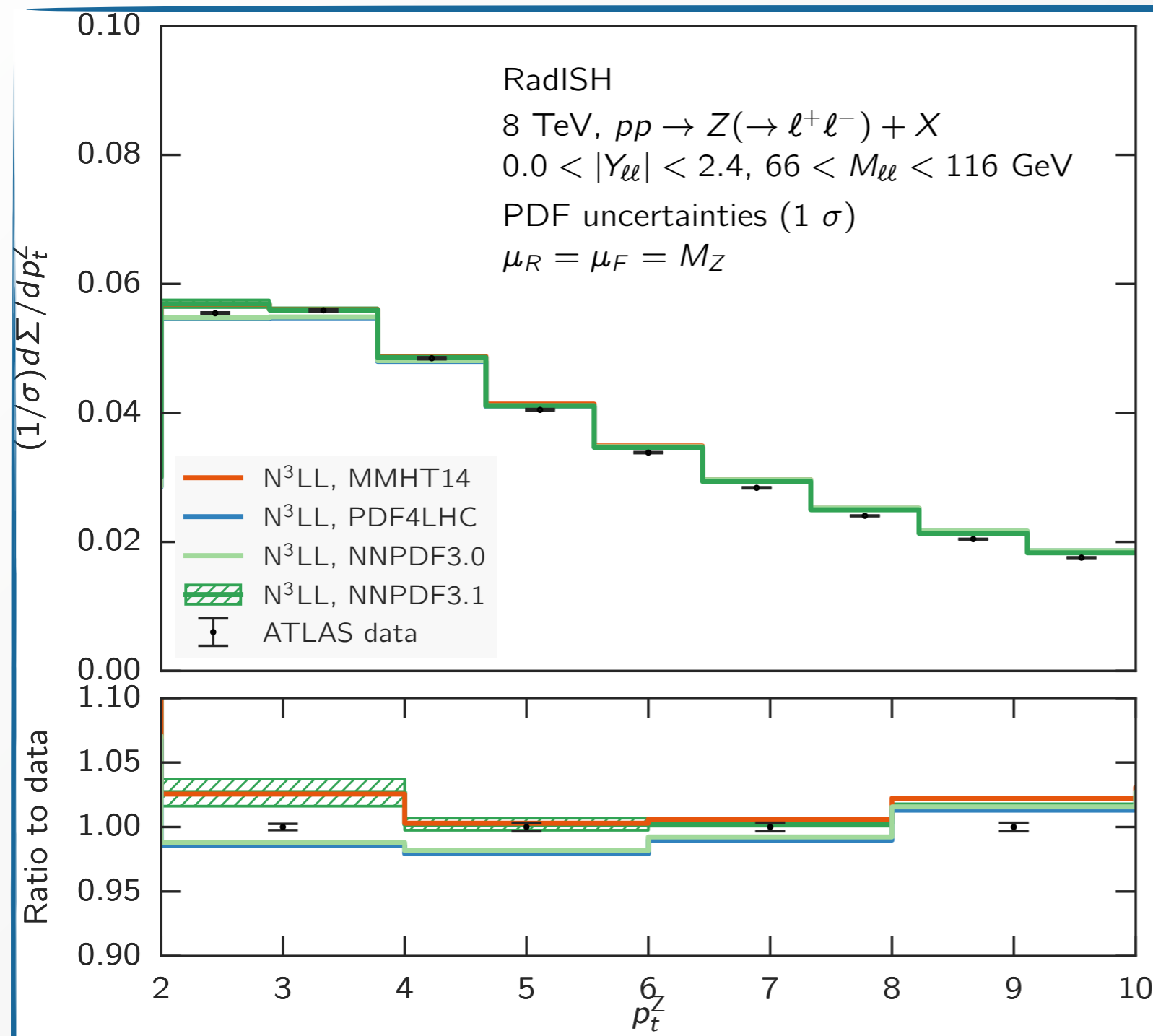


- Matching uncertainties at the sub percent level

- Predictions stable wrt variation of central value of the resummation scale

PDF uncertainties

Beware of different PDFs and central scales

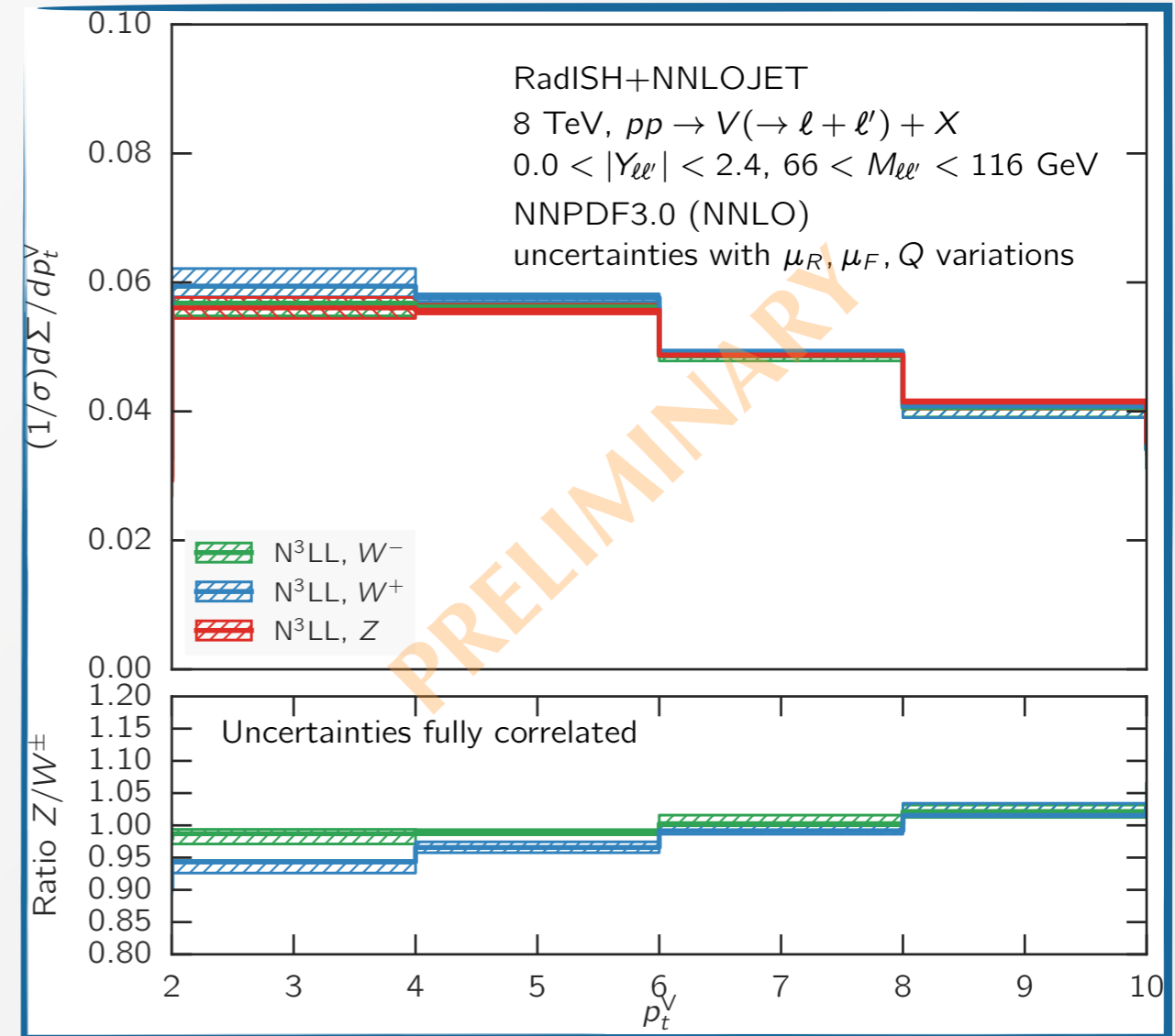
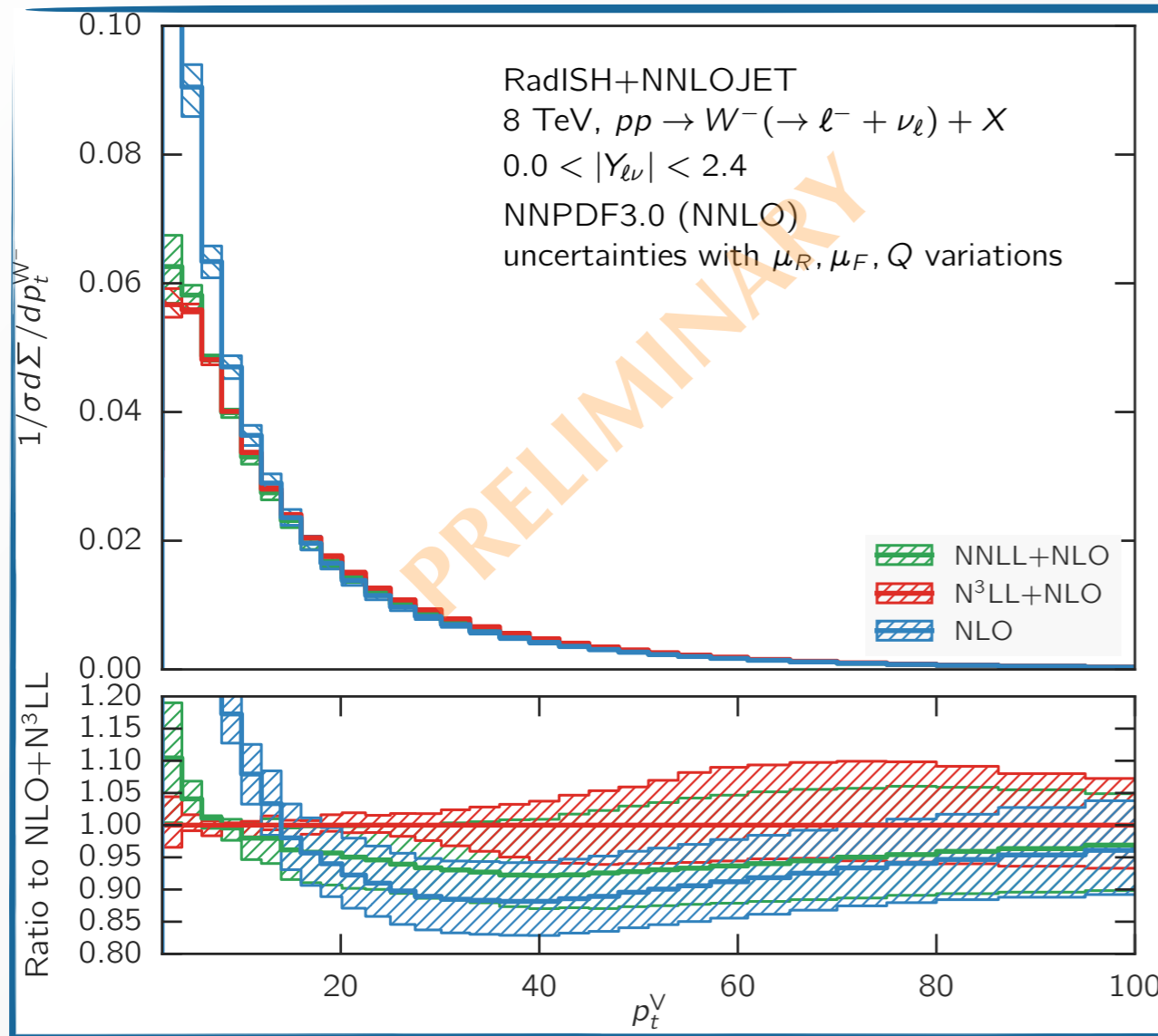


- Uncertainty with state-of-the-art PDFs at the 1-2% level
- Spectrum gets slightly harder than NNPDF3.0 (used in our current studies)

Theoretical predictions for W observables

[ongoing work with A. Huss+NNLOJET]

$$p_t^{\ell^\pm} > 20 \text{ GeV}, \quad |\eta^{\ell^\pm}| < 2.4$$



- W uncertainties similar to the Z case
- Study of correlation of the uncertainty necessary
- NNLO+N³LL for W ongoing

Recapitulation

- No sign of NP at the LHC so far - necessary to perform detailed theory/experimental comparisons, to look for deviations from SM. Perturbation theory must be pushed at its limit
- New formalism formulated in **direct space** for all-order resummation up to **N³LL accuracy** for inclusive, transverse observables.
- Access to **multi-differential information**. This is effectively similar in spirit to a semi-inclusive parton shower, but with higher-order logarithms, and control on formal accuracy
- Results at NNLO+N³LL for Z differential distributions: good description of the data in the **fiducial distributions**, with **uncertainties at the few percent level**. Preliminary results at NLO+N³LL for W
- Difficult to improve further in the theory uncertainty: missing higher orders (few %), QED effects (~1%), PDF uncertainty (1-2%), quark mass corrections (1-2%(?)), NP effects at the same order or less relevant

Backup

Transverse observable resummation with RadISH

Result at NLL accuracy can be written as

$$\Sigma(v) = \sigma^{(0)} \int \frac{dv_1}{v_1} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon v_1)} R'(v_1) \quad v_i = V(k_i), \quad \zeta_i = v_i/v_1$$
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i v_1) \Theta(v - V(\Phi_B, k_1, \dots, k_{n+1}))$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

It contains **subleading effect** which in the original CAESAR approach are disposed of by expanding R and R' around v

~~$$R(\epsilon v_1) = R(v) + \frac{dR(v)}{d \ln(1/v)} \ln \frac{v}{\epsilon v_1} + \mathcal{O}\left(\ln^2 \frac{v}{\epsilon v_1}\right)$$
$$R'(v_i) = R'(v) + \mathcal{O}\left(\ln \frac{v}{v_i}\right)$$~~

Not possible! valid only if the ratio v_i/v remains of order one in the whole emission phase space, but for observables which feature kinematic cancellations there are configurations with $v_i \gg v$. **Subleading effects necessary**

Transverse observable resummation with RadISH

Result at NLL accuracy can be written as

$$\begin{aligned} \Sigma(v) = & \sigma^{(0)} \int \frac{dk_{t1}}{k_{t1}} \int_0^{2\pi} \frac{d\phi_1}{2\pi} e^{-R(\epsilon k_{t1})} R'(k_{t1}) \quad \zeta_i = k_{ti}/k_{t1} \\ & \times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon}^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} R'(\zeta_i k_{t1}) \Theta(v - V(\Phi_B, k_{t1}, \dots, k_{tn+1})) \end{aligned}$$

Formula can be evaluated with Monte Carlo method; dependence on ϵ vanishes exactly and result is finite in four dimensions

Convenient to perform an expansion around k_{t1} (more efficient and simpler implementation)

$$\begin{aligned} R(\epsilon k_{t1}) &= R(k_{t1}) + \frac{dR(k_{t1})}{d \ln(1/k_{t1})} \ln \frac{1}{\epsilon} + \mathcal{O}\left(\ln^2 \frac{1}{\epsilon}\right) \\ R'(k_{ti}) &= R'(k_{t1}) + \mathcal{O}\left(\ln \frac{k_{t1}}{k_{ti}}\right) \end{aligned}$$



Subleading effects retained: no divergence at small v , power-like behaviour respected

Logarithmic accuracy defined in terms of $\ln(M/k_{t1})$

Result formally equivalent to the b -space formulation

Matching to fixed order: multiplicative matching

Cumulative cross section should reduce to the fixed order at large v

$$\Sigma_{\text{matched}}^{\text{mult}}(v) = \Sigma_{\text{res}}(v) \left[\frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{res}}(v)} \right]_{\text{expanded}}$$

- allows to include constant terms from NNLO
- physical suppression at small v cures potential instabilities

To ensure that resummation does not affect the hard region of the spectrum when the matching is performed we introduce **modified logarithms**

This corresponds to restrict the rapidity phase space at large k_t

$$\ln(Q/k_{t1}) \rightarrow \frac{1}{p} \ln \left(1 + \left(\frac{Q}{k_{t1}} \right)^p \right)$$

Q : perturbative **resummation scale** used to probe the size of subleading logarithmic corrections

p : arbitrary matching parameter

Matching improved by normalizing to the **asymptotic value** to avoid **spurious** $\mathcal{O}(\alpha_s^4)$ **contributions**

$$\Sigma_{\text{mult}}^{\text{matched}}(v) = \frac{\Sigma_{\text{res}}(v)}{\Sigma_{\text{asym.}}^{\text{res}}} \left[\Sigma_{\text{asym.}}^{\text{res}} \frac{\Sigma_{\text{f.o.}}(v)}{\Sigma_{\text{exp}}(v)} \right]_{\text{expanded}} \quad \Sigma_{\text{asym.}}^{\text{res}} = \int_{\text{with cuts}} d\Phi_B \left(\lim_{L \rightarrow 0} \mathcal{L}_{\text{N}^{\text{kLL}}} \right)$$