Gauge Coupling Unification & Proton decay

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in collaboration with

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◉ Motivation of SUSY

- **Gauge Hierarchy Problem** ➡ tension with LHC; but better than non-SUSY

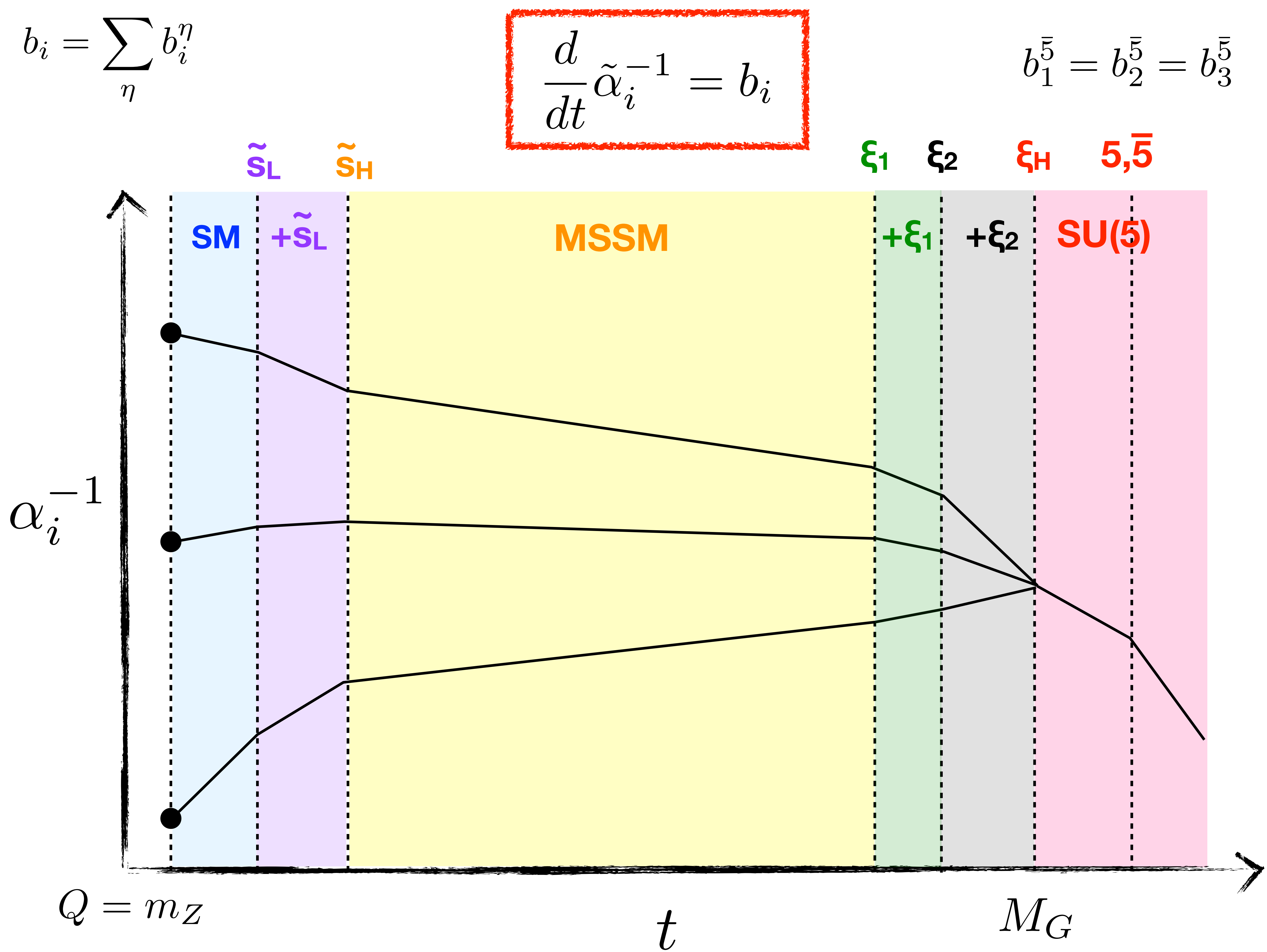
- **Dark Matter** ➡ well studied; consistent if its pure Higgsino (~1TeV) or pure Wino (~3TeV)

- **Gauge Coupling Unification**

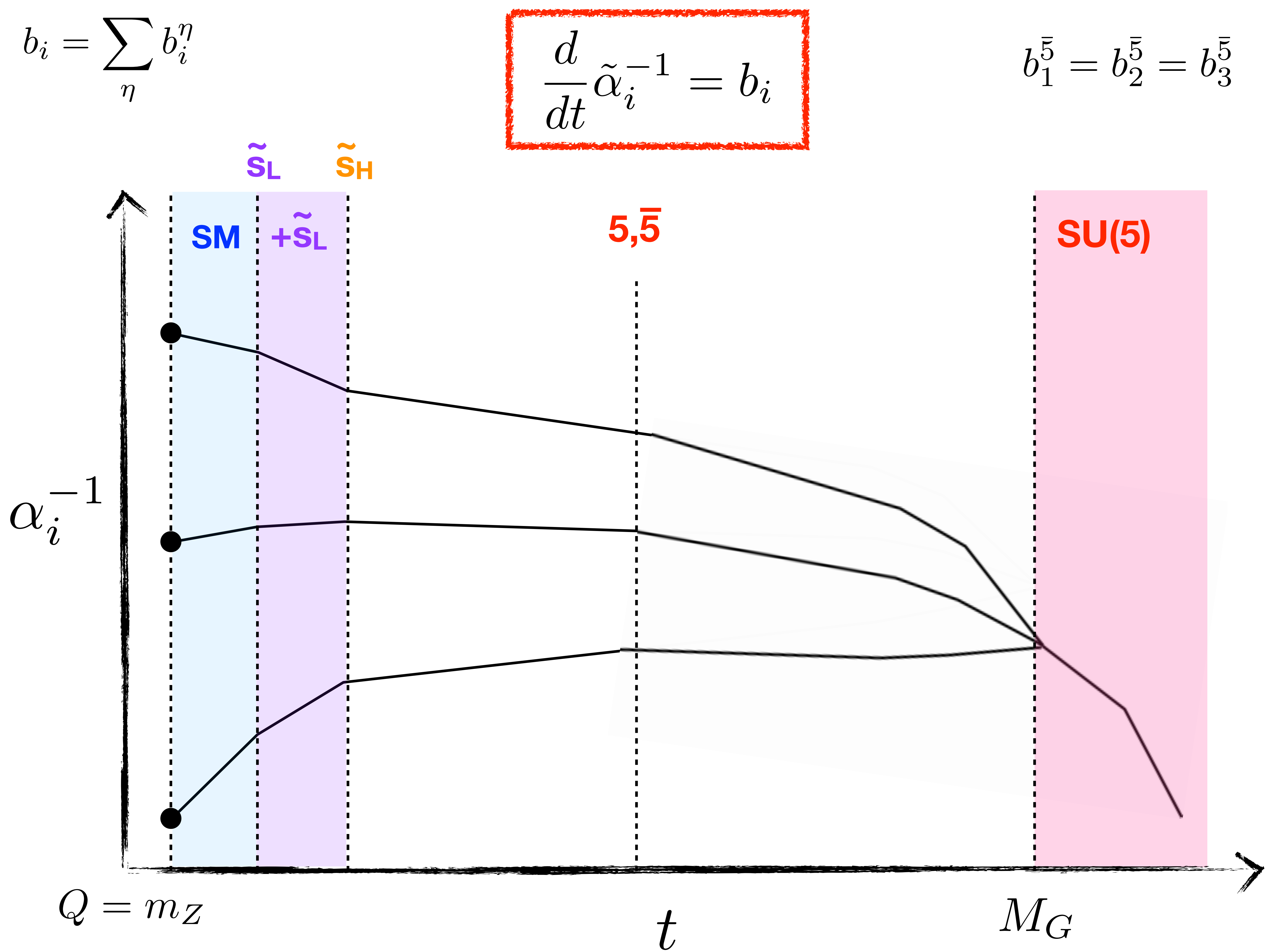
⬇

not well studied compared to the other two

- How is the condition of GCU formulated?
- How light SUSY is required from GCU?
- Any relation between low energy SUSY and proton decay?



$b_i = \sum_\eta b_i^\eta$
$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$
$b_1^{\bar{5}} = b_2^{\bar{5}} = b_3^{\bar{5}}$
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
ξ_2
ξ_H
$5,\bar{5}$
SM
$+\tilde{s}_L$
MSSM
$+\xi_1$
$+\xi_2$
SU(5)
α_i^{-1}
$Q = m_Z$
t
M_G



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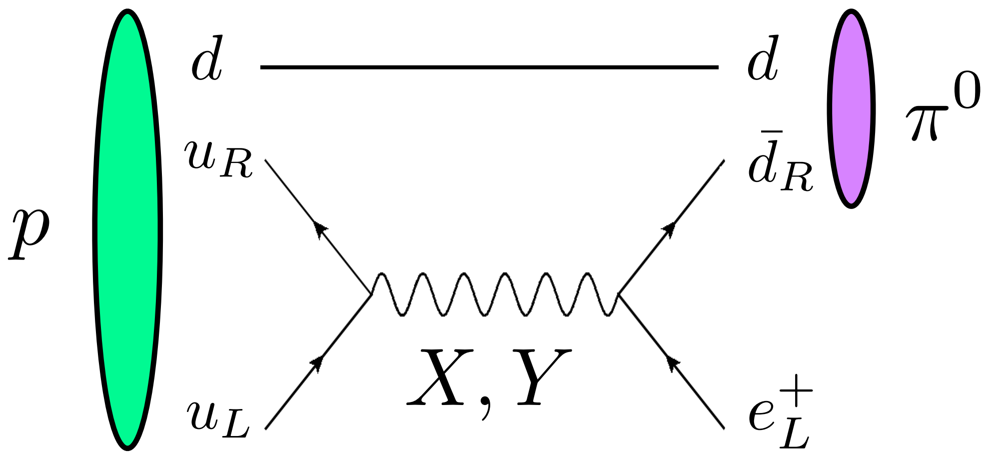
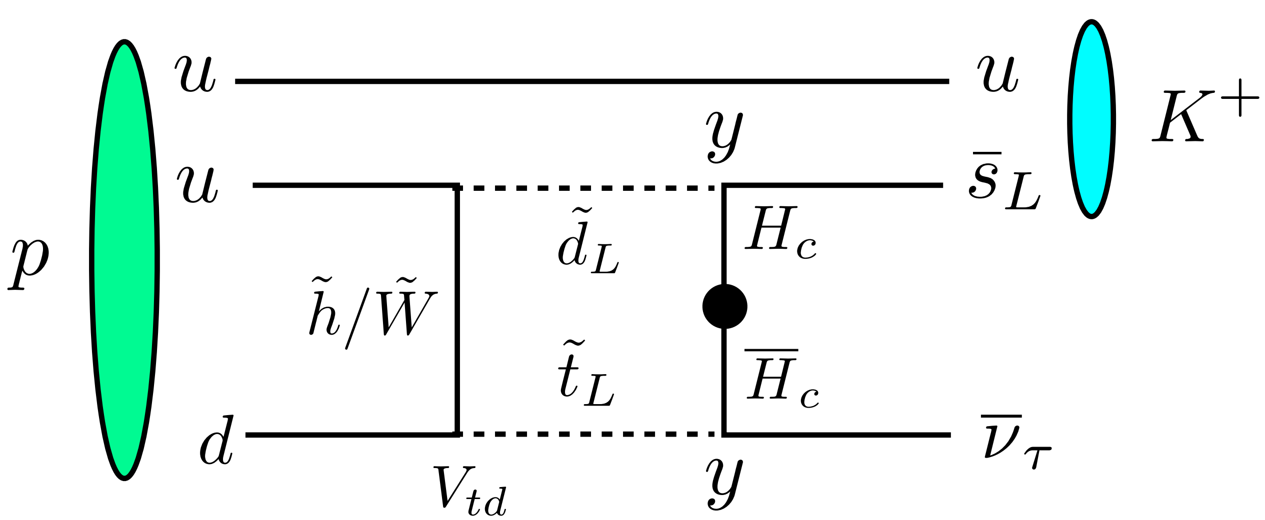
D=6 $\frac{g^2}{\Lambda^2}\,(\mathbf{10}_i^*\mathbf{10}_i)(\mathbf{10}_j^*\mathbf{10}_j)$ $\frac{g^2}{\Lambda^2}\,(\mathbf{10}_i^*\mathbf{10}_i)(\overline{\mathbf{5}}_j^*\overline{\mathbf{5}}_j)$



$$\tau_{p\to e^+\pi^0} \sim \frac{1}{\alpha_G^2}\frac{M_{X,Y}^4}{m_p^5} > 1.7\cdot 10^{34}\ \text{years}$$

$$M_{X,Y} \gtrsim 6\cdot 10^{15}\ \text{GeV}\cdot\left(\frac{\alpha_G}{25.}\right)^2$$

D=5 $\frac{y_i y_j}{\Lambda}\,\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{10}\cdot\overline{\mathbf{5}}$



$$\tau(p\to K^+\bar{\nu}) > 5.9\times 10^{33}\ \text{yrs}$$

$$t_\beta^4\left(\frac{10^3\ \text{GeV}}{M_{\text{SUSY}}}\right)^2\left(\frac{10^{19}\ \text{GeV}}{M_{H_c}}\right)^2 \lesssim 1$$

Is Minimal SU(5) excluded?

[Murayama, Piece '01]

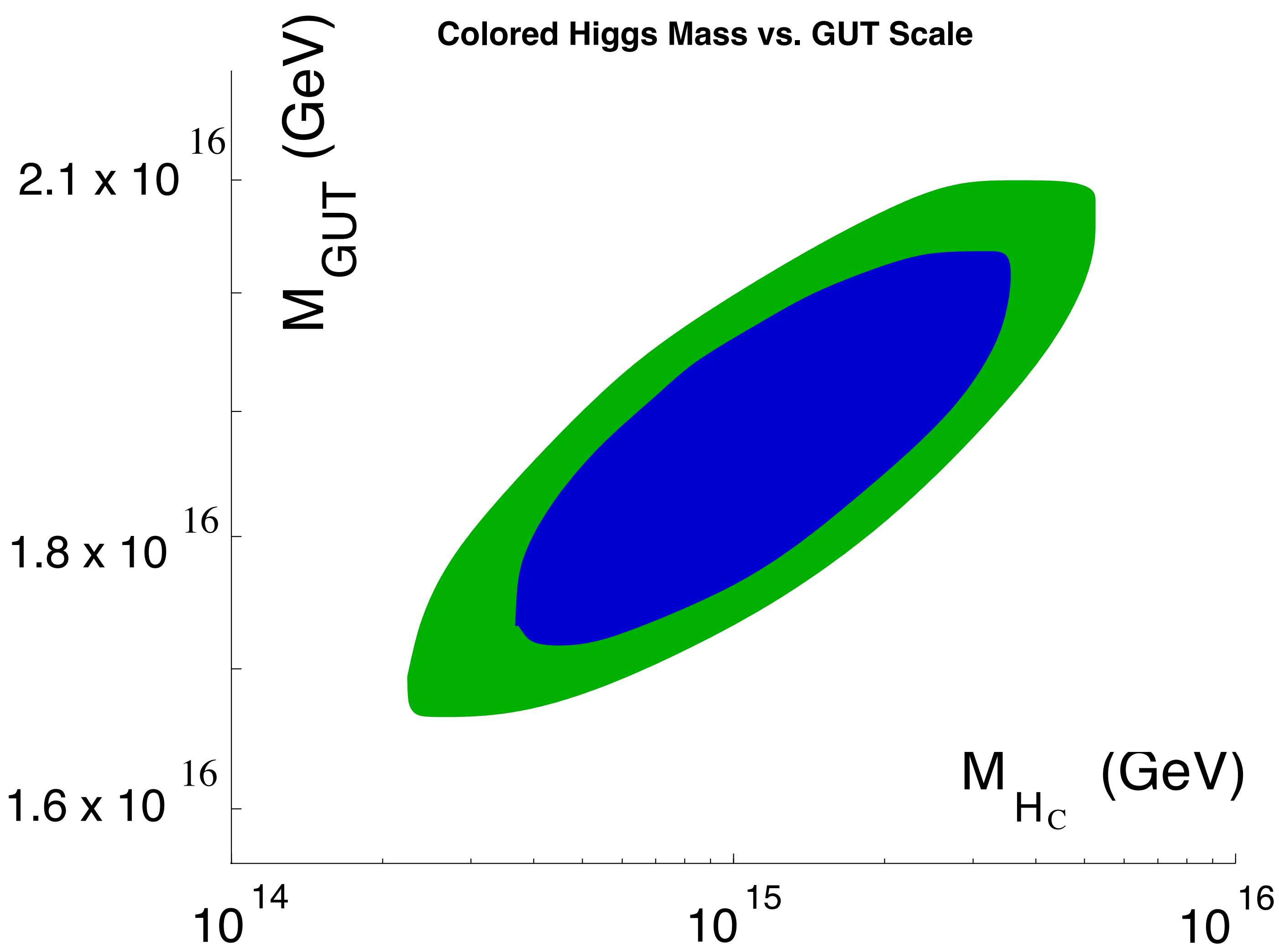


FIG. 2. Plot showing 68% and 90% contours allowed by the renormalization group analysis for the color Higgs triplet mass, M_{H_C}, and the GUT scale, $M_{GUT} \equiv (M_\Sigma M_V^2)^{1/3}$.

assumption:

$$M_3/M_2 = \alpha_3/\alpha_2$$

$$m_{\tilde{f}} = 1\,\mathrm{TeV}$$

$$m_{\tilde{t}} \in (400, 800)\,\mathrm{GeV}$$

$$\tan\beta \in (1.8, 4)$$

$$M_2 \in (100, 400)\,\mathrm{GeV}$$

$$\mu \in (100, 1000)\,\mathrm{GeV}$$

GCU condition

$M_G^* = 1.27 \cdot 10^{16}\,\mathrm{GeV}$

$M_s^* = 2.08\,\mathrm{TeV}$

$$T = M_s^* \Omega' \quad \bigcap \quad T' = M_G^* \Omega$$

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MSSM particles

$$T = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T \right] \qquad X_T \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3} \right) \left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5} \right)$$

$$\Omega = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega \right]^{\frac{1}{288}} \qquad X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8} \right) \left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7} \right)$$

GUT particles

$$\ln \Omega' = \sum_\xi \left(\frac{10}{19} b_1^\xi - \frac{24}{19} b_2^\xi + \frac{14}{19} b_3^\xi \right) \ln \left(\frac{m_\xi}{\Lambda} \right)$$

$$\ln \left(\frac{T'}{\Lambda} \right) = \sum_\xi \left(-\frac{5}{228} b_1^\xi - \frac{15}{76} b_2^\xi + \frac{25}{114} b_3^\xi \right) \ln \left(\frac{m_\xi}{\Lambda} \right)$$

Unified Gauge Coupling

$$\frac{2\pi}{\alpha(\Lambda)} = \frac{2\pi}{\alpha_G^*} + C + C'$$

MSSM particles

$$\begin{aligned} C \;=\; & \frac{125}{19}\ln M_3 - \frac{113}{19}\ln M_2 - \frac{40}{19}\ln\mu - \frac{10}{19}\ln m_A \\ & + \sum_{i=1\ldots 3}\left[\frac{79}{114}\ln m_{\tilde{d}_{Ri}} - \frac{10}{19}\ln m_{\tilde{l}_i} - \frac{121}{114}\ln m_{\tilde{q}_i} + \frac{257}{228}\ln m_{\tilde{u}_{Ri}} + \frac{33}{76}\ln m_{\tilde{e}_{Ri}}\right] \end{aligned}$$

GUT particles

$$C' = \sum_{\xi}\left(\frac{165}{76}b_1^{\xi} - \frac{339}{76}b_2^{\xi} + \frac{125}{38}b_3^{\xi}\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$M_G^* = 1.27\cdot 10^{16}\ \mathrm{GeV}$$

$$M_s^* = 2.08\ \mathrm{TeV}$$

$$\alpha_G^{*-1} = 25.5$$

Minimal SU(5)

$$\mathbf{5}_H + \overline{\mathbf{5}}_H \longrightarrow T, \overline{T} \longrightarrow b_i^T = \left(\frac{2}{5}, 0, 1\right)$$

$$\mathbf{24}_\Sigma \longrightarrow \Sigma \longrightarrow b_i^\Sigma = (0, 2, 3)$$

$$\mathbf{24}_V \longrightarrow X, Y \longrightarrow b_i^X = (-10, -6, -4)$$

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$$T' = M_T^{\frac{4}{19}} \left(M_X^2 M_\Sigma\right)^{\frac{5}{19}} \qquad \Omega' = M_T^{\frac{18}{19}} \left(M_X^2 M_\Sigma\right)^{-\frac{6}{19}}$$

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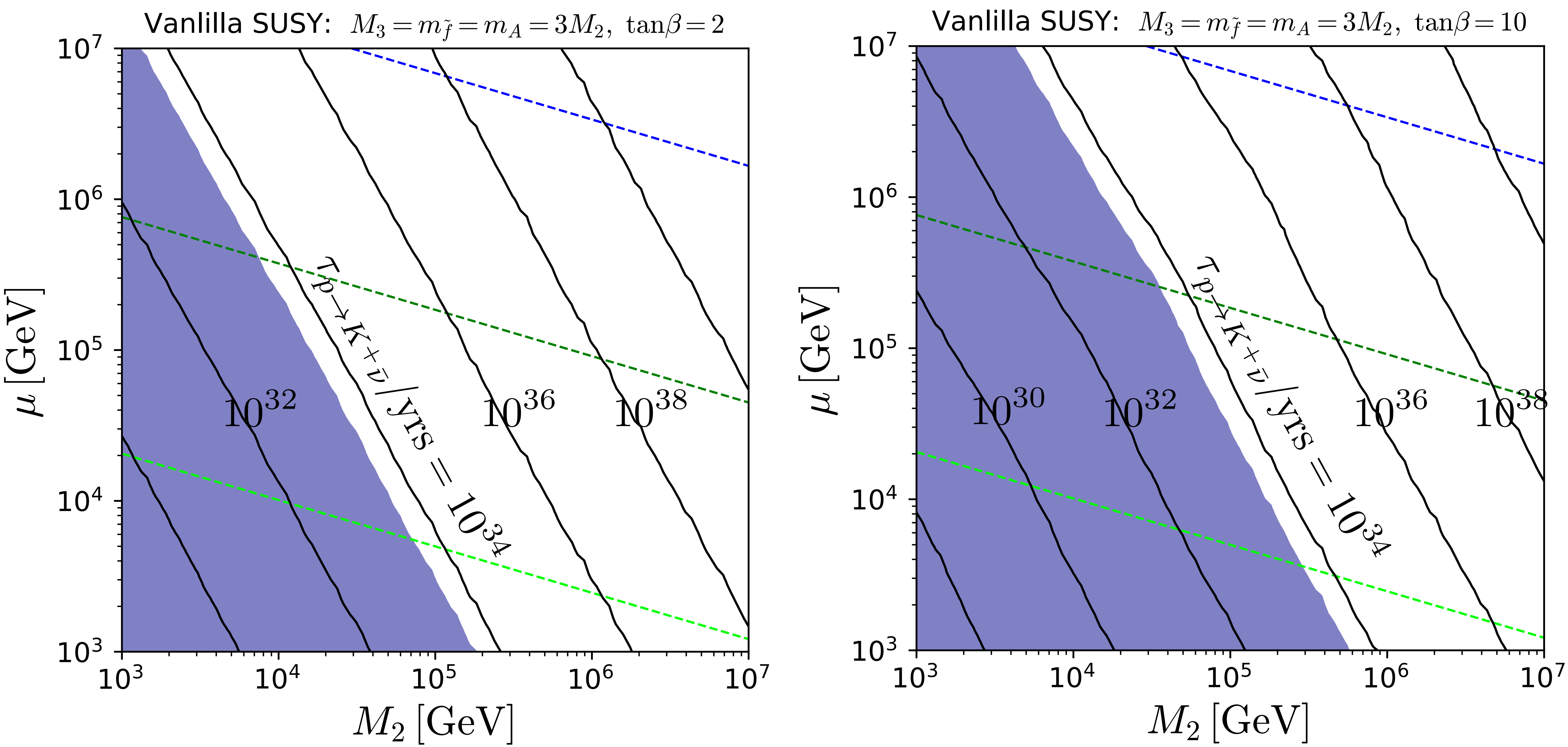
$$T = M_s^* \Omega' \quad \bigcap \quad T' = M_G^* \Omega$$

$$\frac{M_T}{M_G^*} = \left(\frac{T}{M_s^*}\right)^{\frac{5}{6}} \Omega$$

Triplet Higgs mass can be calculated from MSSM spectrum!

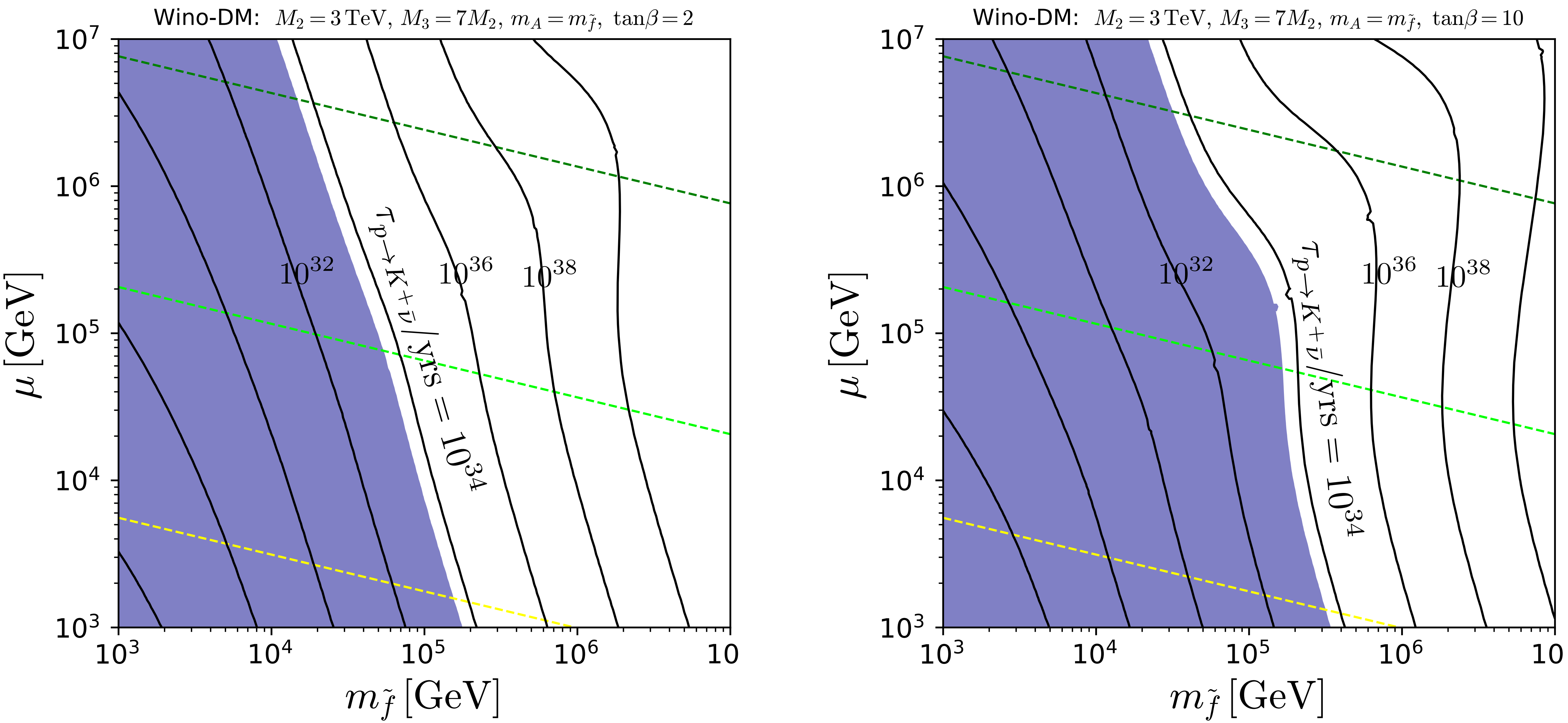
D=5 Proton Decay

Vanlilla SUSY: $M_3 = m_{\tilde{f}} = m_A = 3M_2$



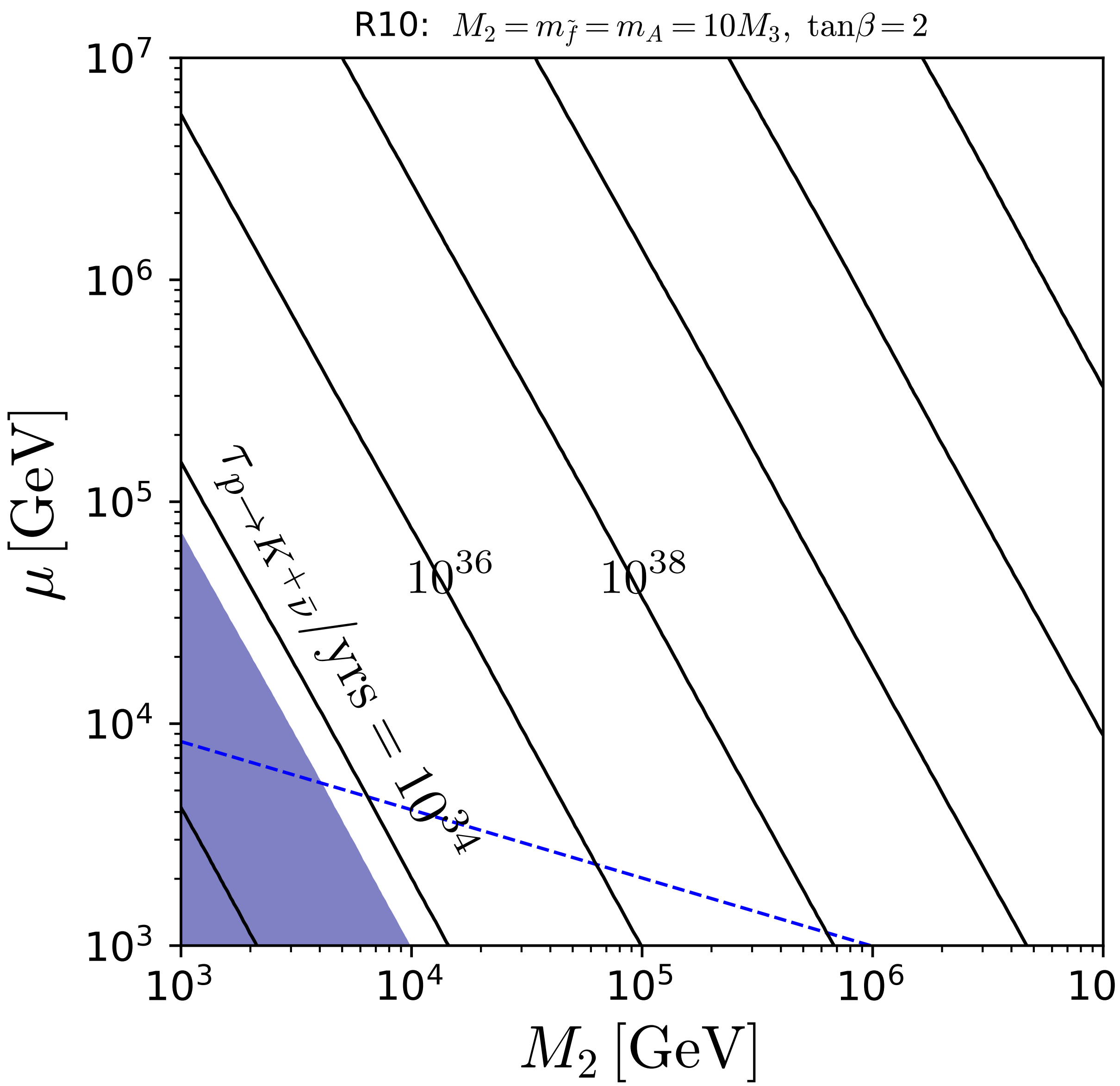
D=5 Proton Decay

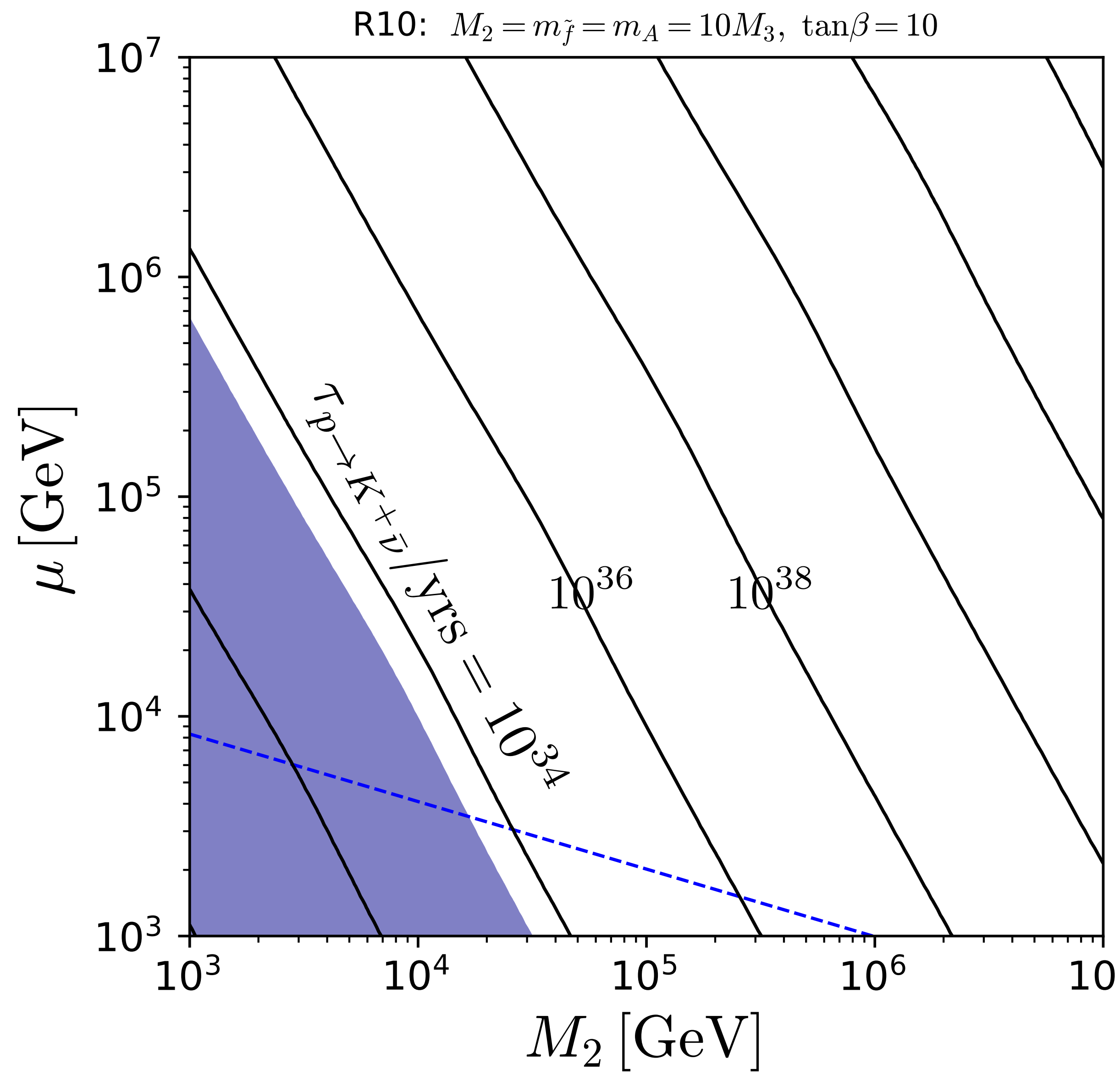
Wino-DM: $M_2 = 3\,\mathrm{TeV},\ M_3 = 7M_2,\ m_A = m_{\tilde{f}}$



D=5 Proton Decay

Non universal gauginos: $M_2 = m_{\tilde{f}} = m_A = 10 M_3$



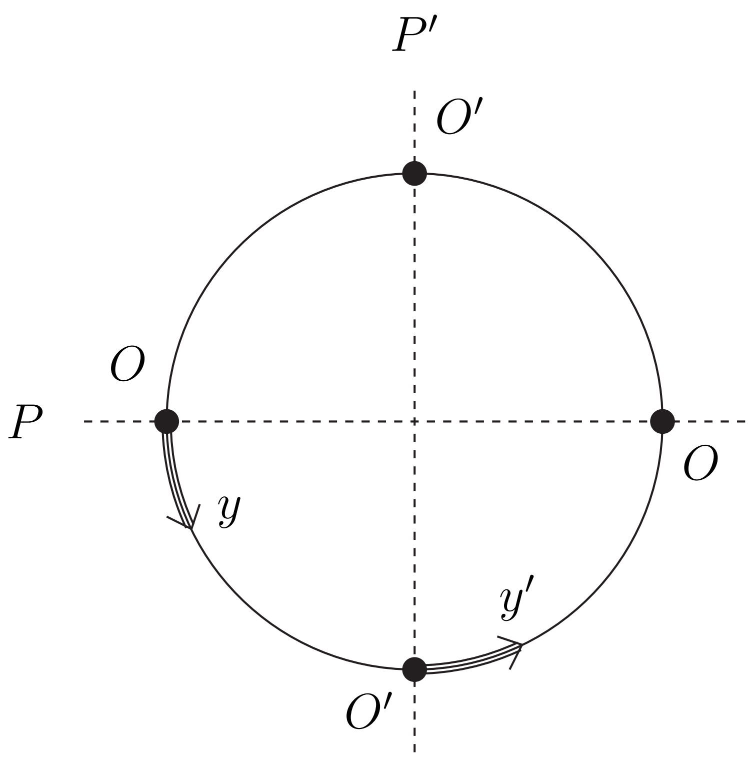


Orbifold SU(5)

[Hall, Nomura ’01]

$S^1/(Z_2 \times Z_2')$ orbifold in the fifth dimension

[Hall, Nomura '01]



two orbifold parities (P, P')

$$\begin{aligned} \phi(x^\mu, y) \to \phi(x^\mu, -y) &= P\phi(x^\mu, y), \\ \phi(x^\mu, y') \to \phi(x^\mu, -y') &= P'\phi(x^\mu, y'), \end{aligned}$$

KK spectrum:

$$\phi_{++}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x^\mu) \cos\frac{2ny}{R},$$

← zero mode + even excitations

$$\phi_{+-}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x^\mu) \cos\frac{(2n+1)y}{R},$$

$$\phi_{-+}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu) \sin\frac{(2n+1)y}{R},$$

← odd excitations

$$\phi_{--}(x^\mu, y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x^\mu) \sin\frac{(2n+2)y}{R},$$

← even excitations

(P, P')	4d $N=1$ superfield	mass
$(+,+)$	$V^a,\ H_F,\ H_{\bar{F}}$	$2n/R$
$(+,-)$	$V^{\hat{a}},\ H_C,\ H^c_C$	$(2n+1)/R$
$(-,+)$	$\Sigma^{\hat{a}},\ H_{\bar{C}},\ H^c_{\bar{C}}$	$(2n+1)/R$
$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\bar{F}}$	$(2n+2)/R$

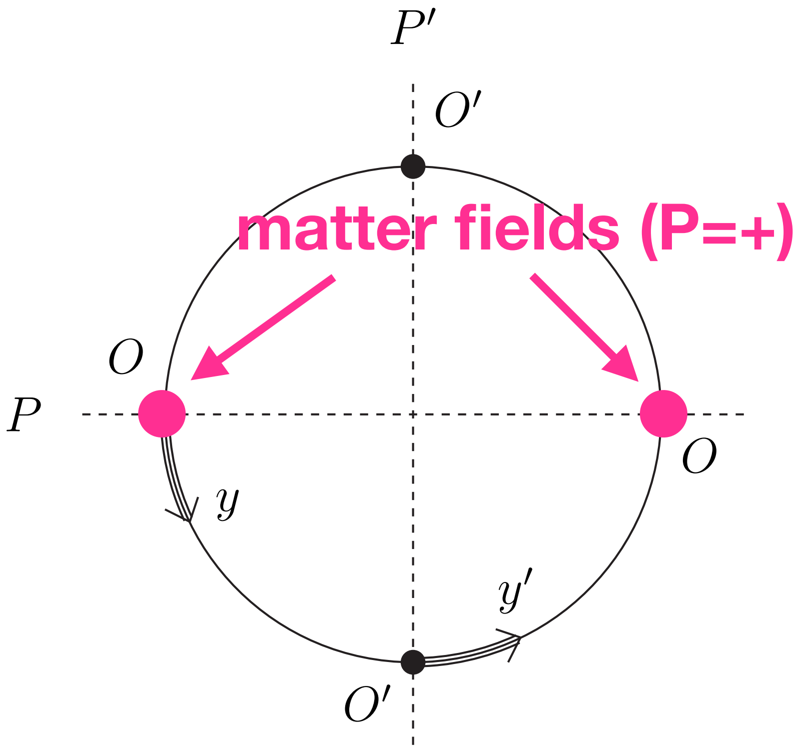
a = unbroken generators

$\hat{a}$ = broken generators

colour triplet Higgs mass terms:

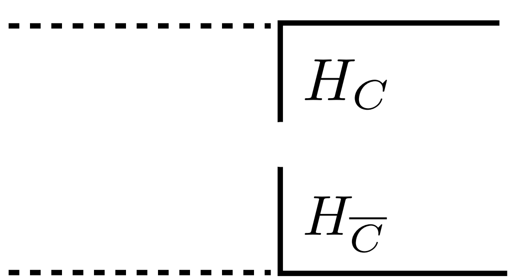
$$H_{\overline{C}} H^c_{\overline{C}} \qquad H_C H^c_C$$

$$\cancel{H^c_C H^c_{\overline{C}}} \qquad \cancel{H_C H_{\overline{C}}}$$



$$P'(Q,U,D,L,E) = \pm(+,-,-,+,-)$$
$$P'(Q,U,D,L,E) = \pm(-,+,-,+,+)$$

$$\mathcal{L}_4 = \sum_{n=0}^{\infty}\int d^2\theta\Bigg[\sqrt{2}y_u\left(\frac{1}{\sqrt{2^{\delta_{n,0}}}}QUH_F^{(2n)} + QQH_C^{(2n+1)} + UEH_C^{(2n+1)}\right) + \sqrt{2}y_d\left(\frac{1}{\sqrt{2^{\delta_{n,0}}}}QDH_{\bar{F}}^{(2n)} + \frac{1}{\sqrt{2^{\delta_{n,0}}}}LEH_{\bar{F}}^{(2n)} + QLH_{\bar{C}}^{(2n+1)} + UDH_{\bar{C}}^{(2n+1)}\right)\Bigg] + \text{h.c.}$$



~~$H_C H_{\overline{C}}$~~

No **D=5 Proton Decay**

$$\text{odd} \quad : \quad m_\xi = (2n+1)M_c, \quad \xi = V^{\hat{a}}, \Sigma^{\hat{a}}, H_T, H_{\bar{T}}, H_T^C, H_{\bar{T}}^C, \quad \sum_\xi b_i^\xi = (-\frac{46}{5}, -6, -2)\,,$$

$$\text{even} \quad : \quad m_\xi = (2n+2)M_c, \quad \xi = V^{a}, \Sigma^{a}, H_F, H_{\bar{F}}, H_F^C, H_{\bar{F}}^C, \quad \sum_\xi b_i^\xi = (\frac{6}{5}, -2, -6)\,,$$

$$r \equiv \Lambda/M_c = \Lambda R > 1$$

$$\ln \Omega' = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right) \ln\left(\frac{m_\xi}{m_Z}\right)$$

$$\ln\left(\frac{T'}{m_Z}\right) = \sum_\xi \left(-\frac{5}{228}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right) \ln\left(\frac{m_\xi}{m_Z}\right)$$

$$\tilde{\Omega}' = \left[\frac{\prod_n (2n+1)(M_c/\Lambda)}{\prod_n (2n+2)(M_c/\Lambda)}\right]^{\frac{24}{19}} = \left[\frac{1}{r}\right]^{\frac{24}{19}}, \left[\frac{1}{2}\right]^{\frac{24}{19}}, \left[\frac{3}{2}\frac{1}{r}\right]^{\frac{24}{19}}, \left[\frac{3}{2\cdot 4}\right]^{\frac{24}{19}}, \cdots,$$

$$\frac{\tilde{T}'}{\Lambda} = \left[\frac{\prod_n (2n+1)(M_c/\Lambda)}{\prod_n (2n+2)(M_c/\Lambda)}\right]^{\frac{18}{19}} = \left[\frac{1}{r}\right]^{\frac{18}{19}}, \left[\frac{1}{2}\right]^{\frac{18}{19}}, \left[\frac{3}{2}\frac{1}{r}\right]^{\frac{18}{19}}, \left[\frac{3}{2\cdot 4}\right]^{\frac{18}{19}}, \cdots,$$

$$\frac{\tilde{T}'}{\Lambda} = (\tilde{\Omega}')^{\frac{7}{3}}$$

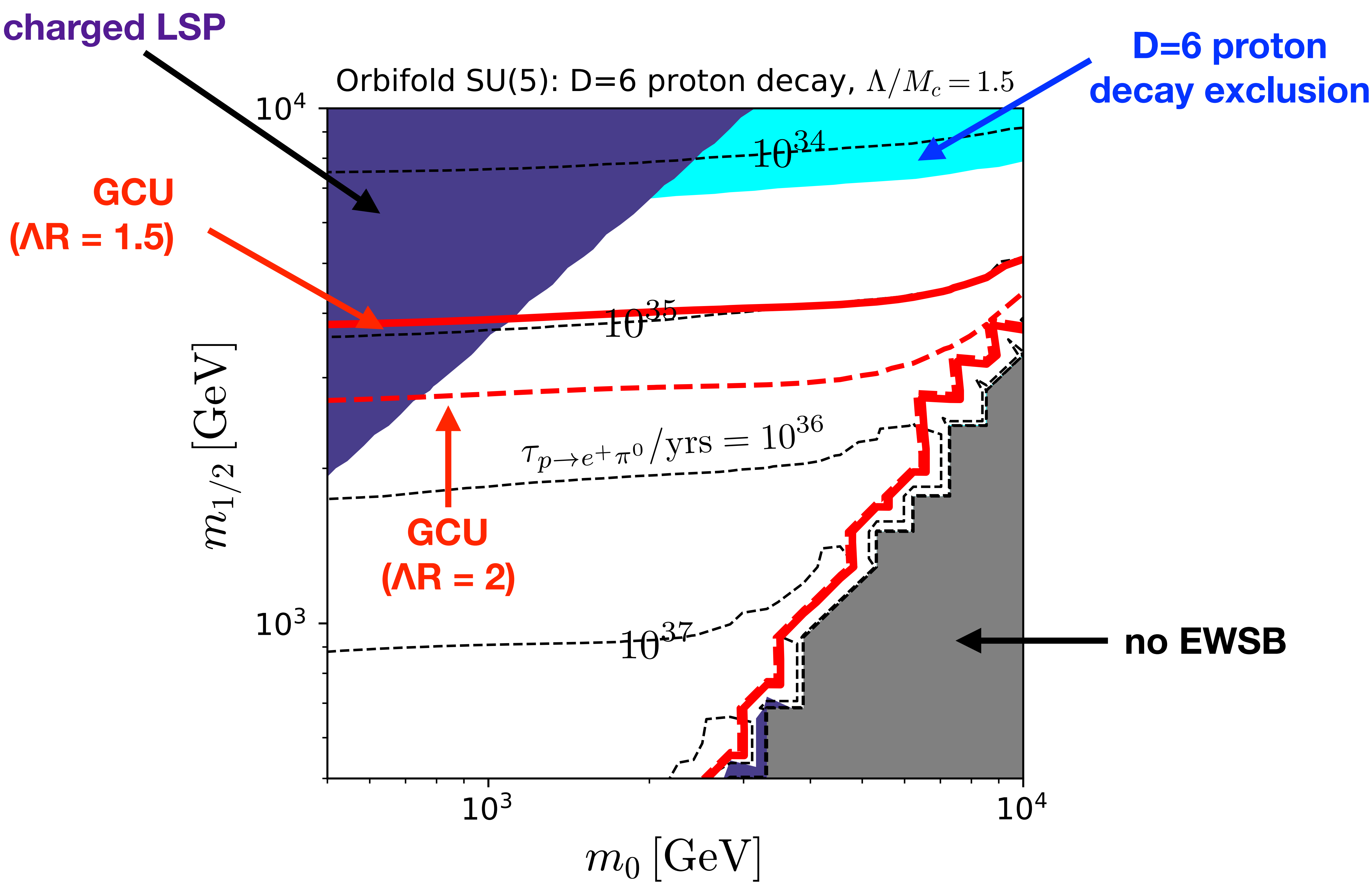
$$T = M_s^* \Omega' \quad \cap \quad T' = M_G^* \Omega$$

$$M_c = \frac{M_G^*}{r} \Omega \left(\frac{T}{M_s^*}\right)^{-\frac{7}{3}}$$

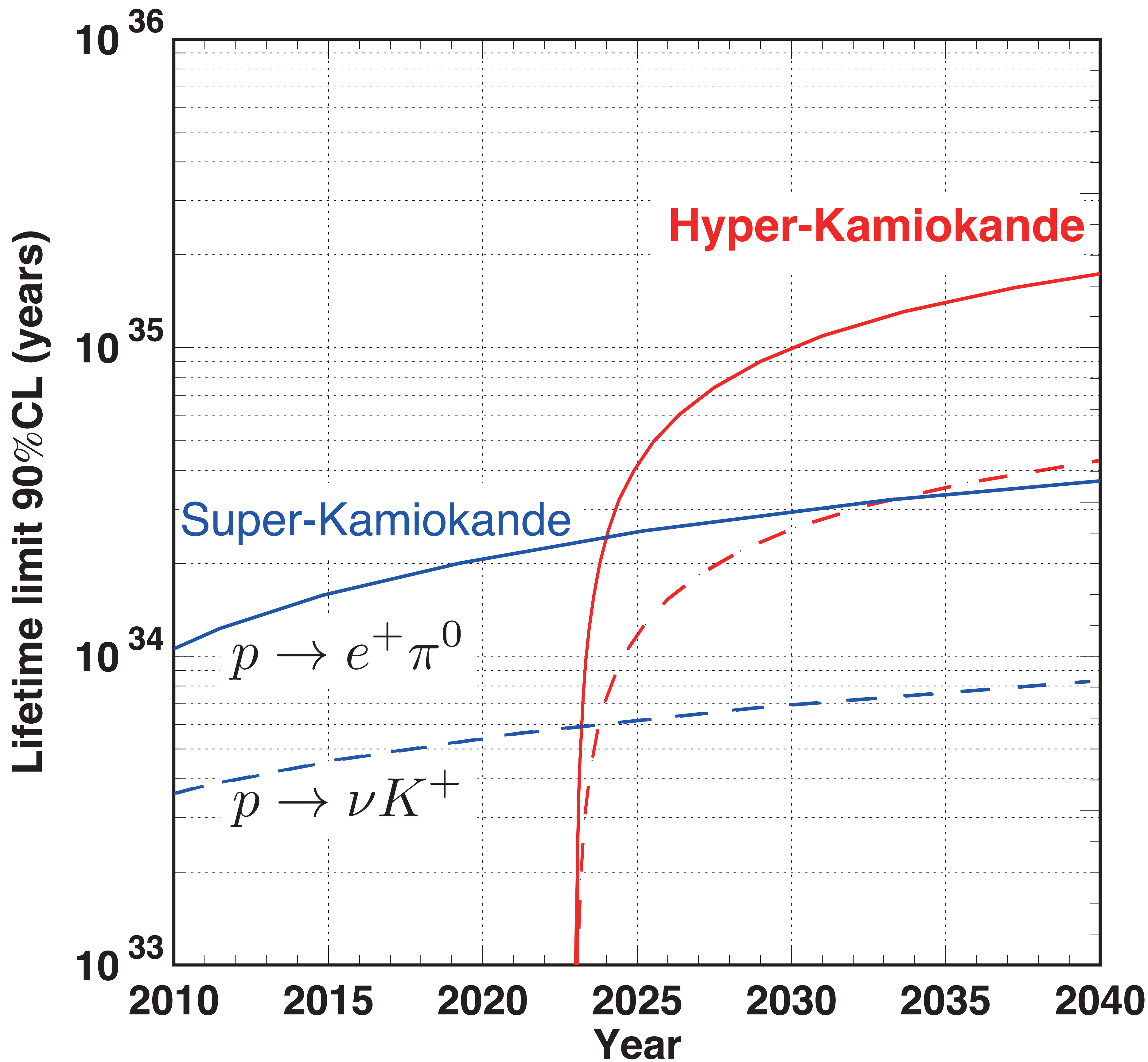
constraint on SUSY masses

X,Y boson mass => D=6 proton decay

CMSSM

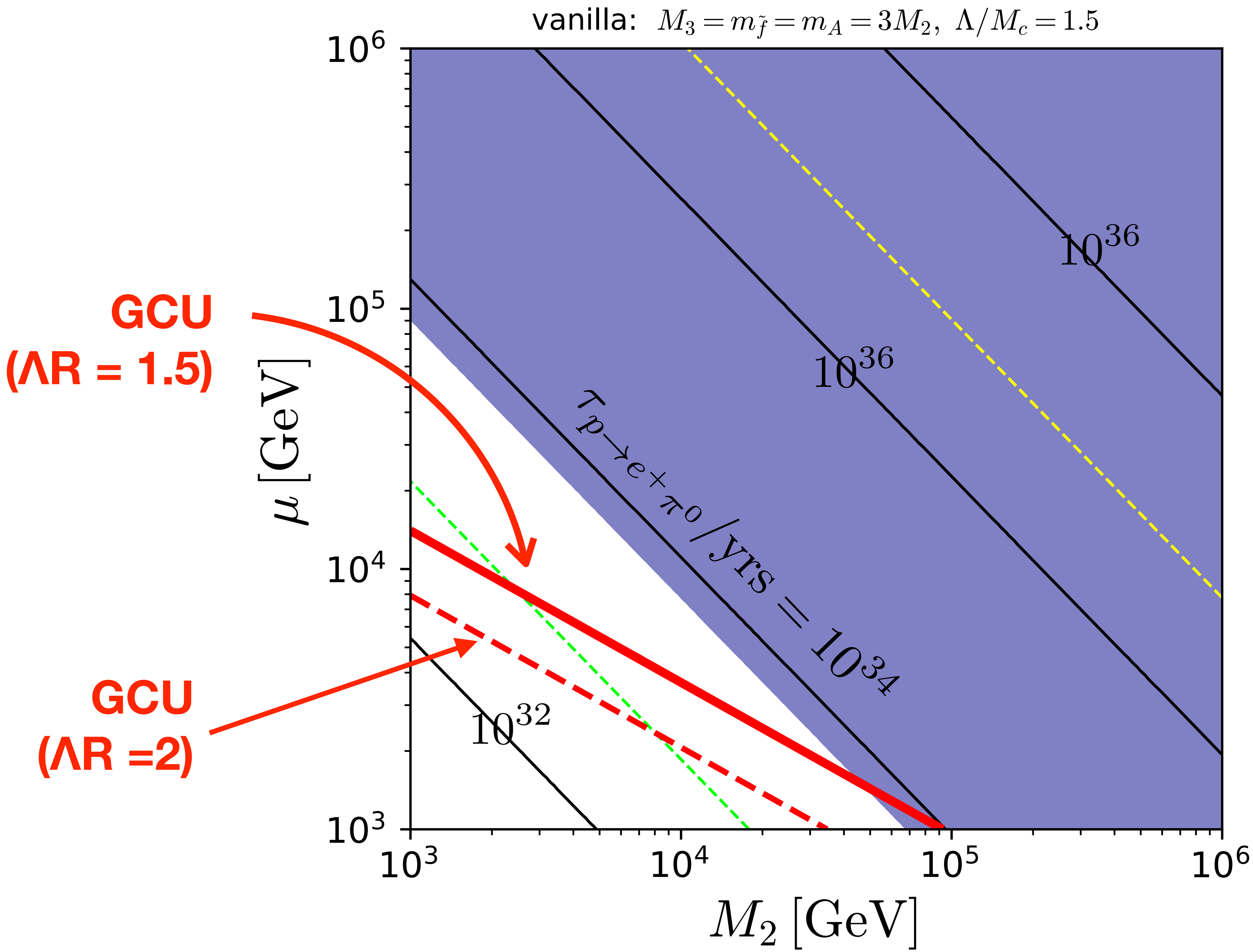


charged LSP
D=6 proton decay exclusion
GCU (ΛR = 1.5)
GCU (ΛR = 2)
no EWSB
Orbifold SU(5): D=6 proton decay, $\Lambda/M_c = 1.5$
10^{34}
10^{35}
$\tau_{p\to e^+\pi^0}/\mathrm{yrs} = 10^{36}$
10^{37}
10^4
10^3
$m_{1/2}\,[\mathrm{GeV}]$
$m_0\,[\mathrm{GeV}]$

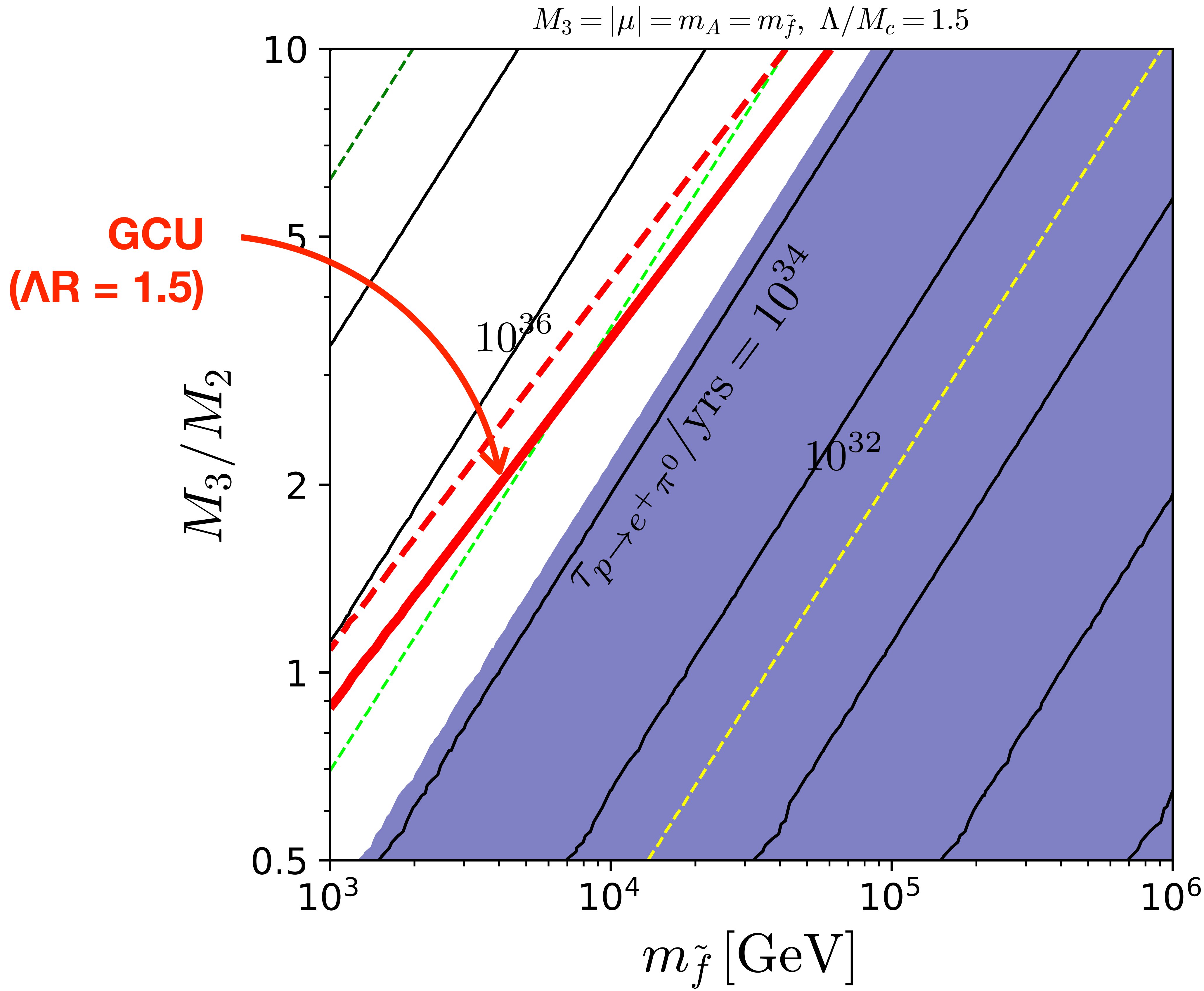


Hyper-Kamiokande
Super-Kamiokande
$p \to e^+\pi^0$
$p \to \nu K^+$
Lifetime limit 90%CL (years)
10^{36}
10^{35}
10^{34}
10^{33}
2010
2015
2020
2025
2030
2035
2040
Year

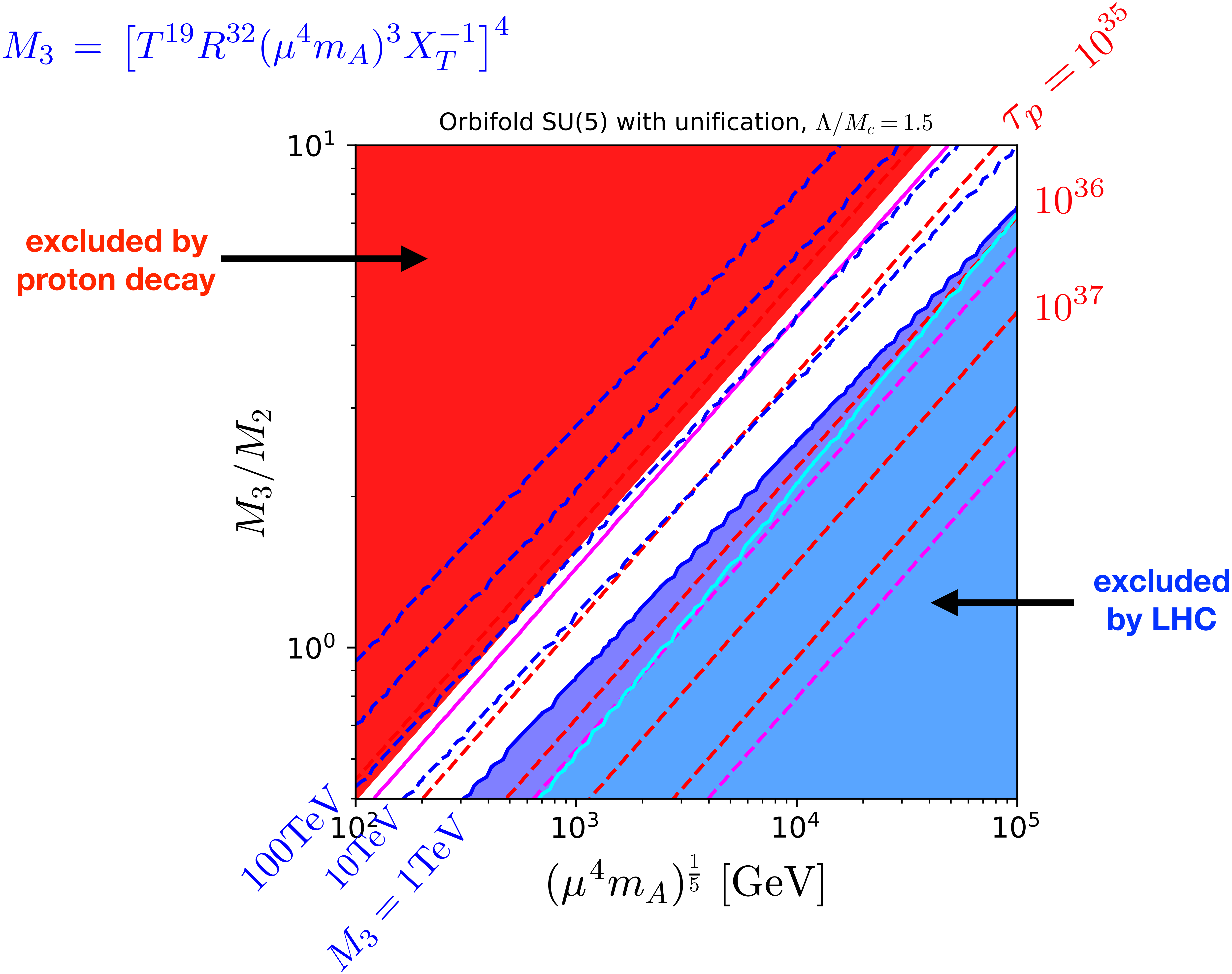
vanilla: $M_3 = m_{\tilde{f}} = m_A = 3M_2,\ \Lambda/M_c = 1.5$

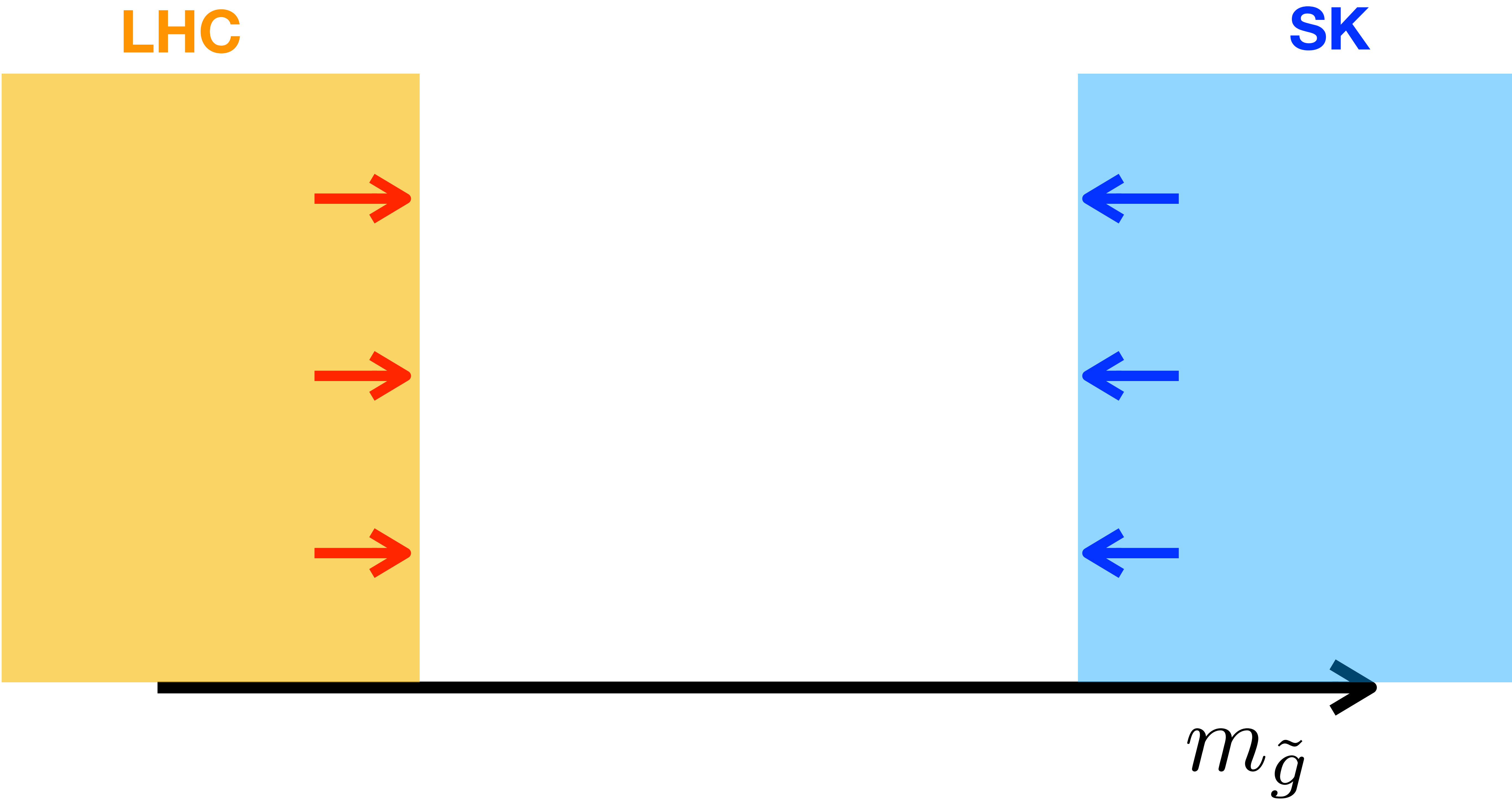


$M_3 = |\mu| = m_A = m_{\tilde{f}},\ \Lambda/M_c = 1.5$

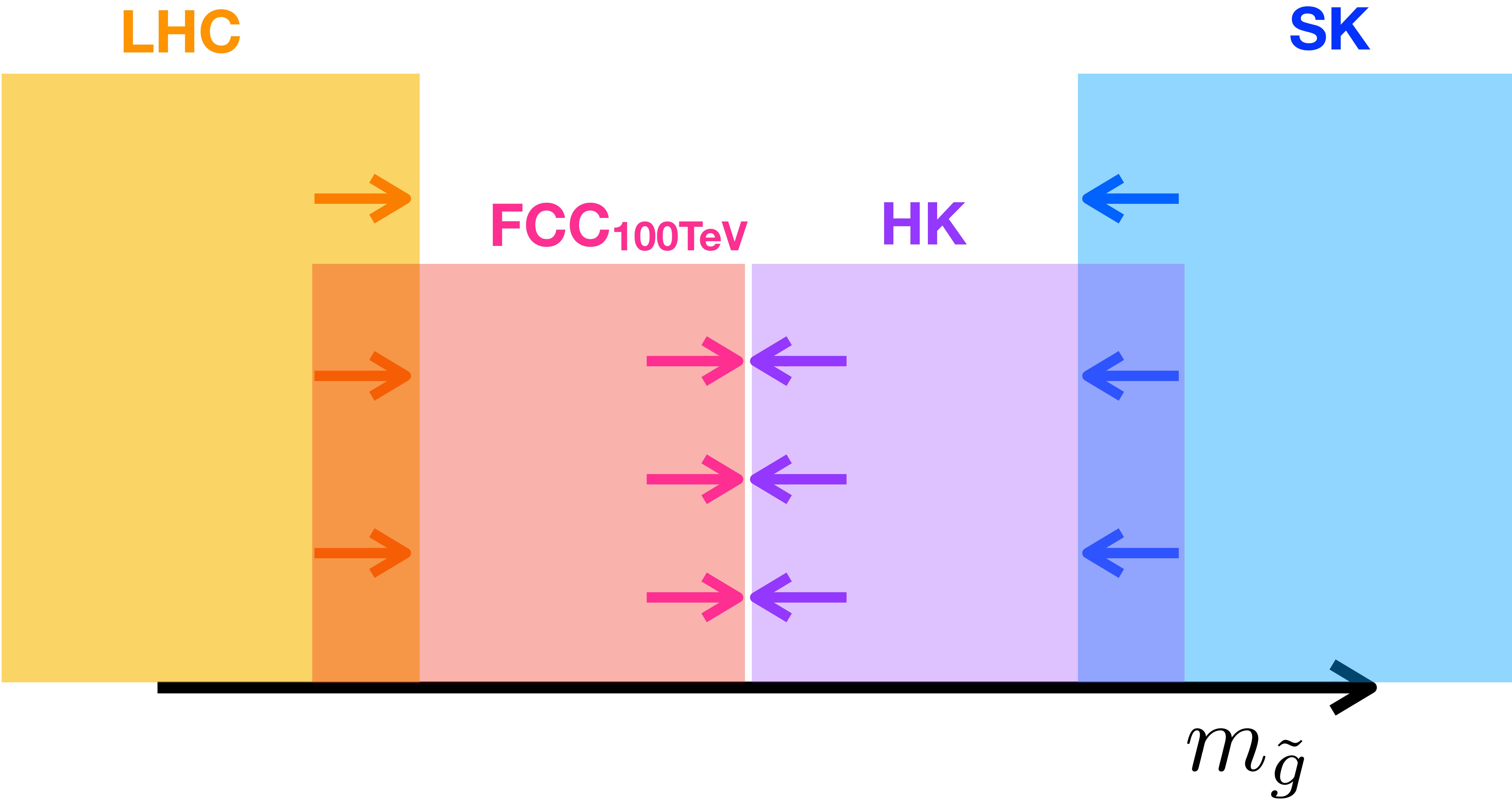


$M_3 = \left[T^{19} R^{32} (\mu^4 m_A)^3 X_T^{-1}\right]^4$





LHC
SK
$m_{\tilde{g}}$



LHC
SK
FCC100TeV
HK
$m_{\tilde{g}}$

Conclusions

- SUSY naturalness has been questioned by LHC results and it may be good time to think/study low energy SUSY from different perspective, for example from GCU
- We have developed a formalism to study GUT models for a given MSSM spectrum and applied it to minimal SU(5) and orbifold SU(5) GUT.

Future works

- Wilson line breaking SU(5)
- Minimal SO(10) [Aulakh, Girdhar ’05]
- Realistic SU(5) [Altarelli, Feruglio, Masina ’00]
- …

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