Singlet Scalar Dark Matter

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Mini-workshop in Warsaw

Work in progress

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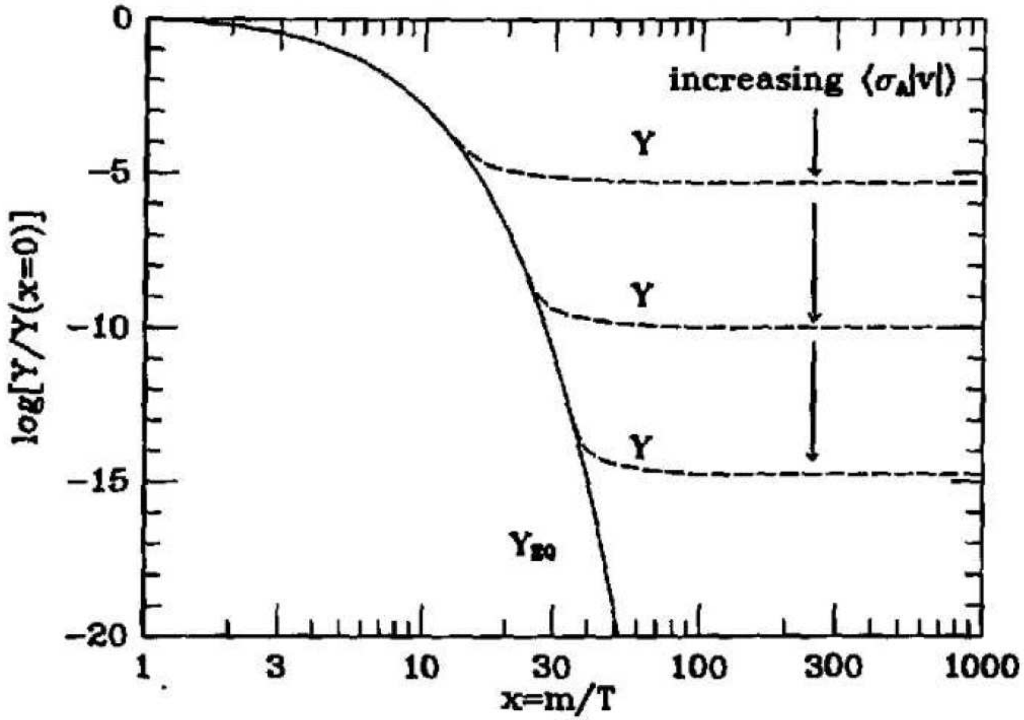
Dark matter

- DM exists in the universe.
- 26% of energy density.
- DM candidates: WIMP, axion, PBH etc.
- Boltzmann equation for WIMP:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm eq}^2\right)$$

where $Y = n/s, \;\; \Gamma_{\rm ann} \equiv \langle \sigma v \rangle n_{\rm eq}$



- Thermal eq. with SM particles $\rightarrow$ freeze-out $\rightarrow$ predictive (do not depend on what happened in early universe.) $x_f = m/T_f \sim 20$ (non-relativstic)

Singlet Scalar DM

Scalar potential

$$\mathcal{V} = \mu_H^2 |H|^2 + \frac{\mu_S^2}{2} S^2 + \frac{\lambda_H}{4} |H|^4 + \frac{\lambda_{HS}}{2} |H|^2 S^2 + \frac{\lambda_S}{4!} S^4$$

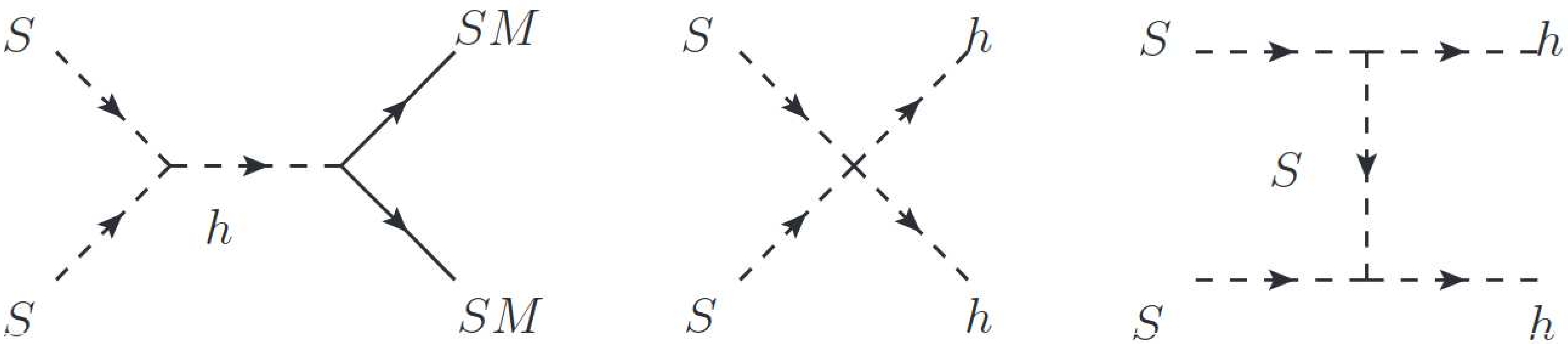
- $\mathbb{Z}_2$ symmetry is impsoed. $\rightarrow$ simplest benchmark model of DM
- 3 parameters: $m_S = \sqrt{\mu_S^2 + \lambda_{HS}\langle H \rangle^2}$, λ_{HS}, λ_S
  Higgs portal coupling λ_{HS}

- If λ_{HS} is not too small ($\lambda_{HS} \gtrsim 10^{-6}$), DM is thermalized with SM.
  $\rightarrow$ compare $\Gamma_{HS} \equiv \langle \sigma_{HS \to HS} v \rangle n_H^{\text{eq}}$ and H (Hubble parameter).

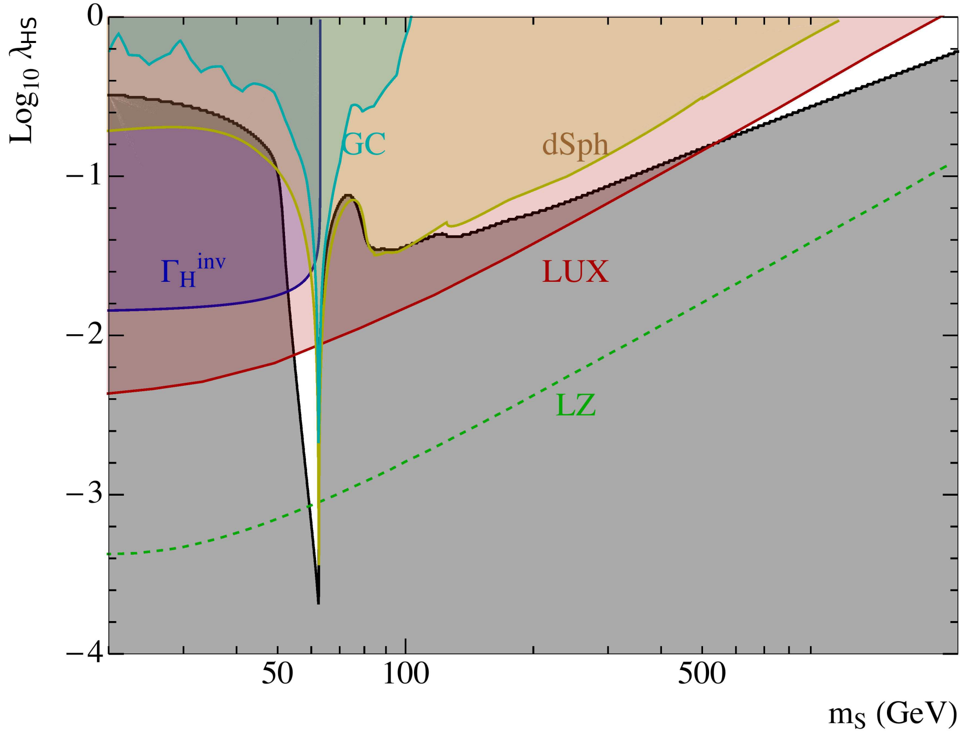
if	$\Gamma_{HS} > H$	$\rightarrow$	thermalized
if	$\Gamma_{HS} < H$	$\rightarrow$	decoupled $(T' \neq T)$

Thermalized case (freeze-out)

J. A. Casas et al., arxiv:1701.08134



- λ_S is irrelevant. $\rightarrow$ 2 parameters
- Strongly constrained by direct, indirect and collider searches.
- Resonance region and $m_S \gtrsim 500$ GeV are still alive.
- Most of parameter space can be explored by LZ future experiment.



Non-thermalized case

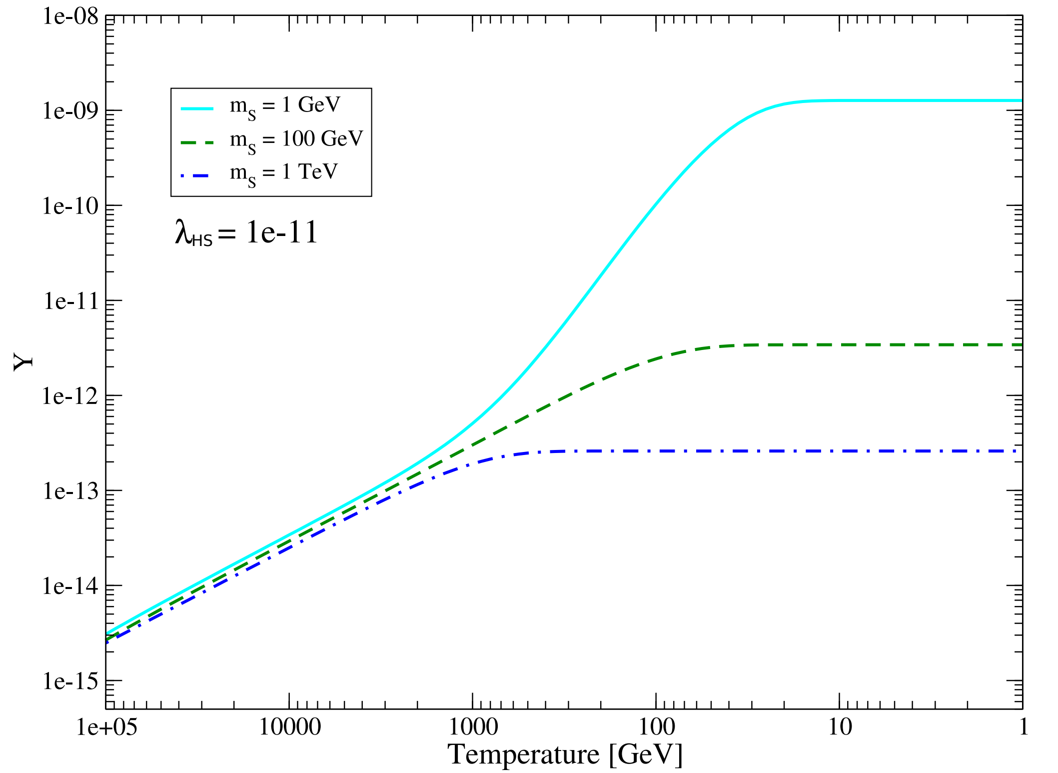
- But thermalized case is not unique option
- If DM is not thermalized with SM, DM may be produced by freeze-in mechanism or freeze-in + self-interaction.

Freeze-in mechanism

- Boltzmann equation

$$\frac{dY}{dT} = \sqrt{\frac{\pi g_*(T)}{45}} m_{\rm pl} \langle \sigma_{SS\to {\rm SM}} v \rangle Y_{S{\rm eq}}^2(T)$$

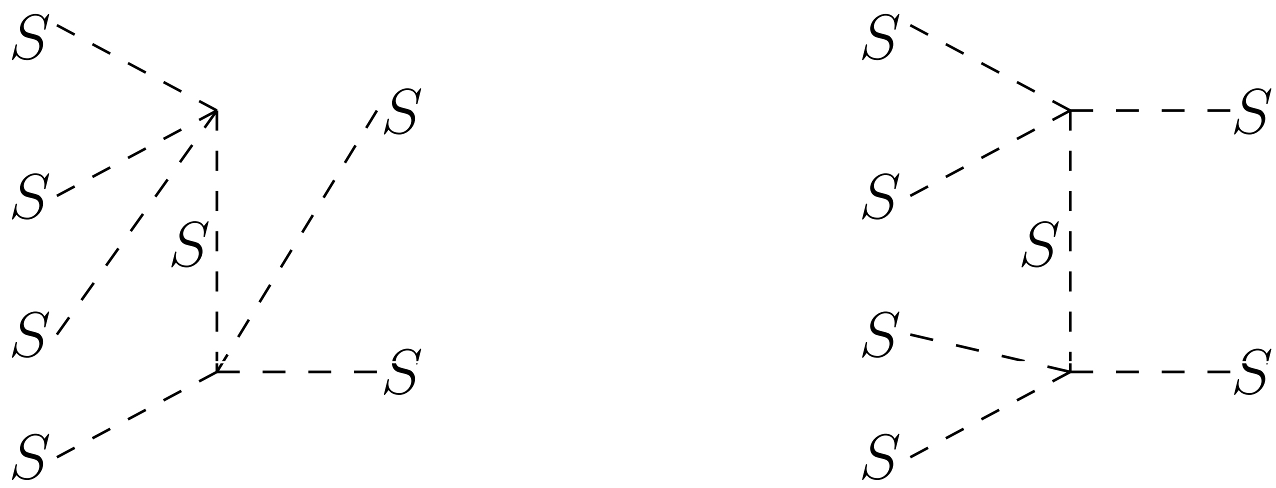
- Initial numder density: $Y = 0$.
- slowly produced by inverse annihilation (SM particles $\to SS$)
  Y is fixed when $T \sim m_S$,
  or when $WW \to SS$ becomes inefficient (T ~30 GeV).



Freeze-in + self-interaction

N. Bernal and X. Chu, arxiv:1510.08527

- If λ_S is sizable, $SSSS \leftrightarrow SS$ process may be relevant.



$\rightarrow$ DM numbder density changes.

- $\Gamma_{4\to 2} \equiv \langle \sigma_{4\to 2} v^3 \rangle n_S > H$
- All computations have been done with non-relativstic approximation.
  - · BE distribution $\rightarrow$ MB distribution
  - · formula of thermal averaged cross sections

Freeze-in + self-interaction

N. Bernal and X. Chu, arxiv:1510.08527

· DM is thermalized in dark sector if $\Gamma_{4\to 2} > H$.

Boltzmann equations

$$\frac{dn_S}{dt} + 3Hn_S = -\langle \sigma v^3 \rangle \left(n_S^4 - n_S^2 n_S^{\mathrm{eq}\,2} \right)$$

Assumptions:

- Quantum statistics is neglected
  $\to$ Assume always Boltzmann statistics
- Initial condition: $n_S = 0$, $\rho_S = 0$.

$\to$ DM abundance is determined by freeze-out temperature ratio.

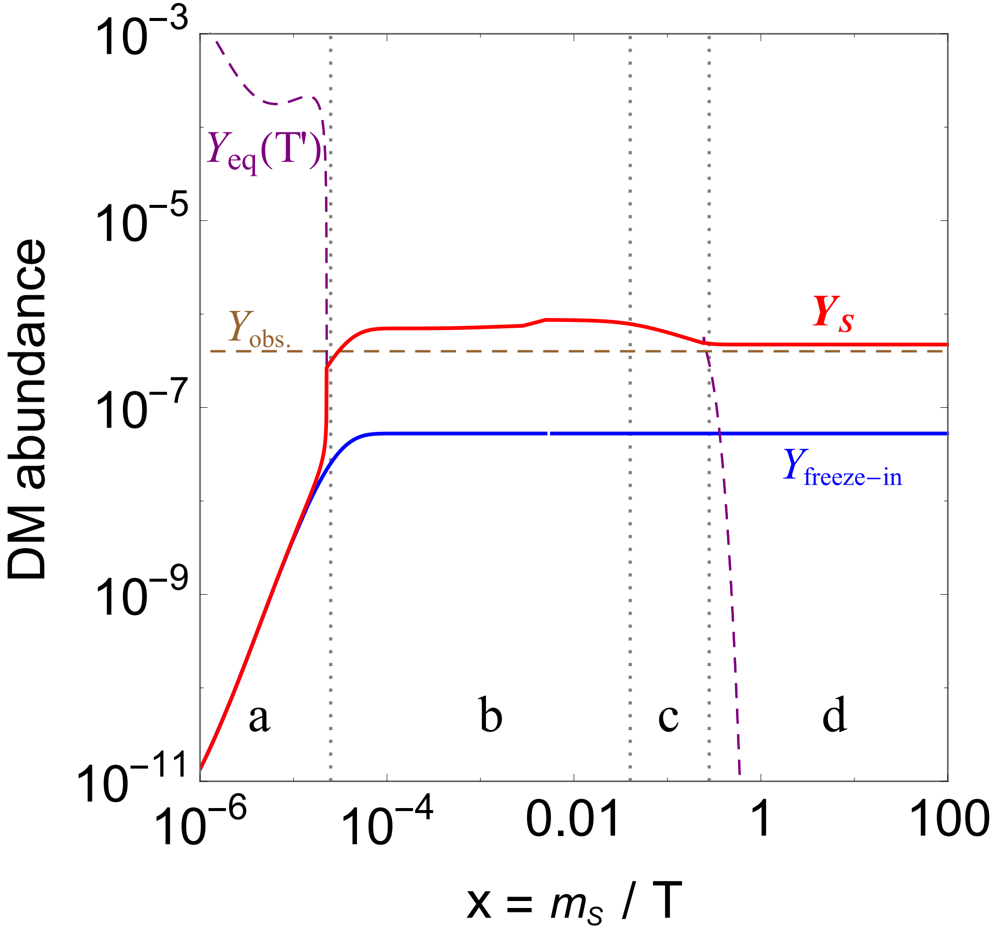
$$\langle \sigma_{4\to 2} v^3 \rangle = \left(\frac{6 \times 10^{10}}{\mathrm{GeV}^8} \right) x_f^7 \, g_*(x_f)^{-3/2} \left(\frac{\mathrm{GeV}}{m_S} \right)^4$$

$$x_f' = 17.4 + \log \left[\left(\frac{x_f}{x_f'} \right)^3 \left(\frac{m_S}{10\ \mathrm{MeV}} \right) \left(\frac{10}{g_*(x_f)} \right) \left(\frac{x_f'}{17.4} \right)^{3/2} \right]$$

Freeze-in + self-interaction

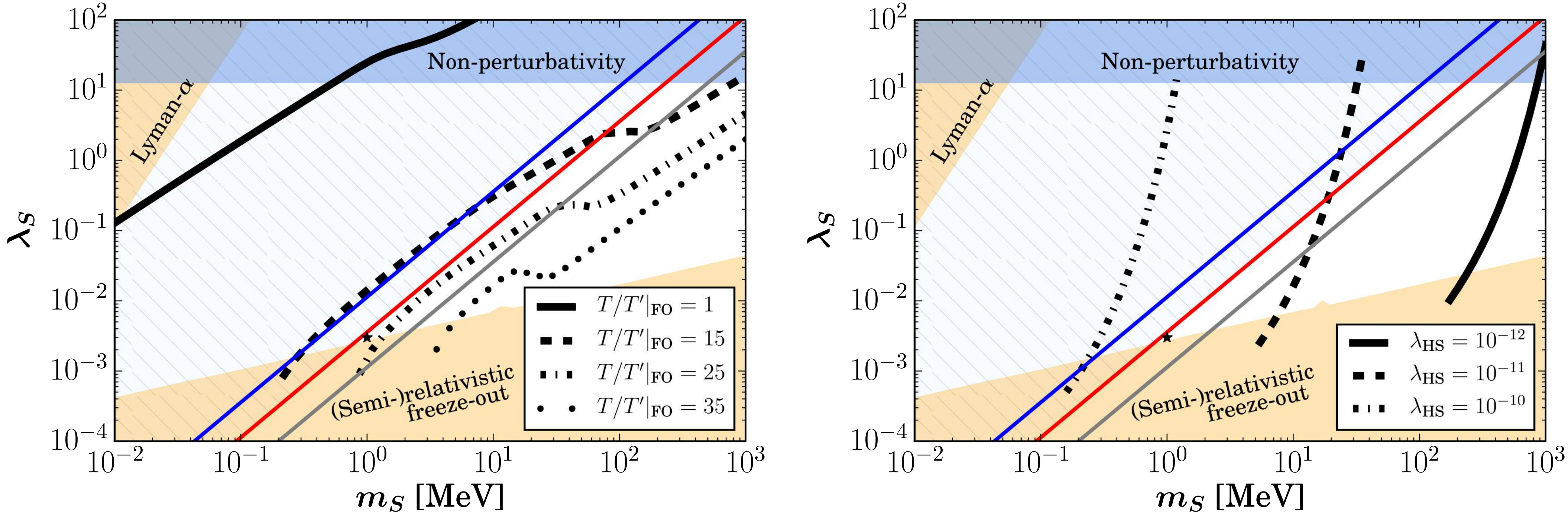
N. Bernal and X. Chu, arxiv:1510.08527

Evolution of DM number density



- Energy injection from SM sector to DM sector due to $h \to SS$.
- $SS \to SSSS$ enters in dark thermal bath. $(\Gamma_{2\to 4} > H)$
  $\to n_S$ rapidly increases, T' decreases.
- DM is in dark thermal bath $\to n_S = n_S^{eq}$.
- When DM is non-relativtsic, freeze-out occurs in DM sector as same as WIMP case. But different temperature $T' \neq T$

Parameter space



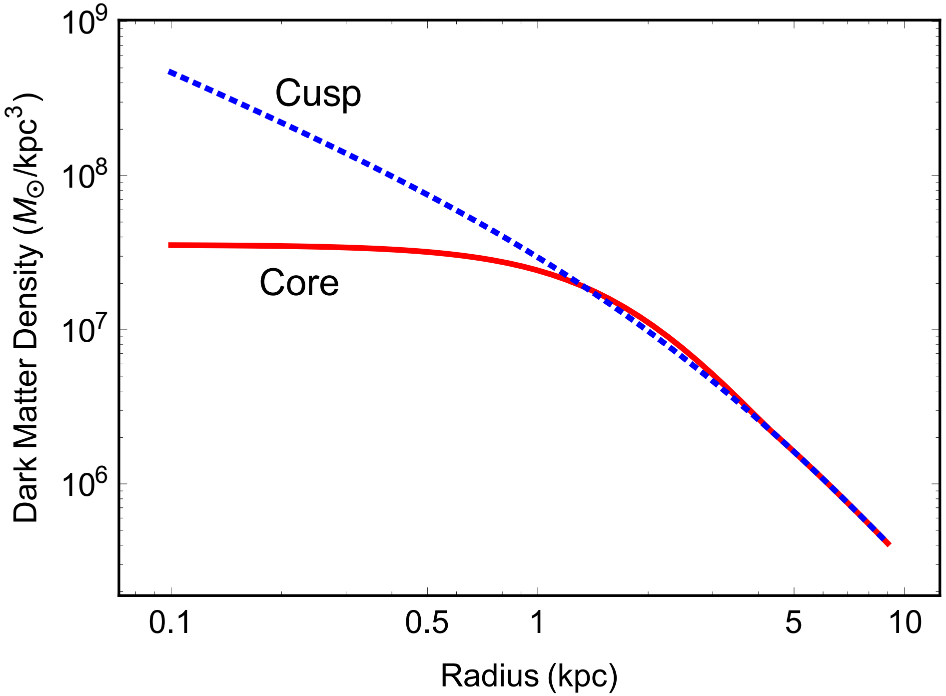
- Constraint of Bullet cluster $\sigma_{\rm self}/m_S \lesssim \mathcal{O}(1)\ {\rm cm}^2/{\rm g}$
- Small scale structure problems $\rightarrow \sigma_{\rm self}/m_S \sim 0.1 - 1\ {\rm cm}^2/{\rm g}$
- Calculated only when DM is non-relativistic.

Small scale problems

- Cusp vs core problem
  N-body simulation infers cusp DM profile at centre of galaxies $\rho_{\rm DM} \propto r^{-1}$.
  But rotation of spiral galaxies prefers core profile $\rho_{\rm DM} \sim$ const.

- Missing satellite problem
  etc

  arXiv:1705.02358, Tulin and Yu



Possible solutions

- Add baryon contribution
- DM self-interaction

We are interested in the following case

to be more natural and extend the parameter space

1. DM is initially produced by inflaton decay etc.
   - previous case: $T' \leftrightarrow \lambda_{HS} \quad \Rightarrow \quad T' \leftrightarrow$ inflation model
   - natural to consider inflaton couples with SM and DM

2. DM abundance is determined at semi-relativistic decoupling. One has to solve the Boltzmann equation numerically.
   $\longrightarrow$ challenging task
   - quantum statistics is not neglected.
   - multi-dimensional integration (in particular $4 \to 2$ annihilation)

integrated Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = \gamma_{2\to4} - \gamma_{4\to2} \quad \to \quad -\langle\sigma_{4\to2}v\rangle\left(n^4 - n^2 n_{\rm eq}^2\right)$$

if non-rela.

Kinetic eq. and chemical eq.

- If $SS \to SS$ scattering is faster than $H \to$ Kinetic eq.

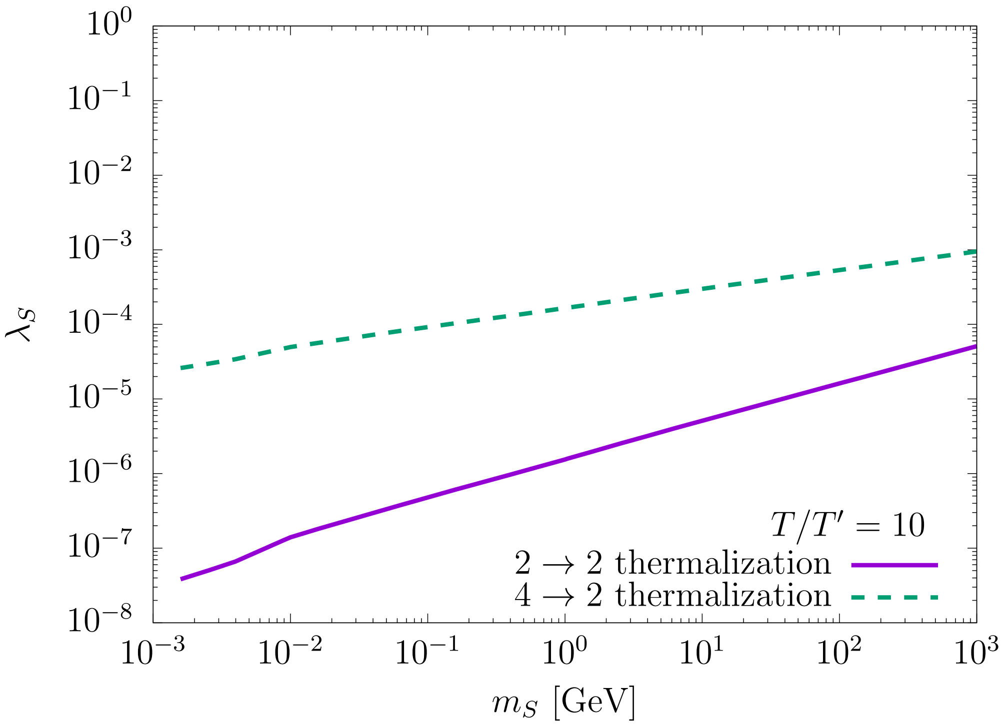
$$\Gamma_{2\to 2} > H$$

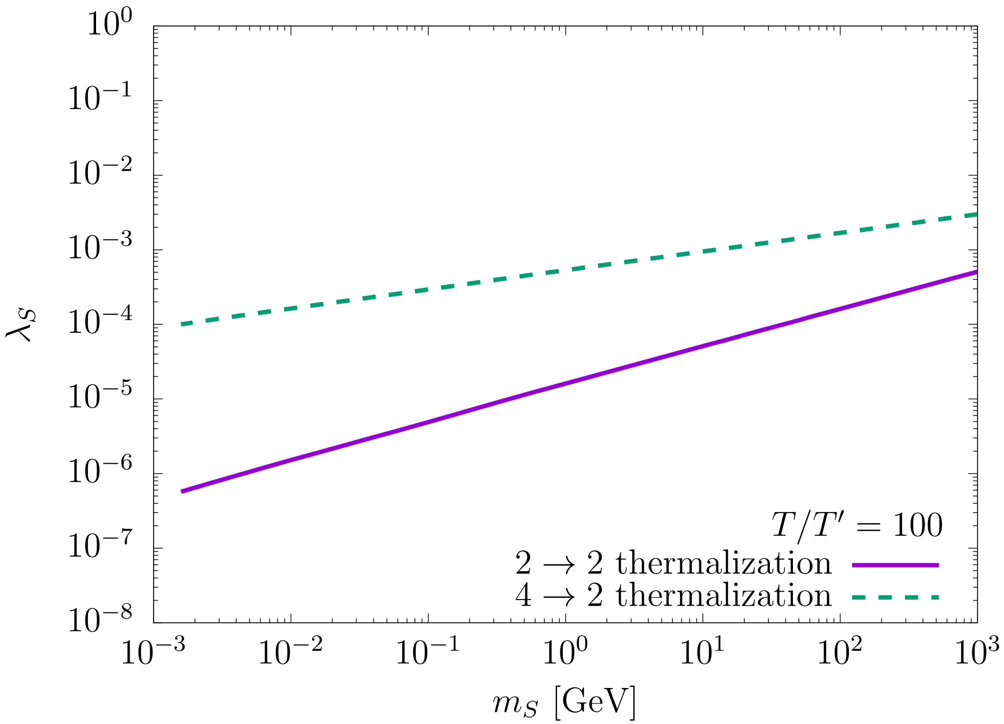
$\to$ phase space distribution function $\quad f = \dfrac{1}{e^{(E-\mu)/T} - 1}$

where μ is "effective" chemical potential

- If $SS \to SSSS$ process is faster than $H \to$ chemical eq. $(\mu = 0)$
- "effective" chemical potential can be understood as deviation from phase space distribution function in thermal eq.
- Reasonable ansatz: $\mu = \begin{cases} 0 & \text{for chemical eq.} \\ m_S\left(1 - \frac{T'}{T'_f}\right) & \text{for chemical dec.} \end{cases}$

Kinetic eq. and chemical eq.





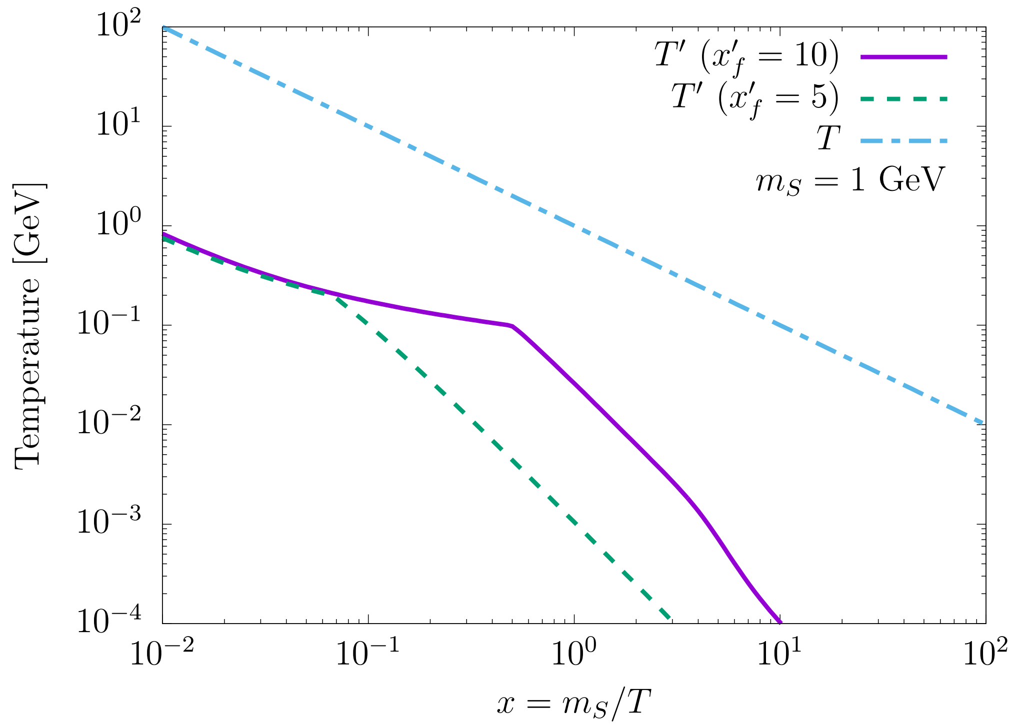
- Compare $\Gamma_{2\to2}$, $\Gamma_{4\to2}$ and H at $T' = 3m_S$
- $\lambda_S \gtrsim 10^{-4}$ for $4 \to 2$ thermalization (chemical eq.)
- $\lambda_S \gtrsim 10^{-8}$ for $2 \to 2$ thermalization (kinetic eq.)

Temperature evolution

Carlson, Machacek, Hall, Astrophys. J. 398 (1992) 43-52

- If each sector is thermalized separately, total entropy in each sector is conserved
  $S_{\rm SM} = s_{\rm SM} a^3 = \text{const}$ and
  $S_{\rm DM} = s_{\rm DM} a^3 = \text{const}$
- Ratio of the entropy densities is also conserved.

$$\frac{s_{\rm SM}(T)}{s_{\rm DM}(T')} = \text{const.}$$



- If the sector is thermalized, $s = \dfrac{\rho + p - \mu n}{T} \to s_{\rm SM} = \dfrac{2\pi^2}{45} g_{*S}(T) T^3$
- Evolution of dark temperature can be followed.

Full integrated Boltzmann equation

$$\frac{dn}{dt} + 3Hn = \gamma_{2\to 4} - \gamma_{4\to 2} \quad \to \quad -\langle \sigma_{4\to 2} v_{\text{Møl}} \rangle \left(n^4 - n^2 n_{\text{eq}}^2 \right)$$

if non-rela.

$$\begin{aligned} \gamma_{2\to 4} &= \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 (2\pi)^4 |\mathcal{M}|^2 f_A f_B \\ &\quad \times (1+f_1)(1+f_2)(1+f_3)(1+f_4) \delta^4 \left(p_A + p_B - k_1 - k_2 - k_3 - k_4 \right) \\ &= \int d\Pi_A d\Pi_B \left(\sigma'_{2\to 4} v_{\text{Møl}} \right) f_A f_B \\ \gamma_{4\to 2} &= \int d\Pi_A d\Pi_B \left(\sigma''_{2\to 4} v_{\text{Møl}} \right) (1+f_A)(1+f_B) \quad \text{where} \quad d\Pi_i \equiv \frac{g_S d^3 p_i}{(2\pi)^3 2E_i} \end{aligned}$$

- $\sigma'_{2\to 4} v_{\text{Møl}}$ and $\sigma''_{2\to 4} v_{\text{Møl}}$ are modified cross sections including phase space distribution function.
- Multi-dimensional integration $\rightarrow$ challenging to solve
- Numerically computed with CalcHEP.

Møller velocity and CM velocity

- Møller velocity $v_{\mathrm{Møl}} \equiv \dfrac{\sqrt{(p_1 \cdot p_2)^2 - m_S^4}}{E_1 E_2}$
- If DM is non-relativistic, $v_{\mathrm{Møl}} \approx v_{\mathrm{CM}}$, and BE and FD distributions can be approximated with MB distribution.

$$\to \gamma_{\mathrm{ann}} = \langle \sigma v_{\mathrm{Møl}} \rangle n_{\mathrm{eq}}^2 = 128\pi^2 T \int \sigma_{\mathrm{ann}}^{\mathrm{CM}} E^2 (E^2 - m_S^2) K_1 \left(2\frac{E}{T} \right) dE$$

Gondolo and Gelmini, Nucl.Phys. B360 (1991) 145-179

where definition of thermal averaged cross section is

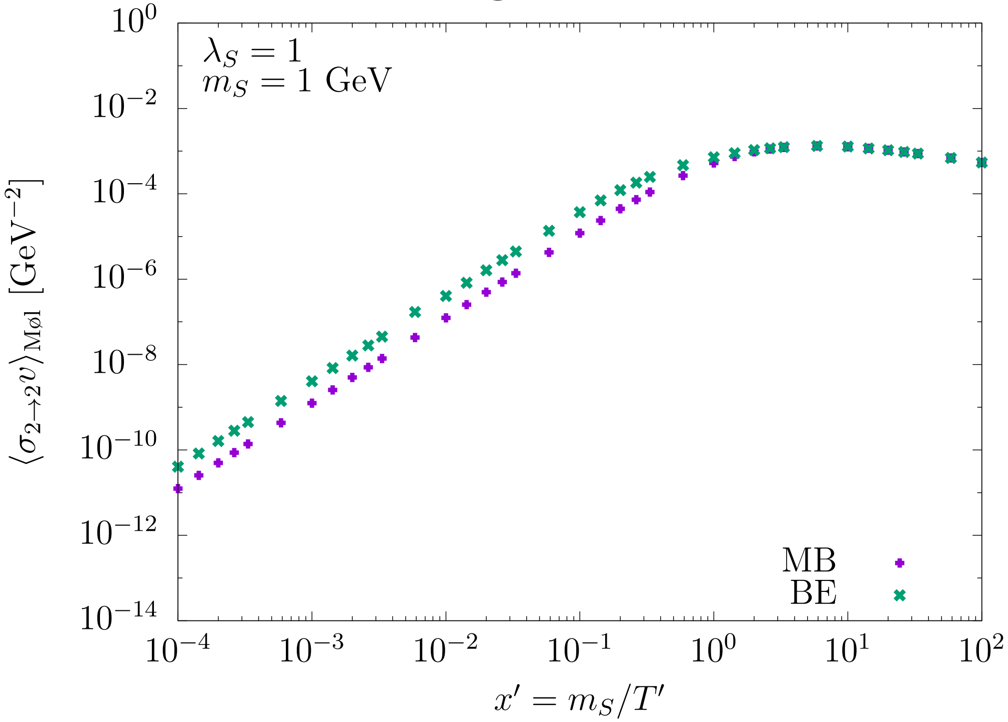
$$\langle \sigma v_{\mathrm{Møl}} \rangle \equiv \frac{\int d^3p_1 d^3p_2 \, (\sigma v_{\mathrm{Møl}}) \, f(p_1) f(p_2)}{\int d^3p_1 d^3p_2 f(p_1) f(p_2)}$$

· $\sigma_{\mathrm{ann}}^{\mathrm{CM}}$ can be computed at CM frame.

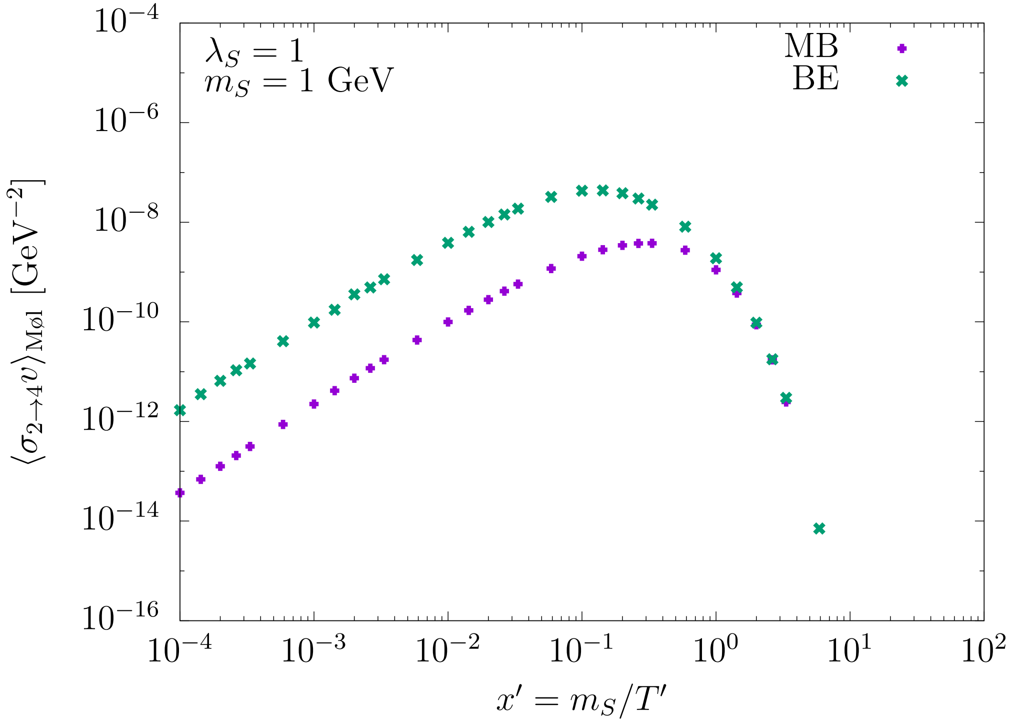
- For our case, we change the frame with Lorentz transformation.

Thermal averaged cross sections

2-to-2 scattering



2-to-4 process



- A few factor difference for 2-to-2 scattering
- An order difference for 2-to-4 process
- Depend on parameters $\lambda_S,\ m_S$
- Thermal mass is included: $m^2 = m_0^2 + m_{\text{th}}^2$ where $m_{\text{th}}^2 = \dfrac{\lambda_S}{24} T'^2$.

Summary

1. Singlet scalar DM is the simplest model.
2. Thermalized case is strongly constrained, and will be killed by future direct detection experiments.
3. DM can be thermalized only in dark sector with large self-interaction.
4. We are interested in the case of dark sector thermalized by inflaton decay etc, and semi-relativistic or relastivistic decoupling.

Future works

1. More sophisticated analysis will be done.