$\alpha_{\text{QED, eff}}(s)$ for precision physics at the FCC-ee/ILC

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11th FCC-ee workshop: Theory and Experiments,
8-11 January 2019, CERN Geneva

Outline of Talk:

- 1. $\alpha_{\rm QED,eff}(M_Z^2)$ in precision physics (precision physics limitations)
- 2. Reducing uncertainties via the Euclidean split trick:
  Adler function controlled pQCD
- 3. Prospects for future improvements
- 4. Need for space-like $\alpha_{\rm QED,eff}(t)$
- 5. Conclusions
- Appendix: The coupling α_2, M_W and $\sin^2\Theta_f$

1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics: besides top Yukawa y_t and Higgs self-coupling λ

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions** $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

50% non-perturbative

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$
\begin{array}{lcccl}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 0.9 \div 1.6 & \times & 10^{-4} \quad (\text{present : lost } 10^5 \text{ in precision!}) \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 5.3 & \times & 10^{-5} \quad (\mathtt{FCC-ee/ILC\ requirement})
\end{array}
$$

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - v_l/a_l)/4 = 0.23148 \pm$ 0.00017

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} =$ 0.00007 **;** $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

affects most precision tests and new physics searches!!!

$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}\,, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}\,, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t$ $\Rightarrow$ Lattice-QCD

❐ Relevance of $\alpha(M_Z^2)$

The Parameters of the Standard Model

– in four fermion and vector boson processes –

in addition QCD coupling α_s, y_t vs. M_t, λ_H vs. M_H

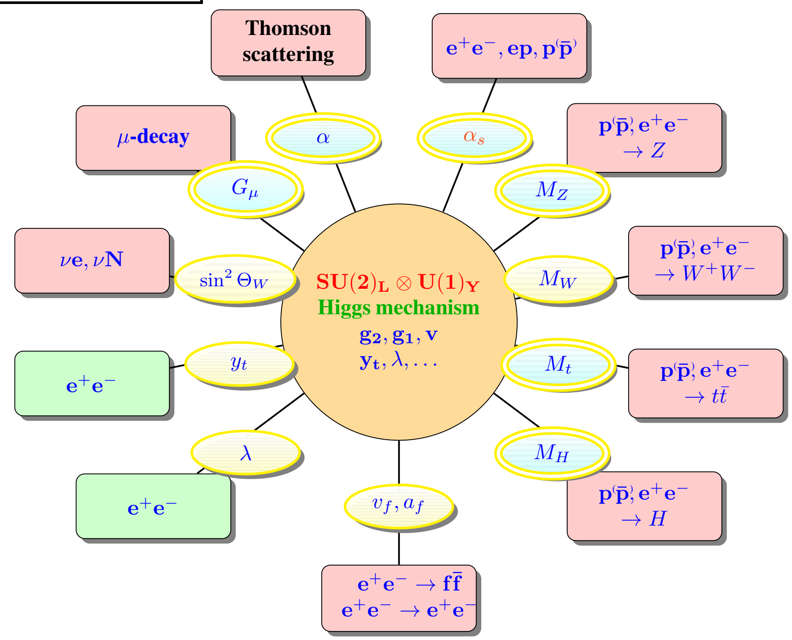
unlike in QED and QCD in SM (SBGT)

parameter interdependence ➠

only 6 independent quantities

(besides fermion masses and mixing parameters)

α, G_μ, M_Z $\Rightarrow$ $\alpha_{\rm eff}(M_Z^2)$ $\Rightarrow$large hadronic correction



$\Downarrow$

$$\sin^2\Theta_i\ \cos^2\Theta_i = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\quad;\quad \Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$$

parameter relationships between very precisely measurable quantities

precision tests, possible sign of new physics

non-perturbative $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is limiting precision predictions

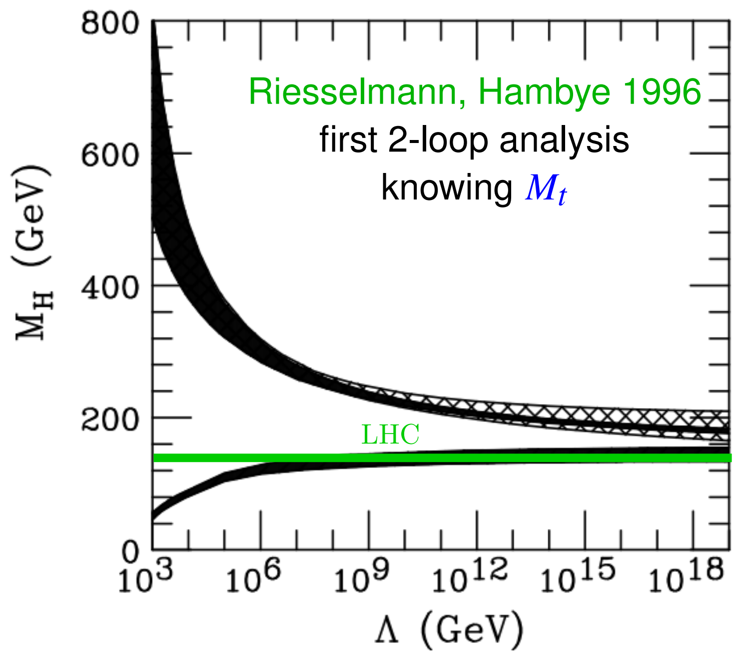
$$\lambda = 3\ \sqrt{2}G_\mu\,M_H^2\,(1+\delta_H(\alpha,\cdots))\ ;\quad y_t^2 = 2\ \sqrt{2}G_\mu\,M_t^2\,(1+\delta_t(\alpha,\cdots)$$

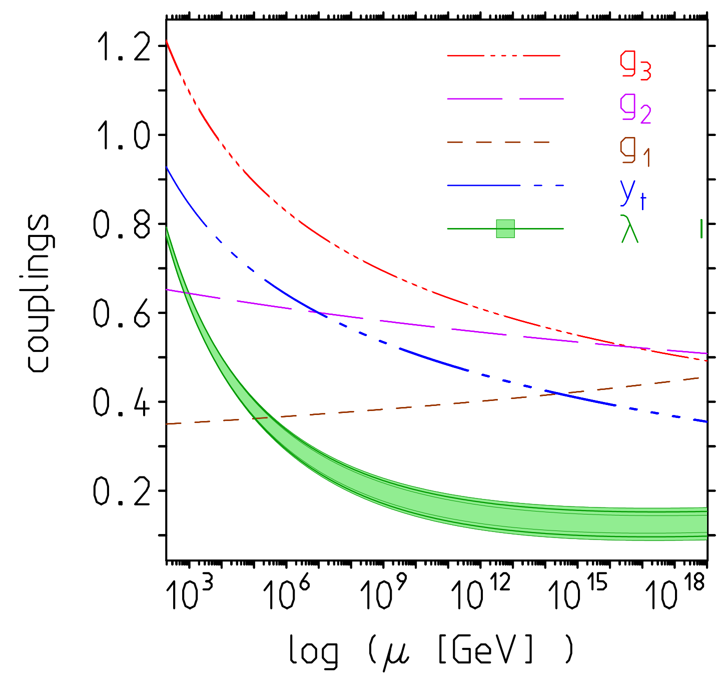
Note: 30 SD disagreement between SM prediction and experiment when subleading corrections are dropped!

SM extrapolation up to Planck scale?

After LHC Higgs discovery: Higgs vacuum stability issue!

⇒Need very precise SM parameters: g', g, g_s, y_t, λ





The SM dimensionless couplings in the $\overline{\mathrm{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 126$ GeV.

- proper effective parameters affect early cosmology, inflation, reheating etc.

- perturbation expansion works up to the Planck scale!
  no Landau pole or other singularities, Higgs potential remains (meta)stable!

❒ $U(1)_Y$ screening (IR free), $SU(2)_L\,,\ SU(3)_c$ antiscreening (UV free): g_1, g_2, g_3

as expected (standard wisdom)

❒ Top Yukawa y_t and Higgs λ: screening if standalone (IR free, like QED)

as part of SM, transmutation from IR free to UV free

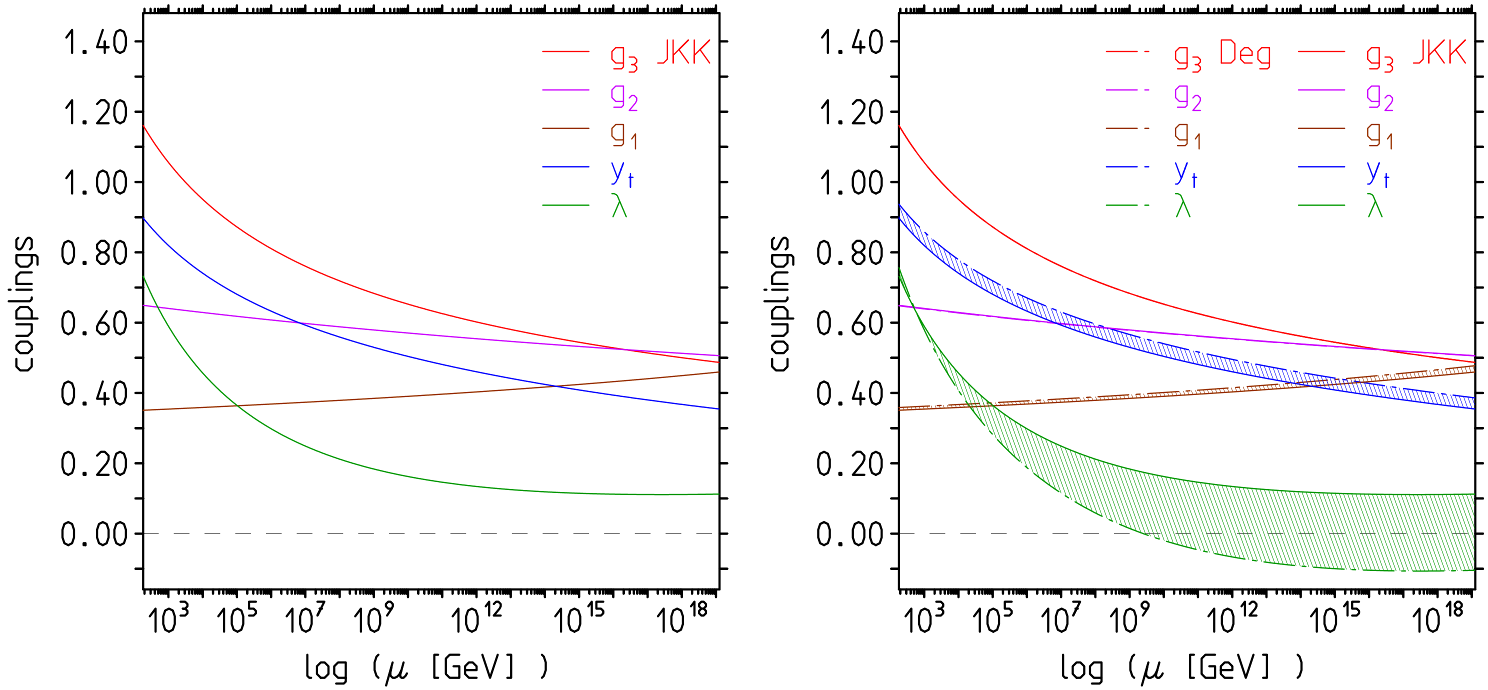
As SM couplings are as they are: QCD dominance in top Yukawa RG requires $g_3 > \frac{3}{4}\,y_t$, top Yukawa dominance in Higgs RG requires $\lambda < \frac{3\,(\sqrt{5}-1)}{2}\,y_t^2$ in the gaugeless ($g_1, g_2 = 0$) limit.

In the focus:

❒ does Higgs self-coupling stay positive $\lambda > 0$ up to Λ_{Pl} ?

❒ the key question/problem concerns the size of the top Yukawa coupling y_t
  decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

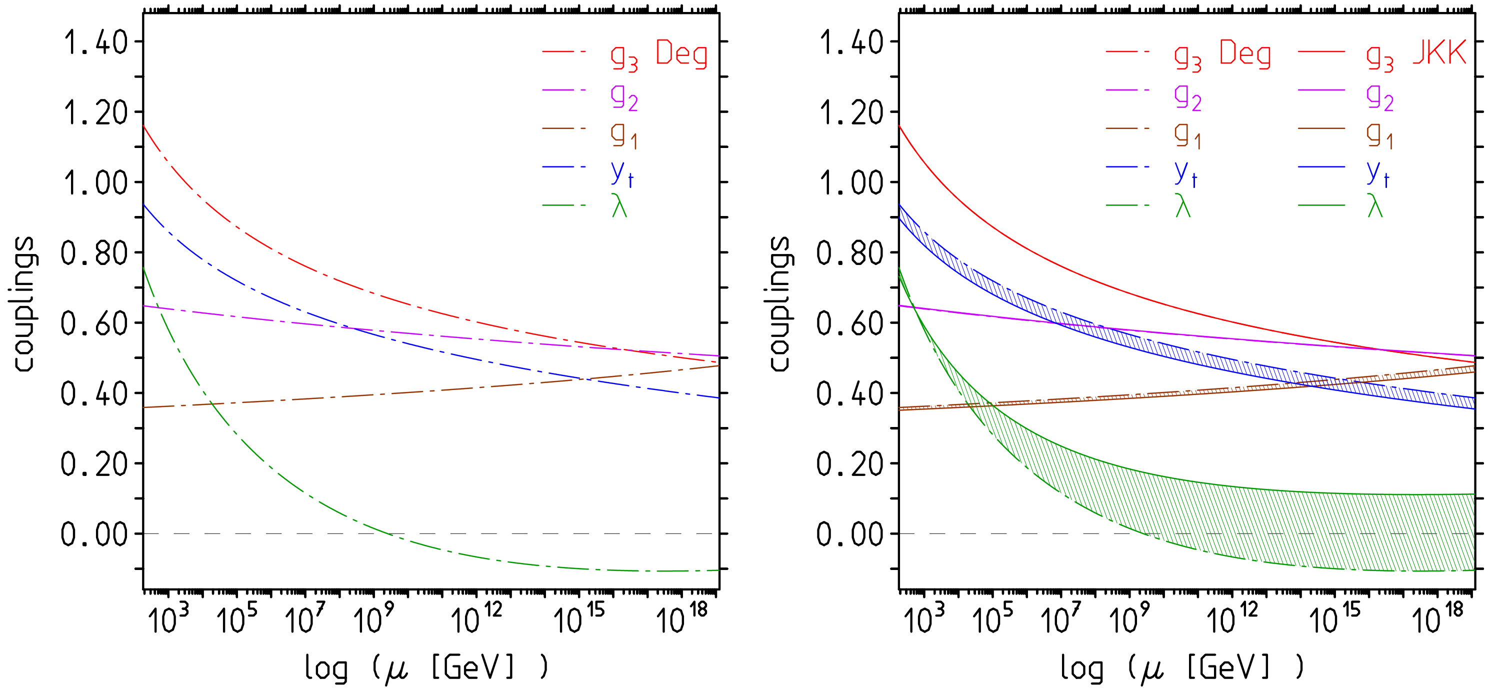
Will be decided by:
- more precise input parameters
- better established EW matching conditions
- direct measurements of y_t and λ at future e^+e^--colliders



JKK On-Shell vs $\overline{\mathrm{MS}}$ parameter matching

- the big issue is the very delicate conspiracy between SM couplings:
  - precision determination of parameters more important than ever $\Rightarrow$
  - the challenge for LHC and ILC/FCC: precision values for λ, y_t and α_s,
  - and for low energy hadron facilities: more precise hadronic cross sections to reduce hadronic uncertainties in $\alpha(M_Z)$ and $\alpha_2(M_Z)$

New gate to precision cosmology of the early universe!



Shaposnikov et al., Degrassi et al. matching

- the big issue is the very delicate conspiracy between SM couplings:
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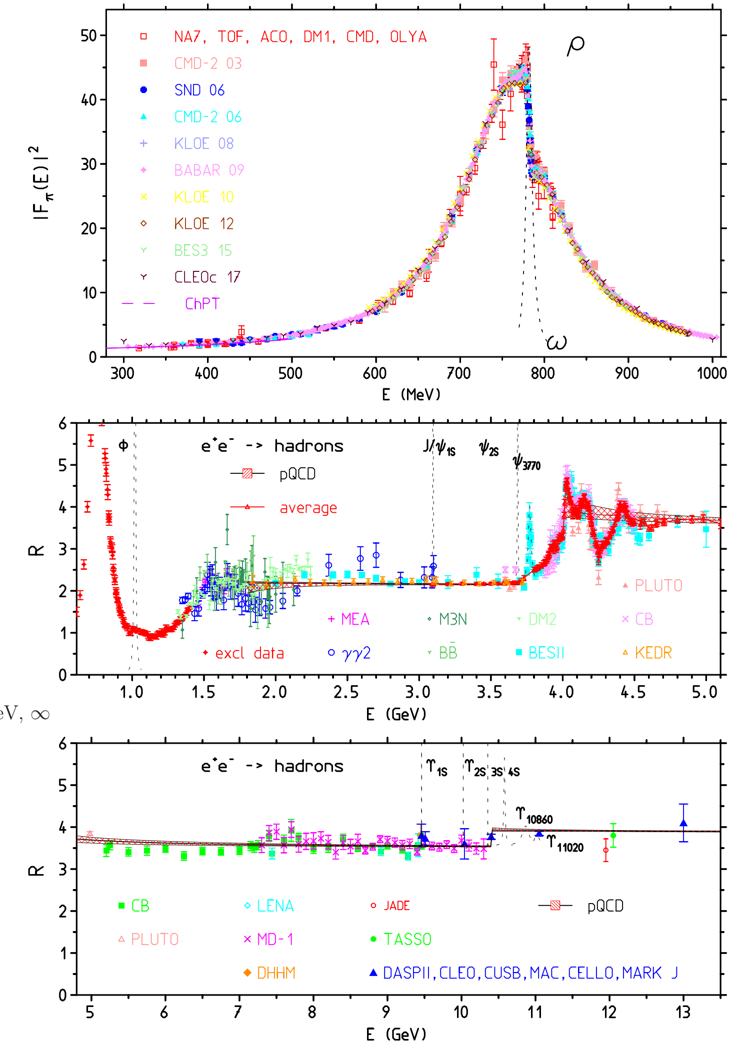
New gate to precision cosmology of the early universe!

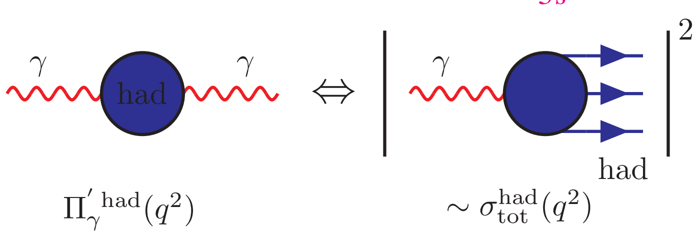
R-data Evaluation of $\alpha(M_Z^2)$

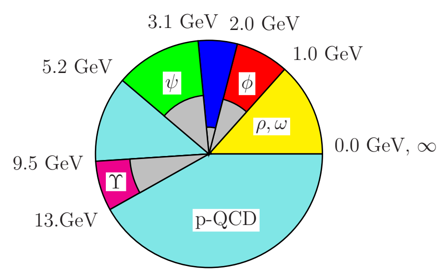
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\;\oint\!\!\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint\!\!\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$



where $\quad R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^-\to\gamma^*\to\text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$



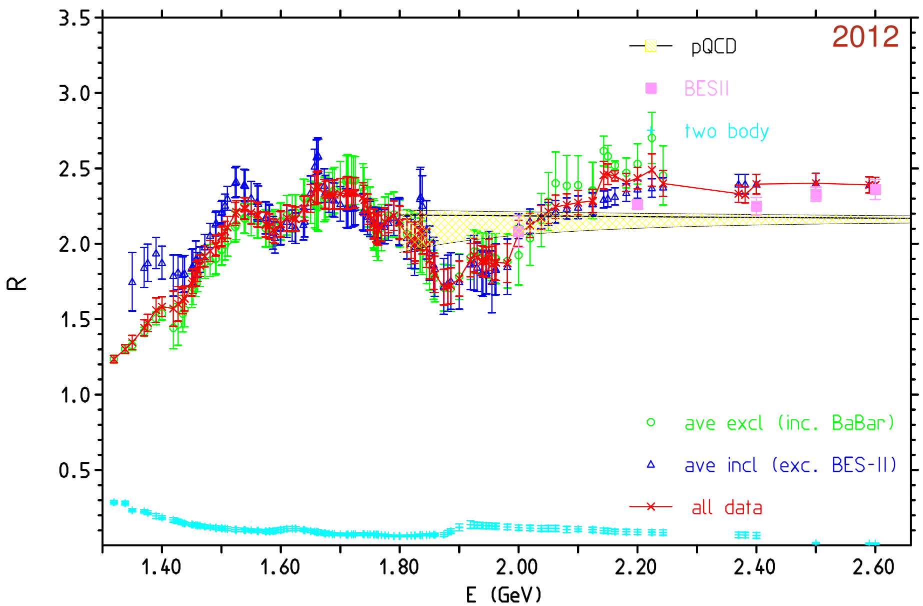


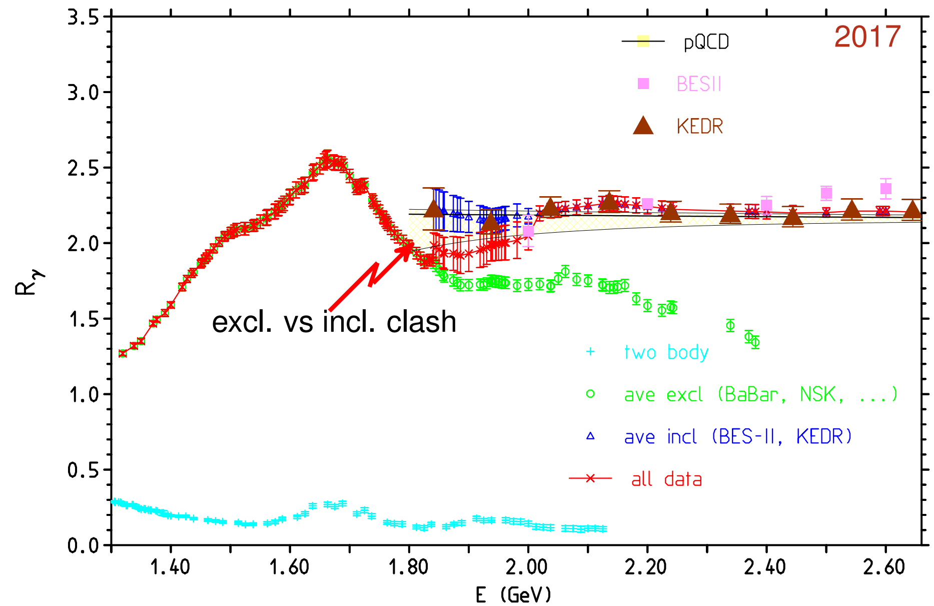
hadronic vacuum polarization

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

Still an issue in HVP

- region 1.2 to 2 GeV data; test-ground exclusive vs inclusive *R* measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty





- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Present situation: (after KLOE, BaBar and first BESIII results)

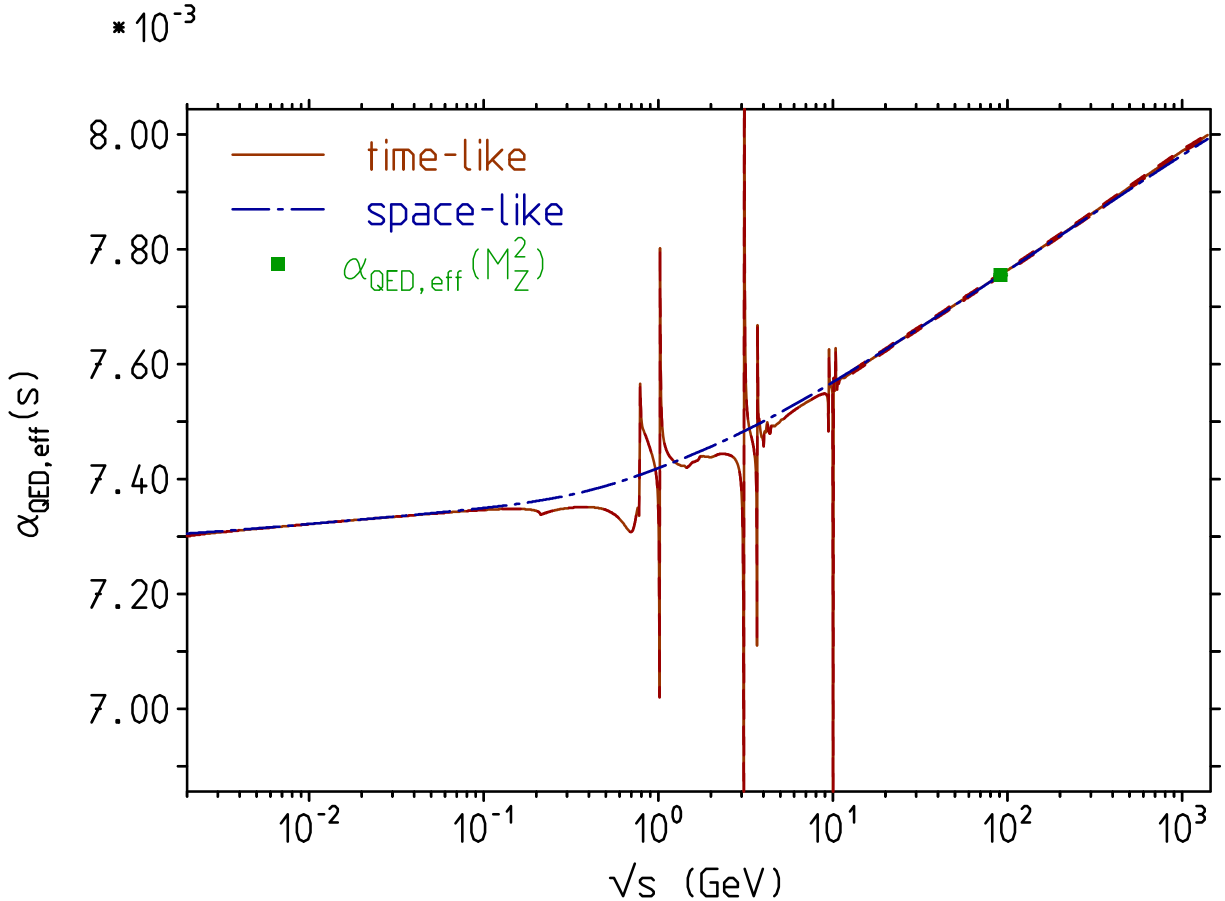
$$\begin{aligned} \Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) &= 0.027756 \pm 0.000157 \\ & \quad 0.027563 \pm 0.000120 \quad \text{Adler} \\ \alpha^{-1}(M_Z^2) &= 128.916 \pm 0.022 \\ & \quad 128.953 \pm 0.016 \quad \text{Adler} \end{aligned}$$

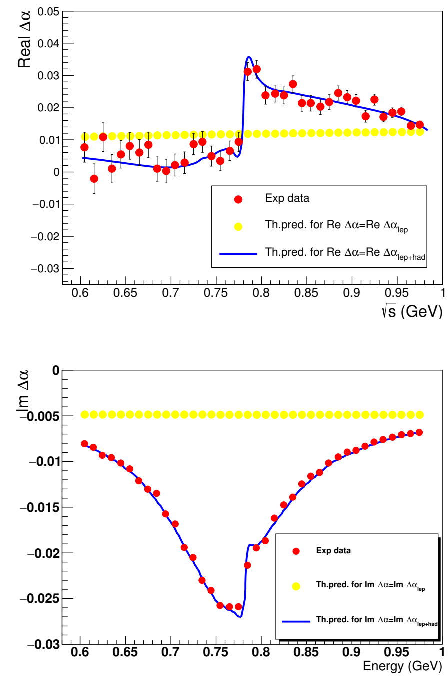
Possible improvements:

- direct dispersion integral requires reducing error of $R(s)$ to 1% up to above Υ resonances
  (likely nobody will do that)

- Euclidean split method (Adler) requires
  - ❏ improvement of 1 to 2 GeV exclusive region (NSK,Belle II can top what BaBar has achieved)
  - ❏ improved pQCD Adler function massive 4-loop, better parameters m_c and m_b besides α_s
    (profiting from activities going on anyway, FCC-ee/ILC further strong motivation)

$\alpha_{\rm QED,eff}$: time-like vs. space-like





KLOE 2016, arXiv:1609.06631

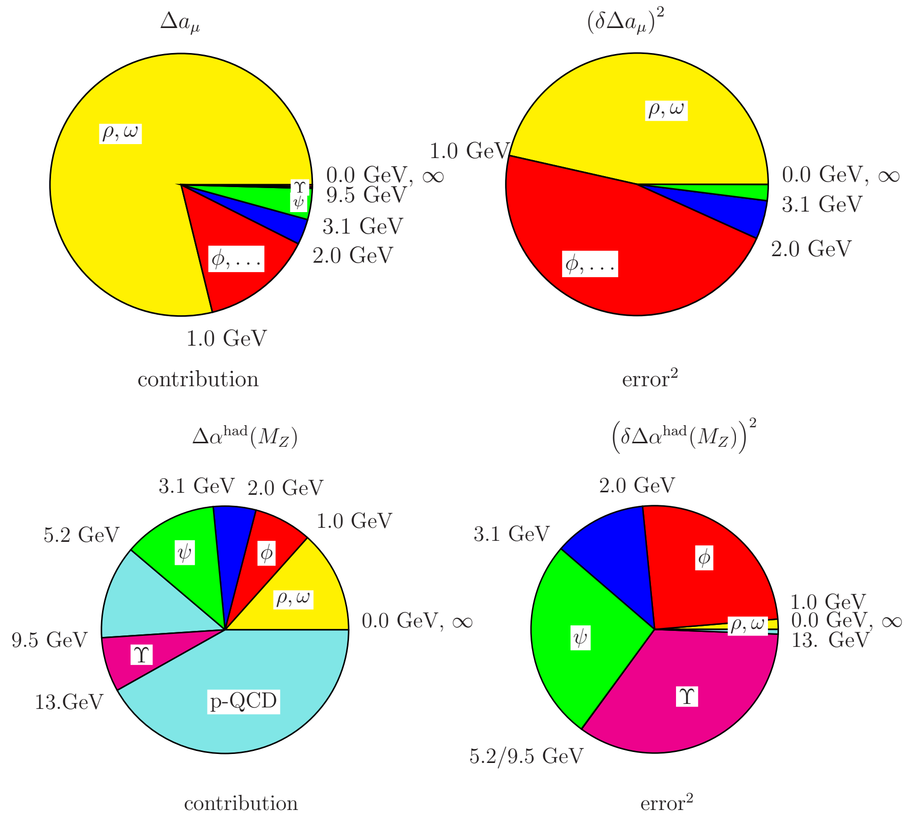
$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version. Locally ill-defined near OZI suppressed meson decays: $J/\psi, \psi_1, \Upsilon_{1,2,3}$! Dyson series not convergent.

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2017 update in terms of e^+e^--data and pQCD. 43% data, 57% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.19]	0.6%	1.4%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.7%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	32.1%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.53 (0.06) (0.83)[0.83]	5.0%	27.4%
had	(2.00, 3.20)	15.34 (0.08) (0.61)[0.62]	4.0%	15.2%
had	(3.10, 3.60)	4.98 (0.03) (0.09)[0.10]	1.9%	0.4%
had	(5.20, 5.20)	16.84 (0.12) (0.21)[0.25]	0.0%	2.4%
pQCD	(5.20, 9.46)	33.84 (0.12) (0.25)[0.03]	0.1%	0.0%
had	(9.46,11.50)	11.12 (0.07) (0.69)[0.69]	6.2%	19.2%
pQCD	(11.50,∞)	123.29 (0.00) (0.05)[0.05]	0.0%	0.1%
data	(0.28,11.50)	120.25 (0.63) (1.45)[1.58]	1.0%	0.0%
total		277.38 (0.63) (1.45)[1.58]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$.

2. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

data   pQCD Adler   pQCD HVP

- the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$; offset $\alpha^{\rm data}(-s_0)$ integrated $R(s)$ data
- the Adler function is i) the monitor to control the applicability of pQCD and ii) pQCD part $\left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD}$ by integrated Adler function $D(Q^2)$
- small remainder $\left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$ by calculation of VP function $\Pi'_\gamma(s)$

❐ $\Delta\alpha^{\rm had}$ Adler function controlled

✓ use old idea: Adler function: Monitor for comparing theory and data

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s \right) .$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

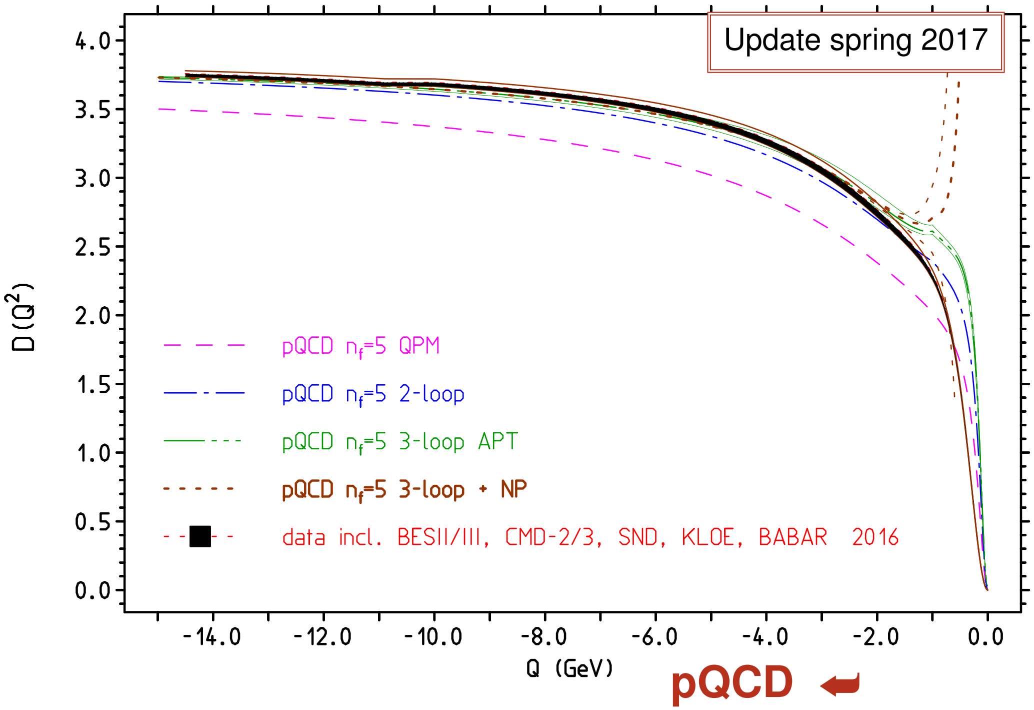
Conclusion:

❖time-like approach: pQCD works well in "perturbative windows"
3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Harlander,Steinhauser

❖space-like approach: pQCD works well for $\sqrt{Q^2 = -q^2} > 2.0$ GeV (see plot)

"Experimental" Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/17 updates)
theory based on results by Chetyrkin, Kühn et al.

$\Rightarrow$ **pQCD works well controlled to predict $D(Q^2)$ down to $s_0 = (2.0\,\text{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int \mathrm{d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\boxed{\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \\ &\quad + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\end{aligned}}$$

and obtain, for $s_0 = (2.0\,\textbf{GeV})^2$: **(FJ 98/17)**

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006409 \pm 0.000063 \\ \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027483 \pm 0.000118 \\ \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027523 \pm 0.000119\end{aligned}$$

❖**shift $+0.000008$ from the 5-loop contribution**

❖**error ± 0.000100 added in quadrature form perturbative part**

QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**

● $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis Kühn et al 2007

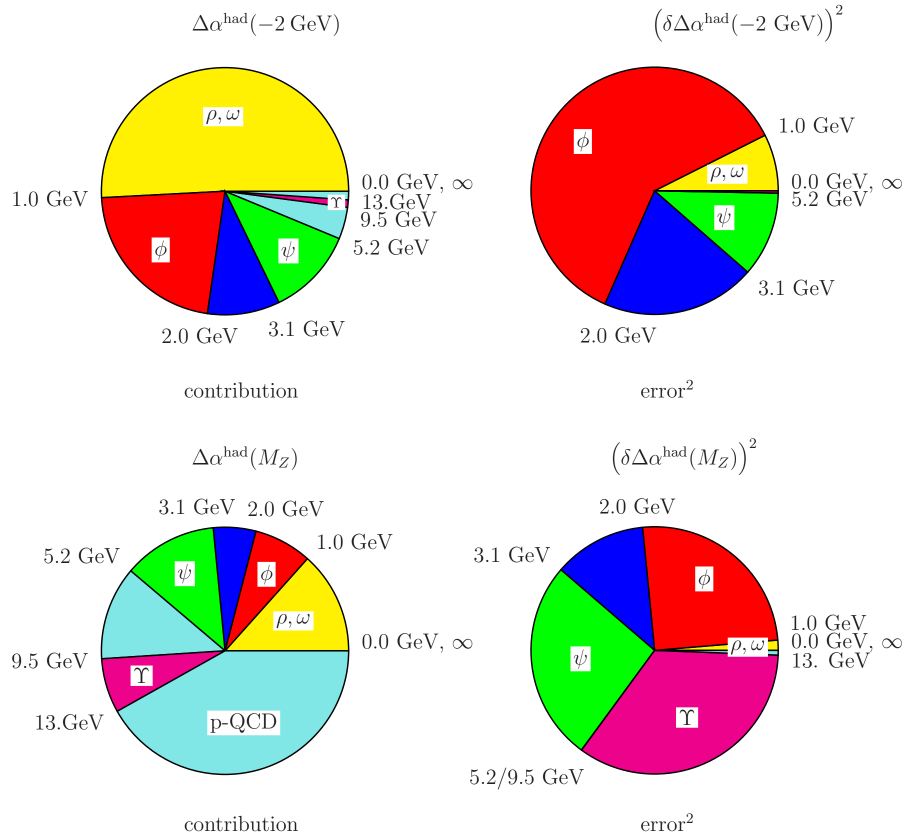
F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

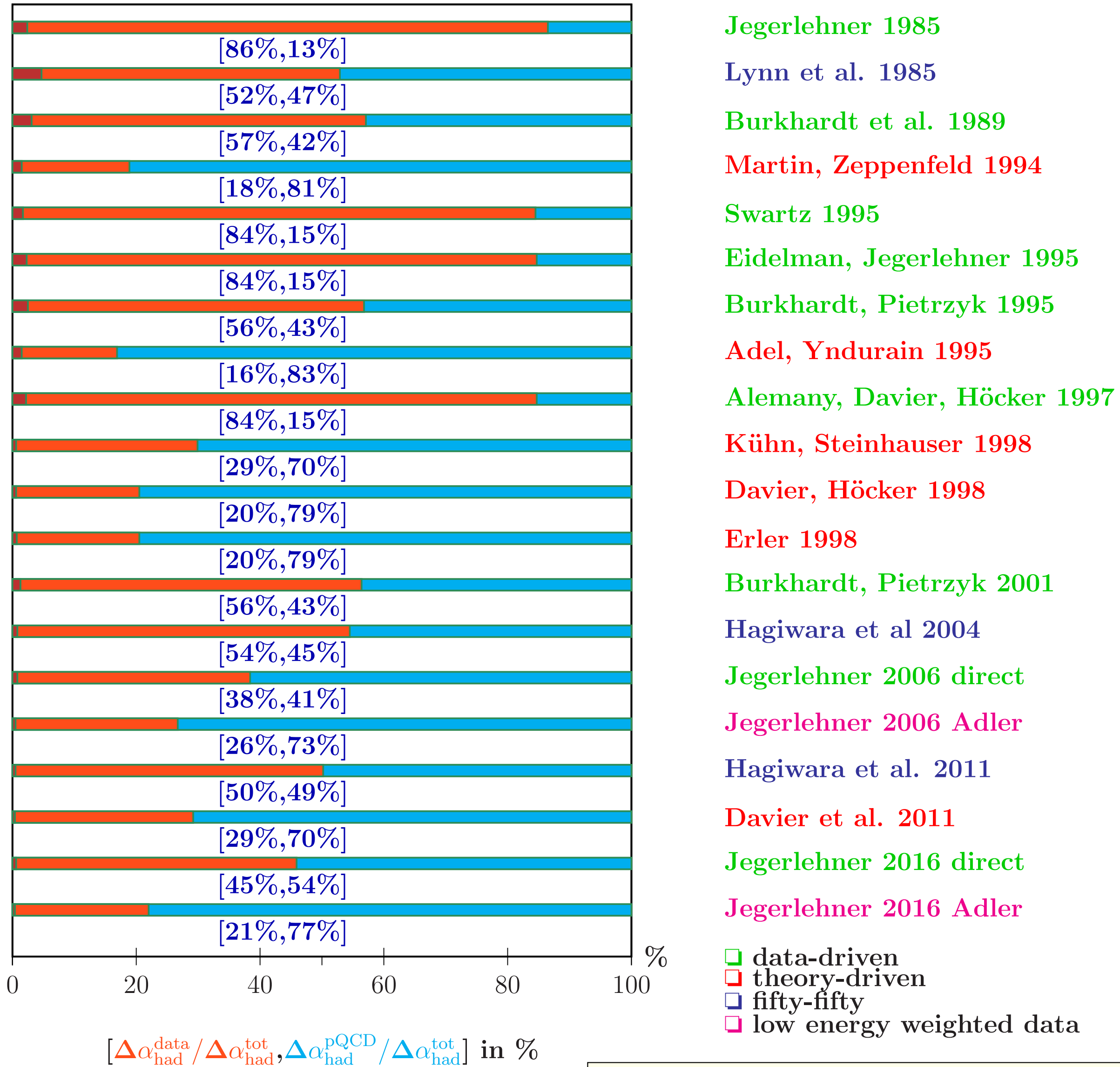
for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 94% data, 6% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2) \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.78 (0.04) (0.16)[0.16]	0.5%	6.6%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.6%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.6%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	11.4%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.00)	10.36 (0.04) (0.49)[0.49]	4.8%	61.2%
had	(2.00, 3.20)	6.06 (0.03) (0.25)[0.25]	4.2%	16.1%
had	(3.10, 3.60)	1.31 (0.01) (0.02)[0.03]	1.9%	0.2%
had	(5.20, 5.20)	2.90 (0.02) (0.02)[0.03]	0.0%	0.2%
pQCD	(5.20, 9.46)	2.66 (0.02) (0.02)[0.00]	0.1%	0.0%
had	(9.46,11.50)	0.39 (0.00) (0.02)[0.02]	5.7%	0.1%
pQCD	(11.50,∞)	0.90 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,11.50)	60.53 (0.18) (0.61)[0.63]	1.0%	0.0%
total		64.09 (0.18) (0.61)[0.63]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ 22% data, 78% pQCD!



Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.



How much pQCD?

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

3. Prospects for future improvements

Note: new muon $g-2$ experiments at Fermilab and JPARC trigger continuation of $e^+e^- \to$ hadrons cross section measurements in low energy region by VEPP 2000 at Novosibirsk, BES III Beijing, Belle II at KEK. This automatically helps improving split trick approach (Adler function controlled)

direct DR approach requires precise data up to much higher energies or heavy reliance on pQCD calculation of time-like $R(s)$!

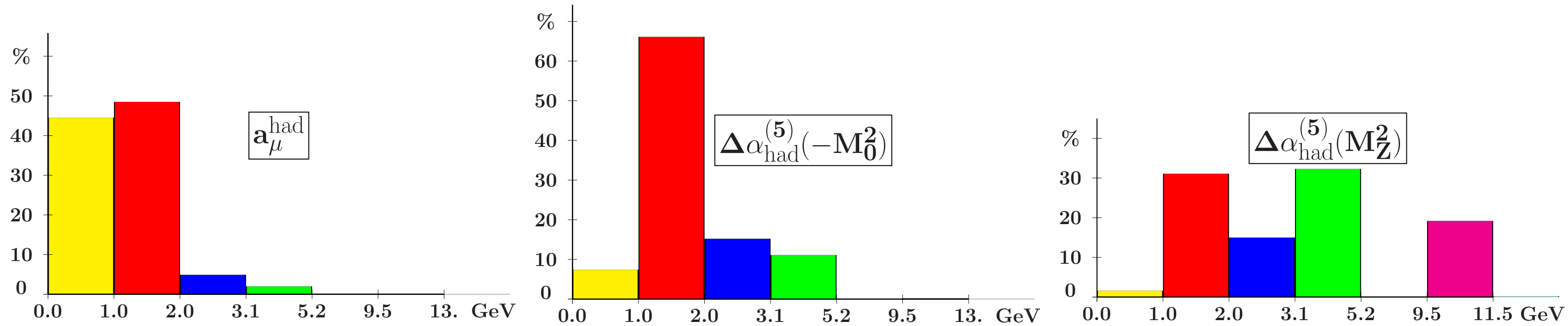
Mandatory pQCD improvements required are:

- **4–loop massive pQCD calculation of Adler function;**
  required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements
  [essentially equivalent to a massive 4–loop calculation of $R(s)$];
- **m_c, m_b improvements by sum rule and/or lattice QCD evaluations;**
- **improved α_s in low Q^2 region above the τ mass.**

Theory: (QCD parameters) has to improve by factor 10 ! $\to \pm 0.20$

Settling the HVP issue for a_μ settles it largely for $\Delta\alpha(-M_0^2)$

Error profiles (standard approach):

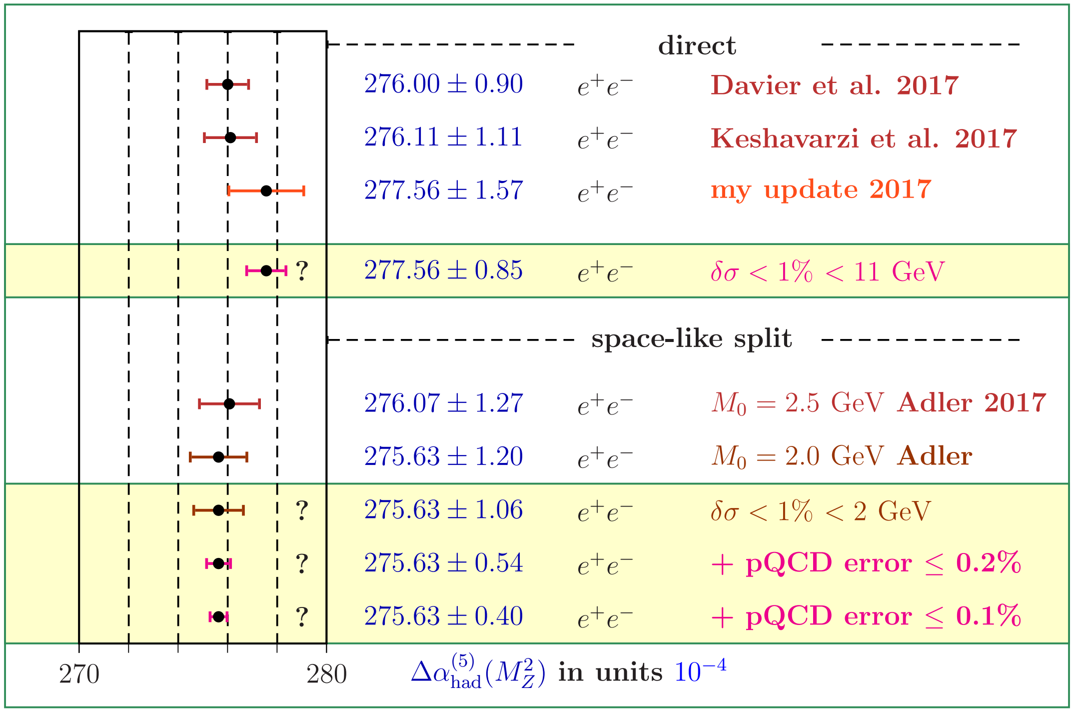


Contributions to the total error from different energy regions to the hadronic lowest order vacuum polarization contribution to a_μ, $\Delta\alpha(M_Z^2)$ and $\Delta\alpha(-M_0^2)$ for $M_0 = 2$ GeV in percent. These errors are to be added in quadrature to get the total uncertainty. The graph illustrates where experimental effort is needed in order to get a better precision.

The virtues of Adler function approach are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.0 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**
- **$\Delta\alpha(-M_0^2)$ would be directly accessible in $\boxed{\textbf{MUonE}}$ experiment (project) and lattice QCD.**

What can we achieve:



Davier et al. 2011: use pQCD above 1.8 GeV

- **no improvement by remeasuring cross sections above 1.8 GeV**
- **no proof that pQCD works at 0.04% precision as adopted**

My analysis is data driven: pQCD $5.2 - 9.5$ and > 11.5 GeV

- × **pQCD at 0.2% Adler function: pQCD error = ½ × present error**
- × **pQCD at 0.1% Adler function: pQCD error = data error ± 0.28**

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in α:	**present**	**direct**	1.7×10^{-4}
		Adler	1.2×10^{-4}
	future	**Adler QCD 0.2%**	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
	future	**via $A_{\mathrm{FB}}^{\mu\mu}$ off Z**	3×10^{-5}

- **Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \to \mu^+\mu^-$**

$$A_{\mathrm{FB}}^{\mu\mu} = A_{\mathrm{FB},0}^{\mu\mu} + \frac{3\,a^2}{4\,v^2}\frac{\mathcal{I}}{\mathcal{Z}+\mathcal{G}}$$

where

$\gamma - Z$ **interference term**	$\mathcal{I} \propto \alpha(s)\,G_\mu$
Z **alone**	$\mathcal{Z} \propto G_\mu^2$
γ **only**	$\mathcal{G} \propto \alpha^2(s)$
v **vector Z coupling**	**also depends on $\alpha(s \sim M_Z^2)$ and $\sin^2\Theta_f(s \sim M_Z^2)$**
a **axial Z coupling**	**sensitive to ρ-parameter (strong M_t dependence)**

❏ **using v, a as measured at Z-peak**

Challenges for direct measurement:

❏ **radiative corrections*** ❏ **needs dedicated off-Z peak running**

* **under way see e.g. Gluza et al. arXiv:1804.10236**

● **Adler function method is much cheaper to get, I think!**

Requirement look to be realistic:

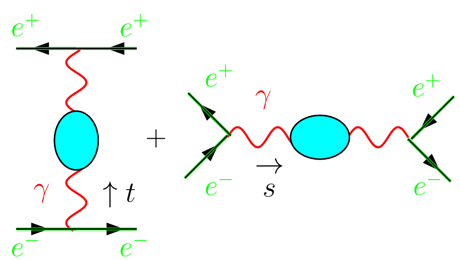
❖ **pin down experimental errors to 1% level in all non-perturbative regions up to 2.0 GeV**

❖ **switch to Euclidean approach, monitored by the Adler function**

❖ **improve on QCD parameters, mainly on m_c and m_b**

4. Need for space-like $\alpha_{\mathrm{QED,eff}}(t)$

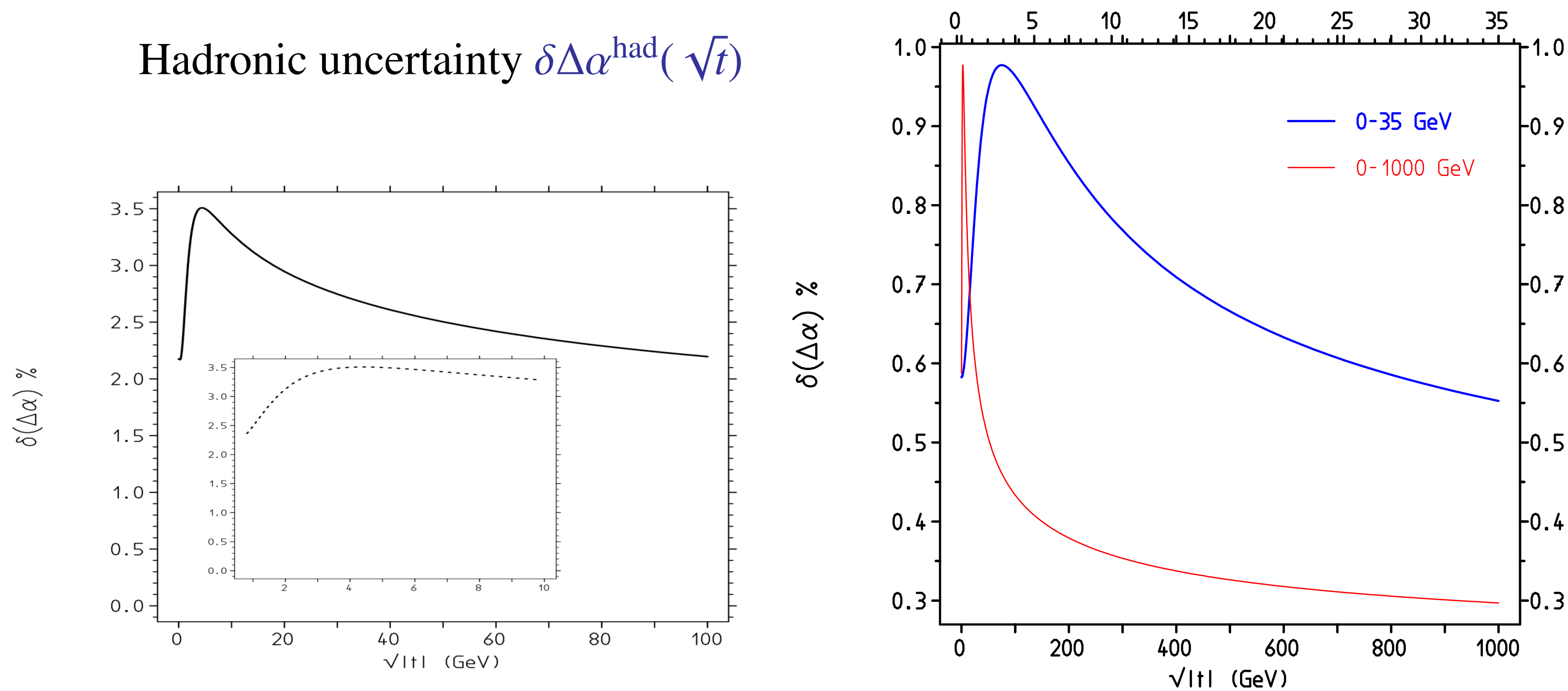
❒ **FCC-ee luminometer: small angle Bhabha scattering**



VP dressed tree level Bhabha scattering in QED

❒ **for small angle Bhabha scattering** $\delta_{\mathrm{HVP}}\sigma/\sigma = 2\,\delta\alpha(\bar{t})/\alpha(\bar{t})$**,**
for FCC-ee luminometer $\sqrt{\bar{t}} \simeq 3.5$ **GeV near** Z **peak and** $\simeq 13$ **GeV at 350 GeV.**

Hadronic uncertainty $\delta\Delta\alpha^{\mathrm{had}}(\sqrt{t})$



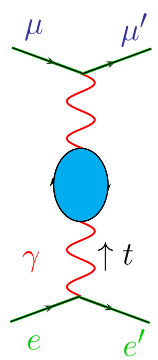
Progress 1996 → 2018 lot of newer low energy data $\pi\pi$ etc.

Jadach et al. arXiv:1812.01004 **Talks B. Ward, C. M. Carloni Calame**

$\sqrt{s}$	$\sqrt{t}$	1996	present	FCC–ee expected
M_Z	3.5 **GeV**	**0.040%**	**0.013%**	0.6×10^{-4}
350 **GeV**	13 **GeV**		1.2×10^{-4}	2.4×10^{-4}

New project: measuring directly low energy $\alpha_{\mathrm{QED}}(t)$

- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information
- use $\mu^- e^-$ scattering **MUonE projects** G. Abbiendi et al., arXiv:1609.08987



$$\frac{d\sigma^{\mathrm{unpol.}}_{\mu^- e^- \to \mu^- e^-}}{dt} = 4\pi\,\alpha(t)^2 \,\frac{1}{\lambda(s, m_e^2, m_\mu^2)} \left\{ \frac{\left(s - m_\mu^2 - m_e^2\right)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right\}$$

- The primary goal determining a_μ^{had} in an alternative way

$$\boxed{a_\mu^{\mathrm{had}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}\left(-Q^2(x)\right)}$$

where $Q^2(x) \equiv \frac{x^2}{1-x} m_\mu^2$ is the space–like square momentum–transfer

- $\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\rm lep}(-Q^2) - 1$ **directly compares with lattice QCD data**

- **My proposal here: determine very accurately**

$$\boxed{\Delta\alpha_{\rm had}\left(-Q^2\right) \text{ at } Q \approx 2.5 \text{ GeV}}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\rm had}\left(M_Z^2\right)$ **as in "Adler function" approach.**

- **direct useful for small angle Bhabha luminometer!**

5. Conclusions

- Muon $g-2$ theory uncertainty remains the key issue and strongly motivates more precise measurements of low energy $e^+e^- \to \mathrm{hadrons}$ cross sections (Novosibirsk VEPP 2000/CMD3,SND, Beijing BEPCII/BESIII, Tsukuba SuperKEKB/BelleII).

- helps to improve $\alpha_{\mathrm{QED}}(t)$ in region relevant for small angle Bhabha process and in calculating $\alpha_{\mathrm{QED}}(s)$ at FCC-ee/ILC energies via Euclidean split trick (Adler function controlled data vs pQCD split)

- the latter method requires pQCD prediction of the Adler-function to improve by a factor 2 (improved parameters mainly m_c and m_b)

- Are presently estimated (essentially agreed) evaluations in terms of R-data reliable? Alternative methods important!

- Patrick Janot's approach certainly is an important alternative method directly accessing $\alpha_{\mathrm{QED}}(M_Z^2)$ with very different systematics. A challenging project.

- Another interesting option is an improved radiative return measurement of $\sigma(e^+e^- \to \mathrm{hadrons})$ at the GigaZ (directly improves dispersion integral incl all resonances and thresholds in one experiment!)

- In any case on paper $e^-\mu^+ \to e^-\mu^+$ looks to be the ideal process to perform an unambiguous measurement of $\alpha(-Q^2)$, which determines the LO HVP to a_μ as well as the non-perturbative part of $\alpha_{\mathrm{QED}}(s)$! Lattice QCD results are very close to become competitive here as well.

❒ at the end we have alternatives available allowing for important crosschecks.

Let's do it!

Thanks you for your attention!

The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2: see also Talk A. Vicini

❖charged current channel M_W ($g \equiv g_2$): $M_W^2 = \frac{g^2 v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu}$

❖neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{\frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}}\right\}\sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \quad \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \quad \hat{\Pi}_V^{3\gamma} \quad - \quad \frac{e^2\, s_\Theta}{c_\Theta} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \quad \hat{\Pi}_{V-A}^{33} \quad - \quad 2\,\frac{e^2}{c_\Theta^2} \quad \hat{\Pi}_V^{3\gamma} \quad + \quad \frac{e^2\, s_\Theta^2}{c_\Theta^2} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \quad \hat{\Pi}_{V-A}^{+-}
\end{aligned}$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$. Leading hadronic contributions:

$$\begin{aligned}
\Delta\alpha_{\mathrm{had}}^{(5)}(s) &= -e^2\; [\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)] \\
\Delta\alpha_{2\,\mathrm{had}}^{(5)}(s) &= -\frac{e^2}{s_\Theta^2}\left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing. $\Delta\alpha_{\mathrm{had}}^{(5)}(s)$ and $\Delta\alpha_{2\,\mathrm{had}}^{(5)}(s)$ via e^+e^--data and isospin arguments

(u,d), s flavor separation: $\Pi^{3\gamma}_{ud} = \frac{1}{2}\,\Pi^{\gamma\gamma}_{ud}\;;\;\;\Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$

$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \quad \Rightarrow \quad \Pi^{3\gamma} = \frac{1}{2}\,\Pi^{(\rho)} + \frac{3}{4}\,\Pi^{(\phi)} + \cdots$$

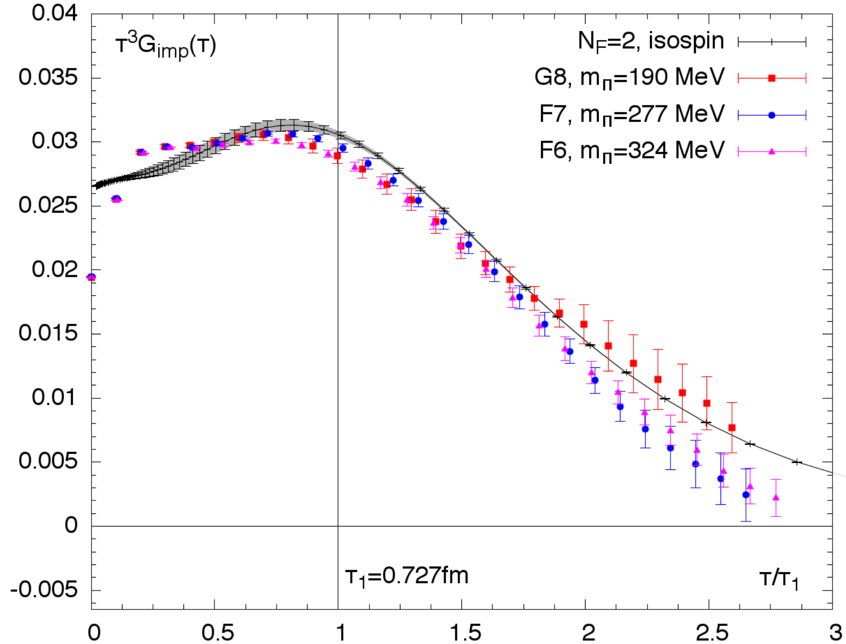
F. J., Z. Phys. C **32** (1986) 195, Nuovo Cim. C **034S1** (2011) 31

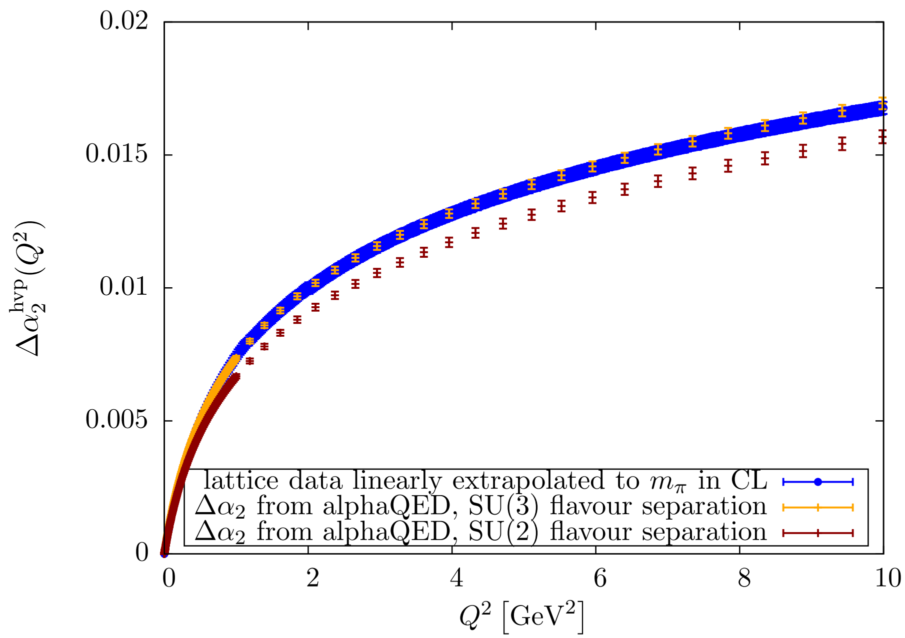
Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)
➠new physics may be obscured by non-perturbative hadronic effects; need to fix this! Flavor separation assuming OZI violating terms to be small $\Rightarrow$ perturbative rewighting $\Rightarrow$ disagrees with lattice QCD results!!!

Note that the "wrong" perturbative weighting

$$\Pi^{3\gamma}_{ud} = \frac{9}{20}\,\Pi^{\gamma\gamma}_{ud}\;;\;\;\Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

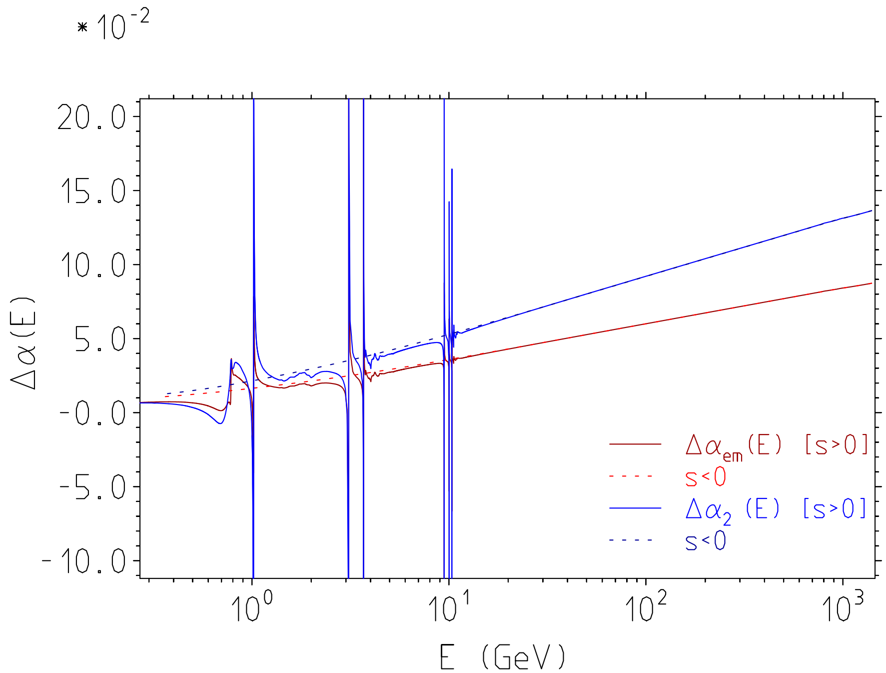
has been proven to clearly mismatch lattice results, while the correction $\frac{9}{20} \Rightarrow \frac{10}{20}$ is in good agreement. This also means the OZI suppressed contributions should be at the 5% level and not negligibly small.



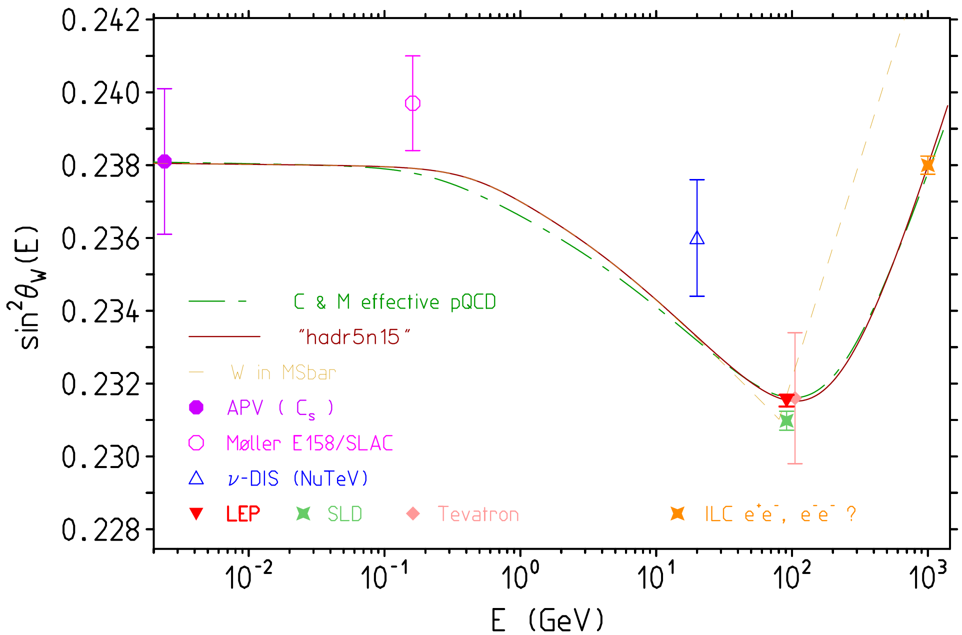


Testing flavor separation in lattice QCD H. Meyer et al. [l], arXiv:1312.0035,arXiv:1811.08669, K. Jansen et al. arXiv:1505.03283[r]

- disproves pQCD reweighting! and pQCD prediction based on effective quark masses.

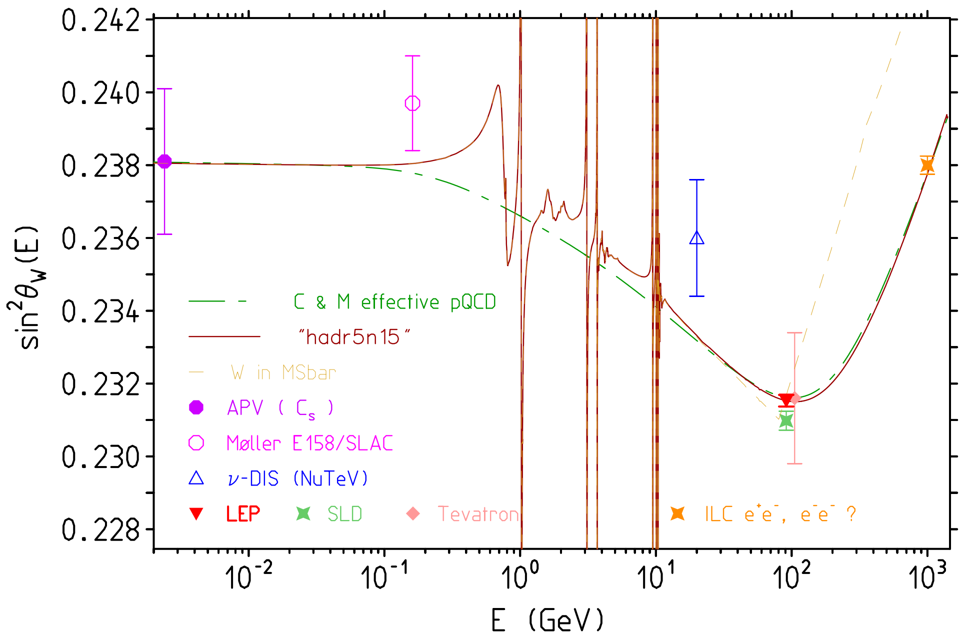


$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23153(16)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements at lower energies are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region. Note that $\sin^2\theta_W(0)/\sin^2\theta_W(M_Z^2) = 1.02876$ a 3% correction established at $6.5\ \sigma$.

$\sin^2\Theta_{\rm eff}$

exhibiting a specific dependence on the gauge boson SEs
is an excellent monitor for New Physics

Details about possible future progress

Goal: ILC/FCC-ee requirement: improve by factor 10 in accuracy

- direct integration of data: 46% from data 54% p-QCD , $\Delta\alpha^{(5)\,\mathrm{data}}_{\mathrm{had}} \times 10^4 = 126.86 \pm 1.78$ (1.4%) 1% overall accuracy ± 1.27
  1% accuracy for each region (divided up as in table)
  added in quadrature: ± 0.40
  Data: [1.78] vs. [0.40] $\Rightarrow$ improvement factor 4.5 , $\Delta\alpha^{(5)\,\mathrm{pQCD}}_{\mathrm{had}} \times 10^4 = 149.57 \pm 0.05$ (0.0%) Theory: no improvement needed !

- integration via Adler function: 22% from data 78% p-QCD
  $$\Delta\alpha^{(5)\,\mathrm{data}}_{\mathrm{had}} \times 10^4 = 063.87 \pm 0.66 \text{ (1.1\%)}$$
  1% overall accuracy ± 0.60
  1% accuracy in region 1.0 to 2.5 GeV
  added in quadrature: ± 0.28
  Data: [1.19] vs. [1.03,0.57,0.37] $\Rightarrow$ improvement factor 2.1-3.2 (Adler vs Adler)
  [1.78] vs. [1.03,0.57,0.37] $\Rightarrow$ improvement factor 3.1-4.8 (Standard vs

Adler)

$$\Delta\alpha_{\rm had}^{(5)\,\rm pQCD} \times 10^4 = 214.48 \pm 1.00 \ (0.05\%)$$

Theory: massive 4-loop needed and more accurate m_c, m_b and α_s!

❖direct measurement (near/off Z peak) Patrick Janot's approach

$\Delta\alpha_{\rm had}(s_0) \times 10^4$: present

s_0	M_Z^2	$-(2.5\ \mathrm{GeV})^2$	$-(2.0\ \mathrm{GeV})^2$
data	126.86 ± 1.78 [1.4%]	73.72 ± 0.79 [1.1%]	63.87 ± 0.66 [1.1%]
pQCD	149.57 ± 0.05 [0.0%]	201.23 ± 0.99 [0.5%]	210.74 ± 1.04 [0.5%
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000178	0.027535 ± 0.000127	0.027501 ± 0.000124
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.024	128.968 ± 0.017	128.972 ± 0.017
accuracy in 10^{-5}	18.92	13.52	13.15
improvement	1.0	1.0	1.0

$\Delta\alpha_{\rm had}(s_0) \times 10^4$. Future I: improving data only a) direct $\delta\sigma \lesssim$ 1% above ϕ to 11.5 GeV, b) low energy space-like cut at $\sqrt{s_0}$, $\delta\sigma \lesssim$ 1% above ϕ to 2.5 GeV

s_0	M_Z^2	$-(2.5\ \mathrm{GeV})^2$	$-(2.0\ \mathrm{GeV})^2$
data	126.86 ± 0.41 [0.3%]	73.72 ± 0.33 [0.4%]	63.87 ± 0.28 [0.4%]
pQCD	149.57 ± 0.05 [0.0%]	201.23 ± 1.03 [0.5%]	210.74 ± 1.04 [0.5%
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000105	0.027501 ± 0.000112
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.014	128.972 ± 0.015
accuracy in 10^{-5}	4.39	11.16	11.95
improvement	4.3	1.2	1.1

$\Delta\alpha_{\rm had}(s_0) \times 10^4$. Future II: improving data as above plus pQCD to 0.2% in case b)

s_0	M_Z^2	$-(2.5\ \mathrm{GeV})^2$	$-(2.0\ \mathrm{GeV})^2$
data	126.86 ± 0.41 [0.3%]	73.72 ± 0.33 [0.4%]	63.87 ± 0.28 [0.4%]
pQCD	149.57 ± 0.05 [0.0%]	201.23 ± 0.40 [0.2%]	210.74 ± 0.42 [0.2%
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000053	0.027501 ± 0.00060
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.007	128.972 ± 0.008
accuracy in 10^{-5}	4.39	5.66	6.37
improvement	4.3	2.4	2.1

$\Delta\alpha_{\rm had}(s_0) \times 10^4$. Future III: improving data as above plus pQCD to 0.1% in case b)

s_0	M_Z^2	$-(2.5\ \mathrm{GeV})^2$	$-(2.0\ \mathrm{GeV})^2$
data	126.86 ± 0.41 [0.3%]	73.72 ± 0.33 [0.4%]	63.87 ± 0.28 [0.4%]
pQCD	149.57 ± 0.05 [0.0%]	201.23 ± 0.20 [0.1%]	210.74 ± 0.21 [0.1%
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000040	0.027501 ± 0.00048
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.006	128.972 ± 0.007
accuracy in 10^{-5}	4.39	4.29	5.06
improvement	4.3	3.1	2.6

a) contributions to the Adler function

$$\Delta^{\mathrm{Adler=pQCD}} = \Delta\alpha^{(5)}_{\mathrm{had}}(-M_Z^2) - \Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)$$

up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) - \Delta\alpha^{(5)}_{\mathrm{had}}(-M_Z^2) = 0.000045 \pm 0.000002$$

which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs

$$\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$$

Since the term is small we can get it as well from direct data integration

$$\Delta\alpha_{\mathrm{had}}(-M_Z^2) = 276.44 \pm 0.64 \pm 1.78$$

$$\Delta\alpha_{\mathrm{had}}(\quad M_Z^2) = 276.84 \pm 0.64 \pm 1.90$$

and taking into account that errors are almost 100% correlated we have

$$\Delta\alpha_{\mathrm{had}}(M_Z^2) - \Delta\alpha_{\mathrm{had}}(-M_Z^2) = 0.40 \pm 0.12$$

ILC/FCC-ee community should actively support these activities as integral part of e^+e^--collider precision physics!!!

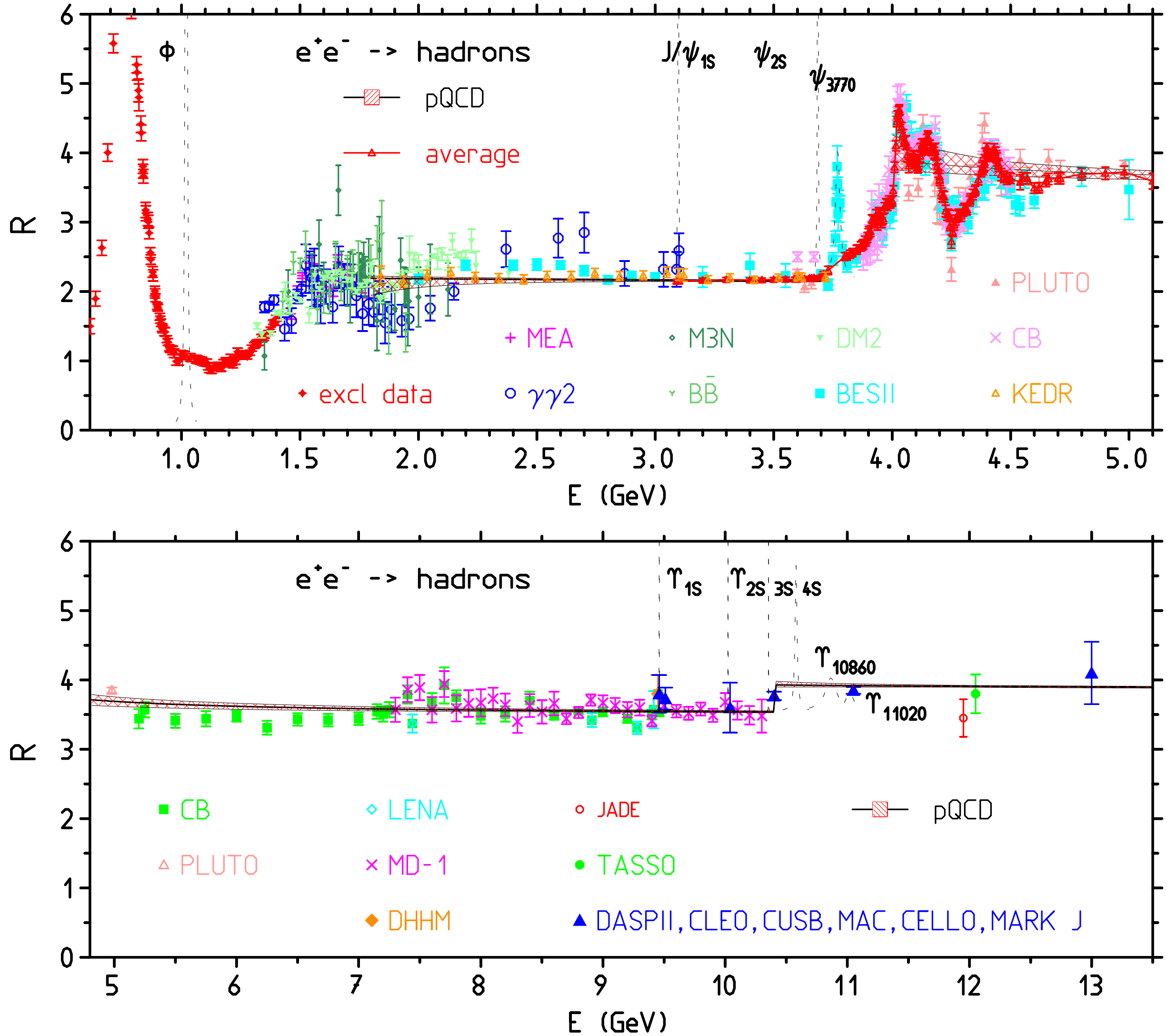
Remember: tremendous progress since middle of 90's

- Novosibirsk VEPP-2M: MD-1, CMD2, SND, KEDR; VEPP-2000: CMD3,SND
- Frascati DAFNE: KLOE
- Beijing BEPC: BES II, BESIII
- Cornell CESR: CLEO
- Stanford SLAC PEP-II: BaBar; KEK Tsukuba: Belle

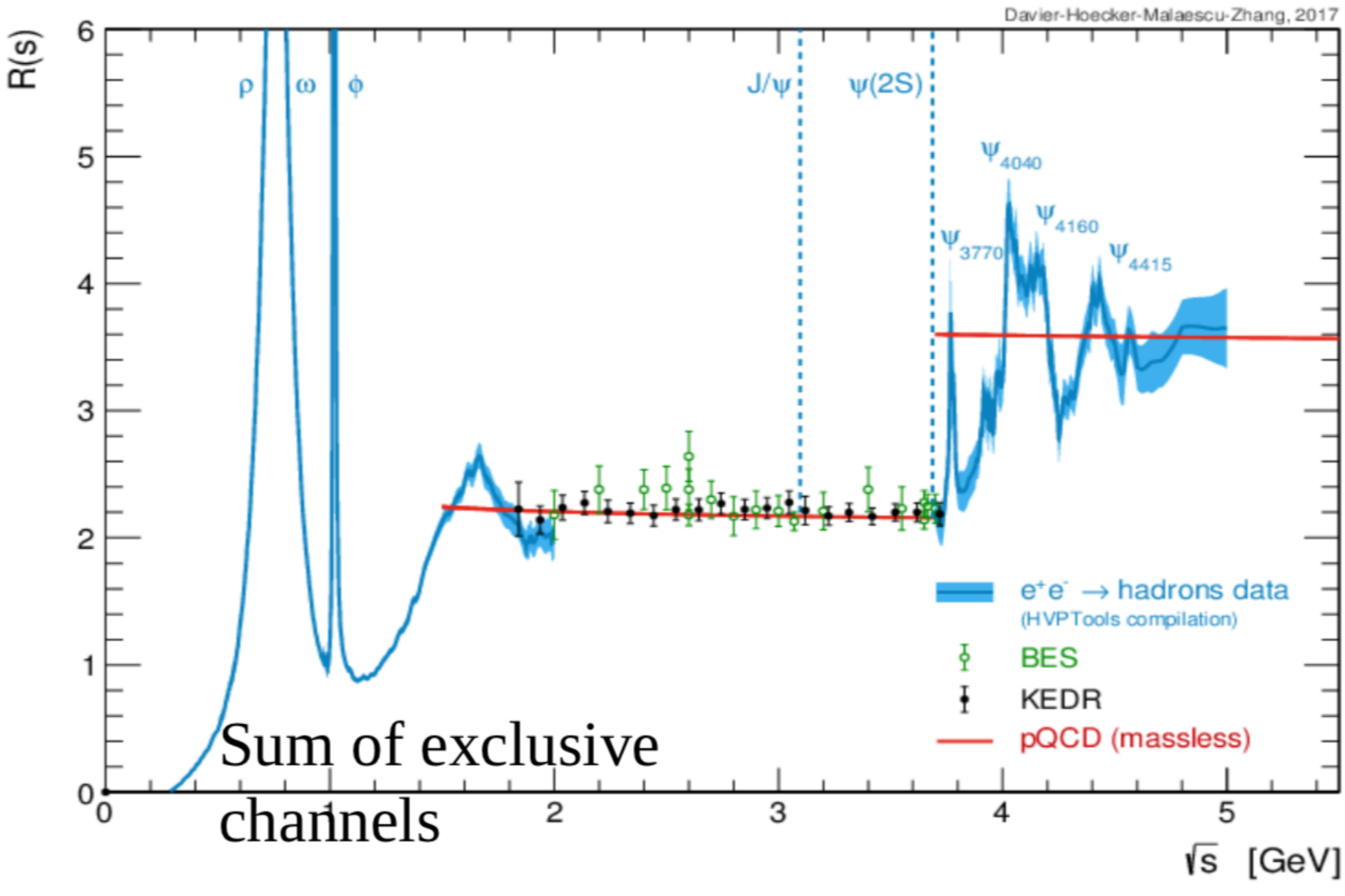
Many analyzes exploiting these results: Davier et al, Hagiwara et al., Burkhardt, Pietrzyk, Yndurain et al....

Indispensable for Muon $g-2$, indirect vs direct LEP Higgs mass etc. and future precision test at ILC/FCC-ee and new physics signals in precision observables. Impact for cosmology!

My $R(s)$ compilation vs. pQCD. Only stat errors shown.



pQCD 4-loops incl mass-effects Harlander,Steinhauser



Compilation by Davier et al.