α_s extractions at FCC-ee

11th FCC-ee workshop: Theory & Experiments

CERN, 8th Jan. 2019

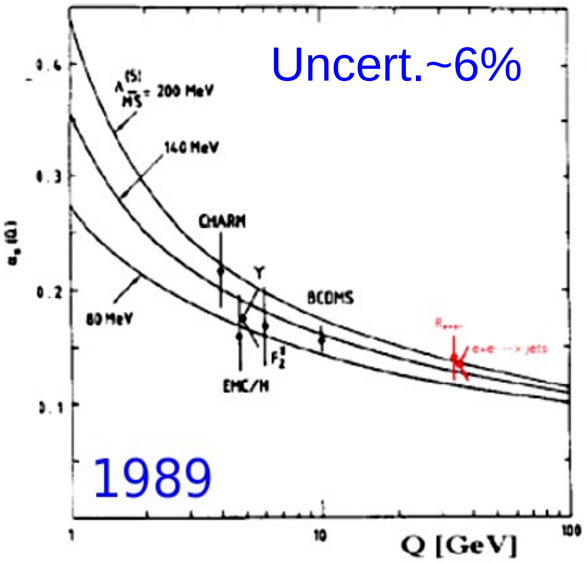
David d'Enterria

CERN

Mostly based on: *D. d'Enterria, P.Z. Skands (eds.), Proceeds. "High-precision α_s from LHC to FCC-ee", CERN Oct. 2015;* *arXiv:1512.05194* *(plus 2017, 2018 updates)*

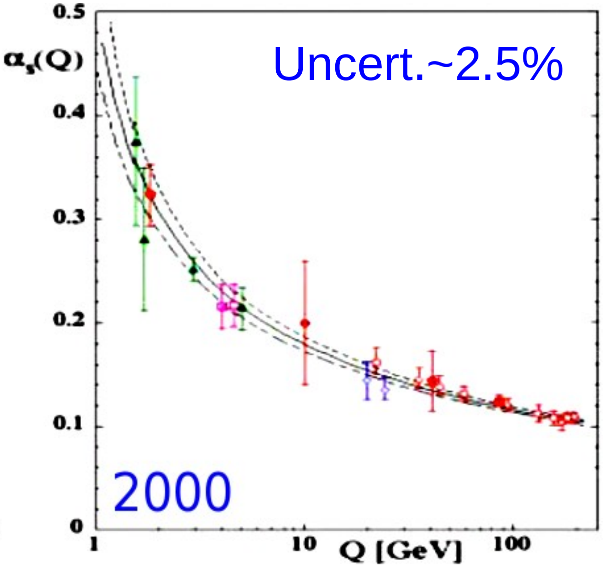
QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter in QCD in the $m_q \rightarrow 0$ limit.
- Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$; $\Lambda \sim 0.2$ GeV



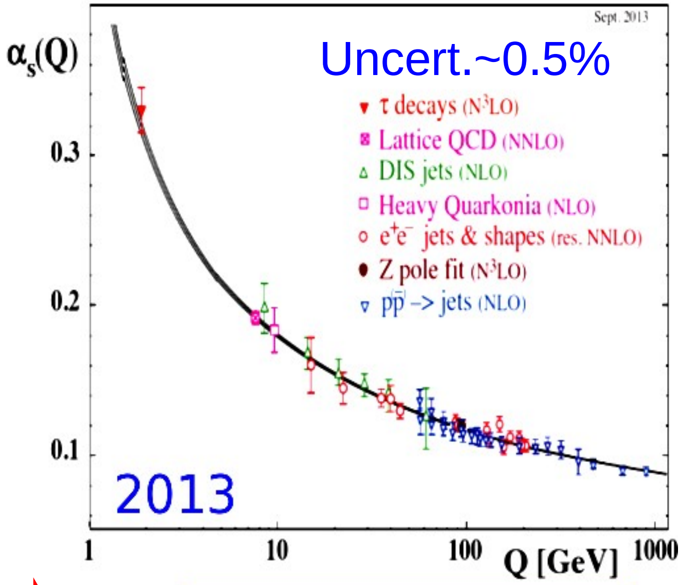
$\alpha_S(M_Z) = 0.110^{+0.006}_{-0.008}$ (NLO)

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$\alpha_S(M_Z) = 0.1184 \pm 0.0031$ (NNLO)

S. B. , J. Phys. G 26, 2000



$\alpha_s(M_Z) = 0.1185 \pm 0.0006$ (NNLO)

QCD coupling α_s

- Determines strength of the strong interaction between quarks & gluons.
- Single free parameter in QCD in the $m_q \rightarrow 0$ limit.
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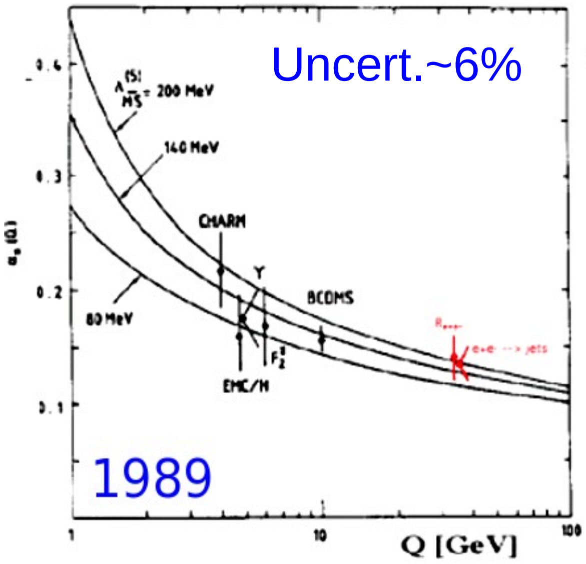

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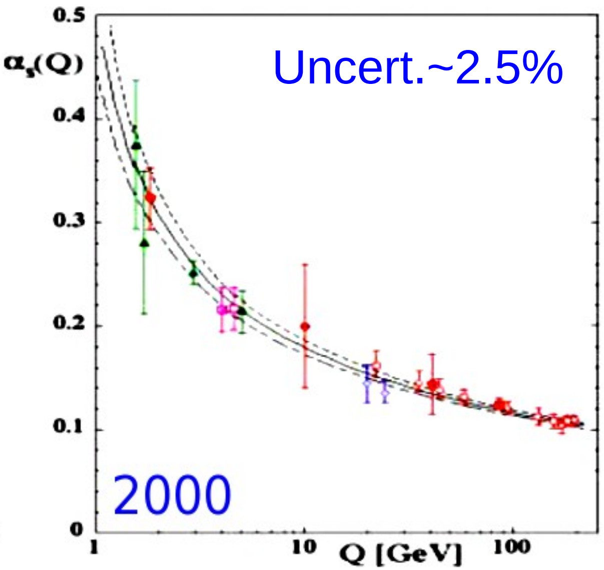

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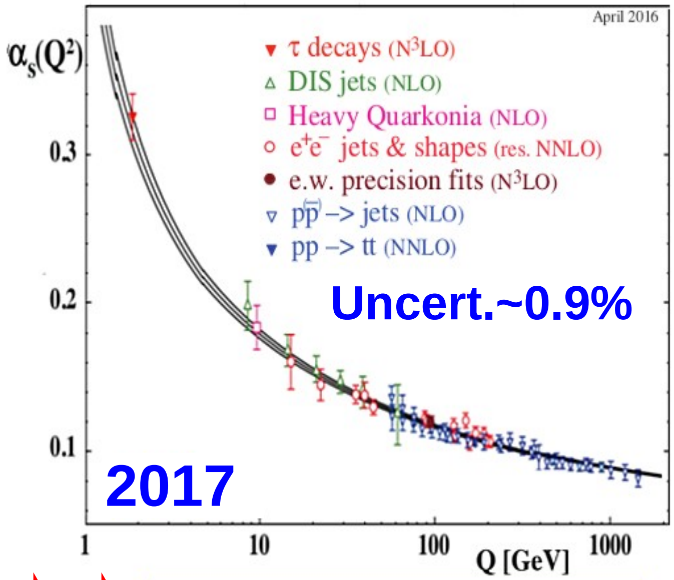


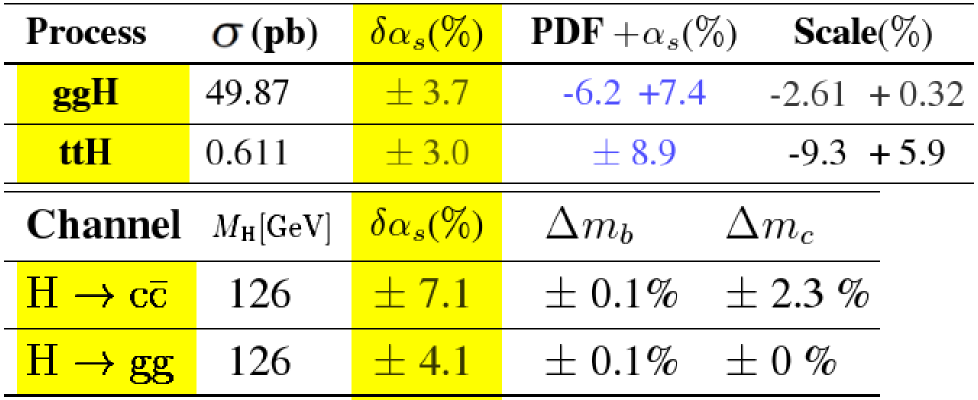
$$\alpha_s(M_Z) = 0.1181 \pm 0.0011 \text{ (NNLO)}$$

- Least precisely known of all interaction couplings !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

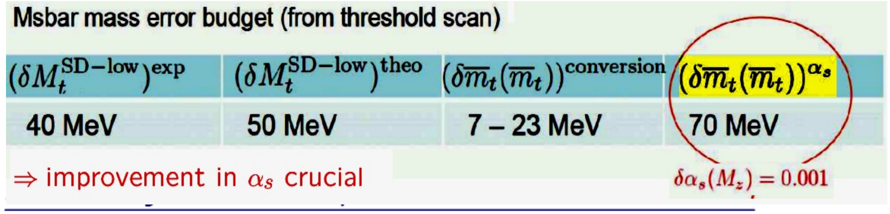
Importance of the QCD coupling α_s

- Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:



Process	σ (pb)	$\delta\alpha_s(\%)$	PDF $+\alpha_s(\%)$	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	± 3.0	± 8.9	-9.3 +5.9

Channel	M_{H}[GeV]	$\delta\alpha_s(\%)$	Δm_b	Δm_c
$\mathrm{H} \to c\bar{c}$	126	± 7.1	$\pm$ 0.1%	$\pm$ 2.3 %
$\mathrm{H} \to gg$	126	± 4.1	$\pm$ 0.1%	$\pm$ 0 %

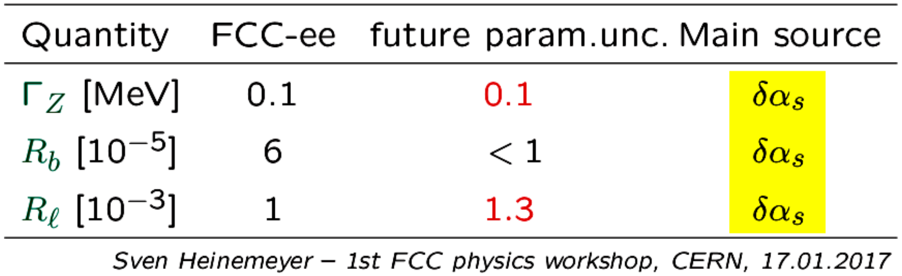


Msbar mass error budget (from threshold scan)

$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{exp}}$	$(\delta M_t^{\mathrm{SD-low}})^{\mathrm{theo}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\mathrm{conversion}}$	$(\delta\overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

⇒ improvement in α_s crucial

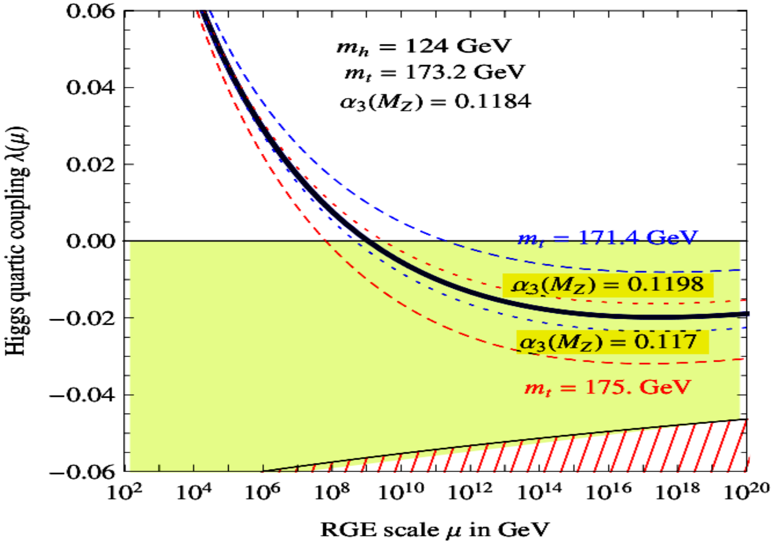
$\delta\alpha_s(M_z) = 0.001$

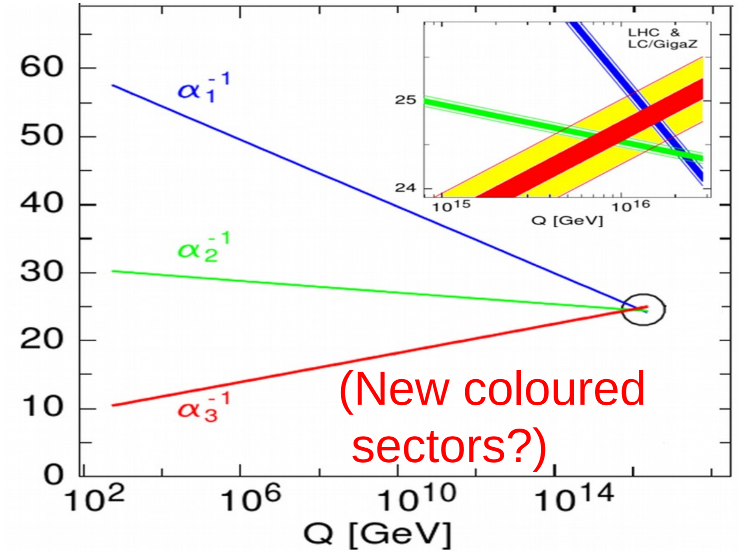


Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$

Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

- Impacts physics approaching Planck scale: EW vacuum stability, GUT



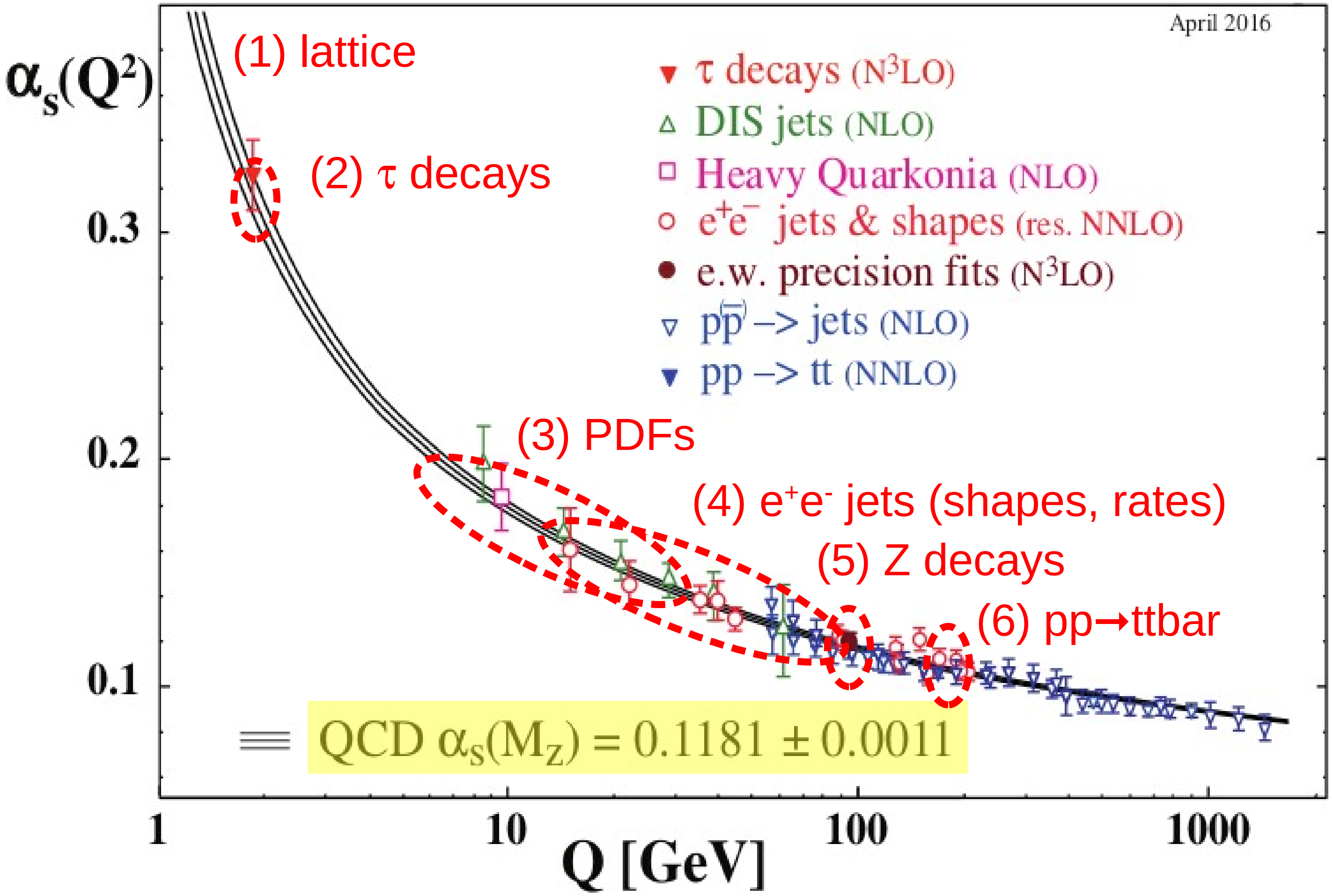


(New coloured sectors?)

World α_s determination

- Determined today by comparing 6 experimental observables to pQCD NNLO,N^{3}LO predictions, plus global average at the Z pole scale:

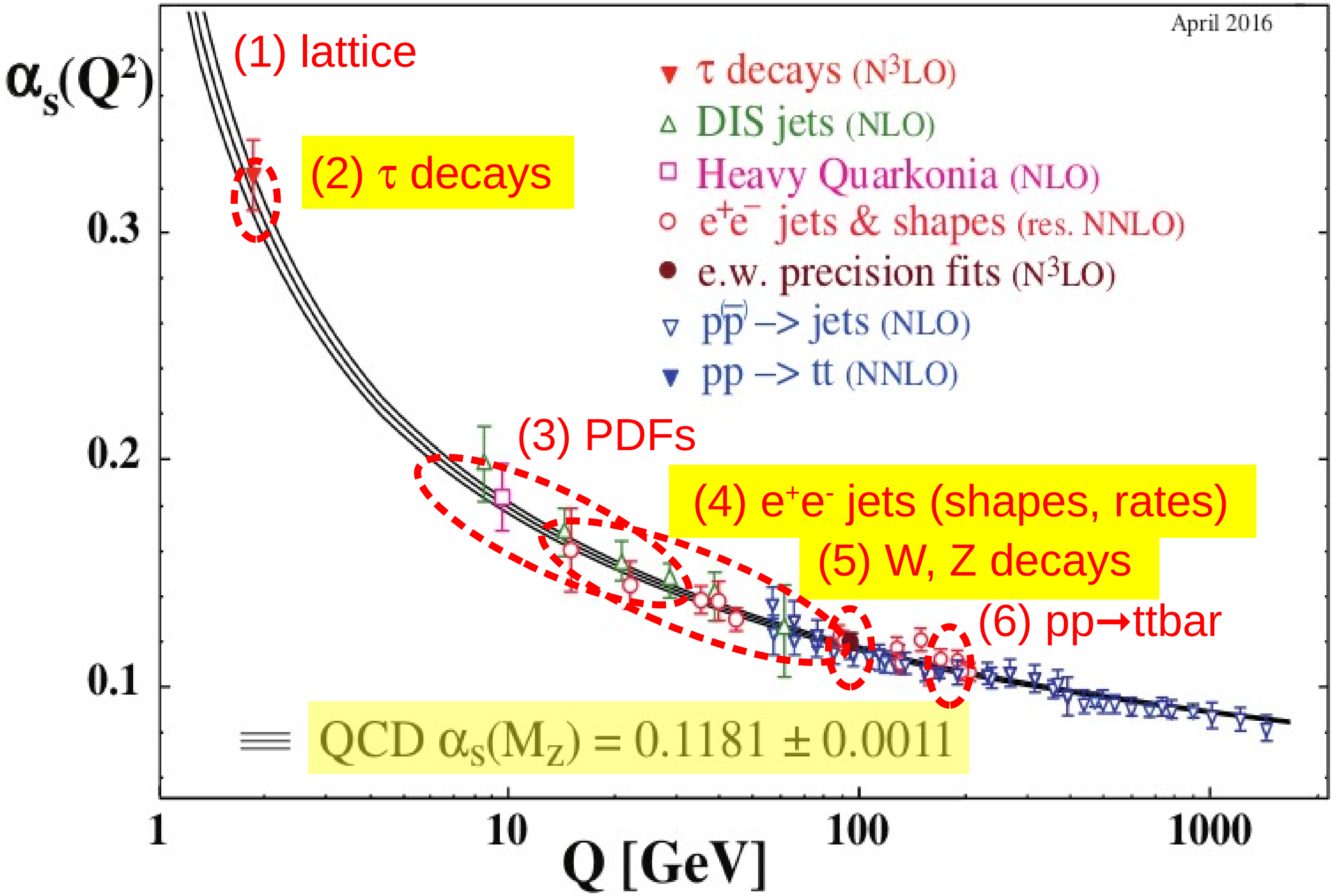
[Bethke/Dissertori/Salam]



World α_s determination (FCC-ee impact)

- Determined today by comparing 6 experimental observables to pQCD NNLO,N^{3}LO predictions, plus global average at the Z pole scale:

[Bethke/Dissertori/Salam]



(1) α_s from lattice QCD

- Comparison of short-distance quantities (Wilson loops, q static potential, vacuum polariz.,...) computed at NNLO in pQCD, to lattice QCD “data”:

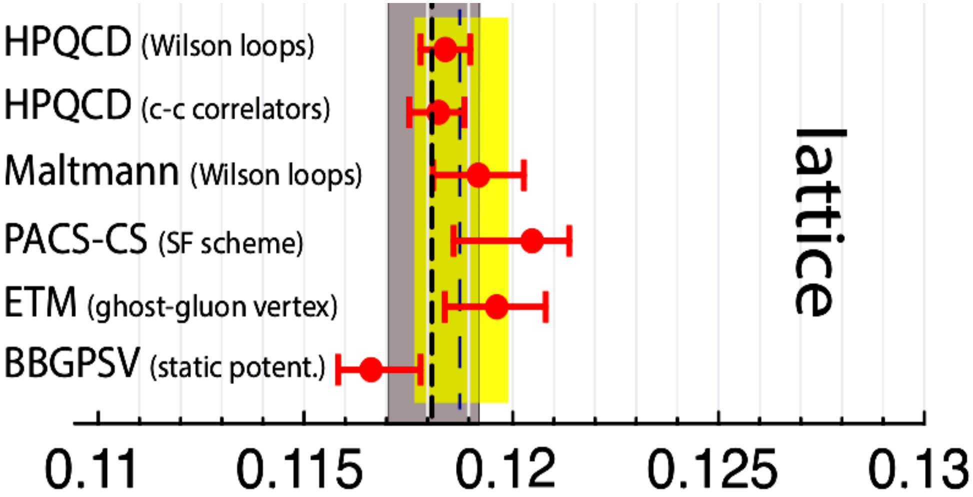
$$K^{\rm NP} = K^{\rm PT} = \sum_{i=0}^{n} c_i \alpha_s^i$$

[FLAG Collab. http://itpwiki.unibe.ch/flag]

- Currently, it's extraction with smallest uncertainties: ±1% (lattice spacing & statistics).

  Extracted value depends on observables:

  Uncertainty increased: 2013 (±0.4%) → 2017 (±1.0%)



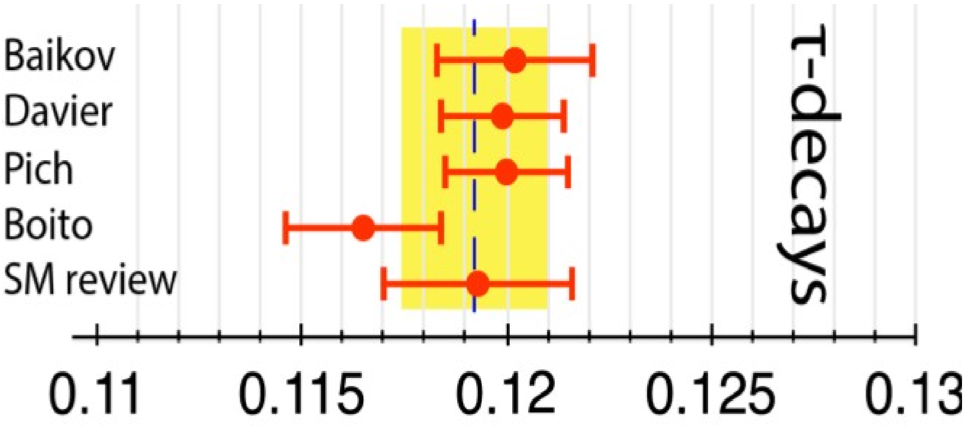
α_s = 0.1184 ± 0.0012 (±1.0%)

- Future prospects:
  - Uncertainty in α_s could be halved with (much) better numerical data.
  - Reaching ±0.1% requires 4th-loop perturbation theory

(2) α_s from hadronic τ-lepton decays

- Computed at N^{3}LO: $$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = S_{\rm EW} N_C \left(1 + \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)\right) + \delta_{\rm np})$$
- Experimentally: $R_{\tau,exp}$ = 3.4697 ± 0.0080 (±0.23%)
- Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections (note: $(\Lambda/m_\tau)^2$ ~2%), yield different results.

Uncertainty slightly increased: 2013 (±1.3%) → 2017 (±1.5%)



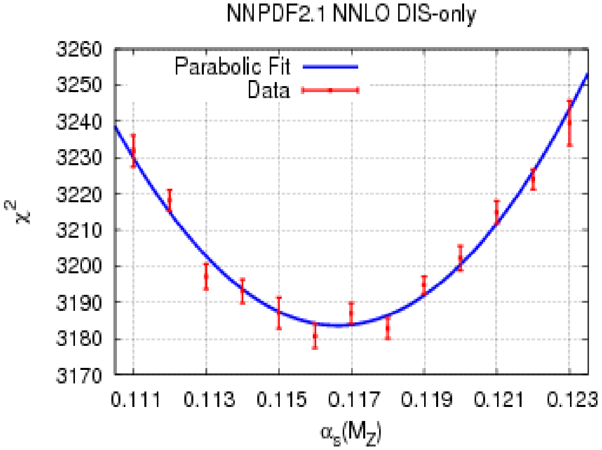
$\alpha_s(M_Z)$= 0.1192 ± 0.0018 (±1.5%)

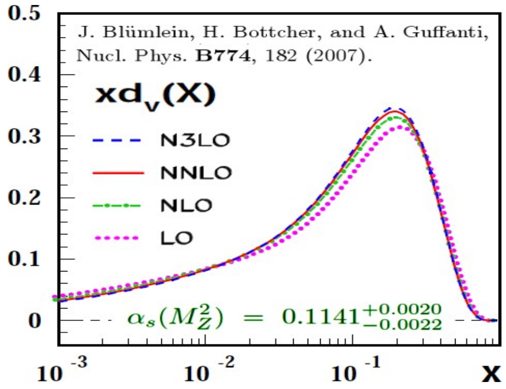
- Future prospects:
  - – Better TH understanding of FOPT vs CIPT differences.
  - – Better EXP spectral functions needed (high stats & better precision): B-factories (BELLE-II?)
  - – FCC-ee: High-stats, $\mathcal{O}(10^{11})$, from Z→ττ: **$\delta\alpha_s/\alpha_s$ < 1%**

(3) α_s from proton structure functions

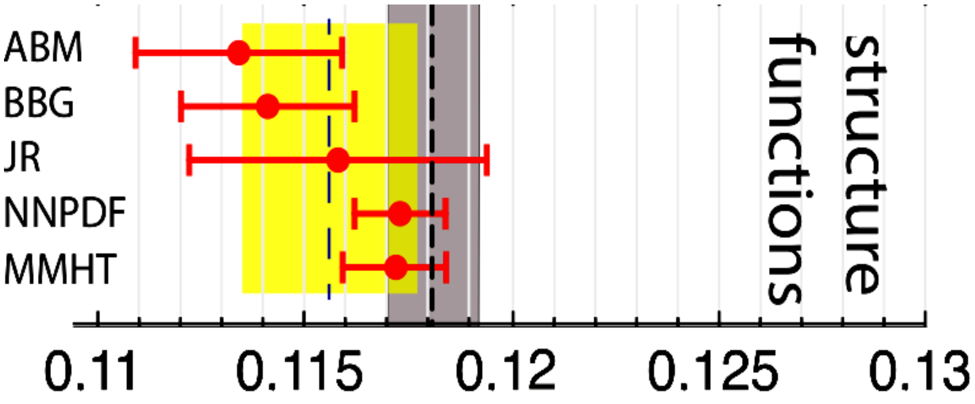
- Computed at N^{2,3}LO: $F_2(x,Q^2) = x \sum_{n=0}^{\infty} \boxed{\frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n}} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\, f_{i/p}\left(\frac{x}{z},\mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- Experimentally: Multiple $F_2(x,Q^2)$, $F^c_2(x,Q^2)$, $F_L(x,Q^2)$, PDFs(x,Q^2)
- Different approaches:

  Non-singlet fits,
  singlet+non-singlet fits,
  global fits of PDFs, …





Uncertainty ~stable:
2013 (±1.7%) → 2015 (±1.8%)

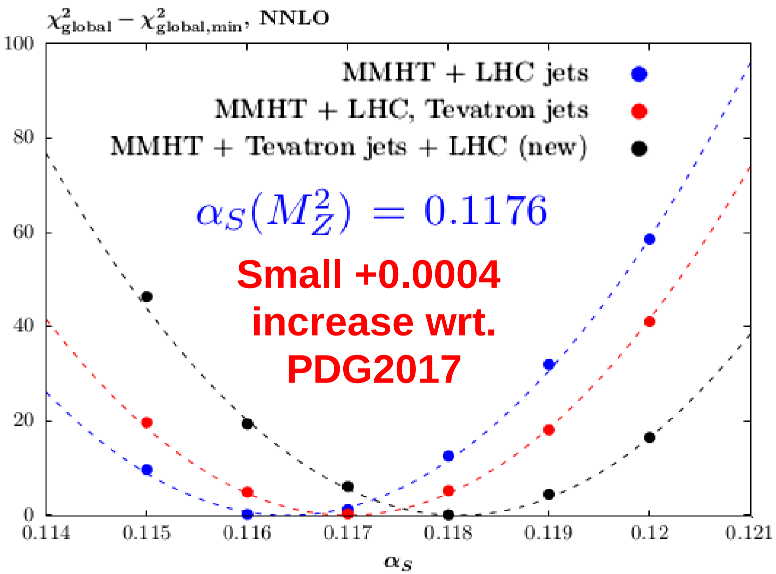


- Lowest central value among all extractions methods.

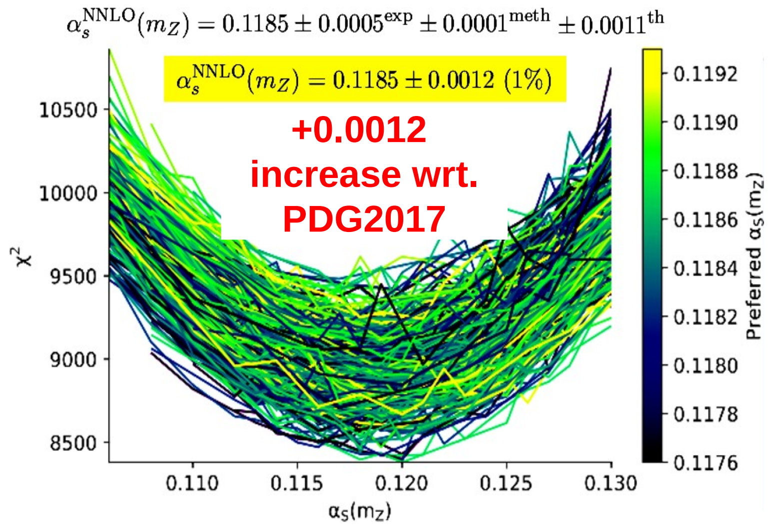
α_s = 0.1156 ± 0.0021 (±1.8%)

(3) α_s from proton structure functions (future)

- Computed at N^{2,3}LO: $F_2(x,Q^2) = x\sum_{n=0}^{\infty}\frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n}\sum_{i=q,g}\int_x^1 \frac{dz}{z}\,C_{2,i}^{(n)}(z,Q^2,\mu_R^2,\mu_F^2)\,f_{i/p}\left(\frac{x}{z},\mu_F^2\right)+\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- Updates by MMHT (R.Thorne, DIS'18) & NNPDF3.1 (N.Hartland, DIS'18)

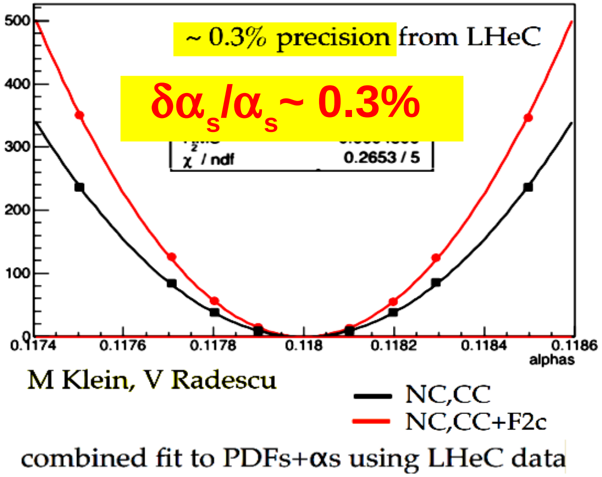


Small +0.0004 increase wrt. PDG2017



+0.0012 increase wrt. PDG2017

- Jets at NNLO included for the 1st time. Small central α_s value increase towards world average.
- Future: LHeC/FCC-eh stats. should lead to 0.3% uncertainty.



~ 0.3% precision from LHeC

$\delta\alpha_s/\alpha_s \sim 0.3\%$

M Klein, V Radescu

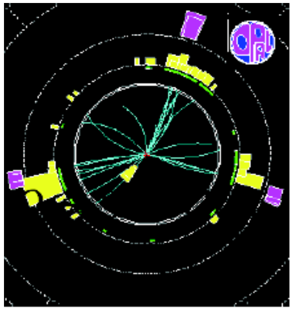
combined fit to PDFs+αs using LHeC data

(4) α_s from e^+e^- event shapes & jet rates

- Computed at $N^{2,3}LO+N^{(2)}LL$ accuracy.
- Experimentally (LEP):
  Thrust, C-parameter, jet shapes
  3-jet x-sections

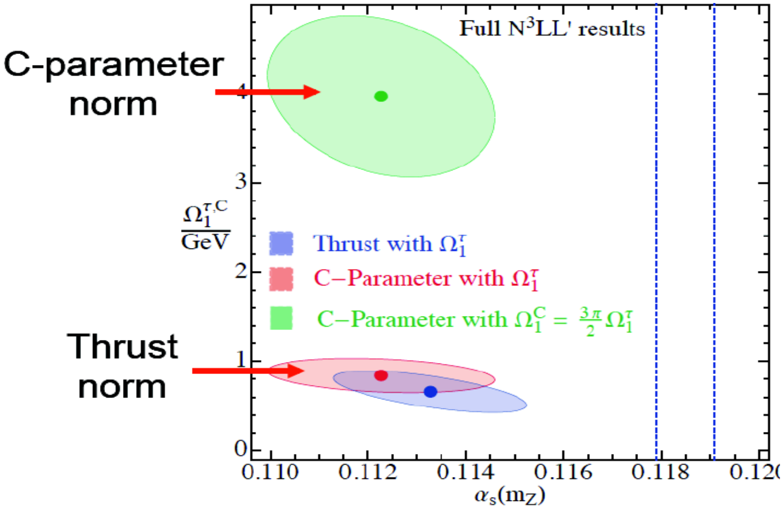
$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

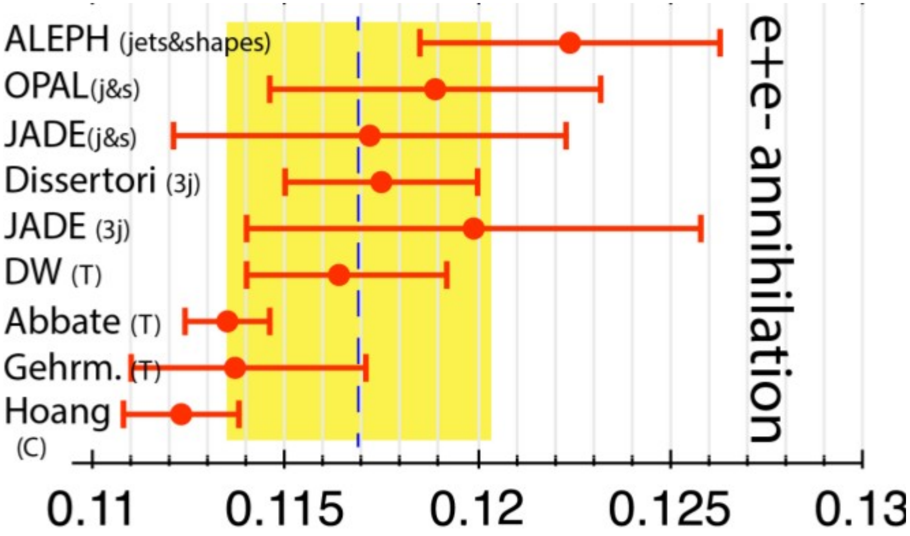
$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



OPAL 3 jet event

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:





$\alpha_s(M_Z) = 0.1169 \pm 0.0034$ (±2.9%)

(4) α_s from e^+e^- event shapes & jet rates (future)

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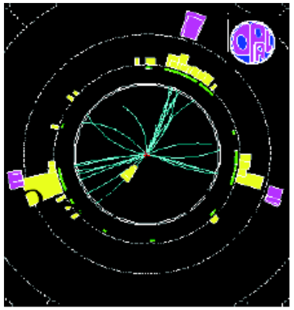

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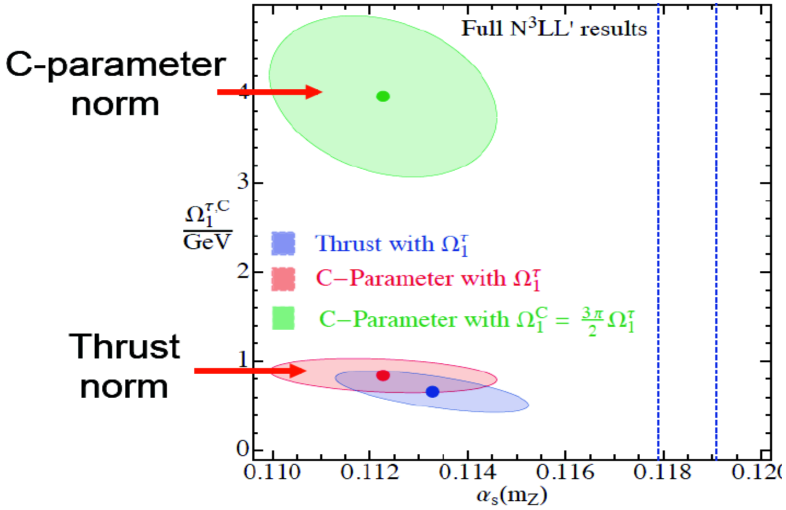
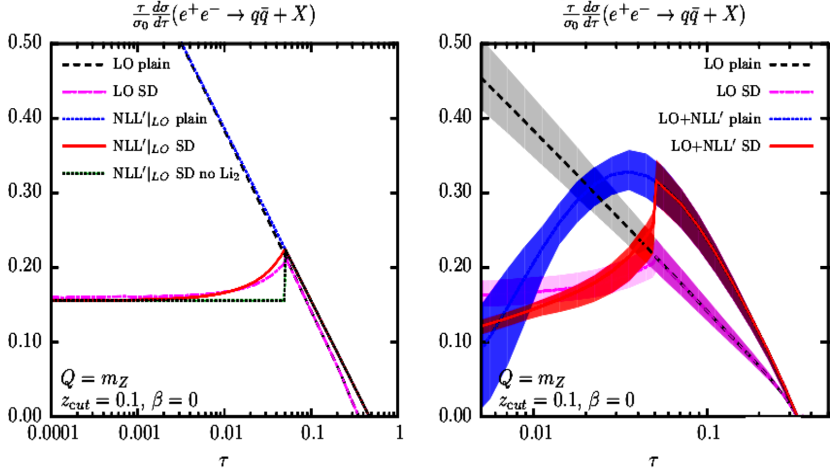


- Modern jet substructure techniques: "Soft drop" can help reduce non-pQCD corrections for thrust:



[Baron, Marzani, VT; '18]

- Future: **$\delta\alpha_s/\alpha_s$ <1%**

– New data: Lower-√s (Belle-II) for evt shapes, higher-√s (FCC-ee) for rates

– TH: Improved hadronization for shapes, (N^{2,3}LL) resummation for rates.

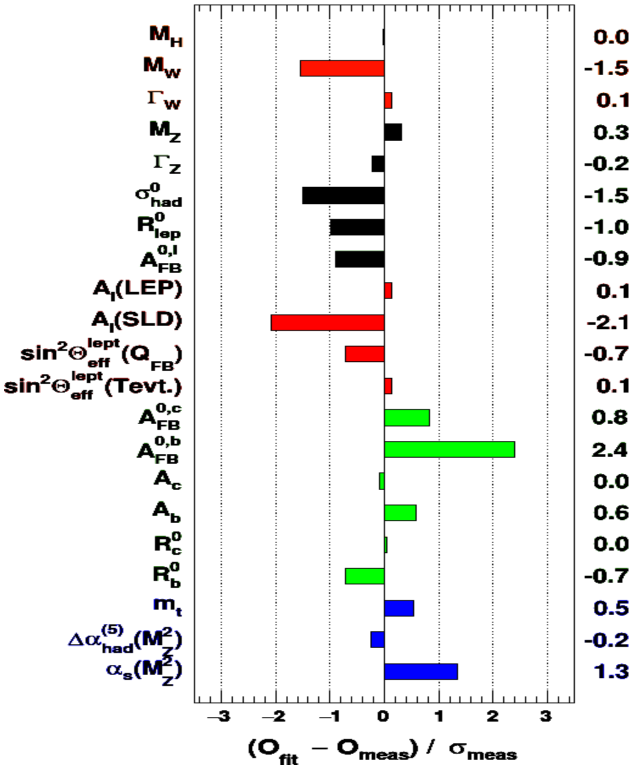
(5) α_s from hadronic Z decays

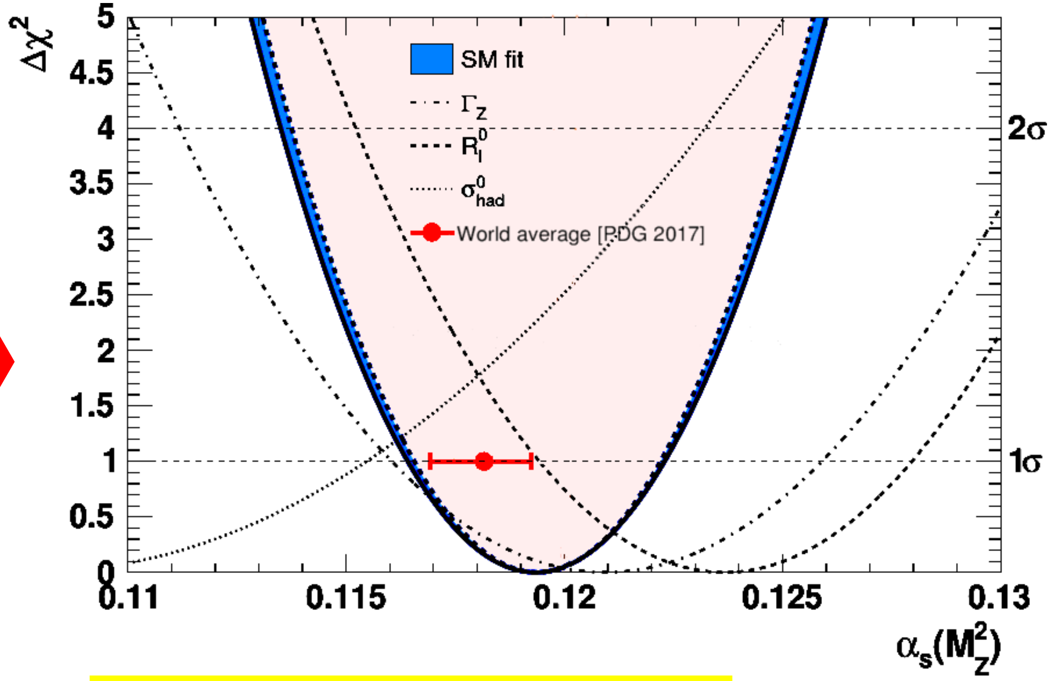
- Computed at N^{3}LO: $R_l^0 \equiv \frac{\Gamma(Z\to h)}{\Gamma(Z\to l)} = R_Z^{\mathrm{EW}} N_C \left(1+\sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)+\delta_{\mathrm{m}}+\delta_{\mathrm{np}}\right)$
- LEP: Extraction from fits to 3 Z-peak pseudo-observables:

$$R_\ell^0=\frac{\Gamma_{\mathrm{had}}}{\Gamma_\ell},\ \sigma_{\mathrm{had}}^0=\frac{12\pi}{m_Z}\frac{\Gamma_e\Gamma_{\mathrm{had}}}{\Gamma_Z^2},\ \sigma_\ell^0=\frac{12\pi}{m_Z}\frac{\Gamma_\ell^2}{\Gamma_Z^2}$$

Γ_Z = 2.4952±0.0023 GeV (±0.1%)

After Higgs discovery, α_s can be also directly determined from full fit of SM:

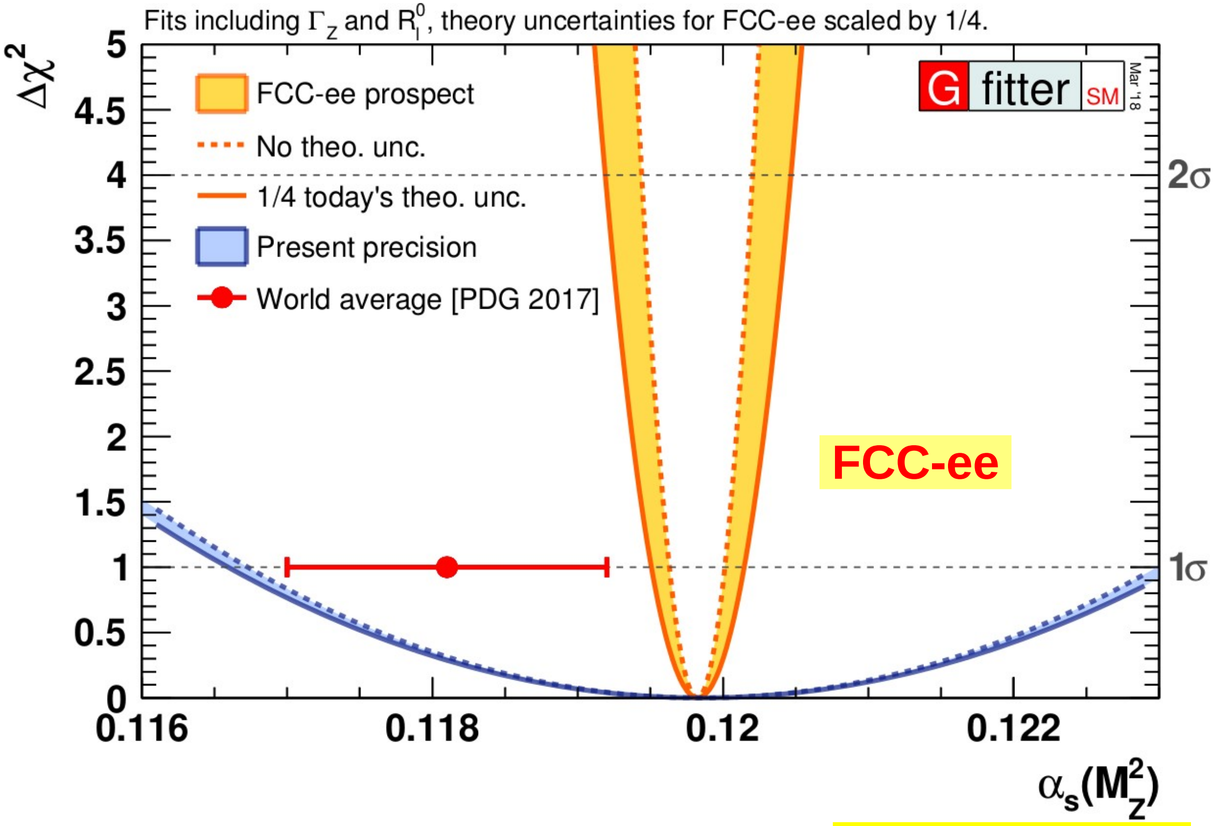




α_s (M_z) = 0.1196 ± 0.0030 **(±2.5%)**

(5) α_s from hadronic Z decays (future)

- Computed at N^{3}LO: $R_l^0 \equiv \frac{\Gamma(Z\to \mathrm{h})}{\Gamma(Z\to l)} = R_Z^{\mathrm{EW}} N_C\left(1+\sum_{n=1}^{4} c_n\left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)\right) + \delta_\mathrm{m} + \delta_\mathrm{np})$
- FCC-ee: Extraction from fits to 3 Z-peak pseudo-observables.



– Huge Z stats (×10^5 LEP) will lead to: **$\delta\alpha_s/\alpha_s < 0.2\%$**

– Full SM-fit extraction: Parallel reduction of parametric ($\sin^2\theta_{\mathrm{eff}}, m_W, m_{\mathrm{top}}$) TH uncertainties.

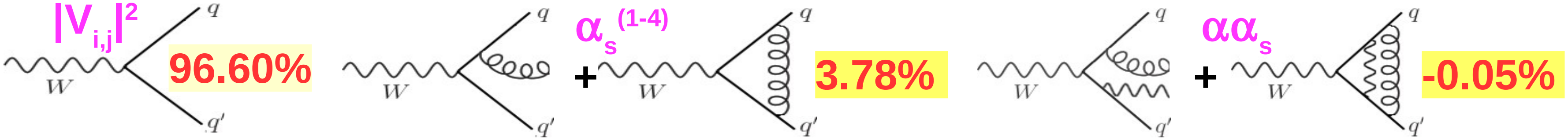
(5b) α_s from hadronic W decays

[D.d'E, M.Srebre, arXiv:1603.06501]

- Width (BR) known at N^3LO (NNLO). Small sensitivity to α_s (beyond Born)

$$\Gamma_{\mathrm{W,had}} = \frac{\sqrt{2}}{4\pi} G_{\mathrm{F}} m_{\mathrm{W}}^3 \sum_{\text{quarks i,j}} |V_{\mathrm{i,j}}|^2 \left[1 + \sum_{\mathrm{k=1}}^{4} \left(\frac{\alpha_{\mathrm{s}}}{\pi}\right)^k + \delta_{\mathrm{electroweak}}(\alpha) + \delta_{\mathrm{mixed}}(\alpha\alpha_{\mathrm{s}}) \right]$$

[EWK: -0.35%]



$|V_{i,j}|^2$ 96.60%

$\alpha_s^{(1-4)}$ + 3.78%

$\alpha\alpha_s$ + -0.05%

- TH improvements: finite quark-mass effects included (LO), updated PDG parameters, careful evaluation of parametric ($V_{i,j}$, m_W) & theoretical uncert.

- Calculation dominated by ±1.5% parametric (mostly V_{cs}) uncertainty:

Γ_W(MeV) = $1428.67 \pm 22.40_{(par)} \pm 0.04_{(th)}$ (exp. CKM)

$1411.40 \pm 0.96_{(par)} \pm 0.04_{(th)}$ (CKM=1)

$BR_W = \Gamma_W/\Gamma_{tot} = 0.6820 \pm 0.0110_{(par)} \pm 0.0002_{(th)}$ (exp. CKM)

$0.6742 \pm 0.0001_{(par)} \pm 0.0002_{(th)}$ (CKM=1)

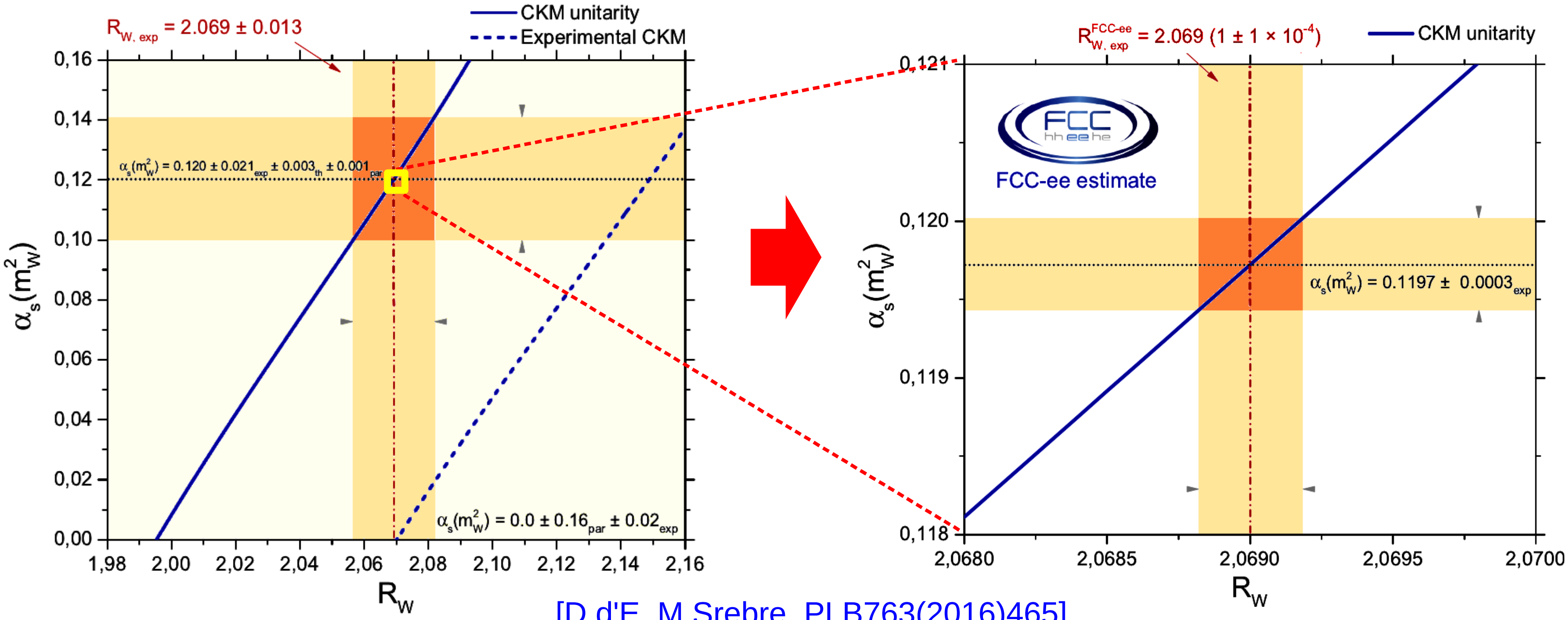
- TH uncertainty (missing α_s^5 terms, non-pQCD $(\Lambda_{QCD}/m_W)^4$ power corrs., finite quark masses beyond LO, CKM matrix renorm. scheme): ±0.03%

(5b) α_s from hadronic W decays (future)

- Computed at N^{2,3}LO: $\Gamma_{\rm W,had} = \frac{\sqrt{2}}{4\pi} G_{\rm F} m_{\rm W}^3 \sum_{\rm quarks\ i,j} |V_{\rm i,j}|^2 \left[1 + \sum_{\rm k=1}^{4} \left(\frac{\alpha_{\rm s}}{\pi}\right)^k + \delta_{\rm electroweak}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_{\rm s})\right]$
- LEP: Γ_W = 1405±29 MeV (±2%), BR$_W$ = 0.6741±0.0027 (±0.4%)

  Extraction with large exp. & parametric (CKM V$_{cs}$) uncertainties today:

  α_s (M$_z$) = 0.117 ± 0.040 **(±35%)**



[D.d'E, M.Srebre, PLB763(2016)465]

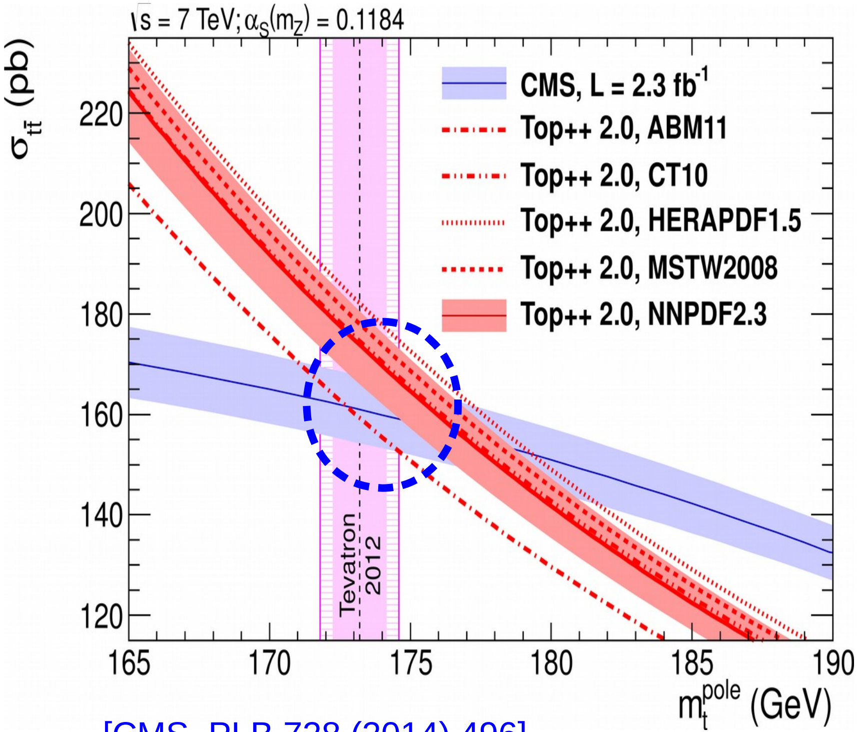
- FCC-ee: – Huge W stats (×10^4 LEP) will lead to: **$\delta\alpha_s/\alpha_s$ < 0.3%**

  – TH (param.) uncertainty: |δV$_{cs}$| to be significantly improved (10^{-4})

(6) α_s from top-pair p-p cross sections

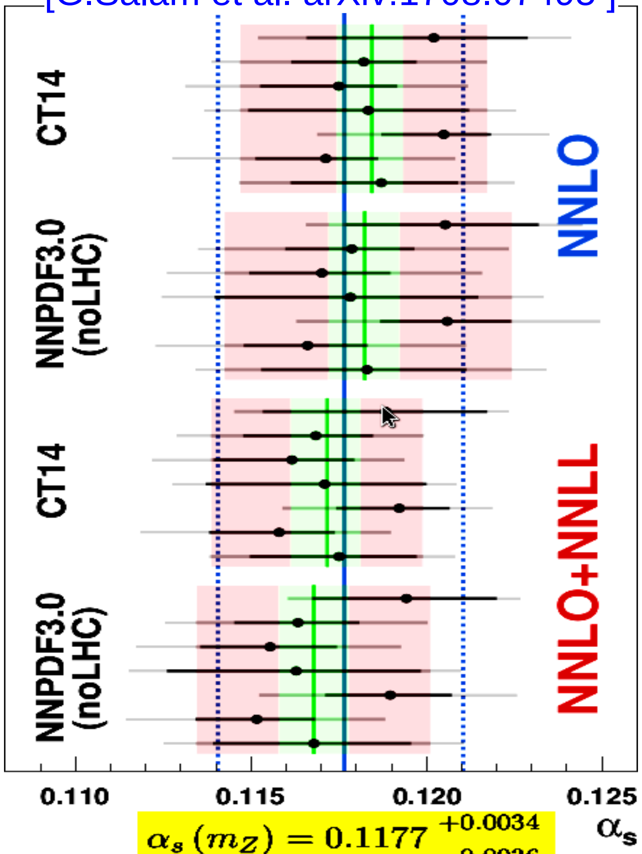
- Total top-antitop cross section (known at NNLO+NNLL) is the 1^{st} p-p collider observable to constrain α_s at NNLO accuracy:

Data-theory x-section comparison for varying PDF+α_s as a function of m_{top}:



[CMS, PLB 728 (2014) 496]

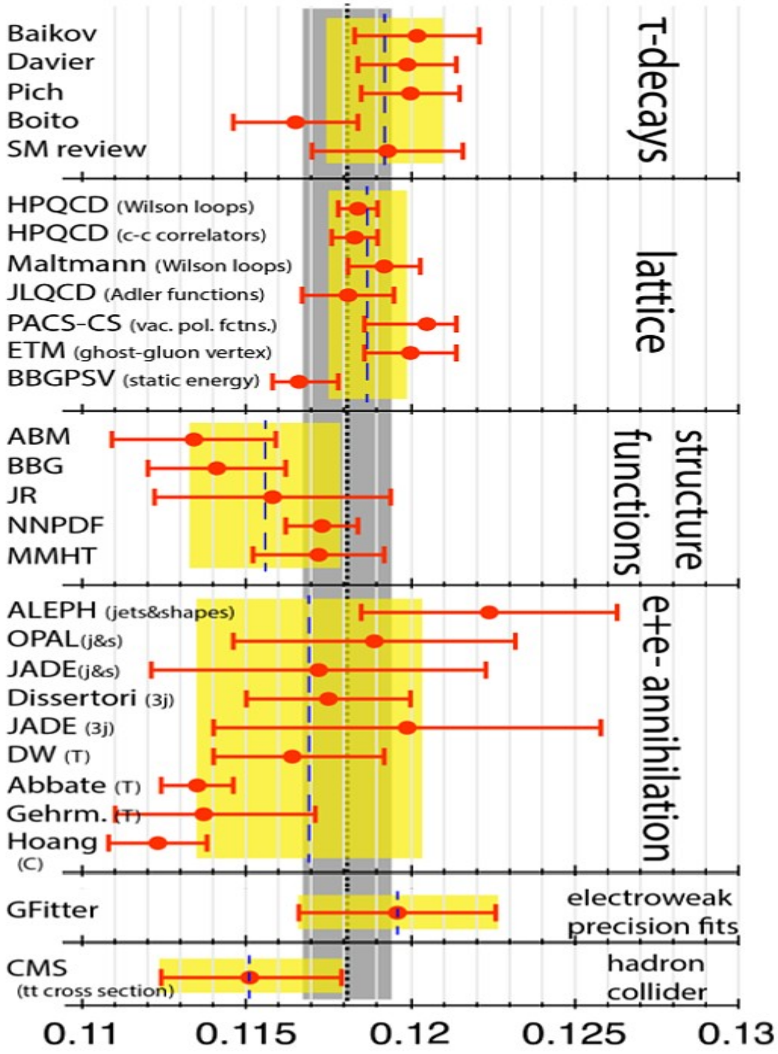
[G.Salam et al. arXiv:1708.07495]



$$\alpha_s(m_Z) = 0.1177^{+0.0034}_{-0.0036}$$

Inclusion of full set of t-tbar data yields $\alpha_s(m_Z)$ with **±2.9%** uncertainty

Current PDG α_s world average (NNLO)



class averages:

$\alpha_s(M_Z) = 0.1192 \pm 0.0018$ (±1.5%)

$\alpha_s(M_Z) = 0.1184 \pm 0.0012$ (±1.0%)

$\alpha_s(M_Z) = 0.1156 \pm 0.0021$ (±1.8%)

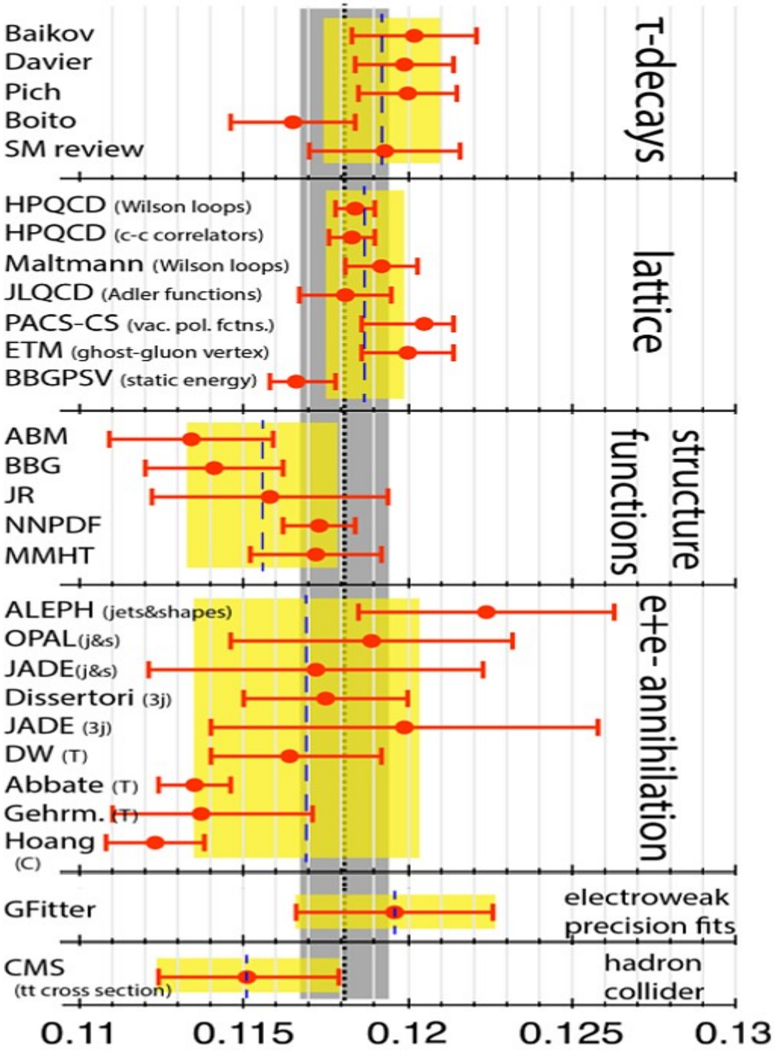
$\alpha_s(M_Z) = 0.1169 \pm 0.0034$ (±2.9%)

$\alpha_s(M_Z) = 0.1196 \pm 0.0030$ (±2.5%)

$\alpha_s(M_Z) = 0.1151 \pm 0.0028$ (±2.5%)

unweighted χ^2 average: $\alpha_s(M_Z) = 0.1181 \pm 0.0011$ (±0.9%)

“Updated” α_s world average (NNLO)



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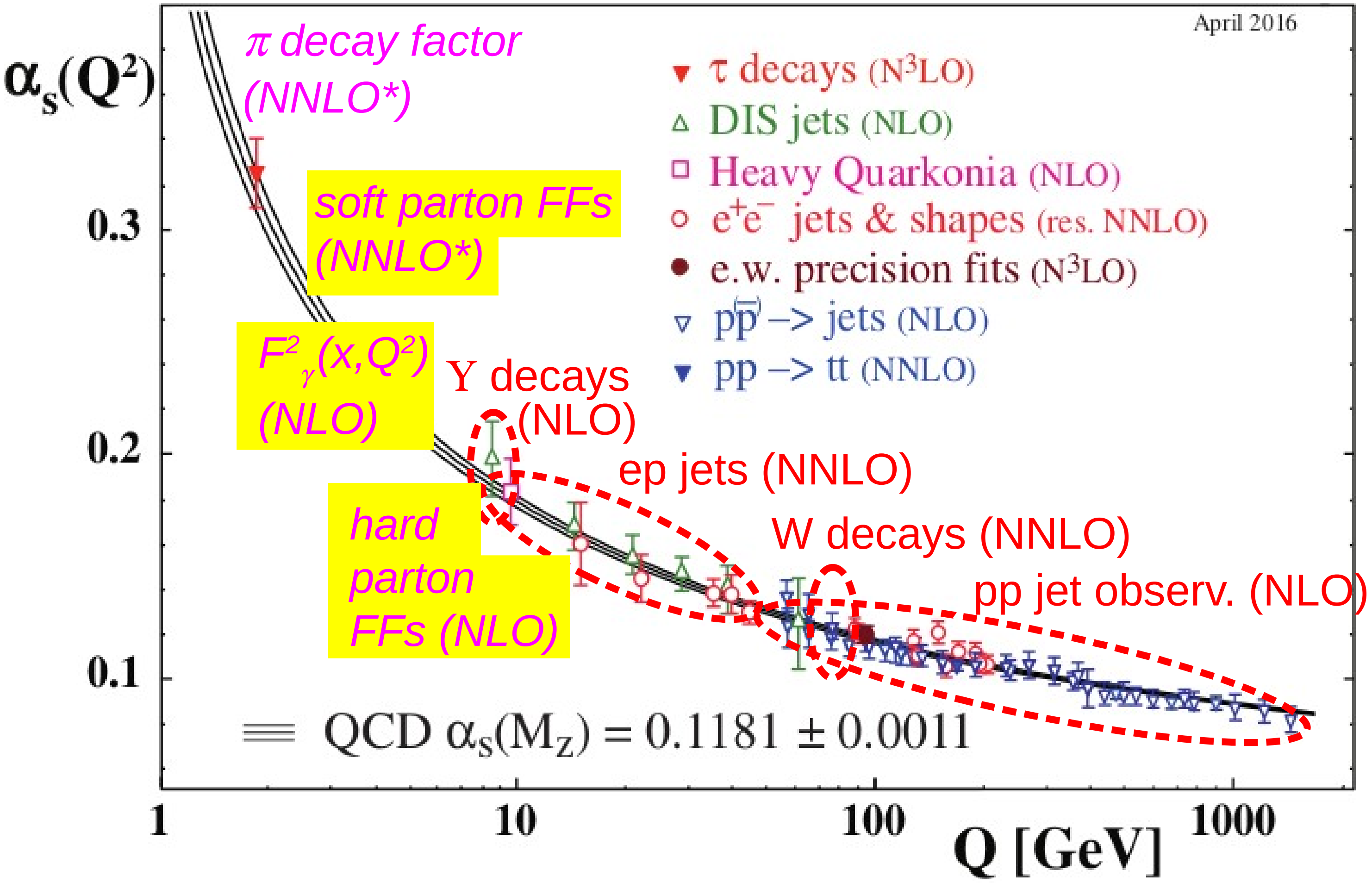
$\alpha_s(M_Z) = 0.1151 \pm 0.0028$ (±2.5%)

$\alpha_s(M_z) = 0.1177 \pm 0.0035$ (±2.9%)

unweighted χ^2 average: $\alpha_s(M_Z) = 0.1182 \pm 0.0011$ (±0.9%)

Other α_s extractions (not *yet* in world average)

- There exist at least 8 other classes of observables, computed today at lower accuracy (NLO, NNLO*), that could be used to extract α_s. Three of them to be exploited at FCC-ee:



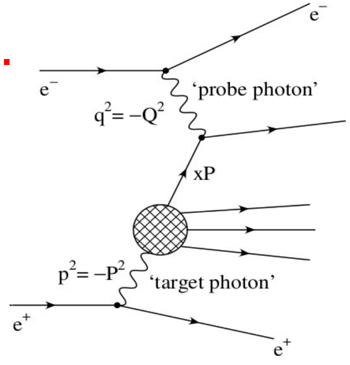
α_s from γ QCD structure function

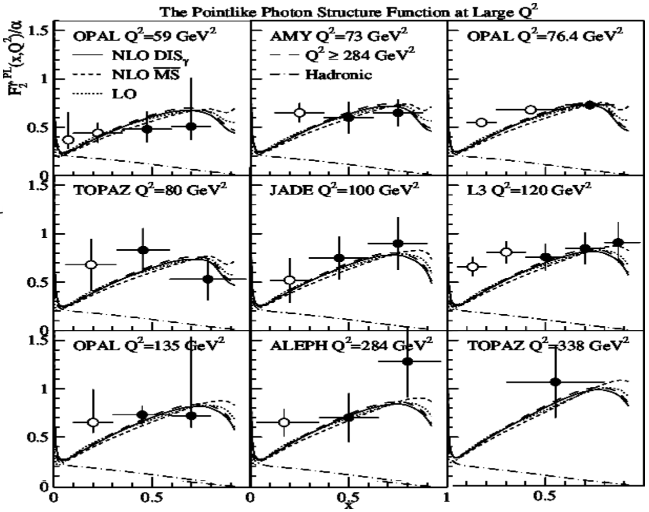
- Computed at NNLO: $$\int_0^1 dx F_2^\gamma(x,Q^2,P^2) = \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{\frac{4\pi}{\alpha_s(Q^2)}c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi}c_{NNLO} + \mathcal{O}(\alpha_s^2)\right\}$$
- Poor $F^2_\gamma(x,Q^2)$ experimental measurements:
- Extraction (NLO) with large exp. uncertainties today:

α_s (M_z) = 0.1198 ± 0.0054

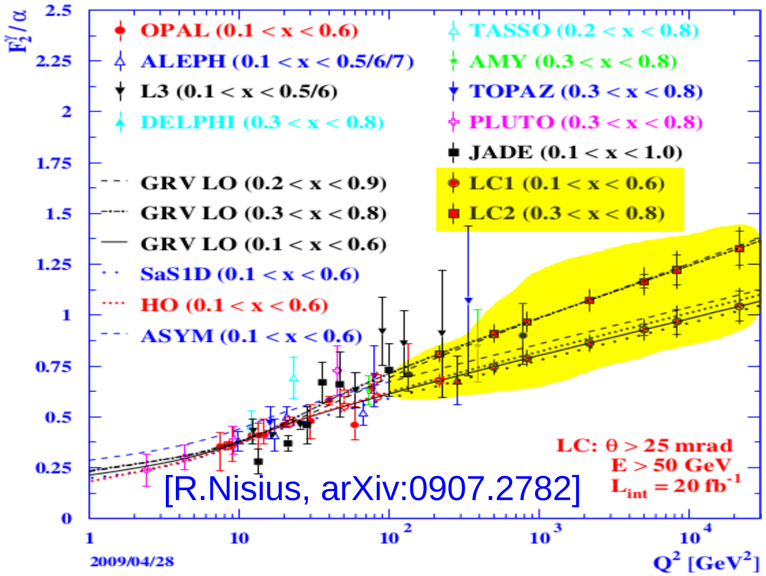
(±4.5%)

[M.Klasen et al. PRL89 (2002)122004]





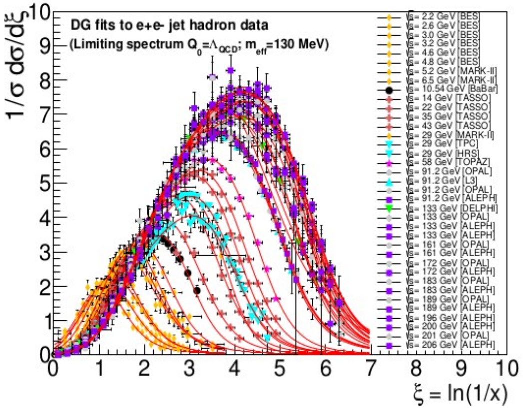
- Future prospects:
  - – Fit with NNLO F^2_γ evolution (ongoing)
  - – Better e^+e^- data badly needed: Belle-II?
  - – Dedicated studies at ILC exist:
  - – Huge $\gamma\gamma$ (EPA) stats at FCC-ee will lead to: **$\delta\alpha_s/\alpha_s$ < 1%**

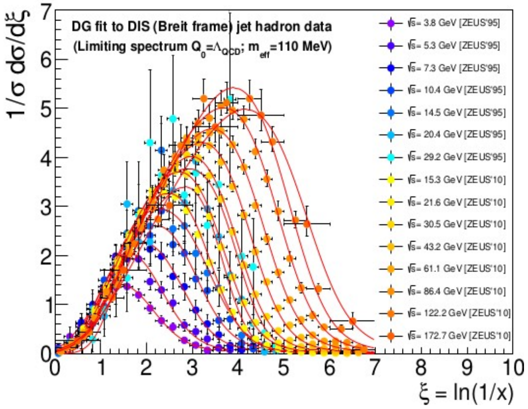


[R.Nisius, arXiv:0907.2782]

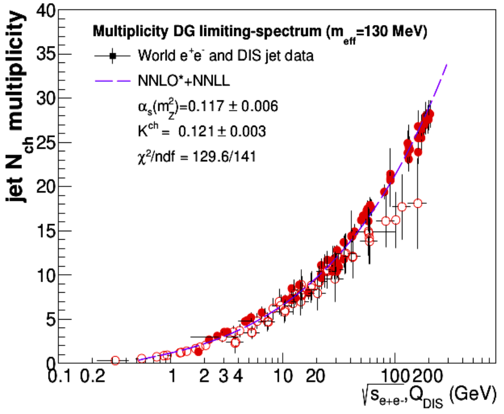
α_s from parton-to-hadron FFs

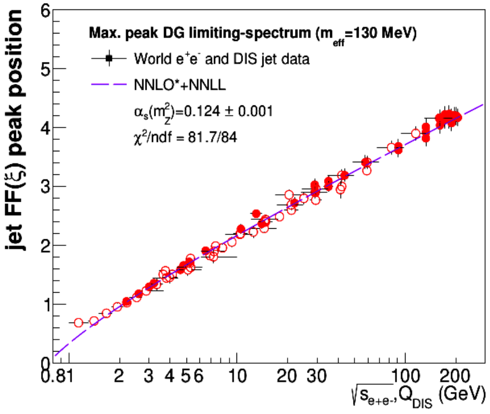
- Soft Fragmentation Functions (NNLO*+NNLL):
- Combined fit of energy evolution of HBP moments:





α_s (NNLO*+NNLL)=0.1205±0.0010$^{+0.0022}$(±2%)

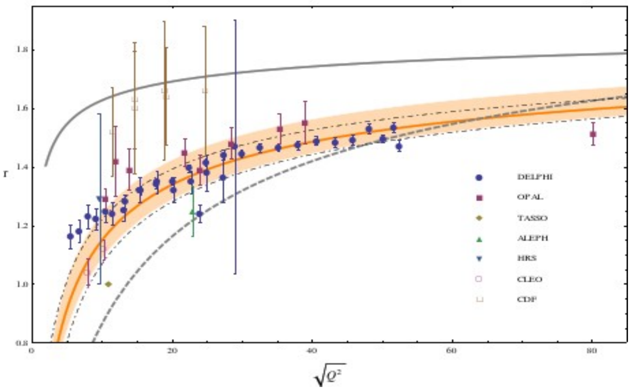




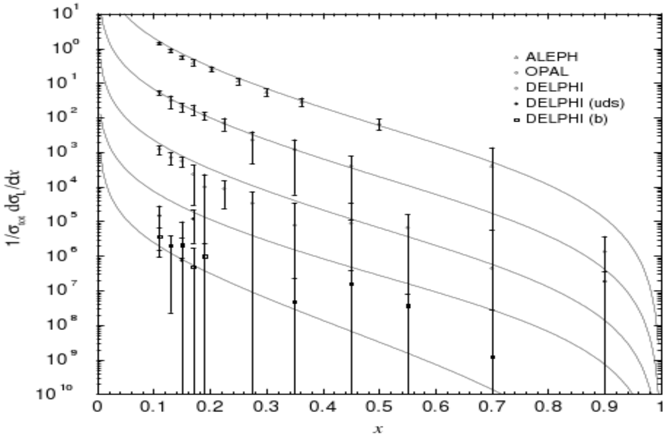
[D.d'E., R.Perez-Ramos, arXiv:1505.02624]

- Hard Frag. Functions (NLO):

$r \quad \langle n_h\rangle_g, \langle n_h\rangle_q$



α_s (NNLO)=0.1199±0.0044 (±3.6%)



[AKK, B.Kniehl et al., NPB 803(2008)42]

α_s (NLO)=0.1176±0.0055±0.0008 (±4.7%)

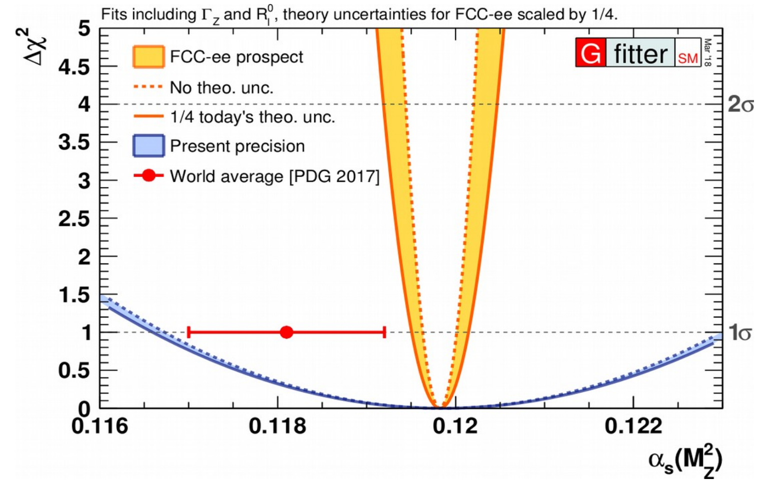
- Future prospects: – Full-NNLO fit for FF (& moments) evolution.
  – FCC-ee: Huge hadron stats. up to ×2 LEP E_{jet}

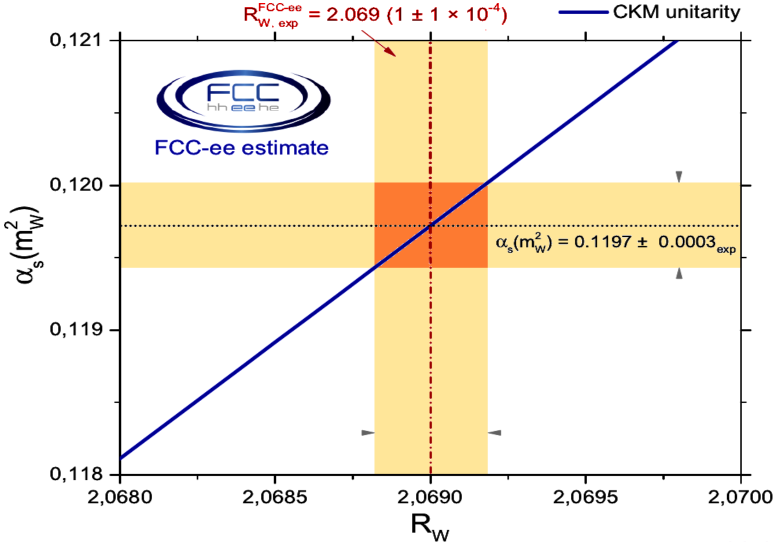
$\delta\alpha_s/\alpha_s$ < 1%

Summary: α_s at FCC-ee

- World-average QCD coupling at N^{2,3}LO:
  - *Determined today from 6 observables with ~1% uncertainty (least well-known coupling).*
  - *Impacts all LHC (& FCC-hh) QCD x-sections & decays.*
  - *Role beyond SM: GUT, EWK vacuum stability New colored sectors?*
- New extractions/updates: PDF fits, e-p jets, full pp→ttbar
- Reduction of hadronization & resummation uncertainties:
  - New TH developments needed
  - New precise e^+e^- data needed
- Other extraction methods proposed: TH work towards NNLO accuracy.

- ~0.1% uncertainty, combining Z,W,τ hadronic decays (plus $F^2_\gamma(x,Q^2)$ & FFs fits) ONLY possible with machine like FCC-ee:



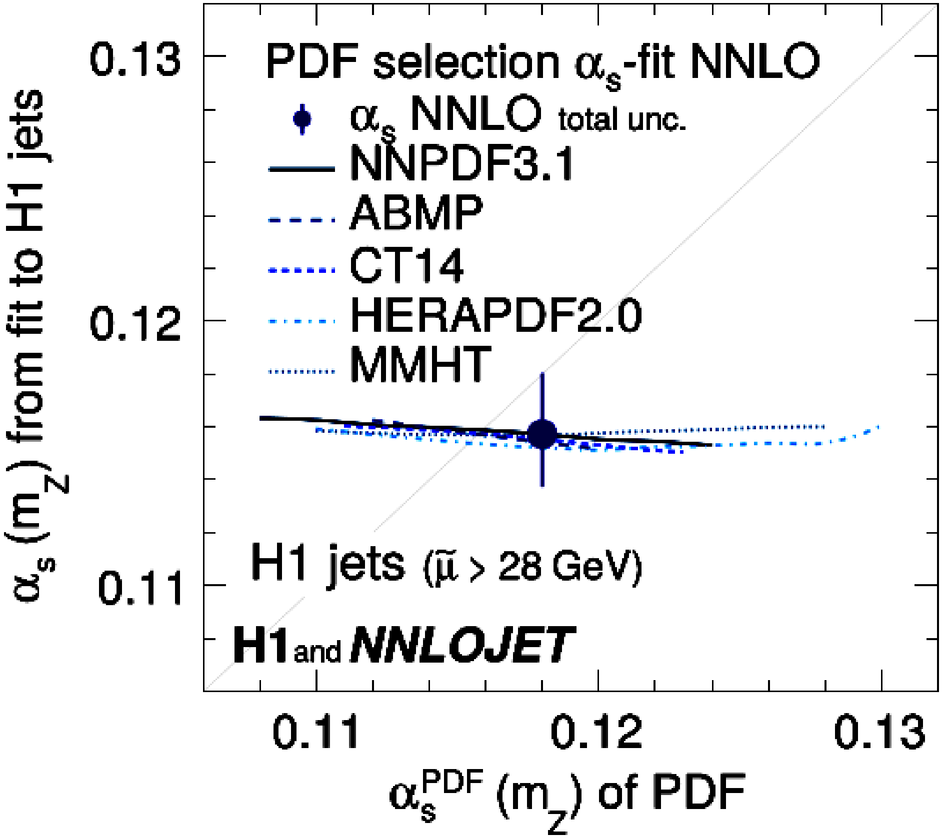


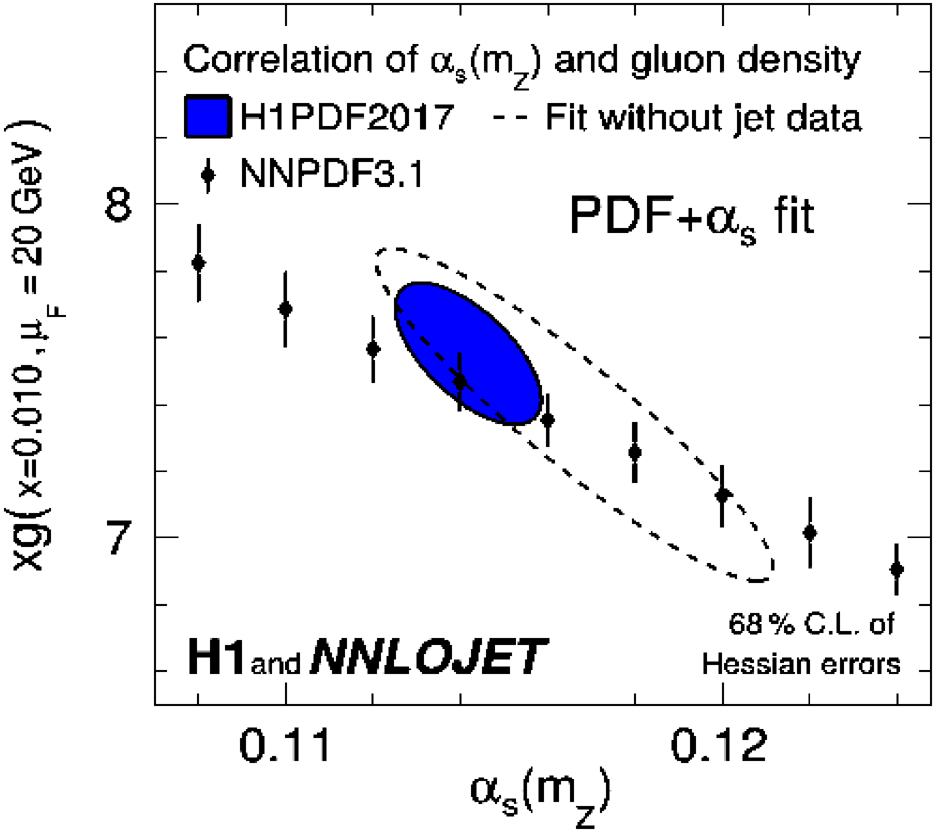
Backup slides

α_s coupling from e-p → jets (NNLO)

- DIS **H1 jet x-sections** and jets+PDF-fit compared for the 1st time to **NNLOjet** calculations:

[Radek Žlebčík, H1, arxiv:1709.07251]





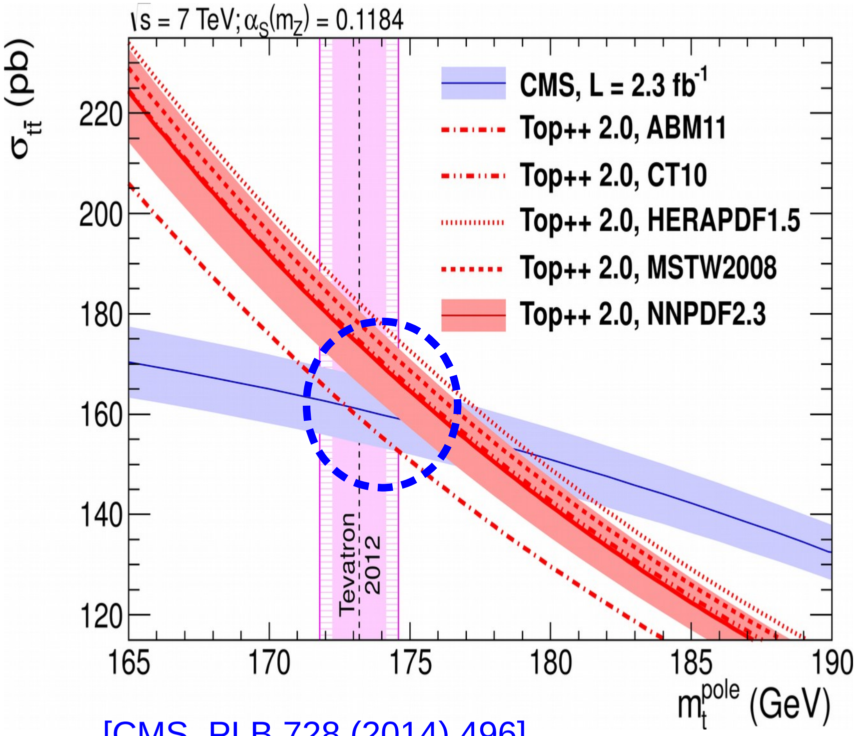
$$\alpha_S^{\mathrm{H1jets},\tilde{\mu}>28\mathrm{GeV}}(m_Z) = 0.1157(20)_{\mathrm{exp}}(28)_{\mathrm{theor.}}$$ **(±3.0%)**

$$\alpha_S^{\mathrm{H1PDF2017}}(m_Z) = 0.1142(11)_{\mathrm{exp}}(26)_{\mathrm{theor.}}$$ **(±2.4%)**

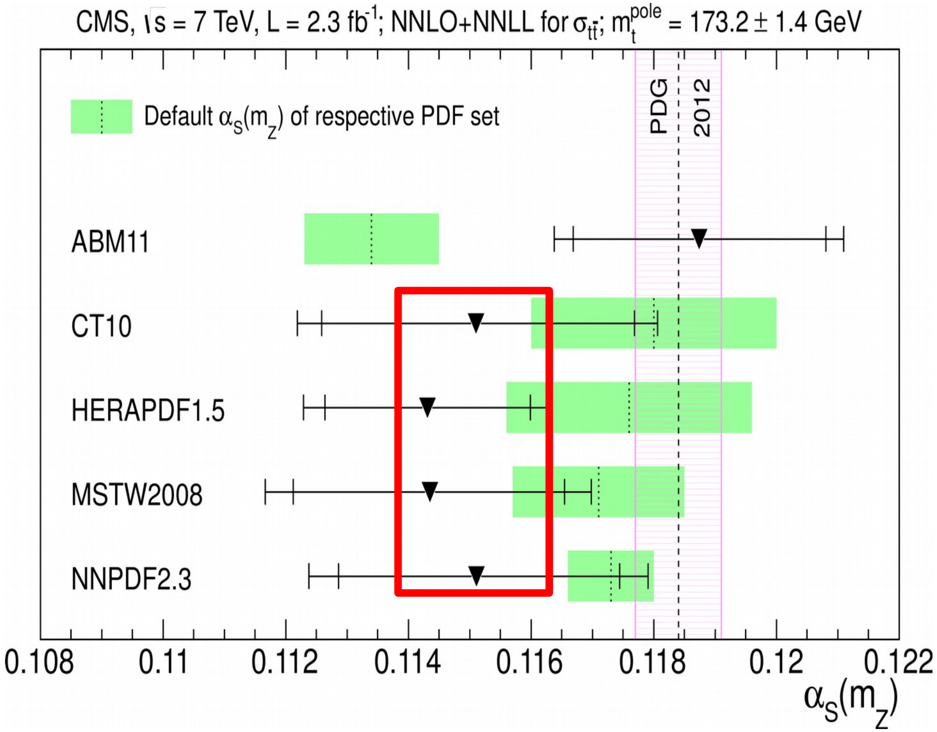
(6) α_s from top-pair p-p cross sections

- Total top-antitop cross section (known at NNLO+NNLL) is the 1st p-p collider observable to constrain α_s at NNLO accuracy:

Data-theory x-section comparison for varying PDF+α_s as a function of m_{top}:



[CMS, PLB 728 (2014) 496]



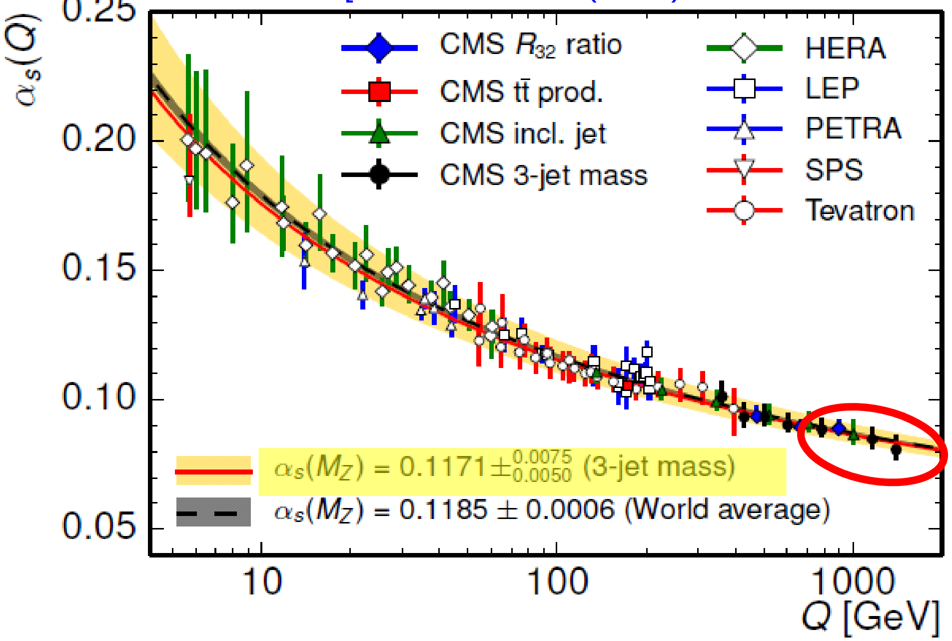
Precise measurement dominated by associated PDF uncertainty (±2.5%)

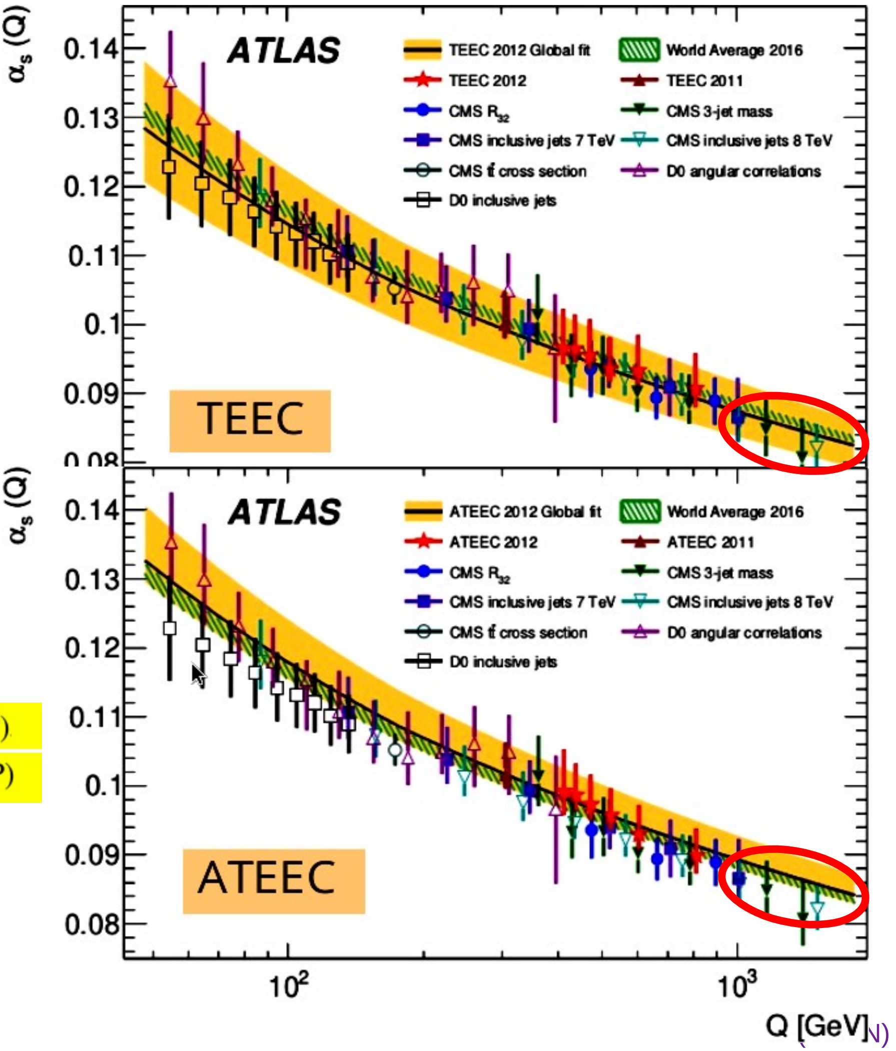
$$\alpha_s(M_Z^2) = 0.1151^{+0.0028}_{-0.0027}$$

α_s coupling from other LHC jet results (NLO)

- Ratio of 3-jets to 2-jets, 3-jet mass x-sections & energy-energy correl. test running α_s (NLO only) up to so-far unprobed scales Q ~ 1.5 TeV:

[CMS, EPJC 75 (2015) 288, and Ibid. 186]





$$\alpha_s(m_Z) = 0.1162 \pm 0.0011\,(\text{exp.})\,{}^{+0.0076}_{-0.0061}\,(\text{scale}) \pm 0.0018\,(\text{PDF}) \pm 0.0003\,(\text{NP})$$

$$\alpha_s(m_Z) = 0.1196 \pm 0.0013\,(\text{exp.})\,{}^{+0.0061}_{-0.0013}\,(\text{scale}) \pm 0.0017\,(\text{PDF}) \pm 0.0004\,(\text{NP})$$

➧ 1st time asymptotic freedom is probed at the TeV scale!

α_s from pion and Υ decays

[Kneur&Neveu,PRD81(2010)125012]

- π decay factor (N^{3}LO, RGOPT):

$$F^2_\pi(pert)_{\overline{\text{MS}}} = N_c \frac{m^2}{2\pi^2}\Big[-L + \frac{\alpha_S}{4\pi}(8L^2 + \tfrac{4}{3}L + \tfrac{1}{6})$$
$$+(\frac{\alpha_S}{4\pi})^2[f_{30}(n_f)L^3 + f_{31}(n_f)L + f_{32}(n_f)L + f_{33}(n_f)] + \mathcal{O}(\alpha_S^3)\Big]$$

$L \equiv \ln\frac{m}{\mu}$, $n_f = 2(3)$

$$\overline{\alpha}_S(m_Z) = 0.1174^{+.0010}_{-.0005}(\text{rgopt th}) \pm .0010|_{(F_\pi/F_0)} \pm .0005_{evol}$$

Issues:

- Too low scale for pQCD? Optimization approach,...
- Intriguing agreement with world average.

α_s = 0.1174±0.0017 (±1.5%)

- Υ decay (NLO):

[Mambrilla et al. PRD75(07)074014]

$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S)\to\gamma X)}{\Gamma(\Upsilon(1S)\to X)} = \frac{36}{5}\frac{e_b^2\alpha}{\alpha_s}\frac{N}{D},$$

$$N,\ D = 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) + \mathcal{O}(\frac{v^4}{\alpha_s})$$
$$+\mathcal{O}(\alpha_s{}^2) + \mathcal{O}(\alpha_s v^2) + \mathcal{O}(\alpha_s\frac{v^4}{\alpha_s}) + \mathcal{O}(v^4) + \mathcal{O}(\frac{v^6}{\alpha_s})$$

α_s(NLO)= 0.1190±0.007 (±6%)

- A NNLO extraction of α_s appears feasible in the coming years, the key ingredients being:
  - More precise data for the $\Upsilon(1S)$ photon spectrum (and total hadronic width)
  - Non-trivial higher order perturbative calculations