NNLO corrections in 4 dimensions

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Outline

1. Four Dimensional Renormalization and the UV problem
2. *NNLO corrections in 4 dimensions*
   Ben Page, R.P., arXiv:1810.00234
3. Conclusions

"Vacuum" subtraction

1. $J(q^2) = \dfrac{1}{(q^2 - M^2)^2}$
2. $q^2 \stackrel{\text{GP}}{\to} \bar{q}^2 := q^2 - \mu^2$
3. $J(q^2) \stackrel{\text{GP}}{\to} \bar{J}(\bar{q}^2) := \dfrac{1}{(\bar{q}^2 - M^2)^2}$

$$\frac{1}{(\bar{q}^2 - M^2)^2} = \left[\frac{1}{\bar{q}^4}\right] + \left(\frac{M^2}{\bar{q}^2(\bar{q}^2 - M^2)^2} + \frac{M^2}{\bar{q}^4(\bar{q}^2 - M^2)}\right)$$

$\Uparrow$

Vacuum

“Vacuum” subtraction

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$\Uparrow$

Vacuum

$$\int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2} := \lim_{\mu \to 0} \int d^4q \left(\frac{M^2}{\bar{q}^2(\bar{q}^2 - M^2)^2} + \frac{M^2}{\bar{q}^4(\bar{q}^2 - M^2)}\right)$$

An integral operator

Action of the linear integral operator $\int [d^4q]$ on $\bar{J}(\bar{q}^2)$:

- subtract the vacuum;
- integrate over q;
- take the asymptotic limit $\mu^2 \to 0$ ($\mu^2 \to \mu_R^2$ in the logs)

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$$\int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2} = -i\pi^2 \ln \frac{M^2}{\mu_R^2}$$

$\Uparrow$

FDR integral

Regularization and Renormalization at once

It can be generalized to more loops (global- and sub-vacua appear)

Examples of two-loop vacua

- Global vacua $(q_{12} := q_1 + q_2)$:

$$\left[\frac{1}{\bar{q}_1^2\bar{q}_2^2\bar{q}_{12}^2}\right], \quad \left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_{12}^2}\right], \quad \left[\frac{1}{\bar{q}_1^4}\right]\left[\frac{1}{\bar{q}_2^4}\right]$$

- Sub-vacua:

$$\frac{M^4}{(\bar{q}_1^2 - M^2)\bar{q}_1^4}\left[\frac{1}{\bar{q}_2^2}\right], \quad \frac{M^4}{(\bar{q}_1^2 - M^2)^2\bar{q}_1^2}\left[\frac{1}{\bar{q}_2^4}\right]$$

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Two core tenets of QFT

1. Gauge invariance

- FDR integrals are invariant under the shift $q \to q + p \ \forall p$
- Cancellations if $q^2 \stackrel{\rm GP}{\to} \bar{q}^2$ in the numerator

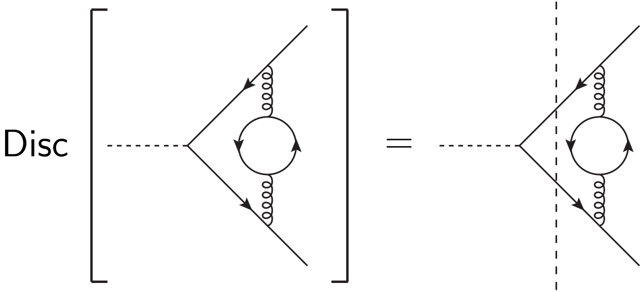
$$\int [d^4q] \frac{\bar{q}^2}{\bar{q}^2(\bar{q}^2 - M^2)^2} = \int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2}$$

$\Rightarrow$ One can prove graphical WI in QFT

2. Unitarity of $S = I + iT$

- It requires $i(T - T^\dagger) = -T^\dagger T$

$\Rightarrow$ The validity of the cutting equations must be enforced



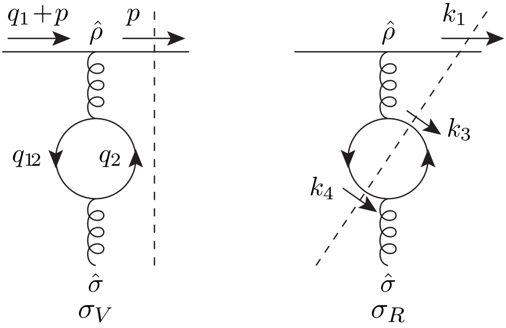
Sub-Integration Consistency (SIC)

In any multi-loop Feynman diagram the divergent sub-diagrams must be treated consistently with the lower loop calculations

We know how to do it for off-shell amplitudes
B. Page, R.P. JHEP 1511 (2015) 183

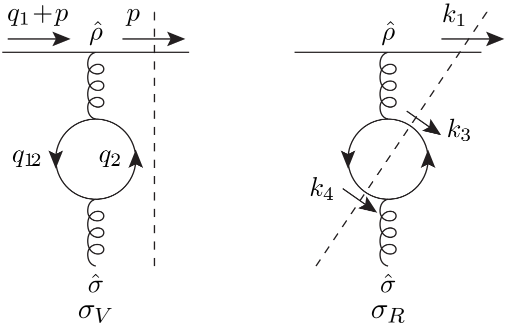
Going on-shell

GP regulates IR divergent virtual cuts, but there are also real cuts:



$$\frac{1}{(\bar q_2^2+i0^+)(\bar q_{12}^2+i0^+)} \leftrightarrow \left(\frac{2\pi}{i}\right)^2 \delta_+(\bar k_3^2)\delta_+(\bar k_4^2)$$

The $\bar q_2^2$ and $\bar q_{12}^2$ propagators in σ_V must correspond to external particles in σ_R obeying $k_{3,4}^2=\mu^2 \quad \Leftarrow \text{regulates real cuts}$



SIC preserved by special treatment of external indices $\hat\rho$ and $\hat\sigma$ and by $\bar{q}_1^2 \to q_1^2$ after subtracting the global vacuum. By doing that

The gluon self-energy has the same form in one-loop, two-loop and cut loop pieces

$\Rightarrow$ Unitarity

if vacuum subtraction does not interfere with cutting rules

But it doesn't, as under $\int [d^4 q_1][d^4 q_2]$

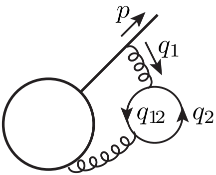
$$\frac{1}{\bar{q}_2^2 \bar{q}_{12}^2} = \left[\frac{1}{\bar{q}_2^4}\right] + \overbrace{\frac{-q_1^2 - 2(q_1 \cdot q_2)}{\bar{q}_2^2}}^{f} \frac{1}{\bar{q}_2^2 \bar{q}_{12}^2}$$

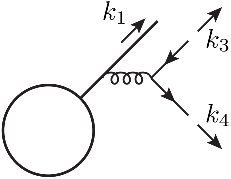
$$\frac{1}{\bar{q}_2^2 \bar{q}_{12}^2} \simeq \frac{f^m}{\bar{q}_2^2 \bar{q}_{12}^2} \leftrightarrow \left(\frac{2\pi}{i}\right)^2 \delta_+(\bar{k}_3^2)\delta_+(\bar{k}_4^2)\ 1^m \qquad \Rightarrow$$

FDR integrals are good objects for QFT calculations

We have used FDR to compute

NNLO final-state quark-pair corrections in four dimensions

$A_{n,\mathrm{IR}}^{(2)} =$ 

$A_{n+2,\mathrm{IR}}^{(0)} =$ 

Our observable

$$\sigma_B \propto \int d\Phi_n \sum_{\text{spin}} |A_n^{(0)}|^2$$

$$\sigma_V \propto \int d\Phi_n \sum_{\text{spin}} \left\{ A_n^{(2)} (A_n^{(0)})^* + A_n^{(0)} (A_n^{(2)})^* \right\}$$

$$\sigma_R \propto \int d\Phi_{n+2} \sum_{\text{spin}} \left\{ A_{n+2}^{(0)} (A_{n+2}^{(0)})^* \right\}$$

$$\sigma^{NNLO} = \sigma_B + \sigma_V + \sigma_R$$

In particular

$\Gamma^{NNLO}(H \to b\bar{b})$ and $\sigma^{\text{NNLO}}_{\gamma^* \to jets}$

Finite renormalization of $a = \frac{\alpha_S}{4\pi}$ and y_b

$$a^0 = a\left(1 + a\delta_a^{(1)}\right) \qquad y_b^0 = y_b\left(1 + a\delta_y^{(1)} + a^2\left(\delta_y^{(2)} + \delta_a^{(1)}\delta_y^{(1)}\right)\right)$$

$$\begin{aligned}
\delta_a^{(1)} &= \frac{2}{3}N_F L \\
\delta_y^{(1)} &= -C_F\left(3L'' + 5\right) \\
\delta_y^{(2)} &= C_F N_F\left(L''^2 + \frac{13}{3}L'' + \frac{2}{3}\pi^2 + \frac{151}{18}\right)
\end{aligned}$$

$$L := \ln(\mu^2/s) \;\; \text{and} \;\; L'' := \ln\frac{\mu^2}{m^2}$$

Renormalization of y_b from bottom quark pole mass

$$\frac{1}{\not p - m^0 + \Sigma^{(1)} + \Sigma^{(2)}}$$

$\Gamma^{\text{NNLO}}(y_b) = \Gamma_2^{(0)}(y_b) + \delta\Gamma^{N_F}$

- $H \to b\bar{b}$ up to two loops
- $H \to b\bar{b}g$ at the tree level
- $H \to b\bar{b}q\bar{q}$ at the tree level

$$\delta\Gamma^{N_F} = \Gamma_2^{N_F} + \Gamma_3^{N_F} + \Gamma_4^{N_F}$$

$$\begin{aligned}
\Gamma_2^{N_F} &= \Gamma_2^0(y_b) a^2 2\Re e \left(\delta V_2^{(2)} + \delta_a^{(1)} \delta V_2^{(1)} + \delta_y^{(2)} + \delta_a^{(1)} \delta_y^{(1)}\right) \\
\Gamma_3^{N_F} &= a^2 \delta_a^{(1)} \Gamma_2^0(y_b) C_F \left(2L^2 + 6L + K_3\right) \\
\Gamma_4^{N_F} &= a^2 C_F N_F \Gamma_2^{(0)}(y_b) \frac{4}{9} \left\{ -L^3 - \frac{19}{2} L^2 - L \left(\frac{155}{3} - 2\pi^2\right) + K_4 \right\}
\end{aligned}$$

- $\delta V_2^{(1,2)}$ from loops and L from phase space integration
- Tree and 1-loop pieces do not change embedded in 2-loops

Collecting all pieces gives an IR finite result ($s = M_H^2$)

$$\Gamma^{\text{NNLO}}(y_b) = \Gamma_2^{(0)}(y_b)\left\{1 + a^2 C_F N_F \left(2\ln^2\frac{m^2}{s} - \frac{26}{3}\ln\frac{m^2}{s} + 8\zeta_3 + 2\pi^2 - \frac{62}{3}\right)\right\}$$

Using the relation between m and $m^{\overline{\text{MS}}}(s)$ one reabsorbs the large logarithms of the ratio m^2/s in a new Yukawa coupling $y_b^{\overline{\text{MS}}}(s)$

$$\Gamma^{\text{NNLO}}(y_b^{\overline{\text{MS}}}(s)) = \Gamma_2^{(0)}(y_b^{\overline{\text{MS}}}(s))\left\{1 + a^2 C_F N_F \left(8\zeta_3 + \frac{2}{3}\pi^2 - \frac{65}{2}\right)\right\}$$

which coincides with the known $\overline{\text{MS}}$ result

$\sigma^{\text{NNLO}}_{\gamma^*\to jets} = \sigma_2^{(0)} + \delta\sigma^{N_F}$

- *Renormalization only involves* α_S
- *Higher rank tensors contribute*
- *Preserving gauge cancellations and unitarity in such an environment provides a more stringent test for our procedures* $\Rightarrow \gamma^* \to jets$ *is* complementary *to* $H \to b\bar{b} + jets$

Gathering all the pieces we reproduce the $\overline{\text{MS}}$ result

$$\sigma^{\text{NNLO}}_{e^+e^-\to jets} = \sigma_2^{(0)} \left\{1 + a^2 C_F N_F \left(8\zeta_3 - 11\right)\right\}$$

Conclusions I

FDR provides a fully four-dimensional framework to compute NNLO quark-pair corrections

- *No (explicit or implicit) UV counterterms have to be included in the Lagrangian*
- *Lower-order substructures are used in higher-order calculations without any modification*
  $\Rightarrow$ better organization of the perturbative approach
- *Renormalization is equivalent to the process of expressing (finite) bare parameters in terms of measurable observables*

Conclusions II

We considered a special class of NNLO corrections but we believe that the basic principles will remain valid also when considering more complicated environments (including ISR)

The intrinsic four-dimensionality of FDR can pave the way to new numerical methods, e.g.

- *infrared divergences in the real component directly show up in terms of logarithms of a small cut-off parameter* μ, *with no need for a prior subtraction of* $1/(d-4)$ *poles*
  $\Rightarrow$ slicing subtraction out of the box
- *one-to-one integrand correspondences can be written down between virtual and real contributions* $\Rightarrow$ local subtraction

There is room for fully exploiting the potential of FDR in NNLO calculations

Backup slides

A two-loop example

$$(\bar{D}_i := \bar{q}_i^2 - m_i^2)$$

$$\begin{aligned}
\frac{1}{\bar{D}_1\bar{D}_2\bar{D}_{12}} &= \left[\frac{1}{\bar{q}_1^2\bar{q}_2^2\bar{q}_{12}^2}\right] \\
&+ m_1^2\left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_{12}^2}\right] + \frac{m_1^4}{(\bar{D}_1\bar{q}_1^4)}\left[\frac{1}{\bar{q}_2^4}\right] - m_1^4\frac{q_1^2+2(q_1\cdot q_2)}{(\bar{D}_1\bar{q}_1^4)\bar{q}_2^4\bar{q}_{12}^2} \\
&+ m_2^2\left[\frac{1}{\bar{q}_1^2\bar{q}_2^4\bar{q}_{12}^2}\right] + \frac{m_2^4}{(\bar{D}_2\bar{q}_2^4)}\left[\frac{1}{\bar{q}_1^4}\right] - m_2^4\frac{q_2^2+2(q_1\cdot q_2)}{\bar{q}_1^4(\bar{D}_2\bar{q}_2^4)\bar{q}_{12}^2} \\
&+ m_{12}^2\left[\frac{1}{\bar{q}_1^2\bar{q}_2^2\bar{q}_{12}^4}\right] + \frac{m_{12}^4}{(\bar{D}_{12}\bar{q}_{12}^4)}\left[\frac{1}{\bar{q}_1^4}\right] - m_{12}^4\frac{q_{12}^2-2(q_1\cdot q_{12})}{\bar{q}_1^4\bar{q}_2^2(\bar{D}_{12}\bar{q}_{12}^4)} \\
&+ \frac{m_1^2m_2^2}{(\bar{D}_1\bar{q}_1^2)(\bar{D}_2\bar{q}_2^2)\bar{q}_{12}^2} + \frac{m_1^2m_{12}^2}{(\bar{D}_1\bar{q}_1^2)\bar{q}_2^2(\bar{D}_{12}\bar{q}_{12}^2)} + \frac{m_2^2m_{12}^2}{\bar{q}_1^2(\bar{D}_2\bar{q}_2^2)(\bar{D}_{12}\bar{q}_{12}^2)} \\
&+ \frac{m_1^2m_2^2m_{12}^2}{(\bar{D}_1\bar{q}_1^2)(\bar{D}_2\bar{q}_2^2)(\bar{D}_{12}\bar{q}_{12}^2)}
\end{aligned}$$

Three-loop logarithmic vacua

$$\left[\frac{1}{\bar{q}_1^2\bar{q}_2^2\bar{q}_3^2\bar{q}_{12}^2\bar{q}_{13}^2((q_2-q_3)^2-\mu^2)}\right]$$

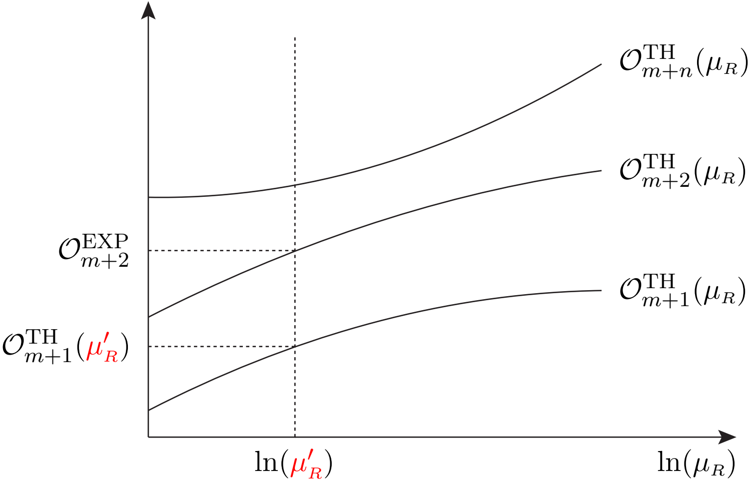
$$\left[\frac{1}{\bar{q}_1^2\bar{q}_3^2\bar{q}_2^4\bar{q}_{12}^2\bar{q}_{23}^2}\right]$$

$$\left[\frac{1}{\bar{q}_1^4\bar{q}_2^2\bar{q}_3^2\bar{q}_{12}^2\bar{q}_{123}^2}\right]$$

$$\left[\frac{1}{\bar{q}_1^4\bar{q}_2^4\bar{q}_3^2\bar{q}_{123}^2}\right]$$

$$\left[\frac{1}{\bar{q}_1^6\bar{q}_2^2\bar{q}_3^2\bar{q}_{123}^2}\right]$$

Fitting μ_R in non-renormalizable QFTs



The virtual master integrals

$$\tilde{I}_j := s^{2-j}\,\frac{i}{\pi^2}\int_0^1 dx\left(\frac{1}{x}-3+4x^2\right)\int d^4q_1\,\frac{(q_1\cdot p_1)^{j-1}}{D_0D_1D_2}$$

$$\begin{aligned}
D_0 &:= q_1^2-\mu_0^2, \quad \mu_0^2 := \frac{\mu^2}{x(1-x)}\\
D_1 &:= (q_1+p_1)^2-\mu^2\\
D_2 &:= (q_1+p_2)^2-\mu^2
\end{aligned}$$

The real integrals

$$w(\mu^2) = \left(1 + 2\frac{\mu^2}{s_{34}}\right)$$

$$\tilde{R}_1 := \frac{1}{s\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{134}}$$

$$\tilde{R}_2 := \frac{1}{\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{134}s_{234}}$$

$$\tilde{R}_3 := \frac{1}{s\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{s_{34}}{s_{134}s_{234}}$$

$$\tilde{R}_4 := \frac{1}{\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{134}^2}$$

$$\tilde{R}_5 := \frac{1}{\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{134}s_{34}}$$

$$\tilde{R}_6 := \frac{1}{s\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{s_{234}}{s_{134}s_{34}}$$

$$\tilde{R}_7 := \frac{s}{\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{34}s_{134}s_{234}}$$

$$\tilde{R}_8 := \frac{1}{s\pi^3}\int d^4\tilde{\Phi}_4\, w(\mu^2)\frac{1}{s_{34}}$$