11th FCC Workshop, CERN, 9th January 2019

The Branco Weiss Fellowship Society in Science

Global SMEFT Fit

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Based on J. Ellis, C. Murphy, V. Sanz and TY, JHEP (1803.03252)

Introduction

- Motivation
- SM EFT framework
- EWPT and TGCs
- Updated Run 2 SM EFT fit
- FCC-ee projections
- Conclusion

Why SM EFT?

Assuming a SM Higgs and **decoupled new physics** at higher energies, the SM EFT is the next phenomenological framework

The TeV Scale

What effective theory captures everything we know experimentally about weak interactions?

1933–1982 4-fermion interactions

$\sim G_F E^2 \qquad \Rightarrow \Lambda \sim \text{TeV}$

1982–2011 SM without Higgs

$\sim \dfrac{g^2 E^2}{m_W^2} \qquad \Rightarrow \Lambda \sim \text{TeV}$

2012-now SM + higher-dimension operators?

$\Rightarrow \Lambda \lesssim M_P$?

Markus Luty PASCOS 2015 slide

SMEFT framework

- New physics appear to be **decoupled at higher energies**
- Given particle content, write down *all* terms **allowed by symmetries**…

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	**3**	**2**	$\frac{1}{6}$
q_R^u	**3**	**1**	$\frac{2}{3}$
q_R^d	**3**	**1**	$-\frac{1}{3}$
L_L	**1**	**2**	$-\frac{1}{2}$
l_R	**1**	**1**	-1
ϕ	**1**	**2**	$\frac{1}{2}$

$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \quad ,$$

$$\mathcal{L}_m = \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R$$
$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}$$
$$\mathcal{L}_H = (D_\mu^L\phi)^\dagger(D^{L\mu}\phi) - V(\phi)$$
$$\mathcal{L}_Y = y_d\bar{Q}_L\phi q_R^d + y_u\bar{Q}_L\phi^c q_R^u + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,$$

- …Including **higher-dimensional** operators!

$$+ \quad \boxed{\mathcal{L}_{\text{SM}}^{\text{dim-6}} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i}$$

- Generated by **new physics at scale $\Lambda \gg v$**

SMEFT framework

- Unique dim-5 Weinberg operator, violates lepton number, predicts neutrino masses
- Modulo flavour structure, there are **59** dim-6 (CP-even) operators in a **non-redundant** basis

Buchmuller and Wyler (Nucl. Phys. B 268 (1986) 621)
Gradkowski et al [arXiv:1008.4884]

- **~19** operators relevant for EWPT, TGC, and Higgs physics

EWPTs	Higgs Physics	TGCs
$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right) D^\nu W^a_{\mu\nu}$		
$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right) \partial^\nu B_{\mu\nu}$		$\mathcal{O}_{3W} = g\frac{\epsilon_{abc}}{3!} W^{a\,\nu}_{\mu} W^b_{\nu\rho} W^{c\,\rho\mu}$
$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$	$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$	
$\mathcal{O}^{(3)\,l}_{LL} = (\bar{L}_L \sigma^a \gamma^\mu L_L)(\bar{L}_L \sigma^a \gamma_\mu L_L)$	$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	
$\mathcal{O}^e_R = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_g = g_s^2 \lvert H\rvert^2 G^A_{\mu\nu} G^{A\mu\nu}$	
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In SILH basis (Giudice et al. hep-ph/0703164), adopted from Pomarol and Riva (1308.1426)

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Combinations of operators constrained in EWPT more easily set to zero in Higgs and TGCs

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Operators constrained by measurements at per cent level or worse

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Operators benefit from per mille precision at LEP

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LEP EWPT Example

- (Pseudo-)Observables

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_\ell + 3\Gamma_\nu \qquad R_\ell = \frac{\Gamma_{had}}{\Gamma_\ell} \qquad \sigma_{had} = 12\pi \frac{\Gamma_e \Gamma_{had}}{\hat{m}_Z^2 \Gamma_Z^2} \qquad A_{FB}^f = \frac{3}{4} A_e A_f \qquad M_W = c_W M_Z$$

$$R_q = \frac{\Gamma_q}{\Gamma_{had}}$$

- Depends on

$$\Gamma_f = \frac{\sqrt{2} G_F M_Z^2 \hat{M}_Z}{6\pi} \left[(g_L^f)^2 + (g_R^f)^2 \right] \qquad A_f = \frac{(g_L^f)^2 - (g_R^f)^2}{(g_L^f)^2 + (g_R^f)^2}$$

$$g^f = T_f^3 - Q_f s_W^2 \qquad s_W^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F M_Z^2}}$$

- Dim-6 operators can modify observables directly through Zff couplings contributions or indirectly through redefinitions of input observables

$$m_Z^2 = (m_Z^2)^0 (1 + \Pi_{ZZ}) \qquad G_F = G_F^0 (1 - \Pi_{WW}^0) \qquad \alpha(m_Z) = \alpha^0(m_Z)(1 + \Pi'_{\gamma\gamma})$$

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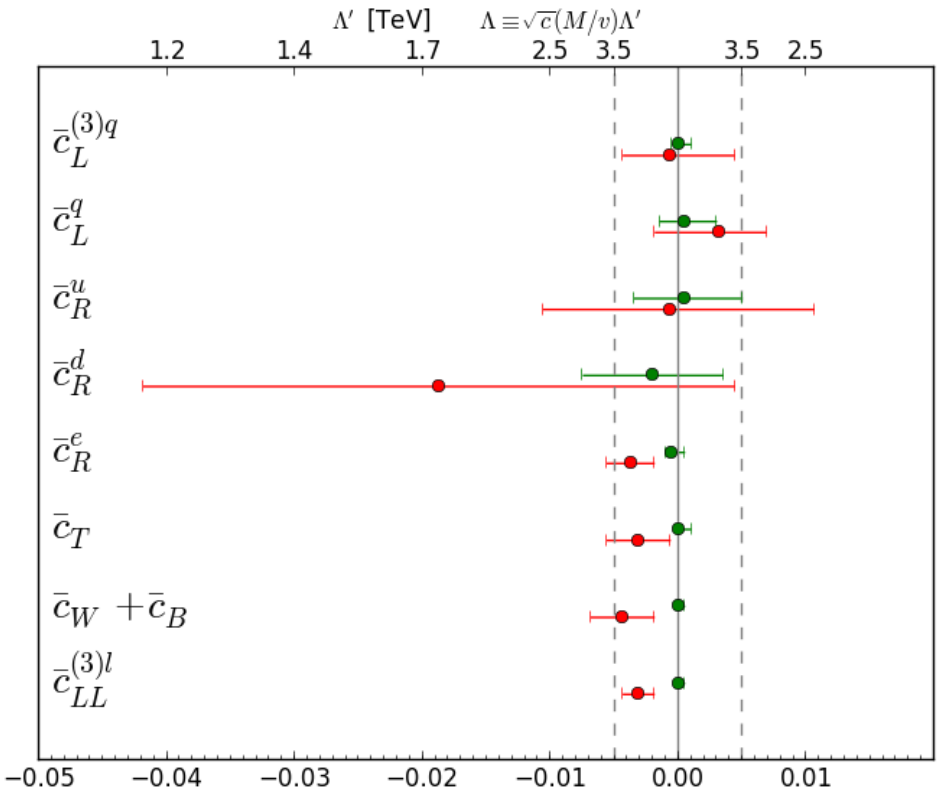
LEP EWPT Example

- Individual (green) and marginalised (red) 95% CL limits



Ellis, Sanz and T.Y. 1410.7703

- 8 (combinations of) operators probed by EWPT

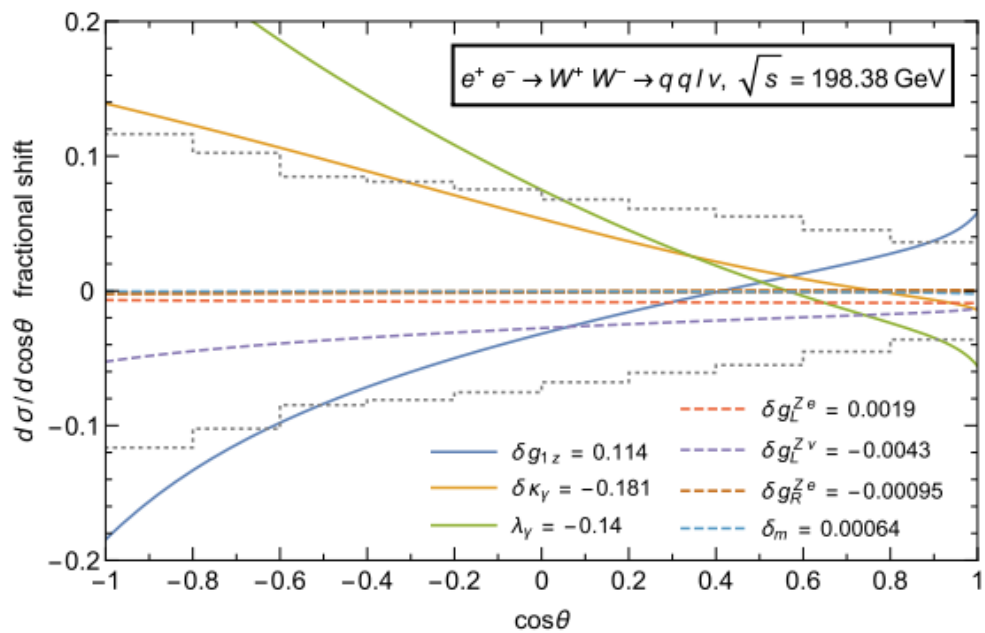
Triple-Gauge-Couplings in Diboson

- Assume SM Z and W couplings to fermions in diboson measurements
- Interpret in anomalous TGC framework:

$$\mathcal{L}_{\rm TGC} = ig\Big\{(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu})\big[(1+\delta g_{1z})\,c_\theta Z^\nu + s_\theta A^\nu\big] + \frac{1}{2}W^+_{[\mu,}W^-_{\nu]}\big[(1+\delta\kappa_z)\,c_\theta Z^{\mu\nu} + (1+\delta\kappa_\gamma)\,s_\theta A^{\mu\nu}\big] + \frac{1}{m_W^2}W^{+\nu}_{\mu}W^{-\rho}_{\nu}(\lambda_z c_\theta Z_\rho{}^\mu + \lambda_\gamma s_\theta A_\rho{}^\mu)\Big\},$$

Z. Zhang, 1610.01618

- Justified at LEP:



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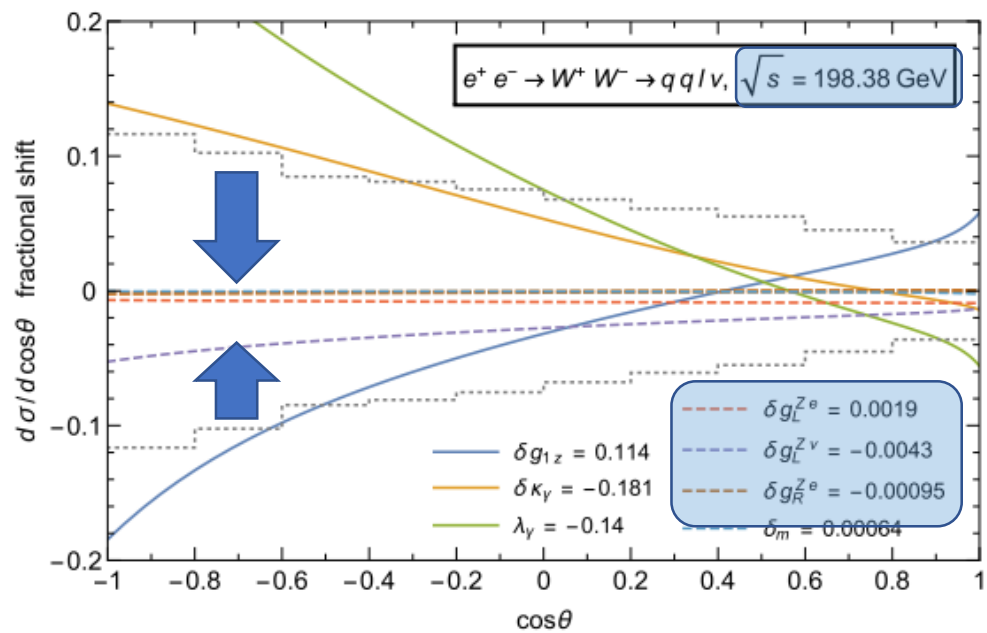

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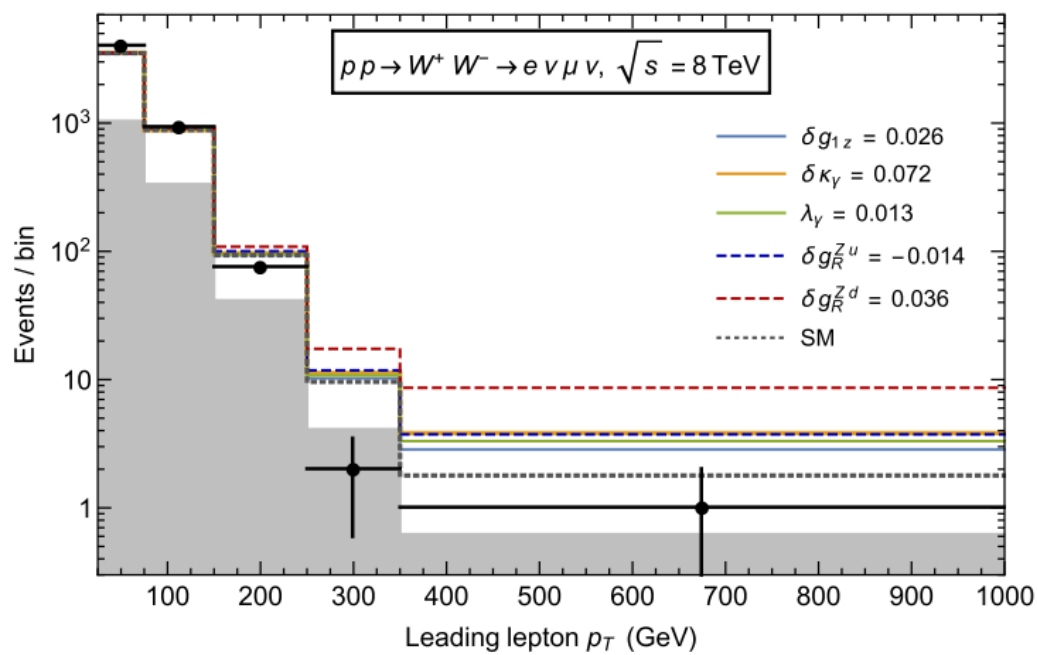
- But not at high pT:



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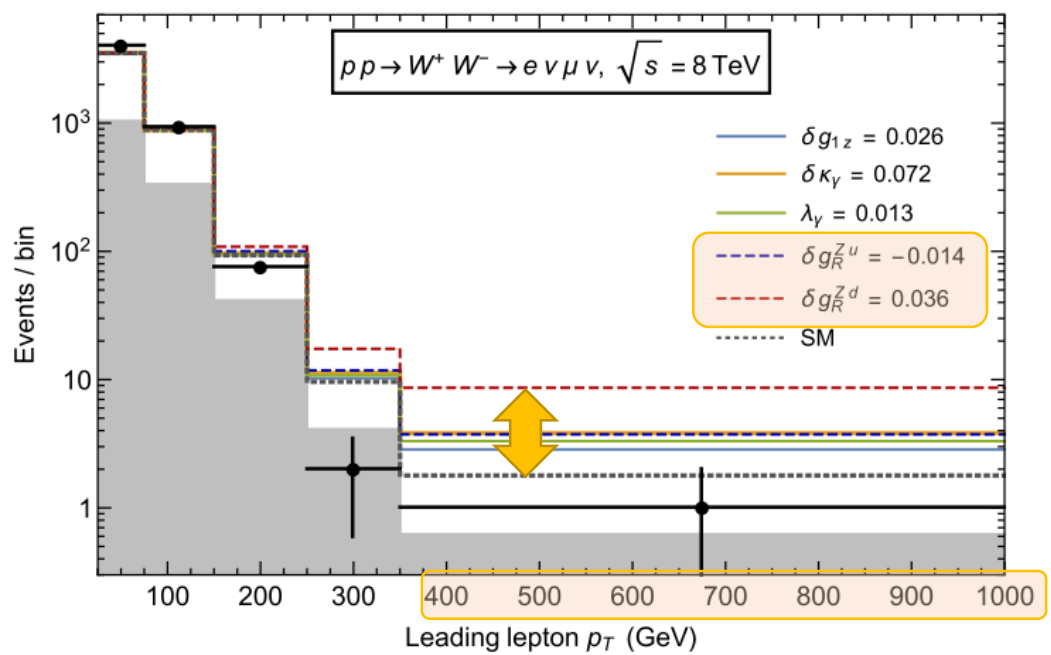


Note: quadratic dim-6 effect unless diboson SM-BSM interference recovered

Azatov et al 1607.05236, 1707.08060, Panico, Riva, Wulzer 1708.07823, Bellazzini, Riva 1806.09640

Updated Global SMEFT Fit

J. Ellis, C. Murphy, V. Sanz and TY, JHEP 1803.03252

- Combine **EWPT, diboson, Higgs** data
- Fit to 20 dim-6 CP-even operators **simultaneously**
- Present results in **Warsaw** and **SILH** basis
- Match to **simplified models**

Updated Global SMEFT Fit

- SILH basis

$$
\begin{aligned}
\mathcal{L}_{\rm SMEFT}^{\rm SILH} \supset\ & \frac{\bar{c}_W}{m_W^2}\frac{ig}{2}\left(H^\dagger\sigma^a\overleftrightarrow{D}^\mu H\right)D^\nu W^a_{\mu\nu} + \frac{\bar{c}_B}{m_W^2}\frac{ig'}{2}\left(H^\dagger\overleftrightarrow{D}^\mu H\right)\partial^\nu B_{\mu\nu} + \frac{\bar{c}_T}{v^2}\frac{1}{2}\left(H^\dagger\overleftrightarrow{D}_\mu H\right)^2 \\
& + \frac{\bar{c}_{ll}}{v^2}2(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L) + \frac{\bar{c}_{He}}{v^2}(iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{e}_R\gamma^\mu e_R) + \frac{\bar{c}_{Hu}}{v^2}(iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{u}_R\gamma^\mu u_R) \\
& + \frac{\bar{c}_{Hd}}{v^2}(iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{d}_R\gamma^\mu d_R) + \frac{\bar{c}'_{Hq}}{v^2}(iH^\dagger\sigma^a\overleftrightarrow{D}_\mu H)(\bar{Q}_L\sigma^a\gamma^\mu Q_L) \\
& + \frac{\bar{c}_{Hq}}{v^2}(iH^\dagger\overleftrightarrow{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L) + \frac{\bar{c}_{HW}}{m_W^2}ig(D^\mu H)^\dagger\sigma^a(D^\nu H)W^a_{\mu\nu} + \frac{\bar{c}_{HB}}{m_W^2}ig'(D^\mu H)^\dagger(D^\nu H)B_{\mu\nu} \\
& + \frac{\bar{c}_{3W}}{m_W^2}g^3\epsilon_{abc}W^{a\,\nu}_{\mu}W^b_{\nu\rho}W^{c\,\rho\mu} + \frac{\bar{c}_g}{m_W^2}g_s^2|H|^2G^A_{\mu\nu}G^{A\mu\nu} + \frac{\bar{c}_\gamma}{m_W^2}g'^2|H|^2B_{\mu\nu}B^{\mu\nu} \\
& + \frac{\bar{c}_H}{v^2}\frac{1}{2}(\partial^\mu|H|^2)^2 - \sum_{f=e,u,d}\frac{\bar{c}_f}{v^2}y_f|H|^2\bar{F}_LH^{(c)}f_R \\
& + \frac{\bar{c}_{3G}}{m_W^2}g_s^3f_{ABC}G^{A\nu}_{\mu}G^{B\rho}_{\nu}G^{C\mu}_{\rho} - \frac{\bar{c}_{uG}}{m_W^2}4g_sy_uH^\dagger\cdot\bar{Q}_L\gamma^{\mu\nu}T_au_RG^A_{\mu\nu}\,. \qquad (6)
\end{aligned}
$$

Updated Global SMEFT Fit

- Warsaw basis

$$\mathcal{L}_{\rm SMEFT}^{\rm Warsaw} \supset \frac{\bar{C}_{Hl}^{(3)}}{v^2}(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}\tau^I\gamma^\mu l) + \frac{\bar{C}_{Hl}^{(1)}}{v^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}\gamma^\mu l) + \frac{\bar{C}_{ll}}{v^2}(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$$
$$+ \frac{\bar{C}_{HD}}{v^2}\left|H^\dagger D_\mu H\right|^2 + \frac{\bar{C}_{HWB}}{v^2}H^\dagger\tau^I H\, W^I_{\mu\nu}B^{\mu\nu}$$
$$+ \frac{\bar{C}_{He}}{v^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}\gamma^\mu e) + \frac{\bar{C}_{Hu}}{v^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}\gamma^\mu u) + \frac{\bar{C}_{Hd}}{v^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}\gamma^\mu d)$$
$$+ \frac{\bar{C}_{Hq}^{(3)}}{v^2}(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}\tau^I\gamma^\mu q) + \frac{\bar{C}_{Hq}^{(1)}}{v^2}(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}\gamma^\mu q) + \frac{\bar{C}_W}{v^2}\epsilon^{IJK}W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$$

$$\mathcal{L}_{\rm SMEFT}^{\rm Warsaw} \supset \frac{\bar{C}_{eH}}{v^2}(H^\dagger H)(\bar{l}eH) + \frac{\bar{C}_{dH}}{v^2}(H^\dagger H)(\bar{q}dH) + \frac{\bar{C}_{uH}}{v^2}(H^\dagger H)(\bar{q}u\widetilde{H})$$
$$+ \frac{\bar{C}_G}{v^2}f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho + \frac{\bar{C}_{H\Box}}{v^2}(H^\dagger H)\Box(H^\dagger H) + \frac{\bar{C}_{uG}}{v^2}(\bar{q}\sigma^{\mu\nu}T^A u)\widetilde{H}\, G^A_{\mu\nu}$$
$$+ \frac{\bar{C}_{HW}}{v^2}H^\dagger H\, W^I_{\mu\nu}W^{I\mu\nu} + \frac{\bar{C}_{HB}}{v^2}H^\dagger H\, B_{\mu\nu}B^{\mu\nu} + \frac{\bar{C}_{HG}}{v^2}H^\dagger H\, G^A_{\mu\nu}G^{A\mu\nu}\,.$$

Observables

- LEP and SLC EWPTs, M_W from ATLAS, Tevatron

Observable	Measurement	Ref.	SM Prediction	Ref.
Γ_Z [GeV]	2.4952 ± 0.0023	[39]	2.4943 ± 0.0005	[38]
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	[39]	41.488 ± 0.006	[38]
R^0_ℓ	20.767 ± 0.025	[39]	20.752 ± 0.005	[38]
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	[39]	0.01622 ± 0.00009	[114]
$\mathcal{A}_\ell\,(P_\tau)$	0.1465 ± 0.0033	[39]	0.1470 ± 0.0004	[114]
$\mathcal{A}_\ell\,(\rm SLD)$	0.1513 ± 0.0021	[39]	0.1470 ± 0.0004	[114]
R^0_b	0.021629 ± 0.00066	[39]	0.2158 ± 0.00015	[38]
R^0_c	0.1721 ± 0.0030	[39]	0.17223 ± 0.00005	[38]
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	[39]	0.1031 ± 0.0003	[114]
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	[39]	0.0736 ± 0.0002	[114]
$\mathcal{A}_b$	0.923 ± 0.020	[39]	0.9347	[114]
$\mathcal{A}_c$	0.670 ± 0.027	[39]	0.6678 ± 0.0002	[114]
M_W [GeV]	80.387 ± 0.016	[40]	80.361 ± 0.006	[114]
M_W [GeV]	80.370 ± 0.019	[94]	80.361 ± 0.006	[114]

- LEP WW measurements
- ATLAS WW high pT overflow bin

Observables

- ATLAS+CMS Higgs Run 1

Production	Decay	Signal Strength	Production	Decay	Signal Strength
ggF	$\gamma\gamma$	$1.10^{+0.23}_{-0.22}$	Wh	$\tau\tau$	-1.4 ± 1.4
ggF	ZZ	$1.13^{+0.34}_{-0.31}$	Wh	bb	1.0 ± 0.5
ggF	WW	0.84 ± 0.17	Zh	$\gamma\gamma$	$0.5^{+3.0}_{-2.5}$
ggF	$\tau\tau$	1.0 ± 0.6	Zh	WW	$5.9^{+2.6}_{-2.2}$
VBF	$\gamma\gamma$	1.3 ± 0.5	Zh	$\tau\tau$	$2.2^{+2.2}_{-1.8}$
VBF	ZZ	$0.1^{+1.1}_{-0.6}$	Zh	bb	0.4 ± 0.4
VBF	WW	1.2 ± 0.4	tth	$\gamma\gamma$	$2.2^{+1.6}_{-1.3}$
VBF	$\tau\tau$	1.3 ± 0.4	tth	WW	$5.0^{+1.8}_{-1.7}$
Wh	$\gamma\gamma$	$0.5^{+1.3}_{-1.2}$	tth	$\tau\tau$	$-1.9^{+3.7}_{-3.3}$
Wh	WW	$1.6^{+1.2}_{-1.0}$	tth	bb	1.1 ± 1.0
pp	$Z\gamma$	$2.7^{+4.6}_{-4.5}$	pp	$\mu\mu$	0.1 ± 2.5

Observables

- ATLAS+CMS Higgs Run 2

	Production	Decay	Sig. Stren.		Production	Decay	Sig. Stren.
[96]	1-jet, $p_T > 450$	$b\bar{b}$	$2.3^{+1.8}_{-1.6}$	[105]	pp	$\mu\mu$	-0.1 ± 1.4
[97]	Zh	$b\bar{b}$	0.9 ± 0.5	[106]	Zh	$b\bar{b}$	$0.69^{+0.35}_{-0.31}$
[97]	Wh	$b\bar{b}$	1.7 ± 0.7	[106]	Wh	$b\bar{b}$	$1.21^{+0.45}_{-0.42}$
[98]	$t\bar{t}h$	$b\bar{b}$	$-0.19^{+0.80}_{-0.81}$	[107]	$t\bar{t}h$	$b\bar{b}$	$0.84^{+0.64}_{-0.61}$
[99]	$t\bar{t}h$	$1\ell + 2\tau_h$	$-1.20^{+1.50}_{-1.47}$	[108]	$t\bar{t}h$	$2\ell os + 1\tau_h$	$1.7^{+2.1}_{-1.9}$
[99]	$t\bar{t}h$	$2\ell ss + 1\tau_h$	$0.86^{+0.79}_{-0.66}$	[108]	$t\bar{t}h$	$1\ell + 2\tau_h$	$-0.6^{+1.6}_{-1.5}$
[99]	$t\bar{t}h$	$3\ell + 1\tau_h$	$1.22^{+1.34}_{-1.00}$	[108]	$t\bar{t}h$	$3\ell + 1\tau_h$	$1.6^{+1.8}_{-1.3}$
[100]	$t\bar{t}h$	$2\ell ss$	$1.7^{+0.6}_{-0.5}$	[108]	$t\bar{t}h$	$2\ell ss + 1\tau_h$	$3.5^{+1.7}_{-1.3}$
[100]	$t\bar{t}h$	3ℓ	$1.0^{+0.8}_{-0.7}$	[108]	$t\bar{t}h$	3ℓ	$1.8^{+0.9}_{-0.7}$
[100]	$t\bar{t}h$	4ℓ	$0.9^{+2.3}_{-1.6}$	[108]	$t\bar{t}h$	$2\ell ss$	$1.5^{+0.7}_{-0.6}$
[101]	0-jet	WW	$0.9^{+0.4}_{-0.3}$	[109]	VBF	WW	$1.7^{+1.1}_{-0.9}$
[101]	1-jet	WW	1.1 ± 0.4	[109]	Wh	WW	$3.2^{+4.4}_{-4.2}$
[101]	2-jet	WW	1.3 ± 1.0	[110]	$\mathrm{B}(h \to \gamma\gamma)/\ \mathrm{B}(h \to 4\ell)$		$0.69^{+0.15}_{-0.13}$
[101]	VBF 2-jet	WW	1.4 ± 0.8	[110]	0-jet	4ℓ	$1.07^{+0.27}_{-0.25}$
[101]	Vh 2-jet	WW	$2.1^{+2.3}_{-2.2}$	[110]	1-jet, $p_T < 60$	4ℓ	$0.67^{+0.72}_{-0.68}$
[101]	Wh 3-lep	WW	-1.4 ± 1.5	[110]	1-jet, $p_T \in (60, 120)$	4ℓ	$1.00^{+0.63}_{-0.55}$
[102]	ggF	$\gamma\gamma$	$1.11^{+0.19}_{-0.18}$	[110]	1-jet, $p_T \in (120, 200)$	4ℓ	$2.1^{+1.5}_{-1.3}$
[102]	VBF	$\gamma\gamma$	$0.5^{+0.6}_{-0.5}$	[110]	2-jet	4ℓ	$2.2^{+1.1}_{-1.0}$
[102]	$t\bar{t}h$	$\gamma\gamma$	2.2 ± 0.9	[110]	"BSM-like"	4ℓ	$2.3^{+1.2}_{-1.0}$
[102]	Vh	$\gamma\gamma$	$2.3^{+1.1}_{-1.0}$	[110]	VBF, $p_T < 200$	4ℓ	$2.14^{+0.94}_{-0.77}$
[103]	ggF	4ℓ	$1.20^{+0.22}_{-0.21}$	[110]	Vh lep	4ℓ	$0.3^{+1.3}_{-1.2}$
[104]	0-jet	$\tau\tau$	0.84 ± 0.89	[110]	$t\bar{t}h$	4ℓ	$0.51^{+0.86}_{-0.70}$
[104]	boosted	$\tau\tau$	$1.17^{+0.47}_{-0.40}$				
[104]	VBF	$\tau\tau$	$1.11^{+0.34}_{-0.35}$				

Observables

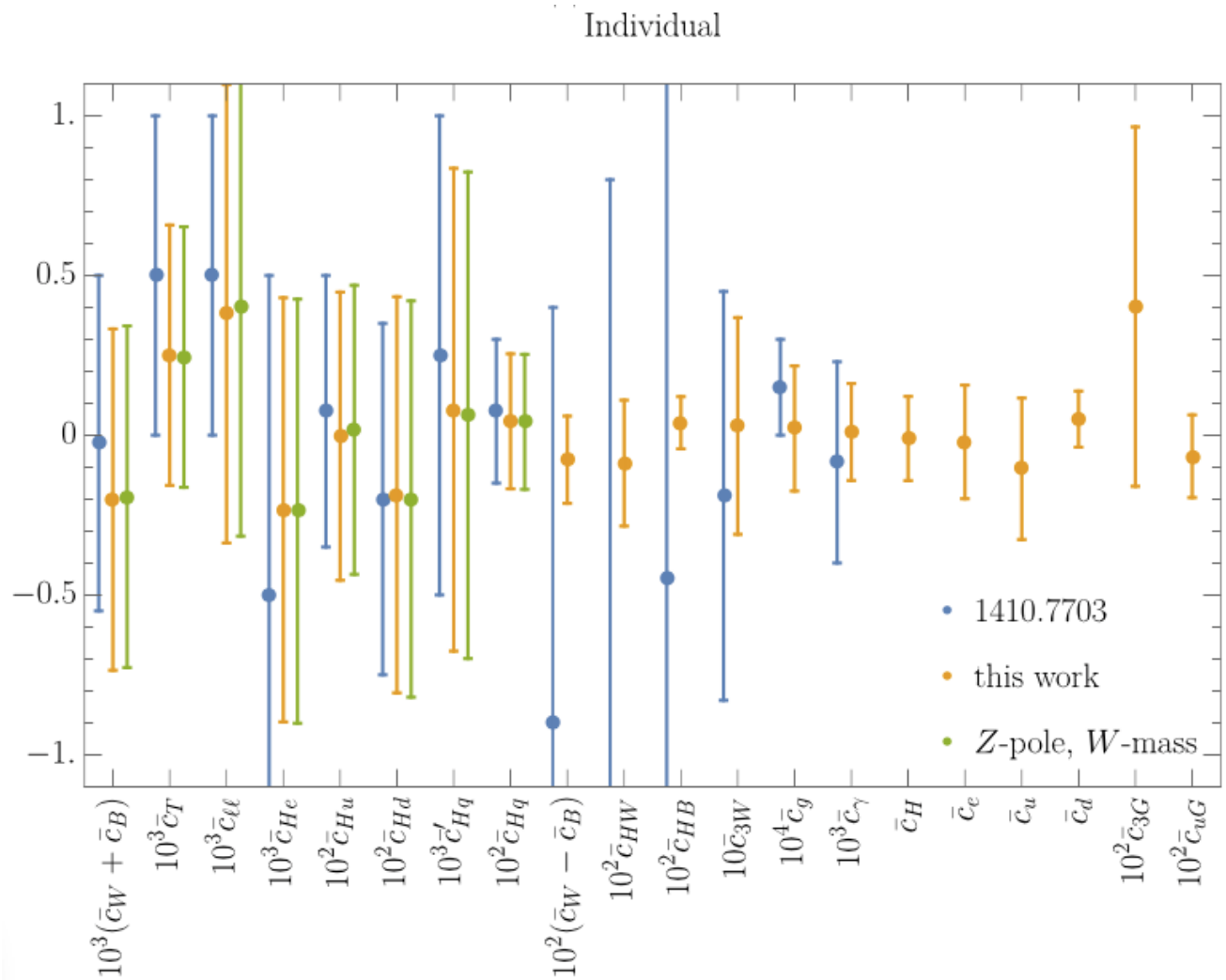
- ATLAS+CMS Higgs Run 2

Including kinematical information facilitated by **STXS**

	Production	Decay	Sig. Stren.		Production	Decay	Sig. Stren.
[96]	1-jet, $p_T > 450$	$b\bar{b}$	$2.3^{+1.8}_{-1.6}$	[105]	pp	$\mu\mu$	-0.1 ± 1.4
[97]	Zh	$b\bar{b}$	0.9 ± 0.5	[106]	Zh	$b\bar{b}$	$0.69^{+0.35}_{-0.31}$
[97]	Wh	$b\bar{b}$	1.7 ± 0.7	[106]	Wh	$b\bar{b}$	$1.21^{+0.45}_{-0.42}$
[98]	$t\bar{t}h$	$b\bar{b}$	$-0.19^{+0.80}_{-0.81}$	[107]	$t\bar{t}h$	$b\bar{b}$	$0.84^{+0.64}_{-0.61}$
[99]	$t\bar{t}h$	$1\ell + 2\tau_h$	$-1.20^{+1.50}_{-1.47}$	[108]	$t\bar{t}h$	$2\ell os + 1\tau_h$	$1.7^{+2.1}_{-1.9}$
[99]	$t\bar{t}h$	$2\ell ss + 1\tau_h$	$0.86^{+0.79}_{-0.66}$	[108]	$t\bar{t}h$	$1\ell + 2\tau_h$	$-0.6^{+1.6}_{-1.5}$
[99]	$t\bar{t}h$	$3\ell + 1\tau_h$	$1.22^{+1.34}_{-1.00}$	[108]	$t\bar{t}h$	$3\ell + 1\tau_h$	$1.6^{+1.8}_{-1.3}$
[100]	$t\bar{t}h$	$2\ell ss$	$1.7^{+0.6}_{-0.5}$	[108]	$t\bar{t}h$	$2\ell ss + 1\tau_h$	$3.5^{+1.7}_{-1.3}$
[100]	$t\bar{t}h$	3ℓ	$1.0^{+0.8}_{-0.7}$	[108]	$t\bar{t}h$	3ℓ	$1.8^{+0.9}_{-0.7}$
[100]	$t\bar{t}h$	4ℓ	$0.9^{+2.3}_{-1.6}$	[108]	$t\bar{t}h$	$2\ell ss$	$1.5^{+0.7}_{-0.6}$
[101]	0-jet	WW	$0.9^{+0.4}_{-0.3}$	[109]	VBF	WW	$1.7^{+1.1}_{-0.9}$
[101]	1-jet	WW	1.1 ± 0.4	[109]	Wh	WW	$3.2^{+4.4}_{-4.2}$
[101]	2-jet	WW	1.3 ± 1.0	[110]	$\mathrm{B}(h \to \gamma\gamma)/\ \mathrm{B}(h \to 4\ell)$		$0.69^{+0.15}_{-0.13}$
[101]	VBF 2-jet	WW	1.4 ± 0.8	[110]	0-jet	4ℓ	$1.07^{+0.27}_{-0.25}$
[101]	Vh 2-jet	WW	$2.1^{+2.3}_{-2.2}$	[110]	1-jet, $p_T < 60$	4ℓ	$0.67^{+0.72}_{-0.68}$
[101]	Wh 3-lep	WW	-1.4 ± 1.5	[110]	1-jet, $p_T \in (60, 120)$	4ℓ	$1.00^{+0.63}_{-0.55}$
[102]	ggF	$\gamma\gamma$	$1.11^{+0.19}_{-0.18}$	[110]	1-jet, $p_T \in (120, 200)$	4ℓ	$2.1^{+1.5}_{-1.3}$
[102]	VBF	$\gamma\gamma$	$0.5^{+0.6}_{-0.5}$	[110]	2-jet	4ℓ	$2.2^{+1.1}_{-1.0}$
[102]	$t\bar{t}h$	$\gamma\gamma$	2.2 ± 0.9	[110]	"BSM-like"	4ℓ	$2.3^{+1.2}_{-1.0}$
[102]	Vh	$\gamma\gamma$	$2.3^{+1.1}_{-1.0}$	[110]	VBF, $p_T < 200$	4ℓ	$2.14^{+0.94}_{-0.77}$
[103]	ggF	4ℓ	$1.20^{+0.22}_{-0.21}$	[110]	Vh lep	4ℓ	$0.3^{+1.3}_{-1.2}$
[104]	0-jet	$\tau\tau$	0.84 ± 0.89	[110]	$t\bar{t}h$	4ℓ	$0.51^{+0.86}_{-0.70}$
[104]	boosted	$\tau\tau$	$1.17^{+0.47}_{-0.40}$				
[104]	VBF	$\tau\tau$	$1.11^{+0.34}_{-0.35}$				

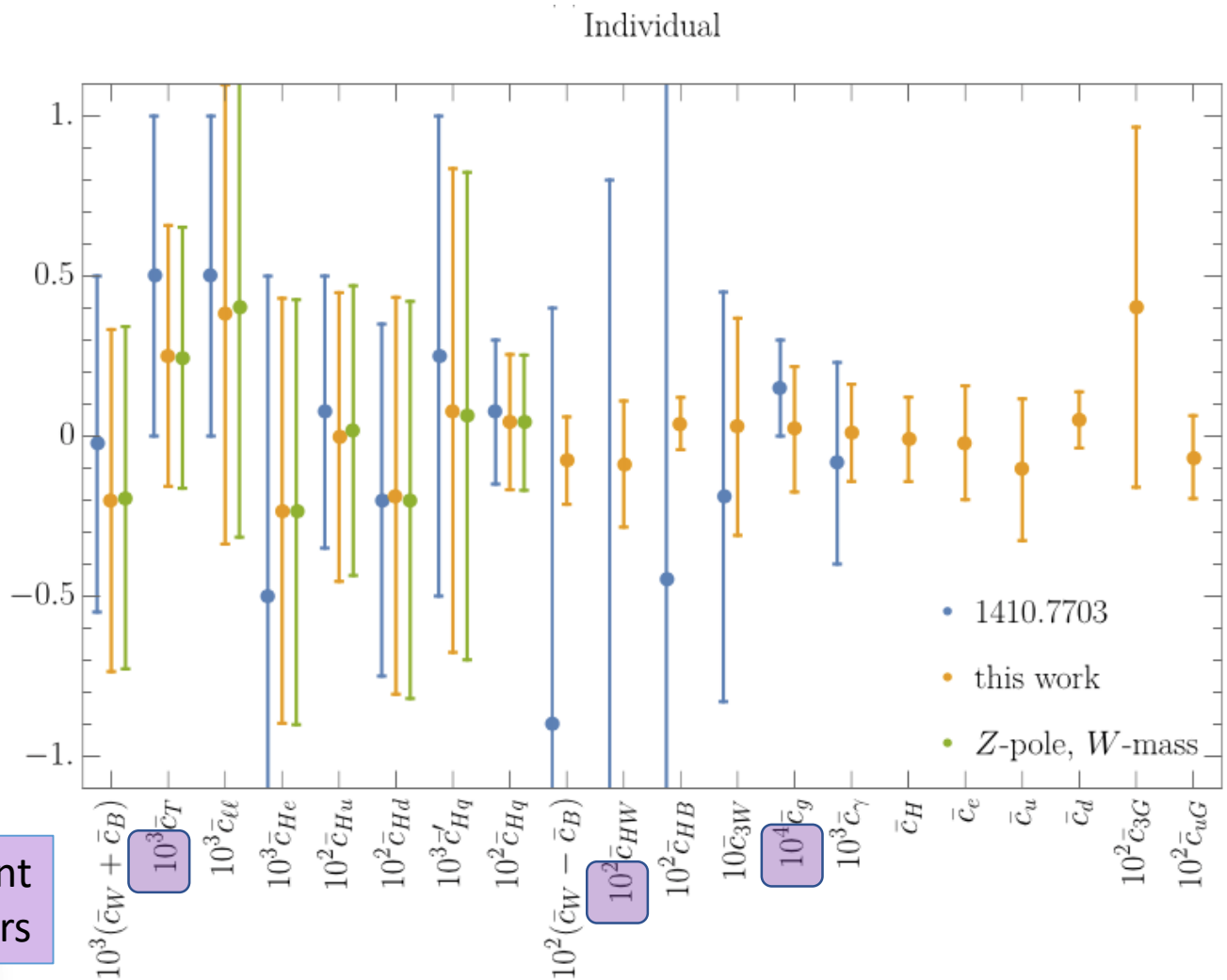
Results

- **SILH** basis, fit each operator **individually**



Results

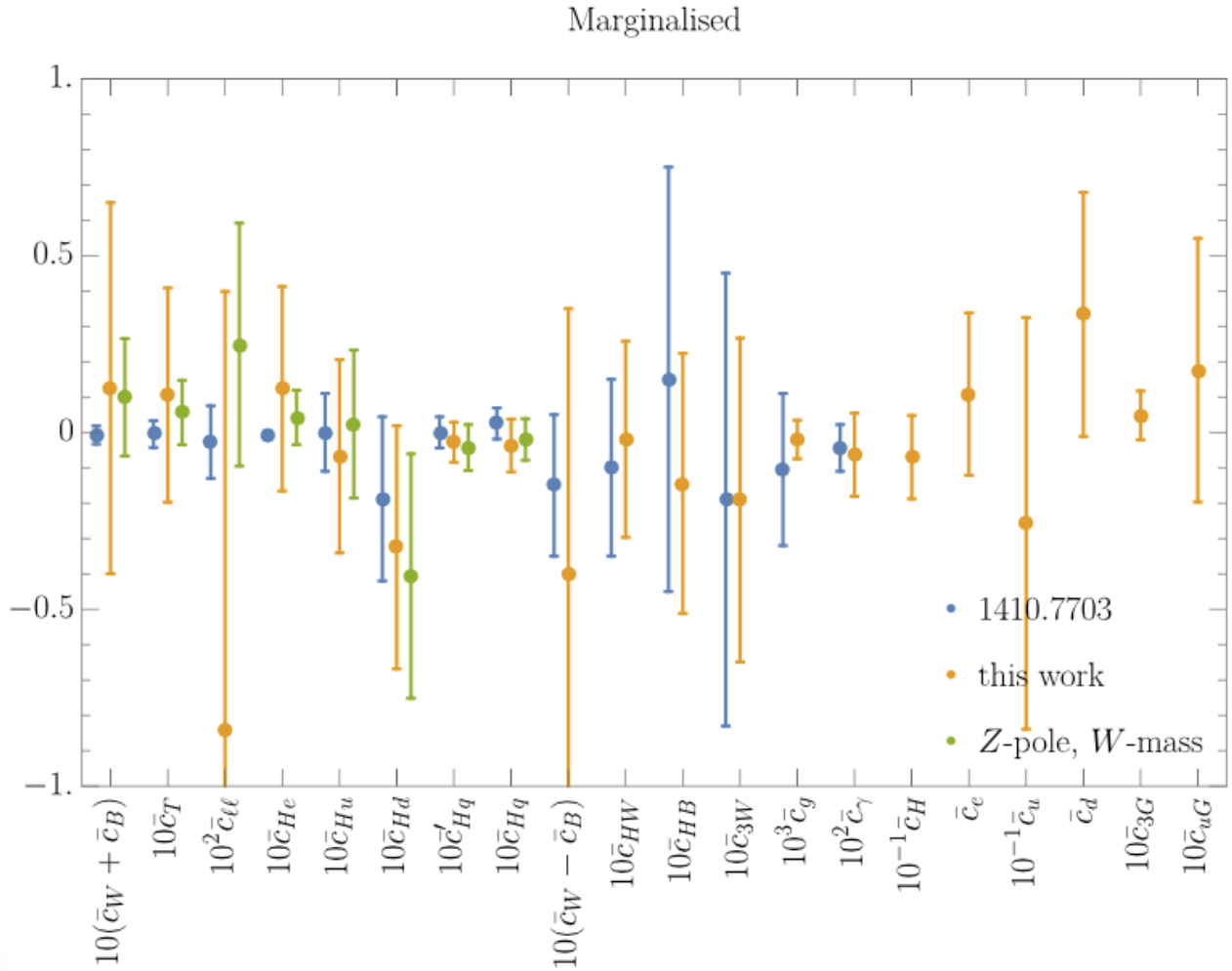
- **SILH** basis, fit each operator **individually**



Note different scaling factors

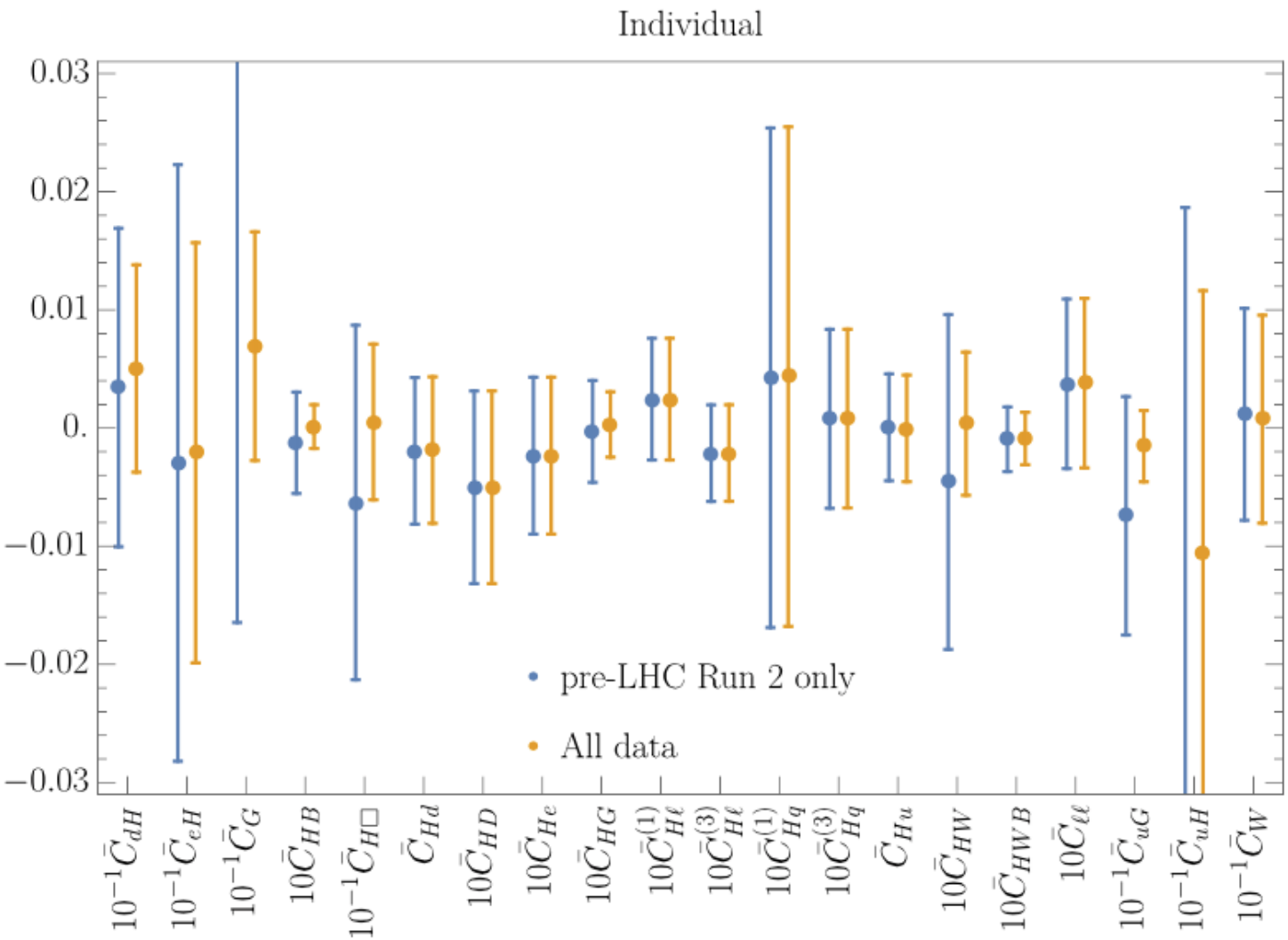
Results

- **SILH** basis, fit *all* operators **simultaneously**



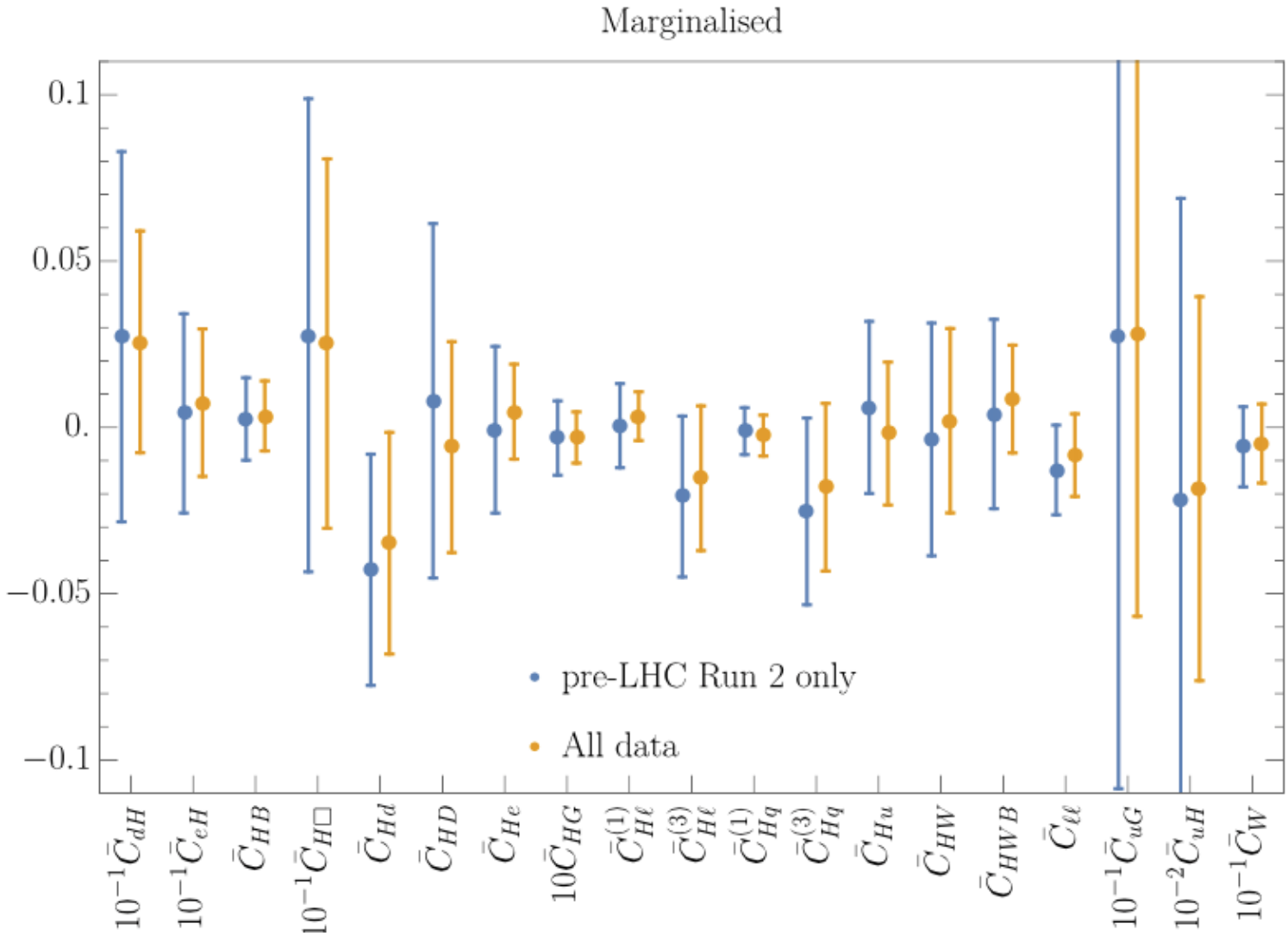
Results

- **Warsaw** basis, fit each operator **individually**



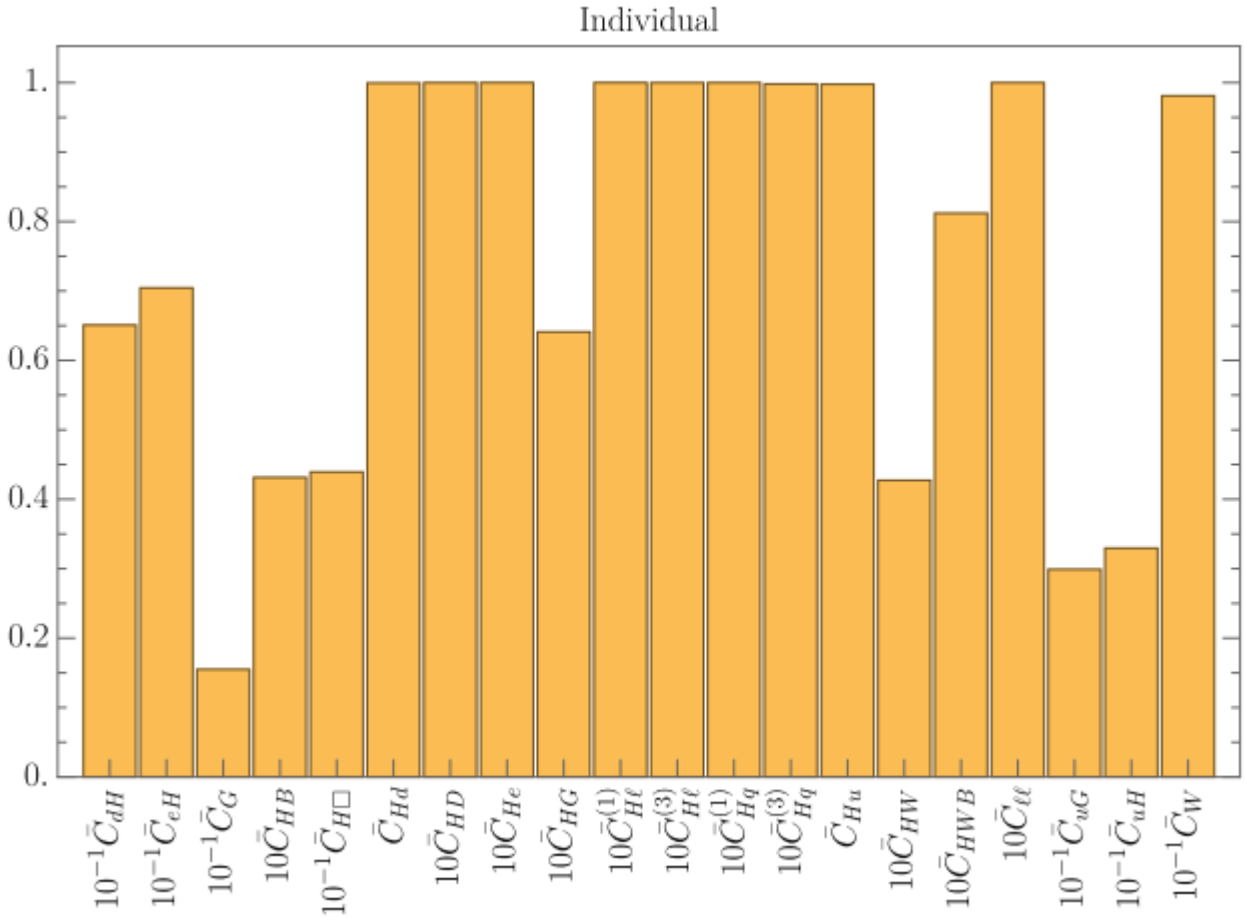
Results

- **Warsaw** basis, fit *all* operators **simultaneously**



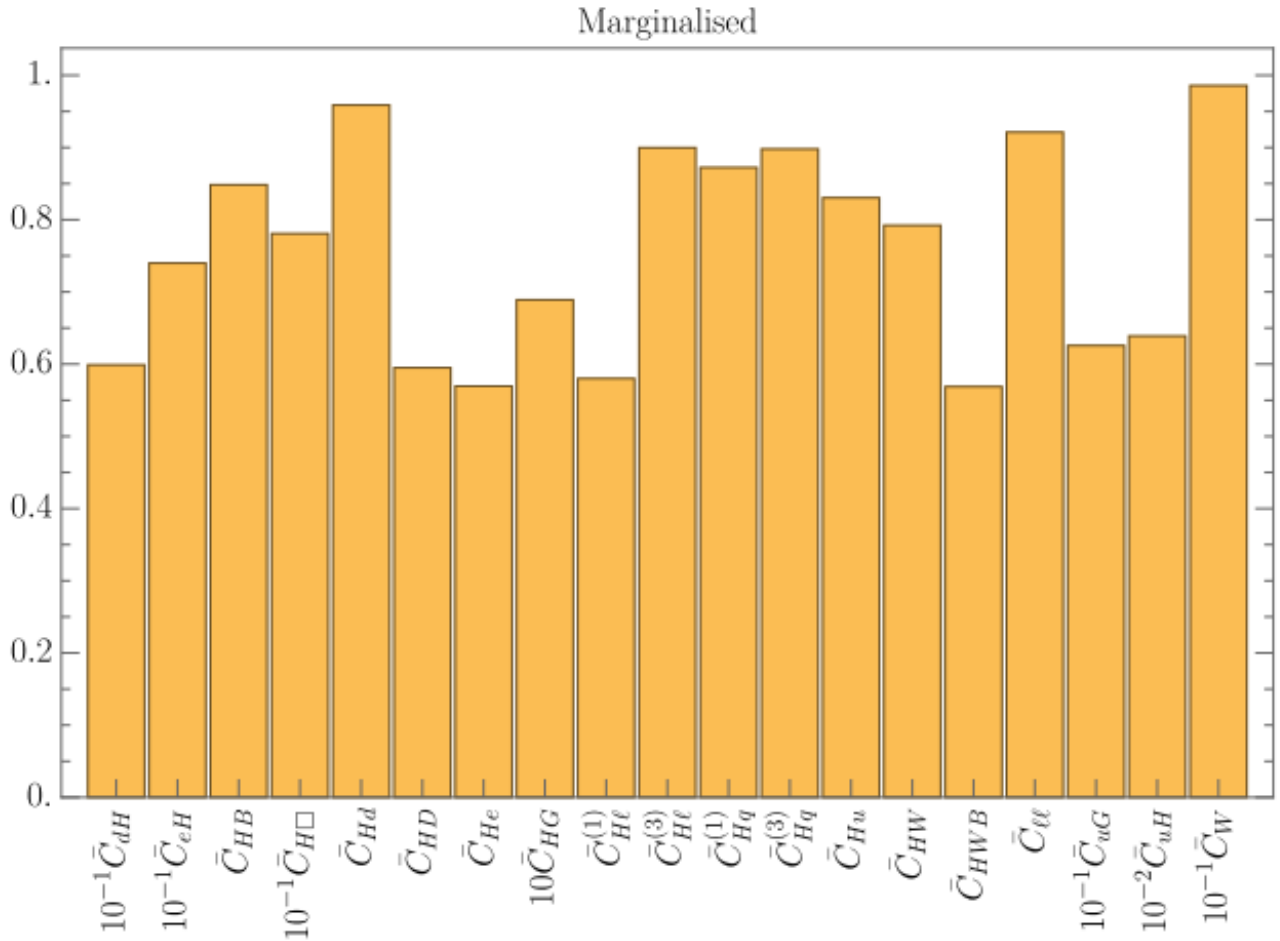
Results

- **Warsaw** basis, improvement from Run 1 to 2 (lower is better) for **individual** fit



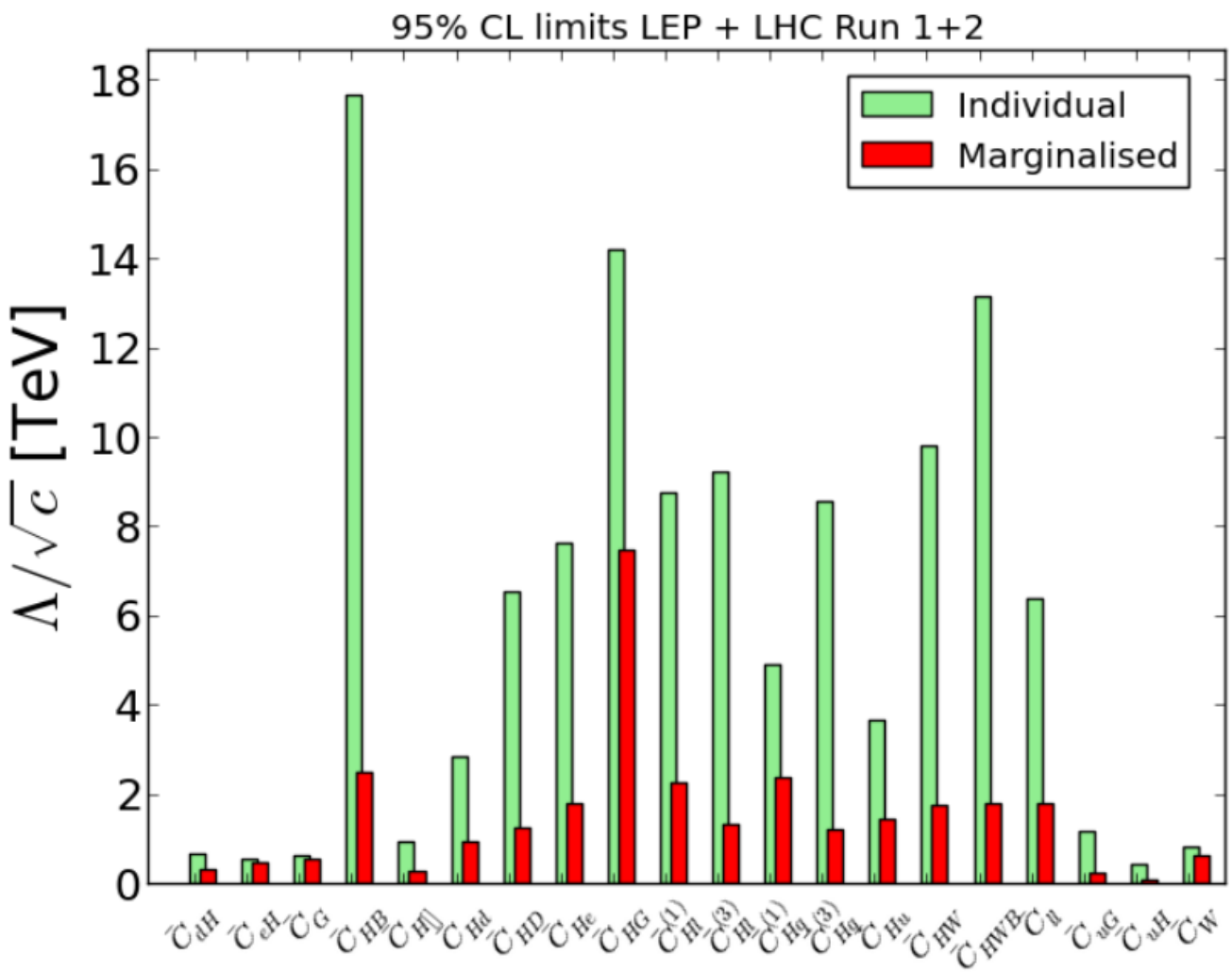
Results

- **Warsaw** basis, improvement from Run 1 to 2 (lower is better) for **marginalised** fit



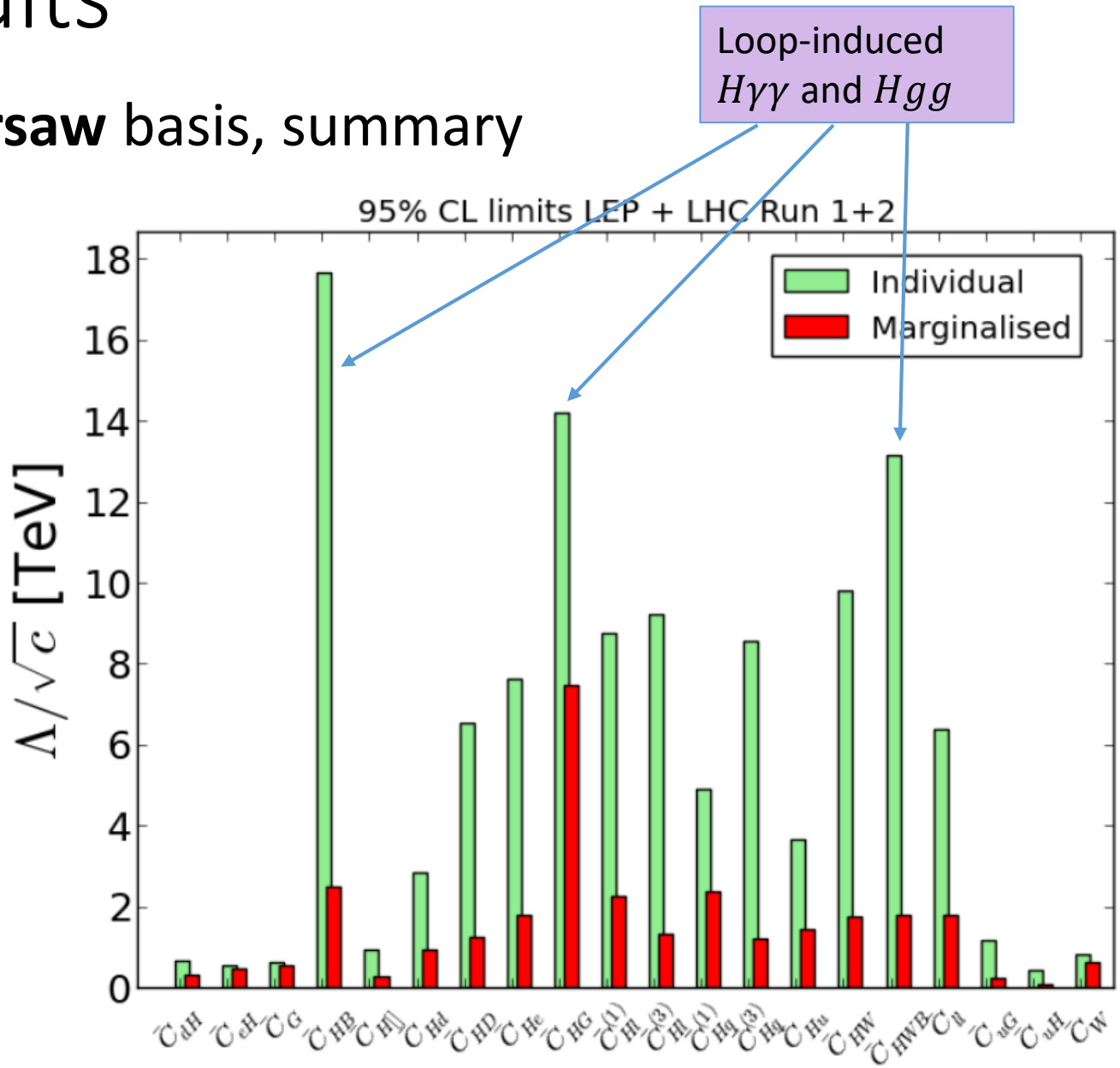
Results

- **Warsaw** basis, summary



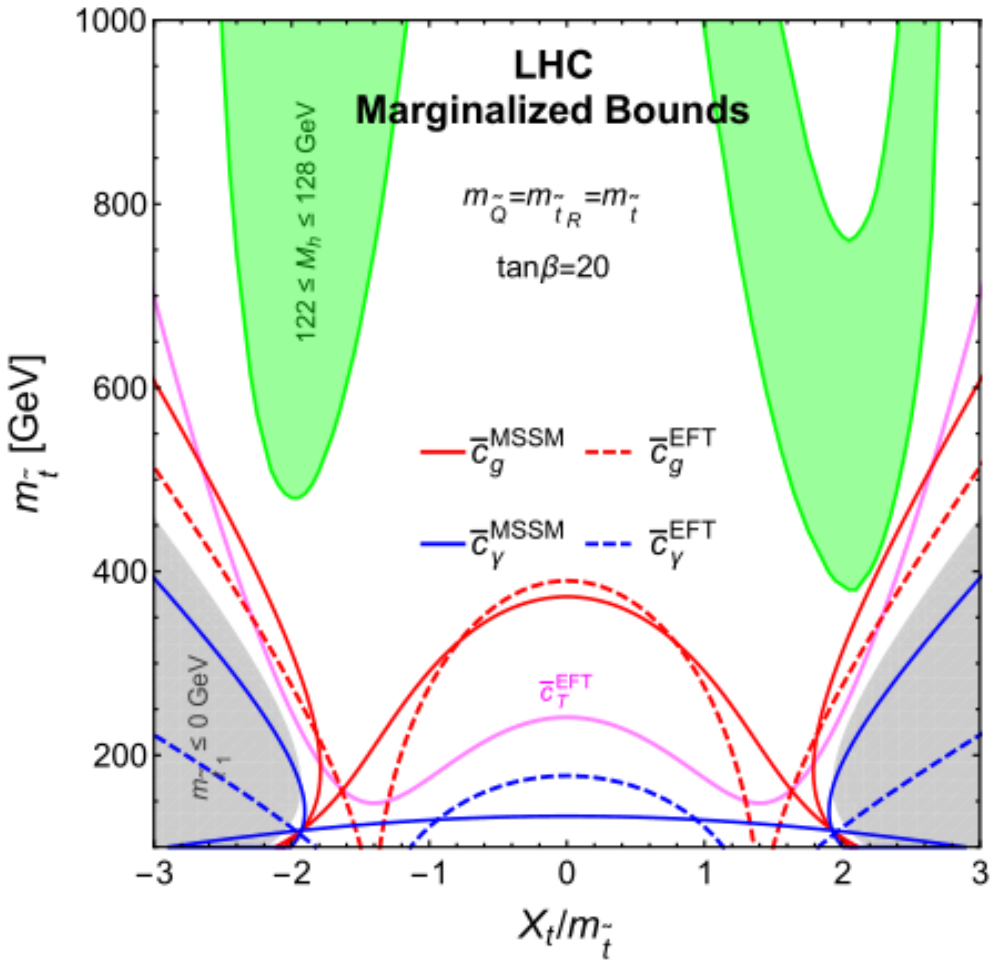
Results

- **Warsaw** basis, summary



Results

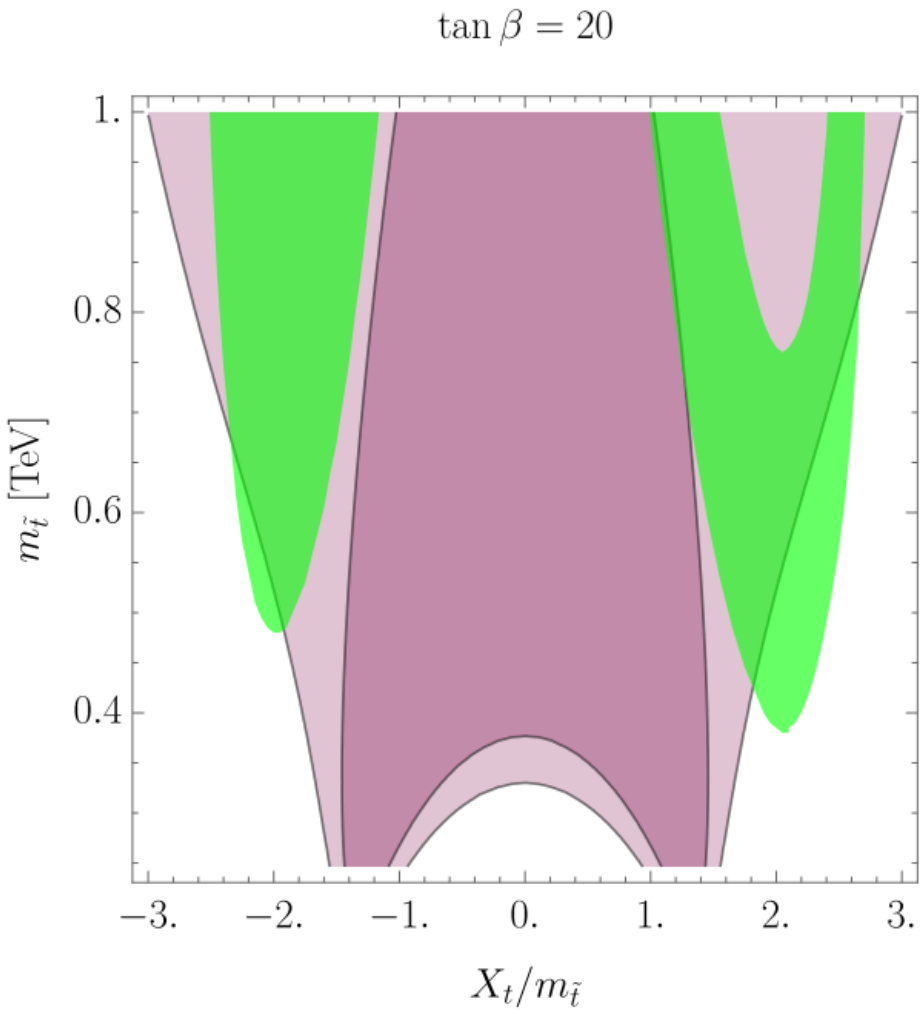
- Simplified models: **stops** (Run 1)



Drozd, Ellis, Quevillon and T.Y.
1504.02409

Results

- Simplified models: **stops** (Run 2)



Results

- Simplified models: **renormalisable SM extensions**

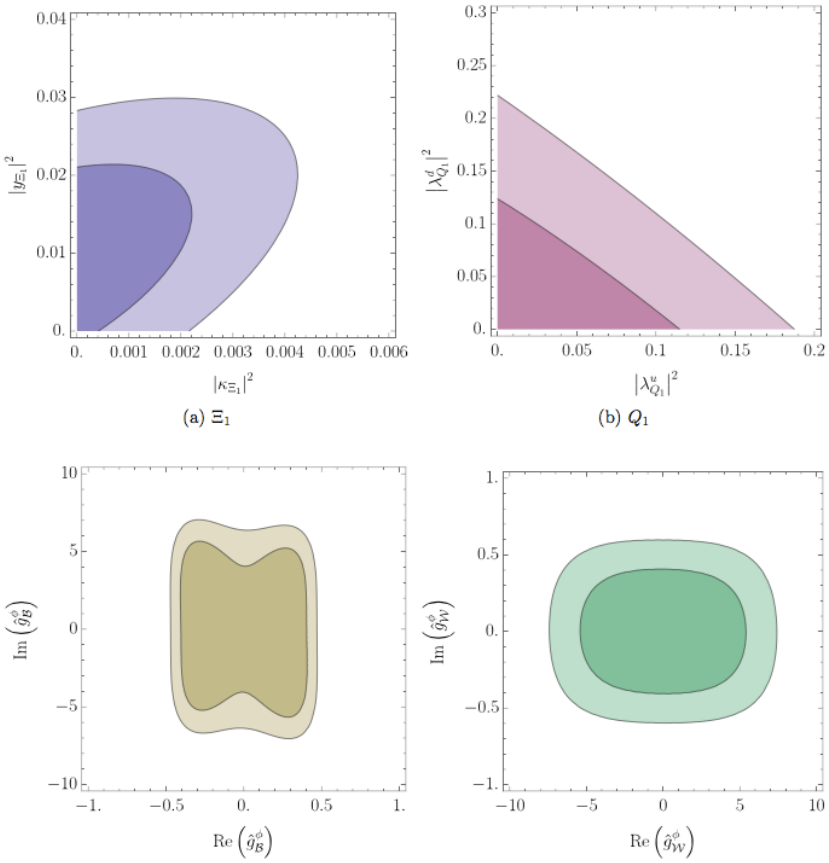
Name	Spin	$SU(3)$	$SU(2)$	$U(1)$	Name	Spin	$SU(3)$	$SU(2)$	$U(1)$
$\mathcal{S}$	0	1	1	0	Δ_1	$\frac{1}{2}$	1	2	$-\frac{1}{2}$
$\mathcal{S}_1$	0	1	1	1	Δ_3	$\frac{1}{2}$	1	2	$-\frac{1}{2}$
φ	0	1	2	$\frac{1}{2}$	Σ	$\frac{1}{2}$	1	3	0
Ξ	0	1	3	0	Σ_1	$\frac{1}{2}$	1	3	-1
Ξ_1	0	1	3	1	U	$\frac{1}{2}$	3	1	$\frac{2}{3}$
$\mathcal{B}$	1	1	1	0	D	$\frac{1}{2}$	3	1	$-\frac{1}{3}$
$\mathcal{B}_1$	1	1	1	1	Q_1	$\frac{1}{2}$	3	2	$\frac{1}{6}$
$\mathcal{W}$	1	1	3	0	Q_5	$\frac{1}{2}$	3	2	$-\frac{5}{6}$
$\mathcal{W}_1$	1	1	3	1	Q_7	$\frac{1}{2}$	3	2	$\frac{7}{6}$
N	$\frac{1}{2}$	1	1	0	T_1	$\frac{1}{2}$	3	3	$-\frac{1}{3}$
E	$\frac{1}{2}$	1	1	-1	T_2	$\frac{1}{2}$	3	3	$\frac{2}{3}$

- Classification and tree-level matching dictionary

De Blas, Criado, Perez-Victoria, Santiago [1711.10391]

Results

- Simplified models: **renormalisable SM extensions**



(a) Ξ_1 (b) Q_1 (c) $\mathcal{B}$, no ψ^4 operators (d) $\mathcal{W}$, no ψ^4 operators

Model	χ^2	χ^2/n_d	Coupling	Mass / TeV
SM	157	0.987	-	-
$\mathcal{S}_1$	156	0.986	$\lvert y_{\mathcal{S}_1}\rvert^2 = (6.3 \pm 5.9) \cdot 10^{-3}$	$M_{\mathcal{S}_1} = (9.0,\ 49)$
φ, Type I	156	0.986	$Z_6 \cdot \cos\beta = -0.64 \pm 0.59$	$M_\varphi = (0.9,\ 4.3)$
Ξ	155	0.984	$\lvert\kappa_\Xi\rvert^2 = (4.2 \pm 3.4) \cdot 10^{-3}$	$M_\Xi = (12,\ 35)$
N	155	0.978	$\lvert\lambda_N\rvert^2 = (1.8 \pm 1.2) \cdot 10^{-2}$	$M_N = (5.8,\ 13)$
$\mathcal{W}_1$	155	0.984	$\left\lvert\hat{g}^\phi_{\mathcal{W}_1}\right\rvert^2 = (3.3 \pm 2.7) \cdot 10^{-3}$	$M_{\mathcal{W}_1} = (4.1,\ 13)$
E	157	0.993	$\lvert\lambda_E\rvert^2 < 1.2 \cdot 10^{-2}$	$M_E > 9.2$
Δ_3	156	0.990	$\lvert\lambda_{\Delta_3}\rvert^2 < 1.9 \cdot 10^{-2}$	$M_{\Delta_3} > 7.3$
Σ	157	0.992	$\lvert\lambda_\Sigma\rvert^2 < 2.9 \cdot 10^{-2}$	$M_\Sigma > 5.9$
Q_5	156	0.990	$\lvert\lambda_{Q_5}\rvert^2 < 0.18$	$M_{Q_5} > 2.4$
T_2	157	0.992	$\lvert\lambda_{T_2}\rvert^2 < 7.1 \cdot 10^{-2}$	$M_{T_2} > 3.8$
$\mathcal{S}$	157	0.993	$\lvert y_\mathcal{S}\rvert^2 < 0.32$	$M_\mathcal{S} > 1.8$
Δ_1	157	0.993	$\lvert\lambda_{\Delta_1}\rvert^2 < 5.7 \cdot 10^{-3}$	$M_{\Delta_1} > 13$
Σ_1	157	0.993	$\lvert\lambda_{\Sigma_1}\rvert^2 < 7.3 \cdot 10^{-3}$	$M_{\Sigma_1} > 12$
U	157	0.993	$\lvert\lambda_U\rvert^2 < 2.8 \cdot 10^{-2}$	$M_U > 6.0$
D	157	0.993	$\lvert\lambda_D\rvert^2 < 1.4 \cdot 10^{-2}$	$M_D > 8.4$
Q_7	157	0.993	$\lvert\lambda_{Q_7}\rvert^2 < 7.7 \cdot 10^{-2}$	$M_{Q_7} > 3.6$
T_1	157	0.993	$\lvert\lambda_{T_1}\rvert^2 < 0.13$	$M_{T_1} > 3.0$
$\mathcal{B}_1$	157	0.993	$\left\lvert\hat{g}^\phi_{\mathcal{B}_1}\right\rvert^2 < 2.4 \cdot 10^{-3}$	$M_{\mathcal{B}_1} > 21$

- Streamlines process of interpreting limits on BSM parameter space

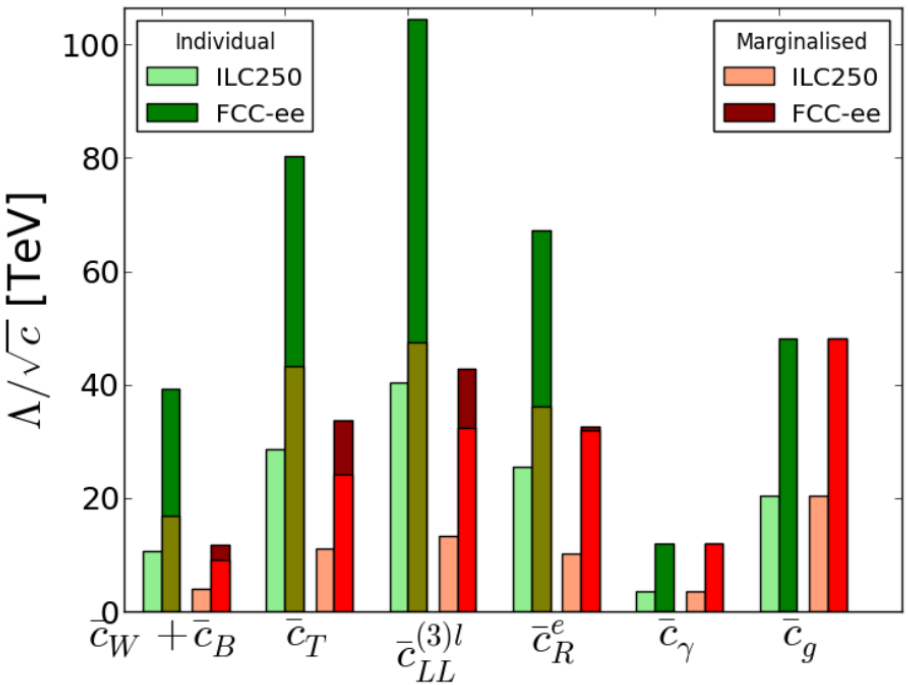
Tevong You

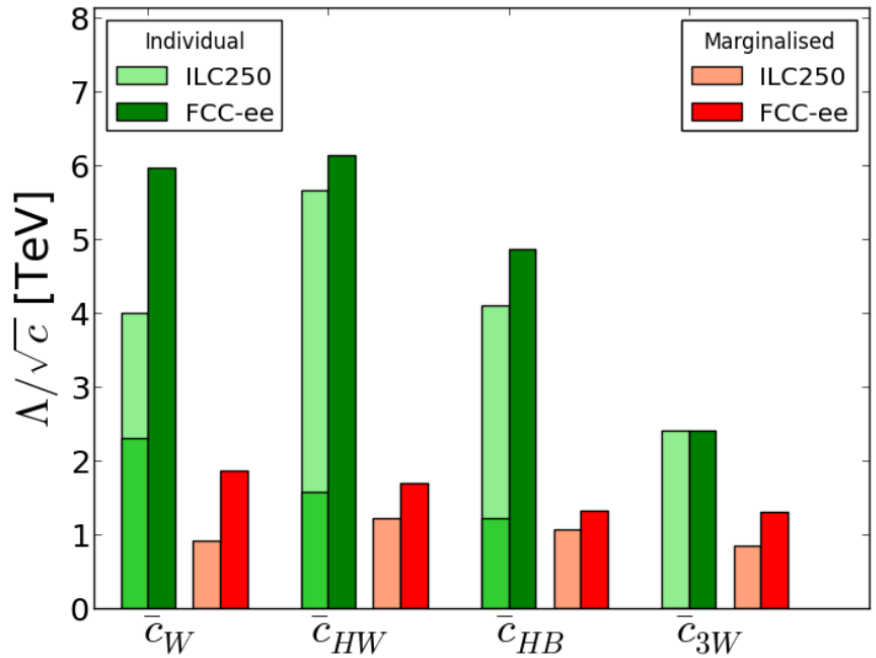
Future e+e- Constraints

J. Ellis and T.Y. [arXiv:1510:04561]

- Simplified FCC-ee projections based on four leptonic observables:

$$\sigma_{m_W} = 0.0005 \text{ GeV} \quad , \quad \sigma_{\Gamma_Z} = 0.0001 \text{ GeV} \quad , \quad \sigma_{R_l} = 0.001 \quad , \quad \sigma_{A_e} = 0.000015$$





- With and without theory uncertainties:

$$\sigma_{\Gamma_Z}^{\text{th}} = 0.0001 \text{ GeV} \quad , \quad \sigma_{m_W}^{\text{th}} = 0.001 \text{ GeV} \quad , \quad \sigma_{A_e}^{\text{th}} = 0.000118$$

Future e+e- Constraints

- Future precision sensitive to TeV scale even for loop-induced operators

- One-loop matching simplified by a **Universal One-Loop Effective Action**

Henning, Lu, Murayama, 1412.1837;
Drozd, J. Ellis, Quevillon, TY, 1512.03003;
S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1604.02445, 1706.07765.

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} [P_\mu, [P_\mu, U_{ij}]] [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} [P_\mu, U_{ij}] [P_\nu, U_{jk}] G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

+ …

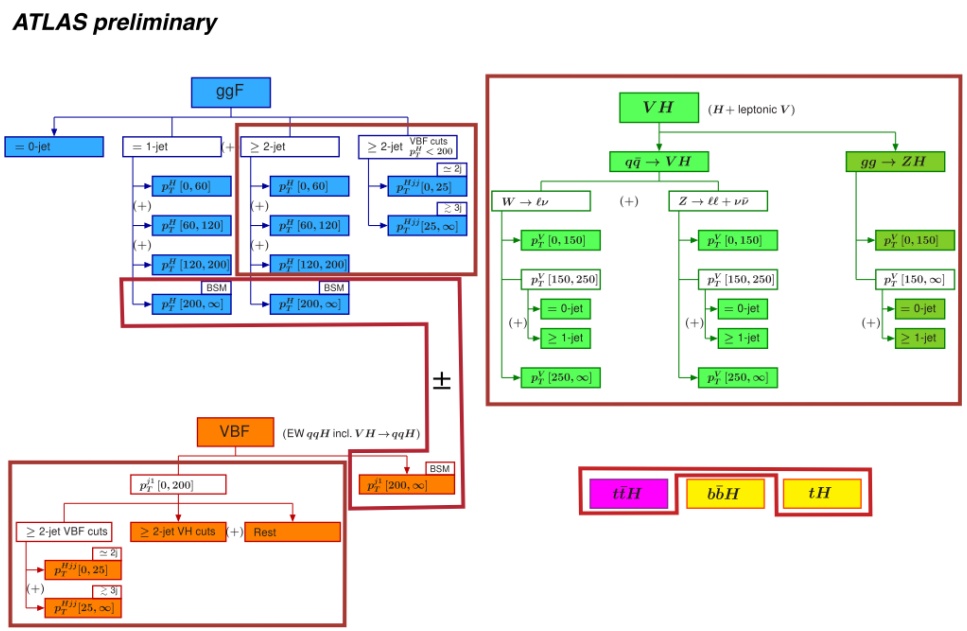
- Enables optimal comparison between BSM and data

Conclusion

- SM EFT framework is the **Fermi theory of the 21st century**
- **Systematic classification** of decoupled new physics
- **Correlates measurements** and **eases interpretation**
- Finding **patterns of deviations** will give clues to a deeper underlying theory at higher energies

STXS

- Simplified Template Cross-Sections
- Sub-division into kinematic regions for production processes

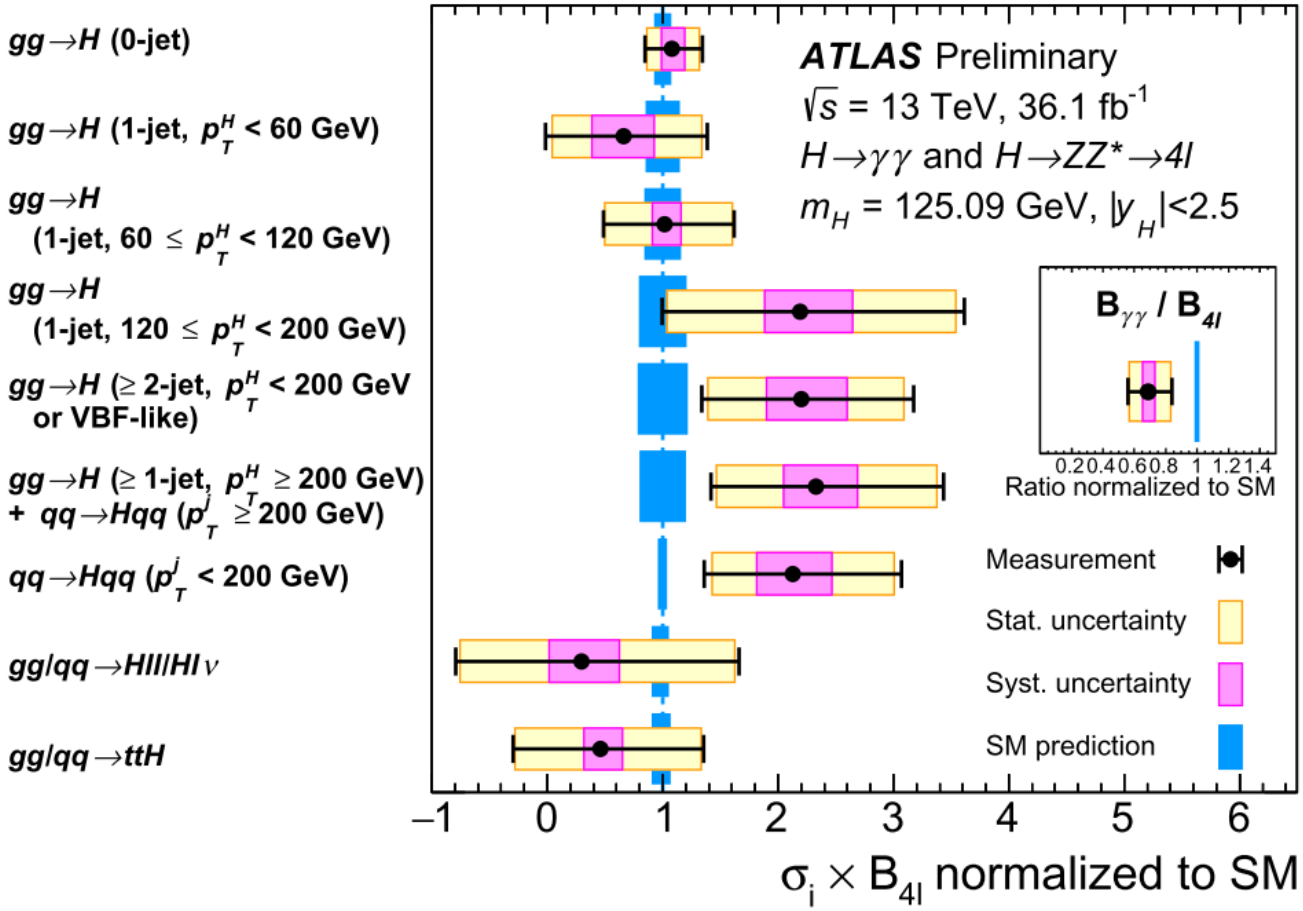


ATLAS-CONF-2017-047

- Facilitates combination and interpretation

STXS

- STXS measurements



ATLAS-CONF-2017-047

STXS

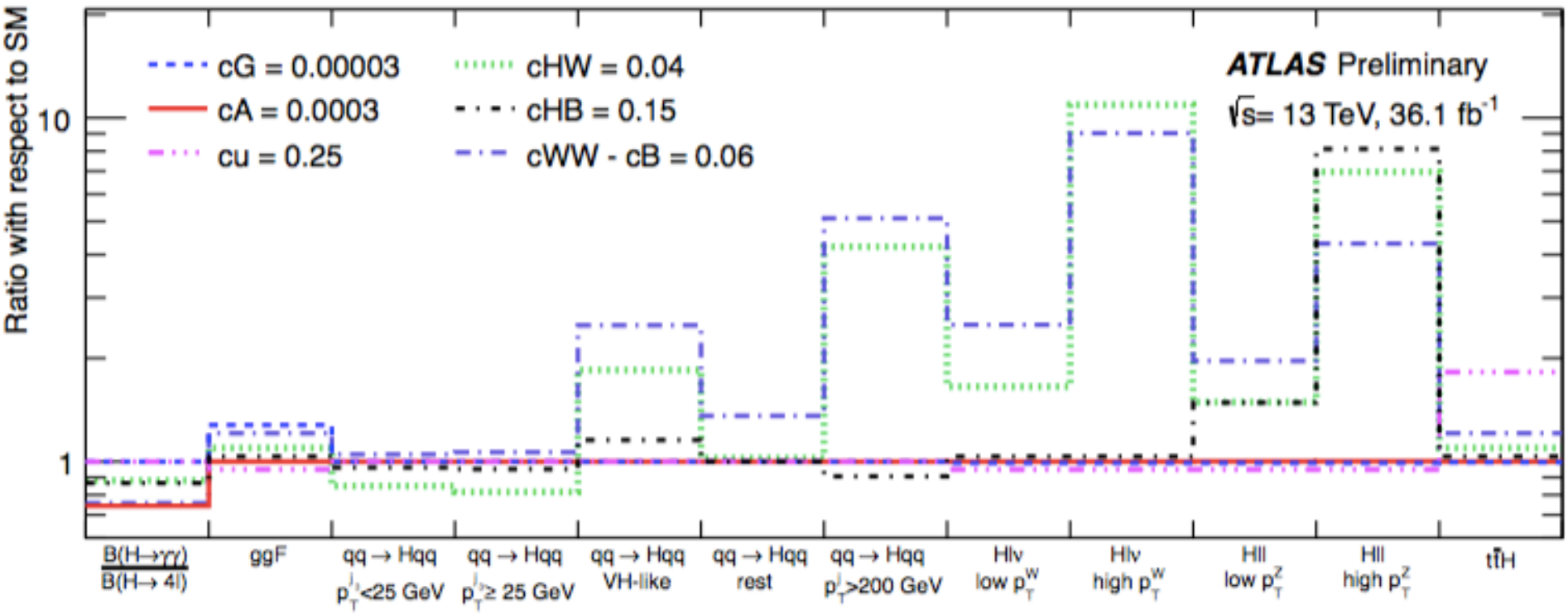
- STXS dim-6 predictions

Cross-section region	$\sum_i A_i c_i$
$gg \to H$ (0-jet) $gg \to H$ (1-jet, $p_T^H < 60$ GeV) $gg \to H$ (1-jet, $60 \le p_T^H < 120$ GeV)	$56c_g'$
$gg \to H$ (1-jet, $120 \le p_T^H < 200$ GeV)	$56c_g' + 18\mathtt{c3G} + 11\mathtt{c2G}$
$gg \to H$ (1-jet, $p_T^H \ge 200$ GeV)	$56c_g' + 52\mathtt{c3G} + 34\mathtt{c2G}$
$gg \to H$ ($\ge$ 2-jet, $p_T^H < 60$ GeV)	$56c_g'$
$gg \to H$ ($\ge$ 2-jet, $60 \le p_T^H < 120$ GeV)	$56c_g' + 8\mathtt{c3G} + 7\mathtt{c2G}$
$gg \to H$ ($\ge$ 2-jet, $120 \le p_T^H < 200$ GeV)	$56c_g' + 23\mathtt{c3G} + 18\mathtt{c2G}$
$gg \to H$ ($\ge$ 2-jet, $p_T^H \ge 200$ GeV)	$56c_g' + 90\mathtt{c3G} + 68\mathtt{c2G}$
$gg \to H$ ($\ge$ 2-jet VBF-like, $p_T^{j_3} < 25$ GeV)	$56c_g'$
$gg \to H$ ($\ge$ 2-jet VBF-like, $p_T^{j_3} \ge 25$ GeV)	$56c_g' + 9\mathtt{c3G} + 8\mathtt{c2G}$
$qq \to Hqq$ (VBF-like, $p_T^{j_3} < 25$ GeV)	$-1.0\mathtt{cH} - 1.0\mathtt{cT} + 1.3\mathtt{cWW} - 0.023\mathtt{cB} - 4.3\mathtt{cHW}$ $-0.29\mathtt{cHB} + 0.092\mathtt{cHQ} - 5.3\mathtt{cpHQ} - 0.33\mathtt{cHu} + 0.12\mathtt{cHd}$
$qq \to Hqq$ (VBF-like, $p_T^{j_3} \ge 25$ GeV)	$-1.0\mathtt{cH} - 1.1\mathtt{cT} + 1.2\mathtt{cWW} - 0.027\mathtt{cB} - 5.8\mathtt{cHW}$ $-0.41\mathtt{cHB} + 0.13\mathtt{cHQ} - 6.9\mathtt{cpHQ} - 0.45\mathtt{cHu} + 0.15\mathtt{cHd}$
$qq \to Hqq$ ($p_T^j \ge 200$ GeV)	$-1.0\mathtt{cH} - 0.95\mathtt{cT} + 1.5\mathtt{cWW} - 0.025\mathtt{cB} - 3.6\mathtt{cHW}$ $-0.24\mathtt{cHB} + 0.084\mathtt{cHQ} - 4.5\mathtt{cpHQ} - 0.25\mathtt{cHu} + 0.1\mathtt{cHd}$
$qq \to Hqq$ ($60 \le m_{jj} < 120$ GeV)	$-0.99\mathtt{cH} - 1.2\mathtt{cT} + 7.8\mathtt{cWW} - 0.19\mathtt{cB} - 31\mathtt{cHW}$ $-2.4\mathtt{cHB} + 0.9\mathtt{cHQ} - 38\mathtt{cpHQ} - 2.8\mathtt{cHu} + 0.9\mathtt{cHd}$
$qq \to Hqq$ (rest)	$-1.0\mathtt{cH} - 1.0\mathtt{cT} + 1.4\mathtt{cWW} - 0.028\mathtt{cB} - 6.2\mathtt{cHW}$ $-0.42\mathtt{cHB} + 0.14\mathtt{cHQ} - 6.9\mathtt{cpHQ} - 0.42\mathtt{cHu} + 0.16\mathtt{cHd}$
$gg/q\bar{q} \to ttH$	$-0.98\mathtt{cH} + 2.9\mathtt{cu} + 0.93\mathtt{cG} + 310\mathtt{cuG}$ $+27\mathtt{c3G} - 13\mathtt{c2G}$

Hays, Sanz, Zemaityte
[LHCHXSWG-INT-2017-01]

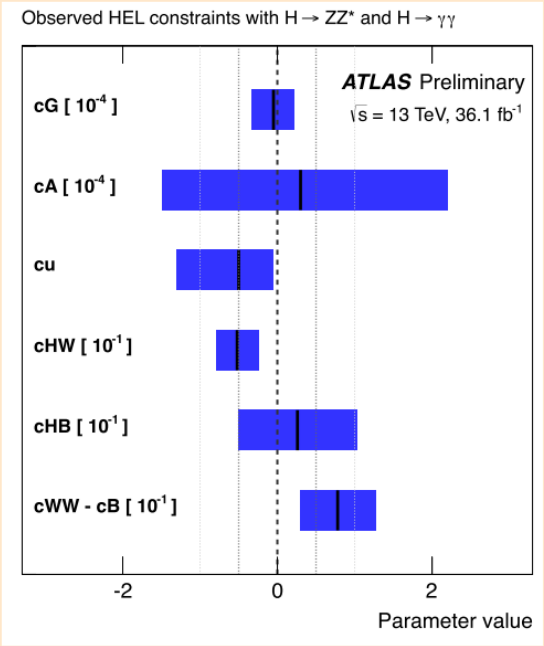
STXS

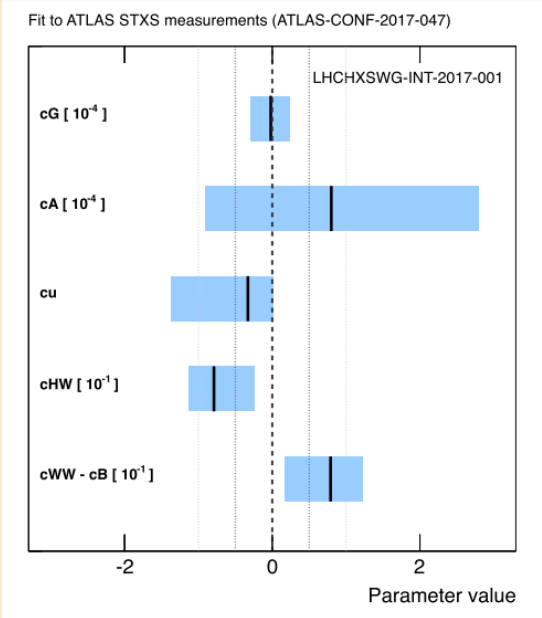
- STXS dim-6 predictions



STXS

- Good agreement with optimised non-STXS fit





ATLAS fit uses more categories than measured STXS

Fit to STXS measurement can only constrain five parameters

https://indico.cern.ch/event/682466/contributions/2796820/attachments/1573120/2482936/WG211Dec2017.pdf

27 Mar 2018 C. Hays, Oxford University 13

- See also VHbb case (ATLAS-CONF-2018-053)

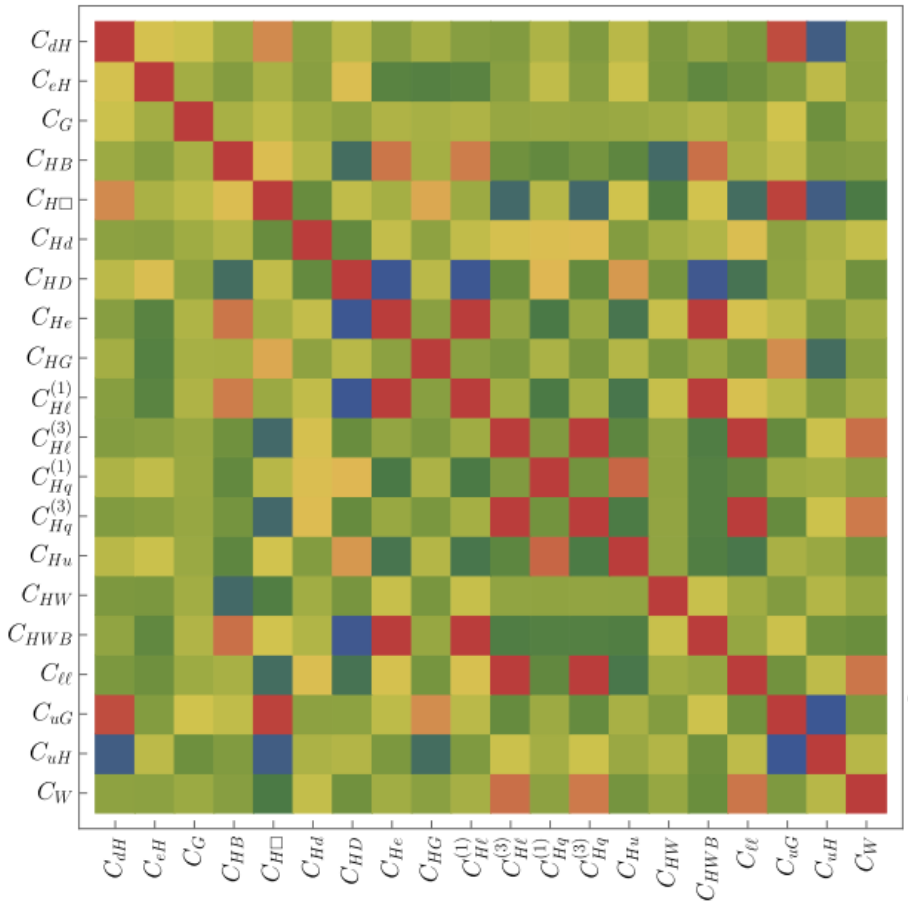
De Blas, Lohwasser, Musella, Mimasu (1803.10379)

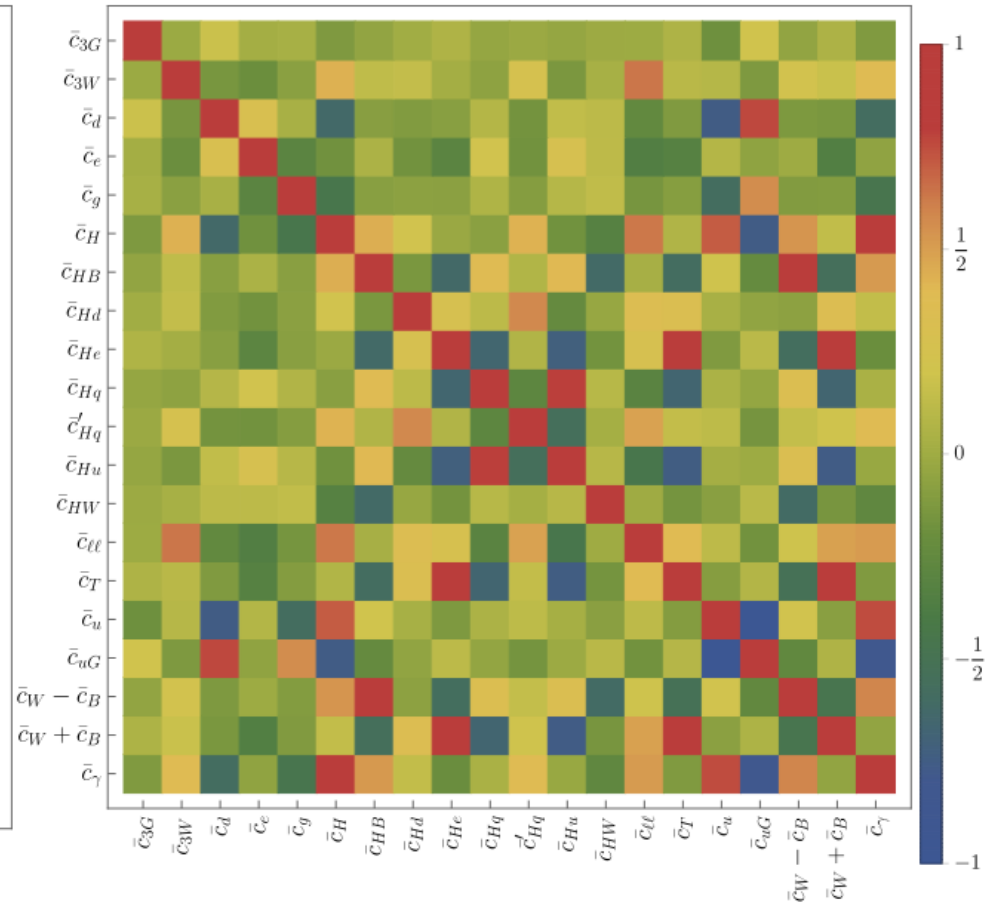
Talks by Douglas Schaefer and Chris Hays

Backup

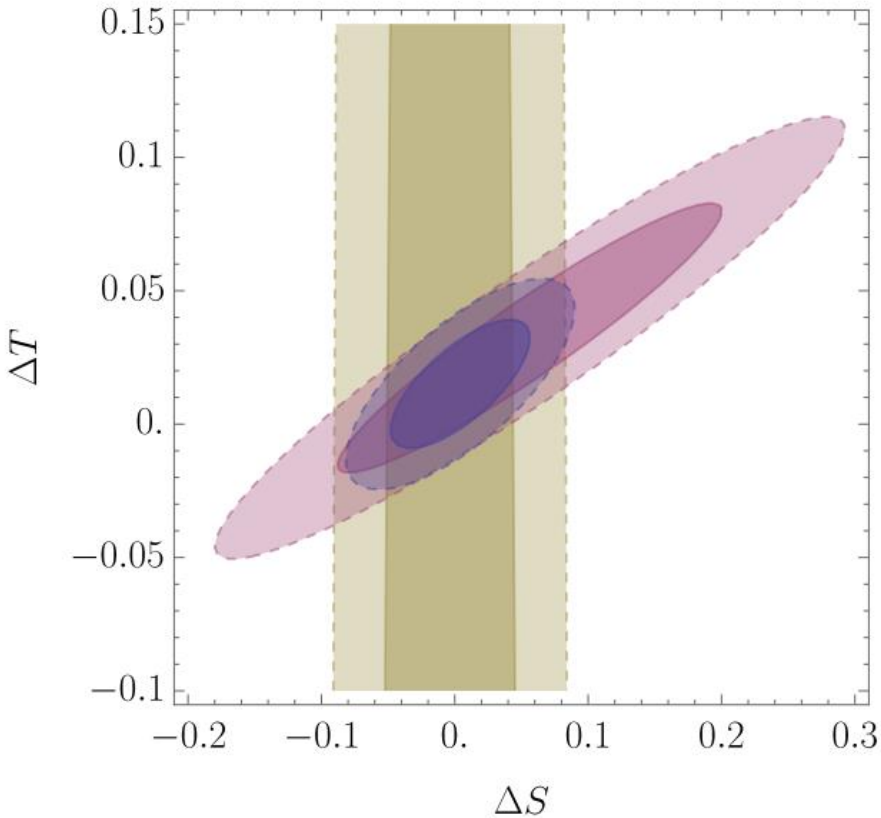
Coefficient	Z-pole + m_W	WW at LEP2	Higgs Run1	Higgs Run2	LHC WW high-p_T
$\bar{C}_{dH}$	×	×	42.4	57.6	×
$\bar{C}_{eH}$	×	×	49.6	50.4	×
$\bar{C}_{G}$	×	×	2.4	97.6	×
$\bar{C}_{HB}$	×	×	18.6	81.4	×
$\bar{C}_{H\Box}$	×	×	19.3	80.7	0.01
$\bar{C}_{Hd}$	99.85	×	0.04	0.1	×
$\bar{C}_{HD}$	99.92	0.06	×	×	×
$\bar{C}_{He}$	99.99	0.01	×	×	×
$\bar{C}_{HG}$	×	×	41.1	58.9	0.03
$\bar{C}^{(1)}_{H\ell}$	99.97	0.03	×	×	×
$\bar{C}^{(3)}_{H\ell}$	99.56	0.41	×	×	0.01
$\bar{C}^{(1)}_{Hq}$	99.98	×	×	×	×
$\bar{C}^{(3)}_{Hq}$	98.5	0.96	0.19	0.31	0.07
$\bar{C}_{Hu}$	99.3	×	0.2	0.42	0.04
$\bar{C}_{HW}$	×	×	18.3	81.7	×
$\bar{C}_{HWB}$	57.7	0.02	8.2	34.1	×
$\bar{C}_{\ell\ell}$	99.66	0.3	×	0.01	×
$\bar{C}_{uG}$	×	×	8.9	91.1	×
$\bar{C}_{uH}$	×	×	10.9	89.1	×
$\bar{C}_{W}$	×	96.2	×	×	3.8

Backup


C_{dH}
C_{eH}
C_G
C_{HB}
$C_{H\Box}$
C_{Hd}
C_{HD}
C_{He}
C_{HG}
$C_{H\ell}^{(1)}$
$C_{H\ell}^{(3)}$
$C_{Hq}^{(1)}$
$C_{Hq}^{(3)}$
C_{Hu}
C_{HW}
C_{HWB}
$C_{\ell\ell}$
C_{uG}
C_{uH}
C_W


$\bar{c}_{3G}$
$\bar{c}_{3W}$
$\bar{c}_d$
$\bar{c}_e$
$\bar{c}_g$
$\bar{c}_H$
$\bar{c}_{HB}$
$\bar{c}_{Hd}$
$\bar{c}_{He}$
$\bar{c}_{Hq}$
$\bar{c}'_{Hq}$
$\bar{c}_{Hu}$
$\bar{c}_{HW}$
$\bar{c}_{\ell\ell}$
$\bar{c}_T$
$\bar{c}_u$
$\bar{c}_{uG}$
$\bar{c}_W - \bar{c}_B$
$\bar{c}_W + \bar{c}_B$
$\bar{c}_\gamma$
1
$\frac{1}{2}$
0
$-\frac{1}{2}$
-1

Backup

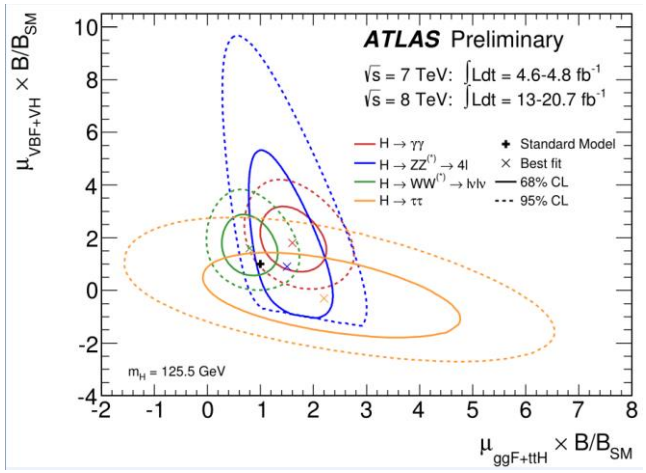


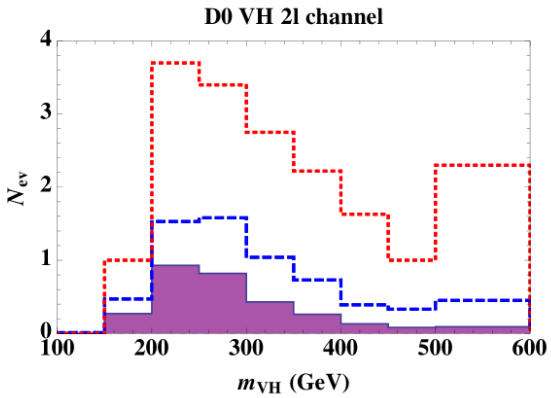
$$\frac{v^2}{\Lambda^2} C_{HWB} = \frac{g_1 g_2}{16\pi} \Delta S, \quad \frac{v^2}{\Lambda^2} C_{HD} = -\frac{g_1 g_2}{2\pi\,(g_1 + g_2)} \Delta T\,,$$

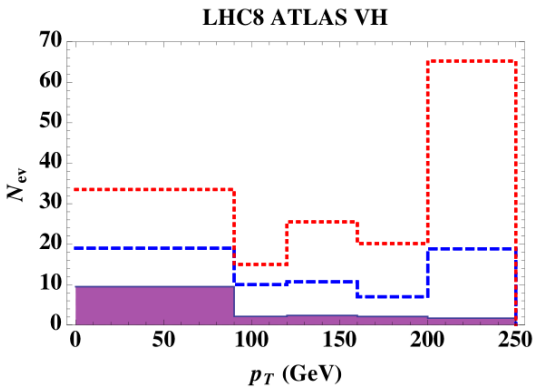
Higgs constraints on dim-6 operators

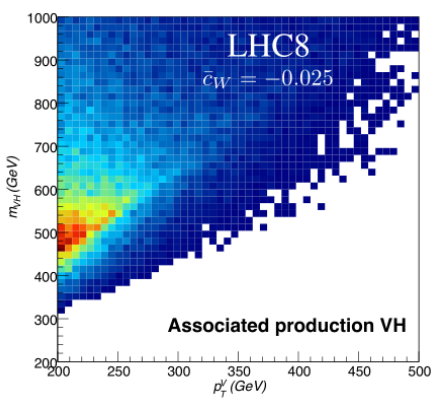
Tevong You

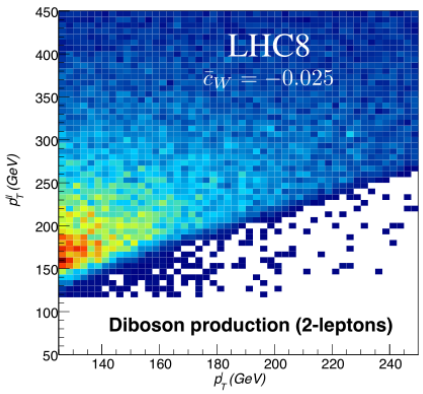
- Operators affect Higgs signal strength measurements, differential distributions











Ellis, Sanz and T.Y. 1410.7703

HKUST IAS, HEP conference 22nd - 25th January 2018

A Universal Approach to One-Loop Matching in EFTs

Tevong You

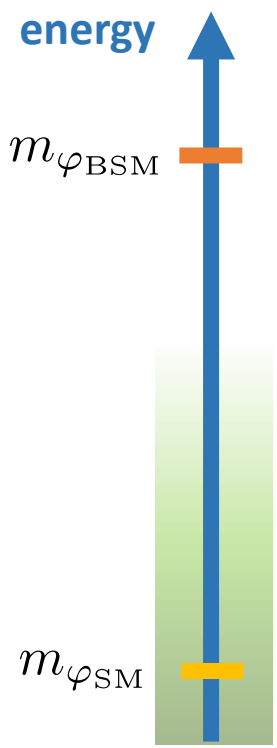
UNIVERSITY OF CAMBRIDGE

Outline

- Overview of recent developments in functional approach to matching
  - Original Gaillard-Cheyette CDE method (though now redundant)
  - Including heavy-light loops
- Universal One-Loop Effective Action (UOLEA)
- Examples of applying the UOLEA
  - MSSM Stops
  - Real Singlet Scalar
- Conclusion

Introduction

- EFT approach to BSM:



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

match *(UV ←→ EFT dictionary)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

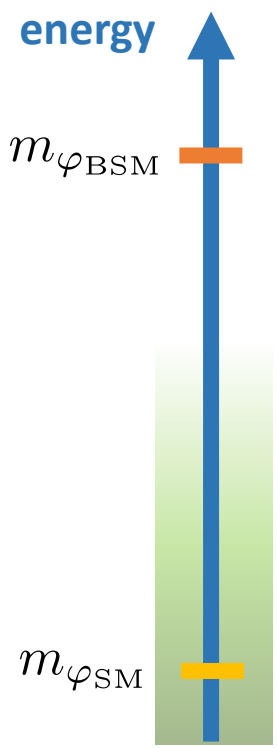
$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Introduction

- Standard approach is to use Feynman diagrams



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Feynman diagrams

$$\langle \varphi_L \varphi_L \cdots \varphi_L \rangle$$

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

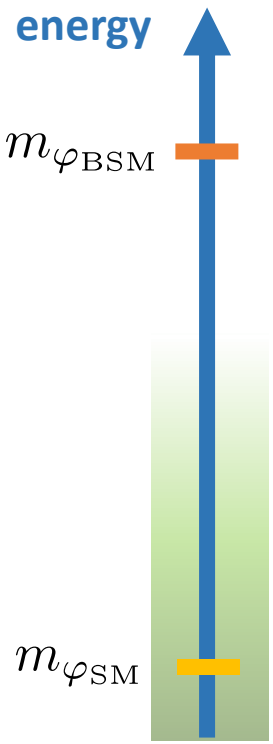
$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim E_{\rm exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But many ways of doing this:



$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Functional path integral evaluation

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

***run** (resum large logs)*

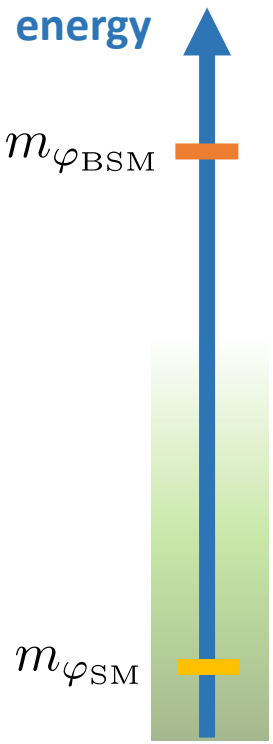
$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

***calculate** (model-independent)*

observables (cross sections, etc.)

Introduction

- Functional method more **elegant** and **direct** way of matching
- But *many* ways of doing this:



$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}$

e.g. Schwinger proper time, Covariant Derivative Expansion methods, Various log expansions, heavy-light subtraction procedures, integration by regions, **covariant diagrams**, etc.

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim m_{\varphi_H})$$

run *(resum large logs)*

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i\, \mathcal{O}_i\, (\mu \sim E_{\rm exp})$$

calculate *(model-independent)*

observables (cross sections, etc.)

Functional methods: Gaillard-Cheyette CDE

- **Gaillard-Cheyette** *('86, '88)* method of doing CDE reviewed/revived in **HLM** *(Henning, Lu, Murayama, 1412.1837)*

- Evaluate the path integral of the action in the usual way:
  - Expand action around minimum
  - Write Gaussian integral as determinant
  - Write determinant as trace of log in exponent

$$
\begin{aligned}
e^{iS_{\text{eff}}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\
&= \int [D\eta] \underline{e^{i\left(S[\phi,\Phi_c]+\frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\eta^2+\mathcal{O}(\eta^3)\right)}} \\
&\approx e^{iS[\phi,\Phi_c]} \underline{\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}}} \\
&\approx e^{iS[\phi,\Phi_c]-\underline{\frac{1}{2}\mathrm{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\Big|_{\Phi=\Phi_c}\right)}},
\end{aligned}
$$

- This is common to all functional methods

- Gaillard-Cheyette also do *momentum shift* before expanding logarithm (see later slide)

- Also, different methods used for expanding log

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger \boxed{F(x)} + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - \boxed{U(x)})\Phi + \mathcal{O}(\Phi^3)\,,$$

Model-dependent light fields encapsulated in F and U

$$\left.\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\Big|_{\Phi=\Phi_c}\right)}\,, \end{aligned}\right\}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= ic_s \text{Tr}\,\ln\left(-P^2 + M^2 + \boxed{U}\right) \\ &= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + \boxed{U}\right) \end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

Heavy fields can be boson or fermion

$$\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\bigg|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\big|_{\Phi=\Phi_c}\right)}\,, \end{aligned}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= i c_s \text{Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= i c_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

Heavy fields can be boson or fermion

$$\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\big|_{\Phi=\Phi_c}\right)}\,, \end{aligned}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= i c_s \text{Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= i c_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$$\mathcal{L}^{\rm eff}_{\text{1-loop}} = i c_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln[e^{P_\mu \partial/\partial q_\mu}(-(P_\mu - q_\mu)^2 + M^2 + U)e^{-P_\mu \partial/\partial q_\mu}]$$

$$= i c_s \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln[-(\tilde{G}_{\nu\mu}\partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]$$

- So covariant derivatives are explicitly in commutators from beginning

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

$$\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [\ldots[P_{\alpha_n}, U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

Functional methods: Gaillard-Cheyette CDE

- For a UV Lagrangian of the form

$P_\mu \equiv iD_\mu$

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

$$\begin{aligned} e^{iS_{\rm eff}[\phi]} &= \int [D\Phi] e^{iS[\phi,\Phi]} \\ &= \int [D\eta] e^{i\left(S[\phi,\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}\big|_{\Phi=\Phi_c}\eta^2 + \mathcal{O}(\eta^3)\right)} \\ &\approx e^{iS[\phi,\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\Big|_{\Phi=\Phi_c}\right)\right]^{-\frac{1}{2}} \\ &\approx e^{iS[\phi,\Phi_c] - \frac{1}{2}\text{Tr}\,\ln\left(-\frac{\delta^2 S}{\delta\Phi}\big|_{\Phi=\Phi_c}\right)}\,, \end{aligned}$$

$$\begin{aligned} S^{\rm eff}_{\text{1-loop}} &= ic_s \text{Tr}\,\ln\left(-P^2 + M^2 + U\right) \\ &= ic_s \int d^4x \int \frac{d^4q}{(2\pi)^4} \text{tr}\,\ln\left(-(P_\mu - q_\mu)^2 + M^2 + U\right) \end{aligned}$$

- Gaillard-Cheyette also do *momentum shift* by inserting $e^{\pm P_\mu \partial/\partial q_\mu}$

$$\mathcal{L}^{\rm eff}_{\text{1-loop}} = \qquad e^{-P_\mu \partial/\partial q_\mu}]$$

$$= \qquad \tilde{J}]$$

- So covariant de ors from beginning

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}} \qquad U = \sum_{n=0} \frac{}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

But **simpler to *avoid* momentum shift!** Instead, gather result into commutators *after* expansion evaluation.

See e.g.
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142;
Z. Zhang, 1610.00710.

Functional methods: Heavy-Light loops?

- Linear coupling = tree-level, quadratic coupling = *heavy-only* one-loop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger \boxed{F(x)} + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - \boxed{U(x)})\Phi + \mathcal{O}(\Phi^3)\,,$$

- What about loops involving both heavy and light fields?
- Naively not accounted for in functional method

See e.g. Bilenky & Santamaria, hep-ph/9310302; Del Aguila, Kunszt, Santiago, 1602.00126.

- Solution: apply background field method to both **heavy** *and* **light** fields

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

$$\mathcal{L}_{\rm quad} = \frac{1}{2}(\Phi', \phi') \begin{pmatrix} P^2 - M^2 - U_{\Phi\Phi} & -U_{\Phi\phi} \\ -U_{\phi\Phi} & P^2 - m^2 - U_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Phi' \\ \phi' \end{pmatrix}$$

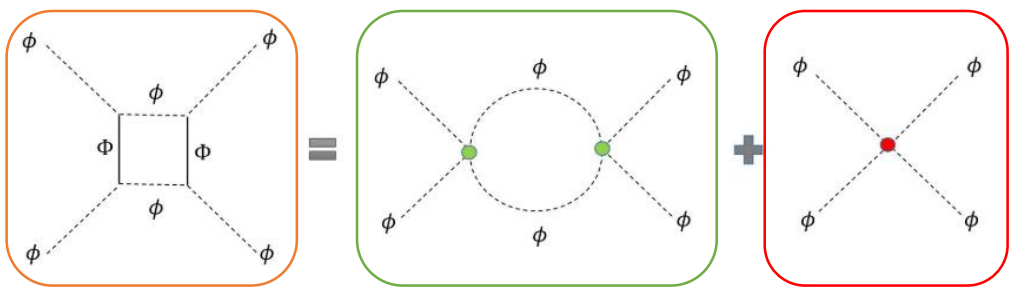
Functional methods: Heavy-Light loops?

- Just apply background field method to both heavy and light fields?

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

- Actually, this gives the one-loop 1PI effective action and **not** $\mathcal{L}_{\rm eff}$

  - Feynman diagram intuition: Heavy-light loops in UV theory match onto both **tree-level-generated** EFT operators *inserted at one-loop*, and **one-loop-generated** EFT operators *inserted at tree-level*



(soft part) *(hard part)*

  - The **former** is not part of $\mathcal{L}_{\rm eff}$, must be subtracted to keep only the **latter**

Functional methods: Heavy-Light subtractions

- Various subtraction procedures proposed

See e.g.
Boggia, Gomez-Ambrosio, Passarino, 1603.03660;
Henning, Lu, Murayama, 1604.01019;
Ellis, Quevillon, TY, Zhang, 1604.02445;
Fuentes-Martin, Portoles, Ruiz-Femenia, 1607.02142.

Universality property also applies to heavy-light case

Integration by regions method avoids subtraction, separates **hard** and **soft** part in integral, greatly simplifies heavy-light treatment

See e.g. Beneke & Smirnov, hep-ph/9711391; Jantzen, 1111.2589;

- Simplification of evaluating CDE from these developments lead to a **Covariant Diagram** formulation *(Z. Zhang, 1610.00710)*

- But **Universality** of CDE results means evaluation via all these different methods *gives same model-independent expression*

Henning, Lu, Murayama, 1412.1837;
Drozd, J. Ellis, Quevillon, TY, 1512.03003;
S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1706.07765.

Universality of the One-Loop Effective Action

- *No need to reinvent the wheel*, every slide up to now can be ignored

- Universality of CDE expansion results first noticed in the **simplified** case of **degenerate mass** for heavy fields *(Henning, Lu, Murayama, 1412.1837)*

- The *general* **Universal One-Loop Effective Action** (UOLEA) subsequently derived without such assumption *(Drozd, J. Ellis, Quevillon, TY, 1512.03003)*

- However, extra structures (**heavy-light terms**, "open" covariant derivatives, momentum-shifted-gamma matrices) in CDE expansion not included in initial UOLEA *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1604.02445)*

- Universal **heavy-light** terms now done *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1706.07765)*
- A *complete* UOLEA, including all possible CDE structures, is in sight…

Universality of the One-Loop Effective Action

- Neglect these extra structures for now; derivation of universal results e.g. in Gaillard-Cheyette CDE starts from

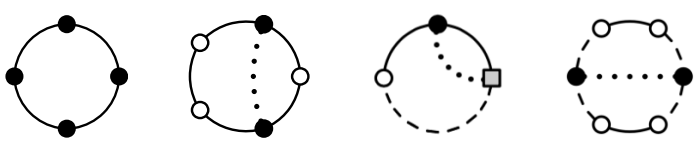
$$\mathcal{L}^{\text{eff}}_{\text{1-loop}} = ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[e^{P_\mu \partial/\partial q_\mu}(-(P_\mu - q_\mu)^2 + M^2 + U)e^{-P_\mu \partial/\partial q_\mu}]$$

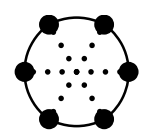
$$= ic_s \int \frac{d^4q}{(2\pi)^4} \text{tr} \ln[-(\tilde{G}_{\nu\mu}\partial/\partial q_\mu + q_\mu)^2 + M^2 + \tilde{U}]$$

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!}[P_{\alpha_1}, [...[P_{\alpha_n}, G'_{\nu\mu}]]] \frac{\partial^n}{\partial q_{\alpha_1}...q_{\alpha_n}}$$

$$\tilde{U} = \sum_{n=0}^{\infty} \frac{1}{n!}[P_{\alpha_1}, [...[P_{\alpha_n}, U]]] \frac{\partial^n}{\partial q_{\alpha_1}...q_{\alpha_n}}$$

- (much easier using Covariant Diagrams, see Z. Zhang talk)





$$= -i\,\frac{1}{6}\,\mathcal{I}[q^6]^6_i \cdot 2^6\,\text{tr}(P^\mu P^\nu P^\rho P_\mu P_\nu P_\rho)$$

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} [P_\mu, [P_\mu, U_{ij}]] [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} [P_\mu, U_{ij}] [P_\nu, U_{jk}] G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- Whatever the method used to obtain it, the resulting UOLEA can be written as

(f3 universal term calculated by 't Hooft '73)

$$
\begin{aligned}
\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2 + f_8^{ijk} (U_{ij} U_{jk} U_{ki}) \\
& + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& + f_{12,a}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\nu, [P_\mu, U_{ji}]] \\
& + f_{12,c}^{ij} [P_\mu, [P_\mu, U_{ij}]] [P_\nu, [P_\nu, U_{ji}]] \\
& + f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} [P_\mu, U_{ij}] [P_\nu, U_{jk}] G'_{\nu\mu,ki} \\
& + \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] \\
& + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .
\end{aligned}
$$

Universal coefficients f *encapsulate dependence on combinations of momentum master integrals*

Drozd, J. Ellis, Quevillon, TY, 1512.03003

Universality of the One-Loop Effective Action

- **Universal coefficients** in terms of standard master integrals:

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots(q^2)^{n_L}} \equiv g^{\mu_1\ldots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]_{ij\ldots 0}^{n_i n_j \ldots n_L}$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

Universality of the One-Loop Effective Action

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$f_6^i = \frac{32}{3}\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
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$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{42} + 2\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

Degenerate limit *(Henning, Lu, Murayama, 1412.1837)*

$$f_5 = -\frac{i}{(4\pi)^2 60m^2}, \quad f_{11} = \frac{i}{(4\pi)^2 12m^4}, \quad f_{15a} = \frac{i}{(4\pi)^2 60m^4},$$
$$f_6 = -\frac{i}{(4\pi)^2 90m^2}, \quad f_{12,a} = 0, \quad f_{15b} = \frac{i}{(4\pi)^2 60m^4},$$
$$f_7 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{12,b} = 0, \quad f_{16} = -\frac{i}{(4\pi)^2 60m^6},$$
$$f_8 = -\frac{i}{(4\pi)^2 6m^2}, \quad f_{12,c} = \frac{i}{(4\pi)^2 120m^4}, \quad f_{17} = -\frac{i}{(4\pi)^2 20m^6},$$
$$f_9 = -\frac{i}{(4\pi)^2 12m^2}, \quad f_{13} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{18} = -\frac{i}{(4\pi)^2 30m^6}$$
$$f_{10} = \frac{i}{(4\pi)^2 24m^4}, \quad f_{14} = \frac{-i}{(4\pi)^2 60m^4}, \quad f_{19} = \frac{i}{(4\pi)^2 120m^8}.$$

Drozd, J. Ellis, Quevillon, TY, 1512.03003;

Simplified form by covariant diagram computation shown here from Z. Zhang, 1610.00710.

Universality of the One-Loop Effective Action

- **Heavy-light** extension also done: *(S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1706.07765)*

$\mathcal{O}(U_H^4 P^2)$ terms	
$f_{17}^{ijkl} = 2\left(\mathcal{I}[q^2]^{1112}_{ijkl} + \mathcal{I}[q^2]^{1212}_{ijkl} + \mathcal{I}[q^2]^{1122}_{ijkl}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{Hkl}][P_\mu, U_{Hli}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]^{2121}_{ijkl} + \mathcal{I}[q^2]^{2112}_{ijkl} + \mathcal{I}[q^2]^{1221}_{ijkl} + \mathcal{I}[q^2]^{1212}_{ijkl}$	$U_{Hij}[P^\mu, U_{Hjk}]U_{Hkl}[P_\mu, U_{Hli}]$

$\mathcal{O}(U_H^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17A}^{ijk} = 2\left(\mathcal{I}[q^2]^{1122}_{ijk0} + \mathcal{I}[q^2]^{1221}_{ijk0} + \mathcal{I}[q^2]^{2121}_{ijk0}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{LHi'k}][P_\mu, U_{Hki}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{HLki'}]$
$f_{17B}^{ijk} = 2\left(\mathcal{I}[q^2]^{1122}_{ijk0} + \mathcal{I}[q^2]^{1212}_{ijk0} + \mathcal{I}[q^2]^{2112}_{ijk0}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{HLki'}][P_\mu, U_{LHi'i}]$
$f_{17C}^{ijk} = 2\left(\mathcal{I}[q^2]^{1122}_{ijk0} + \mathcal{I}[q^2]^{2121}_{ijk0} + \mathcal{I}[q^2]^{1221}_{ijk0}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{Hjk}], [P_\mu, U_{Hki}]$
$f_{18A}^{ijk} = 2\left(\mathcal{I}[q^2]^{1221}_{ijk0} + \mathcal{I}[q^2]^{2121}_{ijk0} + \mathcal{I}[q^2]^{1212}_{ijk0} + \mathcal{I}[q^2]^{2112}_{ijk0}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{LHi'k}[P_\mu, U_{Hki}]$ $+U_{Hij}[P^\mu, U_{Hjk}]U_{HLki'}[P_\mu, U_{LHi'i}]$

$\mathcal{O}(U_H^1 U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17D}^{ij} = 2\left(2\mathcal{I}[q^2]^{123}_{ij0} + \mathcal{I}[q^2]^{222}_{ij0}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{LHj'j}][P_\mu, U_{Hji}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}]$
$f_{17E}^{ij} = 2\left(\mathcal{I}[q^2]^{114}_{ij0} + \mathcal{I}[q^2]^{123}_{ij0} + \mathcal{I}[q^2]^{213}_{ij0}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{Lj'i'}]$
$f_{18B}^{ij} = 2\left(\mathcal{I}[q^2]^{123}_{ij0} + \mathcal{I}[q^2]^{222}_{ij0} + \mathcal{I}[q^2]^{114}_{ij0} + \mathcal{I}[q^2]^{213}_{ij0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{LHj'j}[P_\mu, U_{Hji}]$
$f_{18C}^{ij} = 4\left(\mathcal{I}[q^2]^{123}_{ij0} + \mathcal{I}[q^2]^{213}_{ij0}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{Li'j'}[P_\mu, U_{LHj'i}]$

$\mathcal{O}(U_L^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17F}^{i} = 2\left(2\mathcal{I}[q^2]^{15}_{i0} + \mathcal{I}[q^2]^{24}_{i0}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{Lj'k'}][P_\mu, U_{LHk'i}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{HLik'}][P_\mu, U_{Lk'i'}]$
$f_{17G}^{i} = 2\left(2\mathcal{I}[q^2]^{15}_{i0} + \mathcal{I}[q^2]^{24}_{i0}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{Lj'k'}][P_\mu, U_{Lk'i'}]$
$f_{17H}^{i} = 6\mathcal{I}[q^2]^{24}_{i0}$	$U_{Li'j'}U_{Lj'k'}[P^\mu, U_{LHk'i}][P_\mu, U_{HLii'}]$
$f_{18D}^{i} = 4\left(\mathcal{I}[q^2]^{15}_{i0} + \mathcal{I}[q^2]^{24}_{i0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{Lj'k'}[P_\mu, U_{LHk'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{Lj'k'}[P_\mu, U_{Lk'i}]$

$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
$f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]^{114}_{ij0} + \mathcal{I}[q^2]^{213}_{ij0} + \mathcal{I}[q^2]^{123}_{ij0}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
$f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]^{222}_{ij0} + 2\mathcal{I}[q^2]^{123}_{ij0}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
$f_{18E}^{ij} = \mathcal{I}[q^2]^{114}_{ij0} + 2\mathcal{I}[q^2]^{123}_{ij0} + \mathcal{I}[q^2]^{222}_{ij0}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$

$\mathcal{O}(U_H^3 P^2)$ terms	
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]^{122}_{ijk} + \mathcal{I}[q^2]^{212}_{ijk}\right)$	$U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{Hki}]$

$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]^{122}_{ij0} + \mathcal{I}[q^2]^{212}_{ij0}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}]$
$f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]^{221}_{ij0} + \mathcal{I}[q^2]^{122}_{ij0}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11C}^{ij} = 4\mathcal{I}[q^2]^{23}_{i0}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
$f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]^{14}_{i0} + \mathcal{I}[q^2]^{23}_{i0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U_H^2 P^4)$ terms	
$f_{12}^{ij} = 4\mathcal{I}[q^4]^{33}_{ij}$	$[P^\mu, [P_\mu, U_{Hij}]][P^\nu, [P_\nu, U_{Hji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]^{33}_{ij} + 2\mathcal{I}[q^4]^{33}_{ij} + 2\mathcal{I}[q^4]^{51}_{ij}\right)$	$U_{Hij}U_{Hji}G'^{\mu\nu}_i G'_{\mu\nu i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]^{33}_{ij}$	$[P^\mu, U_{Hij}][P^\nu, U_{Hji}]G'_{\nu\mu i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]^{33}_{ij} + \mathcal{I}[q^4]^{42}_{ij}\right)$	$\left(U_{Hij}[P^\mu, U_{Hji}] - [P^\mu, U_{Hij}]U_{Hji}\right)[P^\nu, G'_{\nu\mu i}]$

$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^4)$ terms	
$f_{12A}^{i} = 8\mathcal{I}[q^4]^{33}_{i0}$	$[P^\mu, [P_\mu, U_{HLii'}]][P^\nu, [P_\nu, U_{LHi'i}]]$
$f_{13A}^{i} = 2\left(\mathcal{I}[q^4]^{24}_{i0} + 2\mathcal{I}[q^4]^{33}_{i0} + 3\mathcal{I}[q^4]^{42}_{i0} + 4\mathcal{I}[q^4]^{51}_{i0}\right)$	$U_{HLii'}U_{LHi'i}G'^{\mu\nu}_i G'_{\mu\nu i}$
$f_{13B}^{i} = 2\left(4\mathcal{I}[q^4]^{15}_{i0} + 3\mathcal{I}[q^4]^{24}_{i0} + 2\mathcal{I}[q^4]^{33}_{i0} + \mathcal{I}[q^4]^{42}_{i0}\right)$	$U_{LHi'i}U_{HLii'}G'^{\mu\nu}_i G'_{\mu\nu i}$
$f_{14A}^{i} = 4\left(-\mathcal{I}[q^4]^{24}_{i0} - 2\mathcal{I}[q^4]^{33}_{i0} + \mathcal{I}[q^4]^{42}_{i0}\right)$	$[P^\mu, U_{HLii'}][P^\nu, U_{LHi'i}]G'_{\nu\mu i}$
$f_{14B}^{i} = 4\left(\mathcal{I}[q^4]^{24}_{i0} - 2\mathcal{I}[q^4]^{33}_{i0} - \mathcal{I}[q^4]^{42}_{i0}\right)$	$[P^\mu, U_{LHi'i}][P^\nu, U_{HLii'}]G'_{\nu\mu i}$
$f_{15A}^{i} = 2\left(\mathcal{I}[q^4]^{24}_{i0} + 2\mathcal{I}[q^4]^{33}_{i0} + \mathcal{I}[q^4]^{42}_{i0}\right)$	$\left(U_{HLii'}[P^\mu, U_{LHi'i}] - [P^\mu, U_{HLii'}]U_{LHi'i}\right)[P^\nu, G'_{\nu\mu i}]$ $+\left(U_{LHi'i}[P^\mu, U_{HLii'}] - [P^\mu, U_{LHi'i}]U_{HLii'}\right)[P^\nu, G'_{\nu\mu i}]$

P-only terms	
$f_3^i = 2\mathcal{I}[q^4]^4_i$	$G'^{\mu\nu}_i G'_{\mu\nu i}$
$f_5^i = 16\mathcal{I}[q^6]^6_i$	$[P^\mu, G'_{\mu\nu i}][P_\rho, G'^{\rho\nu}_i]$
$f_6^i = (32/3)\mathcal{I}[q^6]^6_i$	$G'^{\mu}_{\nu i}G'^{\nu}_{\rho i}G'^{\rho}_{\mu i}$

$\mathcal{O}(U_H^2 P^2)$ terms	
$f_7^{ij} = \mathcal{I}[q^2]^{22}_{ij}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{7A}^{ij} = 2\mathcal{I}[q^2]^{22}_{i0}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

$\mathcal{O}(U_H^1 P^4)$ terms	
$f_9^i = 8\mathcal{I}[q^4]^5_i$	$U_{Hij}G'^{\mu\nu}_i G'_{\mu\nu i}$

$\mathcal{O}(U)$ term		$\mathcal{O}(U^3)$ terms	
$f_2^i = \mathcal{T}_i^1$	U_{Hii}	$f_8^{ijk} = \frac{1}{3}\mathcal{T}^{111}_{ijk}$	$U_{Hij}U_{Hjk}U_{Hki}$
$\mathcal{O}(U^2)$ terms		$f_{8A}^{ij} = \mathcal{T}^{111}_{ij0}$	$U_{Hij}U_{HLji'}U_{LHi'i}$
$f_3^{ij} = \frac{1}{2}\mathcal{T}^{11}_{ij}$	$U_{Hij}U_{Hji}$	$f_{8B}^{i} = \mathcal{T}^{12}_{i0}$	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$f_{4A}^{ij} = \mathcal{T}^{11}_{i0}$	$U_{HLii'}U_{LHi'i}$	$\mathcal{O}(U^6)$ terms	
$\mathcal{O}(U^4)$ terms		$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{T}^{111111}_{ijklmn}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{T}^{1111}_{ijkl}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$	$f_{19A}^{ijklm} = \mathcal{T}^{111111}_{ijklm0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{10A}^{ijk} = \mathcal{T}^{1111}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$	$f_{19B}^{ijkl} = \mathcal{T}^{111112}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{10B}^{ij} = \mathcal{T}^{1112}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{ijk} = \mathcal{T}^{111112}_{ijk00}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{T}^{7112}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{T}^{1113}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{T}^{13}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{T}^{111112}_{ijkl0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{T}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{T}^{11111}_{ijklm}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{T}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{T}^{11111}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{T}^{114}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{T}^{1112}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{T}^{1113}_{ijk0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{T}^{1112}_{ijk0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{T}^{114}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{T}^{113}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{T}^{114}_{ij0}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{T}^{113}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{T}^{15}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{T}^{14}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

Application of the UOLEA: MSSM Stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix, and covariant derivative:

$$\boxed{\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \cancel{(\Phi^\dagger F(x) + \text{h.c.})} + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)}$$

(R-parity)

$$\Phi = (\tilde{Q}\,, \tilde{t}_R^*), \qquad M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix} \qquad G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

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$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix}$$

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Application of the UOLEA: MSSM Stops

- Pick the relevant operators by counting operator dimensions

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$\big[P^\mu, [P_\mu, U_{ij}]\big]\big[P^\nu, [P_\nu, U_{ji}]\big]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji} G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}] G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X \tilde{H} \\ h_t X \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

Application of the UOLEA: MSSM Stops

- Example: $\mathcal{O}_{GG} = g_s^2 \left|H\right|^2 G^a_{\mu\nu} G^{a,\mu\nu}$

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\,\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$\big(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\big)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X \tilde{H} \\ h_t X \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-
c_{WB}	f_9	f_{13}, f_{14}	-	-
c_W	-	f_{15a}, f_{15b}	-	-
c_B	-	f_{15a}, f_{15b}	-	-
c_D	-	f_{12c}	-	-

Application of the UOLEA: MSSM Stops

- Example: $\mathcal{O}_{GG} = g_s^2 \left|H\right|^2 G^a_{\mu\nu} G^{a,\mu\nu}$

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{42} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$(U_{ij}[P^\mu, U_{ij}] - [P^\mu, U_{ij}]U_{ij})[P^\nu, G'_{\nu\mu,i}]$
f_{16}^{ijklm} =	
f_{17}^{ijkl} =	
f_{18}^{ijkl} =	
f_{19}^{ijklmn}	

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X \tilde{H} \\ h_t X \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-

$$c_{GG} = \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m_{\tilde{Q}}^2} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m_{\tilde{t}_R}^2} - \frac{\bar{X}_t^2}{m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2}\right)$$

Application of the UOLEA: MSSM Stops

- Example: $$\mathcal{O}_{GG} = g_s^2 \left|H\right|^2 G^a_{\mu\nu} G^{a,\mu\nu}$$

For full results see Drozd, J. Ellis, Quevillon, TY, 1512.03003.

(note: error in previous results by R. Huo, 1509.05942)

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{ij}]]\,[P^\nu, [P_\nu, U_{ji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{42} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$\left(U_{ij}[P^\mu, U_{ij}] - [P^\mu, U_{ij}]U_{ij}\right)[P^\nu, G'_{\nu\mu,i}]$
f_{16}^{ijklm} =	
f_{17}^{ijkl} =	
f_{18}^{ijkl} =	
f_{19}^{ijklmn}	

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X \tilde{H} \\ h_t X \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

	X_t^0	X_t^2	X_t^4	X_t^6
c_6	f_8	f_{10}	f_{16}	f_{19}
c_H	f_7	f_{11}	f_{17}, f_{18}	-
c_T	f_7	f_{11}	f_{17}, f_{18}	-
c_R	f_7	f_{11}	f_{17}	-
c_{GG}	f_9	f_{13}	-	-
c_{WW}	f_9	f_{13}, f_{14}	-	-
c_{BB}	f_9	f_{13}, f_{14}	-	-

$$c_{GG} = \frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m_{\tilde{Q}}^2} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m_{\tilde{t}_R}^2} - \frac{\bar{X}_t^2}{m_{\tilde{Q}}^2 m_{\tilde{t}_R}^2}\right)$$

Application of the UOLEA: Real Singlet Scalar

$$\Delta\mathcal{L} = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 - \frac{1}{2}m^2\Phi^2 - \underline{A\left|H\right|^2\Phi} - \frac{1}{2}k\left|H\right|^2\Phi^2 - \frac{1}{3!}\mu\Phi^3 - \frac{1}{4!}\lambda_\Phi\Phi^4$$

- U matrix includes heavy-light contributions due to linear coupling

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_\phi & \left(\mathcal{U}^\dagger_{H\phi}\right)_{1\times 2} \\ \left(\mathcal{U}_{H\phi}\right)_{2\times 1} & \left(\mathcal{U}_H\right)_{2\times 2} \end{pmatrix}$$

- Classify possible contributions by counting operator dimensions

$$\mathcal{U}_\phi \equiv -\frac{\delta^2\mathcal{L}}{\delta\phi^2}\bigg|_{\phi_c} = \kappa|H|^2 + \mu\phi_c + \tfrac{1}{2}\lambda_\phi\phi_c^2 \quad \supset \mathcal{O}(H^2, \partial^2H^2, H^4, \partial^2H^4, H^6)$$

$$\mathcal{U}_{H\phi} \equiv -\frac{\delta^2\mathcal{L}}{\delta H^\dagger\delta\phi}\bigg|_{\phi_c} = AH + \kappa H\phi_c \quad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

$$\mathcal{U}_{\tilde{H}\phi} \equiv -\frac{\delta^2\mathcal{L}}{\delta\tilde{H}^\dagger\delta\phi}\bigg|_{\phi_c} = A\tilde{H} + \kappa\tilde{H}\phi_c \quad \supset \mathcal{O}(H, H^3, H^5, H\partial^2H^2)$$

etc…

See S.A.R. Ellis, Quevillon, TY, Z. Zhang, 1705.xxxxx

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2 \qquad \mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \qquad \mathcal{O}_R = |H|^2 \, |D_\mu H|^2$$

- Classify possible contributions by counting operator dimensions

$\mathcal{O}(U)$ term	
✓ $f_2^i = \mathcal{I}_i^1$ ✓	U_{Hii}
$\mathcal{O}(U^2)$ terms	
✓ $f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$ ✓	$U_{Hij}U_{Hji}$
✓ $f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$ ✓	$U_{HLii'}U_{LHi'i}$
$\mathcal{O}(U^4)$ terms	
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$ ✓	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$\mathcal{O}(U^5)$ terms	
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$

$\mathcal{O}(U^3)$ terms	
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$ ✓	$U_{Hij}U_{Hjk}U_{Hki}$
✓ $f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'i}$
✓ $f_{8B}^{i} = \mathcal{I}_{i0}^{12}$ ✓	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$\mathcal{O}(U^6)$ terms	
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$

$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
✓ $f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
✓ $f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
✓ $f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$

$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}]$
✓ $f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U_H^2 P^2)$ terms	
✓ $f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
✓ $f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

$$\mathcal{O}_6 = |H|^6$$

$$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2 \qquad \mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 \qquad \mathcal{O}_R = |H|^2 |D_\mu H|^2$$

- Evaluate sum over each term to get full result for e.g. O6:

$$\begin{aligned}
\mathcal{O}6 \Big(& \left(2 A^2 \kappa^2 - \frac{4 A^3 \kappa \mu}{M^2} + \frac{2 A^4 \mu^2}{M^4}\right) f10A[1,1,1] - \frac{16 A^4 \kappa\, f10C[1,1]}{M^2} + \left(4 A^4 \kappa - \frac{4 A^5 \mu}{M^2}\right) f16C[1,1,1] + 8 A^6 f19I[1,1,1] + \frac{6 A^2 \kappa^2 f4A[1,1]}{M^4} + \\
& \left(\kappa^3 - \frac{3 A \kappa^2 \mu}{M^2} + \frac{3 A^2 \kappa \mu^2}{M^4} - \frac{A^3 \mu^3}{M^6}\right) f8[1,1,1] + f16E[1,1]\left(-\frac{4 A^6}{M^2} + 12 A^4 \lambda_h\right) + f8B[1]\left(\frac{7 A^4 \kappa}{M^4} - \frac{12 A^2 \kappa \lambda_h}{M^2}\right) + f10B[1,1]\left(-\frac{2 A^4 \kappa}{M^2} + \frac{2 A^5 \mu}{M^4} + 6 A^2 \kappa \lambda_h - \frac{6 A^3 \mu \lambda_h}{M^2}\right) + \\
& f10D[1]\left(\frac{2 A^6}{M^4} - \frac{12 A^4 \lambda_h}{M^2} + 18 A^2 \lambda_h^2\right) - \frac{A^2 \kappa\, f2[1] \lambda_\phi}{M^6} + f8A[1,1]\left(-\frac{4 A^2 \kappa^2}{M^2} + \frac{6 A^3 \kappa \mu}{M^4} + \frac{A^4 \lambda_\phi}{M^4}\right) + f4[1,1]\left(\frac{2 A \kappa^2 \mu}{M^4} - \frac{2 A^2 \kappa \mu^2}{M^6} + \frac{A^2 \kappa \lambda_\phi}{M^4} - \frac{A^3 \mu \lambda_\phi}{M^6}\right)\Big)
\end{aligned}$$

$f^{ij}_{10B} = \mathcal{I}^{111}_{ij0}$ ✓	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f^{ijk}_{19C} = \mathcal{I}^{11111}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f^{ij}_{10C} = \frac{1}{2}\mathcal{I}^{112}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f^{ijk}_{19D} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f^{i}_{10D} = \mathcal{I}^{13}_{i0}$ ✓	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f^{ijkl}_{19E} = \frac{1}{2}\mathcal{I}^{11112}_{ijkl0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f^{ijk}_{19F} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f^{ijklm}_{16} = \frac{1}{5}\mathcal{I}^{11111}_{ijklm}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f^{ijk}_{19G} = \mathcal{I}^{1113}_{ijk0}$	$U_{Hij}U_{HLji'}U_{Li'j}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f^{ijkl}_{16A} = \mathcal{I}^{11111}_{ijkl0}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f^{ij}_{19H} = \mathcal{I}^{114}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j}U_{Lj'k}U_{Lk'l'}U_{LHl'i}$
$f^{ijk}_{16B} = \mathcal{I}^{1112}_{ijk0}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j}U_{LHj'i}$	$f^{ijk}_{19I} = \frac{1}{3}\mathcal{I}^{1113}_{ijk0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f^{ijk}_{16C} = \mathcal{I}^{1112}_{ijk0}$ ✓	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f^{ij}_{19J} = \mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k}U_{Lk'l'}U_{LHl'i}$
$f^{ij}_{16D} = \mathcal{I}^{113}_{ij0}$	$U_{Hij}U_{HLji'}U_{Li'j}U_{Lj'k'}U_{LHk'i}$	$f^{ij}_{19K} = \frac{1}{2}\mathcal{I}^{114}_{ij0}$	$U_{HLii'}U_{Li'j}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f^{ij}_{16E} = \mathcal{I}^{113}_{ij0}$ ✓	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f^{i}_{19L} = \mathcal{I}^{15}_{i0}$	$U_{HLii'}U_{Li'j}U_{Lj'k}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f^{i}_{16F} = \mathcal{I}^{14}_{i0}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

✓ $f^{ij}_{11B} = 2\left(\mathcal{I}[q^2]^{221}_{ij0} + \mathcal{I}[q^2]^{122}_{ij0}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$

$\mathcal{O}(U^1_L U^1_{HL} U^1_{LH} P^2)$ terms	
✓ $f^{ij}_{11C} = 4\mathcal{I}[q^2]^{23}_{i0}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
✓ $f^{ij}_{11D} = 2\left(\mathcal{I}[q^2]^{14}_{i0} + \mathcal{I}[q^2]^{23}_{i0}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U^2_H P^2)$ terms	
✓ $f^{ij}_{7} = \mathcal{I}[q^2]^{22}_{ij}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$

$\mathcal{O}(U^1_{HL} U^1_{LH} P^2)$ terms	
✓ $f^{ij}_{7A} = 2\mathcal{I}[q^2]^{22}_{i0}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

Application of the UOLEA: Real Singlet Scalar

- Can (partially) **automate** evaluation of each term, e.g.

f_{10A}

```
sumf10A = NCExpand[Sum[f10A[1, 1, 1] * Uϕ1x1 ** Uϕ1x1 ** UϕH1x2[[1]][[ip]] ** UHϕ2x1[[ip]][[1]], {ip, 1, 2}] /.
   subϕcforO6]
sumf10Adimcount = sumf10A /. subDimCounting
sumf10Adim6only = sumf10Adimcount /. autoremovenondim6step1 /. autoremovenondim6step2 /.
  autoremovenondim6step3
```

$$A^2 \kappa^2 \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H} - \frac{2 A^3 \kappa \mu \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H}}{M^2} + \frac{A^4 \mu^2 \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Hdag} ** \text{H}}{M^4} + A^2 \kappa^2 \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht} - \frac{2 A^3 \kappa \mu \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht}}{M^2} + \frac{A^4 \mu^2 \, \text{f10A}[1,1,1] \, \text{HdagH} ** \text{HdagH} ** \text{Htdag} ** \text{Ht}}{M^4}$$

```
sumf10Adim6op = sumf10Adim6only /. subO6op
Collect[sumf10Adim6op, {f10A[1, 1, 1], O6}]
```

$$O6 \left(2 A^2 \kappa^2 - \frac{4 A^3 \kappa \mu}{M^2} + \frac{2 A^4 \mu^2}{M^4}\right) \text{f10A}[1,1,1]$$

- Substitute operator structure relations for desired basis, worked out by hand
  - This example trivial but in general most of the work involved is in this step
  - Possible automation: dictionary of operator relations, or work out algorithm

Conclusion

- When calculating Feynman diagrams we don't Wick contract and calculate symmetry factors by hand every time
- Similar redundancy in evaluating CDE from the beginning every time we use functional methods for one-loop matching

$$\mathcal{L}_{\rm UV}[\varphi_{\rm BSM}, \varphi_{\rm SM}]$$

Set up path integral

Evaluate CDE expansion

Covariant diagrams

Matrix trace evaluation

Set up correlation functions

Wick contractions

Perturbation series evaluation

Feynman diagrams

$\langle \varphi_L \varphi_L \cdots \varphi_L \rangle$ *UV-EFT matching*

(same process in EFT)

$$\mathcal{L}_{\rm EFT}[\varphi_{\rm SM}] = \mathcal{L}_{\rm SM} + \sum_i c_i \, \mathcal{O}_i \, (\mu \sim m_{\varphi_H})$$

- Standardise functional one-loop matching procedure…

Conclusion

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- **Start directly from UOLEA!**