INCLUSION OF MIXED QCD-QED RESUMMATION EFFECTS AT HIGHER-ORDERS

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in collaboration with L. Cieri, D. de Florian, G. Ferrera and G. Rodrigo

Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10 (2016) 056; PoS (EPS-HEP2017) 398; JHEP 08 (2018) 165 and work in progress

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Outline

- 1- Effects on PDF evolution: mixed QCD-QED DGLAP and splittings functions
- 2- NLO QED: fixed-order effects on $\gamma\gamma$ production
- **3- Mixed H.O. QCD-QED for resummation: effects on Z production**
- Conclusions (and perspectives)

CENTRAL PART OF THE TALK!

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References:

1)- QCD-QED AP kernels: de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

*2)- Collider effects: Cieri, Ferrera and GS, **PoS(EPS-HEP2017)398 and JHEP 08 (2018)165***

Part 1: Effects on PDFs

- Evolution of PDF plays a crucial role for precision measurements, and the importance increases for higher energies/luminosities
- **PHOTON PDF!** We need to think about the presence of photons inside hadrons – ***crucial within the precision physics program!***
- *Modify evolution equations, redefine PDF basis (to solve them) and find the evolution kernels (splittings)*

Splittings and DGLAP within QCD-QED

Extending DGLAP equations

- *DGLAP equations dictate the evolution of PDFs*
- **QED** interactions connects **QCD** partons with **photons and leptons.**

New PDFs (leptons & photon) + *New splitting functions* → ***Modified DGLAP equations***

- **Extend original DGLAP equations to deal with new *objects*:**

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i\bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i\gamma} \otimes \gamma$$

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i\bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i\gamma} \otimes \gamma$$

Kernels with fermions

Photon distributions

Lepton distributions

Kernels with photons

Kernels with leptons

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Splittings and DGLAP within QCD-QED

Splittings: expansions in QCD-QED

- Perturbative expansion in QCD and QED couplings (non-trivial counting…)

$$P_{ij} = a_S\, P_{ij}^{(1,0)} + a\, P_{ij}^{(0,1)} + a_S^2\, P_{ij}^{(2,0)} + a_S\, a\, P_{ij}^{(1,1)} + a^2\, P_{ij}^{(0,2)} + \ldots$$

- Well-known LO results:

$$P_{qq}^{(1,0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] = C_F \left[p_{qq}(x) + \frac{3}{2}\,\delta(1-x)\right],$$

$$P_{qg}^{(1,0)}(x) = T_R \left[x^2 + (1-x)^2\right] = T_R\, p_{qg}(x)\,,$$

$$P_{gq}^{(1,0)}(x) = C_F \left[\frac{1+(1-x)^2}{x}\right] = C_F\, p_{gq}(x)\,,$$

$$P_{gg}^{(1,0)}(x) = 2\, C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{\beta_0}{2}\,\delta(1-x)\,,$$

Standard QCD AP-kernels at LO

$$P_{ff}^{(0,1)}(x) = e_f^2 \left[p_{qq}(x) + \frac{3}{2}\delta(1-x)\right],$$

$$P_{f\gamma}^{(0,1)}(x) = e_f^2\, p_{qg}(x)\,,$$

Includes color degeneration!

$$P_{\gamma f}^{(0,1)}(x) = e_f^2\, p_{gq}(x)\,,$$

$$P_{\gamma\gamma}^{(0,1)}(x) = -\frac{2}{3}\sum_f e_f^2\, \delta(1-x)\,,$$

LO QED splitting functions

- Terms proportional to Dirac's **deltas** are originated by virtual (**loop**) corrections
- **Color** factors in **QCD** replaced with **EM charges** in **QED** → Motivates **Abelianization algorithm**!

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

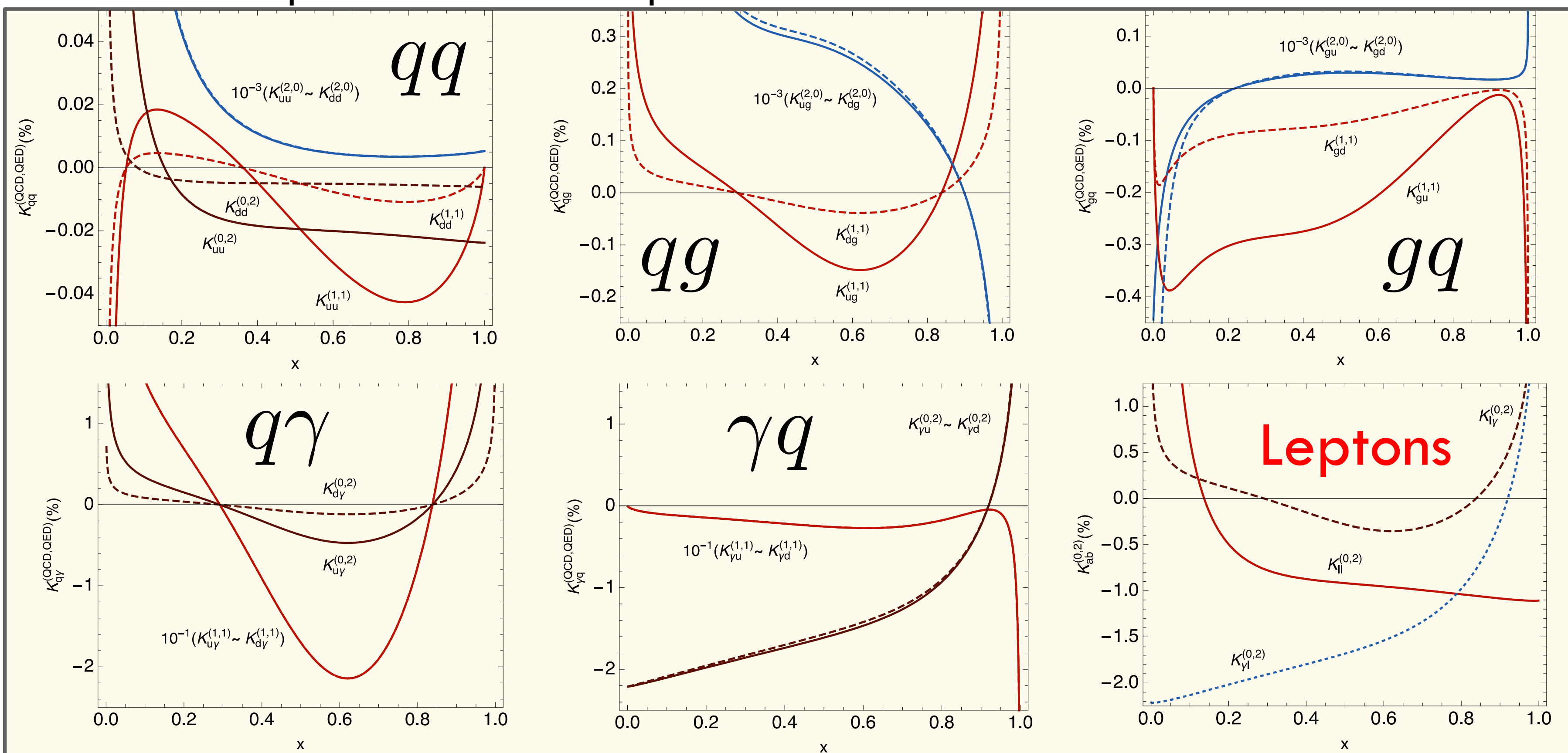
Splittings and DGLAP within QCD-QED

Splittings: quantifying QED effects

- We define a ratio to quantify the effect of H.O. QED corrections

$$K_{ab}^{(i,j)} = a_S^i\, a^j\, \frac{P_{ab}^{(i,j)}(x)}{P_{ab}^{\rm LO}(x)} \qquad \text{with} \qquad P_{ab}^{\rm LO} = a_S\, P_{ab}^{(1,0)} + a\, P_{ab}^{(0,1)}$$

- Some plots to show the impact in the evolution kernels:



Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Part 2: NLO QED fixed-order

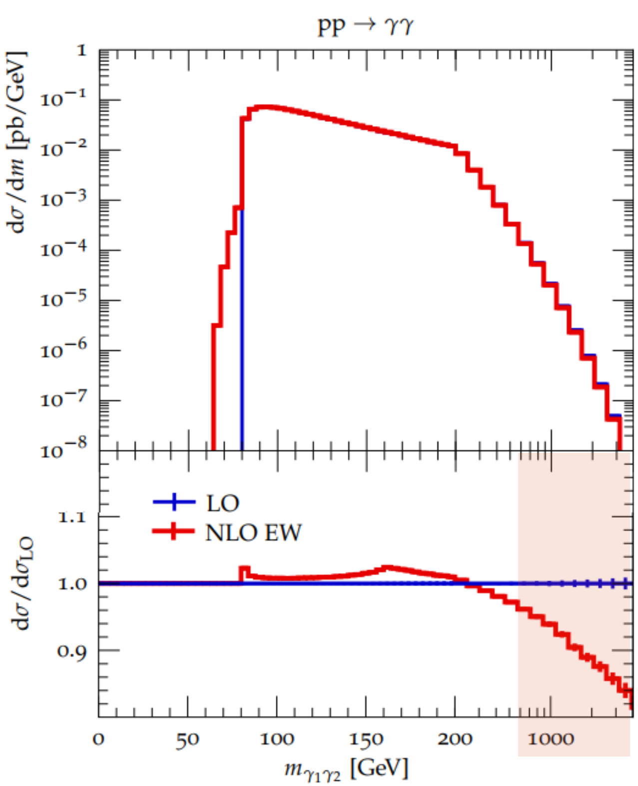
- Effects of QED corrections in hadronic cross-sections
- **I)-Implementation of fixed order NLO QED to diphoton production**
- **II)-Proper treatment of photon radiation**
- **III)- Description of novel features due to mixing QCD-QED**

NLO QED corrections to $pp \to \gamma\gamma$

General remarks

- Application of **Abelianization** techniques to recover NLO QED.
- Full NLO EW corrections recently computed ➡ Non-negligible effects found in the high-invariant mass region!!!

Chiesa et al, arXiv:1706.09022



- Some comments about previous works:
  - Fully automatized but partial inclusion of EW corrections (**only virtuals**).
  - Neglected contributions may play a relevant phenomenological role (in particular, in the high-energy region) ➡ *Z radiation accounts for 2-4% correction!*
  - *Lack of a consistent framework for W,Z radiation.*
- **Our motivation:**
  - Perform a fully **consistent** calculation within QED.
  - Explore the phenomenological consequences of dealing with extra-QED radiation.

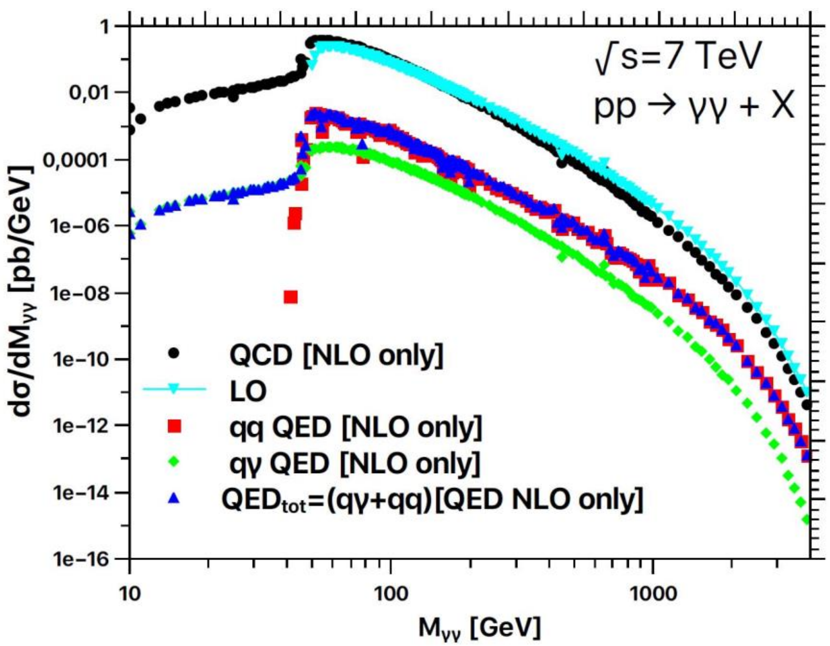
GS, PoS (EPS-HEP2017) 398; work in progress

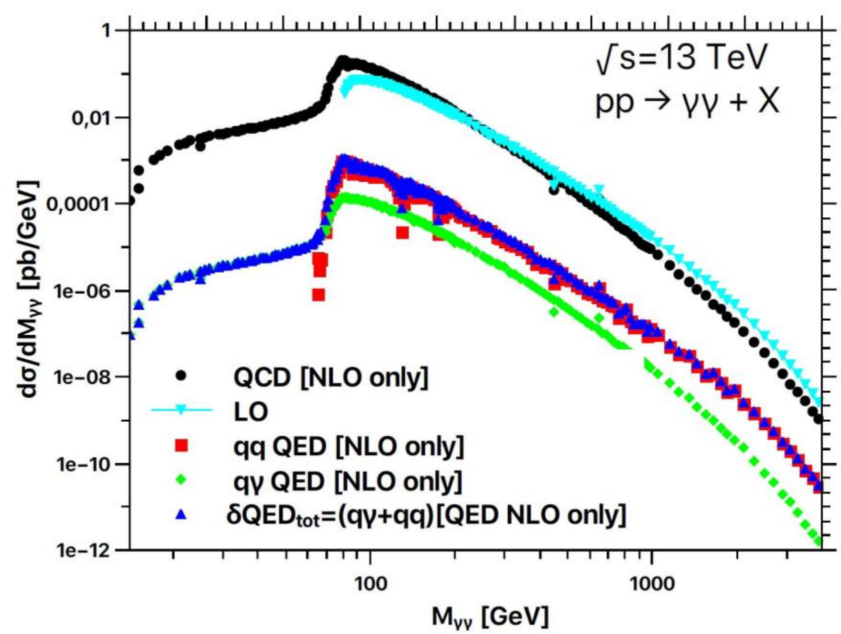
NLO QED corrections to $pp \to \gamma\gamma$

A bit of phenomenology

- Some effects due to QED corrections:
  - *Photon-ordering*: presence of photon radiation, **cuts imposed on the two hardest photons** → *Dynamical constraint, minimum angular separation bigger than 120°!!!!*

$$p_{T,2}^2 \geq p_{T,3}^2 = p_{T,1}^2 + p_{T,2}^2 + 2p_{T,1}p_{T,2}\cos\theta_{12} \quad \leftrightarrow \quad \cos\theta_{12} \leq -\frac{p_{T,1}}{2\,p_{T,2}} \rightarrow \boxed{M_{\gamma\gamma}^{\min} = \sqrt{2\,p_{T,1}\,p_{T,2}\,(1-\cos\theta_{12}^{\min})}}$$



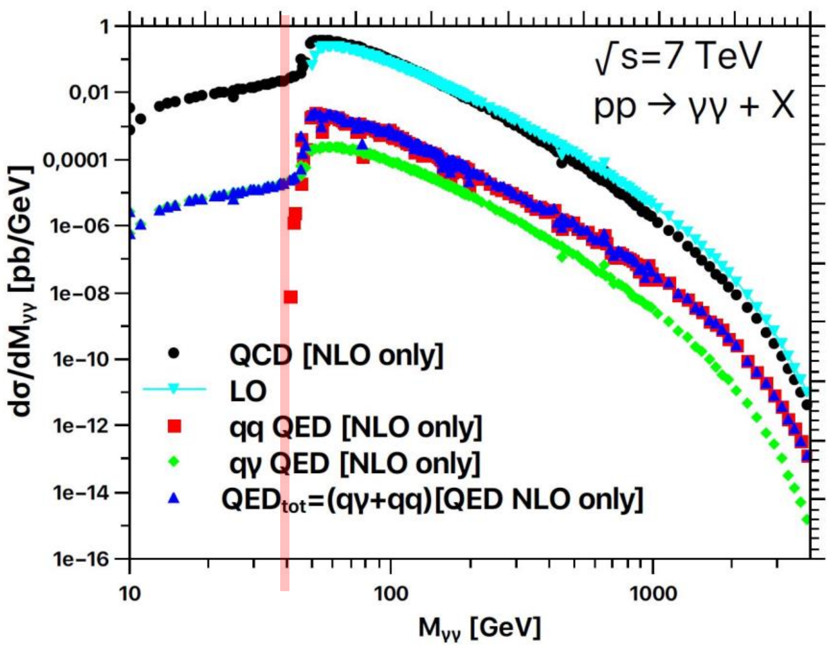


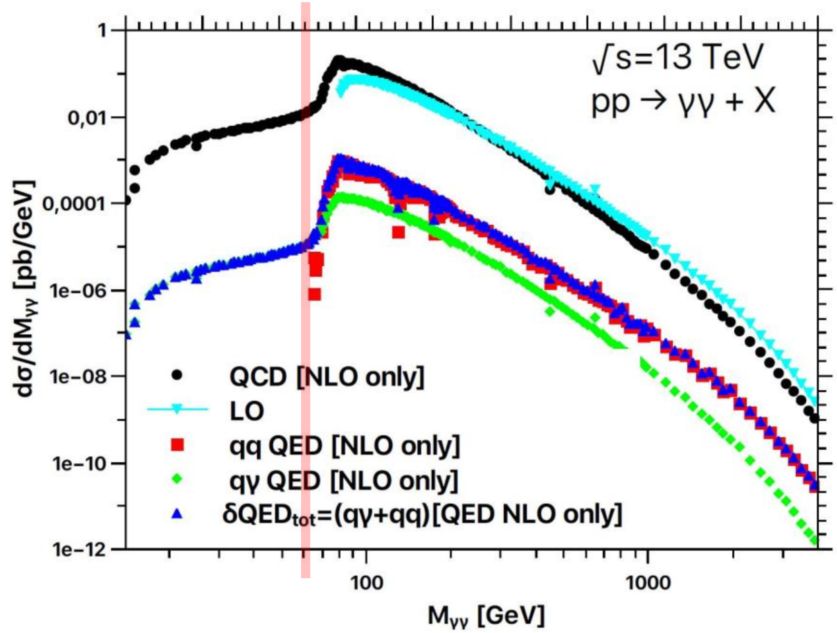
GS, PoS (EPS-HEP2017) 398; work in progress

NLO QED corrections to $pp \to \gamma\gamma$

A bit of phenomenology

- Some effects due to QED corrections:
  - LO and NLO QCD dominates. But… **total NLO QED introduces percent level effects in the high invariant mass region (>1 TeV)**
  - **The qqbar channel dominates QED corrections; BUT the photon initiated one contributes below the threshold!!**





GS, PoS (EPS-HEP2017) 398; work in progress

Part 3: QCD-QED resummation

- Mixed QCD-QED corrections in hadronic cross-sections due to resummation (+matching with f.o.)
- **I)- Development of a formalism to deal with mixed QCD-QED computations**
- **II)- Application to Z production (NNLL+NNLO QCD plus NLL+NLO QED plus *NEW non-trivial mixing*)**

Mixed QCD-QED resummation

Introduction and qt-formalism

- **Case of study:** Drell-Yan (*W,Z,photon production at hadron colliders*)
- Soft photon radiation could provide non-negligible effects in the low q_T region → **Extend qt-resummation to deal with QCD-QED radiation!**
- Some formulae to introduce qt-resummation in QCD:
  - The singular (i.e. divergent) part has an universal structure:

$$\frac{d\sigma_F^{(\text{sing})}(p_1,p_2;\mathbf{q_T},M,y,\mathbf{\Omega})}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2} f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)\,f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)$$

  - The Sudakov factor resums all the soft/collinear-emissions from the incoming legs; it is process independent
  - The **"hard-collinear"** coefficients **H** and **C** are related with the hard-virtual and collinear parts, and also contain the process dependence.

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

Mixed QCD-QED resummation

- **More details about the resummation formula:**
  - The Sudakov factor contains the logarithmically enhanced contributions. It can be resumed to all orders within perturbation theory!

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[A_c(\alpha_S(q^2)) \, \ln \frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$

$$A_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

  - $\mathbf{A_c}$ and $\mathbf{B_c}$ depend on the leg responsible for the emission. *They are related to the splitting functions!*
  - Also, **C** and **H** are calculable within perturbation theory! **C** is process independent (**H** contains the virtuals, i.e. loops):

$$H_q^F(x_1p_1, x_2p_2; \mathbf{\Omega}; \alpha_S) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}(x_1p_1, x_2p_2; \mathbf{\Omega})$$ → Loop information (finite parts)

$$C_{q\,a}(z;\alpha_S) = \delta_{q\,a}\ \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n C_{q\,a}^{(n)}(z)$$ → Radiation from incoming legs (transitions)

Catani et al, Nucl. Phys. B881 (2014) [arXiv:1311.1654]

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Path to QCD-QED resummation:**
- **Step I:** Transform all the QCD coefficients into the QED ones with the Abelianization algorithm (done!). Obtain QED resummation formula (done!).
  - *Subtlety I:* Charge separation effects due to up and down sectors.
  - *Subtlety II:* Photons and leptons must be included (closed loops), as well as the photon PDF → *Non trivial dependence!*

    SOLVED!
- **Step II:** Deal with QCD-QED radiation simultaneously. We need to Abelianizate all the coefficients, and perform the perturbative expansions with two couplings!
  - *Subtlety I:* Check of factorization formulae and its functional structure
  - *Subtlety II:* Compute *all* the coefficients, including the **mixed** ones!
  - *Subtlety III:* Applicable for **color-less neutral** final states…

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - Originally, in the QCD formalism, the resumed component is given by

$$\frac{d\hat{\sigma}^{(\mathrm{res.})}_{a_1a_2\to F}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db\, \frac{b}{2} J_0(b\, q_T)\, \mathcal{W}^F_{a_1a_2}(b, M, \hat{s}; \mu_F)$$

and we extend it by "exponentiating" photon/gluon radiation:

$$\mathcal{W}'^F_N(b, M; \mu_F) = \hat{\sigma}^{(0)}_F(M)\, \mathcal{H}'^F_N(\alpha_S, \alpha; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp\left\{\mathcal{G}'_N(\alpha_S, \alpha, L; M^2/\mu_R^2, M^2/Q^2)\right\}$$

Hard collinear part

Logarithmically-enhanced contributions

  - The hard-collinear part is expanded in a power series:

$$\mathcal{H}'^F_N(\alpha_S, \alpha) = \mathcal{H}^F_N(\alpha_S) + \frac{\alpha}{\pi} \mathcal{H}'^{F\,(1)}_N + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}'^{F\,(n)}_N + \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \mathcal{H}'^{F\,(n,m)}_N$$

Pure QED part

Pure QCD

Mixed QCD-QED

Cieri, Ferrera and GS, JHEP 08 (2018) 165

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Mixed QCD-QED resummation

Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The Sudakov factor is also expanded:

$$\mathcal{G}'_N(\alpha_S,\alpha,L) = \mathcal{G}_N(\alpha_S,L) + L\, g'^{(1)}(\alpha L) + g'^{(2)}_N(\alpha L) + \sum_{n=3}^{\infty}\left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}_N(\alpha L)$$
$$+\, g'^{(1,1)}(\alpha_S L,\alpha L) + \sum_{\substack{n,m=1\\ n+m\neq 2}}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^{n-1}\left(\frac{\alpha}{\pi}\right)^{m-1} g'^{(n,m)}_N(\alpha_S L,\alpha L)$$

Pure QCD

Pure QED

(New) mixed QCD-QED!!

  - The *g-functions* for QED are:

$$\lambda = \frac{1}{\pi}\beta_0\,\alpha_S\, L$$
$$\lambda' = \frac{1}{\pi}\beta'_0\,\alpha\, L$$

Large log!!!

$$g'^{(1)}(\alpha L) = \frac{A'^{(1)}_q}{\beta'_0}\,\frac{\lambda' + \ln(1-\lambda')}{\lambda'}$$
$$g'^{(2)}_N(\alpha L) = \frac{\widetilde{B}'^{(1)}_{q,N}}{\beta'_0}\ln(1-\lambda') - \frac{A'^{(2)}_q}{\beta'^2_0}\left(\frac{\lambda'}{1-\lambda'} + \ln(1-\lambda')\right)$$
$$+\,\frac{A'^{(1)}_q\beta'_1}{\beta'^3_0}\left(\frac{1}{2}\ln^2(1-\lambda') + \frac{\ln(1-\lambda')}{1-\lambda'} + \frac{\lambda'}{1-\lambda'}\right)$$

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED resummation

 Abelianization of the qt-formalism

- **Our (explicit) formulae (in b-space)**
  - The new mixed first-order *g-function*:

$$g'^{(1,1)}(\alpha_S L, \alpha L) = \frac{A_q^{(1)}\beta_{0,1}}{\beta_0^2\beta_0'}\, h(\lambda,\lambda') + \frac{A_q'^{(1)}\beta_{0,1}'}{\beta_0'^2\beta_0}\, h(\lambda',\lambda)$$

$$h(\lambda,\lambda') = -\frac{\lambda'}{\lambda-\lambda'}\ln(1-\lambda) + \ln(1-\lambda')\left[\frac{\lambda(1-\lambda')}{(1-\lambda)(\lambda-\lambda')} + \ln\left(\frac{-\lambda'(1-\lambda)}{\lambda-\lambda'}\right)\right] - \mathrm{Li}_2\left(\frac{\lambda}{\lambda-\lambda'}\right) + \mathrm{Li}_2\left(\frac{\lambda(1-\lambda')}{\lambda-\lambda'}\right),$$

  - New **A, B** and **H** coefficients:

$$A_q'^{(1)} = e_q^2 \qquad A_q'^{(2)} = -\frac{5}{9}\, e_q^2\, N^{(2)}$$

$$\widetilde{B}_{q,N}'^{(1)} = B_q'^{(1)} + 2\gamma_{qq,N}'^{(1)}$$

$$B_q'^{(1)} = -\frac{3}{2}\, e_q^2$$

$$\gamma_{qq,N}'^{(1)} = e_q^2\left(\frac{3}{4} + \frac{1}{2N(N+1)} - \gamma_E - \psi_0(N+1)\right)$$

$$\gamma_{q\gamma,N}'^{(1)} = \frac{3}{2}\, e_q^2\, \frac{N^2+N+2}{N(N+1)(N+2)}$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow q\bar{q},N} = \frac{e_q^2}{2}\left(\frac{2}{N(N+1)} - 8 + \pi^2\right)$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \gamma q,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow q\gamma,N} = \frac{3\, e_q^2}{(N+1)(N+2)}$$

$$\mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \gamma\gamma,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow qq,N} = \mathcal{H}'^{F\,(1)}_{q\bar{q}\leftarrow \bar{q}\bar{q},N} = 0$$

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Mixed QCD-QED coupling evolution

Mixed RGE equations

- **Coupled differential equations:** Crucial to recover non-trivial mixed terms in *g-functions*

$$\frac{d\ln\alpha_S(\mu^2)}{d\ln\mu^2} = \beta(\alpha_S(\mu^2),\alpha(\mu^2)) = -\sum_{n=0}^{\infty}\beta_n\left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta_{n,m}\left(\frac{\alpha_S}{\pi}\right)^{n+1}\left(\frac{\alpha}{\pi}\right)^{m}$$

$$\frac{d\ln\alpha(\mu^2)}{d\ln\mu^2} = \beta'(\alpha(\mu^2),\alpha_S(\mu^2)) = -\sum_{n=0}^{\infty}\beta'_n\left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty}\beta'_{n,m}\left(\frac{\alpha}{\pi}\right)^{n+1}\left(\frac{\alpha_S}{\pi}\right)^{m}$$

- **Mixed beta function coefficients:**

$$\beta_0 = \frac{1}{12}(11\,C_A - 2\,n_f)\,, \qquad \beta_{0,1} = -\frac{N_q^{(2)}}{8}\,,$$

$$\beta'_0 = -\frac{N^{(2)}}{3}\,, \qquad \beta'_1 = -\frac{N^{(4)}}{4}\,, \qquad \beta'_{0,1} = -\frac{C_F C_A N_q^{(2)}}{4},$$

Cieri, Ferrera and GS, JHEP 08 (2018) 165

QCD-QED corrections to Drell-Yan (inclusive)

Motivation & some previous results

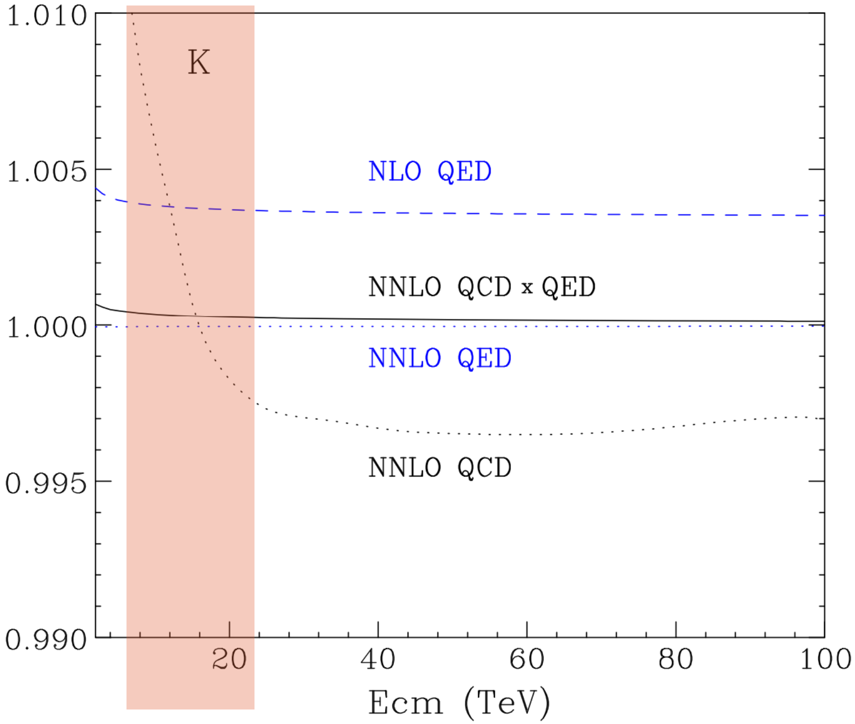
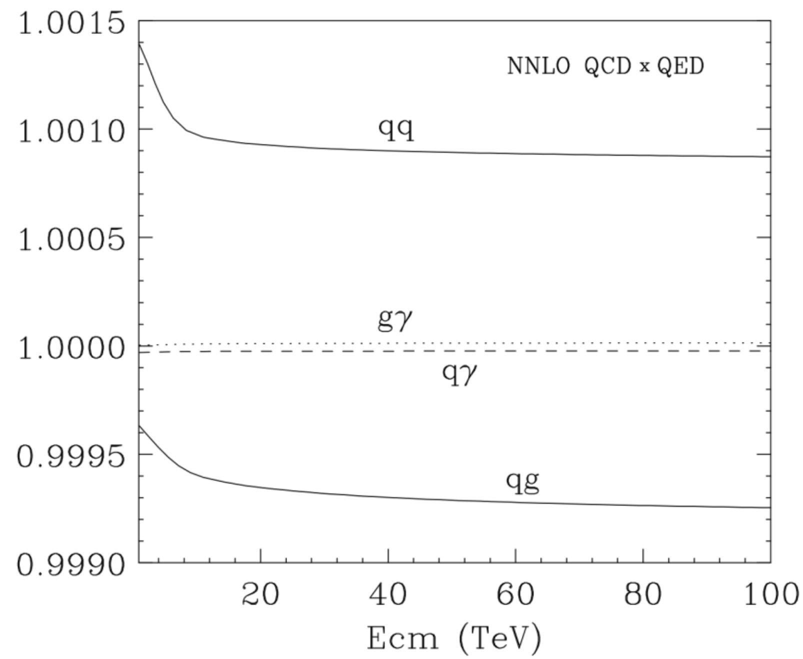


FIG. 3. *K*-factors for the different distributions as defined in Eq. (6). The (blue) dashed line corresponds to $K^{\mathrm{NLO}}_{\mathrm{QED}}$, the (blue) dotted line to $K^{\mathrm{NNLO}}_{\mathrm{QED}}$, the solid line to the mixed $K^{\mathrm{NNLO}}_{\mathrm{QCD\times QED}}$ and the (black) dotted line to the pure NNLO QCD corrections $K^{\mathrm{NNLO}}_{\mathrm{QCD}}$.



- Tiny photon initiated contribution
- Dominated by qq and qg

De Florian, Der and Fabre, Phys.Rev.D98, 094008

Extracted from the talk "QCD⊕QED NNLO corrections to Drell Yan production", by D. de Florian, (LHC EW precision sub-group meeting, Jun 20th 2018, CERN)

QCD-QED corrections to Drell-Yan (inclusive)

Conclusions

- Full QED+QCD NNLO corrections to DY (on-shell Z production)
- QED NLO ~ QCD NNLO (opposite sign) around 5 per-mille
- Mixed QEDxQCD below the per-mille level

Cancellation between qq and qg channels

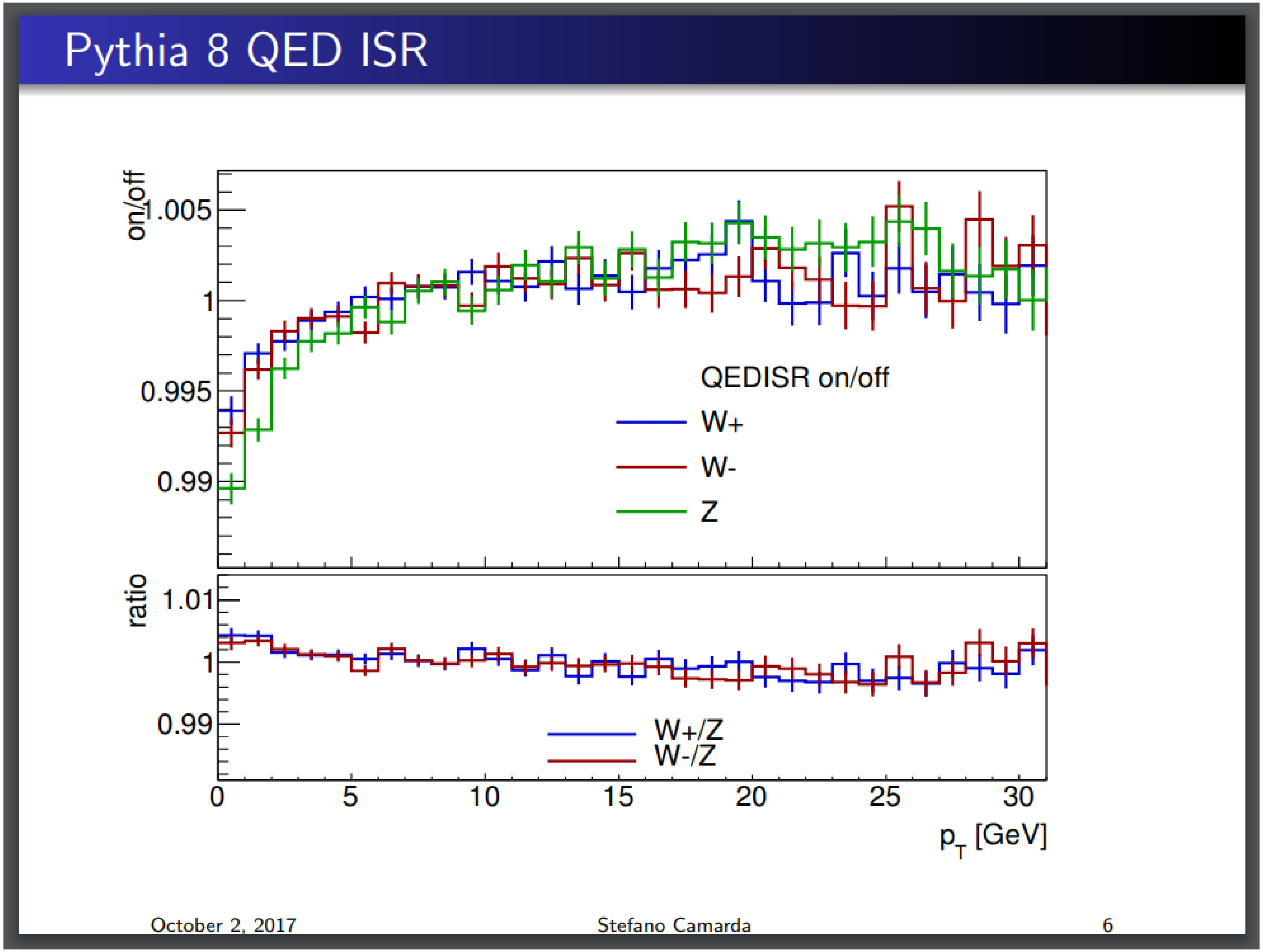
- At 14 TeV QCD NNLO ~ 3.5 mixed QEDxQCD (QCD cancellation)
- Factorization approach for mixed QEDxQCD fails by factor of 2
- Very stable under scale variations at NNLO

De Florian, Der and Fabre, Phys.Rev.D98, 094008

Extracted from the talk "QCD⊕QED NNLO corrections to Drell Yan production", by D. de Florian, (LHC EW precision sub-group meeting, Jun 20th 2018, CERN)

Z production with *mixed NLL QED*

Motivation & some previous results

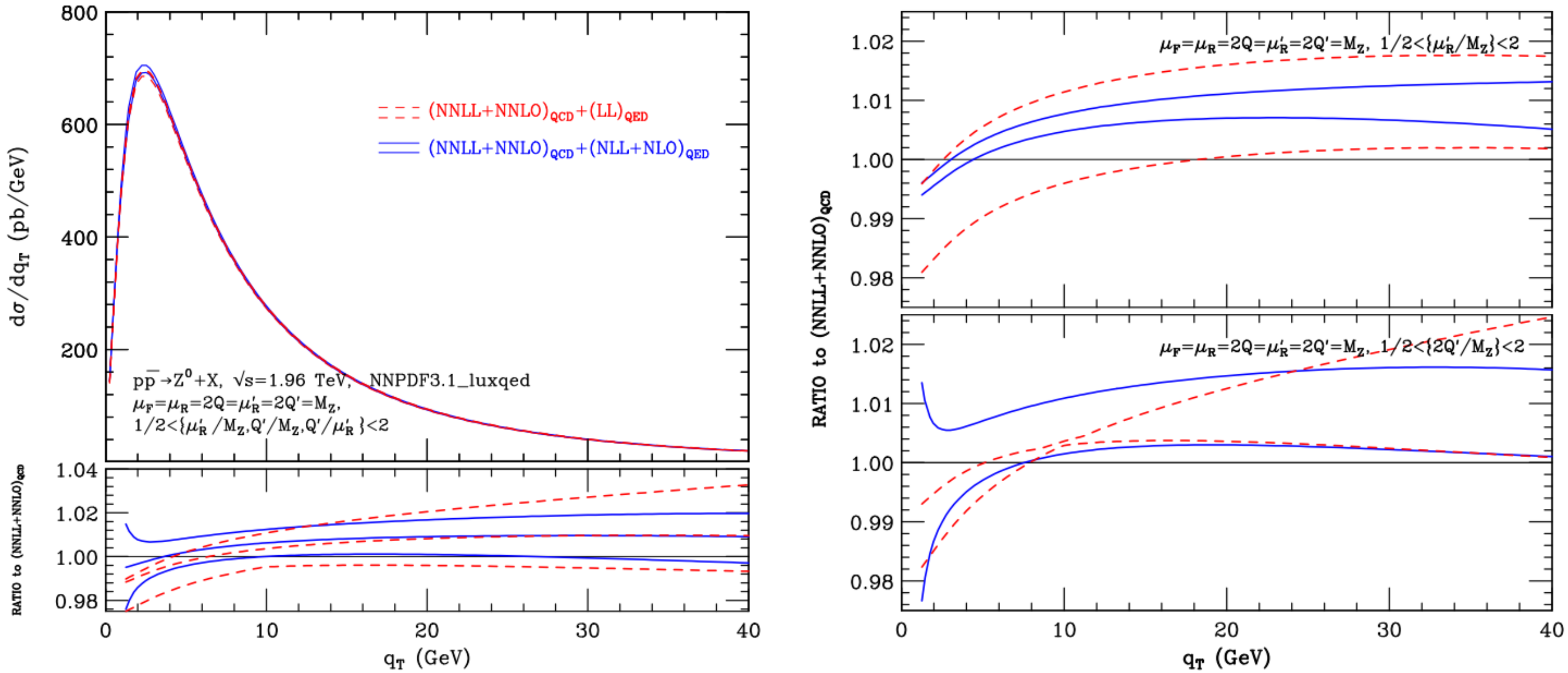


S. Camarda, "Studies of W/Z pT", Oct. 2017

Z production with *mixed NLL QED*

Some plots

- **Case of study:** Z production (implemented in DYqt)



- Collider: Tevatron at 1.96 TeV
- Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Z production with *mixed NLL QED*

Some plots

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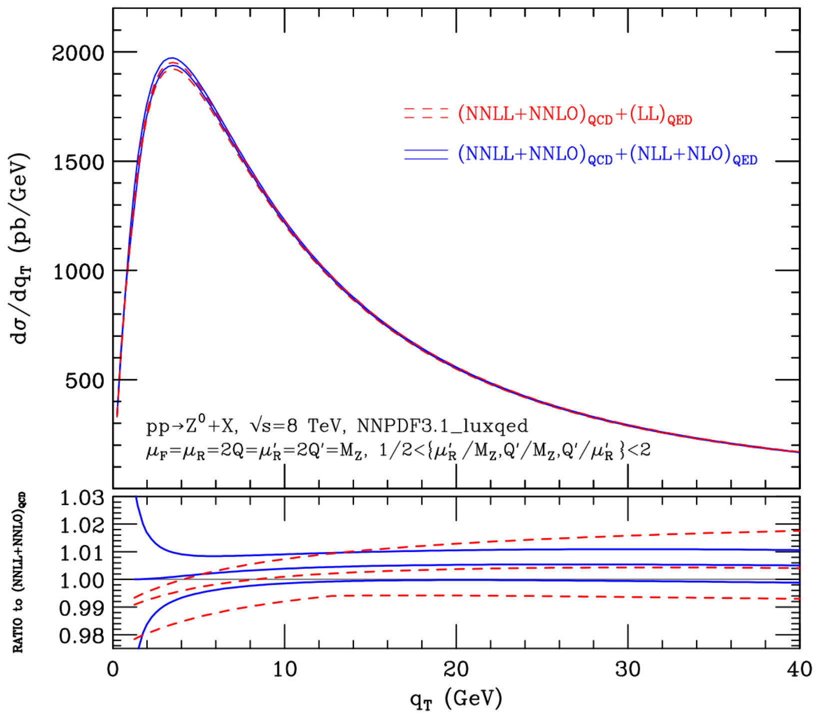

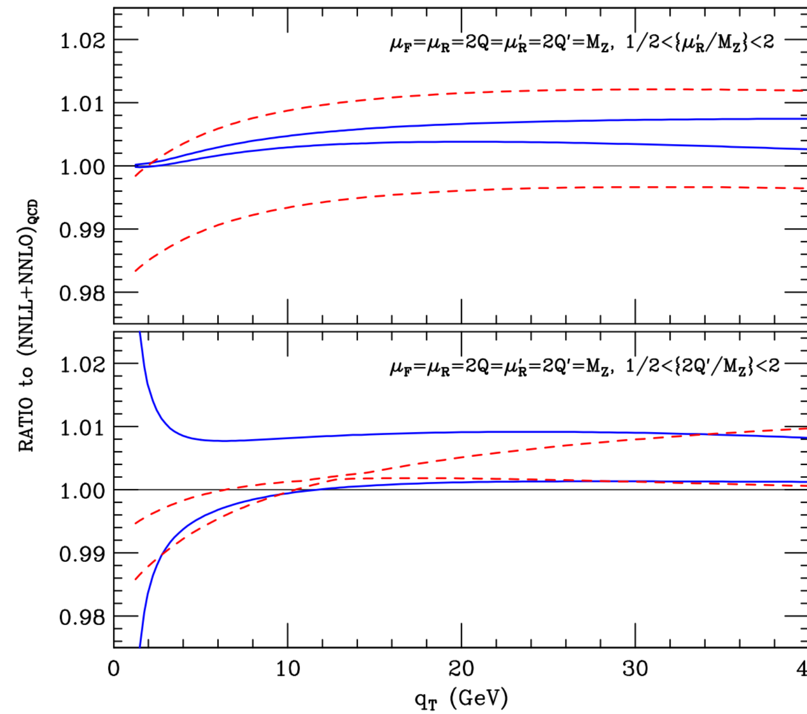

- Collider: LHC at 8 TeV
- Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Z production with *mixed NLL QED*

Some plots

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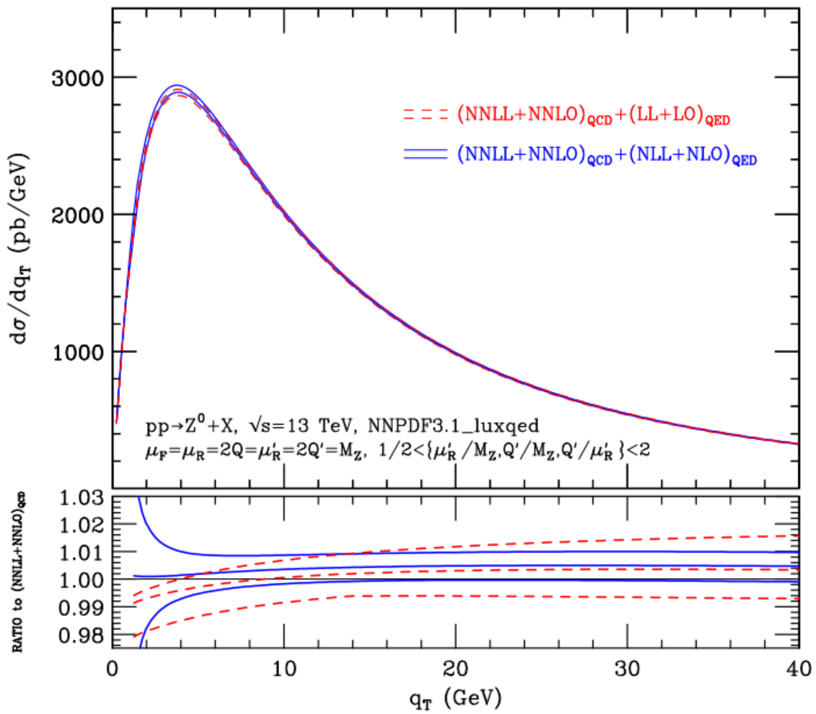
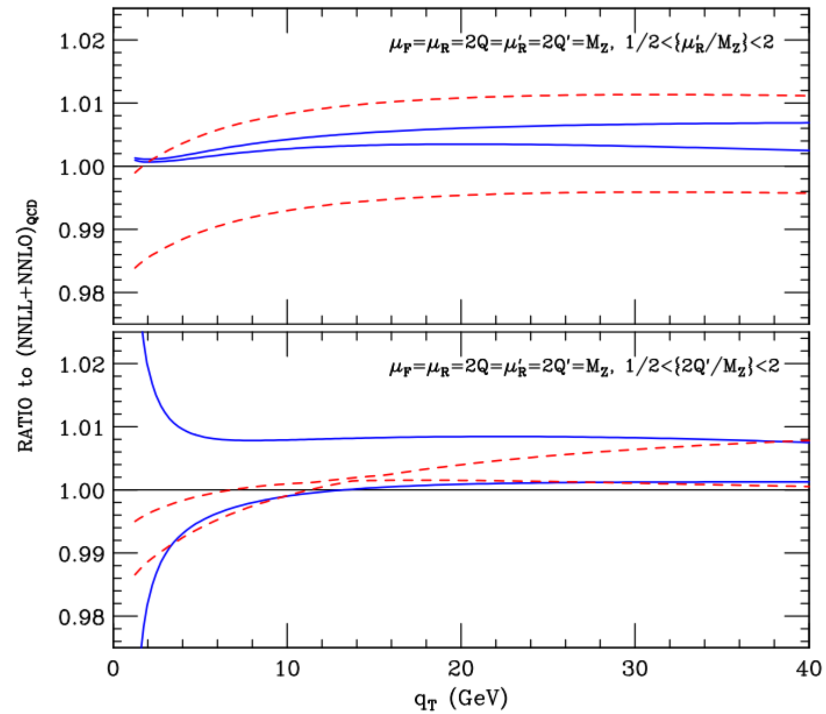




- Collider: LHC at 13 TeV
- Z production, using the narrow with approximation, with NNLL + NNLO QCD as reference to compare the QED effects. NEW NNPDF3.1QED (uses LUX's method)

Cieri, Ferrera and GS, JHEP 08 (2018) 165

Conclusions

- In the context of precision HEP, mixed QCD-QED are crucial!!!
- **Step 1:** splittings and DGLAP equations within QCD-QED
  - Fully consistent treatment of IR factorization
  - *Percent level contributions to PDF evolution and crucial effects in the determination of photon PDF*
- **Step 2:** fixed order effects
  - Physical example: NLO QED corrections to diphoton production
  - *Additional subtleties due to photon radiation (**ordering, merging, identification**)*
- **Step 3:** resummation within QCD-QED (mixed effects)
  - Physical example: Z production
  - Non negligible (few percent) effects at low pt!!!

Thanks for the attention!!

BACKUP SLIDES

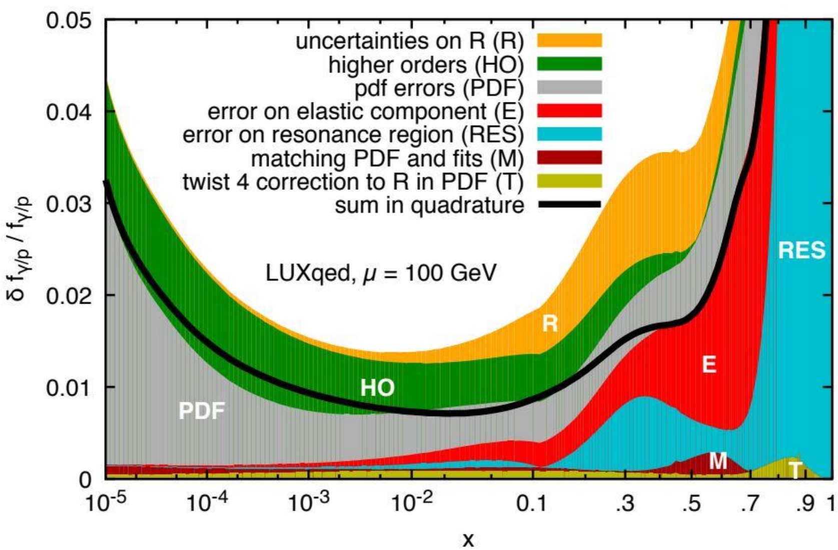
About photon PDFs

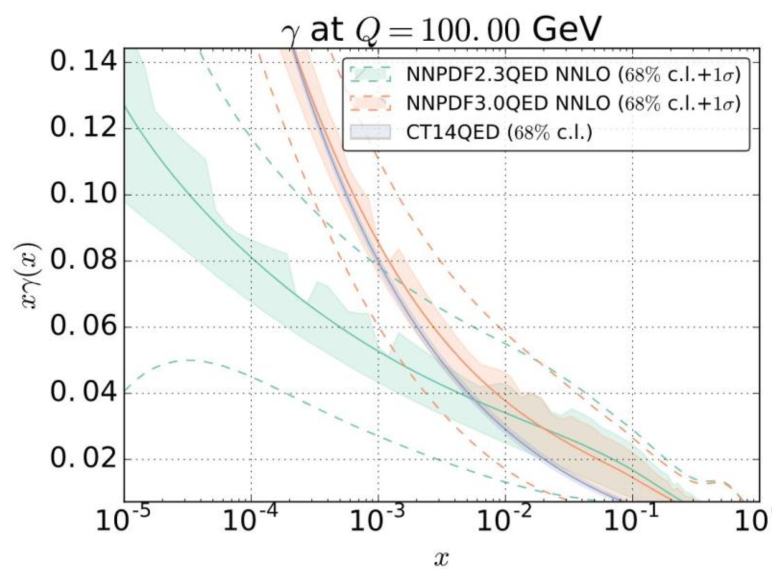
PDF dependence: explanation

- **Diphoton production is sensitive to photon PDF (at NLO QED)**
- Originally, **NNPDF** and **LUXqed** use(d) very different approaches. **NNPDF** does a full **global fit with NN** (no assumptions), whilst **LUXqed** uses an **analytical formula** to describe photon PDF (modeling structure functions)

More info available in Zanderighi et al' 17

- *Recently,* **NNPDF3.1QED** *adopted* **LUXqed** *strategy to reduce errors, and both sets leads to compatible results.*





Splittings and DGLAP within QCD-QED

Extended DGLAP equations (easiest ones)

- New optimized DGLAP equations! They become completely diagonal at some perturbative orders (due to vanishing kernels).

$$\frac{dq_{v_i}}{dt} = P^{-}_{q_i} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^{S}_{q_i q_j} \otimes q_{v_j} + \Delta P^{S}_{q_i l} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right),$$

$$\frac{dl_{v_i}}{dt} = P^{-}_{l} \otimes l_{v_i} + \sum_{j=1}^{n_F} \Delta P^{S}_{l q_j} \otimes q_{v_j} + \Delta P^{S}_{ll} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right),$$

Valence PDFs

$$\frac{d\{\Delta_{uc}, \Delta_{ct}\}}{dt} = P^{+}_{u} \otimes \{\Delta_{uc}, \Delta_{ct}\},$$

$$\frac{d\{\Delta_{ds}, \Delta_{sb}\}}{dt} = P^{+}_{d} \otimes \{\Delta_{ds}, \Delta_{sb}\},$$

$$\frac{d\Delta^{l}_{\{2,3\}}}{dt} = P^{+}_{l} \otimes \Delta^{l}_{\{2,3\}},$$

Diagonal equations

There are some remaining equations to describe the full coupled system, but they are more complicated (… much more complicated…)

- These equations are usually solved with Mellin transformations. The coupled differential system is reduced to an algebraic one for the Mellin momenta.

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Splittings and DGLAP within QCD-QED

Introducing new PDFs

- Change **PDFs basis** to simplify the system of coupled integro-differential equations *Roth, Weinzierl '04*

New lepton distributions!

$$\mathcal{B}_c = \{u, \bar{u}, \ldots, t, \bar{t}, e, \bar{e}, \ldots, \tau, \bar{\tau}, g, \gamma\}$$

Photon distribution!

$$\Delta_{uc} = u + \bar{u} - c - \bar{c}\,,$$
$$\Delta_{ds} = d + \bar{d} - s - \bar{s}\,,$$
$$\Delta_{sb} = s + \bar{s} - b - \bar{b}\,,$$
$$\Delta_2^l = e + \bar{e} - \mu - \bar{\mu}\,,$$
$$\Delta_{UD} = u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b}$$
$$\Delta_3^l = e + \bar{e} + \mu + \bar{\mu} - 2(\tau + \bar{\tau})\,,$$

$$\Sigma = \sum_{i=1}^{n_F} (q_i + \bar{q}_i)\,,$$
$$\Sigma^l = \sum_{i=1}^{n_L} (l_i + \bar{l}_i)\,,$$

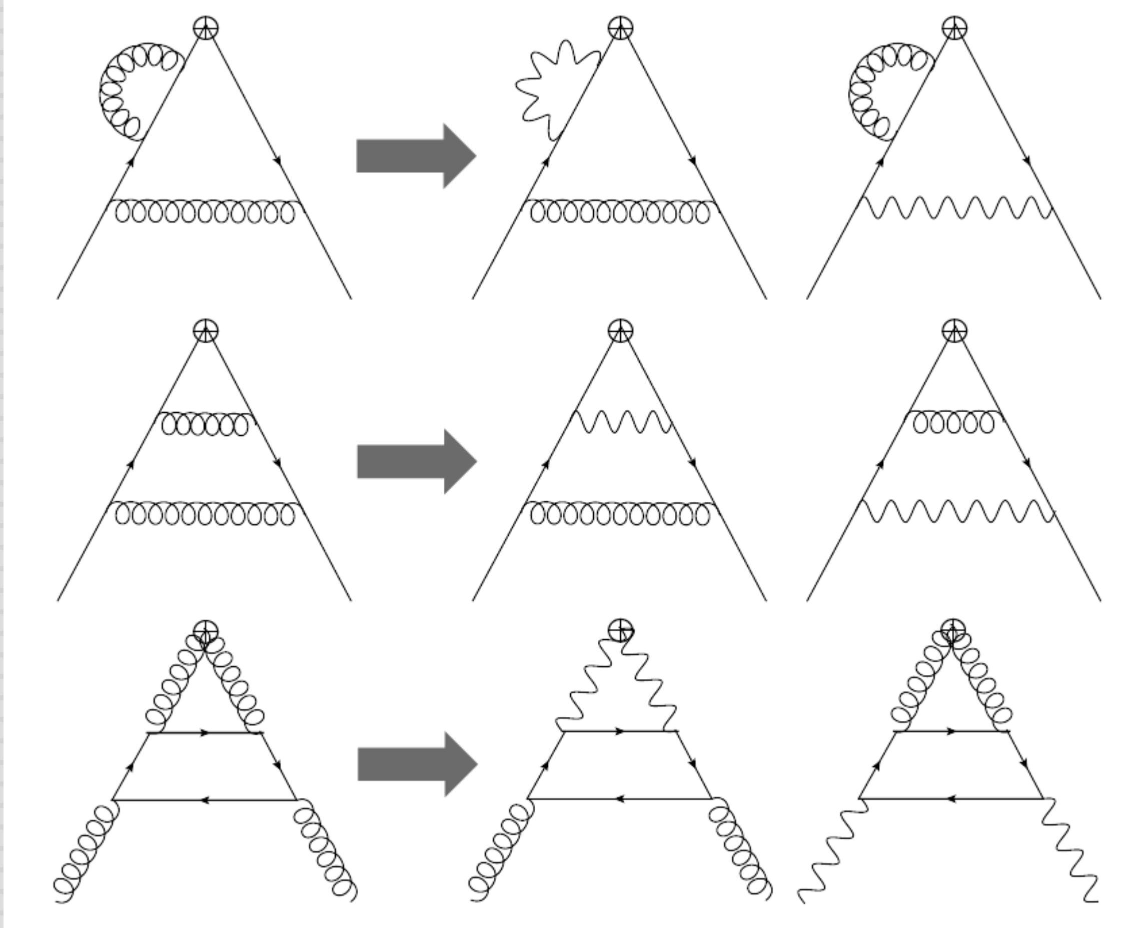
CANONICAL BASIS

$$q_v = q_i - \bar{q}_i\,,\; l_v = l_i - \bar{l}_i\,,$$

Valence distributions

- Photon and gluon distributions are not altered
- Straightforward extension to deal with $n_F=6$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056



$P_{qq}^{(2,0)} \to P_{qq}^{(1,1)}$

Non-observable gluon leads to non-equivalent diagrams contributing to the same kernel

$P_{gg}^{(2,0)} \to P_{g\gamma}^{(1,1)} \oplus P_{\gamma g}^{(1,1)}$

Replacement of external gluons leads to different kernels (no need of factor 2)

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What does "**Abelianization**" mean?

The *Abelianization* is an algorithm that we defined to extract QED corrections from QCD ones. **Moreover, mixed QCD-QED corrections can be recovered with the same strategy**. Even if it seems easy, the structure of **mixed corrections is not trivial** (involves expanding in two different couplings, potential crossed terms might appear…)

- Use two-loop QCD results as starting point; keeping track of the different topologies contributing to the splittings is crucial to check the results

> Curci, Furmanski and Petronzio, Nucl. Phys. B 175 (1980) 27
> Furmanski and Petronzio, Phys. Lett. B 97 (1980) 437
> Ellis and Vogelsang, hep-ph/9602356

- Mixed QCD-QED contributions (i.e. $\mathcal{O}(\alpha\,\alpha_S)$) obtained through the replacement of one gluon with one photon.
- Two-loop QED contributions (i.e. $\mathcal{O}(\alpha^2)$) involve replacing two gluons; **internal fermion loops could contain leptons:**

$$n_F \rightarrow \sum_f e_f^2 \qquad \text{with} \qquad \sum_f e_f^a = N_C \sum_{j=1}^{n_F} e_{q_j}^a + \sum_{j=1}^{n_L} e_{l_j}^a$$

- Results have been cross-checked independently by another group!

Manohar, Nason, Salam and Zanderighi, '16 and '17

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