CoLoRFulNNLO at work: determination of $\alpha_s$

Adam Kardos

University of Debrecen

based on work in collaboration with

S. Kluth, G. Somogyi, Z. Trócsányi, Z. Tulipánt, A. Verbytskyi

11th FCC-ee workshop: Theory and Experiments
Motivation:

The theory of pQCD has 7 free parameters, one is the strong coupling, $\alpha_s$ → its most precise determination is of fundamental importance!

Technology and theory allows for multifold ways to measure it: from decays through lattice to event shapes in $e^+ e^-$ collisions

Measurements at LHC are coming up fast but lack theoretical precision (so far they are at most @ NLO)

At the moment lattice computations seem unbeatable: per mil uncertainty combined with consistent results favoring a high $\alpha_s$ value

How can we regain dominance?
Motivation:

Refining results coming from pQCD is not a walk in the park:

1) We need quality data:
   a) We can reanalyze old data (like Verbytskyi, arXiv:1804.01019)
   b) We need more data (but we need a new machine for it… like the FCC-ee)
   c) Coming up with new observables less sensitive to factors we do not have too much control over (like hadronization)

2) We have to put more pressure on us:
   a) Increasing theoretical precision of predictions:
      Basically one more N in front of LO (fixed order) and/or LL (resummation):
      - Currently at NNLO, going to N3LO would take half a decade or more…
      - Mostly we have NNLL but N3LL seems feasible in the near future due to SCET…
   b) Increasing numerical precision of predictions (preventing fitting to noise)
   c) New observables

See also David d’Enterria’s talk from Tuesday!
One more N: Energy-Energy Correlation
EEC:

EEC is the normalized energy-weighted cross section defined in terms of the angle between two particles $i$ and $j$ in the event:

$$
\frac{1}{\sigma_t} \frac{d\Sigma(\chi)}{d \cos \chi} \equiv \frac{1}{\sigma_t} \int \sum_{i,j} \frac{E_i E_j}{Q^2} d\sigma_{e^+e^- \rightarrow ij + X} \delta(\cos \chi + \cos \theta_{ij})
$$

With CoLoRFuI NNLO the NNLO became available (Del Duca, Duhr, AK, Somogyi and Trócsányi, arXiv:1603.08927):

![Graph showing the comparison of LO, NLO, and NNLO predictions for $\sin^2 \chi$ versus $d\sigma/d\cos \chi$. The graph includes lines for different values of $\xi_R$ and highlights the variation with $Q^2 = 91.2$ GeV and $\alpha_s(Q^2) = 0.118$.](attachment:graph.png)
NNLL resummation is available for EEC (De Florian & Grazzini, arXiv: 0407241) making it possible to have NNLL + NNLO result as well:

EEC is sufficiently precise at NNLL + NNLO to be a worthy candidate for $\alpha_S$ extraction

Caveat: the prediction is on parton level but hadrons are observed!
Meaningful comparison is only possible if prediction is on the hadron level as well!

Yet so far only phenomenological models are available for hadronization:

Two approaches are possible to model non-perturbative effects:

1) Using an analytical model
2) Extraction of non-perturbative corrections from MC tools

Option 1:

We applied the analytical model of Dokshitzer, Marchesini and Webber, (arXiv:9905339) to our NNLL+NNLO result:

The original Sudakov gets multiplied by an additional factor:

\[ S_{NP} = e^{-\frac{1}{2} a_1 b^2} (1 - 2a_2 b) \]
Extracted $\alpha_s (@ \text{NNLL+NNLO})$:

$$\alpha_s(M_Z) = 0.121^{+0.001}_{-0.003}, \quad a_1 = 2.47^{+0.48}_{-2.38} \text{GeV}^2, \quad a_2 = 0.31^{+0.27}_{-0.05} \text{GeV}.$$  

$\chi^2 / \text{d.o.f.} = 1.18$

Fit was done to 91.2 GeV OPAL and SLD data

Large anticorrelation between $\alpha_s$ and $a_2$

$\longrightarrow$ The hadronization is not fully under control in this scenario

Can we improve on this?
Option 2:

MC tools also provide fragmentation and hadronization models. These can also be used to extract the $H/P$ ratio

(AK, Kluth, Somogyi, Tulipánt, Verbytskyi, arXiv:1804.09146)

We can use SHERPA and Herwig to obtain predictions both at parton and hadron level ⟹ the bin by bin ratios give the correction factor from parton to hadron level

\[
\text{SHERPA2.2.4} \rightarrow \text{Lund string frag. model (} S_L \text{)} \rightarrow \text{cluster frag. model (} S_C \text{)} \\
\text{Herwig7.1.1} \rightarrow \text{cluster frag. model (} H_M \text{)}
\]
Having the prediction at hadron and parton level the bin-by-bin H/P ratio can be obtained:

In the $\alpha_S$ fit $S^L$ was used as reference hadronization and $S^C$ to obtain systematics.
To extract $\alpha_s$ several measurements were used but keeping an eye on:

- high precision of differential distributions
- charged and neutral particles considered in the full $\chi$ range
- corrections for detector effects
- corrections for initial state photon radiation
Resulting $\alpha_s$ values:

**NNLL+NNLO:**

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018 \text{(exp.)} \pm 0.00102 \text{(hadr.)} \pm 0.00257 \text{(ren.)} \pm 0.00078 \text{(res.)}$$

**NNLL+NLO:**

$$\alpha_s(M_Z) = 0.12200 \pm 0.00023 \text{(exp.)} \pm 0.00113 \text{(hadr.)} \pm 0.00433 \text{(ren.)} \pm 0.00293 \text{(res.)}$$

Uncertainties are dominated by the truncation of the perturbative series and hadronization (for NNLL+NNLO).

Bringing down the uncertainty from scale variation is unlikely in the next couple of years. We need one more order (N3LO)!
New Observables: Soft-Dropped Event Shapes
Main uncertainties are coming from renormalization (truncation of perturbative series) and non-perturbative effects.

Going from NNLO to N3LO for $e^+e^-$ event shapes is unrealistic for at least half a decade.

What can be done with the non-perturbative effects?

Decreasing hadronization corrections also decreases their large uncertainty.

One interesting prospect is the modification of well-established event shapes to tune down NP effects.

Objective criticism from experimenters: modification results in a decreased yield. If statistics is limited we are dropping useful events!
New Observables:

Even if altering event shape definitions is considered malicious for the extremely limited data we collected in $e^+e^-$ they can be of interest when the plans for a new experiment are laid down.

Other possibility: getting to know the problematic part better

NP contributions can be shrunk by grooming away problematic tracks from the events.

A typical roadmap to grooming an event is:

1) Cluster all the hadronic tracks book-keeping merging info on pseudojets.

2) When clustering is done undo the last merging and apply a condition on the pseudojets.

3) Pseudojets (and subsequent tracks) are kept/discarded according to the condition.
Soft-Dropped Observables:

In soft-drop (Larkoski, Marzani, Soyez, Thaler, Frye, Schwartz, Yan) the condition is:

\[
\frac{\min \left[ E_i, E_j \right]}{E_i + E_j} > z_c \left( \frac{1 - \cos \theta_{ij}}{1 - \cos R} \right)^{\beta/2}
\]

or

\[
z_c \left( 1 - \cos \theta_{ij} \right)^{\beta/2}
\]

When applied to jets with radius R

If False: softer pseudojet is dropped and continue with harder one

If True: there is no soft content to be stripped off

For details, see Baron et al., arXiv:1803.04719

Typically the analysis is run on tracks surviving the grooming procedure

In case of thrust (\(\tau_{SD}'\)) the original thrust axis is used!
Soft-Dropped Observables:

Soft-dropped observables have two additional parameters: \( z_c \) and \( \beta \) ⟹ these can be used to optimize for least loss of yield:

\[ Q = 91.2 \text{ GeV}, \ \alpha_s(Q) = 0.118 \]

Soft-dropped thrust distribution for \( z_c=0.1, \ \beta=1 \) (L) and K-factors for different \((z_c, \ \beta)\) pairings (R)

AK. Somogyi, Trócsányi arXiv:1807.11472
Soft-Dropped Observables:

The groomed event can be used to calculate the hemisphere mass as well:

Soft-dropped hemisphere mass distribution for $z_c = 0.1$, $\beta = 1$ (L) and $K$-factors for different $(z_c, \beta)$ pairings (R)
Soft-Dropped Observables:

Soft-drop can also be used in conjunction with jet clustering with a radius $R$:

Although there is a region very perturbative stability is good at low values this is completely lost!
Summary and Conclusions:

- At the moment it seems lattice QCD computations are unbeatable in the determination of strong coupling.
- $e^+e^-$ collisions provide the perfect environment for $\alpha_S$ extraction.
- First NNLO computation with non-trivial final-state appeared for lepton colliders. We can expect the same with N3LO.
- Dominant uncertainties are from NP effects and the perturbative series. A new collider would allow for new observables where NP effects can be aggressively suppressed.
- We already have a bunch of observables where NP effects are minor. Need a machine where these could be measured with high stat!
- The next decade will be fantastic!
Summary and Conclusions:

Build a new collider to keep us keeping on!
Thank you for your attention!