The shower-cut dependence of the top quark mass

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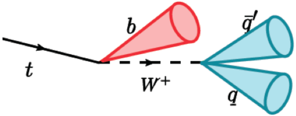
universität wien

$\int dk \prod$ Doktoratskolleg Particles and Interactions

FWF Der Wissenschaftsfonds.

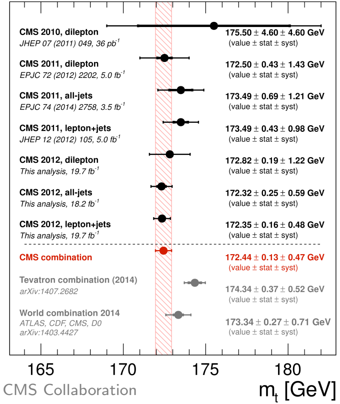
Motivation

- Parton showers as part of Monte Carlo (MC) event generators widely used in collider phenomenology



- Most precise top mass measurements based on reconstruction rely heavily on MCs

$$m_t^{\mathrm{MC}} = 172.44 \pm 0.49\,\mathrm{GeV}\ \text{(CMS)}$$
$$m_t^{\mathrm{MC}} = 172.84 \pm 0.70\,\mathrm{GeV}\ \text{(ATLAS)}$$
$$m_t^{\mathrm{MC}} = 172.44 \pm 0.64\,\mathrm{GeV}\ \text{(Tevatron)}$$



- Which mass scheme is determined in these measurements is still unsettled

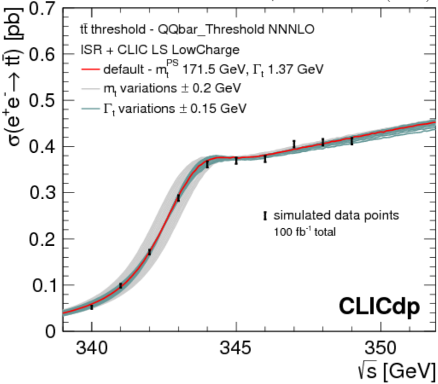
- Shower cut is expected to have impact on the IR behavior of the parton shower and the MC $\rightarrow$ mass scheme?

Future Measurements

Projections of precision of top mass measurements:

- at the LHC (reconstruction): $\delta m \sim 200$ MeV
- at future lepton collider:
  - $t\bar{t}$-threshold: $\delta m \lesssim 50$ MeV
    mass scheme well defined!
  - reconstruction: $\delta m \lesssim 200$ MeV

CLICdp Collaboration (2018)



$\Rightarrow$ $t\bar{t}$-threshold: major measurement
reconstruction: cross check

agreement means excellent control of QCD + EW theory in the top quark sector!

MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of $\mathbf{m_t^{MC}}$?

- picture of "top quark particle" does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
  $\rightarrow$ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\mathrm{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\mathrm{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...
- should be covered by "MC uncertainty" or better negligible

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this talk

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Previous Quantitative Examinations of m_t^{MC}

- Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart (2017), arxiv:1608.01318
  - **numerical** relation between Pythia MC top mass and MSR mass using 2-jettiness in e^+e^- in the resonance region from calibration fits
  - "MC top mass calibration"
  - $m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(1\,\mathrm{GeV}) + (0.18 \pm 0.22)\,\mathrm{GeV}$

    $m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + (0.57 \pm 0.28)\,\mathrm{GeV}$
  - universality conjectured but not proven

$$m_t^{\mathrm{MC}} = m_t^{\mathsf{pole}} + \underbrace{\Delta_m^{\mathsf{pert}} + \Delta_m^{\mathsf{non\text{-}pert}} + \Delta_m^{\mathrm{MC}}}_{\text{numerical calibration cannot distinguish the three contributions}}$$

Recent work on related issues (selection)

- Ravasio, Jezo, Nason, Oleari (2018), arxiv:1801.03944
  - POWHEG study: NLO corrections in various approximations
- Corcella, Franceschini, Kim (2017), arxiv:1712.05801
  - Dependence of m_t^{MC} from kinematic decay distributions on fragmentation parameters
- Heinrich, Maier, Nisius, Schlenk, Winter (2017), arxiv:1709.08615
  - Effects of off-shell top production compared to narrow width approximation

Aim of our work

- want to examine theoretical properties of **parton showers** (Δ_m^{pert}) with respect to dependence of shower cut Q_0

  to avoid infrared singularities every parton shower has to terminate at infrared cutoff here: cutoff on transverse momentum in splitting $q_\perp > Q_0$

- want to understand mass of the top quark state ($=$ top $+$ gluons around) that is produced in the hard interaction by the parton shower

  do not address issues related to **decay** (1. restriction)

- adopt narrow width approximation as used in state of the art MCs

  we do not address **finite lifetime** issues (2. restriction)
  (factorization of production and decay)

- parton showers for top quarks only conceptually valid in the quasi-coll. limit

  consider only **boosted** tops (3. restriction)

What is the effect of the shower cutoff on the generator mass scheme?

Overview of our work

- study coherent branching (CB) - basis of the Herwig 7 angular ordered parton shower
- shower cut on the transverse momentum in the splitting $q_\perp > Q_0$
- study 2-jettiness distribution in the peak region for e^+e^- for boosted tops
- can be calculated analytically in QCD factorization (SCET+bHQET) and CB
- model hadronization by convolution with non-perturbative shape function

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q, m) = \int_0^{Q\tau} \mathrm{d}\ell \, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\big(\tau - \frac{\ell}{Q}, Q, m\big) S_{\mathrm{mod}}(\ell - \Delta)$$

$Q_0 = 0$ (strict perturbative expansion in α_s)

- SCET+bHQET: in the resonance region the partonic cross section is factorized into hard, (bHQET-) jet and soft functions

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}(\tau, Q, m) = H_Q(Q,\mu_H) \times U_H(\mu_H,\mu_m) U_m(\mu_m,\mu_H) H_m(\mu_m)$$
$$\times \Big[U_{J_B}(\mu_H,\mu_{J_B}) \otimes J_B(\mu_{J_B}) \otimes U_S(\mu_H,\mu_S) \otimes S(\mu_s) \Big](\tau)$$

ultra-collinear radiation   wide angle soft radiation

[Fleming, Hoang, Mantry, Stewart (2008))]

$$\mu_H^2 \sim Q^2 \qquad \mu_m^2 \sim m^2 \qquad \mu_{J_B}^2 \sim \frac{Q^4\tau^2}{m^2} \qquad \mu_S^2 \sim Q^2\tau^2$$

mass scheme fixed in the jet function

- coherent branching: analytic solution in the resonance region in Laplace space

[Catani, Marchesini, Webber (1991); Gieseke, Stephens, Webber (2003)]

$$\mathcal{L}\left[\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}(\tau, Q, m)\right](\nu) = \exp\left[2\int_{m^2}^{Q^2}\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\int_{\frac{m}{\tilde{q}}}^{1}\mathrm{d}z\, P_{qq}\left[\alpha_s\big((1-z)\tilde{q}\big), z, \frac{m^2}{\tilde{q}^2}\right]\left(\mathrm{e}^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}}-1\right)\right]$$
$$\underset{\mathrm{NLL}}{\approx} \mathcal{L}\Big[U_H(\mu_H,\mu_m)\times U_m(\mu_m,\mu_H)\times U_{J_B}(\mu_H,\mu_{J_B})\otimes U_S(\mu_H,\mu_S)\Big](\nu)$$

- **coherent branching with $\mathbf{Q_0 = 0}$ and SCET+bHQET with $\mathbf{m = m^{pole}}$ equivalent at NLL** (known for the massless case, new for the massive case)

NLO precision in the peak region

- partonic cross section in SCET at NLO (massless)

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi}\left\{ \underbrace{-8\left[\frac{\ln\,\tau}{\tau}\right]_+}_{\mathrm{LL}} \underbrace{-\,6\left[\frac{1}{\tau}\right]_+}_{\mathrm{NLL}} + \underbrace{\delta(\tau)\left(\frac{2\pi^2}{3} - 2\right)}_{\mathrm{N^2LL}} \right\} + \mathcal{O}(\alpha_s^2)$$

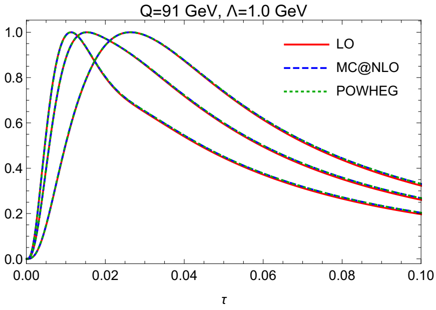
this piece is not correctly reproduced by CB

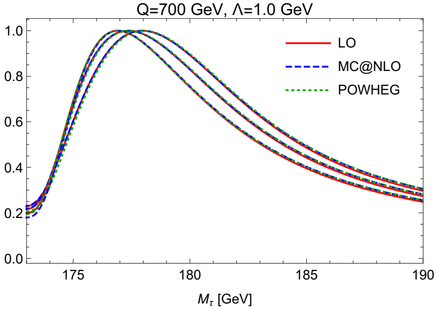
- $\mathrm{N^2LL}$ term at NLO is proportional to LO order cross section
- contributes only at higher orders to the position of the resonance peak τ_{peak}

$$\left.\frac{\mathrm{d}^2\sigma^{\mathrm{NLO}}(\tau)}{\mathrm{d}\tau^2}\right|_{\tau=\tau_{\mathrm{peak}}^{\mathrm{NLO}}} = 0$$

- $\tau_{\mathrm{peak}}^{\mathrm{NLO}}$ fully determined by NLL terms
- NLL sufficient for full NLO information in the peak
- **mass scheme of coherent branching without shower cut is pole mass**

NLO matched shower





massless

massive

$Q_0 = 1.0\,\mathrm{GeV}$ right bunch of curves

$Q_0 = 1.5\,\mathrm{GeV}$ middle bunch of curves

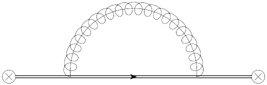
$Q_0 = 2.0\,\mathrm{GeV}$ left bunch of curves

NLO matching does not increase precision for simulations in the resonance region

Effects of a cutoff $Q_0 > 0$

- pole of the top quark propagator $= m_t^{\mathrm{CB}}(Q_0) \neq m_t^{\mathrm{pole}}$ **(coherent branching mass)**

$$\boxed{m_t^{\mathrm{CB}}(Q_0) = m_t^{\mathrm{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)}$$



- In the presence of the shower cut the **ultra-collinear radiation** generated by CB produces exactly the mass scheme change correction that is required so that the generator mass is exactly the coherent branching mass $m_t^{\mathrm{CB}}(Q_0)$

$$\sigma(m_1, Q, ...) = \sigma(m_2, Q, ...) + \delta m \times \frac{\mathrm{d}}{\mathrm{d}m} \sigma(m, Q, ...)\Big|_{m=m_1} + \ldots$$

$$\delta m = m_2 - m_1$$

- The shower cut also affects **large-angle soft radiation**. The corresponding effects are directly tied to the amount of hadronization effects that are supposed to be fixed by tuning.

- All conclusions explicitly cross checked by correspondence between analytic QCD factorization calculations and analytic solutions of the CB algorithm

- All results checked directly by comparing with Herwig 7 event generator.

$Q_0 > 0$: coherent branching (angular ordered parton shower)

- we can now work out the leading effects of introducing a shower cut $q_\perp > Q_0$
- keep only terms linear in Q_0 and m, only NLO in α_s
- with these expansions the difference of the distributions with and without cutoff can be calculated analytically
- leading effect of Q_0 is a shift in the partonic cross section with contributions coming from the soft and ultra-coll. regions

$$\frac{\mathrm{d}\sigma^{\mathrm{cb}}}{\mathrm{d}\tau}(\tau, Q, m, Q_0) = \frac{\mathrm{d}\sigma^{\mathrm{cb}}}{\mathrm{d}\tau}\left(\tau + \frac{\alpha_s(Q_0)}{4\pi}\left[16 C_F \frac{Q_0}{Q} - 8\pi C_F \frac{Q_0 m}{Q^2}\right], Q, m, Q_0 = 0\right)$$

$Q_0 > 0$: QCD factorization theorem

- introduce the cutoff $q_\perp > Q_0$ in the one-loop diagrams for the soft and jet functions in the QCD factorization theorem
- keep only terms linear in Q_0 and m, multipole expansion for real radiation terms
- SCET soft function at one-loop with $q_\perp$ cut:

$$S(\ell, Q_0) = S(\ell) + \frac{\alpha_s(Q_0)}{4\pi} 16 C_F Q_0 S'(\ell) + \mathcal{O}(\alpha_s^2)$$

extra term needs to be absorbed into a change in the non-pert. shape function

- bHQET jet function at one-loop with $q_\perp$ cut: **off-shell**

$$J_B^{\text{off}}(\hat{s}, m^{\text{pole}}, Q_0) = J_B^{\text{off}}(\hat{s}, m^{\text{pole}}) - \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{off}\prime}(\hat{s}, m^{\text{pole}}) + \mathcal{O}(\alpha_s^2)$$

extra term gets absorbed by change of mass scheme

- bHQET jet function at one-loop with $q_\perp$ cut: **on-shell self energy**

$$J_B^{\text{os}}(\hat{s}, m^{\text{pole}}, Q_0) = J_B^{\text{os}}(\hat{s}, m^{\text{pole}}) + \frac{\alpha_s(Q_0)}{4\pi} 8\pi C_F m Q_0 J_B^{\text{os}\prime}(\hat{s}, m^{\text{pole}}) + \mathcal{O}(\alpha_s^2)$$
$$= J_B^{\text{os}}\left(\hat{s}, m^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0)\right) + \mathcal{O}(\alpha_s^2)$$

change of mass scheme: $\mathbf{m}^{\text{pole}} \to \mathbf{m}^{\text{pole}} - \frac{\mathbf{2}}{\mathbf{3}} \mathbf{Q_0} \alpha_{\mathbf{s}}(\mathbf{Q_0})$

Comparison with CB

- if NOT compensated by retuning of hadronization model and redefinition of mass scheme, change of cutoff leads to shift that gets contributions from soft and ultra-collinear radiation

$$\tau_{\text{peak}}(Q_0) = \tau_{\text{peak}}(Q_0') - \frac{1}{Q}\left(16C_F - 8\pi C_F \frac{m}{Q}\right) \int_{Q_0'}^{Q_0} \mathrm{d}R \, \frac{\alpha_s(R)}{4\pi}$$

- agrees with result for shift of peak position obtained from coherent branching

- mass reduces leading coefficient of R-evolution

- dependence of the peak position on shower cut Q_0 can be compared to actual angular ordered parton shower in MC

Comparison with Herwig

- compare our predictions for peak shift with real parton shower
- used Herwig 7 with angular ordered shower for $e^+e^- \to t\bar{t}$
- modifications:
  - set all constituent masses of light quarks and gluon to zero
  - unrestricted kinematics in evolution of CB
  - on-shell top production
  - only leptonic W-decays
  - switched off: QED radiation, hadronization
- only partonic distribution from Herwig

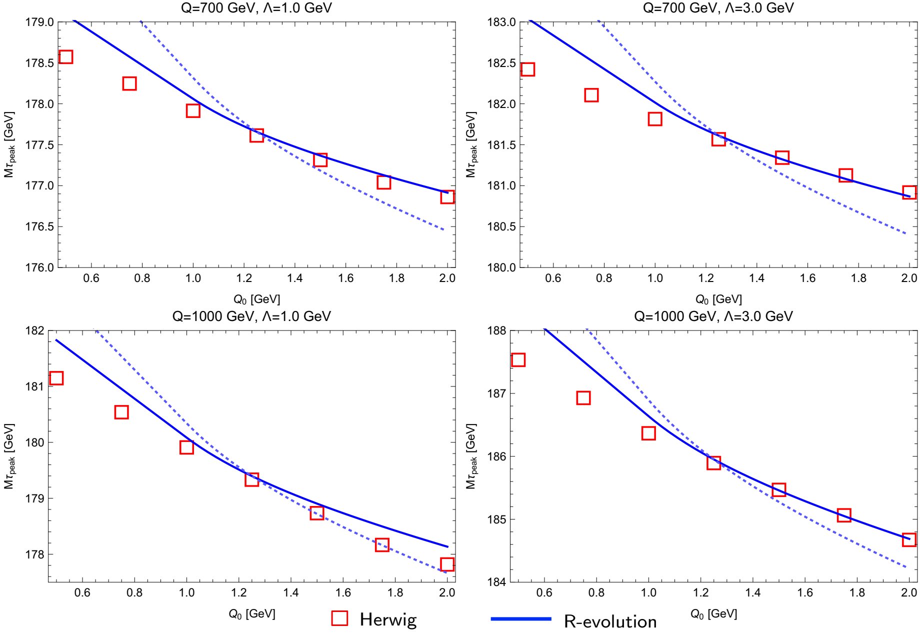
  "hadronization": convolution with model function $\to$ disentangle parton shower and hadronization model

$$S_{\text{mod}}(k,\lambda) = \frac{128\,k^3 \mathrm{e}^{-\frac{4k}{\tilde{\lambda}}}}{3\tilde{\lambda}^4} \qquad\qquad \tilde{\lambda} = \lambda + \frac{4m_t\Gamma_t}{Q}$$

- $m_t = 173\,\mathrm{GeV} \qquad \Gamma_t = 1.5\,\mathrm{GeV}$
- run for different values of Q_0, Q and λ.
- use rescaled τ variable $M_\tau = \frac{Q^2\tau}{m_t}$ (partonic threshold at m_t)
- compare cutoff dependence of peak position with R-evolution

$$M_{\tau,\text{peak}}(Q_0) = M_{\tau,\text{peak}}(Q_0') - \left(8C_F\,\frac{Q}{m_t} - 4\pi C_F\right)\int_{Q_0'}^{Q_0} \mathrm{d}R\,\frac{\alpha_s(R)}{4\pi}$$

Comparison with Herwig



Relation of m_t^{CB} to other Masses

Herwig 7: $Q_0 = 1.25\ \mathrm{GeV} \qquad \rightarrow \qquad \mathrm{m_t^{Herwig}} = \mathrm{m_t^{CB}}(1.25\ \mathrm{GeV})$

<u>MSR Mass:</u>

$$m_t^{\mathrm{MSR}}(Q_0) = m_t^{\mathrm{CB}}(Q_0) + 0.24\, Q_0\, \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow \boxed{m_t^{\mathrm{MSR}}(Q_0) = m_t^{\mathrm{CB}}(Q_0) + (0.190 \pm 0.070)\ \mathrm{GeV}}$$

- CB and MSR masses do not suffer from $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon (due to IR cut) $\rightarrow$ good convergence
- uncertainty estimated from difference between α_s in $\overline{\mathrm{MS}}$ and MC schemes
- precision sufficient for all possible applications at the LHC! (recall restriction 1-3)
- more precision may be needed for a future e^+e^- collider

Relation of $m_t^{\rm CB}$ to other Masses

Herwig 7: $Q_0 = 1.25$ GeV $\quad\rightarrow\quad$ $\mathrm{m_t^{Herwig}} = \mathrm{m_t^{CB}}(1.25\ \mathrm{GeV})$

Pole Mass:

$$m_t^{\rm pole} = m_t^{\rm MSR}(Q_0) + (0.350 \pm 0.250)\ \mathrm{GeV}$$

[Hoang, Lepenik, Preisser (2017)]
[± 110 MeV: Beneke, Marquard, Nason, Steinhauser (2017)]

$$\Rightarrow \boxed{m_t^{\rm pole} = m_t^{\rm CB}(Q_0) + (0.540 \pm 0.260)\ \mathrm{GeV}}$$

- pole mass suffers from $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon → irreducible ambiguity of 250 MeV
- difference between $m_t^{\rm pole}$ and $m_t^{\rm Herwig} \sim 500$ MeV > ambiguity!

  ⇒ important to study $\Delta_m^{\rm pert}$ beyond current set up (lift restrictions 1-3)
- shift as large as current experimental uncertainty from direct methods

Conclusions/Outlook

- for angular ordered parton showers (Herwig) one can derive the **perturbative contributions** between generator mass and pole masse (Δ_m^{pert})

$$m^{\mathrm{CB}}(Q_0) = m^{\mathrm{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0 + \mathcal{O}(\alpha_s^2)$$

  this corresponds the **pole of the quark propagator** in presence of a shower cut

- current restrictions: needed to remove restriction:
  - boosted top quarks $\rightarrow$ parton shower algorithm for slow tops
  - narrow width approximation $\rightarrow$ parton shower for unstable tops
  - top production (2-jettiness) $\rightarrow$ factorized predictions including top decay

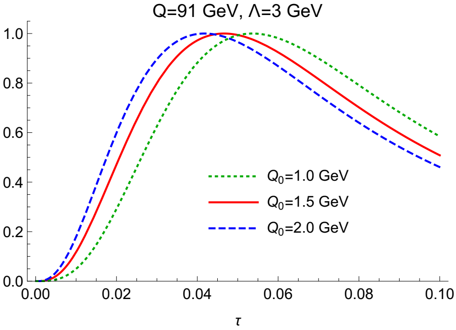
- for all three new conceptual developments are required (w.i.p.)

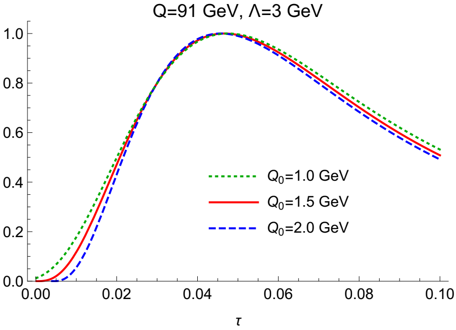
- study of non-perturbative contributions to the relation between generator mass and pole mass ($\Delta_m^{\mathsf{non\text{-}pert}}$) can be carried out by dedicated MC simulations (w.i.p.)

- numerical calibration still important tool for consistency checks

Backup

Massless τ Distributions

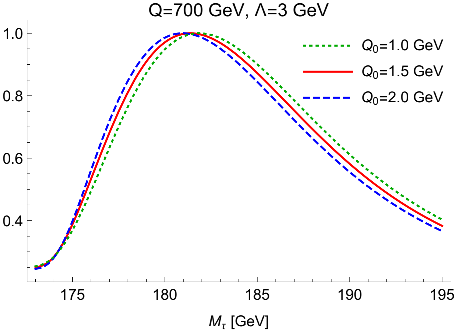


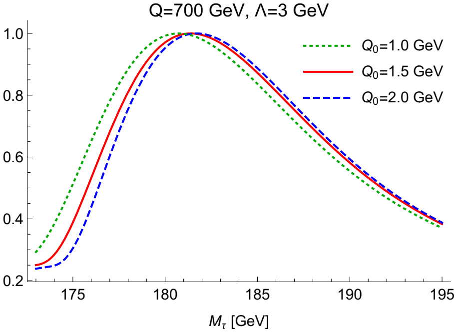


no gap

$$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5\text{GeV}}^{Q_0} \mathrm{d}R\, \alpha_s(R)$$

Massive M_τ Distributions





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$m = 173\,\mathrm{GeV}$

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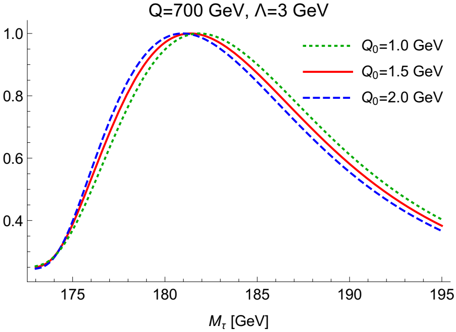
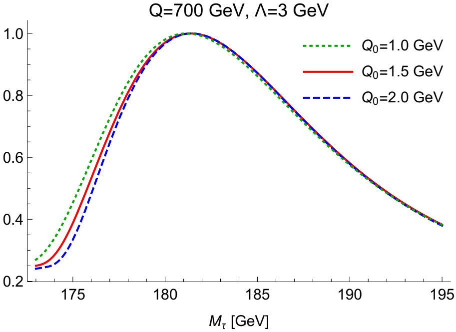
Massive M_τ Distributions





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$$m = 173\,\mathrm{GeV}$$

$$\Delta(Q_0) = \frac{16}{3\pi} \int_{1.5\mathrm{GeV}}^{Q_0} \mathrm{d}R\, \alpha_s(R)$$

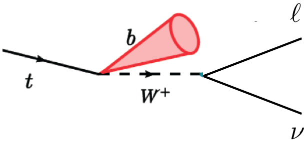
$$m(Q_0) = 173\ \mathrm{GeV} - \frac{2}{3} \int_{1.5\mathrm{GeV}}^{Q_0} \mathrm{d}R\, \alpha_s(R)$$

$$m(1.0\,\mathrm{GeV}) = 173.22\,\mathrm{GeV}$$
$$m(2.0\,\mathrm{GeV}) = 172.86\,\mathrm{GeV}$$

Reconstructed Observables: $m_{b_j l}$ and $m_{b_j W}$

- studied two different observables (for boosted tops)

$$m_{b_j l} = \sqrt{(p_{b_j} + p_\ell)^2}$$
$$m_{b_j W} = \sqrt{(p_{b_j} + p_\ell + p_\nu)^2}$$



- jet distance measures

$$d_{ij} = \min(E_i^{2p}, E_j^{2p}) \frac{1-\cos\theta_{ij}}{1-\cos R} \qquad d_{iB} = E_i^{2p} \qquad p = \{-1, 0, 1\}$$

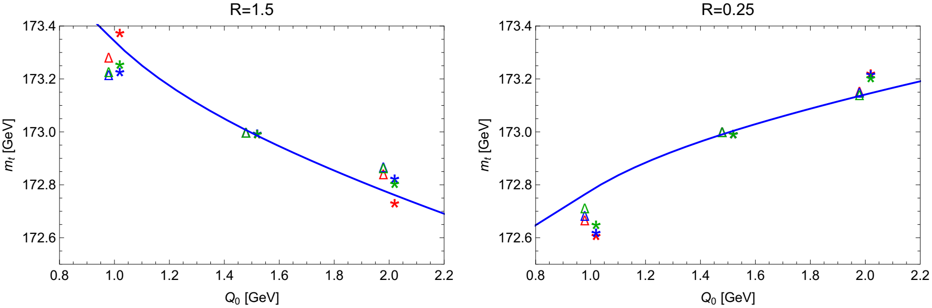
- for $R = \pi/2$: recover full hemisphere mass

$$m_{t,\mathrm{fit}}^{R=\pi/2}(Q_0) = m_{t,\mathrm{fit}}^{R=\pi/2}(Q_0') - \left[4C_F \frac{Q}{m_t} - 2\pi C_F\right] \int_{Q_0'}^{Q_0} \mathrm{d}R \, \frac{\alpha_s(R)}{4\pi}$$

- for $R \sim m_t/Q$: wide angle soft radiation is cut away, only ultra-collinear (boosted with top quark) radiation inside the cone

$$m_{t,\mathrm{fit}}^{R\sim m_t/Q}(Q_0) = m_{t,\mathrm{fit}}^{R\sim m_t/Q}(Q_0') + 2\pi C_F \int_{Q_0'}^{Q_0} \mathrm{d}R \, \frac{\alpha_s(R)}{4\pi}$$

Reconstructed Observables: $m_{b_j l}$ and $m_{b_j W}$: comparison with Herwig

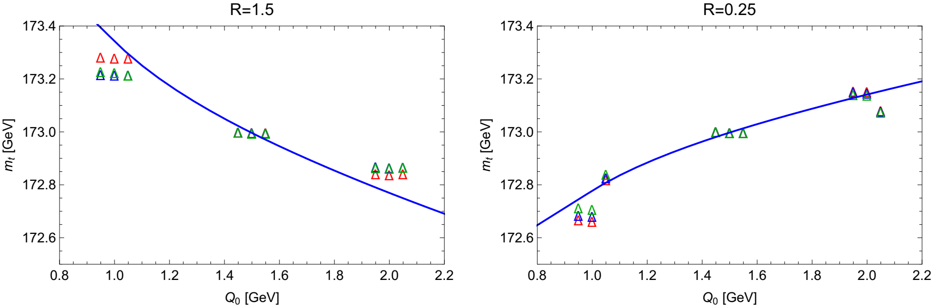


generate distributions for given Q_0 for $m_t = 173\,\mathrm{GeV}$
fit top mass with distribution for reference cutoff $Q_0 = 1.5\,\mathrm{GeV}$

$Q = 700\,\mathrm{GeV} \Rightarrow m_t/Q \sim 0.25$

$m_{b_j W}$	$m_{b_j l}$	
△ $p = -1$	$*$ $p = -1$	anti-k_t type
△ $p = 0$	$*$ $p = 0$	Cambridge-Aachen type
△ $p = +1$	$*$ $p = +1$	k_t type

Reconstructed Observables: $m_{b_j l}$ and $m_{b_j W}$: NLO matched shower

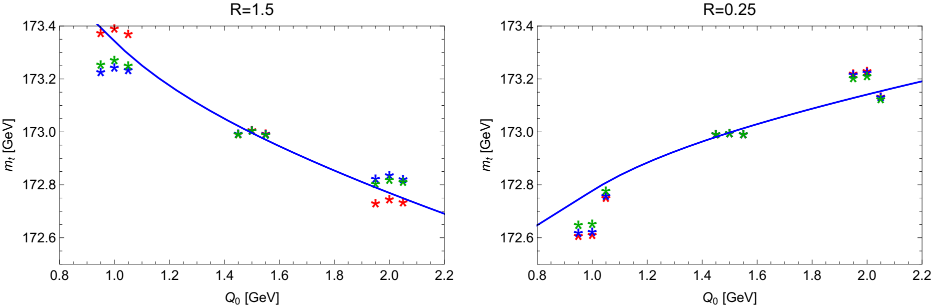


for each cutoff:

left: LO
middle: MC@NLO type
right: POWHEG type

symbols and colors as in previous slide

Reconstructed Observables: $m_{b_j l}$ and $m_{b_j W}$: NLO matched shower



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