KoralW and YFSWW3 – lesson from LEP2 for FCCee

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in collaboration with

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In a nutshell

- KoralW Monte Carlo contains complete process $e^+e^- \to 4fermions$ at the Born level. Radiation: multiphoton ISR YFS-type
- YFSWW3 Monte Carlo generates signal process $e^+e^- \to W^+W^- \to 4fermions$ with up to $\mathcal{O}(\alpha)$ electroweak corrs. in production of W^+W^-. It includes multiphotonic radiation from the production part in the YFS framework.
- KandY = KoralW$\oplus$YFSWW3 combined 4-fermion and $\mathcal{O}(\alpha)$ WW
- KandY: is precision 0.02% at FCCee feasible ????

KoralW ($e^+e^- \to 4$ fermions)

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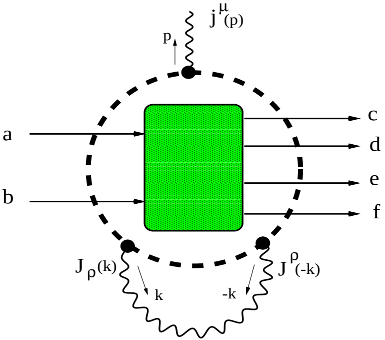
- Multiple soft ISR photons with finite transverse momenta generated according to YFS MC method
- FSR handled by PHOTOS
- t-channel radiation emulated for t-channel dominated final states
- Third order LO QED ISR matrix element
- Fully massive kinematics
- Fully massive four-fermion matrix element ($e^+e^- \to 4$ fermions) generated by Grace system
- Two independent presamplers for the four-fermion phase space
- Anomalous couplings in CC03 graphs ($e^+e^- \to W^+W^- \to 4$ f)
- Coulomb correction (multiplicative)
- Semianalytical routine KORWAN for CC03

Yennie-Frautschi-Suura-1961 Exponentiation



$$j^\mu(k) = ie \sum_{X=a,b,c,d,e,f} Q_X \theta_X \frac{2p_X^\mu}{2p_X k}$$

$$J^\mu(k) = \sum_{X=a,b,c,d,e,f} \hat{J}_X^\mu(k),$$

$$\hat{J}_X^\mu(k) \equiv Q_X \theta_X \frac{2p_X^\mu + k^\mu \theta_X}{k^2 + 2p_X k \theta_X + i\varepsilon}$$

Virtual lines are pair-contracted giving S-factors:

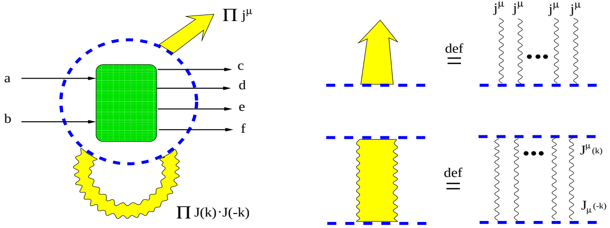
$$S(k) = J(k) \circ J(k) = \sum_{\substack{X=a,b,c,d,e,f \\ Y=a,b,c,d,e,f}} J_X(k) \circ J_Y(k),$$

where Q_X is charge, $\theta = +1, -1$ for initial, final state and

$$J_X(k) \circ J_Y(k) \equiv J_X(k) \cdot J_Y(-k), \text{ for } X \neq Y,$$

$$J_X(k) \circ J_X(k) \equiv J_X(k) \cdot J_X(k).$$

YFS, 6 external legs



$$M^{\mu_1\mu_2\cdots\mu_m}(k_1,k_2,...,k_m) =$$

$$= \mathcal{M}\prod_{l=1}^{m} j^{\mu}(k_l)\ \sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\int\frac{i}{(2\pi)^3}\ \frac{d^4k_i}{k_i^2-\lambda^2+i\varepsilon}\ J^{\mu}(k_i)\circ J_{\mu}(k_i)$$

$$= \mathcal{M}\prod_{l=1}^{m} j^{\mu}(k_l)\ e^{\alpha B_6},$$

$$B_6 = \int\frac{i}{(2\pi)^3}\ \frac{d^4k}{k^2-\lambda^2+i\varepsilon}\ J(k)\circ J(k).$$

KoralW algorithm

$$e^+ + e^- \longrightarrow f_1 + \bar{f}_2 + f_3 + \bar{f}_4 + n\gamma\,,\ (n = 0, 1, \ldots)$$

Write down exact analytical all order master formula with exclusive Yennie-Frautschi-Suura exponentiation:

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{j=1}^{4} \frac{d^3 q_j}{q_j^0} \left\{ \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0} \tilde{S}(\{p\},\{q\},k_i) \Theta\left(\frac{2k_i^0}{\sqrt{s}} - \epsilon \right) \right\} e^{Y(\{p\},\{q\};\epsilon)}$$

$$\times \delta^{(4)}\left(p_1 + p_2 - \sum_{j=1}^{4} q_i - \sum_{j=1}^{n} k_i \right) \times \left[\bar{\beta}_0^{(m)}(\{p\},\{q\}) + \sum_{i=1}^{n} \frac{\bar{\beta}_1^{(m)}(\{p\},\{q\},k_i)}{\tilde{S}(\{p\},\{q\},k_i)} + \ldots \right]$$

where

$\tilde{S}(\{p\},\{q\},k)$ — Soft Photon Radiation Factor

$Y(\{p\},\{q\};\epsilon)$ — YFS FormFactor

$\bar{\beta}_n^{(m)}(\ldots)$ — $\mathcal{O}(\alpha^m)$ YFS Residuals for n Real Photons

Four-fermion matrix el. and EW loop corrs. enter through $\bar{\beta}$'s.

KoralW algorithm II

Simplify master formula until it is analytically integrable, construct crude photonic distrib:

$$\sigma_{crude,1} = \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ \prod_{i=1}^{n} \int \frac{d^3 k_i}{k_i^0} \tilde{S}(\{p\}, k_i) \Theta(k_i^0 - k_\epsilon) \right\} e^{Y(\{p\};\epsilon)} \int \prod_{j=1}^{4} \frac{d^3 q_j}{q_j^0} \sigma_{Born}$$

$$Y(\{p\}, k_\epsilon) = 2\alpha \Re B(p_1, p_2) + \int \frac{d^3 k}{k^0} \tilde{S}(p_1, p_2, k) \theta(k_\epsilon - k^0)$$

Simplify 4-fermion matrix el., integrate analytically down to 1-dim. photonic distrib.:

$$\sigma_{crude,2} = e^{Y(\{p\};\epsilon)} \int_0^{v_{max}} dv \gamma v^{\gamma-1} \rho(v) \sigma_{Born}^{crude}((1-v)s)$$

Generate this distribution and then generate "backwards" step by step all the photonic variables

Generate fermionic final state momenta with the multi-branching algorithm and simplified Born

Undo all the simplifications by means of weights

t-channel dominated radiation in $e^+e^- \to e^+e^- x\bar{x}$

Approximate solution by reweighting:

Real radiation weight: $W_{\mathrm{ECS}} = \prod_{photons} \frac{\tilde{S}_{abCD}(k_i)}{\tilde{S}_{ab}(k_i) + \tilde{S}_{CD}(k_i)}$

Virtual formfactor weight:

$$W_{ECS}^{norm} = \exp\left[-\int_{\epsilon\sqrt{s}}^{\sqrt{s}} \frac{d^3k}{k^0} \tilde{S}_{ab}(k) \frac{\tilde{S}_{aC}(k) + \tilde{S}_{bD}(k) + \tilde{S}_{aD}(k) + \tilde{S}_{bC}(k)}{\tilde{S}_{ab}(k) + \tilde{S}_{CD}(k)} \right]$$

We also correct the normalisation (in Leading-log approximation):

$$W_{ECS}^{LLST} = \exp\left(\frac{3}{4}(\bar{\gamma}_t - \gamma_s)\right), \quad \gamma_s = \frac{2\alpha}{\pi}\left(\log\frac{s}{m_{el}^2} - 1\right)$$

And modify the coupling constant: $W_{Run} = \left(\frac{\alpha(t^+)\alpha(t^-)}{\alpha_{G_\mu}^2}\right)^2$

YFSWW3

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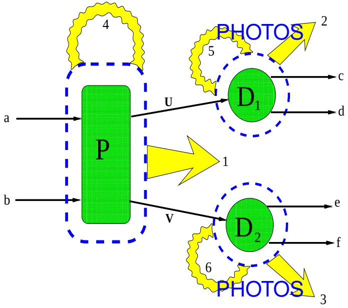
$e^+e^- \to W^+W^- \to 4\,fermions$

- Multiphotonic radiation from the production part in the YFS framework
- Hard ISR corrected to the third order in the LO approximation
- FSR in W-decay handled by PHOTOS $(\star)$
- $\mathcal{O}(\alpha)$ in production based on calculations of J. Fleischer, F. Jegerlehner, K. Kołodziej, M. Zrałek, in R_ξ gauge (ξ independence checked). Two gauge-inv. versions of Leading Pole Approximation implemented in YFSWW3.



$(\star)$ Note that WINHAC Monte Carlo code for $q\bar{q} \to W^\pm \to f_1 f_2$ already includes YFS exponentiation in W-decays with $\mathcal{O}(\alpha)$ EW corrections from SANC and can be implemented in YFSWW3

YFSWW3 Master formula

$$
\begin{aligned}
d\sigma(Y) = \\
&\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{4}\frac{d^3q_i}{q_i^0}\left(\prod_{i=1}^{n}\frac{d^3k_i}{k_i^0}\tilde{S}(p_1,p_2,Q_1,Q_2,k_i)\,\theta(k_i^0-k_\epsilon)\right)\\
&\delta^{(4)}\left(p_1+p_2-\sum_{i=1}^{4}q_i-\sum_{i=1}^{n}k_i\right)\exp\left(Y'(p_1,p_2,Q_1,Q_2,k_\epsilon)\right)\\
&(1+\delta_C)\left(1+\delta_{An}^{TGC}\right)\Bigg\{\bar\beta^{(3)}_{0,ISR}\left(\{p,Q,q\}^{\mathcal{R}}\right)+\Delta\bar\beta^{(1)}_{0,NL}(\{p,Q,q\}^{\mathcal{R}})\\
&+\sum_{i=1}^{n_I}\frac{\bar\beta^{(3)}_{1,ISR}\left(\{p,Q,q\}^{\mathcal{R}},k_i^I\right)}{\tilde{S}(k_i^I)}+\sum_{i=1}^{n}\frac{\Delta\bar\beta^{(1)}_{1,NL}\left(\{p,Q,q\}^{\mathcal{R}},k_i^{\mathcal{R}}\right)}{\tilde{S}(k_i^{\mathcal{R}})}\\
&+\sum_{i>j}^{n_I}\frac{\bar\beta^{(3)}_{2,ISR}\left(\{p,Q,q\}^{\mathcal{R}},k_i^I,k_j^I\right)}{\tilde{S}(k_i^I)\tilde{S}(k_j^I)}+\sum_{i>j>l}^{n_I}\frac{\bar\beta^{(3)}_{3,ISR}\left(\{p,Q,q\}^{\mathcal{R}},k_i^I,k_j^I,k_l^I\right)}{\tilde{S}(k_i^I)\tilde{S}(k_j^I)\tilde{S}(k_l^I)}\Bigg\}
\end{aligned}
$$

YFSWW3 Master formula

Formula similar to KoralW's one, but exponentiation of real and virtual photonic radiation from W-pair makes the $\tilde{S}$-factors and YFS-formfactor Y more complicated:

$$Y'(p_1, p_2, Q_1, Q_2, k_\epsilon) = 2\alpha \Re B(p_1, p_2, Q_1, Q_2) - \frac{\pi\theta(\beta_t - \beta)}{4\beta} + \int \frac{d^3k}{k^0} \tilde{S}(p_1, p_2, Q_1, Q_2, k)\theta(k_\epsilon - k^0)$$

where

$$\tilde{S}(p_1, p_2, Q_1, Q_2, k) = -\frac{\alpha}{4\pi^2}\left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2} - \frac{Q_1}{kQ_1} + \frac{Q_2}{kQ_2}\right)^2$$

$Q_1 = q_1 + q_2$ and $Q_2 = q_3 + q_4$ are W^- and W^+ four-momenta
The complications in $\tilde{S}$ factors and YFS-formfactor are included by reweighting
The inclusion of $\mathcal{O}(\alpha)$ EW corrections is done in $\bar{\beta}$ functions

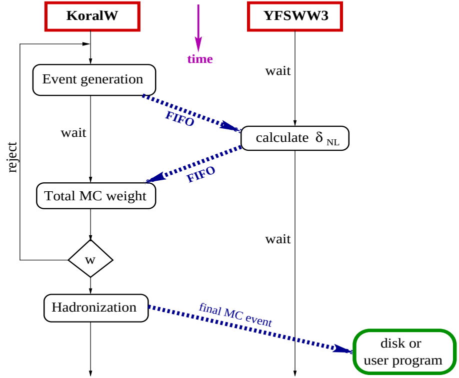
Technical details of YFSWW3

- The YFS scheme, originally derived for fermions, has been extended to bosons because soft photons are “blind” to spin.
- The YFS form factors have been derived for the case of heavy massive particles (W)
- Since photons are radiated from the W-bosons with a finite width, one must do it in a way that respects gauge invariance. It has been done by adding compensating loop corrections that restore gauge invariance
- To avoid double counting of Coulomb effect in the matrix element and in YFS virtual B-function, both arising from the same type of loop corrections, a proper redefinition of the B-function was necessary

Merge of KoralW and YFSWW3 = KandY

Possible because the underlying photonic distribution is the same YFS-ISR in both codes. All other photonic effects are included as weights. So are the $\mathcal{O}(\alpha)$ EW corrs.

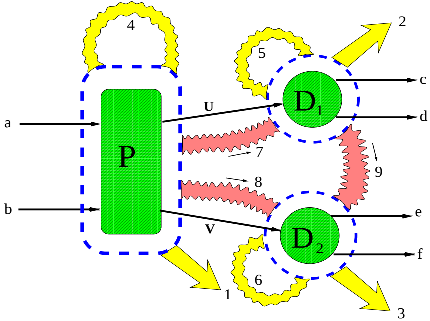
Concurent realization of $\sigma_{K/Y}$ with ”named pipes”



Works effectively as a single MC event generator

New! Exponentiation of interferences in YFS scheme

The YFS Monte Carlo scheme can be extended to exponentiate also the soft non-factorizable corrections (pink lines)



Not implemented in KoralW+YFSWW3 yet!

New extended YFS scheme will exponentiate *all* types of soft photons: Coulomb-like, non-factorizable interferences etc.

It can be a good starting point to improve threshold behavior

For now the Coulomb-like corrections are by hand removed from exponentiation in YFSWW3 – standard Coulomb corr. is added into hard residuals

Technical precision of KoralW and YFSWW3

we focus on the *WW* total cross-section

At LEP2 the target precision level was 0.5%. At FCCee it is 0.02%

- *Technical precision of KoralW* has been estimated at 0.2% based on:
  - internal comparisons of two presamplers,
  - comparisons with other codes,
  - comparisons with semi-analytical results (for CC03 matrix el.).
- *Technical precision of YFSWW3* has been estimated at 0.2% based on:
  - comparison of two technically different implementations of the code: YFSWW-2 and YFSWW-3
  - comparison with KoralW
  - comparison with RacoonWW. The difference between the two implementations (DPA vs. LPA) of the virtual plus soft $\mathcal{O}(\alpha)$ corrections to W-pair production is below 0.01% for the total cross section.

Technical improvement factor of 20 is necessary !!

Numerical instabilities in 4-ferm. matrix el. of KoralW

The ratio m_e^2/s is at WW-tresh. of the order of 10^{-12}. Together with gauge and unitarity cancellations it leads to numerical instability in the matrix element calculations. Example in the $e^-\bar{\nu}_e\nu_e e^+$ channel:

pdg	p_x	p_y	p_z	E
11	-.0000003412784	-.0000016147681	92.98316568221636	92.9831656836208
-12	-.6330003297103	.1098628636344	-11.98532440377251	12.0025314550677
12	.7715733269089	.3584158265593	-70.47596293969590	70.4810977538014
-11	-.1385726559201	-.4682770754257	-10.521878338748095	10.5332051075099

four-fermion weight = 913570469940928

Now modify by hand the last two digits of the p_z components of 4-mom. and rerun the event:

pdg	p_x	p_y	p_z	E
11	-.0000003412784	-.0000016147681	92.98316568221622	92.9831656836208
-12	-.6330003297103	.1098628636344	-11.98532440377231	12.0025314550677
12	.7715733269089	.3584158265593	-70.47596293969576	70.4810977538014
-11	-.1385726559201	-.4682770754257	-10.521878338748115	10.5332051075099

four-fermion weight = 25094

4-ferm. weight (matrix el.) changed by 11 orders of magnitude!

Physical precision of KoralW and YFSWW3

Physical precision of KoralW has been estimated at 2%
for WW physics, based on the size of the $\mathcal{O}(\alpha)$ correction calculated by the YFSWW3 and RacoonWW codes.

Physical precision of KoralW$\oplus$YFSWW3 has been estimated at 0.5%

- The overall agreement of YFSWW3 and RacoonWW, including all physical effects is $\sim$ 0.3% for the total cross section at 200 GeV.
- Complete $\mathcal{O}(\alpha)$ corrs. to $e^+e^- \to 4f$ have been calculated for selected final states [Denner et al. arXiv:0502063] with the conclusion that above the threshold difference DPA/LPA vs. full 4f $\mathcal{O}(\alpha)$ is below 0.5$\%$ of Born. At 161 GeV this difference grows to $\sim$ 2$\%$ of Born.

Physical improvement factor of 50 is necessary !!

Other $\mathcal{O}(\alpha)$ WW Monte Carlo code – RacoonWW

A. Denner, S. Dittmaier, M. Roth, D. Wackeroth

- $e^+e^- \to 4f$ Born-level process and complete real correction $e^+e^- \to 4f\gamma$ in massless approx.
- $\mathcal{O}(\alpha)$EW virtual corrections in Double Pole Approx. based on one-loop calculations for on-shell WW production and decay.
- ISR radiation based on LO QED structure functions to second order with soft photon exponentiation.
- Soft and collinear photon singularities treated in two ways: by dipole subtraction and phase-space slicing
- Coulomb correction for off-shell W's.
- Anomalous Triple and Quartic gauge-boson couplings.
- Multichannel MC algorithm for integration and event generation

Overall precision of RacoonWW: **0.5%** for WW physics

For the record – semianalytical $\mathcal{O}(\alpha)$ result

Complete semianalytical $\mathcal{O}(\alpha)$ EW calculation of $e^+e^- \to 4f$ for selected final states

A. Denner, S. Dittmaier, M. Roth, L.H. Wieders
arXiv:0502063, arXiv:0505042

Overall EW precision of $\mathcal{O}(\alpha)$ semianalytical result

total cross section (161–500 GeV): few $\times$ 0.1%

Based on estimates of missing corrections

- Higher order ISR
- Higher order EW corrs. dominated by $\alpha^2 \log(m_e^2/s) \leqslant 0.1\%$
- Higher order Coulomb effect $\sim 0.2\%$

The QCD effects must be also included: $\mathcal{O}(\alpha_S)$ corrections including matching with Parton Shower, B–E and colour reconnection.

Calculations at the threshold

S. Actis, M. Beneke, P. Falgari, C. Schwinn

arXiv:08070102

See Christian Schwinn's talk for details

- With the method of **unstable particle effective theory** the dominant NNLO corrections to four fermion process ($\mu^- \bar{\nu}_\mu u \bar{d}$) were calculated **near the WW threshold**
- These corrections are related to the Coulomb effect
- They are nick-named $N^{3/2}LO^{EFT}$ because in EFT a different expansion parameter, non-relativistic W velocity $v^2 \sim (s - 4M_W^2)/4M_W^2$, is used to count the strength of particular corrections.
- The effect of $N^{3/2}LO^{EFT}$ corrections on W mass is 3 MeV and on total cross-section **0.2 %** near the threshold, thanks to some cancellations
- The drawback of the EFT method is that it provides **inclusive results only**.

Towards high precision MC for W's

LEP2 case: MC code with $\mathcal{O}(\alpha)$ 4fermion processes was not feasible, so *pragmatic* strategy of expanding in Γ_W/M_W was used and only numerically important terms were included:

Born:	Double Pole	+ Single/No Pole
$\mathcal{O}(\alpha)$:	Double Pole	

FCCee case: MC code with $\mathcal{O}(\alpha^2)$ 4fermion processes – likely calculation out of reach, so use pragmatic approach:

Born:	Double Pole	+ Single/No Pole
$\mathcal{O}(\alpha)$:	Double Pole	+ Single/No Pole
$\mathcal{O}(\alpha^2)$:	Double Pole	

- MC code with $\mathcal{O}(\alpha)$ 4fermion process – feasible, calculations and automated tools already exist
- MC with $\mathcal{O}(\alpha^2)$ separate corrections in WW production and in W decays, i.e. calculations $\mathcal{O}(\alpha^2)$ in Double Pole approx. feasible ??

see talks by: Christian: some parts of calculations already exist

Costas: $\mathcal{O}(\alpha^2)$ automated programs may appear in near future

How big are pragmatically omitted terms?

How big omitted Single Pole $\mathcal{O}(\alpha^2)$ corrs. could be?

Naive math. estimate

$\mathcal{O}(\alpha)_{DP} \sim 30\%\ (9\%)$ *of Born at* 161 (200) *GeV* LO ISR(α) included

$\mathcal{O}(\alpha)_{SP} \sim \mathcal{O}(\alpha)_{4f} - \mathcal{O}(\alpha)_{DP} \sim 8\%(0.8\%)$ *of* $\mathcal{O}(\alpha)_{DP}$ *at* 161 (200) *GeV*

LO ISR(α) included, *cancells* in ratio SP/DP ?

[calculation by Denner et al, no clear separation DP/SP/ISR given]

If whole $\mathcal{O}(\alpha^2) \leqslant 0.2\%$

LO ISR(α^2) excluded [estimate by Denner et al, Actis et al]

then $\mathcal{O}(\alpha^2)_{SP} \sim 0.2\% \times 8\% \sim 0.016\%$ (161 *GeV*)

$\mathcal{O}(\alpha^2)_{SP} \sim 0.2\% \times 0.8\% \sim 0.0016\%$ (200 *GeV*)

Conclusions: Above threshold OK,
At threshold a more detailed discussion may be needed

$\mathcal{O}(\alpha^3)$ EW corrections

- How big omitted $\mathcal{O}(\alpha^3)$ can be?

If $\mathcal{O}(\alpha)_{DP-(ISR\ LO)} \sim 2\%$ Born   Jadach et al hepph/9705429

and $\mathcal{O}(\alpha^2)_{EW-(ISR\ LO)} \leqslant 0.2\%$ Born   estimate by Denner et al, Actis et al

then $\mathcal{O}(\alpha^3)_{EW-(ISR\ LO)} \leqslant 0.02\%$ Born

Conclusion: $\mathcal{O}(\alpha^3)_{EW}$ seems to be just negligible.

How much of the leading $\mathcal{O}(\alpha^3)$ DP corrs. would be picked up by the extended YFS exponentiation ??

Conclusions and outlook

- KoralW+YFSWW3: LEP2 precision is 0.5%.
  Factor of 20 $\div$ 50 improvement is needed for FCCee
- Lesson from LEP2: be pragmatic, split into Double- and Single-Pole, pick only numerically dominant terms:
  - $\mathcal{O}(\alpha^1)$ for $e^- e^+ \to 4f$ must be implemented in MC with explicit split into Double Pole and Single Pole. Calculations exist
  - $\mathcal{O}(\alpha^2)_{DP}$ calculations for the Double-Pole production and decay parts are needed! Feasible?
  - $\mathcal{O}(\alpha^2)_{SP}$ and $\mathcal{O}(\alpha^3)$ seem to be negligible
- More detailed analysis at the threshold may be instrumental
  - EFT methods promising, but for now inclusive results only
  - Non-factorizable soft interferences can be exponentiated within YFS scheme. How much of the higher order corrs. would be reproduced this way?

The overall precision tag $\sim 2 \times 10^{-4}$ feasible (?)

YFSWW3$\oplus$KoralW with new exponentiation
look like a good starting point