

tau event* generators – for precision tests of SM at FCC

* Stress is on the τ decay.

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- (1) There is nothing particular in τ polarization as SM precision test
- (2) It seem to be even simpler than A_{FB} because it is less dependent of \sqrt{s}
- (3) Tau polarization is not a primary datapoint.
- (4) One has to measure spectra of τ decay products. Then \rightarrow
- (5) Beware: measurements unctainties, decay channels cross-contamination.
- (6) QED FSR is sizable, also of non-QED photon production ME.
- (7) That is why subject deserves attention
- (8) τ lepton decay event generators: in-between
carrier of theory predictions and carrier of other experiments data.
- (9) Topic of optimal variables is skipped, and only partly covered in extra slides.

Motivation → pre-LEP and LEP times

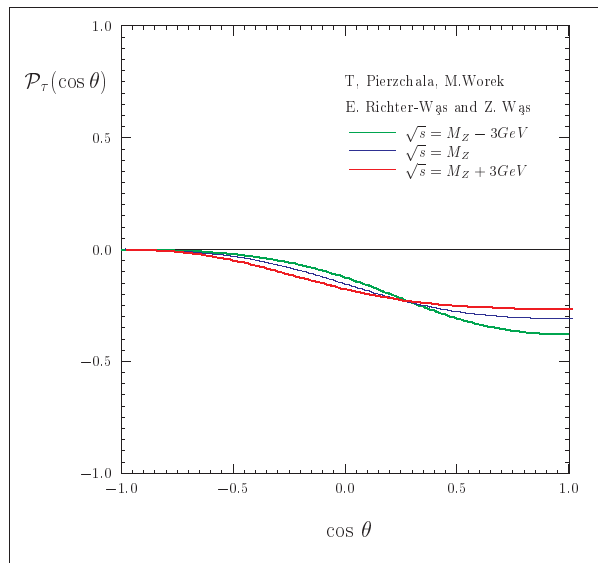
1. F. Boillot and Z. Was, “Uncertainties in τ Polarization Measurement at SLC / LEP and QED / Electroweak Radiative Corrections,” Z. Phys. C **43**, 109 (1989).
2. P. H. Eberhard *et al.*, “The Tau Polarization Measurement At Lep,” In *Geneva 1989, Proceedings, Z physics at LEP 1, vol. 1* 235-265.
3. A. Heister *et al.* [ALEPH Collaboration], “Measurement of the tau polarization at LEP,” Eur. Phys. J. C **20** (2001) 401
4. A. Blondel *et al.*, “Standard Model Theory for the FCC-ee: The Tera-Z,” arXiv:1809.01830 [hep-ph].
5. Then I will comment on evaluation of systematic uncertainties for FCC.
6. Effort to reduce uncertainties substantially may need input from on-going measurements, e.g. from Belle.
7. Presentation of Tauola and Photos.
8. Interfaces to KKMC/KORALW and general purpose universal interface.

Nothing special, all as for A_{FB} . See loop section of 1809.01830

Born cross-section, for $e^+e^- \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$:

$$\frac{d\sigma_{Born}}{d\cos\theta}(s, \cos\theta) = \frac{\pi\alpha^2}{2s} \left\{ (1 + \cos^2\theta) F_0(s) + 2\cos\theta F_1(s) - p[(1 + \cos^2\theta) F_2(s) + 2\cos\theta F_3(s)] \right\}$$

Tests of the TAUOLA universal interface, preliminary results.
The τ lepton polarization as a function of $\cos\theta$ for the $e^+e^- \rightarrow \tau^+\tau^-$ process.



Average sensitive to $1/4 - \sin^2\theta_W^{eff, \tau\tau}$

Asymm. sensitive to $1/4 - \sin^2\theta_W^{eff, ee}$

p - τ polarization, $\cos\theta$ in the frame of τ pair.

Form-factors read:

$$F_0(s) = [q_f^2 q_\ell^2 \cdot \chi_\gamma^2(s) + 2 \cdot \chi_\gamma(s) \text{Re}\chi_Z(s) q_f q_\ell v_f v_\ell + |\chi_Z^2(s)|^2 (v_f^2 + a_f^2)(v_\ell^2 + a_\ell^2)],$$

$$F_1(s) = [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 2v_f a_f 2v_\ell a_\ell],$$

$$F_2(s) = [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 (v_f^2 + a_f^2) 2v_\ell a_\ell],$$

$$F_3(s) = [2\chi_\gamma(s) \text{Re}\chi(s) q_f q_\ell v_f v_\ell + |\chi^2(s)|^2 (v_f^2 + a_f^2) 2v_\ell a_\ell],$$

$$\chi_Z(s) = \frac{G_\mu M_Z^2 \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha_{QED}(0)} \frac{s}{s - M_Z^2 + i\Gamma_Z \cdot M_Z}, \quad \chi_\gamma(s) = 1$$

The following definitions are often used and are very helpful:

$$\mathcal{A}_f = \dots$$

$$\mathcal{A}_{FB}^{pol} = \dots$$

$$\mathcal{P}_f(\cos \theta) =$$

Important building blocks for pseudo-observables. But:

For FCC precision is too high, the above quantities depend on approximations, or require careful definition of reference frames. Too much details for my talk.

This is because of chiralities rather than helicities.

Relation $\left(\frac{2m_\tau}{M_Z}\right)^2 \ll \textit{precision tag}$ does not hold.

For precise FCC regime spin amplitude formalism is anyway required, because of coherent exclusive exponentiation, like in KKMC.

Easy to obtain from spin ampl. where EW corr. can be added

$$\begin{aligned}
 ME_{Born+EW} = & \quad [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu u] \cdot (q_e \cdot q_f) \cdot \Gamma_{V\Pi} \cdot \frac{\chi_\gamma(s)}{s} \\
 & + [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (v_e \cdot v_f \cdot vv_{ef}) + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{\nu}\gamma^\nu \gamma^5 u \cdot (v_e \cdot a_f) \\
 & + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu u \cdot (a_e \cdot v_f) + \bar{u}\gamma^\mu \gamma^5 v g_{\mu\nu} \bar{\nu}\gamma^\nu \gamma^5 u \cdot (a_e \cdot a_f)] \cdot Z_{V\Pi} \cdot \frac{\chi_Z(s)}{s}
 \end{aligned} \tag{2}$$

$$v_e = (2 \cdot T_3^e - 4 \cdot q_e \cdot s_W^2 \cdot K_e(s, t)) / \Delta$$

$$v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s, t)) / \Delta$$

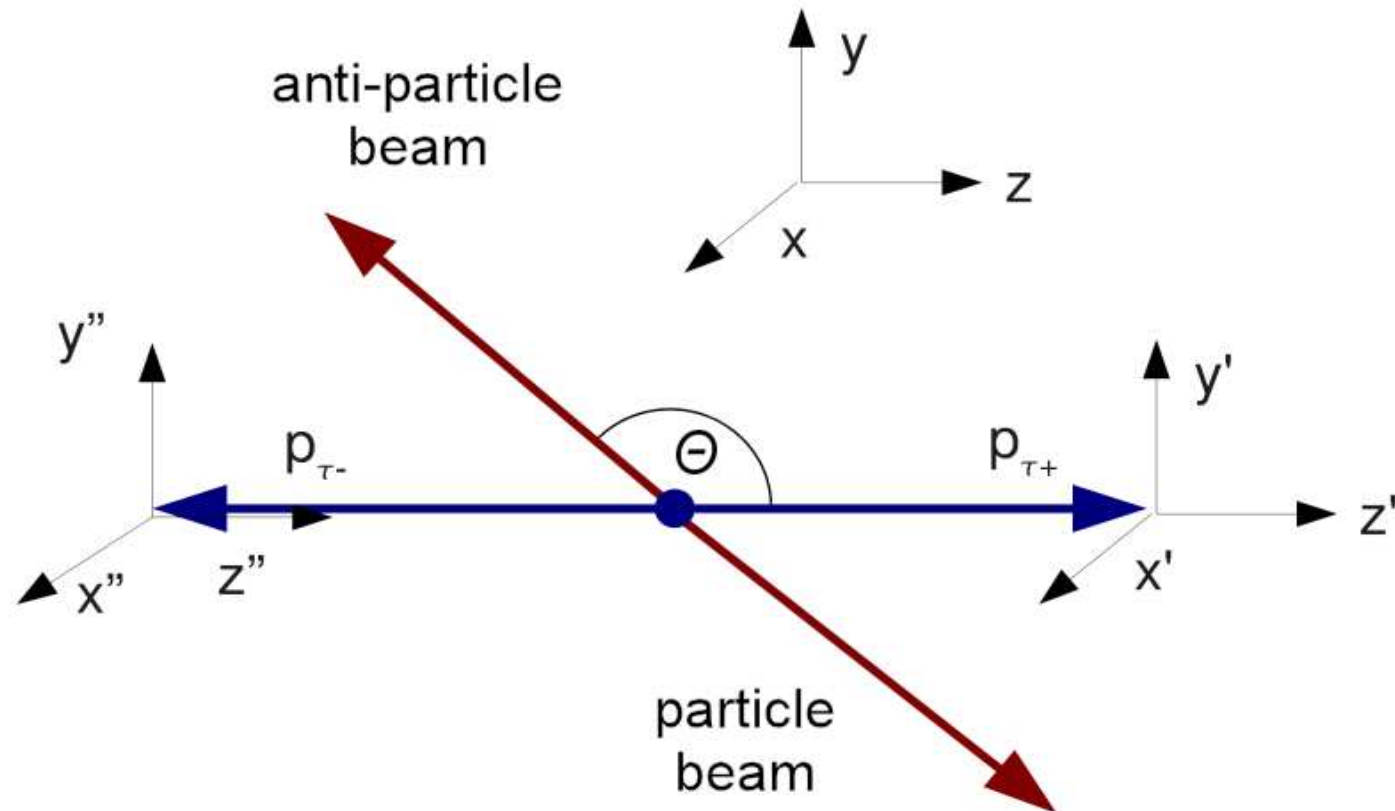
$$a_e = (2 \cdot T_3^e) / \Delta$$

$$a_f = (2 \cdot T_3^f) / \Delta$$

$$\Gamma_{V\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma})}$$

$$\begin{aligned}
 vv_{ef} = & \quad \frac{1}{v_e \cdot v_f} [(2 \cdot T_3^e)(2 \cdot T_3^f) - 4 \cdot q_e \cdot s_W^2 \cdot K_f(s, t) - 4 \cdot q_f \cdot s_W^2 \cdot K_e(s, t) \\
 & + (4 \cdot q_e \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{ef}(s, t)] \frac{1}{\Delta^2}
 \end{aligned}$$

Frame defs. for KKMC, TauSpinner (Higgs CP-weights), ...

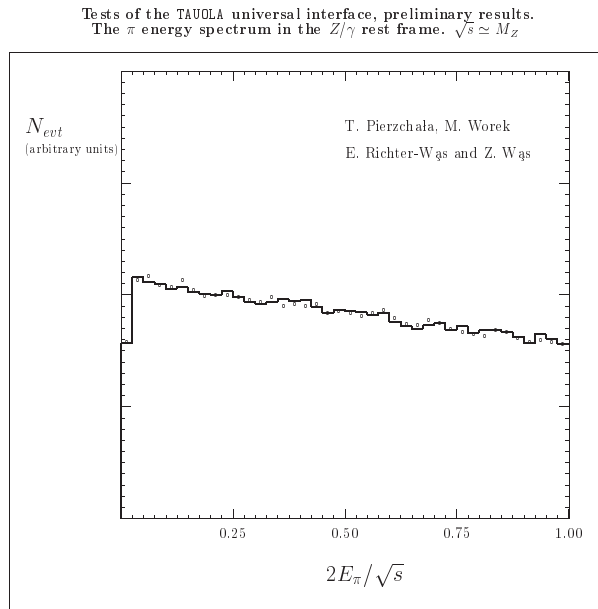


Matching $H/Z/\gamma^*$, τ^+ and τ^- frames essential for transverse spin **and** for CEEX exponentiation of KKMC.

IF TAU SPIN WAS MEASURABLE THAT
WOULD BE THE END OF THE STORY

BUT IT IS NOT ...

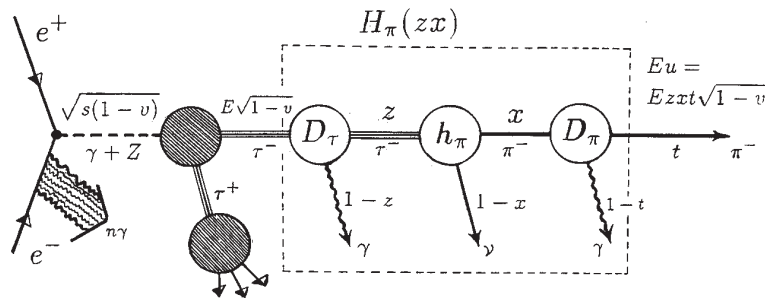
Spectra of τ decay products are only measured...



- Slope of this E_π/E_τ spectrum proportional to longitudinal τ spin, thus to $1/4 - \sin^2 \theta_W^{eff}$.
- It requires precision measurement of E_π and precision simulation of E_τ that is of (QED ISR/FSR/in decay) bremsstrahlung
- **Bremsstrahlung “in decay” is hadronic structure dependednt, even for $\tau \rightarrow \pi\nu$ decay.**
- If left without care, bring sizable systematic ambiguity $\simeq \frac{2\alpha}{\pi}$ on v/a couplings.
- The τ decay channels cross contamination bring ambiguities from $\pi^0 \rightarrow 2\gamma$.

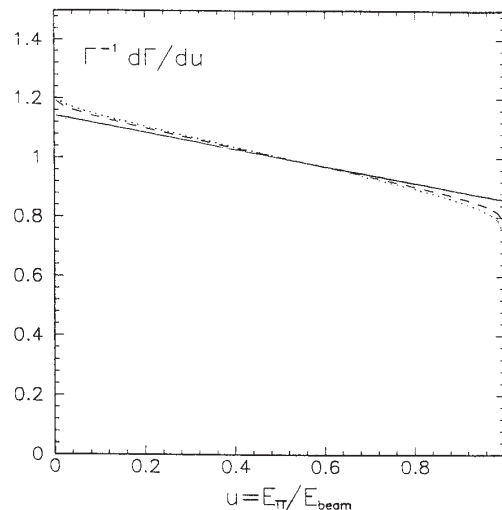
Bremsstrahlung bring extra effects known since late 80's.

Let us consider the τ production and decay process. We shall speak in the following about τ decay into $\pi\nu$ but our discussion will apply equally to $e\nu\bar{\nu}$ and $\mu\nu\bar{\nu}$ decays.

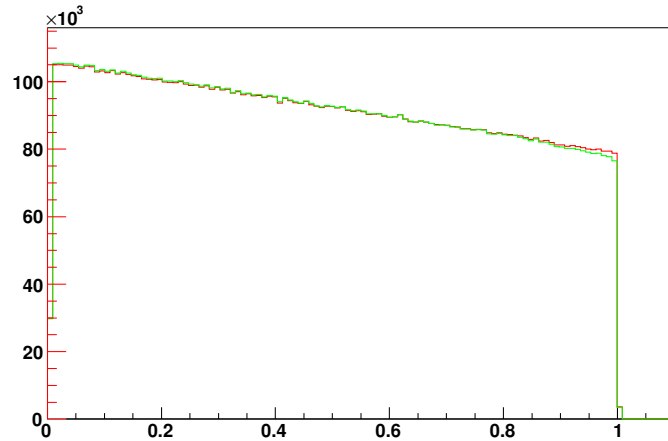


- Effect of the τ polarization manifests itself thanks to $\tau \rightarrow \pi\nu$ matrix element.
- This manifestation is deformed because of QED bremsstrahlung:
- ISR, energy shift by a factor $\sqrt{1-v}$
- FSR, factor D_τ
- Bremsstrahlung in decay, factor D_π
- The first two terms (here in leading logarithm approximation) can be controlled perfectly, thanks to $\mathcal{O}(\alpha^2)$ CEEEX scheme of QED exponentiation used in KKMC.
- The D_π depend in non-leading part on electromagnetic structure of the pion.

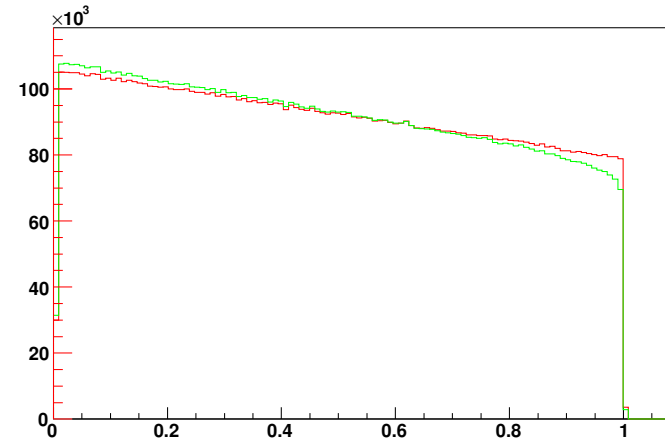
Bremsstrahlung bring extra effects known since late 80's.



- Numerically effects of ISR FSR dominate
- But it is bremsstrahlung in decay, which bring new source of systematic uncertainty.
- The effect, nearly invisible dotted line on the scanned plot (of 1989), is not large ~ 0.03 times the τ polarization.
- Its non dominant part may in principle bring ambiguity of the order of $20 \cdot 10^{-5}$ on $\sin^2 \theta_W^{eff}$.
- This can be minimized thanks to the analysis of $\tau \rightarrow \pi \nu$ data from other experiments (other energies), or from theoretical considerations.



(a) bremsstrahlung from τ^+ decay only



(b) bremsstrahlung from Z , τ^+ and τ^- decays

Figure 1: Bremsstrahlung effects for longitudinal spin observables for the cascade decay: $Z \rightarrow \tau^+ \tau^-$, $\tau^\pm \rightarrow \pi^\pm \nu$. The π^+ energy spectrum in the Z rest-frame is shown. The red line is for bremsstrahlung switched off and green (light grey) when its effect is included. In the left plot, bremsstrahlung is in τ^+ decay only. In the right plot, bremsstrahlung from Z and τ^\pm decays is taken into account. These plots have been prepared using a custom `UserTreeAnalysis` of `MC-TESTER`. They can be recreated with the test located in the `examples/testing/Ztautau` directory, see `examples/testing/README-plots` for technical details. Results are consistent with Fig. 5 of Ref. Eberhard:1989ve i.e. of previous slide.

$20 \cdot 10^{-5}$ ambiguity on $\sin^2 \theta_W^{eff}$ is very pessimistic. 12

1. This estimate relies on size of bremsstrahlung effect and assumption that only leading logarithm terms are properly managed in simulation (e.g. by Photos Monte Carlo). That lead to $20 \cdot 10^{-5}$ on $\sin^2 \theta_W^{eff}$
2. This is known to be too pessimistic since a long time:
R. Decker and M. Finkemeier, "Short and long distance effects in the decay $\tau \rightarrow \pi \tau$ -neutrino (γ)," Nucl. Phys. B **438**, 17 (1995)
3. $\sim 5 \cdot 10^{-5}$ ambiguity on $\sin^2 \theta_W^{eff}$ is straightforward to achieve. Some verification on interplay of detector response and collinear photons should be sufficient. Old and new upgrades as for Photos Monte Carlo, hep-ph/0607019, 1706.05571 are available.
4. No fundamental theory for the emission of photons from scalars (not point like scalars) exist.
5. Verification with τ decay data would be necessary to go beyond $\sim 5 \cdot 10^{-5}$.

1. I have discussed simplest to present case of $\tau^\pm \rightarrow \pi^\pm \nu$
2. The case $\tau^\pm \rightarrow \pi^\pm \pi^0 \nu$ is also attractive.
3. Br. is bigger.
4. Spin sensitivity as good, if π^0 can be reconstructed with sufficient precision.
5. But, τ decay modelling, Use of fits to Belle data, more important.
6. QED corrections in decay smaller, but uncertainty not necessarily.
7. Hopefully success story of ALDO monumental work (hep-ex/0104038 arXiv:1302.3415 Phys.Rept. 532 (2013) 119) can be repeated.
8. See follow up slides, where this decay mode and optimal variables were used.

1. Photos <http://photospp.web.cern.ch/photospp/> Monte Carlo for bremsstrahlung in decays enriched with emission of pairs, and tests:
S. Antropov, A. Arbuzov, R. Sadykov and Z. Was, “Extra lepton pair emission corrections to Drell-Yan processes in PHOTOS and SANC,” Acta Phys. Polon. B **48** (2017) 1469
Option to model matrix elements to improve $\tau \rightarrow \pi\nu(\gamma)$ available for Belle.
2. TauSpinner <http://tauolapp.web.cern.ch/tauolapp/>, algorithm for re-weighting τ production. This program was used to obtain results presented in earlier parts of my talk. References:
 - a T. Przedzinski, E. Richter-Was and Z. Was, “Documentation of TauSpinner algorithms – program for simulating spin effects in tau-lepton production at LHC,” arXiv:1802.05459.
 - b E. Richter-Was and Z. Was, “The TauSpinner approach for electroweak corrections in LHC Z to ll observable,” arXiv:1808.08616
 - c E. Barberio, B. Le, E. Richter-Was, Z. Was, D. Zanzi and J. Zaremba, “Deep learning approach to the Higgs boson CP measurement in $H \rightarrow \tau\tau$ decay and associated systematic,” Phys. Rev. D **96** (2017) no.7, 073002

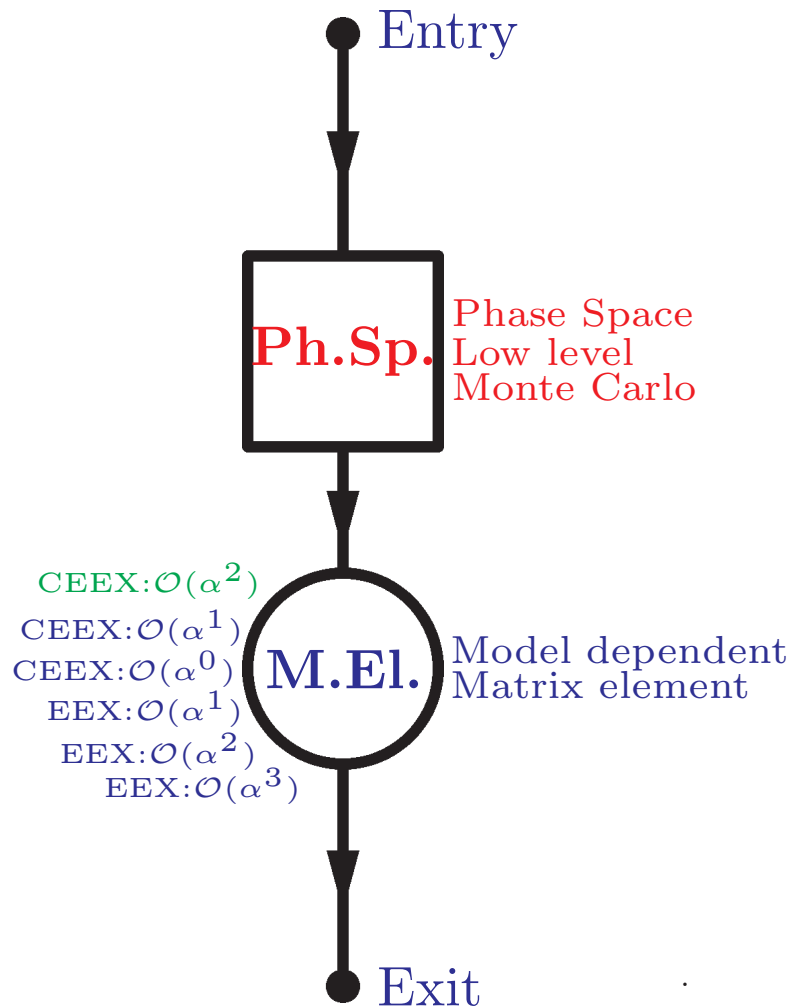
News on the programs: TAUOLA, τ decay Monte Carlo 15

1. TAUOLA for τ decay simulations is upgraded with new hadronic currents. Up to 500 decay channels, which can be manipulated by the user is prepared:
[M. Chrzaszcz, T. Przedzinski, Z. Was and J. Zaremba, Comput. Phys. Commun. **232**, 220 \(2018\) .](#)
2. For archivization purposes initialization of hadronic decay channels is compatible with defaults as BaBar was using.
3. This is mainly prepared for the work in Belle collaboration for new parametrizations of tau decay hadronic currents.
4. **Beneficial to improve on FCC ambiguities for τ decay channels cross-contamination.**
5. Program is prepared to be translated piece after piece into C++, or other language. Whenever a need will arrive.
6. Interface to KKMC is adopted.

- I have addressed the main difference in list of uncertainties for precision tests of SM with τ lepton polarization.
- `Tauola`, `Photos`, `TauSpinner` projects evolve slowly but some new, useful for precision evaluation features were presented.
- Synergy with Belle was pointed. It may be needed to reduce substantially uncertainty in $\sin^2 \theta_W^{eff}$ below $5 \cdot 10^{-5}$, the estimation possible with present day calculations.
- Example of application with the use of Machine Learning techniques is in follow-up slides.
- Also in this case, as for evaluation of systematic ambiguities use of event weights was helpful.
- Essential for future developments will be thus control of systematic errors for multi-dimensional distributions.
- If experimental data should have background subtracted, or if dominant backgrounds will be fitted simultaneously with the signature is technically less important.
- Question of manpower and training as well as motivation of involved people is very important. Competition for talent from other fields can not be ignored.

Follow up slides on Machine Learning application.

“matrix element \times exact phase space” useful for ML too.



- Phase-space Monte Carlo module producing “raw events”.
- Library of models for provides input for “model weight”
- **The scalar from pseudo-scalar distinguished by M.E. weight attributed to each event**
- Ratios define probability that event could be scalar or pseudoscalar Higgs.
- Convenient for ML training sample.

The Higgs boson's parity is imprinted in M.E.

- H/A parity information can be extracted from the correlations between τ^+ and τ^- spin components which are further reflected in correlations between the τ decay products in the plane transverse to the $\tau^+\tau^-$ axes.

- The decay probability

$$\Gamma(H/A \rightarrow \tau^+\tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} \pm s_{\perp}^{\tau^+} s_{\perp}^{\tau^-}$$

is sensitive to the τ^{\pm} polarization vectors s^{τ^-} and s^{τ^+} (defined in their respective rest frames). The symbols \parallel, \perp denote components parallel/transverse to the Higgs boson momentum as seen from the respective τ^{\pm} rest frames.

- This idea and its practical refinements are universal: 'Higgs spin' is blind on Higgs origin. But it is not true for the background DY processes .

Phenomenology Of Mixed Parity: also from M.E.

- Higgs boson Yukawa coupling expressed with the help of the scalar–pseudo-scalar mixing angle ϕ

$$\bar{\tau} N (\cos \phi + i \sin \phi \gamma_5) \tau$$

- *Decay probability for the mixed scalar–pseudo-scalar case*

$$\Gamma(h_{mix} \rightarrow \tau^+ \tau^-) \sim 1 - s_{\parallel}^{\tau^+} s_{\parallel}^{\tau^-} + s_{\perp}^{\tau^+} R(2\phi) s_{\perp}^{\tau^-}$$

- *$R(2\phi)$ – operator for the rotation by angle 2ϕ around the \parallel direction.*

$$R_{11} = R_{22} = \cos 2\phi \quad R_{12} = -R_{21} = \sin 2\phi$$

- *Pure scalar case is reproduced for $\phi = 0$.*
- *For $\phi = \pi/2$ we reproduce the pure pseudo-scalar case.*

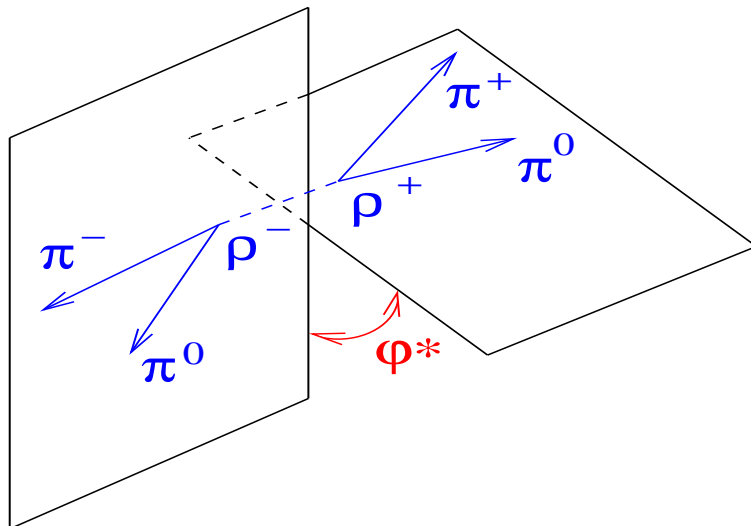
Transverse spin correlations through τ decays

- Case of $\tau \rightarrow \rho\nu_\tau$ decay, $\mathcal{BR}(\tau \rightarrow \rho\nu_\tau) = 25\%$, also M.E. expressed.
- Polarimeter vector h^i is (where q for $\pi^\pm - \pi^0$ and N for ν_τ four momenta).

$$h^i = \mathcal{N} \left(2(q \cdot N)q^i - q^2 N^i \right)$$

$$q \cdot N = (E_{\pi^\pm} - E_{\pi^0})m_\tau$$

- Acoplanarity of ρ^+ and ρ^- decay prod. (in $\rho^+ \rho^-$ r.f.) and events separation.



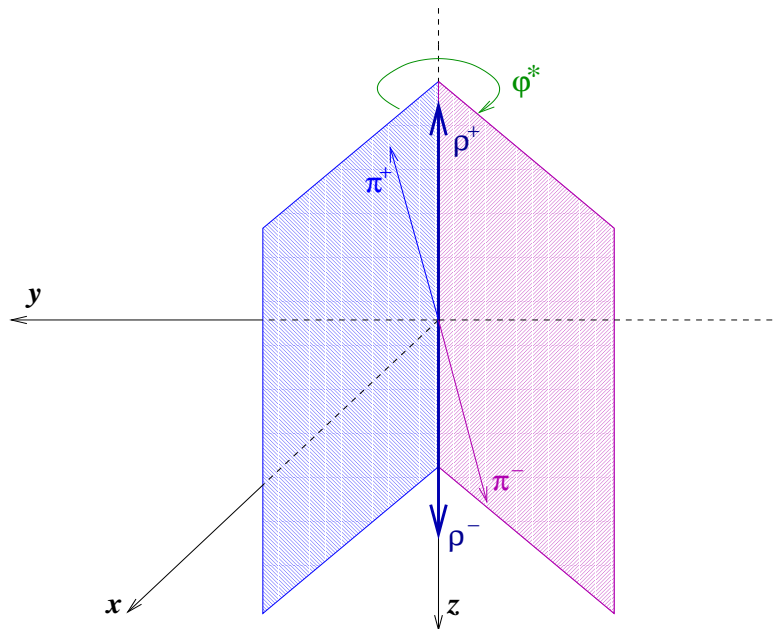
$$y_1 y_2 > 0; \quad y_1 y_2 < 0 \text{ (in } \tau^\pm \text{ r.f.'s)}$$

$$y_1 = \frac{E_{\pi^+} - E_{\pi^0}}{E_{\pi^+} + E_{\pi^0}}; \quad y_2 = \frac{E_{\pi^-} - E_{\pi^0}}{E_{\pi^-} + E_{\pi^0}}$$

Observable of visible products in $H \rightarrow \tau^+ \tau^- \rightarrow \pi^+ \pi^0 \pi^- \pi^0$

Optimal Observable Mixed Scalar–Pseudoscalar Case

- For mixing angle ϕ , transverse component of τ^+ spin polarization vector is correlated with the one of τ^- rotated by angle 2ϕ .
- Acoplanarity $0 < \varphi^* < 2\pi$ is of physical interest, not just $\arccos \mathbf{n}_- \cdot \mathbf{n}_+$.
- Distinguish between the two cases $0 < \varphi^* < \pi$ and $2\pi - \varphi^*$
- If no separation made the parity effect would wash itself out.



Normal to planes: $\mathbf{n}_{\pm} = \mathbf{p}_{\pi^{\pm}} \times \mathbf{p}_{\pi^0}$

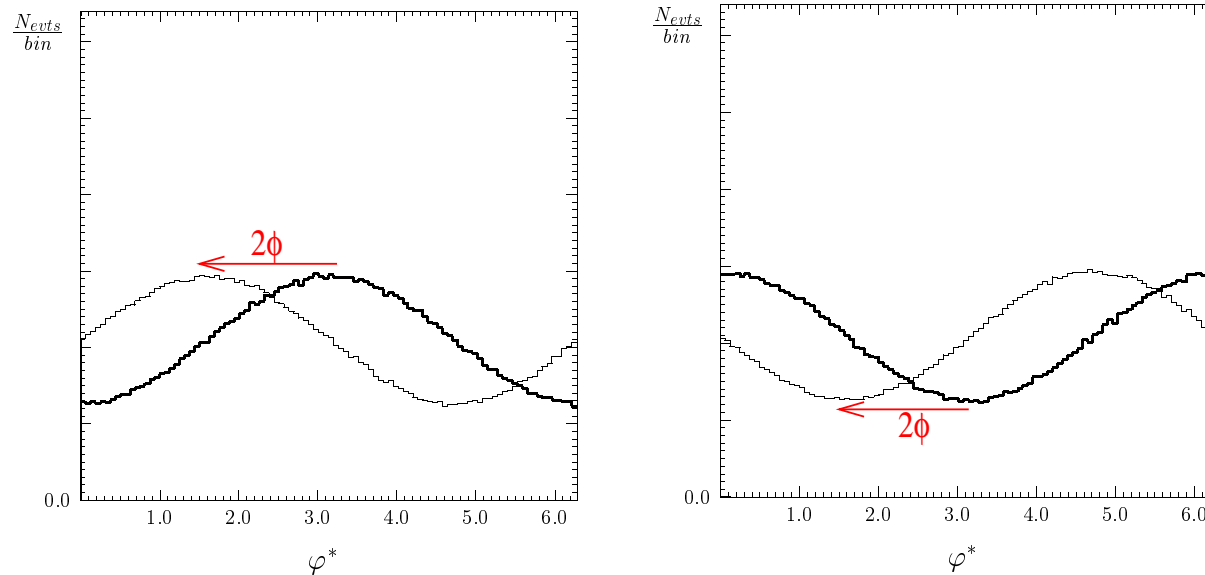
Find the sign of $\mathbf{p}_{\pi^-} \cdot \mathbf{n}_+$

Negative $0 < \varphi^* < \pi$

Otherwise $2\pi - \varphi^*$

Observable of visible products in $H \rightarrow \tau^+ \tau^- \rightarrow \pi^+ \pi^0 \pi^- \pi^0$

Old attempts, at the end 1-dim plot 'easy' to understand



- Only events where the signs of y_1 and y_2 are the same whether calculated using the method without or with the help of the τ impact parameter.
- Tesla-like set-up SIMDET used, [K. Desch](#), A. Imhof, ZW, M. Worek, Phys.Lett. B579 (2004) 157.
- The thick line corresponds to a scalar Higgs boson, the thin line to a mixed one.

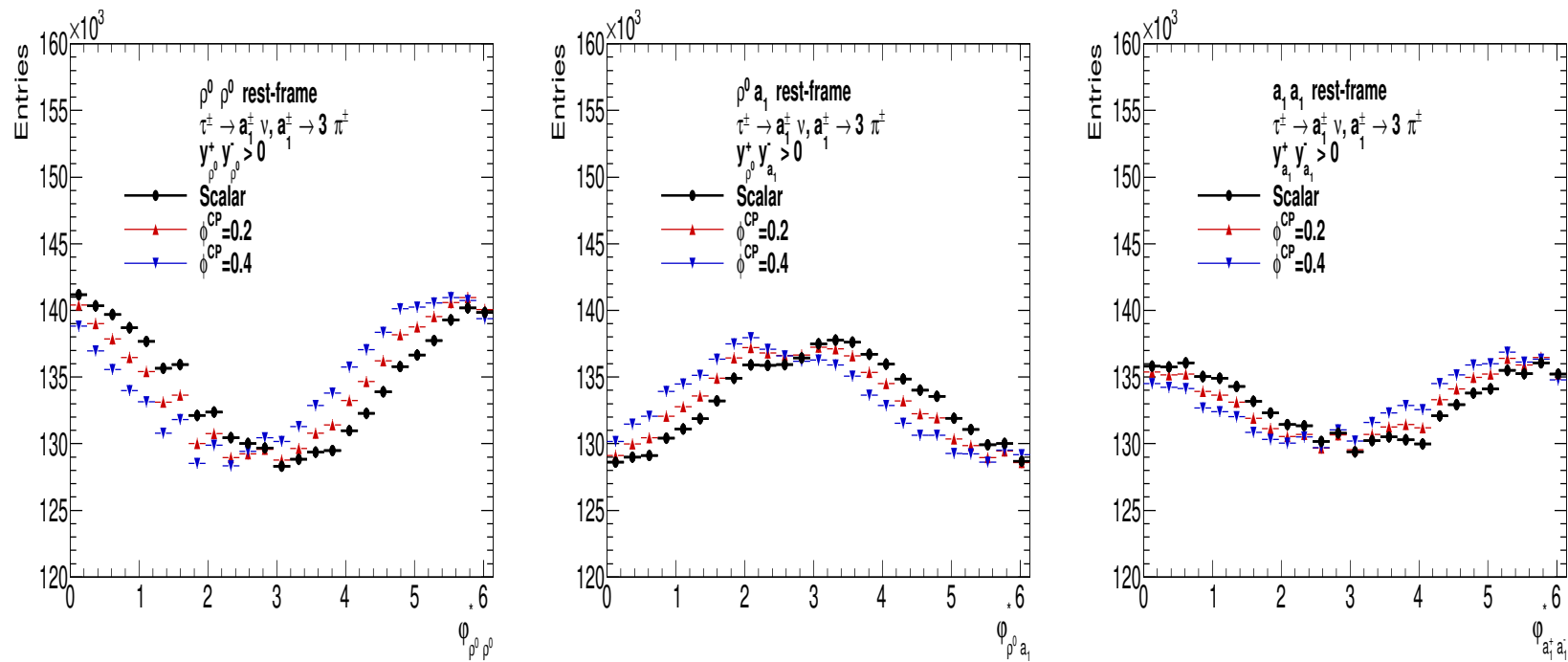
Precision on $\phi \sim 6^\circ$, for $1ab^{-1}$ and 350 GeV CMS.

Motivations:

- $\mathcal{BR}(\tau \rightarrow \rho\nu_\tau) = 25\%$, that mean 6% of $H \rightarrow \tau\tau$. Why not use other decay modes? They all have (in principle) the same sensitivity to spin: J. H. Kuhn, Phys. Rev. D52 (1995) 3128, but in practice ν_τ is not observable and:
- from the π^-, π^0 of $\tau \rightarrow \rho\nu$ we can define one plane for acoplanarity, nice CP sensitive variable
- from the π^-, π^-, π^+ of $\tau \rightarrow 3\pi\nu$ we can define **four** such planes.
- Each plane bring its own y_i variable to avoid cancellations due to properties of τ decay ME.

Many distributions of little content.

Acoplanarity angles of oriented half decay planes: $\varphi_{\rho^0\rho^0}^*$ (left), $\varphi_{a_1\rho^0}^*$ (middle) and $\varphi_{a_1a_1}^*$ (right), for events grouped by the sign of $y_{\rho^0}^+ y_{\rho^0}^-$, $y_{a_1}^+ y_{\rho^0}^-$ and $y_{a_1}^+ y_{a_1}^-$ respectively. Three CP mixing angles $\phi^{CP} = 0.0$ (scalar), 0.2 and 0.4. Note scale, effect on individual plot is so much smaller now. But up to **16 plots like that** have to be measured, correlations understood. Physics model depends on 1 parameter only ϕ^{CP} mixing scalar pseudo-scalar angle, which brings linear shift. **I remained frustrated for 15 years, how to digest...**



Q: what is ML?

A: I can not cover this topic, I assume that basics are widely known, see later in slide some educative references.

Basic principle:

- We start with the training sample where events belong to two classes (and are labeled as such).
- Network is supposed to identify/learn the differences.
- We use the same sample of events for hypothesis A and B. They differ with attributed weights (matrix elements²).
- Once training complete we ask the network to identify if a given even of tested samle is rathe A or B.
- **Of course our sample to identify will never be A or B but will feature some bias** → systematic errors is the main issue.
- Particularly dangerous; covered by new class of technology and math.

- I can not present ML technology. It is a huge domain now.
- I will present results: probabilities that Network will identify event to be scalar, when it was scalar.
- 0.5 means random choice. 1.0 would mean certainty. Anything in-between is already useful.
- Usually more sophisticated area under curve (AUC) is used, because we have four possibilities: Event A identified as A (A as B, B as B, B as A).
- Getting sensitivity was not automatic. I had to:
- boost events to rest frame of all visible objects combined,
- rotate all to set τ^+ primary decay resonance along z axis.
- Quest on what is needed and what is not needed to get results:
- use of classical *optimal variables*,
- what is essential in their definition and what ML can do better.
- How much it may depend on choice of ML classifiers.

Features/var- iables	$\rho^\pm - \rho^\mp$ $\rho^\pm \rightarrow \pi^0 \pi^\pm$	$a_1^\pm - \rho^\mp$ $a_1^\pm \rightarrow \rho^0 \pi^\mp, \rho^0 \rightarrow \pi^+ \pi^-$ $\rho^\mp \rightarrow \pi^0 \pi^\mp$	$a_1^\pm - a_1^\mp$ $a_1^\pm \rightarrow \rho^0 \pi^\pm,$ $\rho^0 \rightarrow \pi^+ \pi^-$
True classification	0.782	0.782	0.782
$\varphi_{i,k}^*$	0.500	0.500	0.500
$\varphi_{i,k}^*$ and y_i, y_k	0.624	0.569	0.536
4-vectors	0.638	0.590	0.557
$\varphi_{i,k}^*$, 4-vectors	0.638	0.594	0.573
$\varphi_{i,k}^*$, y_i, y_k and m_i^2, m_k^2	0.626	0.578	0.548
$\varphi_{i,k}^*$, y_i, y_k, m_i^2, m_k^2 and 4-vectors	0.639	0.596	0.573

Table 1: Average probability p_i that a model predicts correctly event x_i to be of a type A (scalar), with training being performed for separation between type A and B (pseudo-scalar).

$\varphi_{i,k}^*$ and y_i : expert variables In rest frame of all visible, aligned along z . Essential for measure of event distance.

Features				Ideal \pm (stat)	Smearred \pm (stat) \pm (syst)
ϕ^*	4-vec	y_i	m_i		
$a_1 - \rho$ Decays					
✓	✓	✓	✓	0.6035 ± 0.0005	$0.5923 \pm 0.0005 \pm 0.0002$
✓	✓	✓	-	0.5965 ± 0.0005	$0.5889 \pm 0.0005 \pm 0.0002$
✓	✓	-	✓	0.6037 ± 0.0005	$0.5933 \pm 0.0005 \pm 0.0003$
-	✓	-	-	0.5971 ± 0.0005	$0.5892 \pm 0.0005 \pm 0.0002$
✓	✓	-	-	0.5971 ± 0.0005	$0.5893 \pm 0.0005 \pm 0.0002$
✓	-	✓	✓	0.5927 ± 0.0005	$0.5847 \pm 0.0005 \pm 0.0002$
✓	-	✓	-	0.5819 ± 0.0005	$0.5746 \pm 0.0005 \pm 0.0002$
$a_1 - a_1$ Decays					
✓	✓	✓	✓	0.5669 ± 0.0004	$0.5657 \pm 0.0004 \pm 0.0001$
✓	✓	✓	-	0.5596 ± 0.0004	$0.5599 \pm 0.0004 \pm 0.0001$
✓	✓	-	✓	0.5677 ± 0.0004	$0.5661 \pm 0.0004 \pm 0.0001$
-	✓	-	-	0.5654 ± 0.0004	$0.5641 \pm 0.0004 \pm 0.0001$
✓	✓	-	-	0.5623 ± 0.0004	$0.5615 \pm 0.0004 \pm 0.0001$
✓	-	✓	✓	0.5469 ± 0.0004	$0.5466 \pm 0.0004 \pm 0.0001$
✓	-	✓	-	0.5369 ± 0.0004	$0.5374 \pm 0.0004 \pm 0.0001$

Table 2: AUC for NN to separate scalar and pseudo-scalar hypotheses. Inputs with a ✓ used. Results in column "Ideal" - from NNs trained/used with particle-level simulation, in column "Smearred" - from NNs trained/used with smearing. NN trained on smeared samples, for used on exact samples give similar results as "Ideal".

1. We have played with input, and we have observed:
2. Our precious expert variables were not necessary from some point
3. But seemingly trivial overall boosts and rotations were indispensable
4. Only some time later we understood why: network required help to separate longitudinal from transverse degrees of freedom.
5. There was not problem that some variables were then systematically big or small. Such properties were easy for NN to understand. Re scaling was in the system.
6. It does not need to be always like that. It will be application domain dependent.
7. My training case was in a sense easy, we could get help from ME. calculation and adjust variable set accordingly.
8. Only some time after finishing work and after some studies of literature I understood this ML contexts.