Tau Physics at FCC-ee

Mogens Dam
Niels Bohr Institute
Copenhagen

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The Future Circular Colliders

International collaboration to Study Colliders fitting in a new ~100 km infrastructure, in the Geneva region

- Ultimate goal:  
  100 TeV pp-collider: FCC-hh  
  - Defining infrastructure requirements

- Possible first step:  
  e^+e^- collider: FCC-ee  
  - High Lumi, $E_{cm} = 90$-400 GeV

CDR and cost review to appear Q4 2018 for European Strategy Update

Resources:
- First Look at the Physics Case of TLEP
- Physics at the FCC-hh, a 100 TeV pp collider
- 1st FCC Physics Workshop, Jan 2017
- 2nd FCC Physics Workshop, Jan 2018
Outline

a. FCC-ee
b. $\tau$-lepton Properties and Lepton Universality
c. Lepton Flavour Violating Z decays
d. Lepton Flavour Violating $\tau$ decays
FCC-ee

Lepton beams must cross over through the common RF to enter the IP from inside. Only a half of each ring is filled with bunches.

Max. separation of 3(4) rings is about 12 m:

wider tunnel or two tunnels are necessary around the IPs, for ±1.2 km.
Luminosity & Statistics

Enormous statistics. Also for τ-leptons

<table>
<thead>
<tr>
<th>Process</th>
<th>Energy (GeV)</th>
<th>Luminosity ($10^{34}$ cm$^{-2}$s$^{-1}$)</th>
<th>Cross Section ($10^{-1}$ cm$^{-2}$)</th>
<th>Time (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z decays</td>
<td>91</td>
<td>$4.0 - 4.6 \times 10^{36}$</td>
<td>$5 \times 10^{12}$</td>
<td>4</td>
</tr>
<tr>
<td>WW threshold</td>
<td>161</td>
<td>$5.0 - 5.6 \times 10^{36}$</td>
<td>$10^{8}$</td>
<td>1</td>
</tr>
<tr>
<td>ZH threshold</td>
<td>240</td>
<td>$1.4 - 1.7 \times 10^{35}$</td>
<td>$10^{6}$</td>
<td>3</td>
</tr>
<tr>
<td>tt threshold</td>
<td>350</td>
<td>$3.4 - 3.8 \times 10^{34}$</td>
<td>$10^{6}$</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.8 - 3.1 \times 10^{34}$</td>
<td></td>
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</tr>
</tbody>
</table>

Z → τ+τ- 

$\tau$-leptons

1 vs. 3 prongs: $4.2 \times 10^{10}$
3 vs. 3 prong: $3.6 \times 10^{9}$
1 vs. 5 prong: $2.8 \times 10^{8}$
1 vs. 7 prong: < 87,000
1 vs. 9 prong: ?

Enormous statistics.
**FCC-ee Detector Designs**

- **Baseline detector #1: CLD**
  - The CLIC detector is being adapted for FCC-ee
  - Changeover mostly straightforward
    - Smaller beam pipe radius (15 mm)
      - Inner pixel layer closer to IP
    - Smaller B field
      - Larger tracker radius (1.5 → 2.2 m)
    - Lower collision energies
      - Thinner HCAL (4.2 → 3.7 m)
    - Continuous operation (no power pulsing)
      - Increased cooling
      - Thicker pixel/tracker layers
      - Reduced calorimeter granularity

- **Baseline detector #2: IDEA Concept**
  - Main “peculiarities”
    - Extremely light drift chamber
    - Dual readout calorimeter
    - Coil inside calorimeters
### Electroweak

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</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ (MeV)</td>
<td>Lineshape</td>
<td>91187.5 ± 2.1</td>
<td>0.005</td>
<td>&lt; 0.1</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>Lineshape</td>
<td>2495.2 ± 2.3</td>
<td>0.008</td>
<td>&lt; 0.1</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$R_l$</td>
<td>Peak</td>
<td>20.767 ± 0.025</td>
<td>0.00001</td>
<td>&lt; 0.001</td>
<td>Statistics</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Peak</td>
<td>0.21629 ± 0.00066</td>
<td>0.0000003</td>
<td>&lt; 0.00006</td>
<td>$g \rightarrow bb$</td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>Peak</td>
<td>2.984 ± 0.008</td>
<td>0.000004</td>
<td>&lt; 0.004</td>
<td>Lumi meast</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{eff}$</td>
<td>$A_{FB}^{\mu\mu}$ (peak)</td>
<td>0.23148 ± 0.00016</td>
<td>0.0000003</td>
<td>0.0000006</td>
<td>Beam energy</td>
</tr>
<tr>
<td>$1/\alpha_{QED}(m_Z)$</td>
<td>$A_{FB}^{\mu\mu}$ (off-peak)</td>
<td>128.952 ± 0.014</td>
<td>0.004</td>
<td>&lt; 0.004</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$\alpha_s(m_Z)$</td>
<td>$R_l$</td>
<td>0.1190 ± 0.0025</td>
<td>0.000001</td>
<td>0.0001</td>
<td>New Physics</td>
</tr>
<tr>
<td>$m_W$ (MeV)</td>
<td>Threshold scan</td>
<td>80385 ± 15</td>
<td>0.3</td>
<td>&lt; 0.5</td>
<td>EW Corr.</td>
</tr>
<tr>
<td>$N_\nu$</td>
<td>$e^+e^-\rightarrow\gamma Z, Z\rightarrow\nu\nu, l$</td>
<td>2.92 ± 0.05</td>
<td>0.001</td>
<td>&lt; 0.001</td>
<td>?</td>
</tr>
<tr>
<td>$\alpha_s(m_W)$</td>
<td>$B_{had} = (\Gamma_{had}/\Gamma_{tot})_W$</td>
<td>$B_{had} = 67.41 ± 0.27$</td>
<td>0.000018</td>
<td>&lt; 0.0001</td>
<td>CKM Matrix</td>
</tr>
<tr>
<td>$m_{top}$ (MeV)</td>
<td>Threshold scan</td>
<td>173340 ± 760 ± 500</td>
<td>10</td>
<td>20</td>
<td>QCD corr.</td>
</tr>
<tr>
<td>$\Gamma_{top}$ (MeV)</td>
<td>Threshold scan</td>
<td>?</td>
<td>25</td>
<td>?</td>
<td>$\alpha_s(m_Z)$</td>
</tr>
<tr>
<td>$\lambda_{top}$</td>
<td>Threshold scan</td>
<td>$\mu = 1.2 ± 0.4$</td>
<td>15%</td>
<td>?</td>
<td>$\alpha_s(m_Z)$</td>
</tr>
</tbody>
</table>

### Higgs

<table>
<thead>
<tr>
<th>Coupling</th>
<th>HL-LHC</th>
<th>FCC-ee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{HWW}$</td>
<td>3.5%</td>
<td>0.47%</td>
</tr>
<tr>
<td>$g_{HZZ}$</td>
<td>3.5%</td>
<td>0.22%</td>
</tr>
<tr>
<td>$g_{Hbb}$</td>
<td>8.2%</td>
<td>0.68%</td>
</tr>
<tr>
<td>$g_{Hcc}$</td>
<td>SM</td>
<td>1.2%</td>
</tr>
<tr>
<td>$g_{Htt}$</td>
<td>6.5%</td>
<td>0.80%</td>
</tr>
<tr>
<td>$g_{H\mu\mu}$</td>
<td>5.0%</td>
<td>8.6%</td>
</tr>
<tr>
<td>$g_{H\gamma\gamma}$</td>
<td>3.6%</td>
<td>3.8%</td>
</tr>
<tr>
<td>$g_{Hgg}$</td>
<td>3.9%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$g_{HHY}$</td>
<td>~12%</td>
<td>?</td>
</tr>
<tr>
<td>$BR_{EXOT}$</td>
<td>SM</td>
<td>&lt; 1.1%</td>
</tr>
<tr>
<td>$\Gamma_H$</td>
<td>~50%</td>
<td>1.6%</td>
</tr>
<tr>
<td>$g_{Htt}$</td>
<td>4.2%</td>
<td>10% (*)</td>
</tr>
<tr>
<td>$g_{HHH}$</td>
<td>30-50% ?</td>
<td>40% (*)</td>
</tr>
</tbody>
</table>

And on top, we can also do some tau physics...
τ-lepton properties and Lepton Universality

a) Mass
b) Lifetime
c) Leptonic branching fractions
**Tau Mass (i)**

- **Current world average:** $m_\tau = 1776.86 \pm 0.12$ MeV
- **Best in world:** BES3 (threshold scan) $m_\tau = 1776.91 \pm 0.12$ (stat.) $^{+0.10}_{-0.13}$ (syst.) MeV
- **Best at LEP:** OPAL
  - About factor 10 from world’s best
  - Main result from endpoint of distribution of pseudo-mass in $\tau \to 3\pi^\pm(\pi^0\nu_\tau$
  - Dominant systematics:
    - Momentum scale: 0.9 MeV
    - Energy scale: 0.25 MeV (including also $\pi^0$ modes)
    - Dynamics of $\tau$ decay: 0.10 MeV

- **Same method from Belle**
  - Main systematics
    - Beam energy & tracking system calib.: 0.26 MeV
    - Parameterisation of the spectrum edge: 0.18 MeV

  $m_\tau = 1776.61 \pm 0.13$ (stat.) $\pm 0.35$ (syst.) MeV

**Pseudo-mass:**

$$M_{\text{min}} = \sqrt{M_{3\pi}^2 + 2(E_{\text{beam}} - E_{3\pi})(E_{3\pi} - P_{3\pi})}$$
Prospects for FCC-ee:

- 3 prong, 5 prongs, (perhaps even 7 prongs?)
- Statistics $10^5$ times OPAL: $\delta_{\text{stat}} = 0.004$ MeV
- Systematics:
  - At FCC-ee, $E_{\text{BEAM}}$ known to better than 0.1 MeV (~ 1 ppm) from resonant depolarisation
    - Negligible effect on $m_\tau$
  - Likely dominant experimental contribution comes from understanding of the mass scale
    - Use high stats $e^+e^- \rightarrow \mu^+\mu^-$ sample to fix momentum scale. Extrapolate down to momenta typical for $\tau \rightarrow 3\pi$.
    - Use $D^0 \rightarrow K^-\pi^+ / K^-\pi^+\pi^-\pi^-$ and $D^+ \rightarrow K^-\pi^+\pi^+$ to fix mass scale ($m_D$ known to 50 keV)
  - Reduce uncertainty from parametrisation of spectrum edge by use of theoretical spectrum checked against high statistics data
  - Cross checks using 5-prongs
- Suggested overall systematics: $\delta_{\text{syst}} = 0.1$ MeV
  - Could potentially touch current precision but probably no substantial improvement?
Current world average: $\tau_T = 290.3 \pm 0.5 \text{ fs}$

Best in world (Belle): $\tau_T = 290.17 \pm 0.53 \text{ stat} \pm 0.22 \text{ syst} \text{ fs}$

- Large statistics: 711 fb$^{-1}$ @ $\Upsilon(4S)$: $6.3 \times 10^8 \tau^+\tau^-$ events
- Use 3 vs. 3 prong events; reconstruct 2 secondary vertices + primary vertex
- Measure flight distance $\Rightarrow$ proper time
- Dominant systematics: Vertex detector alignment to $\sim 0.25 \mu$m
  - Vertex detector outside 15 mm beam pipe

Best at LEP (DELPHI): $\tau_T = 290.0 \pm 1.4 \text{ stat} \pm 1.0 \text{ syst} \text{ fs}$

- "Low" statistics: $\sim 250,000 \tau^+\tau^-$ events
- Three methods:
  - Decay length ($1v3 + 3v3$), impact parameter difference ($1v1$), miss distance ($1v1$)
  - Lowest systematics from decay length ($1v3$)
    - Dominant systematics: Vertex detector alignment to 7.5 $\mu$m
      - Alignment with data ($q\bar{q}$ events): statistics limited
    - Vertex detector: 7.5 $\mu$m point resolution at 63, 90, and 109 mm
Prospects at FCC-ee

- Small beam-pipe radius (15 mm): Vertex detector with 3 μm space points at 18, 38, 58 mm
  [DELPHI: 7.5 μm @63, 90, 109 mm]

- Impact parametre resolution ~5 times better than at LEP for relevant momenta
  - DELPHI: a = 20 μm, b = 65 μm
  - Belle: a = 19 μm, b = 50 μm
  - FCC-ee: a = 3 μm, b = 15 μm

- Assume same alignment uncertainty as Belle:
  - 0.25 μm, i.e. factor 30 improvement wrt DELPHI.
  - Possible systematics on flight distance method: 1.3/30 fs
    \[ \delta_{\text{syst}} = 0.04 \text{ fs} ; \quad \delta_{\text{stat}} = 0.001 \text{ fs} \]

Further prospects: lifetime can be measured with different systematics in many modes

- 1v1: impact parameter difference, miss distance
- 1v3: flight distance
- 3v3 (4x10⁹ events): flight distance sum
World average

- $B(\tau \to e\nu\nu) = 17.82 \pm 0.05 \%$  \hspace{1cm}  ;  \hspace{1cm}  B(\tau \to \mu\nu\nu) = 17.39 \pm 0.05 \%$

Dominated by ALEPH

- $B(\tau \to e\nu\nu) = 17.837 \pm 0.072_{\text{stat}} \pm 0.036_{\text{syst}} \%$  \hspace{1cm}  ;  \hspace{1cm}  B(\tau \to \mu\nu\nu) = 17.319 \pm 0.070_{\text{stat}} \pm 0.032_{\text{syst}} \%$

Three uncertainty contributions dominant in the Aleph measurement

- Selection efficiency: 0.021 / 0.020 \%
- Non-$\tau^+\tau^-$ background: 0.029 / 0.020 \%
- Particle ID: 0.019 / 0.021 \%
- All of these were limited by statistics: size of test samples, etc.

Prospects at FCC-ee

- Enormous statistics:

  $\delta_{\text{stat}} = 0.0001 \%$

- Systematic uncertainty is hard to (gu)estimate at this point.
  - Depends intimately on the detailed performance of the detector(s)
    - At the end of the day, between LEP experiments, $\delta_{\text{syst}}$ varied by factor $\sim 3$
    - Lesson: **Design your detector with care!**

With the large statistics we will learn a lot. Suggest a factor 10 improvement wrt ALEPH:

$\delta_{\text{syst}} = 0.003 \%$
Summary of Precisions & Lepton Universality

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</thead>
<tbody>
<tr>
<td>$m_\tau$ [MeV]</td>
<td>Threshold / inv. mass endpoint</td>
<td>1776.86 ± 0.12</td>
<td>0.004</td>
<td>0.1</td>
<td>Mass scale</td>
</tr>
<tr>
<td>$\tau_\tau$ [fs]</td>
<td>Flight distance</td>
<td>290.3 ± 0.5 fs</td>
<td>0.001</td>
<td>0.04</td>
<td>Vertex detector alignment</td>
</tr>
<tr>
<td>$B(\tau\rightarrow e\nu\nu)$ [%]</td>
<td>Selection of $\tau^+\tau^-$, identification of final state</td>
<td>17.82 ± 0.05</td>
<td>0.0001</td>
<td>0.003</td>
<td>Efficiency, bkg, Particle ID</td>
</tr>
<tr>
<td>$B(\tau\rightarrow \mu\nu\nu)$ [%]</td>
<td></td>
<td>17.39 ± 0.05</td>
<td>0.0001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Lepton Universality Tests:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measurement</th>
<th>Current precision</th>
<th>FCC-ee precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>g_\mu/g_e</td>
<td>\Gamma_{\tau\rightarrow \mu}/\Gamma_{\tau\rightarrow e}$</td>
<td>1.0018 ± 0.0014</td>
</tr>
<tr>
<td>$</td>
<td>g_\tau/g_\mu</td>
<td>\Gamma_{\tau\rightarrow \tau}/\Gamma_{\mu\rightarrow e}$</td>
<td>1.0030 ± 0.0015</td>
</tr>
</tbody>
</table>

With the precise FCC-ee measurements of lifetime and BRs, $m_\tau$ could become the limiting measurement in the universality test.

$\left(\frac{g_\tau}{g_\mu}\right)^2 \approx \frac{\tau_\mu}{\tau_\tau} \frac{BF(\tau^- \rightarrow e^-\nu_e\nu_\tau)}{\left(\frac{m_\mu}{m_\tau}\right)^5}$

Lepton universality with $m_\tau = 1776.86 \pm 0.12$ MeV
LFV Z decays
Z → e\(\tau\) and Z → \(\mu\tau\)

- **Current limits:**
  - \(\text{Br}(Z \rightarrow e\tau) < 9.8 \times 10^{-6}\) LEP/OPAL \((4 \times 10^6 Z\) decays\)
  - \(\text{Br}(Z \rightarrow \mu\tau) < 12. \times 10^{-6}\) LEP/DELPHI \((4 \times 10^6 Z\) decays\)

- **Method:**
  - Identify **clear tau decay** in one hemisphere
  - Look for **“beam-energy” lepton** (electron or muon) in other hemisphere

- **Limitation:** How to define **“beam-energy” lepton**
  - Unavoidable background from \(\tau \rightarrow e\nu\nu / \tau \rightarrow \mu\nu\nu\) with two (very) soft neutrinos
  - How much background depends on energy/momentum resolution
  - Example DELPHI

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Z.Phys. C73

\[
\frac{dN}{d(p/E_{beam})}
\]

**DELPHI**

- \(\mu\) from \(Z \rightarrow \mu\mu\)
- \(\tau \rightarrow \mu\nu\nu\)
Z → ℓτ - Study of Sensitivity

- Generate very upper part of μ momentum spectrum from τ → μνν decays
  - Luminosity equivalent to 5x10^{12} Z decays
- Inject LFV signal of adjustable strength
  - Here for illustration, Br(Z → τμ) = 10^{-7}, i.e. 500,000 e/μ
- Smear momentum by variable amounts, here 1.8 x 10^{-3}
- Define x > 1 as signal region
- Derive 95% confidence limit on excess in signal region
- Findings:
  - Sensitivity scales linear with momentum resolution
  - FCC-ee detectors have a momentum resolution at p=45.6 GeV of about 1.5 x 10^{-3}
    - Ten times better than for LEP detectors
  - Add contribution from beam-energy spread (0.9 x 10^{-3}). Total: 1.8 x 10^{-3}
- Sensitivity for 5 x 10^{12} Z decays, δp/p = 1.8x10^{-3}, 25% signal and bkg efficiency (clear tau)
  - For Z → τμ, sensitivity down to BRs of 10^{-9}
  - For Z → τe, similar sensitivity 10^{-9}
    - Momentum resolution of electrons tend to be slightly worse than muons due to bremsstrahlung.
      However, downwards smearing is not a major concern.
Z → eμ

- **Current limit:**
  - $7.5 \times 10^{-7}$ LHC/ATLAS (20 fb$^{-1}$; no candidates)
  - $1.7 \times 10^{-6}$ LEP/OPAL (4.0 x 10$^6$ Z decays: no candidates)

- **Clean experimental signature:**
  - Beam energy electron vs. beam energy muon

- **Main experimental challenge:**
  - **Catastrophic bremsstrahlung energy loss** of muon in electromagnetic calorimeter
    - Muon would deposit (nearly) full energy in ECAL: Misidentification $μ → e$
    - NA62: Probability of muon to deposit more than 95% of energy in ECAL: $4 \times 10^{-6}$
    - Possible to reduce by
      - ECAL longitudinal segmentation: Require energy > mip in first few radiation lengths
      - Aggressive veto on HCAL energy deposit and muon chamber hits
  - If $dE/dx$ measurement available, (some) independent $e/μ$ separation at 45.6 GeV
    - Could give handle to determine misidentification probability $P(μ → e)$
    - Notice: ATLAS uses transition radiation as part of electron ID.

- **FCC-ee:**
  - Misidentification from catastrophic energy loss corresponds to limit of about $Br(Z \rightarrow eμ) \approx 10^{-8}$
  - Possibly do $O(10)$ better than that $Br(Z \rightarrow eμ) \sim 10^{-9}$ (probably even $10^{-10}$ with IDEA $dE/dx$)
LFV $\tau$ decays

Signal side

Tag side

Signal side

Tag side
**τ^- → e^-γ, τ^- → μ^-γ**

- **Current limits:**
  - $\text{Br}(\tau^- \rightarrow e^-\gamma) < 3.3 \times 10^{-8}$  
    BaBar, 10.6 GeV; $4.8 \times 10^8 \, e^+e^- \rightarrow \tau^+\tau^- : 1.6 \text{ expected bckg}$
  - $\text{Br}(\tau^- \rightarrow \mu^-\gamma) < 4.4 \times 10^{-8}$  
    3.6 expected bckg

- **Main background:** Radiative events (IRS+FSR), $e^+e^- \rightarrow \tau^+\tau^-\gamma$
  - $\tau \rightarrow \mu\gamma$ faked by combination of $\gamma$ from ISR/FSR and $\mu$ from $\tau \rightarrow \mu\nu\nu$

- **At FCC-ee, with $1.7 \times 10^{11}$ $\tau^+\tau^-$ events, what can be expected?**
  - Boost 4 - 5 times higher than at superKEKB
  - Detector resolutions rather different, especially ECAL
  - Parametrised study of signal and the main background, $e^+e^- \rightarrow \tau^+\tau^-\gamma$, performed
    - See two following pages
  - From this study (assuming a 25% signal and background efficiency), projected BR sensitivity: $2 \times 10^{-9}$
**Generate signal events** with pythia8: $e^+e^- \to Z \to \tau^+\tau^-(\gamma)$, with $\tau^- \to \mu^-\gamma$

In order to de-corrrelate the $E$ and $m$ variables, this mass, $m_{\gamma\mu}$, is in fact the measured mass scaled by measured energy over beam energy:

$$m_{\gamma\mu} = m_{\text{raw}} \times (E_{\gamma\mu}/E_{\text{beam}})$$

Smear with assumed FCC-ee detector resolutions:
- Muon momentum [GeV]
  $$\sigma(p_T)/p_T = 2 \times 10^{-5} \times p_T \oplus 1 \times 10^{-3}$$
- Photon ECAL energy [GeV]
  $$\sigma(E)/E = 0.165/\sqrt{E} \oplus 0.010/E \oplus 0.011$$
- Photon ECAL spatial
  $$\sigma(x) = \sigma(y) = (6/E \oplus 2) \text{ mm}$$

From this, determine **FCC-ee** effective detector resolution for $\tau \to \mu\gamma$

$$\sigma(m_{\gamma\mu}) = 26 \text{ MeV}; \quad \sigma(E_{\gamma\mu}) = 850 \text{ MeV}$$
**τ → μγ Study – The background**

- **Background:** Generate $5 \times 10^8$ events $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-(\gamma) \rightarrow (\mu^+\nu\nu)(\mu^-\nu\nu)(\gamma)$
  - $1 \times 10^9 \tau \rightarrow \mu\nu\nu$ decays corresponding to
    - $5.7 \times 10^9 \tau$ decays from $8.4 \times 10^{10}$ $Z$ decays
  - Study all $\mu$ and $\gamma$ combinations

![Plot showing $E_\gamma - E_\mu$ vs $m_{\mu\gamma}$](image-url)

- $1.7 \times 10^7 \mu\gamma$ combinations
- 6080 $\mu\gamma$ combinations
- 2σ contours
- 500 combinations
- FCC-ee detector resolution
- Flat linear rise
\[ \tau^- \rightarrow \ell^- \ell^+ \ell^- \]

- **Current limits:**
  - All 6 combs. of $e^\pm$, $\mu^\pm$: $\text{Br} \lesssim 2 \times 10^{-8}$ Belle@10.6 GeV; $7.2 \times 10^8 e^+e^- \rightarrow \tau^+\tau^-$: no cand.
  - $\mu^-\mu^+\mu^-$: $\text{Br} < 4.6 \times 10^{-8}$ LHCb 2.0 fb$^{-1}$: background candidates

- **FCC-ee prospects**
  - Expect this search to have very low background, even with FCC-ee like statistics
  - Should be able to have sensitivity down to BRs of $\lesssim 10^{-10}$

- Many more decay modes to search when time comes...
Summary

- With an unrivalled luminosity, the four stage FCC-ee programme foresees the production of $5 \times 10^{12}$ Z decays in its first stage
- A treasure trove for precision measurements and discoveries
- Of most direct relevance to this conference is the production of $1.7 \times 10^{11}$ $\tau^+\tau^-$ pairs:
  - Improved lepton universality test by $O(10)$ or more. Down to $10^{-4}$ level on coupling ratios
    - Substantial improvement in $\tau$ lifetime measurement: $O(300)$ statistical, $O(10)$ systematic
    - Substantial improvement in $\tau$ branching fractions: $O(300)$ statistical, $O(-10)$ systematic
    - Possibly competitive measurement of $\tau$ mass
  - Searches for lepton flavour violating $Z$ decays more sensitive than today by factor $O(10^4)$
    - Sensitivities down to $10^{-9}$
  - Searches for lepton flavour violating $\tau$ decays with sensitivities comparable with recent Belle2 projections (arxiv:1808.10567)
    - $\lesssim 10^{-10}$ (for channels with no background) to few $\times 10^{-9}$
- Plus, of course
  - Tau polarisation measurement for $\sin^2\theta_W$, $\alpha_S$, $\tau$ neutrino mass, etc., etc.
Extra Slides
Scaling of $Z \rightarrow \ell \tau$ sensitivity with #events

With backgrounds $1/\sqrt{N}$ scaling

If no backgrounds $1/N$ scaling
**τ → μγ Study – Check of method**

**Cross check:** Perform similar study at B-factory, \( \sqrt{s} = 10.6 \text{ GeV} \)

- Again \( 5 \times 10^8 \) events \( e^+e^- \rightarrow Z \rightarrow \tau^+\tau^- (\gamma) \rightarrow (\mu^+\nu\nu)(\mu^-\nu\nu)(\gamma) \)

From this study, estimated limit: \( 1.9 \times 10^{-9} \)

Compare to my extrapolation of current BaBar limit: \( \sim 3-4 \times 10^{-9} \)

Agrees within a factor 2

Not too bad