

Summary of precision QCD issues

**11th FCC-ee workshop:
Theory & Experiments**
CERN, 11th Jan. 2019

David d'Enterria

CERN

Precision QCD & FCC-ee physics

- FCC-ee uniquely small EXP uncertainties is key to $\Lambda_{\text{NP}} > 50\text{TeV}$ limits.
- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables.

Precision QCD theory & FCC-ee physics

- FCC-ee uniquely small EXP uncertainties is key to $\Lambda_{NP} > 50\text{TeV}$ limits.
- Precise QCD theory (small intrinsic uncertainty) is required in SM stress tests & BSM searches for most FCC-ee observables.
- ➔ New analytical+numerical concepts & tools needed to compute (faster) H.O. EWK+QCD multi-loop,-legs,-scales corrections for heavy particles (Z,W,H,t):

09:00	Numerical calculations for elliptic Feynman integrals Speaker: Stefan Weinzierl (Universität Mainz) cern7.pdf	25m
09:25	Unsubtractions at NNLO Speaker: German Rodrigo (IFIC CSIC-UV) 2019_01_rodrigo_F...	25m
09:50	NNLO corrections in 4 dimensions Speaker: Roberto Pittau (Universidad de Granada (ES)) talk_pittau.pdf	
10:15	Recent progress on the calculation of two-loop five-point Master Integrals Speaker: Konstantinos Papadopoulos (Nat. Cent. for Sci. Res. Demokritos (GR)) fcc-cern-2019.pdf	
10:40	coffee	
11:10	1-loop Feynman integrals at arbitrary space-time d Speaker: Tord Riemann (University of Silesia (PL)) riemann11FCCee...	
11:35	Analytics from Numerics: 5-Point QCD Amplitudes at Two Loops Speakers: Ben Page (University of Freiburg), Benjamin John Page (Universidad de Granada (ES)) FCC2019.pdf	
12:00	Laporta algorithm for multi-loop vs multi-scale problems Speakers: Mr Johann Usovitsch (Trinity College Dublin), Johann Usovitsch Usovitsch.pdf	25m
12:25	Numerical Multi-loop Calculations: Sector Decomposition & QMC Integration in pySecDec Speakers: Stephen Jones, Stephen Jones (MPI, Munich) sjones_fccee.pdf	20m

[S. Weinzierl]
[G. Rodrigo]
[R. Pittau]
[K. Papadopoulos]
[T. Riemann]
[B. Page]
[J. Usovitsch]
[S. Jones]

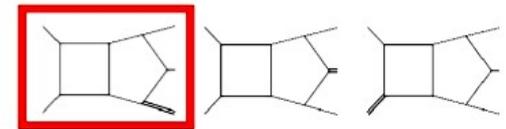


Figure 1. The three planar pentaboxes of the families P_1 (left), P_2 (middle) and P_3 (right) with one external massive leg.

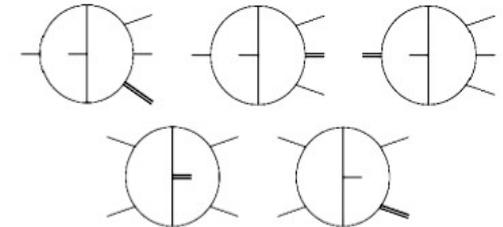
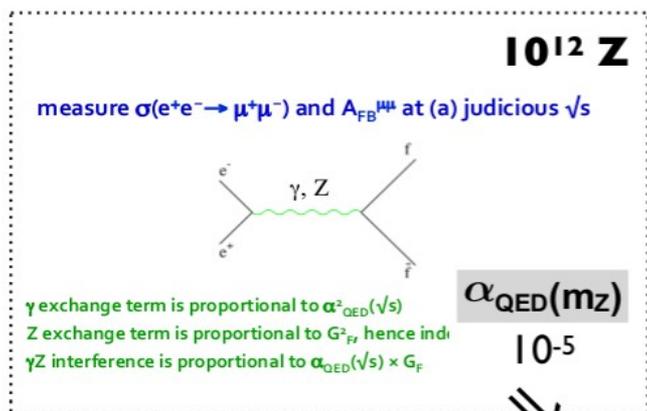


Figure 2. The five non-planar families with one external massive leg.

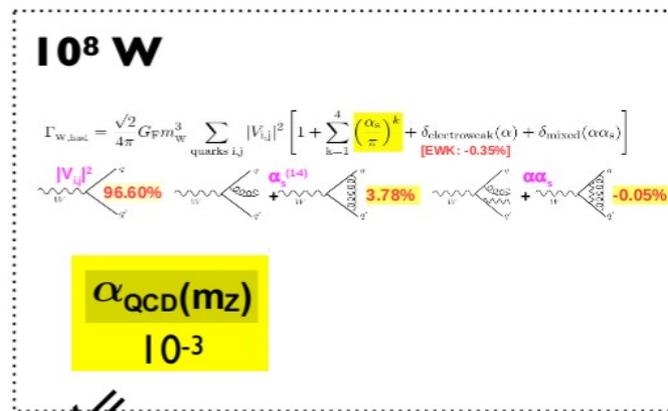
[See J. Gluza next]

Precision QCD coupling & FCC-ee physics

- FCC-ee uniquely small EXP uncertainties is key to $\Lambda_{NP} > 50\text{TeV}$ limits.
- Precise QCD coupling (small parametric uncertainty) is required in SM stress tests & BSM searches for most FCC-ee observables:

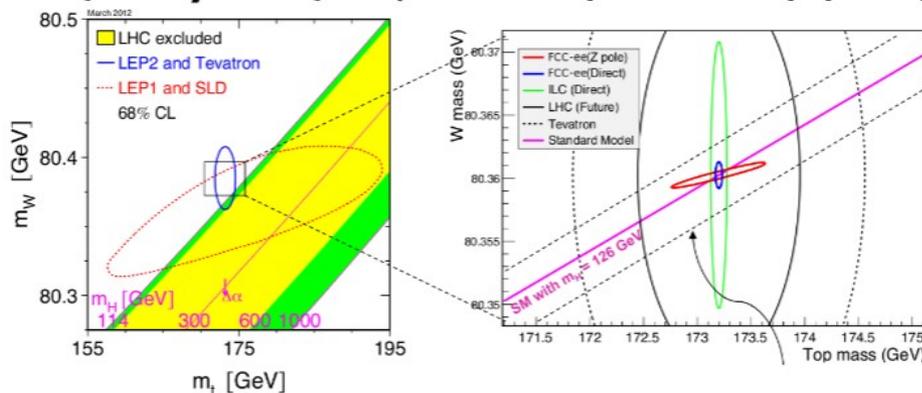


Material from P. Janot



Material from D. d'Enterria

best test of QM beyond QED (and indirect probe of new physics up to $\sim 40\text{TeV}$)



[C. Grojean]

Christophe Grojean

ee-learning

18

CERN, Jan. 9, 2019

Precision QCD & FCC-ee physics

- FCC-ee **uniquely small EXP uncertainties** is key to $\Lambda_{\text{NP}} > 50\text{TeV}$ limits.
- **Precise QCD theory & QCD coupling** are required in SM stress tests & BSM searches for most FCC-ee observables:
 - (1) pQCD Higher-Order (H.O.) theoretical predictions of **EWPO at Z pole**.
 - (2) pQCD H.O. & coupling impact on **Higgs decay widths** ($H \rightarrow gg, \dots$)
 - (3) pQCD H.O. theoretical predictions of **WW cross sections**.
 - (4) pQCD H.O. & coupling & non-pQCD impact on **top properties** (m_t, λ_t, Γ_t).
 - (5) pQCD H.O. & coupling impact on **SM parameters** extractions ($m_{c,b}, \alpha$)
 - (6) pQCD H.O. theoretical predictions of **flavour** physics.
 - (7) pQCD H.O. & coupling impact on **(N)MSSM Higgs**.
- **Unique permille-precision** extractions of the **QCD coupling** at FCC-ee.

Precision QCD & FCC-ee Z boson physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(1) pQCD H.O. theoretical predictions of EWPO at Z pole.

Here we are with Tera-Z physics:

	$\delta\Gamma_Z$ [MeV]	δR_l [10^{-4}]	δR_b [10^{-5}]	$\delta \sin_{eff}^{2,l} \theta$ [10^{-6}]
Present EWPO uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO uncertainties when FCC-ee will start				
EXP-FCC-ee	0.1	10	2 ÷ 6	6
TH-FCC-ee	0.07	7	3	7

[J. Gluza]

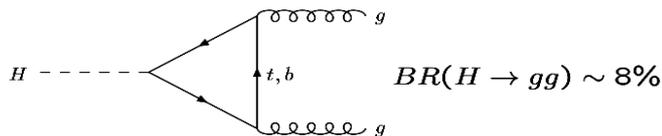
Electroweak 3-loop and the dominant 4-loop EW-QCD corrections will be needed at the start of FCC-ee: $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$

For more details, see Executive Summary and Chapter 2 in arXiv:1809.01830

Precision QCD & FCC-ee Higgs physics

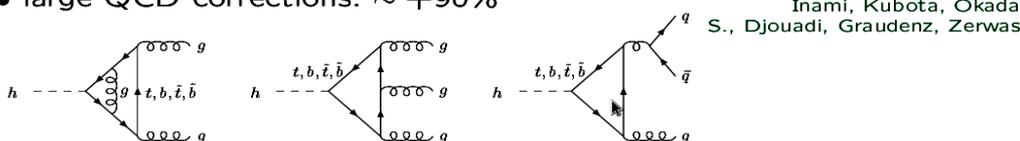
- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(2) pQCD H.O. & coupling impact on Higgs decay widths ($H \rightarrow gg, \dots$)



[M. Spira]

- large QCD corrections: $\sim +90\%$



[S. Heinemeyer]

$$\Gamma(H \rightarrow gg) \approx \Gamma_{LO} \{1 + 0.67 + 0.20 + 0.02\}$$

- 3/4/5-loop corrections ($M_H \ll 4m_t$): $\mathcal{O}(20\%) \Rightarrow$ perturbatively stable
Chetyrkin, Kniehl, Steinhauser
Baikov, Chetyrkin
Herzog, Ruijl, Ueda, Vermaseren, Vogt

Partial Width	QCD	Electroweak	Total	on-shell Higgs
$H \rightarrow b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	NNNNLO / NLO
$H \rightarrow \tau^+\tau^-/\mu^+\mu^-$		$\sim 0.5\%$	$\sim 0.5\%$	NLO
$H \rightarrow gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	NNNLO approx. / NLO
$H \rightarrow \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \rightarrow Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	(N)LO / LO
$H \rightarrow WW/ZZ \rightarrow 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	(N)NLO

Precision QCD theory & FCC-ee W boson physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:
- (3) pQCD H.O. theoretical predictions of **WW cross sections**.

Theoretical methods for WW

EFT expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$ (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

Full NNLO-EW in EFT for total $e^+e^- \rightarrow 4f$ cross section

- Soft $\log \beta$ terms can be adapted from QCD results
- NNLO $\log(m_e/M_W)$ terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections

[C. Schwinn]

(from amplitudes for $e^+e^- \rightarrow W^+W^-$ at threshold: border of current capabilities)

resulting uncertainty from cross-section calculation

$$\Delta\sigma_{\text{hard}}^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\% \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

Full NNLO for $e^+e^- \rightarrow 4f$: completely new methods needed

Precision QCD theory & FCC-ee top physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(4) pQCD H.O. & coupling & non-pQCD impact on top properties (m_t , λ_t , Γ_t):

Observable	Measurement	Current precision	FCC-ee stat.	FCC-ee syst.	Dominant exp. error
m_Z (keV)	Z Lineshape	91187500 ± 2100	5	< 100	Beam energy
Γ_Z (MeV)	Z Lineshape	2495200 ± 2300	8	< 100	Beam energy
R_l (×10 ³)	Z Peak ($\Gamma_{had}/\Gamma_{lep}$)	20767 ± 25	0.06	0.2 – 1	Detector acceptance
R_b (×10 ⁶)	Z Peak (Γ_{bb}/Γ_{had})	216290 ± 660	0.3	< 60	g → bb
N_ν (×10 ³)	Z Peak (σ_{had})	2984 ± 8	0.005	1	Lumi measurement
$\sin^2\theta_W^{eff}$ (×10 ⁶)	$A_{FB}^{l\mu}$ (peak)	231480 ± 160	3	2 – 5	Beam energy
$1/\alpha_{QED}(m_Z)$ (×10 ³)	$A_{FB}^{l\mu}$ (off-peak)	128952 ± 14	4	< 1	Beam energy
$\alpha_s(m_Z)$ (×10 ⁴)	R_l	1196 ± 30	0.1	0.4 – 1.6	Same as R_l
m_W (MeV)	WW Threshold scan	80385 ± 15	0.6	0.3	Beam energy
Γ_W (MeV)	WW Threshold scan	2085 ± 42	1.5	0.3	Beam energy
N_ν (×10 ³)	$e^+e^- \rightarrow \gamma Z, Z \rightarrow \nu\nu, ll$	2920 ± 50	0.8	small	?
$\alpha_s(m_W)$ (×10 ⁴)	$B_l = (\Gamma_{had}/\Gamma_{lep})_W$	1170 ± 420	2	small	CKM Matrix
m_{top} (MeV)	Top Threshold scan	173340 ± 760 ± 500	17	< 40	QCD corr.
Γ_{top} (MeV)	Top Threshold scan	?	45	< 40	QCD corr.
λ_{top}	Top Threshold scan	$\mu = 1.28 \pm 0.25$	0.10	< 0.05	QCD corr.
ttZ couplings	$\sqrt{s} = 365$ GeV	± 30%	0.5 – 1.5%	< 2%	QCD corr

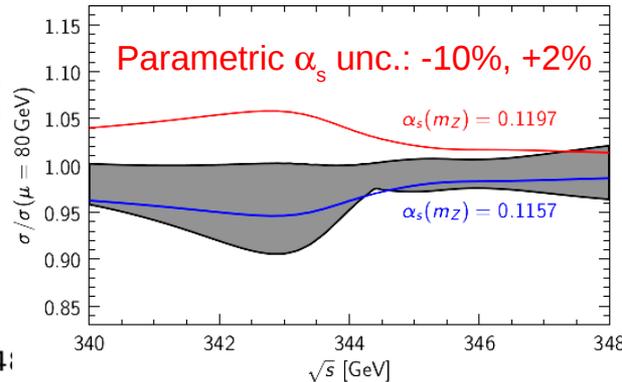
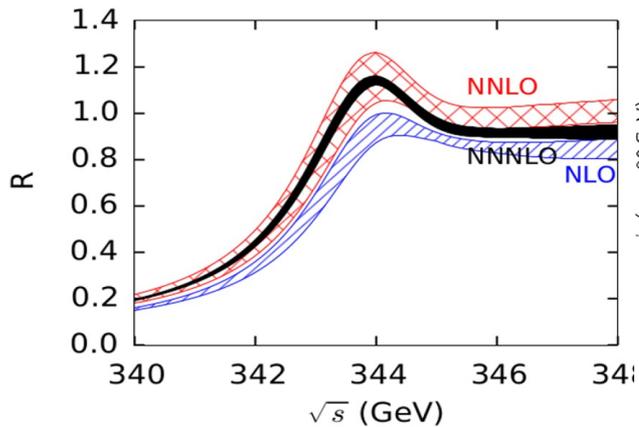
[P. Janot]



Precision QCD theory & FCC-ee top physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(4) pQCD H.O. & coupling & non-pQCD impact on top x-section & mass:



$$\delta^{(5+)} m_t^{OS} = 0.304^{+0.012}_{-0.063} (N) \pm 0.030 (m_{b,c}) \pm 0.009 (\alpha_s) \pm 0.108 \text{ (ambiguity) GeV}$$

input #loops	$m_t^{\overline{MS}}(m_t^{\overline{MS}})$				
	$m_t^{PS} =$	$m_t^{IS} =$	$m_t^{RS} =$	$m_t^{RS'} =$	
1	164.174	164.904	163.702	164.226	
2	163.580	163.727	163.520	163.591	1-2 GeV
3	163.492	163.519	163.490	163.500	$\lesssim 200$ MeV
4	163.508	163.508	163.508	163.508	$\lesssim 20$ MeV

- Apparent convergence at **NNNLO, 3% scale uncertainty**
- Similar convergence at NNLO + NNLL [Hoang, Stahlhofen 2013]

[A. Maier]

[M. Steinhauser]

$$m_t^{MC} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{MC}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

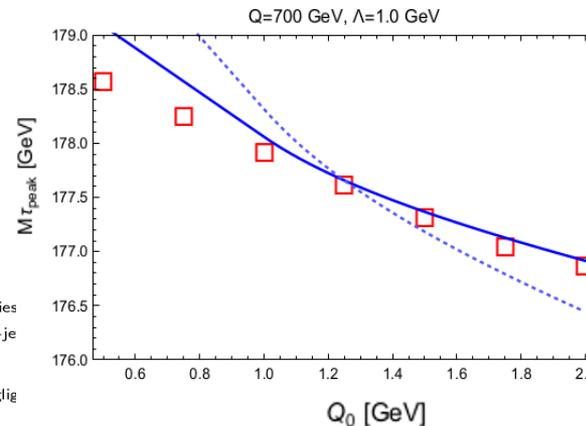
this talk

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...
- should be covered by "MC uncertainty" or better neglig



MSR Mass: Herwig 7: $Q_0 = 1.25$ GeV

$$m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + 0.24 Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow m_t^{\text{MSR}}(Q_0) = m_t^{\text{CB}}(Q_0) + (0.190 \pm 0.070) \text{ GeV}$$

Pole Mass:

$$m_t^{\text{pole}} = m_t^{\text{MSR}}(Q_0) + (0.350 \pm 0.250) \text{ GeV}$$

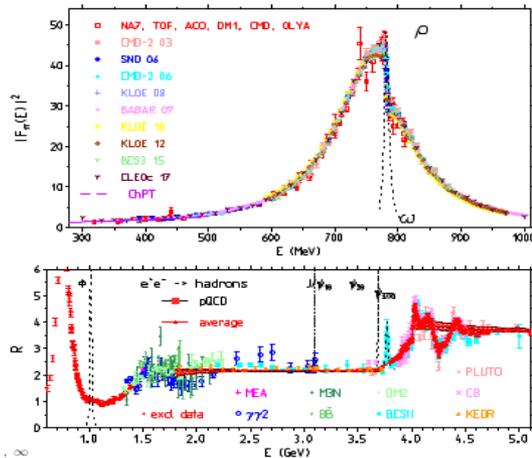
$$\Rightarrow m_t^{\text{pole}} = m_t^{\text{CB}}(Q_0) + (0.540 \pm 0.260) \text{ GeV}$$

[D. Samnitz]

Precision QCD theory & FCC-ee α_{QED} extraction

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(5) pQCD H.O. & coupling impact on α_{QED} extraction: [F. Jegerlehner]



Mandatory pQCD improvements required are:

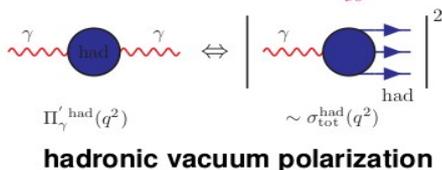
- 4-loop massive pQCD calculation of Adler function; required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements [essentially equivalent to a massive 4-loop calculation of $R(s)$];
- m_c, m_b improvements by sum rule and/or lattice QCD evaluations;
- improved α_s in low Q^2 region above the τ mass.

Theory: (QCD parameters) has to improve by factor 10! $\rightarrow \pm 0.20$

Note: theory-driven standard analyses ($R(s)$ integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

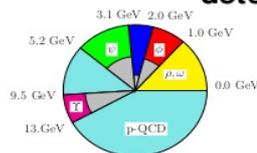
$$\Delta\alpha_{\text{had}}^{(5)}(s) = -\frac{\alpha s}{3\pi} \left(\int_0^{E_{\text{cut}}^2} ds' \frac{R_{\gamma}^{\text{data}}(s')}{s'(s'-s)} + \int_0^{\infty} ds' \frac{R_{\gamma}^{\text{pQCD}}(s')}{s'(s'-s)} \right)$$

where $R_{\gamma}(s) \equiv \frac{\sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$



precision in α :	present	direct	
			1.7×10^{-4}
		Adler	1.2×10^{-4}
future		Adler QCD 0.2%	5.4×10^{-5}
		Adler QCD 0.1%	3.9×10^{-5}
future		via $A_{\text{FB}}^{\mu\mu}$ off Z	3×10^{-5}

- Adler function method is competitive with Patrick Janot's direct near Z pole determination via forward backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$



Precision QCD theory & FCC-ee $m_{c,b}$ extraction

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(5) pQCD H.O. & coupling impact on charm, bottom mass extractions:

$$m_c(\mu) = \frac{1}{2} \left(\frac{\bar{C}_n}{\mathcal{M}_n^{\text{exp}}} \right)^{1/(2n)}$$

latest development: \bar{C}_4 analytically to 4 loops [Marquard, Maier'17]

n	$m_c(3 \text{ GeV})$	exp	α_s	μ	np	total
1	993	7	4	2	1	8
2	982	4	7	5	1	10
3	982	3	8	6	1	10
4	1003	2	5	28	1	29

[M. Steinhauser]

$$m_c(3 \text{ GeV}) = 0.993(8) \text{ GeV}$$

$$m_c(m_c) = 1.279(8) \text{ GeV}$$

[Kühn, Steinhauser, Sturm'07; Chetyrkin, Kühn, Maier, Maierhöfer, Marquard, Steinhauser, Sturm'09'17]

[Uncertainties: $\delta \mathcal{M}_n^{\text{exp}} \mid \alpha_s(M_Z) = 0.1181 \pm 0.0011 \mid \mu = (3 \pm 1) \text{ GeV}$]

$$\mathcal{M}_n^{\text{th}} \stackrel{!}{=} \mathcal{M}_n^{\text{exp}}$$

n	$m_b(10 \text{ GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

$$m_b(10 \text{ GeV}) = 3.610(16) \text{ GeV}$$

$$m_b(m_b) = 4.163(16) \text{ GeV}$$

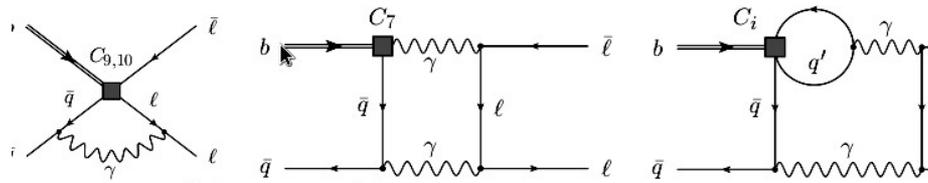
[Kühn, Steinhauser, Sturm'07; Chetyrkin, Kühn, Maier, Maierhöfer, Marquard, Steinhauser, Sturm'10]

[Uncertainties: $\delta \mathcal{M}_n^{\text{exp}} \mid \alpha_s(M_Z) = 0.1189 \pm 0.0020$ ([Bethke'06]: $\delta \alpha_s \times 2$) $\mid \mu = (10 \pm 5) \text{ GeV}$]

Precision QCD theory & FCC-ee flavour physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

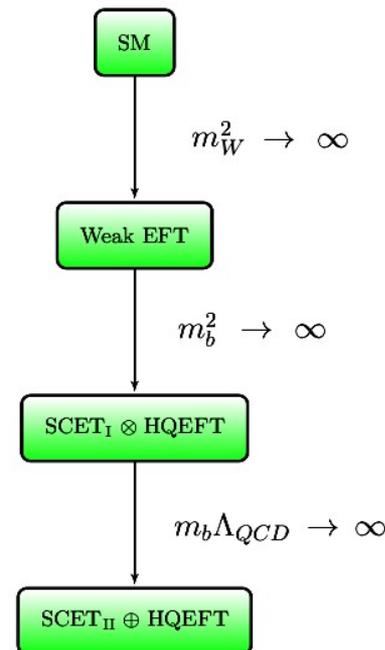
(6) pQCD H.O. theoretical predictions of flavour physics:



Leptonic decay of B_s is a multi-scale problem

- ▶ Electroweak scale m_W
- ▶ Hard scale m_b
- ▶ Hard-collinear scale $\sqrt{m_b \Lambda_{\text{QCD}}}$
- ▶ Soft scale Λ_{QCD}
- ▶ Collinear scale m_μ

We take $\Lambda_{\text{QCD}} \sim m_\mu$ so the soft scale of HQEFT is also a soft scale of SCET_I



[R. Szafron]

Precision QCD & FCC-ee BSM Higgs physics

- Precise QCD theory & QCD coupling are required in SM stress tests & BSM searches for most FCC-ee observables:

(7) pQCD H.O. & coupling impact on (N)MSSM Higgs:

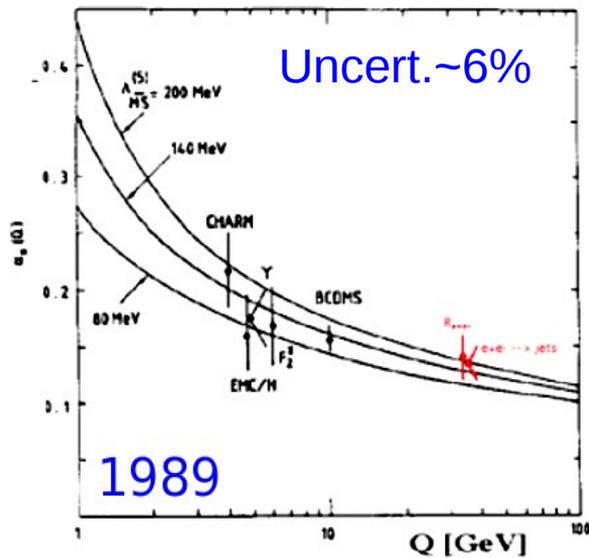
Overall (N)MSSM Higgs decay uncertainty estimates

- $h_i \rightarrow q\bar{q}$: SM-like: SM NNLO QCD, EW NNLO, SUSY 2L: $\sim 5\%$
heavy: as SM-like, Sudakov logs: $\sim 5 - 10\%$
- $h_i \rightarrow \ell\bar{\ell}$: SM-like: $\lesssim 1\%$
heavy: Sudakov logs for very heavy Higgses $\lesssim 10\%$
- $h_i \rightarrow WW^{(*)}, ZZ^{(*)}$: SM-like: $\lesssim 1\%$
heavy: missing 2L (very small width): $\lesssim 50\%$
- $h_i \rightarrow \gamma\gamma, gg, \gamma Z$: $\gamma\gamma$: NNLO QCD, EW: $\lesssim 4\%$
 gg : NNLO QCD, EW: $\lesssim 4\%$
 γZ : NLO: $\sim 5\%$

[S. Heinemeyer]

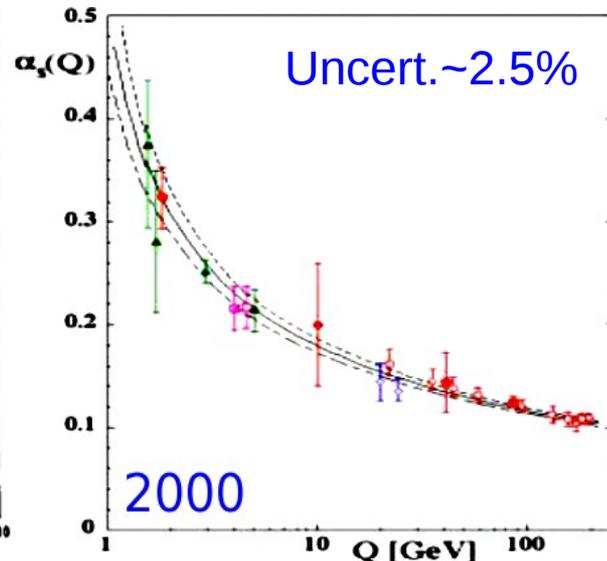
QCD coupling α_s

- ➔ Determines **strength of the strong interaction** between quarks & gluons.
- ➔ **Single free parameter in QCD** in the $m_q \rightarrow 0$ limit.
- ➔ Determined at a ref. scale ($Q=m_Z$), decreases as $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$, $\Lambda \sim 0.2$ GeV



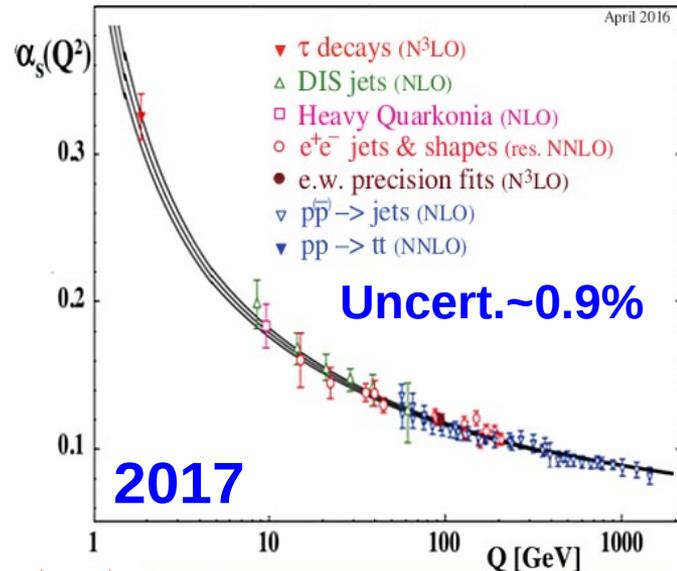
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

S. B., J. Phys. G 26, 2000



$$\alpha_s(M_Z) = 0.1181 \pm 0.0011 \text{ (NNLO)}$$

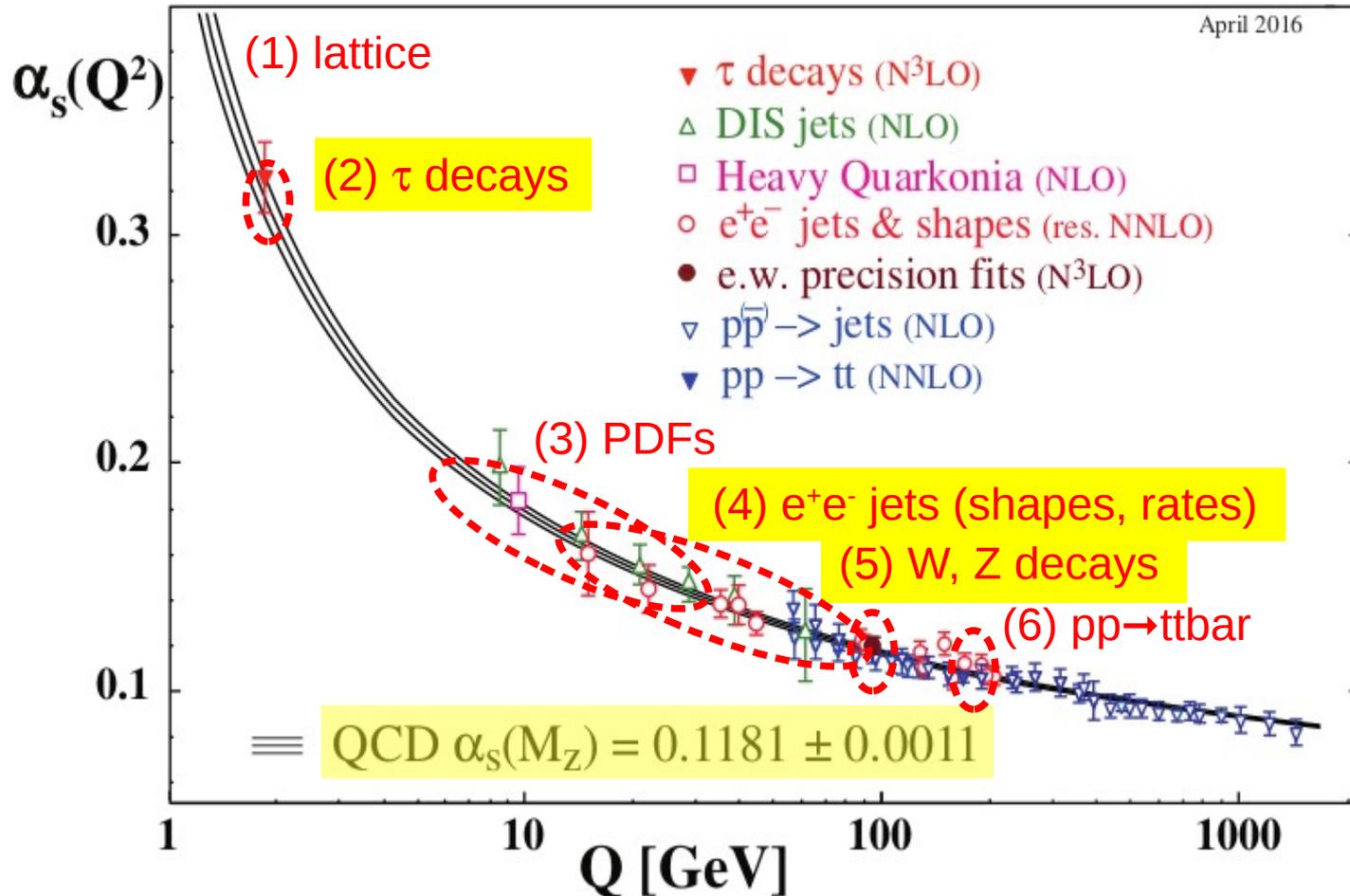
- ➔ **Least precisely known** of all interaction **couplings** !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

α_s determination at FCC-ee

- Determined today by comparing 6 experimental observables to pQCD NNLO, N³LO predictions, plus global average at the Z pole scale:

[Bethke/Dissertori/Salam]



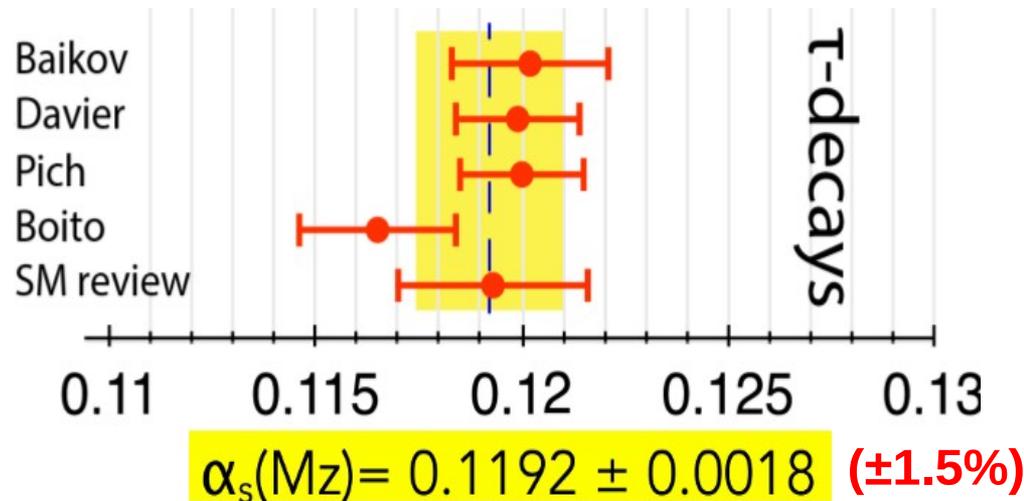
α_s from hadronic τ -lepton decays

➔ Computed at **N³LO**: $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\text{np}})$

➔ Experimentally: $R_{\tau, \text{exp}} = 3.4697 \pm 0.0080$ ($\pm 0.23\%$)

➔ Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections (note: $(\Lambda/m_\tau)^2 \sim 2\%$), yield different results.

Uncertainty slightly increased:
2013 ($\pm 1.3\%$) \rightarrow 2017 ($\pm 1.5\%$)



➔ Future prospects:

- Better TH understanding of FOPT vs CIPT differences.
- Better EXP spectral functions needed (high stats & better precision): B-factories (BELLE-II?)
- FCC-ee: High-stats, $\mathcal{O}(10^{11})$, from $Z \rightarrow \tau\tau$:

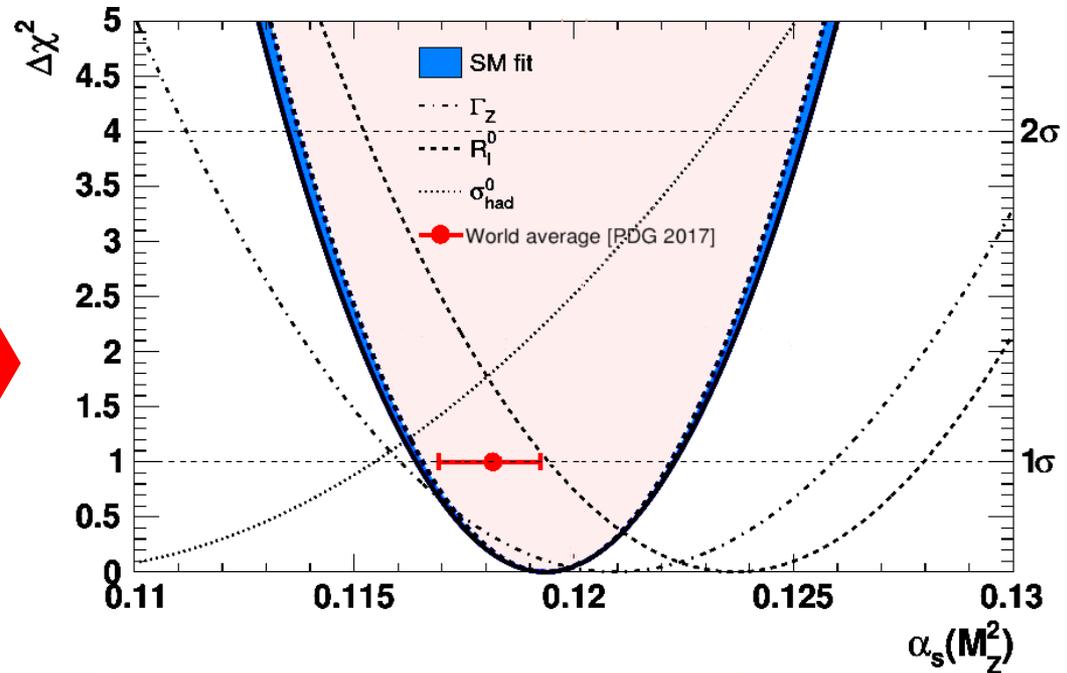
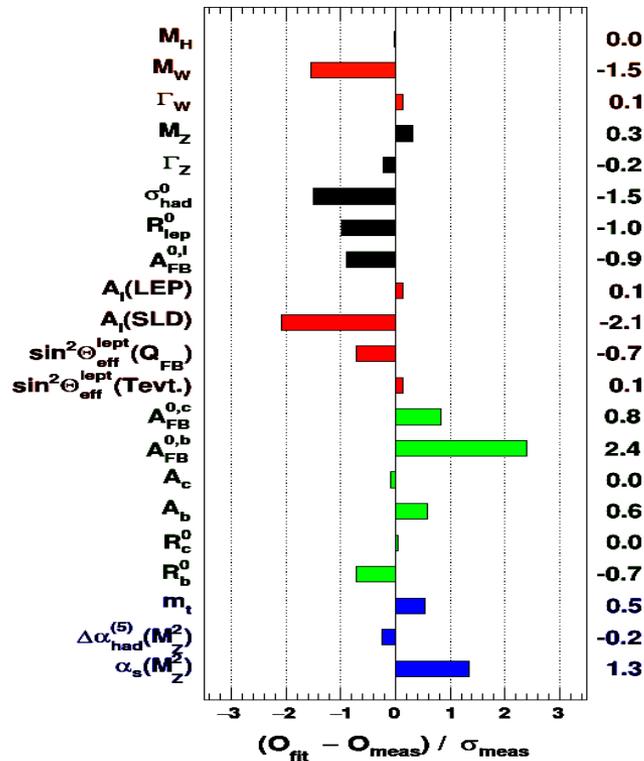
$\delta\alpha_s/\alpha_s < 1\%$

α_s from hadronic Z decays (today)

- ▶ Computed at **N³LO**: $R_l^0 \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_m + \delta_{\text{np}})$
- ▶ **LEP**: Extraction from fits to 3 Z-peak pseudo-observables:

$$R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}, \quad \sigma_{\text{had}}^0 = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}, \quad \sigma_\ell^0 = \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2} \quad \Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV } (\pm 0.1\%)$$

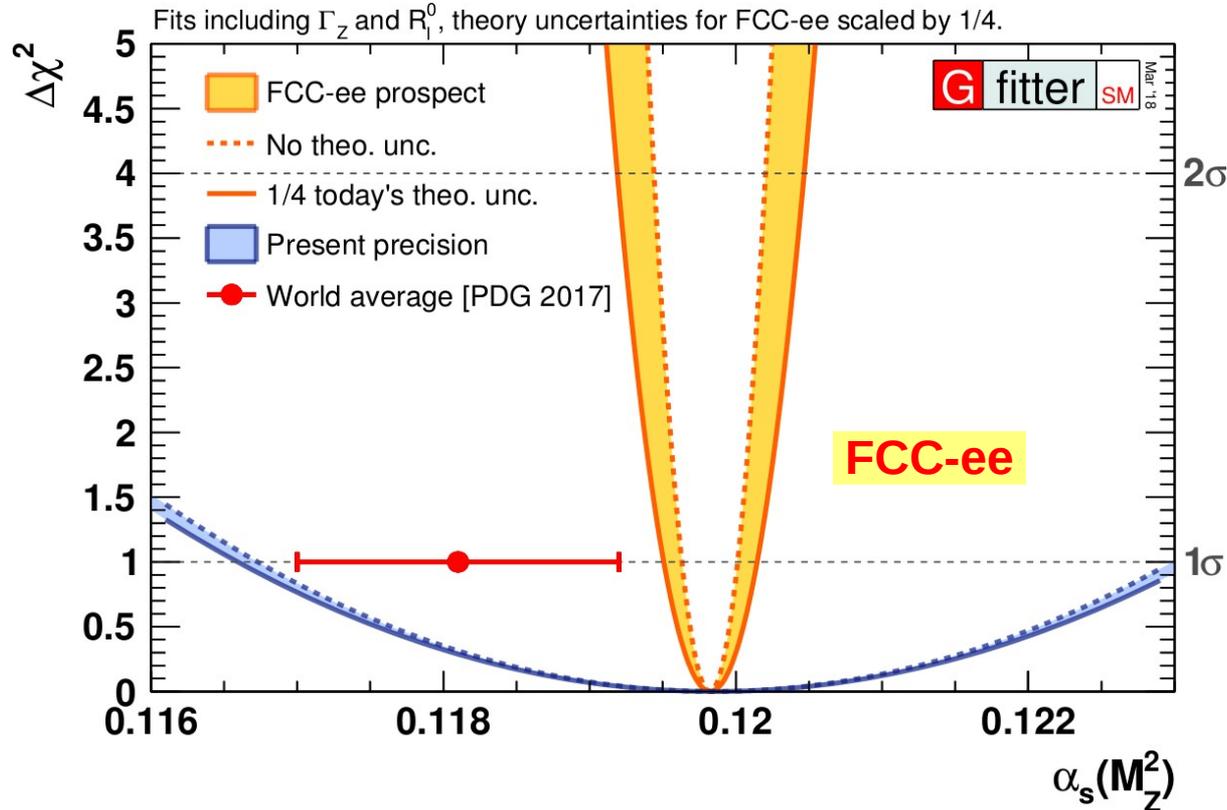
After Higgs discovery, α_s can be also directly determined from full fit of SM:



$$\alpha_s(M_Z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

α_s from hadronic Z decays (FCC-ee)

- ➔ Computed at **N³LO**: $R_l^0 \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)) + \delta_m + \delta_{\text{np}}$
- ➔ **FCC-ee**: Extraction from fits to 3 Z-peak pseudo-observables.



- **Huge Z stats** ($\times 10^5$ LEP) will lead to: $\delta\alpha_s/\alpha_s < 0.2\%$
- Full SM-fit extraction: Parallel reduction of parametric ($\sin^2\theta_{\text{eff}}, m_W, m_{\text{top}}$) TH uncertainties.

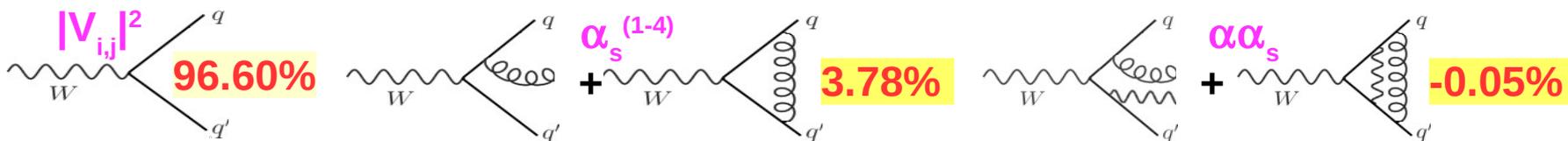
α_s from hadronic W decays

[D.d'E, M.Srebre, PLB763 (2016) 465]

- Width (BR) known at N³LO (NNLO). Small sensitivity to α_s (beyond Born)

$$\Gamma_{W,\text{had}} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\text{quarks } i,j} |V_{i,j}|^2 \left[1 + \sum_{k=1}^4 \left(\frac{\alpha_s}{\pi} \right)^k + \delta_{\text{electroweak}}(\alpha) + \delta_{\text{mixed}}(\alpha\alpha_s) \right]$$

[EWK: -0.35%]



- TH improvements: finite quark-mass effects included (LO), updated PDG parameters, careful evaluation of parametric (V_{ij} , m_W) & theoretical uncert.

- Calculation dominated by $\pm 1.5\%$ parametric (mostly V_{cs}) uncertainty:

$$\Gamma_W (\text{MeV}) = 1428.67 \pm 22.40_{(\text{par})} \pm 0.04_{(\text{th})} \quad (\text{exp. CKM})$$

$$1411.40 \pm 0.96_{(\text{par})} \pm 0.04_{(\text{th})} \quad (\text{CKM}=1)$$

$$\text{BR}_W = \Gamma_W / \Gamma_{\text{tot}} = 0.6820 \pm 0.0110_{(\text{par})} \pm 0.0002_{(\text{th})} \quad (\text{exp. CKM})$$

$$0.6742 \pm 0.0001_{(\text{par})} \pm 0.0002_{(\text{th})} \quad (\text{CKM}=1)$$

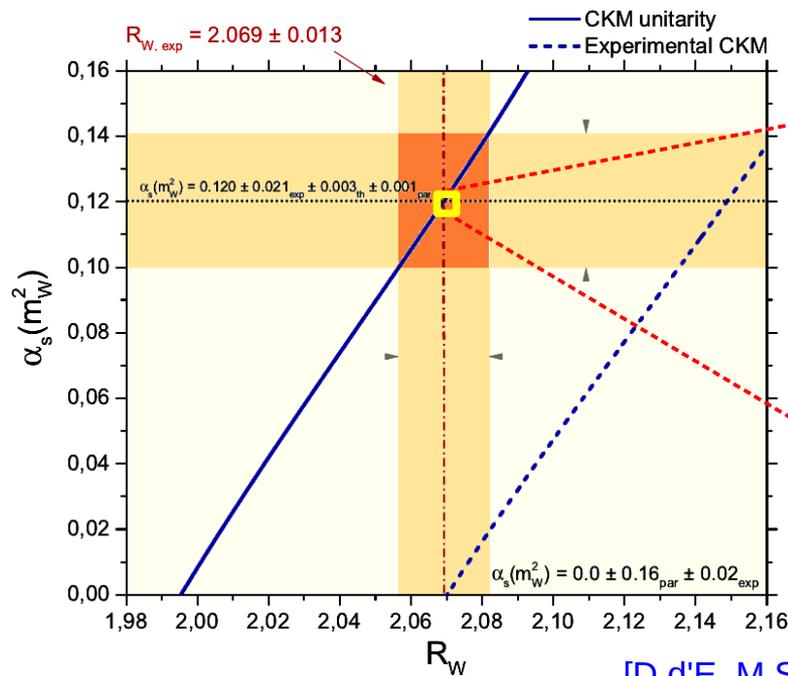
- TH uncertainty (missing α_s^5 terms, non-pQCD $(\Lambda_{\text{QCD}}/m_W)^4$ power corr., finite quark masses beyond LO, CKM matrix renorm. scheme): $\pm 0.03\%$

α_s from hadronic W decays (today \rightarrow FCC-ee)

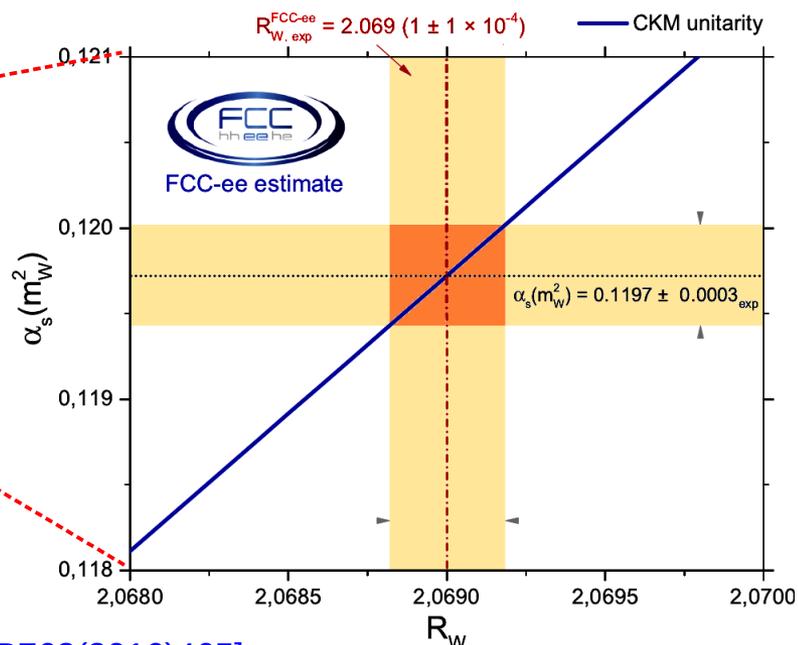
- Computed at **N^{2,3}LO**: $\Gamma_{W,\text{had}} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\text{quarks } i,j} |V_{i,j}|^2 \left[1 + \sum_{k=1}^4 \left(\frac{\alpha_s}{\pi} \right)^k + \delta_{\text{electroweak}}(\alpha) + \delta_{\text{mixed}}(\alpha\alpha_s) \right]$
- LEP**: $\Gamma_W = 1405 \pm 29$ MeV ($\pm 2\%$), $\text{BR}_W = 0.6741 \pm 0.0027$ ($\pm 0.4\%$)

Extraction with **large exp. & parametric** (CKM V_{CS}) **uncertainties** today:

$$\alpha_s(M_Z) = 0.117 \pm 0.040 \quad (\pm 35\%)$$



[D.d'E, M.Srebre, PLB763(2016)465]



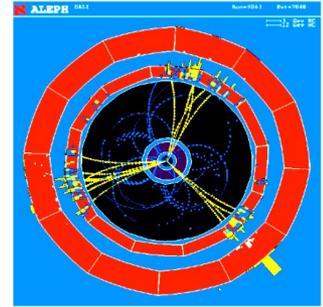
- FCC-ee**: – Huge W stats ($\times 10^4$ LEP) will lead to: $\delta\alpha_s/\alpha_s < 0.3\%$
- TH (param.) uncertainty: $|\delta V_{CS}|$ to be significantly improved (10^{-4})

α_s from e^+e^- event shapes & jet rates

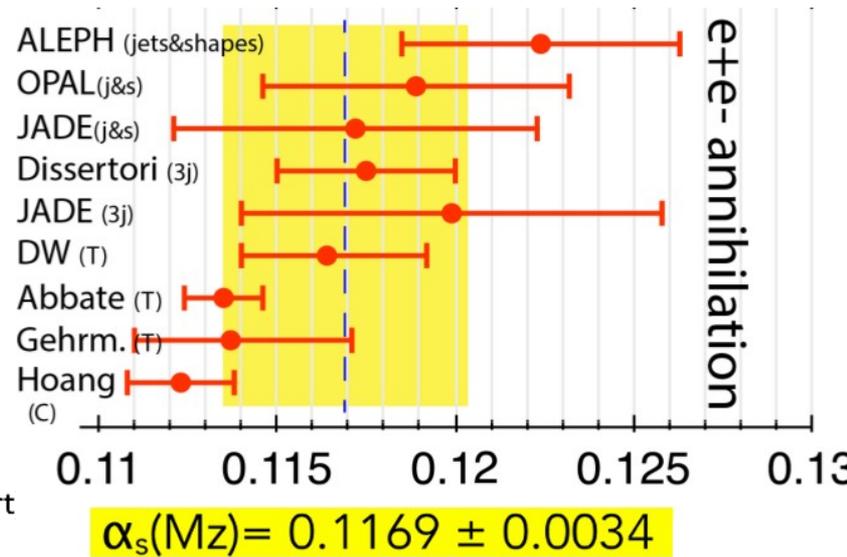
- Computed at $N^{2,3}LO+N^{(2)}LL$ accuracy.
- Experimentally (LEP):
 - Thrust, C-parameter, jet shapes
 - 3-jet x-sections
- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



- Analytic hadronization models
 - predict larger corrections than MC models
- Result typically in lower values of α_s
 - Event shape moments (M. Jaquier, G. Luisoni, TG)
 - $\alpha_s(M_Z) = 0.1153 \pm 0.0017(stat) \pm 0.0023(theo)$
 - Thrust (R. Abbate, M. Fickinger, A. Hoang, V. Mateu, I. Stewart)
 - $\alpha_s(M_Z) = 0.1135 \pm 0.0002(stat) \pm 0.0005(had) \pm 0.0009(pert)$
 - C-Parameter (A. Hoang, D. Kolodrubetz, V. Mateu, I. Stewart)
 - $\alpha_s(M_Z) = 0.1128 \pm 0.0002(stat) \pm 0.0005(had) \pm 0.0011(pert)$
- Re-visit hadronization for full distributions
 - Need precision data at multiple (higher) energies



(±2.9%)

[T. Gehrmann]

α_s from e^+e^- event shapes & jet rates (future)

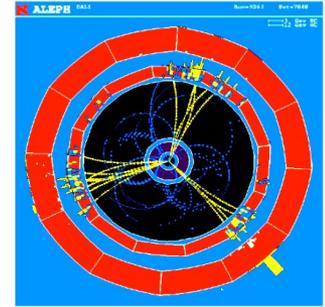
➤ Computed at $N^{2,3}LO+N^{(2)}LL$ accuracy.

➤ Experimentally (LEP):
Thrust, C-parameter, jet shapes
3-jet x-sections

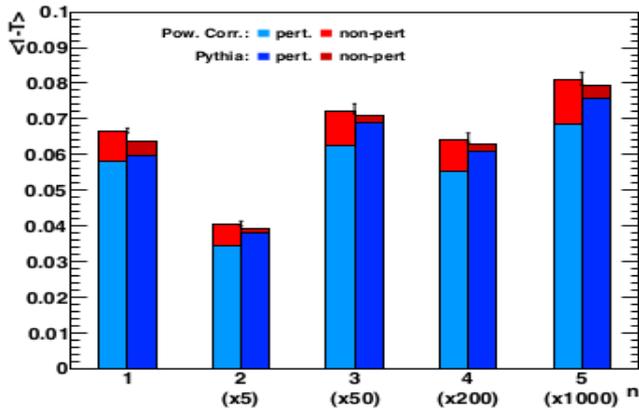
➤ Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically:

$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$



➤ Modern jet substructure techniques:
“Soft drop”/grooming to reduce non-pQCD corrections for evt shapes:

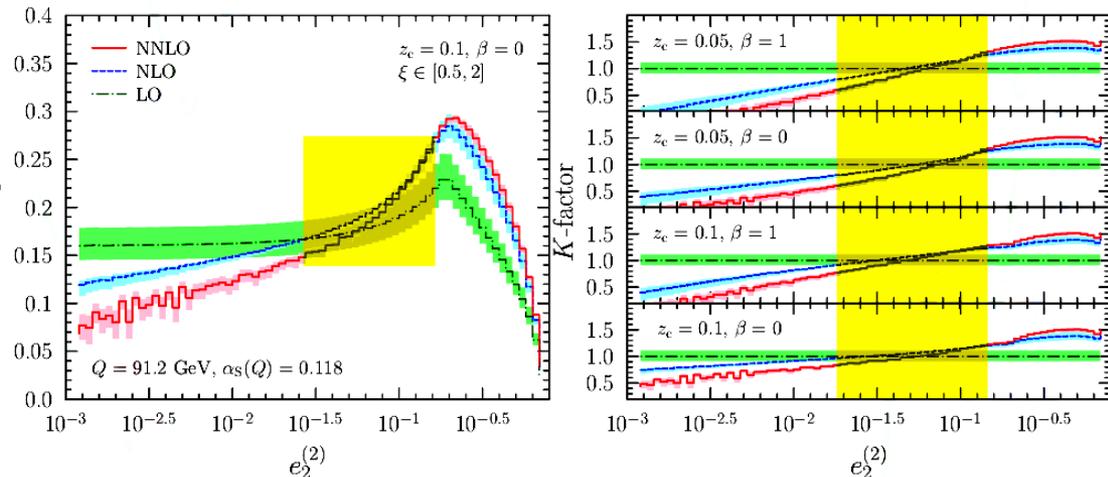


[T. Gehrmann]

$\Delta\alpha_s/\alpha_s < 1\%$

➤ Future:

- New data: Lower- \sqrt{s} (Belle-II) for evt shapes, higher- \sqrt{s} (FCC-ee) for rates
- TH: Improved hadronization for shapes, ($N^{2,3}LL$) resummation for rates.



[A. Kardos]

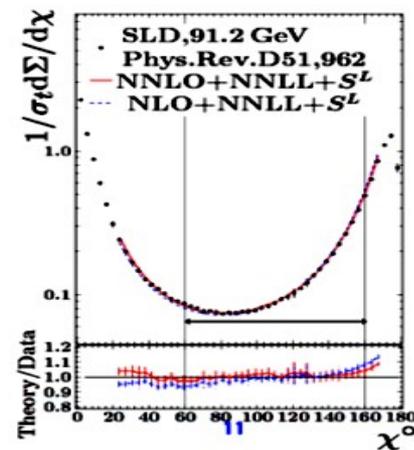
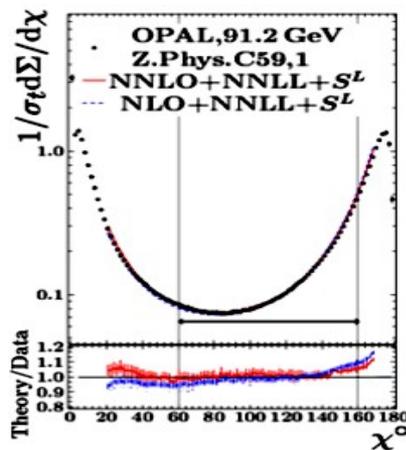
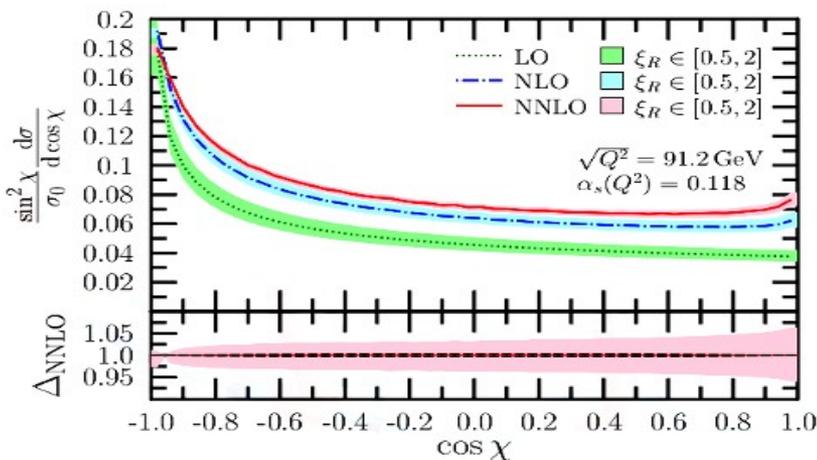
α_s from e^+e^- energy-energy correlations (new)

EEC is the normalized energy-weighted cross section defined in terms of the angle between two particles i and j in the event:

$$\frac{1}{\sigma_t} \frac{d\Sigma(\chi)}{d \cos \chi} \equiv \frac{1}{\sigma_t} \int \sum_{i,j} \frac{E_i E_j}{Q^2} d\sigma_{e^+e^- \rightarrow ij+X} \delta(\cos \chi + \cos \theta_{ij})$$

With CoLoRFulNNLO the NNLO became available (Del Duca, Duhr, AK, Somogyi and Trócsányi, arXiv:1603.08927):

NNLL resummation is available for EEC (De Florian & Grazzini, arXiv: 0407241) making it possible to have NNLL + NNLO result as well:



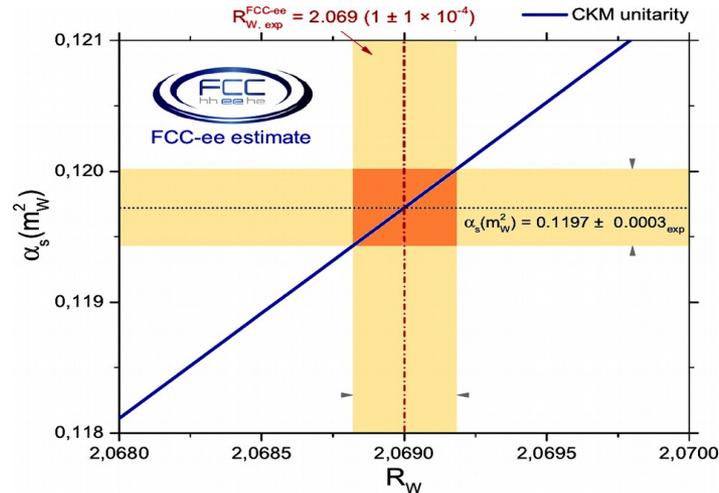
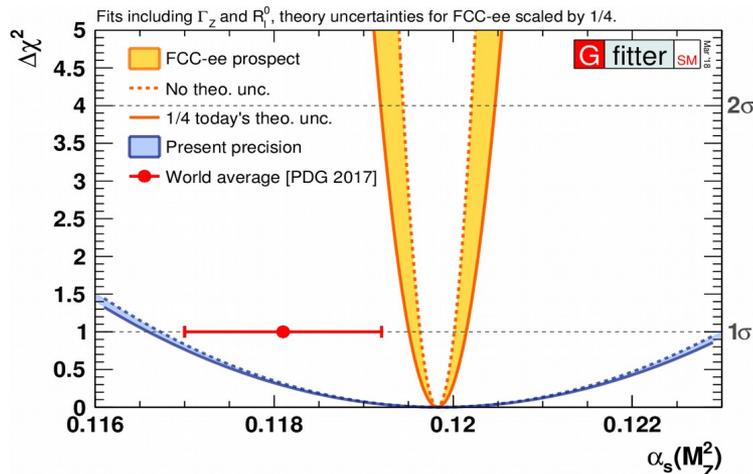
NNLL+NNLO:

[A. Kardos]

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018(\text{exp.}) \pm 0.00102(\text{had.}) \pm 0.00257(\text{ren.}) \pm 0.00078(\text{res.})$$

Summary

- FCC-ee **uniquely small EXP uncertainties** is key to $\Lambda_{\text{NP}} > 50\text{TeV}$ limits.
- **Precise QCD theory & QCD coupling** are required in SM stress tests & BSM searches for most FCC-ee observables:
 - (1) pQCD Higher-Order (H.O.) theoretical predictions of **EWPO at Z pole**.
 - (2) pQCD H.O. & coupling impact on **Higgs decay widths** ($H \rightarrow gg, \dots$)
 - (3) pQCD H.O. theoretical predictions of **WW cross sections**.
 - (4) pQCD H.O. & coupling & non-pQCD impact on **top properties** (m_t, λ_t, Γ_t).
 - (5) pQCD H.O. & coupling impact on **SM parameters** extractions ($m_{c,b}, \alpha$)
 - (6) pQCD H.O. theoretical predictions of **flavour physics**.
 - (7) pQCD H.O. & coupling impact on **(N)MSSM Higgs**.
- **Unique permille-precision** extraction of the **QCD coupling** at FCC-ee:

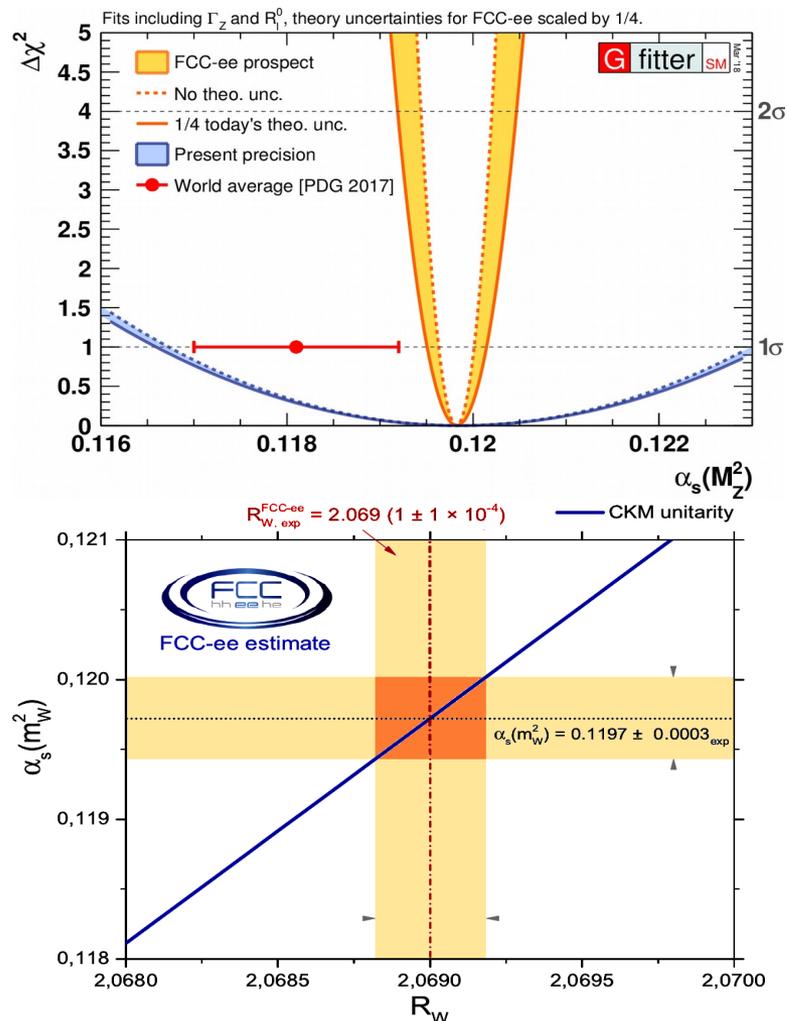


Backup slides

Summary: α_s at FCC-ee

- World-average QCD coupling at $N^{2,3}LO$:**
 - Determined today from **6 observables** with **$\sim 1\%$ uncertainty** (least well-known coupling).
 - Impacts **all LHC (& FCC-hh) QCD x-sections & decays**.
 - Role **beyond SM**: GUT, EWK vacuum stability, New colored sectors?
- New extractions/updates:** PDF fits, e-p jets, full pp \rightarrow ttbar
- Reduction of hadronization & resummation uncertainties:**
 - New TH developments needed
 - New precise e^+e^- data needed
- Other extraction methods proposed:** TH work towards NNLO accuracy.

- $\sim 0.1\%$ uncertainty, combining Z,W, τ hadronic decays (plus $F_\gamma^2(x,Q^2)$ & FFs fits)**
- ONLY possible with machine like FCC-ee:**



Importance of the QCD coupling α_s

Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

Process	σ (pb)	$\delta\alpha_s$ (%)	PDF + α_s (%)	Scale (%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 + 0.32
ttH	0.611	± 3.0	± 8.9	-9.3 + 5.9

Channel	M_H [GeV]	$\delta\alpha_s$ (%)	Δm_b	Δm_c
H $\rightarrow c\bar{c}$	126	± 7.1	$\pm 0.1\%$	$\pm 2.3\%$
H $\rightarrow gg$	126	± 4.1	$\pm 0.1\%$	$\pm 0\%$

Msbar mass error budget (from threshold scan)

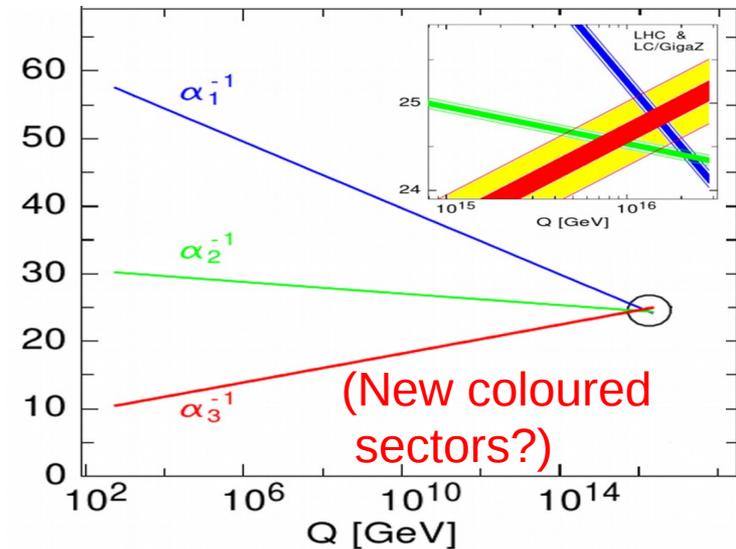
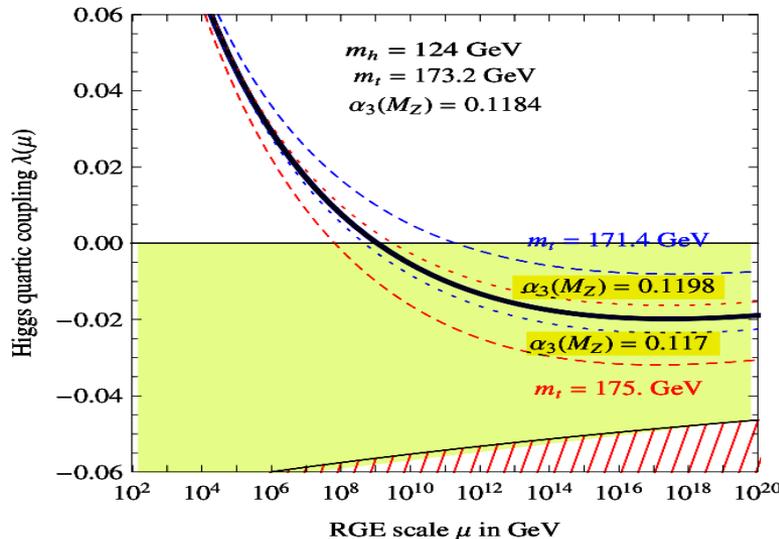
$(\delta M_t^{\text{SD-low}})^{\text{exp}}$	$(\delta M_t^{\text{SD-low}})^{\text{theo}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\text{conversion}}$	$(\delta \overline{m}_t(\overline{m}_t))^{\alpha_s}$
40 MeV	50 MeV	7 – 23 MeV	70 MeV

\Rightarrow improvement in α_s crucial $\delta\alpha_s(M_Z) = 0.001$

Quantity	FCC-ee	future param.unc.	Main source
Γ_Z [MeV]	0.1	0.1	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_ℓ [10^{-3}]	1	1.3	$\delta\alpha_s$

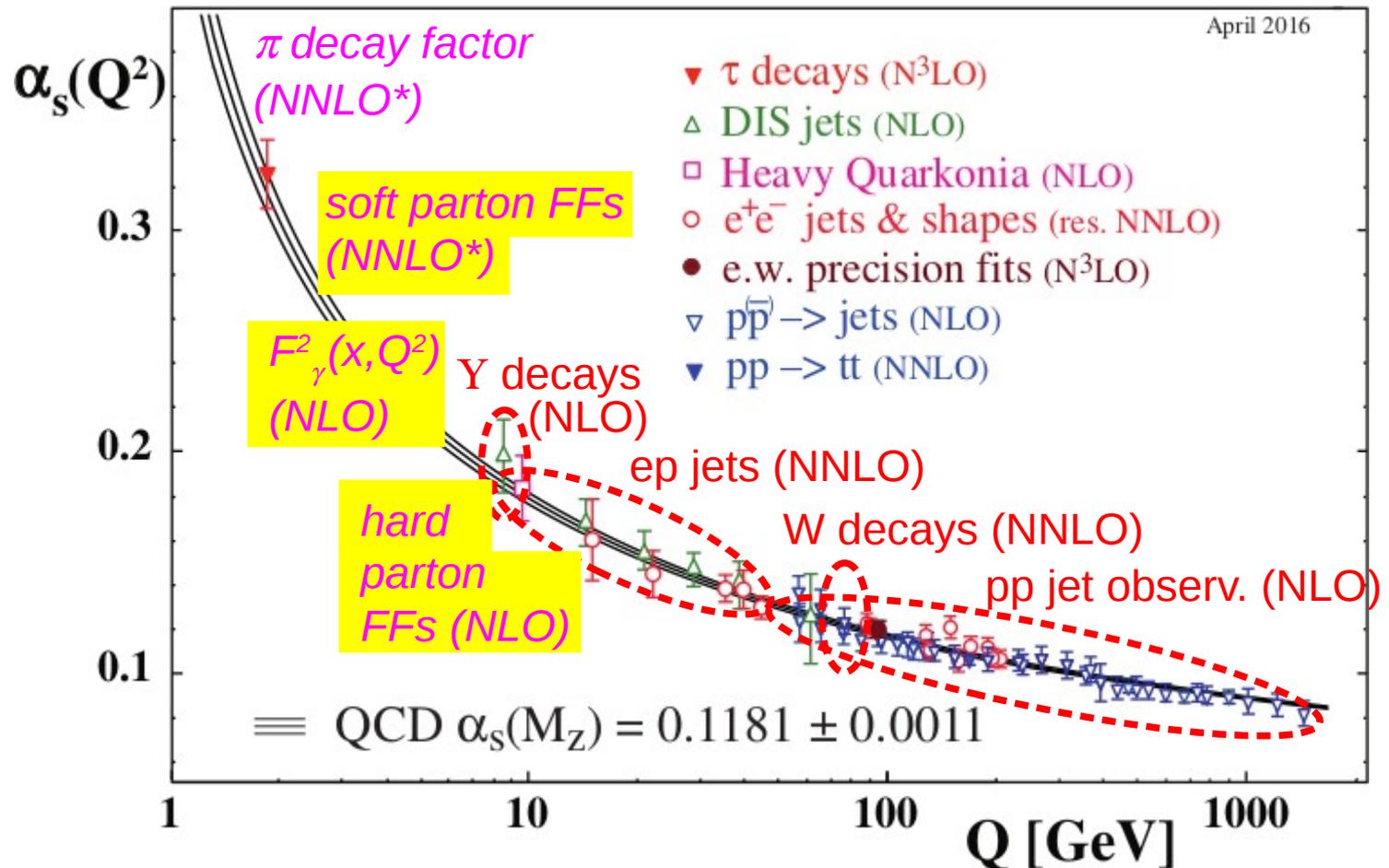
Sven Heinemeyer – 1st FCC physics workshop, CERN, 17.01.2017

Impacts physics approaching Planck scale: EW vacuum stability, GUT



Other α_s extractions (not yet in world average)

- There exist at least 8 other classes of observables, computed today at lower accuracy (NLO, NNLO*), that could be used to extract α_s . Three of them to be exploited at FCC-ee:



α_s from γ QCD structure function

➔ Computed at NNLO: $\int_0^1 dx F_2^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left\{ \frac{4\pi}{\alpha_s(Q^2)} c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi} c_{NNLO} + \mathcal{O}(\alpha_s^2) \right\}$

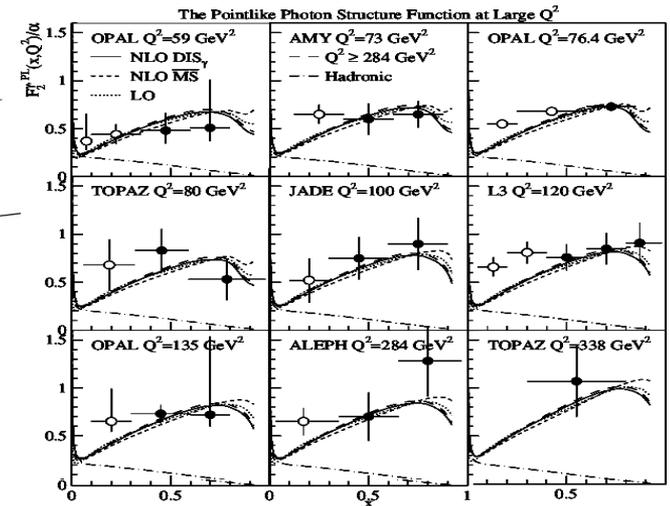
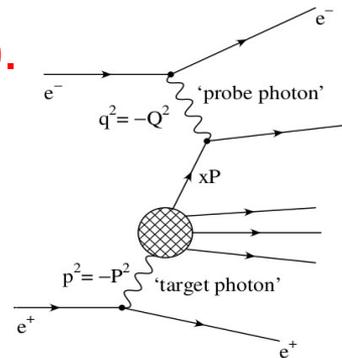
➔ Poor $F_2^\gamma(x, Q^2)$ experimental measurements:

➔ Extraction (NLO) with large exp. uncertainties today:

$$\alpha_s(M_Z) = 0.1198 \pm 0.0054$$

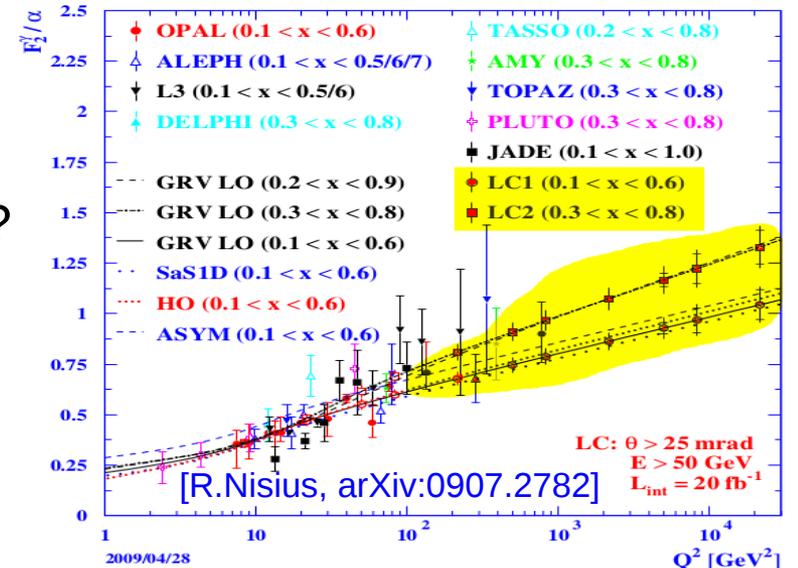
($\pm 4.5\%$)

[M.Klasen et al. PRL89 (2002)122004]



➔ Future prospects:

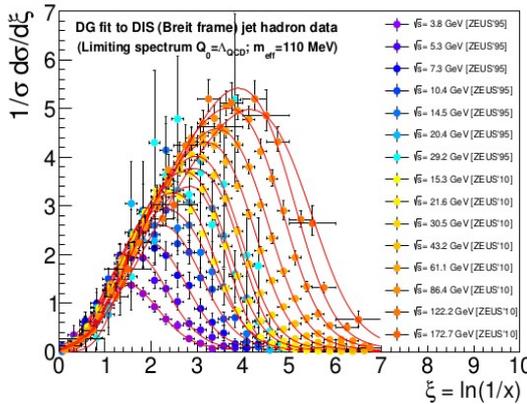
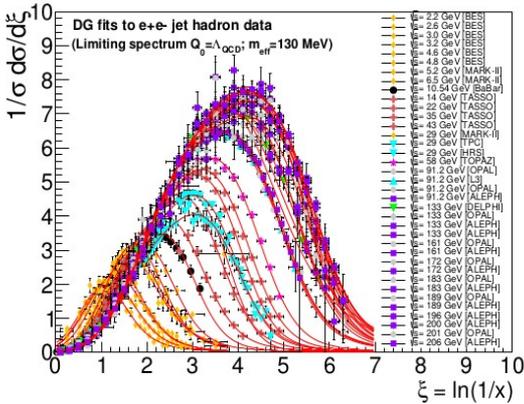
- Fit with NNLO F_2^γ evolution (ongoing)
- Better e^+e^- data badly needed: Belle-II?
- Dedicated studies at ILC exist:
- Huge $\gamma\gamma$ (EPA) stats at FCC-ee will lead to: $\delta\alpha_s/\alpha_s < 1\%$



α_s from parton-to-hadron FFs

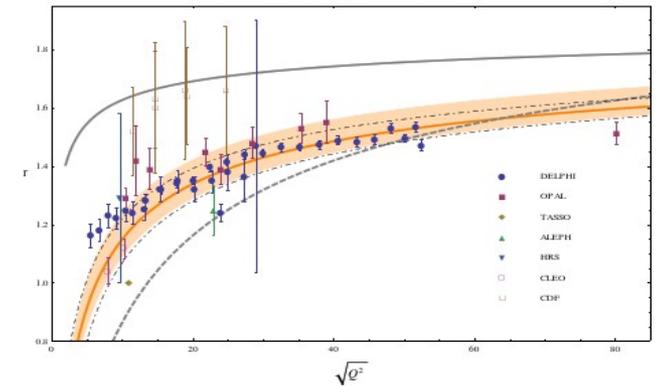
➔ **Soft** Fragmentation Functions (NNLO*+NNLL):

➔ Combined fit of energy evolution of **HBP** moments:



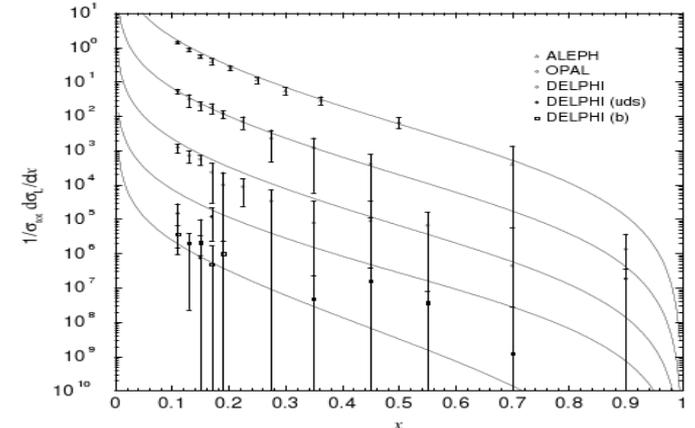
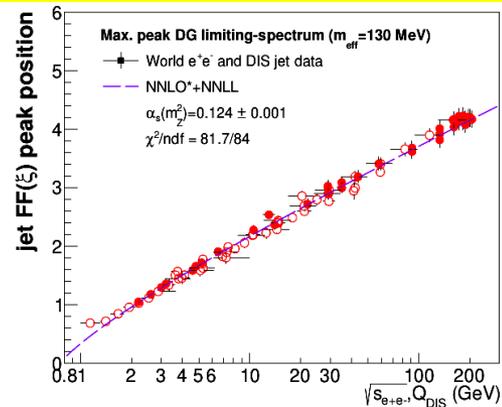
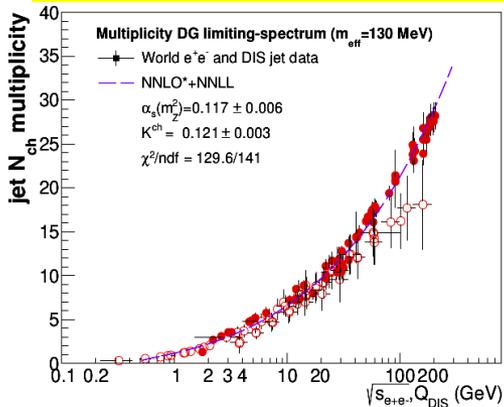
➔ **Hard** Frag. Functions (NLO):

$$r \langle n_h \rangle_g, \langle n_h \rangle_q$$



α_s (NNLO)= 0.1199 ± 0.0044 ($\pm 3.6\%$)

α_s (NNLO*+NNLL)= $0.1205 \pm 0.0010^{+0.0022}$ ($\pm 2\%$)



[AKK, B.Kniehl et al., NPB 803(2008)42]

α_s (NLO)= $0.1176 \pm 0.0055 \pm 0.0008$ ($\pm 4.7\%$)

➔ **Future** prospects: – Full-NNLO fit for FF (& moments) evolution.

– FCC-ee: Huge hadron stats. up to $\times 2$ LEP E_{jet}

$\delta\alpha_s/\alpha_s < 1\%$

David d'Enterria (CERN)