Next-to-leading power corrections and resummation

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Outline

- Power corrections and SCET framework
- QED corrections to $B_s \rightarrow \mu^+\mu^-$
- Threshold resummation
Why study power-corrections?

\[ \mathcal{O} = \sum_{k=0}^{\infty} \sum_{i=i_{\text{min}}}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^k \lambda^i a_{ki}(\ln \lambda, \ldots) \]

with \( \lambda \) being some small parameter, e.g. \( \lambda \sim \frac{\Lambda_{\text{QCD}}}{m_b} \) or \( \lambda \sim 1 - \frac{Q^2}{s^2} \) such that observable \( \mathcal{O} \) simplifies (factorizes) in the limit \( \lambda \rightarrow 0 \)

We want to go beyond leading power in order to

- increase precision of theory predictions (understand new effects such as QED corrections in flavor physics, or how to match fixed order and resummed result)
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- better understand all order structure of QFT (universality, new structures, ...)
- use a bootstrap procedure to reconstruct full amplitudes from its limits
- improve the accuracy of numerical computations – fixed order subtraction methods based on factorization [M. Ebert, I. Moult, I.Stewart, F. Tackmann, G. Vita, H. Zhu, JHEP 1812 (2018) 084]
Power corrections in SCET

Effective field theory approach allows to systematically compute power-corrections and perform resummation.

- Power suppressed Lagrangian

\[ \mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{LP}} + \mathcal{L}^{(1)}_{\xi} + \mathcal{L}^{(2)}_{\xi} + \mathcal{L}^{(1)}_{\xi q} + \mathcal{L}^{(2)}_{\xi q} + \mathcal{O} (\lambda^3) \]
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\]

\[
\mathcal{L}^{(1)}_{\xi} = \bar{\xi} \left( x_\perp n_\perp W_c g_s F^{s}_{\mu\nu} W_c^\dagger \right) \frac{\gamma^\perp}{2} \xi,
\]

\[
\mathcal{L}^{(2)}_{\xi} = \frac{1}{2} \bar{\xi} \left( (n_- x) n_+ n_\perp W_c g_s F^{s}_{\mu\nu} W_c^\dagger + x_\perp x_\perp \rho n_\perp W_c \left[ D^\rho_s, g_s F^{s}_{\mu\nu} \right] W_c^\dagger \right) \frac{\gamma^\perp}{2} \xi
\]

\[
+ \frac{1}{2} \bar{\xi} \left( i \tilde{\pounds} \perp \frac{1}{i n_+ D} x_\perp \gamma_\perp W_c g_s F^{s}_{\mu\nu} W_c^\dagger + x_\perp \gamma_\perp W_c g_s F^{s}_{\mu\nu} W_c^\dagger \frac{1}{i n_+ D} i \tilde{\pounds} \perp \right)
\]

\[
\mathcal{L}^{(1)}_{\xi q} = \bar{q} W_c^\dagger i \tilde{\pounds} \perp \xi - \bar{\xi} i \tilde{\pounds} \perp W_c q,
\]

\[
\mathcal{L}^{(2)}_{\xi q} = \bar{q} W_c^\dagger (i n_- D + i \tilde{\pounds} \perp (i n_+ D)^{-1} i \tilde{\pounds} \perp) \frac{\gamma^\perp}{2} \xi + \bar{q} \tilde{D}^\mu_s x_\perp W_c^\dagger i \tilde{\pounds} \perp \xi
\]

\[
- \bar{\xi} \frac{\gamma^\perp}{2} \left( i n_- \tilde{D} + i \tilde{\pounds} \perp (i n_+ \tilde{D})^{-1} i \tilde{\pounds} \perp \right) W_c q - \bar{\xi} i \tilde{\pounds} \perp W_c x_\perp D^\mu s q.
\]

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▶ Power suppressed currents

\[ \chi = W^\dagger \xi \sim \lambda \]
\[ A_\mu^\perp = W^\dagger D_\mu^\perp W \sim \lambda \]
\[ J^{A0}(s) = \chi(n+s) \]
\[ J^{A1}(s) = \partial_\perp \chi(n+s) \]
\[ J^{B1}_{A\chi}(s, t) = A_\perp(n+t) \chi(n+s) \]
\[ J^{B1}_{\chi\chi}(s, t) = \chi(n+t) \chi(n+s) \]

Power corrections in SCET

Effective field theory approach allows to \textit{systematically} compute power-corrections and perform resummation.

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- Power suppressed currents
- Kinematic corrections
QED corrections to $B_s \rightarrow \mu^+\mu^-$
Why do we need to know the QED corrections in flavor physics?

- Large logarithmic $\ln \left( \frac{m_b^2}{m_\ell^2} \right)$ enhancements can mimic lepton-flavor universality violation
- Soft photon approximation employed so far is not suitable for virtual photons that can resolve the B-meson.
- Factorization theorems do not yet exist for QED
- Expected precision of measurements may require the inclusion of QED corrections or at least a proof that no effects above 1% exist
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Our starting point

$$B_s \rightarrow \mu^+\mu^-$$

In the SM the process is

- loop suppressed (FCNC)
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Our starting point

$$ B_s \rightarrow \mu^+ \mu^- $$

In the SM the process is

- loop suppressed (FCNC)
- helicity suppressed (scalar meson decaying into energetic muons, vector interaction), $\mathcal{A} \sim m_\mu$
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Our starting point

\[ B_s \rightarrow \mu^+\mu^- \]

In the SM the process is

- loop suppressed (FCNC)
- helicity suppressed (scalar meson decaying into energetic muons, vector interaction), \( A \sim m_\mu \)
- purely leptonic final state allows for a precise SM prediction, QCD contained in the meson decay constant \( f_{B_s} \) [C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou, M. Steinhauser, Phys.Rev.Lett. 112 (2014) 101801]
The final state has no strong interaction – QCD is contained in the decay constant

\[ \langle 0 | \bar{q}(0) \gamma^\mu \gamma_5 b(0) | \bar{B}_q (p) \rangle = i f_{B_q} p^\mu \]

This is no longer true when QED effects are included – non-local time ordered products have to be evaluated

\[ \langle 0 | \int d^4 x \ e^{i q x} T \{ j_{\text{QED}}(x), \mathcal{L}_{\Delta B=1}(0) \} | \bar{B}_q \rangle \]

This can be done for QED bound-states but QCD is non-perturbative at low scales
Helicity suppression

Can the helicity suppression be relaxed?

Without QED:

\[ u(p_\ell) = u_c(p_\ell) + \mathcal{O}\left(\frac{m_\ell}{E_\ell}\right) \]

For \( m_\ell \to 0 \) the amplitude has to vanish

Annihilation and helicity flip take place at the same point \( r \lesssim \frac{1}{m_b} \)
Helicity suppression
Can the helicity suppression be relaxed?

With QED:

Annihilation and helicity flip can be separated by \( r \sim \frac{1}{\sqrt{m_b \Lambda_{QCD}}} \)

It is still a short distance effect since the size of the meson is \( r \sim \frac{1}{\Lambda_{QCD}} \)

For \( m_\ell \to 0 \) the amplitude still vanishes

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Scales in the problem

Leptonic decay of $B_s$ is a multi-scale problem

- Electroweak scale $m_W$
- Hard scale $m_b$
- Hard-collinear scale $\sqrt{m_b\Lambda_{QCD}}$
- Soft scale $\Lambda_{QCD}$
- Collinear scale $m_\mu$

We take $\Lambda_{QCD} \sim m_\mu$ so the soft scale of HQEFT is also a soft scale of SCET_I

\[
\begin{align*}
&\text{SM} \\
\downarrow \\
&\text{Weak EFT} \\
\downarrow \\
&\text{SCET}_I \otimes \text{HQEFT} \\
\downarrow \\
&\text{SCET}_II \oplus \text{HQEFT} \\
\end{align*}
\]

\[
\begin{align*}
m_W^2 &\rightarrow \infty \\
m_b^2 &\rightarrow \infty \\
m_b\Lambda_{QCD} &\rightarrow \infty
\end{align*}
\]

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**C_9** contribution, hard matching

Typical SCET I → SCET II problem but with \( m_\ell \neq 0 \)

\[
\sum_i C_i (\mu_b) \mathcal{O}_i \rightarrow \sum_i \int du \mathcal{H}_i (u) \bar{J}_i (u),
\]

Weak operators are matched on B-type SCET I currents

\[
J_9 (s, t) = \left[ \overline{\chi C} (n+s) \gamma^\mu P_L h_\nu (0) \right] \left[ \overline{\ell C} (n+t) \gamma^\mu \ell_C (0) \right]
\]

*In the first step, the quark is hard-collinear rather than soft!*

\[
\chi_C \sim \lambda; \quad q_s \sim \lambda^3
\]

We work with Fourier transformed currents and coefficients

\[
\bar{J}_9 (u) = n+p_\ell \int \frac{dr}{2\pi} \exp \left[ -iun + p_\ell r \right] J_9 (0, r)
\]

\[
\mathcal{H}_9 (u) = C_9^{\text{eff}} (u) + \mathcal{O} (\alpha_{em})
\]
We need $\mathcal{L}_{\xi q}^{(1)}$ to convert the hc-quark into a soft quark.

For pure hc-interaction, mass is power suppressed $m_\ell \sim \lambda^2$, $p_{hc}^\perp \sim \lambda$, $\mathcal{L}_{hc,m}^{(1)}$.

For pure c-interaction, mass is included in the leading power Lagrangian, $m_\ell \sim \lambda^2 \sim p_c^\perp$, $\mathcal{L}_{c,m}^{(0)}$. 
Two operators are relevant for the power-enhanced contribution

\[ \mathcal{J}_9(u) \rightarrow \int d\omega \mathcal{J}_a(u;\omega) \mathcal{J}_{m\chi}^{A1}(\omega) + \int d\omega \int dw \mathcal{J}_b(u;\omega,w) \mathcal{J}_{A\chi}^{B1}(\omega,w) \]

- Hard-collinear jet functions \( \sim \frac{1}{\omega} \)

\[ J_{m\chi}^{A1}(v) = \left[ \overline{q_s} (vn_−) \frac{\phi}{2} \Gamma_{PL} h_v (0) \right] \left[ m_\ell \overline{\ell}_c (0) \Gamma_{\perp \ell_c} (0) \right] \left[ Y_+ (0) Y_\uparrow (0) \right] \]

\[ J_{A\chi}^{B1}(t,v) = \left[ \overline{q_s} (vn_−) \frac{\phi}{2} \Gamma_{PL} h_v (0) \right] \left[ A_{\perp (n+t)} \overline{\ell}_c (0) \Gamma_{\perp \ell_c} (0) \right] \left[ Y_+ (0) Y_\uparrow (0) \right] \]

\[ \mathcal{J}_a(u;\omega) \sim \mathcal{O}(\alpha_{em}) \]
\[ \mathcal{J}_b(u;\omega,w) \sim \mathcal{O}(1) \]

We use soft gauge QED-QCD invariant building blocks e.g.

\[ q_s(x) = Y_+^\dagger(x)Y_+^\dagger_{QCD}(x)q(x) \]
Two operators are relevant for the power-enhanced contribution

\[ \overline{J}_9 (u) \rightarrow \int d\omega \overline{J}_a (u; \omega) \overline{J}^{A1}_{m\chi} (\omega) + \int d\omega \int dw \overline{J}_b (u; \omega, w) \overline{J}^{B1}_{A\chi} (\omega, w) \]

- Hard-collinear jet functions \( \sim \frac{1}{\omega} \)
- Additional QED soft Wilson lines

\[ J^{A1}_{m\chi} (v) = \left[ \overline{q}_s (vn_-) \frac{\phi}{2} \Gamma_{\perp} P_L h_v (0) \right] \left[ m_\ell \overline{\ell}_c (0) \Gamma_{\perp} \ell_\bar{c} (0) \right] \begin{bmatrix} Y_+ (0) \ Y_\uparrow (0) \end{bmatrix} \]

\[ J^{B1}_{A\chi} (t, v) = \left[ \overline{q}_s (vn_-) \frac{\phi}{2} \Gamma_{\perp} P_L h_v (0) \right] \left[ A'_{\perp} (n+t) \overline{\ell}_c (0) \Gamma_{\perp} \ell_\bar{c} (0) \right] \begin{bmatrix} Y_+ (0) \ Y_\uparrow (0) \end{bmatrix} \]

\[ \overline{J}_a (u; \omega) \sim \mathcal{O} (\alpha_{\text{em}}) \]

\[ \overline{J}_b (u; \omega, w) \sim \mathcal{O} (1) \]

We use soft gauge QED-QCD invariant building blocks e.g.

\[ q_s (x) = Y_+^\dagger (x) Y_+^\dagger_{\text{QCD}} (x) q (x) \]
Factorization and resummation

\[ \mathcal{A} = -if_{B_q} m_b m_\ell Z_\ell Z_{\bar{\ell}} T_+ \int du \mathcal{H}_9(u) \]

\[ \times \int d\omega \phi_+(\omega) \left( \mathcal{J}_a(u;\omega) + \int d\omega \mathcal{J}_b(u;\omega,w) M(w) \right) \]

Modified B-meson LCDA

\[ \phi_+(\omega) \sim \langle 0 | [\bar{q}(n-v)]_\beta [h_v(0)]_\alpha Y_+(0) Y_+^\dagger(0) | \bar{B}_q(p) \rangle \]

The anticollinear contribution is symmetric \( \rightarrow \) multiply by 2 to get the total result

**LL resummation**

\[ \frac{d}{d\ln \mu} \begin{pmatrix} \mathcal{J}_b(u;\omega,w;\mu) \\ \mathcal{J}_a(u;\omega;\mu) \end{pmatrix} = \]

\[ - \begin{pmatrix} \Gamma_{\text{cusp}}^b \ln \frac{\mu}{\mu_{hc}} & 0 \\ -Q_\ell \frac{\alpha_{em}}{2\pi} \bar{w} \Gamma_{\text{cusp}}^a \ln \frac{\mu}{\mu_{hc}} \end{pmatrix} \begin{pmatrix} \mathcal{J}_b(u;\omega,w;\mu) \\ \mathcal{J}_a(u;\omega;\mu) \end{pmatrix} \]
Numerical prediction: the power-enhanced correction

Total power-enhanced correction changes the branching fraction by:

$$-(0.3 - 1.1)\%$$


Cancellation between $C_7$ and $C_9$ part

central value:

$$-0.6\% = 1.1\% - 1.7\% \ (C_7, C_9 \text{ parts})$$

Uncertainty comes from $\lambda_B$, $\sigma_1$, $\sigma_2$. $C_7$ part is surprisingly large thanks to double logs enhancement.

The previous estimate of QED uncertainty was 0.3%, obtained by scale variation method. This uncertainty is still present due to non-enhanced corrections.
Threshold resummation
Drell-Yan production, $q\bar{q}$ channel
Breakdown of perturbative description

In certain regions of the phase space, perturbative expansion breaks down!

Consider threshold expansions $z \to 1 \left( z = \frac{Q^2}{\hat{s}} \right)$ of DY production

$$A(p_A)B(p_B) \to \text{DY}(Q) + X$$

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \left[ c_n \delta(1 - z) \right. + \sum_{m=0}^{2n-1} \left( c_{nm} \left[ \ln^m(1 - z) \right] \right) + \left. + d_{nm} \ln^m(1 - z) \right] + \ldots$$

▶ Expansion parameter is not $\frac{\alpha_s}{4\pi}$ but $\frac{\alpha_s}{4\pi} \ln^2(1 - z)$
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\]

- Expansion parameter is not \( \frac{\alpha_s}{4\pi} \) but \( \frac{\alpha_s}{4\pi} \ln^2(1-z) \)
- Resummation is necessary to get meaningful results
Breakdown of perturbative description

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▶ Leading power

▶ Next-to-leading power

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Leading power factorization in SCET

Drell-Yan production process $pp \rightarrow \ell^+ \ell^- + X$
Leading power factorization in SCET

Leading power Drell-Yan factorization

Parton Distribution Functions

Hard Contribution

Soft Contribution
The Drell-Yan process - the leading power result

\[
\frac{d\sigma_{\text{DY}}}{dQ^2} = \frac{4\pi\alpha_{\text{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab}(z)
\]

where

\[
\hat{\sigma}(z) = |C(Q^2)|^2 QS_{\text{DY}}(Q(1 - z))
\]

\[
S_{\text{DY}}(\Omega) = \int \frac{dx^0}{4\pi} e^{ix^0\Omega/2} \frac{1}{N_c} \text{Tr} \langle 0 | \bar{T}(Y_+^\dagger(x^0)Y_-(x^0)) T(Y_+^\dagger(0)Y_+(0)) |0 \rangle
\]

Factorization formula at NLP

A schematic formula

\[
\frac{d\sigma_{DY}}{dQ^2} = \frac{4\pi\alpha_{em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \ f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab}(z)
\]

The \( \hat{\sigma}_{ab}(z) \) is now

\[
\hat{\sigma} = \sum_{\text{terms}} [C \otimes J \otimes \bar{J}]^2 \otimes S
\]

- \( C \) is the hard Wilson matching coefficient \( \mu_h \sim Q \)

arXiv:1809.10631]

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\]

- \(C\) is the hard Wilson matching coefficient \(\mu_h \sim Q\)
- \(S\) is the \textit{generalized} soft function \(\mu_s \sim Q(1 - z)\)
- \(J\) is the collinear function \(\mu_c \sim Q(1 - z)^{1/2}\)

Each object depends only on a single scale!

Renormalization scale

\[
\mu \leftrightarrow \text{physical scale, e.g.}
\]

\[
S(\mu_s, \mu) = S\left(\ln\left(\frac{\mu_s}{\mu}\right)\right)
\]

\[
J(\mu_c, \mu) = J\left(\ln\left(\frac{\mu_c}{\mu}\right)\right)
\]

arXiv:1809.10631]

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Collinear function is a matching coefficient

Lagrangian insertions at various positions each with its own $\omega_i$

We can separate the Lagrangian insertions

$$L_V^{(n)}(z) = L_c^{(n)}(z) \otimes L_s^{(n)}(z_-)$$

$$i^n \left( \sum_{i=1}^{n} \int d^4 z_j e^{i \omega_j \frac{n+z_j}{2}} \right) \times T \left[ \chi_c(tn_+) \times L_c^{(n)}(z_1) \times L_c^{(n)}(z_2) \times ... \right]$$

$$= J(t; \omega_1, \omega_2, ...) \chi_c^{PDF}(tn_+)$$

Collinear function is a non-local object
Next to leading power soft function

Soft operator in position space is a non-local object

\[ \tilde{S}_{2\xi}(x, z_-) = \tilde{T} \left[ Y_+^\dagger(x)Y_-^{}(x) \right] T \left[ Y_+^\dagger(0)Y_+^{}(0) \frac{i\partial_{\perp}^\nu}{in_-\partial} B_{\perp\nu}(z_-) \right] \]

with decoupled soft fields

\[ B_\perp^\mu = Y_\perp^\dagger [iD_s^\mu Y_\perp^{}] \]

Lagrangian is already multipole expanded → soft fields depend only on \( z_- \)

\[ \mathcal{L}^{(2)}_{2\xi} = \frac{1}{2} \bar{\chi}_c z_\perp^\mu z_\perp^\nu \left[ i\partial_{\nu} in_-\partial B_\mu^+ \right] \frac{\eta_+}{2} \chi_c \]
Next to leading power soft function

Soft operator in position space is a non-local object

\[ \tilde{S}_{2\xi}(x, z_-) = \hat{T} \left[ Y_+(x)Y_-(x) \right] \hat{T} \left[ Y_+^0(0)Y_+^0(0) \frac{i\partial_\perp}{in_\perp} B_\perp^+(z_-) \right] \]

In the factorization theorem, we need only vacuum matrix element

\[ S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n+z)}{4\pi} e^{ix^0\Omega/2 - i\omega(n+z)/2} \frac{1}{N_c} \text{Tr} \langle 0 | \tilde{S}_{2\xi}(x^0, z_-) | 0 \rangle \]
Next to leading power soft function

Soft operator in position space is a non-local object

$$\tilde{S}_{2\xi}(x, z^-) = \tilde{T} \left[ Y_+^\dagger(x) Y_-(x) \right] T \left[ Y_-^\dagger(0) Y_+(0) \frac{i \partial_\perp}{i n^- \partial} B_\perp^+(z^-) \right]$$

In the factorization theorem, we need only vacuum matrix element

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n^+ z^-)}{4\pi} e^{ix^0 \Omega/2 - i\omega(n^+ z^-)/2} \frac{1}{N_c} \text{Tr} \langle 0 | \tilde{S}_{2\xi}(x^0, z^-) | 0 \rangle$$
Next to leading power soft function

Soft operator in position space is a non-local object

$$\tilde{S}_{2\xi}(x, z_{-}) = \mathbf{T} \left[ Y_{+}^{\dagger}(x)Y_{-}(x) \right] \mathbf{T} \left[ Y_{-}^{\dagger}(0)Y_{+}(0) \frac{i\partial_{\nu}}{i n_{-}\partial} B_{\nu}^{+}(z_{-}) \right]$$

In the factorization theorem, we need only vacuum matrix element

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx_{0}^{0}}{4\pi} \int \frac{d(n+z)}{4\pi} e^{ix_{0}\Omega/2-i\omega(n+z)/2} \frac{1}{N_{c}} \text{Tr} \langle 0|\tilde{S}_{2\xi}(x_{0}, z_{-})|0 \rangle$$

$$S_{2\xi}(\Omega, \omega) = \frac{\alpha_{s}C_{F}}{2\pi} \left\{ \theta(\Omega)\delta(\omega) \left( -\frac{1}{\epsilon} + \ln \frac{\Omega^{2}}{\mu^{2}} \right) + \left[ \frac{1}{\omega} \right] + \theta(\omega)\theta(\Omega - \omega) \right\}$$

arXiv:1809.10631]
Renormalization and resummation

Operator mixing → we need to introduce a new soft function whose tree-level matrix element is non-zero

\[
\tilde{S}_{x_0}(x^0) = \frac{-2i}{x^0 - i\epsilon N_c} \frac{1}{N_c} \text{Tr} \langle 0 | \bar{T}(Y_+^\dagger(x^0)Y_-(x^0)) T(Y_+^\dagger(0)Y_+(0)) | 0 \rangle
\]

At the LL we have

\[
\frac{d}{d \ln \mu} \begin{pmatrix} S_{2\xi}(\Omega, \omega) \\ S_{x_0}(\Omega) \end{pmatrix} = \frac{\alpha_s}{\pi} \begin{pmatrix} 4C_F \ln \frac{\mu}{\mu_s} & -C_F \delta(\omega) \\ 0 & 4C_F \ln \frac{\mu}{\mu_s} \end{pmatrix} \begin{pmatrix} S_{2\xi}(\Omega, \omega) \\ S_{x_0}(\Omega) \end{pmatrix}
\]

\[
S_{2\xi}^{LL}(\Omega, \omega, \mu) = \frac{2C_F}{\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_s)} \exp \left[ -4S_{2\xi}^{LL}(\mu_s, \mu) \right] \theta(\Omega) \delta(\omega)
\]

\[
= C_F \frac{\alpha_s}{\pi} \ln \frac{\mu_s}{\mu} \exp \left[ -2C_F \frac{\alpha_s}{\pi} \ln^2 \frac{\mu_s}{\mu} \right] \theta(\Omega) \delta(\omega)
\]

Log from the mixing        Sudakov double log
Resummed result

- Kinematic corrections can be treated in a similar way
  \[ \rightarrow \text{LL cancel for } \frac{\hat{\sigma}^{\text{LL}}_{\text{NLP}}(z, \mu)}{z} \]

- Collinear function associated with the \( \frac{i \partial^\perp}{\partial n_\perp} B^+_{\perp \nu}(z_-) \) operator does not have LL

- Resummed hard function is equal to the LP one

Resummed partonic cross-section for the \( q\bar{q} \) channel

\[
\frac{\hat{\sigma}^{\text{LL}}_{\text{NLP}}(z, \mu)}{z} = \exp \left[ 2 \frac{\alpha_s C_F}{\pi} \ln^2 \frac{\mu}{\mu_s} - 2 \frac{\alpha_s C_F}{\pi} \ln^2 \frac{\mu}{\mu_h} \right] \\
\times (-4) \frac{\alpha_s C_F}{\pi} \ln \frac{\mu_s}{\mu} \theta(1 - z)
\]

arXiv:1809.10631]
Application of effective field theory methods allows for a systematic study of power corrections in QED and QCD for processes involving energetic particles.

QED corrections in flavor physics should be further analyzed to achieve necessary precision. Methods based on soft photon approximation or LL are not good enough to achieve the necessary accuracy.

We performed NLP LL threshold resummation, further studies are necessary to go beyond LL and extend results to different channels/observables.

Many applications for future colliders.
Auxiliary slide: Operators at the hard scale

\[ Q_9 = \frac{\alpha_{\text{em}}}{4\pi} (\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \bar{\ell}) \]

\[ Q_{10} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \bar{\ell}) \]

\[ Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \]

Four-quark operators can be treated as a modification of \( C_9 \) and \( C_7 \).
Auxiliary slide: The QED correction at the amplitude level

\[ iA = m_\ell f_{Bq} N C_{10} \bar{\ell} \gamma_5 \ell + \frac{\alpha_{em}}{4\pi} Q_\ell Q_q m_\ell m_B f_{Bq} N \bar{\ell} (1 + \gamma_5) \ell \]

\[ \times \left\{ \int_0^1 du \left( 1 - u \right) C_{9}^{\text{eff}} (u m_b^2) \left( \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \right) \left[ \ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1 - u} \right] \right\} \]

\[ - Q_\ell C_7^{\text{eff}} \left( \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \right) \left[ \ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \]
Auxiliary slide: The QED correction at the amplitude level

\[ iA = m_\ell f_{Bq} N C_{10} \bar{\ell} \gamma_5 \ell + \frac{\alpha_{\text{em}}}{4\pi} Q_\ell Q_q \frac{m_\ell m_B}{m_\ell} f_{Bq} N \bar{\ell}(1 + \gamma_5) \ell \]

\[ \times \left\{ \int_0^1 du (1 - u) C_9^{\text{eff}} (u m_\ell^2) \int_0^\infty d\omega \phi_{B+}(\omega) \left[ \ln \frac{m_\ell \omega}{m_\ell^2} + \ln \frac{u}{1 - u} \right] - Q_\ell C_7^{\text{eff}} \int_0^\infty d\omega \omega \phi_{B+}(\omega) \left[ \ln^2 \frac{m_\ell \omega}{m_\ell^2} - 2 \ln \frac{m_\ell \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\} \]

▶ Tree level amplitude

Robert Szafron
Auxiliary slide: The QED correction at the amplitude level

\[ iA = m_\ell f_{Bq} N C_{10} \bar{\gamma}_5 \ell + \frac{\alpha_\text{em}}{4\pi} Q_\ell Q_q m_\ell m_B f_{Bq} N \bar{\ell}(1 + \gamma_5) \ell \]

\[ \times \left\{ \int_0^1 du \ (1 - u) C_{9}^{\text{eff}} (um_b^2) \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \left[ \ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1 - u} \right] \right\} \]

\[ - Q_\ell C_{7}^{\text{eff}} \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \left[ \ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \}

- Helicity suppression \times power enhancement factor

Robert Szafron
Auxiliary slide: The QED correction at the amplitude level

\[ iA = m_\ell f_{Bq} N C_{10} \bar{\ell} \gamma_5 \ell + \frac{\alpha_{\text{em}}}{4\pi} Q_\ell Q_q \ m_\ell m_B \ f_{Bq} N \bar{\ell}(1 + \gamma_5)\ell \]

\[ \times \left\{ \int_0^1 du \ (1 - u) \ C_9^{\text{eff}} (u m_\ell^2) \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1 - u} \right\} \]

\[ - Q_\ell C_7^{\text{eff}} \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \left[ \ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \}

Convolution with the light-cone distribution function
Auxiliary slide: The QED correction at the amplitude level

\[ i A = m_\ell f_{Bq} N C_{10} \bar{\ell} \gamma_5 \ell + \frac{\alpha_{em}}{4\pi} Q_\ell Q_q \ m_\ell m_B \ f_{Bq} N \bar{\ell}(1 + \gamma_5) \ell \]

\[ \times \left\{ \int_0^1 du \ (1 - u) \ C_{eff}^9 (u m_\ell^2) \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \left[ \ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1 - u} \right] 
- Q_\ell C_{eff}^7 \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \left[ \ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \right\} \]

- Double logarithmic enhancement due to endpoint singularity

\[ \frac{1}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega, \mu), \]

\[ \frac{\sigma_n(\mu)}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\mu_0}{\omega} \phi_{B+}(\omega, \mu) \]

$\lambda_B(1 \text{ GeV}) = (275 \pm 75) \text{ MeV}$
$\sigma_1(1 \text{ GeV}) = 1.5 \pm 1$
$\sigma_2(1 \text{ GeV}) = 3 \pm 2$

Power-enhancement factor

\[ m_B \int_0^\infty \frac{d\omega}{\omega} \phi_{B+}(\omega) \ln^k \omega \sim \frac{m_B}{\lambda_B} \times \sigma_k \]
At the one-loop order in dimensional regularization with $d = 4 - 2\epsilon$, the bare soft function must have a simple dependence

$$\tilde{S}_{0,\text{bare}}(x) = 1 + \frac{\alpha_s}{\pi} \left(-n_- x n_+ x \mu^2\right)^\epsilon f \left(\epsilon, \frac{x^2}{n_+ x n_-}\right)$$

Explicit evaluation gives

$$\tilde{S}_{0,\text{bare}}(x) = 1 + \frac{\alpha_s C_F}{\pi} \frac{\Gamma(1 - \epsilon)}{\epsilon^2} e^{-\epsilon \gamma_E} \times$$

$$\left(-\frac{1}{4} n_- x n_+ x \mu^2 e^{2\gamma_E}\right)^\epsilon \left(\frac{x^2}{n_- x n_+ x}\right)^{1+\epsilon} {}_2F_1\left(1, 1, 1 - \epsilon; 1 - \frac{x^2}{n_- x n_+ x}\right)$$

$$= 1 + \frac{\alpha_s C_F}{\pi} \left(\frac{1}{\epsilon^2} + \frac{L}{\epsilon} + \frac{L^2}{2} + \frac{\pi^2}{12} + \text{Li}_2\left(1 - \frac{x^2}{n_- x n_+ x}\right) + \mathcal{O}(\epsilon)\right)$$

where we defined

$$L \equiv \ln\left(-\frac{1}{4} n_- x n_+ x \mu^2 e^{2\gamma_E}\right).$$
Auxiliary slide: The Drell-Yan Process

\[ A(p_A) B(p_B) \rightarrow \text{DY}(Q) + X \]

Light-cone coordinates

\[
\begin{align*}
n^\mu_+ &= (1, 0, 0, +1) \\
n^\mu_- &= (1, 0, 0, -1)
\end{align*}
\]

hard

\[
p_h = (n^+ p_h, n^- p_h, p_{h\perp}) \sim Q(1, 1, 1)
\]
collinear

\[
p_c = (n^+ p_c, n^- p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda)
\]
anticollinear

\[
p_{\bar{c}} = (n^+ p_{\bar{c}}, n^- p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda)
\]
soft

\[
p_s = (n^+ p_s, n^- p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2)
\]

\[ z = \frac{Q^2}{\hat{s}} \quad \lambda = \sqrt{1 - z} \]

\[ p_c - \text{PDF} \sim (Q, \Lambda^2/Q, \Lambda) \]
When considering power corrections we have to be careful about kinematic factors. Consider the hard function

$$\bar{\psi} \gamma_{\mu} \psi(0) = \int dt d\bar{t} \tilde{C}^{A_0}(t, \bar{t}) J^{A_0}_{\mu}(t, \bar{t}), \quad H(\hat{s}, \mu_h) = |C^{A_0}(-\hat{s})|^2$$

we can obtain power corrections from expansion

$$H(\hat{s}) = H(Q^2) + Q^2 (1 - z) H'(Q^2) + \ldots$$

The LP factorization is

$$\hat{\sigma}(z) = H(Q^2) Q S_{DY}(Q(1 - z))$$

with

$$H(\hat{s}) = 1 + \mathcal{O}(\alpha_s) \quad \text{and} \quad S_{DY}(\Omega) = \delta(\Omega) + \mathcal{O}(\alpha_s)$$

so at the LL accuracy it is enough to consider $H(Q^2)$
Auxiliary slide: Hard function running

Well known RGE for two-jet operator

\[ \frac{d}{d \ln \mu} H(Q^2, \mu) = \left( 2 \Gamma_{\text{cusp}} \ln \frac{Q^2}{\mu^2} + 2\gamma \right) H(Q^2, \mu) \]

\[ \Gamma_{\text{cusp}} = \frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2), \quad \gamma = -\frac{3}{2} \frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2), \]

The general solution RGE reads

\[ H(Q^2, \mu) = \exp \left[ 4S(\mu_h, \mu) - 2a_\gamma(\mu_h, \mu) \right] \left( \frac{Q^2}{\mu_h^2} \right)^{-2a_\Gamma(\mu_h, \mu)} H(Q^2, \mu_h) \]

where

\[ S(\nu, \mu) = - \int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} d\alpha' \frac{1}{\beta(\alpha')} \]

\[ a_\Gamma(\nu, \mu) = - \int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)}, \quad a_\gamma(\nu, \mu) = - \int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma(\alpha)}{\beta(\alpha)} \]
There is no collinear function present at LP because of decoupling transformation.

This is no longer true at NLP. Consider an example of subleading SCET Lagrangian:
\[ \mathcal{L}_{2\xi}^{(2)} = \frac{1}{2} \bar{\chi}_c z_\perp^\mu z_\perp^\rho \left[ i\partial_\rho \imath n - \partial \mathcal{B}_\mu^+ (z_-) \right] \frac{\eta^+}{2} \chi_c \]

Crucially, an insertion of a piece of a subleading Lagrangian comes with an integral over its position,
\[ \int d^4 z \]

\[ \left( J_{A0,2\xi}^T (s, t) \right)^\mu = i \int d^4 x \mathbf{T} \left[ J_{A0}^\mu (s, t) \mathcal{L}_{2\xi}^{(2)} (x) \right] \]