Next-to-leading power corrections and resummation

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Outline

- Power corrections and SCET framework
- QED corrections to $B_s \to \mu^+\mu^-$
- Threshold resummation

Why study power-corrections?

$$\mathcal{O} = \sum_{k=0}^{\infty} \sum_{i=i_{\min}}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^k \lambda^i \, a_{ki}(\ln\lambda, \ldots)$$

with λ being some small parameter, e.g. $\lambda \sim \frac{\Lambda_{\rm QCD}}{m_b}$ or $\lambda \sim 1 - \frac{Q^2}{s^2}$
such that observable $\mathcal{O}$ simplifies (factorizes) in the limit $\lambda \to 0$

We want to go beyond leading power in order to

- increase precision of theory predictions (understand new effects such as QED corrections in flavor physics, or how to match fixed order and resummed result)

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- better understand all order structure of QFT (universality, new structures, ...)
- use a bootstrap procedure to reconstruct full amplitudes from its limits
- improve the accuracy of numerical computations – fixed order subtraction methods based on factorization [M. Ebert, I. Moult, I.Stewart, F. Tackmann, G. Vita, H. Zhu, JHEP 1812 (2018) 084]

Power corrections in SCET

Effective field theory approach allows to systematically compute power-corrections and perform resummation.

- Power suppressed Lagrangian

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_{\text{LP}} + \mathcal{L}_{\xi}^{(1)} + \mathcal{L}_{\xi}^{(2)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\xi q}^{(2)} + \mathcal{O}\left(\lambda^3\right)$$

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$$\begin{aligned}
\mathcal{L}_{\xi}^{(1)} &= \bar{\xi}\left(x_\perp^\mu n_-^\nu\, W_c\, g_s F_{\mu\nu}^s W_c^\dagger\right)\frac{\not{n}_+}{2}\xi, \\
\mathcal{L}_{\xi}^{(2)} &= \frac{1}{2}\bar{\xi}\left((n_-x)\, n_+^\mu n_-^\nu\, W_c\, g_s F_{\mu\nu}^s W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu W_c \left[D_s^\rho, g_s F_{\mu\nu}^s\right] W_c^\dagger\right)\frac{\not{n}_+}{2}\xi \\
&\quad + \frac{1}{2}\bar{\xi}\left(i\not{D}_\perp \frac{1}{in_+D}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, g_s F_{\mu\nu}^s W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu\, W_c\, g_s F_{\mu\nu}^s W_c^\dagger \frac{1}{in_+D}\, i\not{D}_\perp\right) \\
\mathcal{L}_{\xi q}^{(1)} &= \bar{q}\, W_c^\dagger i\not{D}_\perp \xi - \bar{\xi}\, i\overleftarrow{\not{D}}_\perp W_c q, \\
\mathcal{L}_{\xi q}^{(2)} &= \bar{q}\, W_c^\dagger \left(in_-D + i\not{D}_\perp\, (in_+D)^{-1}\, i\not{D}_\perp\right)\frac{\not{n}_+}{2}\xi + \bar{q}\, \overleftarrow{D}_s^\mu x_{\perp\mu} W_c^\dagger\, i\not{D}_\perp \xi \\
&\quad - \bar{\xi}\frac{\not{n}_+}{2}\left(in_-\overleftarrow{D} + i\overleftarrow{\not{D}}_\perp \left(in_+\overleftarrow{D}\right)^{-1} i\overleftarrow{\not{D}}_\perp\right) W_c\, q - \bar{\xi}\, i\overleftarrow{\not{D}}_\perp W_c\, x_{\perp\mu} D_s^\mu q.
\end{aligned}$$

[M. Beneke, T. Feldmann, Phys.Lett. B553 (2003) 267]

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- Power suppressed currents

$$\chi = W^\dagger \xi \sim \lambda$$
$$\mathcal{A}_\perp^\mu = W^\dagger D_\perp^\mu W \sim \lambda$$

$$\begin{aligned}
J^{A0}(s) &= \chi(n_+ s) \\
J^{A1}(s) &= \partial_\perp \, \chi(n_+ s) \\
J^{B1}_{\mathcal{A}\chi}(s,t) &= \mathcal{A}_\perp(n_+ t) \, \chi(n_+ s) \\
J^{B1}_{\chi\chi}(s,t) &= \chi(n_+ t) \, \chi(n_+ s)
\end{aligned}$$

[M. Beneke, M. Garny, RS, J. Wang, JHEP 1803 (2018) 001]
[M. Beneke, M. Garny, RS, J. Wang, JHEP 1811 (2018) 112]

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- Power suppressed currents
- Kinematic corrections

QED corrections to $B_s \to \mu^+\mu^-$

Why do we need to know the QED corrections in flavor physics?

- Large logarithmic $\ln\left(m_b^2/m_\ell^2\right)$ enhancements can mimic lepton-flavor universality violation
- Soft photon approximation employed so far is not suitable for virtual photons that can resolve the B-meson.
- Factorization theorems do not yet exist for QED
- Expected precision of measurements may require the inclusion of QED corrections or at least a proof that no effects above 1% exist

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Our starting point

$$B_s \to \mu^+\mu^-$$

In the SM the process is

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- loop suppressed (FCNC)
- helicity suppressed (scalar meson decaying into energetic muons, vector interaction), $\mathcal{A} \sim m_\mu$
- purely leptonic final state allows for a precise SM prediction, QCD contained in the meson decay constant f_{B_s} [C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou, M. Steinhauser, Phys.Rev.Lett. 112 (2014) 101801]

QED corrections in QCD bound-states

The final state has no strong interaction – QCD is contained in the decay constant

$$\langle 0|\, \bar{q}(0)\gamma^\mu\gamma_5 b(0)\, |\bar{B}_q(p)\rangle = i f_{B_q} p^\mu$$

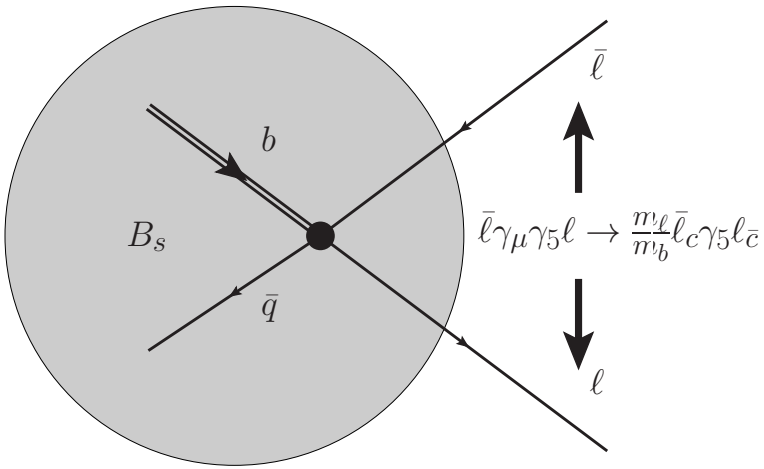
This is no longer true when QED effects are included – non-local time ordered products have to be evaluated

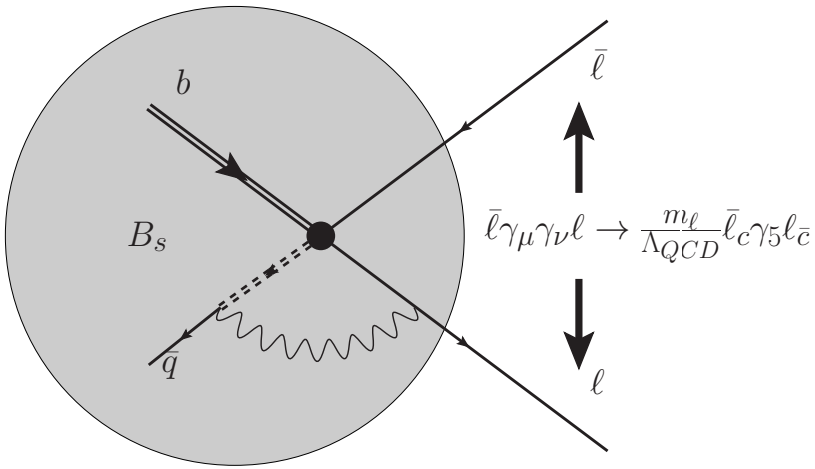
$$\langle 0| \int d^4x\, e^{iqx} T\{j_{\rm QED}(x), \mathcal{L}_{\Delta B=1}(0)\}|\bar{B}_q\rangle$$

This can be done for QED bound-states but QCD is non-perturbative at low scales

Helicity suppression

Can the helicity suppression be relaxed?





Without QED:

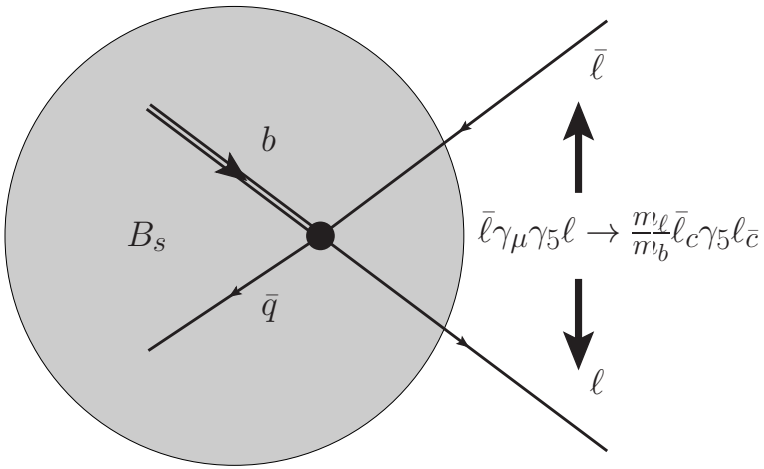
$$u(p_\ell) = u_c(p_\ell) + \mathcal{O}\left(\frac{m_\ell}{E_\ell}\right)$$

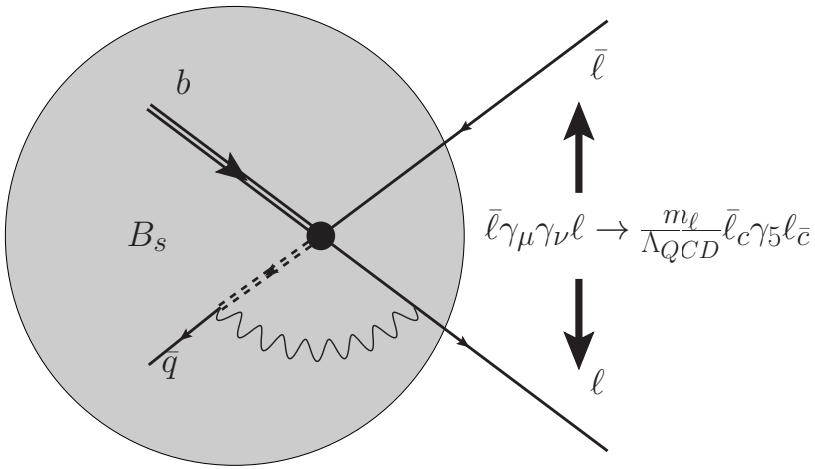
For $m_\ell \to 0$ the amplitude has to vanish

Annihilation and helicity flip take place at the same point $r \lesssim \frac{1}{m_b}$

Helicity suppression

Can the helicity suppression be relaxed?





With QED:

Annihilation and helicity flip can be separated by $r \sim \frac{1}{\sqrt{m_b \Lambda_{\rm QCD}}}$

It is still a short distance effect since the size of the meson is $r \sim \frac{1}{\Lambda_{\rm QCD}}$

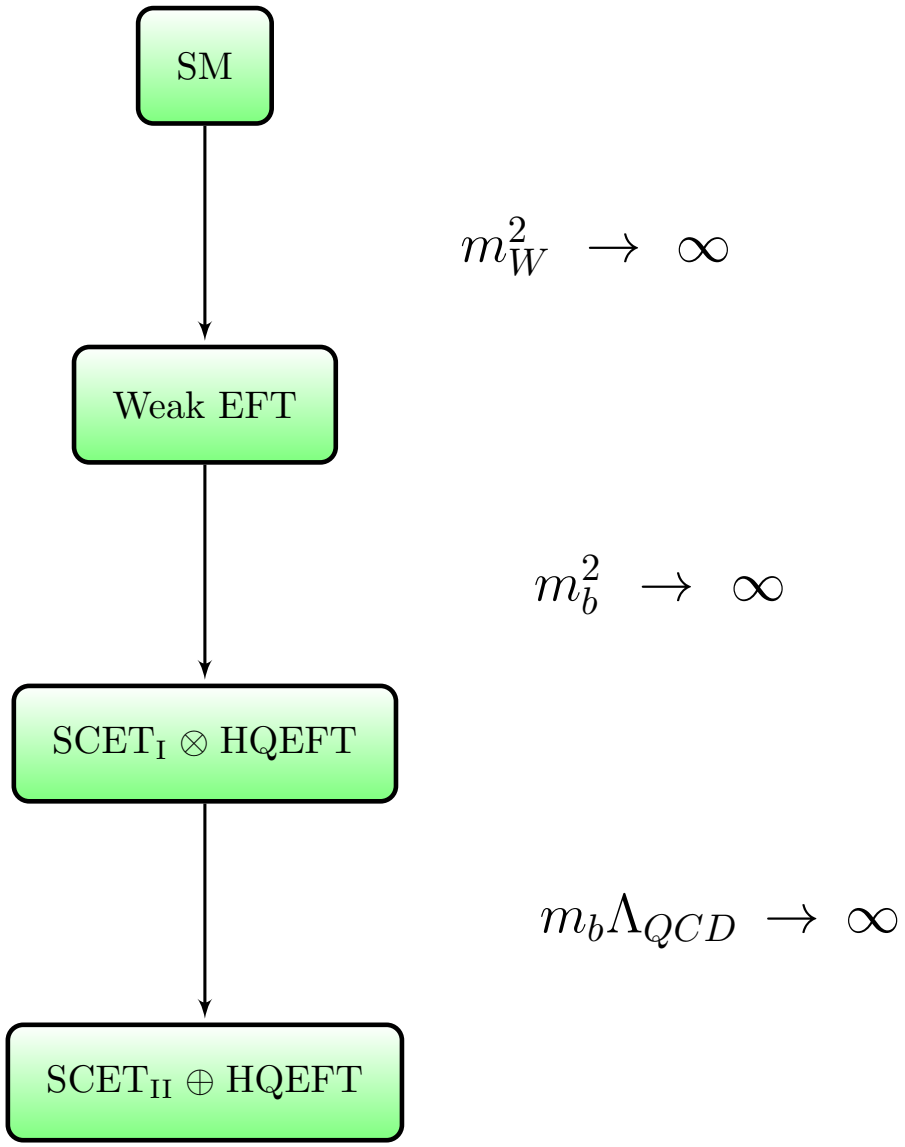
For $m_\ell \to 0$ the amplitude still vanishes

Scales in the problem

Leptonic decay of B_s is a multi-scale problem

- Electroweak scale m_W
- Hard scale m_b
- Hard-collinear scale $\sqrt{m_b \Lambda_{\rm QCD}}$
- Soft scale $\Lambda_{\rm QCD}$
- Collinear scale m_μ

We take $\Lambda_{\rm QCD} \sim m_\mu$ so the soft scale of HQEFT is also a soft scale of $\text{SCET}_{\rm I}$



C_9 contribution, hard matching

Typical SCET I $\to$ SCET II problem but with $m_\ell \neq 0$

$$\sum_i C_i\left(\mu_b\right)\mathcal{O}_i \to \sum_i \int du\, \mathcal{H}_i\left(u\right)\overline{J}_i\left(u\right),$$

Weak operators are matched on B-type SCET I currents

$$J_9\left(s,t\right) = \left[\overline{\chi_C}\left(n_+ s\right)\gamma_\perp^\mu P_L h_\nu\left(0\right)\right]\left[\overline{\ell_C}\left(n_+ t\right)\gamma_\mu^\perp \ell_{\overline{C}}\left(0\right)\right]$$

In the first step, the quark is hard-collinear rather than soft!

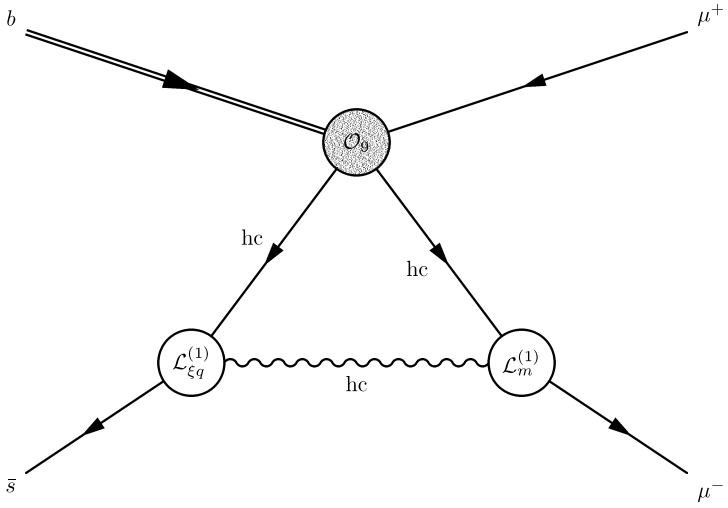
$$\chi_C \sim \lambda; \quad q_s \sim \lambda^3$$

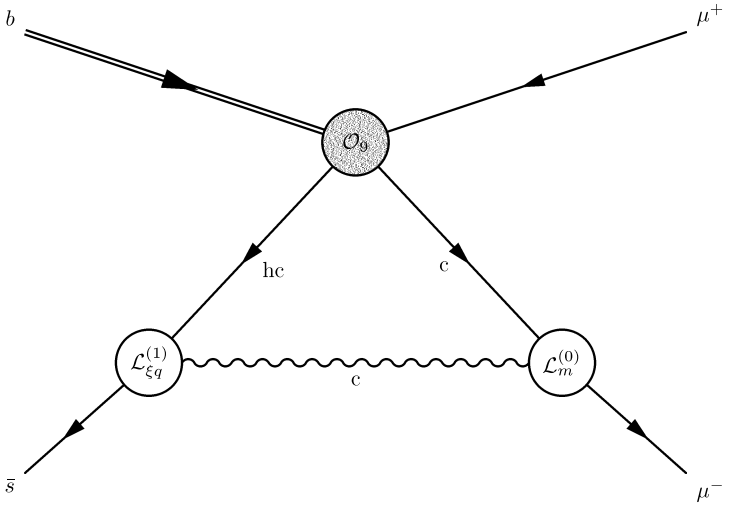
We work with Fourier transformed currents and coefficients

$$\overline{J}_9\left(u\right) = n_+ p_\ell \int \frac{dr}{2\pi}\exp\left[-iun_+p_\ell r\right] J_9\left(0,r\right)$$

$$\mathcal{H}_9\left(u\right) = C_9^{\text{eff}}\left(u\right) + \mathcal{O}\left(\alpha_{em}\right)$$

SCET I diagrams





- We need $\mathcal{L}^{(1)}_{\xi q}$ to convert the hc-quark into a soft quark
- For pure hc-interaction, mass is power suppressed $m_\ell \sim \lambda^2$, $p_\perp^{hc} \sim \lambda$, $\mathcal{L}^{(1)}_{hc,m}$
- For pure c-interaction, mass is included in the leading power Lagrangian, $m_\ell \sim \lambda^2 \sim p_\perp^c$, $\mathcal{L}^{(0)}_{c,m}$

SCET II

Two operators are relevant for the power-enhanced contribution

$$\overline{J}_9(u) \to \int d\omega\, \mathcal{J}_a(u;\omega)\, \overline{J}^{A1}_{m\chi}(\omega) + \int d\omega \int dw\, \mathcal{J}_b(u;\omega,w)\, \overline{J}^{B1}_{\mathcal{A}\chi}(\omega,w)$$

- Hard-collinear jet functions $\sim \frac{1}{\omega}$

$$\begin{aligned} J^{A1}_{m\chi}(v) &= \left[\overline{q_s}(vn_-)\frac{\not{n}_-}{2}\Gamma_\perp P_L h_v(0)\right]\left[m_\ell\, \overline{\ell}_c(0)\,\Gamma_\perp \ell_{\overline{c}}(0)\right]\left[Y_+(0)\,Y_-^\dagger(0)\right] \\ J^{B1}_{\mathcal{A}\chi}(t,v) &= \left[\overline{q_s}(vn_-)\frac{\not{n}_-}{2}\Gamma_\perp P_L h_v(0)\right]\left[\mathcal{A}^\nu_\perp(n_+t)\,\overline{\ell}_c(0)\,\Gamma_\perp \ell_{\overline{c}}(0)\right]\left[Y_+(0)\,Y_-^\dagger(0)\right] \end{aligned}$$

$$\begin{aligned} \mathcal{J}_a(u;\omega) &\sim \mathcal{O}(\alpha_{\rm em}) \\ \mathcal{J}_b(u;\omega,w) &\sim \mathcal{O}(1) \end{aligned}$$

We use soft gauge QED-QCD invariant building blocks e.g.

$$q_s(x) = Y_+^\dagger(x) Y^\dagger_{+\rm QCD}(x) q(x)$$

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- Hard-collinear jet functions $\sim \frac{1}{\omega}$
- Additional QED soft Wilson lines

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Factorization and resummation

$$\begin{aligned}\mathcal{A} &= -if_{B_q} m_b m_\ell Z_\ell \, Z_{\overline{\ell}} \, T_+ \int du \, \mathcal{H}_9 \left(u\right) \\ &\times \int d\omega \, \phi_+ \left(\omega\right) \left(\mathcal{J}_a \left(u;\omega\right) + \int dw \mathcal{J}_b \left(u;\omega,w\right) M \left(w\right) \right)\end{aligned}$$

Modified B-meson LCDA

$$\phi_+(\omega) \sim \langle 0| \left[\overline{q} \left(n_- v\right)\right]_\beta \left[h_v \left(0\right)\right]_\alpha Y_+ \left(0\right) Y_-^\dagger \left(0\right) \left|\overline{B_q} \left(p\right)\right\rangle$$

The anticollinear contribution is symmetric $\rightarrow$ multiply by 2 to get the total result

LL resummation

$$\frac{d}{d \ln \mu} \begin{pmatrix} \mathcal{J}_b \left(u;\omega,w;\mu\right) \\ \mathcal{J}_a \left(u;\omega;\mu\right) \end{pmatrix} =$$

$$- \begin{pmatrix} \Gamma^b_{\text{cusp}} \ln \mu/\mu_{hc} & 0 \\ -Q_\ell \frac{\alpha_{em}}{2\pi} \overline{w} & \Gamma^a_{\text{cusp}} \ln \mu/\mu_{hc} \end{pmatrix} \begin{pmatrix} \mathcal{J}_b \left(u;\omega,w;\mu\right) \\ \mathcal{J}_a \left(u;\omega;\mu\right) \end{pmatrix}$$

Numerical prediction: the power-enhanced correction

Total power-enhanced correction changes the branching fraction by:

$$-(0.3 - 1.1)\%$$

[M. Beneke, C. Bobeth, RS, Phys.Rev.Lett. 120 (2018) no.1, 011801]
Cancellation between C_7 and C_9 part
central value:

$$-0.6\% = 1.1\% - 1.7\% \ (C_7, C_9 \text{ parts})$$

Uncertainty comes from λ_B, σ_1, σ_2. C_7 part is surprisingly large thanks to double logs enhancement.

The previous estimate of QED uncertainty was 0.3%, obtained by scale variation method. This uncertainty is still present due to non-enhanced corrections.

Threshold resummation
Drell-Yan production, $q\overline{q}$ channel

Breakdown of perturbative description

In certain regions of the phase space, perturbative expansion breaks down!

Consider threshold expansions $z \to 1$ $\left(z = \frac{Q^2}{\hat{s}}\right)$ of DY production

$$A(p_A)B(p_B) \to \mathrm{DY}(Q) + X$$

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \alpha_s^n \Bigg[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + +d_{nm} \ln^m(1-z) \right) + \ldots \Bigg]$$

- Expansion parameter is not $\frac{\alpha_s}{4\pi}$ but $\frac{\alpha_s}{4\pi} \ln^2(1-z)$

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- Expansion parameter is not $\frac{\alpha_s}{4\pi}$ but $\frac{\alpha_s}{4\pi} \ln^2(1-z)$
- Resummation is necessary to get meaningful results

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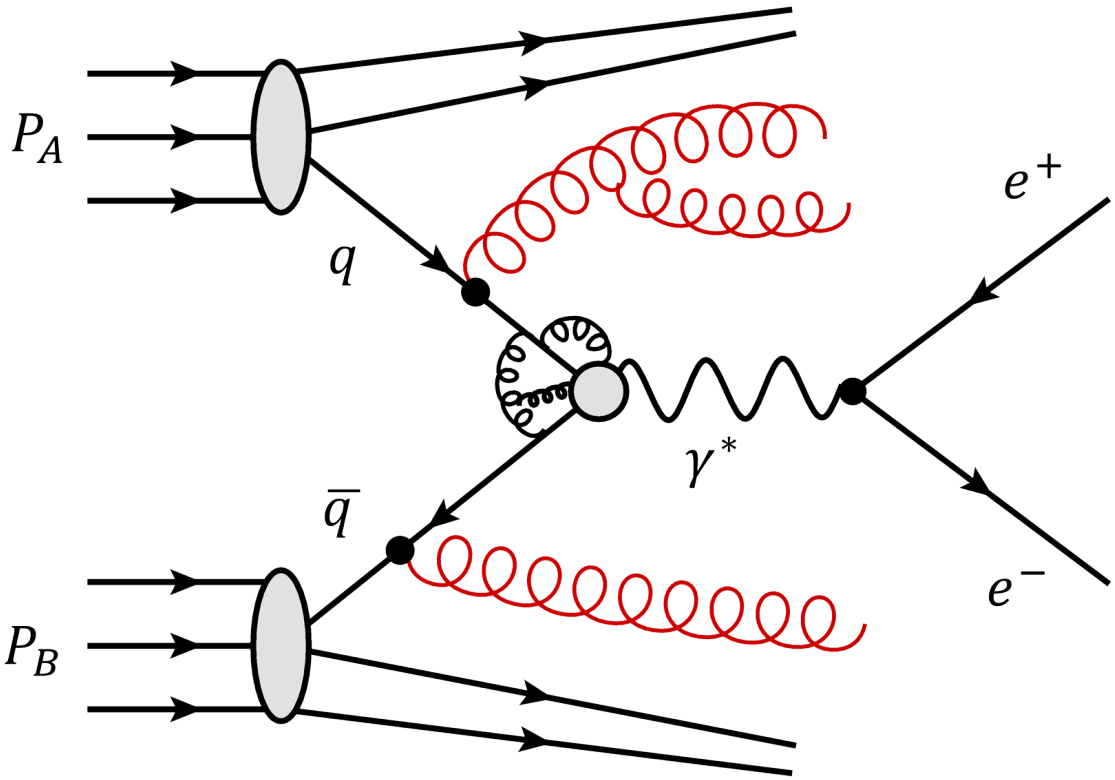
- Leading power
- Next-to-leading power

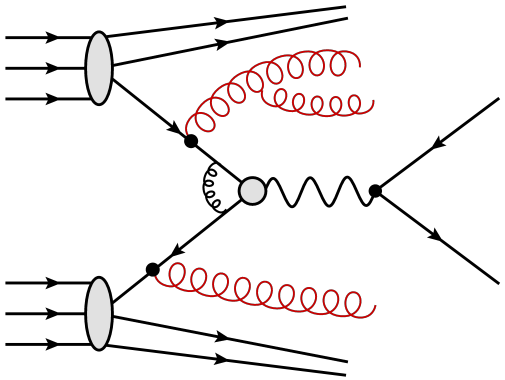
Leading power factorization in SCET

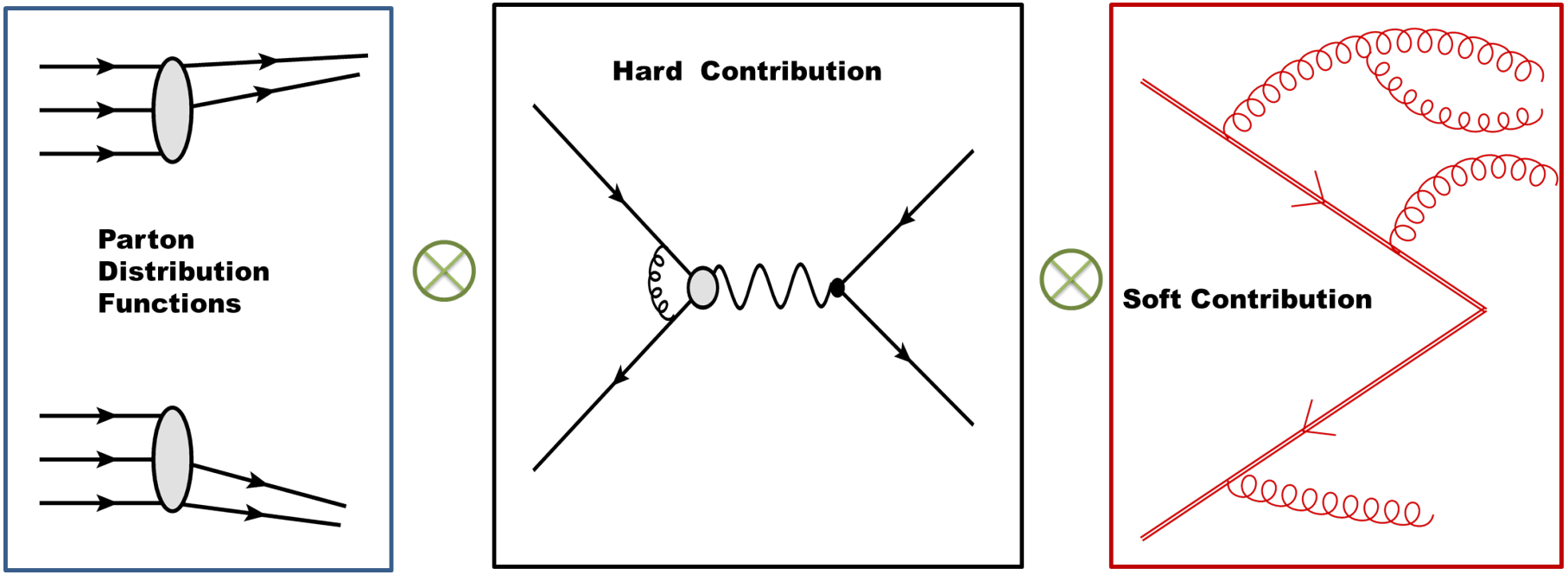
Drell-Yan production process $pp \to \ell^+\ell^- + X$



Leading power factorization in SCET

Leading power Drell-Yan factorization





The Drell-Yan process - the leading power result

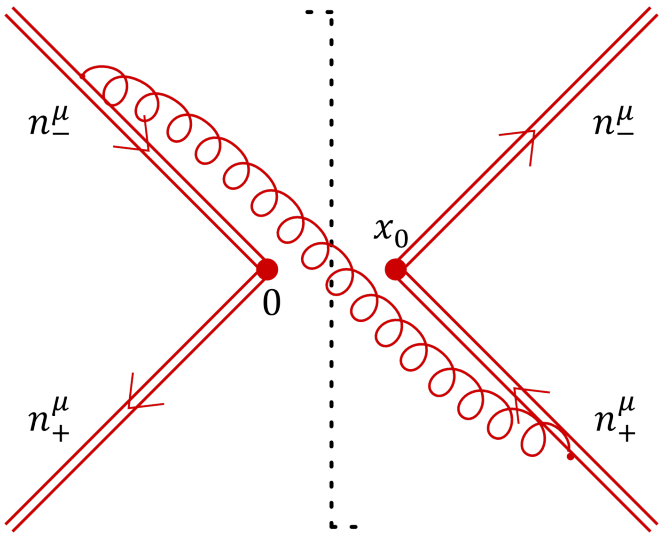
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

where

$$\hat{\sigma}(z) = |C(Q^2)|^2 \, Q S_{\rm DY}(Q(1-z))$$

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi} \, e^{ix^0\Omega/2} \, \frac{1}{N_c} \, {\rm Tr} \, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0)) \, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

[G. Korchemsky, G. Marchesini, Phys.Lett. B313 (1993) 433]
[T. Becher, M. Neubert and G. Xu, JHEP 07 (2008) 030]



Factorization formula at NLP

A schematic formula

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\hat{\sigma} = \sum_{\rm terms} [C \otimes J \otimes \bar{J}]^2 \otimes S$$

- C is the hard Wilson matching coefficient $\mu_h \sim Q$

[M. Beneke, A. Broggio, M. Garny, S. Jaśkiewicz, RS, L. Vernazza, J. Wang arXiv:1809.10631]

Factorization formula at NLP

A schematic formula

$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm em}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$\hat{\sigma} = \sum_{\rm terms} [C \otimes J \otimes \bar{J}]^2 \otimes S$$

- C is the hard Wilson matching coefficient $\mu_h \sim Q$
- S is the *generalized* soft function $\mu_s \sim Q(1-z)$

[M. Beneke, A. Broggio, M. Garny, S. Jaśkiewicz, RS, L. Vernazza, J. Wang arXiv:1809.10631]

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- S is the *generalized* soft function $\mu_s \sim Q(1-z)$
- J is the collinear function $\mu_c \sim Q(1-z)^{\frac{1}{2}}$

Each object depends only on a single scale!

Renormalization scale $\mu \leftrightarrow$ physical scale, e.g.

$$S(\mu_s, \mu) = S\left(\ln(\tfrac{\mu_s}{\mu})\right)$$

$$J(\mu_c, \mu) = J\left(\ln(\tfrac{\mu_c}{\mu})\right)$$

[M. Beneke, A. Broggio, M. Garny, S. Jaśkiewicz, RS, L. Vernazza, J. Wang arXiv:1809.10631]

Physics at NLP

- Collinear function is a matching coefficient
- Lagrangian insertions at various positions each with its own ω_i

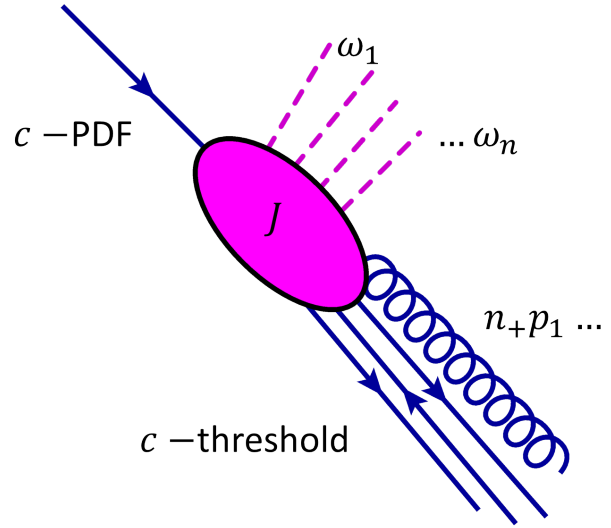
We can separate the Lagrangian insertions

$$\mathcal{L}_V^{(n)}(z) = \mathcal{L}_c^{(n)}(z) \otimes \mathcal{L}_s^{(n)}(z_-)$$

$$i^n \left(\sum_{i=1}^{n} \int d^4 z_j \, e^{i\,\omega_j \, \frac{n_+ z_j}{2}} \right)$$

$$\times \, \mathbf{T} \left[\chi_c(t n_+) \times \mathcal{L}_c^{(n)}(z_1) \times \mathcal{L}_c^{(n)}(z_2) \times \ldots \right]$$

$$= J\,(t; \omega_1, \omega_2, \ldots)\, \chi_c^{\mathrm{PDF}}(t n_+)$$



Collinear function is a non-local object

Next to leading power soft function

Soft operator in position space is a non-local object

$$\widetilde{\mathcal{S}}_{2\xi}\left(x, z_-\right) = \bar{\mathbf{T}}\left[Y_+^\dagger(x) Y_-(x)\right] \mathbf{T}\left[Y_-^\dagger(0) Y_+(0) \frac{i\partial_\perp^\nu}{in_-\partial} \mathcal{B}_{\perp\nu}^+(z_-)\right]$$

with decoupled soft fields

$$\mathcal{B}_\pm^\mu = Y_\pm^\dagger \left[iD_s^\mu Y_\pm\right]$$

Lagrangian is already multipole expanded $\rightarrow$ soft fields depend only on z_-

$$\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2} \bar{\chi}_c z_\perp^\mu z_\perp^\nu \left[i\partial_\nu in_-\partial \mathcal{B}_\mu^+\right] \frac{\not{n}_+}{2} \chi_c$$

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In the factorization theorem, we need only vacuum matrix element

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+ z)}{4\pi}\, e^{ix^0\Omega/2 - i\omega(n_+ z)/2}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

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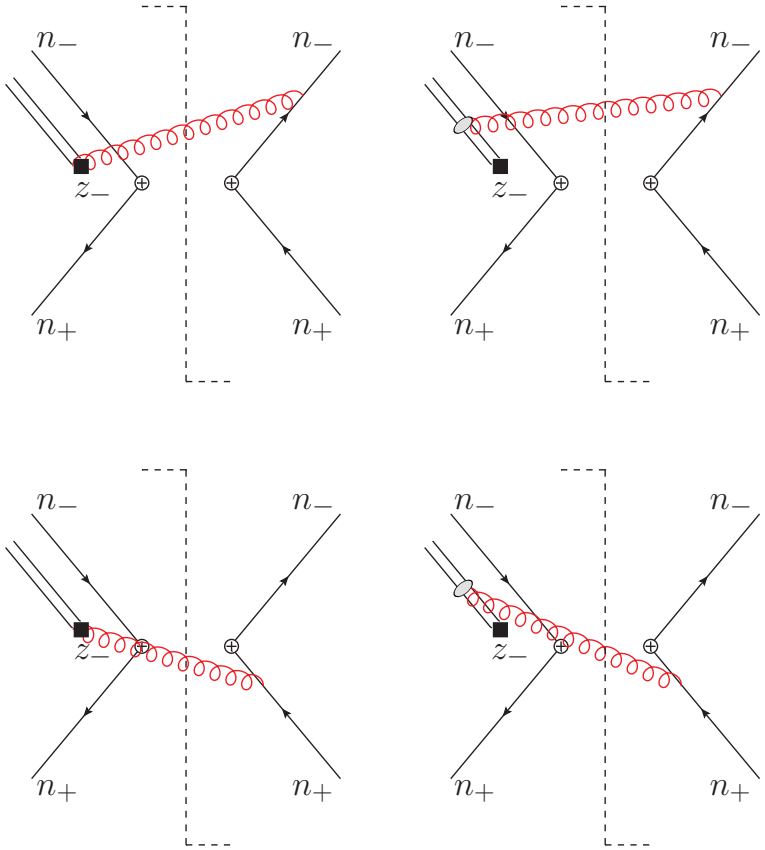

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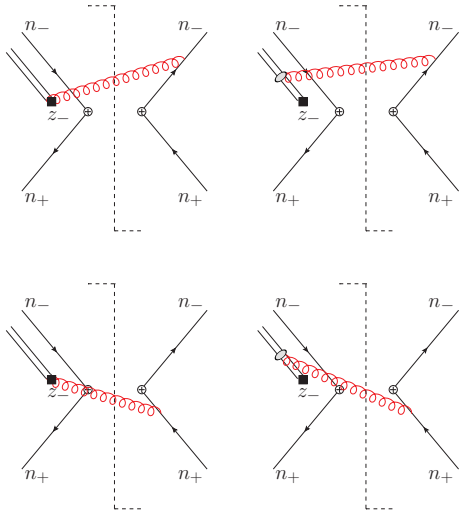


[M. Beneke, A. Broggio, M. Garny, S. Jaśkiewicz, RS, L. Vernazza, J. Wang arXiv:1809.10631]

$$S_{2\xi}(\Omega, \omega) = \frac{\alpha_s C_F}{2\pi} \left\{ \theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon} + \ln\frac{\Omega^2}{\mu^2}\right) + \left[\frac{1}{\omega}\right]_+ \theta(\omega)\theta(\Omega - \omega) \right\}$$

Renormalization and resummation

Operator mixing $\rightarrow$ we need to introduce a new soft function whose tree-level matrix element is non-zero

$$\widetilde{S}_{x0}(x^0) = \frac{-2i}{x^0 - i\epsilon}\frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\,\mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

At the LL we have

$$\frac{d}{d\ln\mu}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x_0}(\Omega) \end{pmatrix} = \frac{\alpha_s}{\pi}\begin{pmatrix} 4C_F\ln\frac{\mu}{\mu_s} & -C_F\delta(\omega) \\ 0 & 4C_F\ln\frac{\mu}{\mu_s} \end{pmatrix}\begin{pmatrix} S_{2\xi}(\Omega,\omega) \\ S_{x_0}(\Omega) \end{pmatrix}$$

$$\begin{aligned} S_{2\xi}^{\mathrm{LL}}(\Omega,\omega,\mu) &= \frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\exp\left[-4S^{\mathrm{LL}}(\mu_s,\mu)\right]\theta(\Omega)\delta(\omega) \\ &= C_F\frac{\alpha_s}{\pi}\ln\frac{\mu_s}{\mu}\exp\left[-2C_F\frac{\alpha_s}{\pi}\ln^2\frac{\mu_s}{\mu}\right]\theta(\Omega)\delta(\omega) \end{aligned}$$

Log from the mixing Sudakov double log

Resummed result

- Kinematic corrections can be treated in a similar way
  $\rightarrow$ LL cancel for $\frac{\hat{\sigma}^{\rm LL}_{\rm NLP}(z,\mu)}{z}$
- Collinear function associated with the $\frac{i\partial^\nu_\perp}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)$ operator does not have LL
- Resummed hard function is equal to the LP one

Resummed partonic cross-section for the $q\bar{q}$ channel

$$\frac{\hat{\sigma}^{\rm LL}_{\rm NLP}(z,\mu)}{z} = \exp\left[2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s} - 2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right] \times(-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

[M. Beneke, A. Broggio, M. Garny, S. Jaśkiewicz, RS, L. Vernazza, J. Wang arXiv:1809.10631]

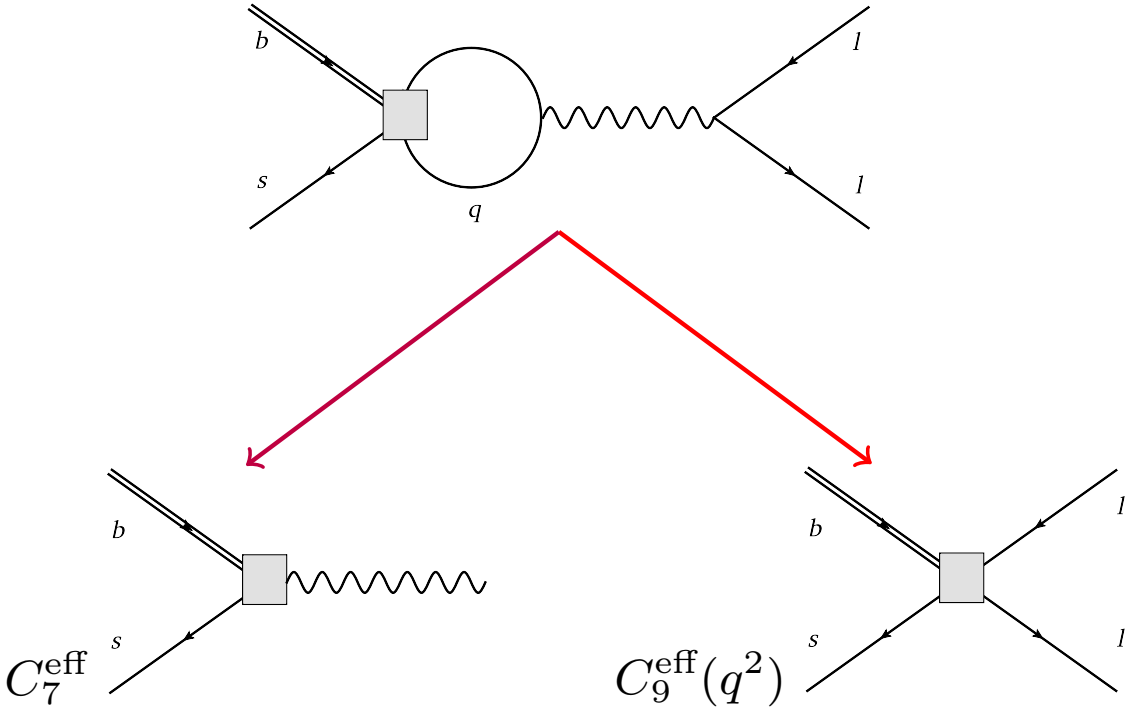
Summary and outlook

- Application of effective field theory methods allows for a systematic study of power corrections in QED and QCD for processes involving energetic particles
- QED corrections in flavor physics should be further analyzed to achieve necessary precision. Methods based on soft photon approximation or LL are not good enough to achieve the necessary accuracy
- We performed NLP LL threshold resummation, further studies are necessary to go beyond LL and extend results to different channels/observables
- Many applications for future colliders

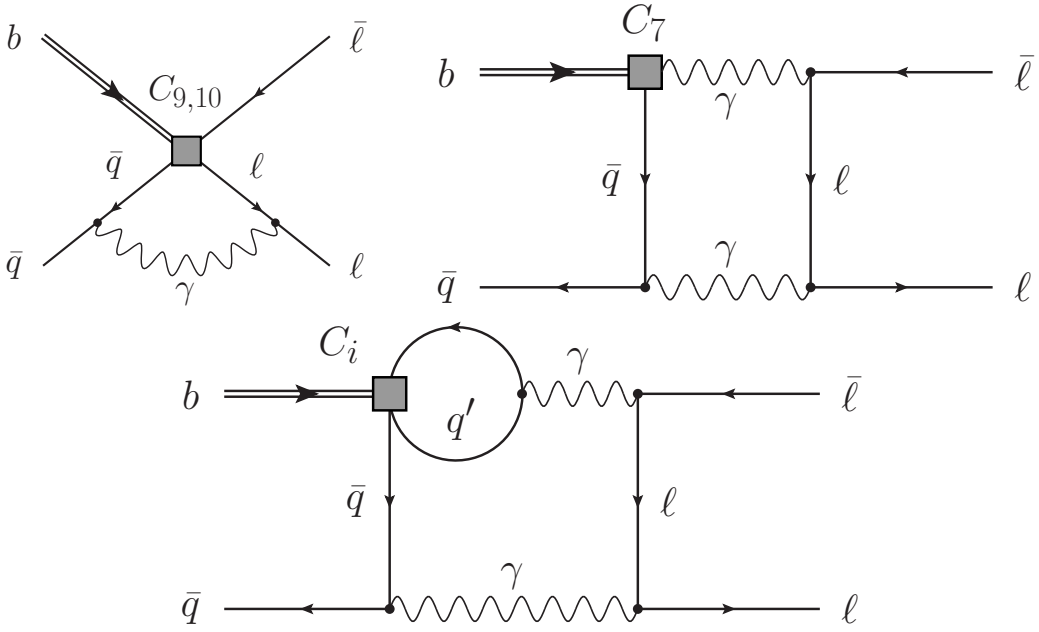
Auxiliary slide: Operators at the hard scale

$$
\begin{aligned}
Q_9 &= \frac{\alpha_{\rm em}}{4\pi}(\bar{q}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell) \\
Q_{10} &= \frac{\alpha_{\rm em}}{4\pi}\left(\bar{q}\gamma^\mu P_L b\right)\left(\bar{\ell}\gamma_\mu\gamma_5 \ell\right) \\
Q_7 &= \frac{e}{16\pi^2} m_b \left(\bar{q}\sigma^{\mu\nu} P_R b\right) F_{\mu\nu}
\end{aligned}
$$

Four-quark operators can be treated as a modification of C_9 and C_7

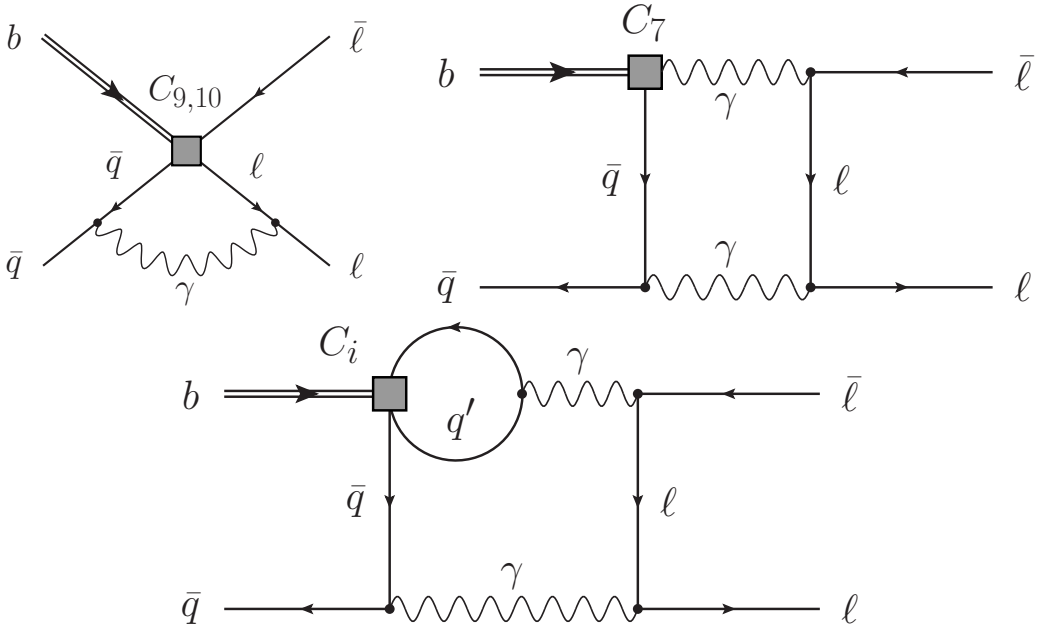


Auxiliary slide: The QED correction at the amplitude level



$$
\begin{aligned}
i\mathcal{A} \quad = \quad & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \; m_\ell m_B \; f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\, (1-u)\, C_9^{\rm eff}(u m_b^2) \; \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \\
& - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \Bigg\}
\end{aligned}
$$

Auxiliary slide: The QED correction at the amplitude level



$$
\begin{aligned}
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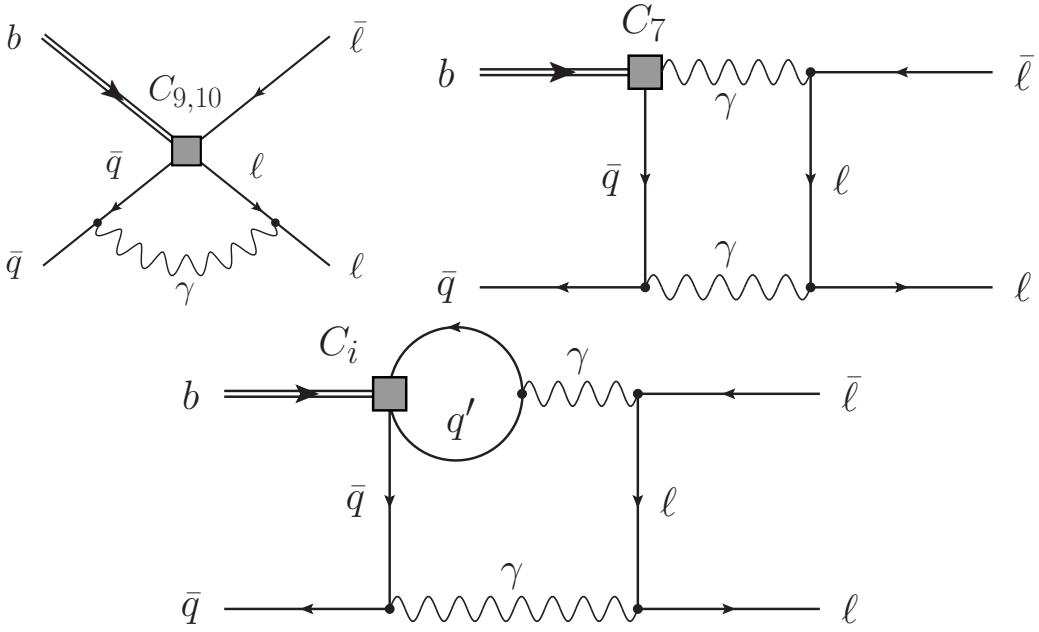
\end{aligned}
$$

- Tree level amplitude

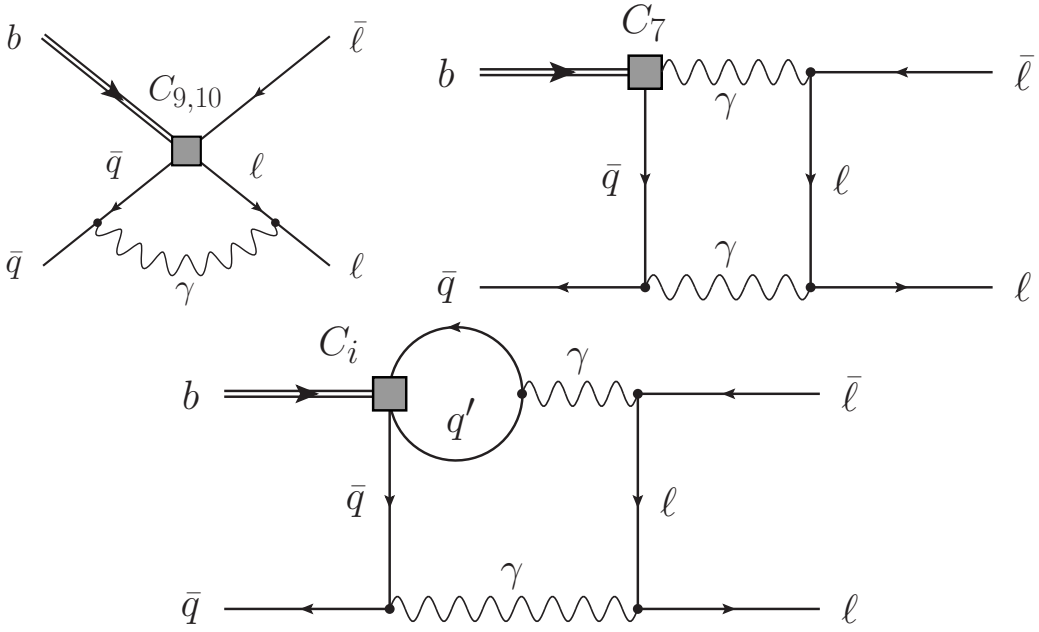
Auxiliary slide: The QED correction at the amplitude level



$$
\begin{aligned}
i\mathcal{A} \quad = \quad & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \; m_\ell m_B \; f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\, (1-u)\, C_9^{\rm eff}(u m_b^2) \; \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \\
& - Q_\ell C_7^{\rm eff} \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln^2 \frac{m_b \omega}{m_\ell^2} - 2 \ln \frac{m_b \omega}{m_\ell^2} + \frac{2\pi^2}{3} \right] \Bigg\}
\end{aligned}
$$

- Helicity suppression $\times$ power enhancement factor

Auxiliary slide: The QED correction at the amplitude level



$$
\begin{aligned}
i\mathcal{A} \quad = \quad & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \; m_\ell m_B \; f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
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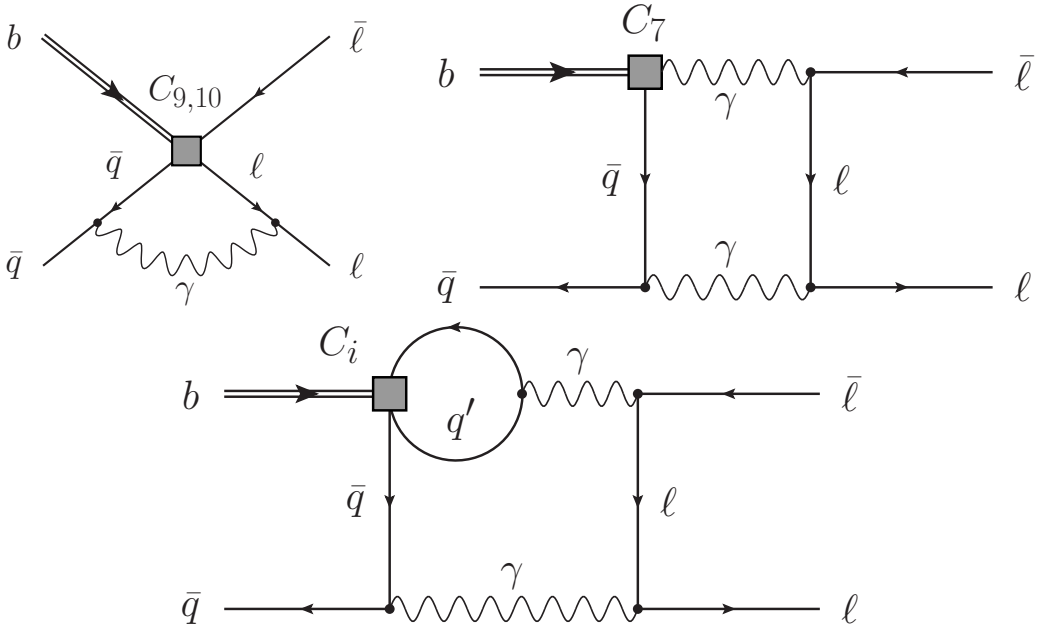
\end{aligned}
$$

- Convolution with the light-cone distribution function

Auxiliary slide: The QED correction at the amplitude level



$$
\begin{aligned}
i\mathcal{A} \quad = \quad & m_\ell f_{B_q} \mathcal{N}\, C_{10}\, \bar{\ell}\gamma_5 \ell + \frac{\alpha_{\rm em}}{4\pi} Q_\ell Q_q \; m_\ell m_B \; f_{B_q} \mathcal{N}\, \bar{\ell}(1+\gamma_5)\ell \\
& \times \Bigg\{ \int_0^1 du\,(1-u)\, C_9^{\rm eff}(u m_b^2) \; \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega) \left[\ln \frac{m_b \omega}{m_\ell^2} + \ln \frac{u}{1-u} \right] \\
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\end{aligned}
$$

- Double logarithmic enhancement due to endpoint singularity

Auxiliary slide: Non-perturbative contribution

Non-perturbative physics is encoded in the moments of B-meson light-cone distribution function [M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, 1999]

$$\frac{1}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega,\mu),$$

$$\frac{\sigma_n(\mu)}{\lambda_B(\mu)} \equiv \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{\mu_0}{\omega}\, \phi_{B+}(\omega,\mu)$$

$\lambda_B(1\ \text{GeV}) = (275 \pm 75)\ \text{MeV}$
$\sigma_1(1\ \text{GeV}) = 1.5 \pm 1$
$\sigma_2(1\ \text{GeV}) = 3 \pm 2$
Power-enhancement factor

$$m_B \int_0^\infty \frac{d\omega}{\omega}\, \phi_{B+}(\omega)\, \ln^k \omega \sim \frac{m_B}{\lambda_B} \times \sigma_k$$

Auxiliary slide: soft function in position space

At the one-loop order in dimensional regularization with $d = 4 - 2\epsilon$, the bare soft function must have a simple dependence

$$\tilde{S}_{0,\text{bare}}(x) = 1 + \frac{\alpha_s}{\pi}\left(-n_- x n_+ x \mu^2\right)^\epsilon f\left(\epsilon, \frac{x^2}{n_+ x n_- x}\right)$$

Explicit evaluation gives

$$\begin{aligned}
\widetilde{S}_{0,\text{bare}}(x) &= 1 + \frac{\alpha_s C_F}{\pi}\frac{\Gamma(1-\epsilon)}{\epsilon^2} e^{-\epsilon\gamma_E} \times \\
&\quad \left(-\frac{1}{4} n_- x n_+ x \mu^2 e^{2\gamma_E}\right)^\epsilon \left(\frac{x^2}{n_- x n_+ x}\right)^{1+\epsilon} {}_2F_1\left(1, 1, 1-\epsilon; 1 - \frac{x^2}{n_- x n_+ x}\right) \\
&= 1 + \frac{\alpha_s C_F}{\pi}\left(\frac{1}{\epsilon^2} + \frac{L}{\epsilon} + \frac{L^2}{2} + \frac{\pi^2}{12} + \text{Li}_2\left(1 - \frac{x^2}{n_- x n_+ x}\right) + \mathcal{O}(\epsilon)\right)
\end{aligned}$$

where we defined

$$L \equiv \ln\left(-\frac{1}{4} n_- x n_+ x \mu^2 e^{2\gamma_E}\right).$$

Auxiliary slide: The Drell-Yan Process

$$A(p_A)B(p_B) \to \mathrm{DY}(Q) + X$$

Light-cone coordinates

$$\begin{aligned} n_+^\mu &= (1,0,0,+1) \\ n_-^\mu &= (1,0,0,-1) \end{aligned}$$

$$z = \frac{Q^2}{\hat{s}} \qquad \lambda = \sqrt{(1-z)}$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$

hard

$$p_h = (n_+p_h, n_-p_h, p_{h\perp}) \sim Q(1,1,1)$$

collinear

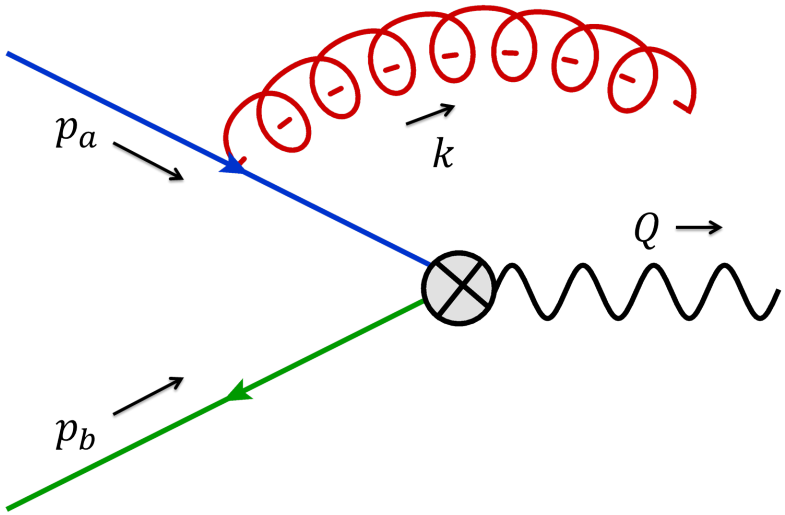
$$p_c = (n_+p_c, n_-p_c, p_{c\perp}) \sim Q(1,\lambda^2,\lambda)$$

anticollinear

$$p_{\bar{c}} = (n_+p_{\bar{c}}, n_-p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2,1,\lambda)$$

soft

$$p_s = (n_+p_s, n_-p_s, p_{s\perp}) \sim Q(\lambda^2,\lambda^2,\lambda^2)$$



Auxiliary slide: Hard function

When considering power corrections we have to be careful about kinematic factors. Consider the hard function

$$\bar{\psi}\gamma_\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t})\, J_\mu^{A0}(t,\bar{t}), \qquad H(\hat{s},\mu_h) = |C^{A0}(-\hat{s})|^2$$

we can obtain power corrections from expansion

$$H(\hat{s}) = H(Q^2) + Q^2(1-z)H'(Q^2) + \ldots$$

The LP factorization is

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

with

$$H(\hat{s}) = 1 + \mathcal{O}(\alpha_s) \quad \text{and} \quad S_{\mathrm{DY}}(\Omega) = \delta(\Omega) + \mathcal{O}(\alpha_s)$$

so at the LL accuracy it is enough to consider $H(Q^2)$

Auxiliary slide: Hard function running

Well known RGE for two-jet operator

$$\frac{d}{d\ln\mu} H(Q^2,\mu) = \left(2\Gamma_{\rm cusp}\ln\frac{Q^2}{\mu^2} + 2\gamma\right) H(Q^2,\mu)$$

$$\Gamma_{\rm cusp} = \frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2), \qquad \gamma = -\frac{3}{2}\frac{\alpha_s}{\pi} C_F + \mathcal{O}(\alpha_s^2),$$

The general solution RGE reads

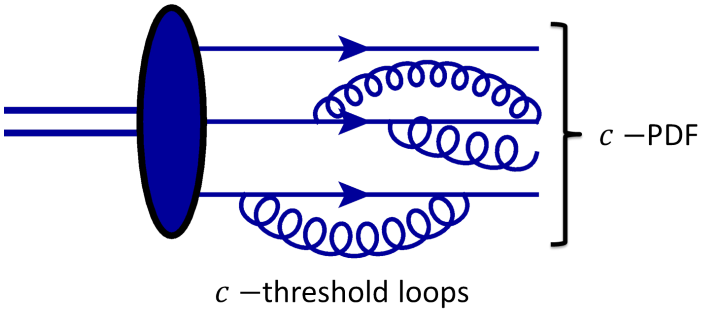
$$H(Q^2,\mu) = \exp\left[4S(\mu_h,\mu) - 2a_\gamma(\mu_h,\mu)\right]\left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu)} H(Q^2,\mu_h)$$

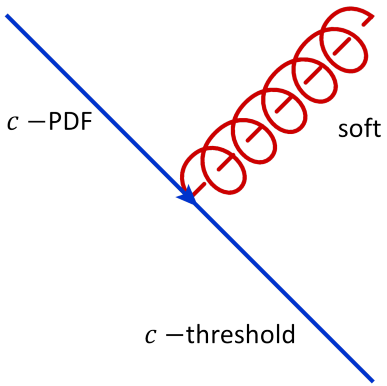
where

$$\begin{aligned} S(\nu,\mu) &= -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}, \\ a_\Gamma(\nu,\mu) &= -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}, \qquad a_\gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\gamma(\alpha)}{\beta(\alpha)} \end{aligned}$$

Auxiliary slide: Collinear function at NLP

- There is no collinear function present at LP because of decoupling transformation
- This is no longer true at NLP. Consider an example of subleading SCET Lagrangian: $\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar{\chi}_c z^\mu_\perp z^\rho_\perp \left[i\partial_\rho i n_- \partial \mathcal{B}^+_\mu(z_-) \right] \frac{\not{n}_+}{2} \chi_c$
- Crucially, an insertion of a piece of a subleading Lagrangian comes with an integral over its position, $\int d^4 z$





$$\left(J^{T2}_{A0,2\xi}(s,t) \right)^\mu = i \int d^4x \, \mathbf{T} \left[J^\mu_{A0}(s,t) \, \mathcal{L}^{(2)}_{2\xi}(x) \right]$$