Non-renormalizable interactions of the Standard Model fields

Renato Fonseca

fonseca@ipnp.mff.cuni.cz

Institute of Particle and Nuclear Physics
Charles University, Prague, Czech Republic

COST workshop on Higgs and Flavour Physics: Present and Future

The SMEFT

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SO(3,1)$
Q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	$\left(\frac{1}{2},0\right)$
u^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	$\left(\frac{1}{2},0\right)$
d^c	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	$\left(\frac{1}{2},0\right)$
L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$\left(\frac{1}{2},0\right)$
e^c	$\mathbf{1}$	$\mathbf{1}$	1	$\left(\frac{1}{2},0\right)$
H	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	$(0,0)$
F_G	$\mathbf{8}$	$\mathbf{1}$	0	$(1,0)$
F_W	$\mathbf{1}$	$\mathbf{3}$	0	$(1,0)$
F_B	$\mathbf{1}$	$\mathbf{1}$	0	$(1,0)$

¿ What are the effective interactions between the Standard Model fields allowed by gauge and Lorentz symmetries? ?

The SM supplemented by these interactions is often called the "Standard Model effective field theory" (SMEFT)

SMEFT: What do we know? (I)

All operators up to dimension 6 are known
(partial results exist for higher dimensions)

but it took some time ...

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Using an effective lagrangian technique we have studied systematically deviations from the standard model which could be the low-energy manifestations of $SU(3) \times SU(2) \times U(1)$ invariant new interactions with scales Λ in the TeV range. The effective lagrangian contains 1 dimension-five operator, which violates lepton number, and 80 baryon- and lepton-number conserving dimension-six operators.

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Several papers contributed to this positive development in our understanding of SMEFT

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1 complex = 2 real parameters encode the Weinberg operator LLHH for 1 generation

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The Molien/Hilbert series

All the papers mention earlier use the Molien series (also referred to as the Hilbert series)

For a representation R of a group G, it indicates how many times the trivial representation is in the (symmetric) product $R \times R \times R \times \cdots$

Works also for continuous groups

$$\frac{1}{|G|}\sum_i \frac{1}{\det\left(\mathbb{1} - tR_i\right)} = \sum_{n=0}^{\infty} x_n t^n$$

$|G|$ = group size
x_n = numbers we want

Example

Take the natural representation of S_3

$$R_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\frac{1}{|G|}\sum_i \frac{1}{\det\left(\mathbb{1} - tR_i\right)} = \frac{1}{(1-t)\left(1-t^2\right)\left(1-t^3\right)} = 1 + t + 2t^2 + 3t^3 + 4t^4 + 5t^5 + 7t^6 + 8t^7 + 10t^8 + 12t^9 + \cdots$$

Meaning that there is 1,2,3,4,5,7,8,10,12, ... invariants in R^n, n=1,2,3,4,5,6,7,8,9,...

(caveat: on the symmetric part only of these tensor products)

The Molien/Hilbert series

But the natural representation of S_3 is isomorphic
to 1+2 in terms of irreducible representations

$$S = (1), (1), (1), (1), (1), (1)$$

$$D \equiv \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

$$R_i = \begin{pmatrix} S_i & 0 \\ 0 & D_i \end{pmatrix}$$

Let us use this to extract more information from the multi-graded Molien/Hilbert series

$$\frac{1}{|G|} \sum_i \frac{1}{\det\left(\mathbb{1} - \epsilon \begin{pmatrix} t_S \mathbb{1} & 0 \\ 0 & t_D \mathbb{1} \end{pmatrix} \begin{pmatrix} S_i & 0 \\ 0 & D_i \end{pmatrix}\right)} = \frac{1}{|G|} \sum_i \frac{1}{\det\left(\mathbb{1} - \epsilon t_S S_i\right) \det\left(\mathbb{1} - \epsilon t_D D_i\right)}$$

$$= \frac{1}{\left(1 - \epsilon t_S\right) \left[1 - \left(\epsilon t_D\right)^2\right] \left[1 - \left(\epsilon t_D\right)^3\right]}$$

$$= 1 + \epsilon t_S + \epsilon^2 \left({t_D}^2 + {t_S}^2\right) + \epsilon^3 \left({t_D}^3 + t_D^2 t_S + {t_S}^3\right) + \epsilon^4 \left({t_D}^4 + {t_D}^3 t_S + {t_D}^2 {t_S}^2 + {t_S}^4\right) + \cdots$$

$S \quad D^2 \quad S^2 \quad D^3 \quad D^2 S \quad S^3 \quad D^4 \quad D^3 S \quad D^2 S^2 \quad S^4$

If S and D were fields, we would now know how many terms of each type appear in the Lagrangian

The Molien/Hilbert series

Real case

Henning, Lu, Melia, Murayama (2017)

SMEFT dim 5

$$6 H^2 L^2 + 6 H^{*2} L^{*2}$$

SMEFT dim 6

$$\begin{aligned}
&G^3 + 57 L Q^3 + 45 d^2 d^{*2} + 81 d e d^* e^* + 36 e^2 e^{*2} + G^{*3} + B^2 H H^* + G^2 H H^* + 9 B e L H^* + 9 B d Q H^* + 9 d G Q H^* + \\
&H B^{*2} H^* + H G^{*2} H^* + 9 e H L H^{*2} + 9 d H Q H^{*2} + H^3 H^{*3} + 81 d L d^* L^* + 81 e L e^* L^* + 81 d Q e^* L^* + 9 H B^* e^* L^* + \\
&9 H^2 e^* H^* L^* + 45 L^2 L^{*2} + 81 e L d^* Q^* + 162 d Q d^* Q^* + 9 H B^* d^* Q^* + 81 e Q e^* Q^* + 9 H d^* G^* Q^* + 9 H^2 d^* H^* Q^* + \\
&162 L Q L^* Q^* + 90 Q^2 Q^{*2} + 57 L^* Q^{*3} + 81 L Q d^* u^* + 54 Q^2 e^* u^* + 9 B^* H^* Q^* u^* + 9 G^* H^* Q^* u^* + 9 H H^{*2} Q^* u^* + \\
&162 e^* L^* Q^* u^* + 162 d^* Q^{*2} u^* + 81 d^* e^* u^{*2} + H B^* H^* W^* + 9 H e^* L^* W^* + 9 H d^* Q^* W^* + 9 H^* Q^* u^* W^* + H H^* W^{*2} + W^{*3} + \\
&9 B H Q u + 9 G H Q u + 162 e L Q u + 162 d Q^2 u + 9 H^2 Q H^* u + 81 d L^* Q^* u + 54 e Q^{*2} u + 162 d d^* u^* u + 81 e e^* u^* u + \\
&81 L L^* u^* u + 162 Q Q^* u^* u + 81 d e u^2 + 45 u^{*2} u^2 + B H H^* W + 9 e L H^* W + 9 d Q H^* W + 9 H Q u W + H H^* W^2 + W^3 + \\
&9 d H d^* H^* \partial + 9 e H e^* H^* \partial + 18 H L H^* L^* \partial + 18 H Q H^* Q^* \partial + 9 d H^{*2} u^* \partial + 9 H^2 d^* u \partial + 9 H H^* u^* u \partial + 2 H^2 H^{*2} \partial^2
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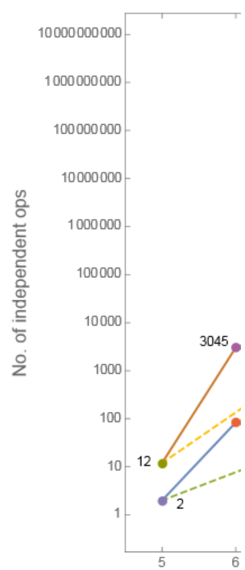

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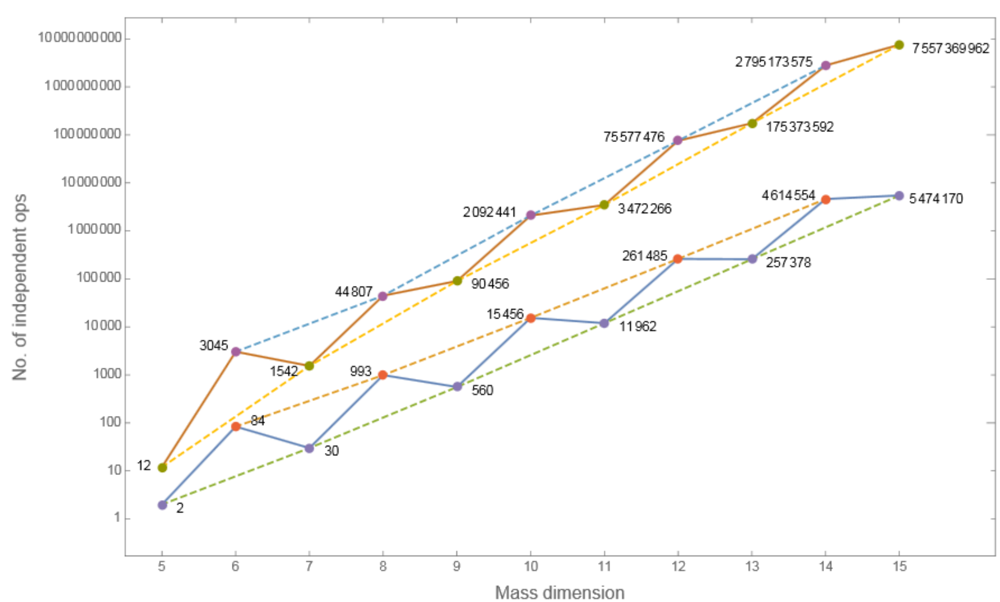
This kind of calculation was performed up to dim 12; furthermore, the total number of operators was calculated up to dim 15

The dim 15 operators of SMEFT are encoded by 7557369962 real numbers according to these authors



Applications

B anomalies

Model independent (EFT) approach to B anomalies

Consider $B^+ \to K^+ \ell^- \ell^+$. This is often seen as $b \to s \ell^- \ell^+$, i.e. a 4-fermion process

$$\mathscr{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_k \left(C_k \mathcal{O}_k + C'_k \mathcal{O}'_k \right)$$

$$\mathcal{O}_9^{\ell\ell'} = (\bar{s}\gamma_\mu P_L b)\left(\bar{\ell}\gamma^\mu \ell'\right)$$
$$\mathcal{O}_{10}^{\ell\ell'} = (\bar{s}\gamma_\mu P_L b)\left(\bar{\ell}\gamma^\mu \gamma_5 \ell'\right)$$
...

However, strictly speaking this is a 6-fermion dimension 9 operator (u quark might not be spectating): $b\bar{s}u\bar{u}\ell^-\ell^+$

What if we ever need to apply an EFT approach to other observables involving more particles?

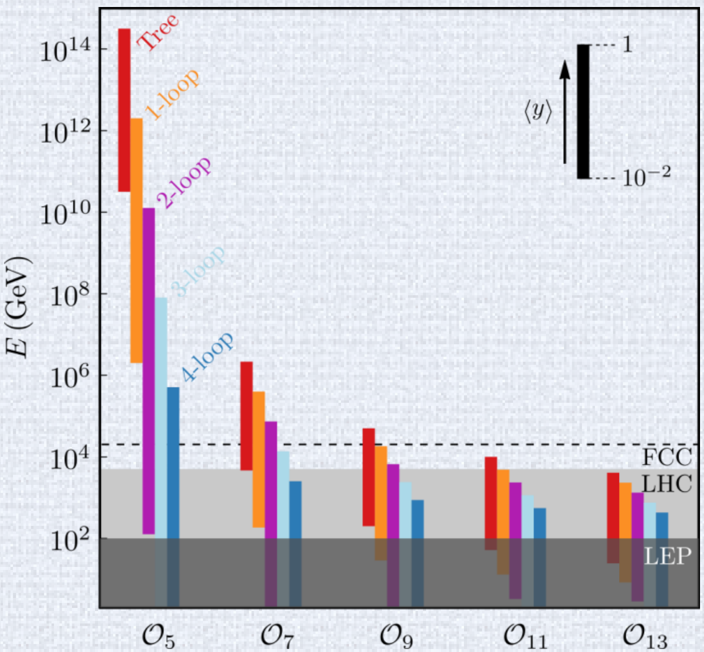
Example $\quad \mathrm{Br}\left(B^+ \to K^+\pi^+\pi^-\mu^+\mu^-\right) = (4.3 \pm 0.4) \times 10^{-7} \quad$ (PDG)

This is a 10-fermion process ...

Applications

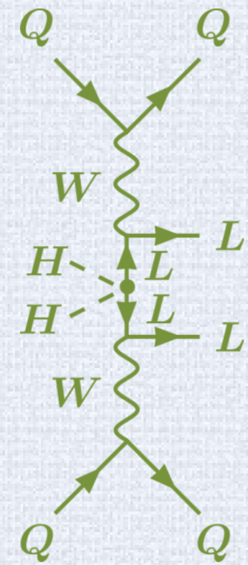
Lepton/Baryon number violating processes

Neutrino masses via the operators $LL\,(H^*H)^{1/2(d-3)}$



Anamiati *et al.* (2018)

$0\nu2\beta$ is a 6-fermion process (dim 9 at least)



List of lepton number violating operators up to dim 11 available (but it is not complete)

Babu, Leung (2001)

Proton decay operators up to dim 13 mediated by TeV particles can be observed

Fonseca, Hirsch, Srivastava (2018)

Applications

Correction to predictions using only lower order operators

process within the dimension ≤ 8 Standard Model Effective Theory. We quantify the effects of dimension-8 by turning on one dimension-6 operator at a time and setting all dimension-8 operator coefficients to the same magnitude. Under this procedure and given the current accuracy on $\sigma(pp \to h\,W^+)$, we find the effect of dimension-8 operators on the inferred new physics scale to be small, $\mathcal{O}(\text{few}\,\%)$, with some variation depending on the relative signs of the dimension-8 coefficients and on which dimension-6 operator is considered. The impact of the dimension-8 terms grows as $\sigma(pp \to h\,W^+)$ is measured more accurately or (more significantly) in high-mass kinematic regions. We provide a FeynRules implementation of our

Hays, Martin, Sanz, Setford (2018)

Basic idea

Amplitudes: $\mathcal{A} \sim \mathcal{A}_4 + \frac{\mathcal{A}_6}{\Lambda^2} + \frac{\mathcal{A}_8}{\Lambda^4} + \cdots$

Observables: $\left(\frac{\mathcal{A}_6}{\Lambda^2}\right)^2$ naively comparable to $\mathcal{A}_4 \times \frac{\mathcal{A}_8}{\Lambda^4}$

My approach

The brute force approach ... with delicacy

1. Multiply together all SM fields up to desired order, in all possible ways, taking into account the equations of motion (EOM)
2. Retain only the gauge and Lorentz invariant combinations
3. Deal with the integration by parts (IBP) redundancies

With this, it should be possible to check the counting of operators obtained with the Hilbert series. In fact, more information on the flavor structure is obtained

(spoiler)

Confirms the Hilbert series counting up to dim 15

To do:

4. Aided by this information, build the operators explicitly

   (it is very hard to do so in a fully automated way)

Delicacy is needed in this apparent brute force approach

Complications

Repeated fields in an interaction

Repeated fields in an interaction (such as L and H in LLHH) **lead to symmetries in the coupling parameters**.
For example, there are only 6 independent parameters in $L_i L_j H H$ (not $3 \times 3 = 9$)

Derivatives

Some operators with derivatives are redundant because they are related either by a total derivative or by the classical equations of motion of the fields

Repeated fields

We go to reference texts on group theory and we find things like this:

$$2 \times 2 = 1_A + 3_S \qquad SU(2)$$

In the Weinberg operator,

$$\underbrace{L \times L}_{\cancel{1_A} + 3_S} \times \underbrace{H \times H}_{\cancel{1_A} + 3_S}$$

As a consequence, introducing flavors,

$$\kappa_{ij} L_i L_j H H$$

$\boldsymbol{\kappa_{ij}}$ is symmetric: $\boldsymbol{\kappa_{ij} = \kappa_{ji}}$

So ...
we need to track all S's and A's in the product of representations and that is it?

No! A's and S's are not the end of the story. In general one has to deal with more complicated symmetries

No time to go into the details (see instead talk R.F., "Plethysms and their applications in particle physics", Bonn 2017)

Repeated fields

Food for thought

Consider the following:

Repeated fields

Food for thought

Consider the following:

? ? ?

I

Suppose there is a **scalar ϕ, doublet under $SU(2)$** (and no hypercharge)

How many quartic couplings $\phi\phi\phi\phi$ can we write down?

$$(\phi\phi\phi\phi)\,,(\phi\phi\phi\phi)'\,,\cdots?$$

<u>Harder version:</u> if there are n flavours $\phi_{i=1,\cdots,n}$, how many couplings are there?

$$\phi_i\phi_j\phi_k\phi_l$$

Repeated fields

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? I ? II ?

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There is the Weinberg operator *LLHH*

For one flavor (say, e), how **many couplings of the form $L_eL_eL_eL_eHHHH$ are there?**

$$(L_eL_eL_eL_eHHHH)$$
$$(L_eL_eL_eL_eHHHH)'$$

... ?

Harder version: what about in the general case

$$L_iL_jL_kL_lHHHH$$

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Harder version: what about in the general case $L_iL_jL_kL_lHHHH$

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III

?

Add **one right-handed neutrino N^c** to the SM

How many couplings $N^cN^cN^cN^c$?

$$(N^cN^cN^cN^c)$$
$$(N^cN^cN^cN^c)'$$
$$(N^cN^cN^cN^c)''$$

... ?

Harder version: what about the multi-favour case $N_i^cN_j^cN_k^cN_l^c$?

Repeated fields

Food for thought

Consider the following:

I

Suppose there is a **scalar ϕ, doublet under $SU(2)$** (and no hypercharge)

Ho

$\phi\phi$

(

II

There is the Weinberg operator $LLHH$

For one flavor (say, e), how

... ?

III

Add **one right-handed neutrino N^c** to the SM

... ?

0

(these couplings do not exist if there is just one flavor of these fields)

<u>Harder version:</u> if there are n flavours $\phi_{i=1,\cdots,n}$, how many couplings are there?

$$\phi_i\phi_j\phi_k\phi_l$$

<u>Harder version:</u> what about in the general case

$$L_iL_jL_kL_lHHHH$$

<u>Harder version:</u> what about the multi-favour case $N_i^cN_j^cN_k^cN_l^c$?

Repeated fields

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For n flavors, there are $\dfrac{n^2\left(n^2-1\right)}{12}$ couplings in all three cases

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For n flavors, there are $\frac{n^2(n^2-1)}{12}$ couplings in all three cases

To handle the SMEFT operators, one needs of course to tackle the multi-flavor case

Repeated fields

SMEFT example

Take the 4-fermion, dimension 6 interaction $Q_i Q_j Q_k L_l$

Do we need $3^4 = 81$ complex parameters?

Repeated fields

SMEFT example

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Do we need $3^4 = 81$ complex parameters?

No. Only 57.

$G^3 + 57\, L\, Q^3 + 45\, d^2\, d^{*2} + 81\, d\, e\, d^*\, e^* +$

$H\, B^{*2}\, H^* + H\, G^{*2}\, H^* + 9\, e\, H\, L\, H^{*2} + 9\, d\, H$

$9\, H^2\, e^*\, H^*\, L^* + 45\, L^2\, L^{*2} + 81\, e\, L\, d^*\, Q^*$

$162\, L\, Q\, L^*\, Q^* + 90\, Q^2\, Q^{*2} + 57\, L^*\, Q^{*3} +$

Repeated fields

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How to get this peculiar number $57 = 3 \times 19$?

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How to get this peculiar number $57 = 3 \times 19$?

Looking at the quantum number of the fields, there are 4 gauge and Lorentz invariant combinations that can be made, but they are not the same!

Effect of permuting the Q's

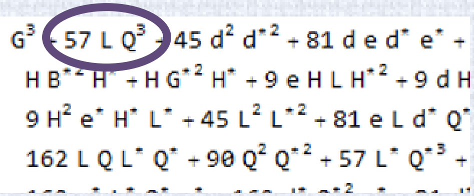
Combination 1	Combination 2	Combination 3	Combination 4
$+1$	$(-1)^{\pi}$		
S	A	2	

Repeated fields

SMEFT example

Take the 4-fermion, dimension 6 interaction $Q_i Q_j Q_k L_l$

Do we need $3^4 = 81$ complex parameters?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Looking at the quantum number of the fields, there are 4 gauge and Lorentz invariant combinations that can be made, but they are not the same!

Combination 1	Combination 2	Combination 3	Combination 4

Effect of permuting the Q's

$+1$ $\qquad$ $(-1)^\pi$

$S \equiv \{3\}$ $\qquad$ $A \equiv \{1,1,1\}$ $\qquad$ $\mathbf{2} = \{2,1\}$

The various combinations are organized in irreducible representations of the discrete group S3: {3}, {2,1} and {1,1,1}

Repeated fields

SMEFT example

How many parameters then? $t_{ijkl} Q_i Q_j Q_k L_l$

Combination 1

$+1$

S

A symmetric matrix $t_{ij} = t_{ji}$ has $\frac{1}{2!} n\,(n+1)$ components

Fully symmetric tensor t_{ijk}: $\frac{1}{3!} n\,(n+1)\,(n+2)$

So ... $$\# t^{(1)}_{ijkl} = \frac{1}{6} n_Q\,(n_Q+1)\,(n_Q+2)\,n_L$$

Repeated fields

SMEFT example

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Combination 1

$\circlearrowleft +1$

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Fully symmetric tensor t_{ijk}: $\frac{1}{3!}n\left(n+1\right)\left(n+2\right)$

So ... $$\#t_{ijkl}^{(1)}=\frac{1}{6}n_Q\left(n_Q+1\right)\left(n_Q+2\right)n_L$$

A similar exercise can be done for combination 2:

$$\#t_{ijkl}^{(2)}=\frac{1}{6}n_Q\left(n_Q-1\right)\left(n_Q-2\right)n_L$$

Repeated fields

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For combinations 3 and 4 the math does not get more complicated. But I'll just quote the formula

$$\#t_{ijkl}^{(3+4)}=\frac{1}{3}n_Q\left(n_Q^2-1\right)n_L$$

Repeated fields

SMEFT example

How many parameters then? $t_{ijkl}Q_iQ_jQ_kL_l$

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$+1$

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For **combinations 3 and 4** the math does not get more complicated. But I'll just quote the formula

$$\#t_{ijkl}^{(3+4)}=\frac{1}{3}n_Q\left(n_Q^2-1\right)n_L$$

$$\left[\#t_{ijkl}^{(1)}\right]+\left[\#t_{ijkl}^{(2)}\right]+\left[\#t_{ijkl}^{(3+4)}\right]=\frac{1}{3}n_Q\left(2n_Q^2+1\right)n_L\overset{n_Q=n_L=3}{=}57$$

Derivatives*

⇒ redundancies

*Note: all derivatives must be covariant. Still, I will use the partial derivative symbol in these slides.

Equations of motion

$$\frac{\delta \mathscr{L}_{ren.}}{\delta\psi} - \partial_\mu\left[\frac{\delta \mathscr{L}_{ren.}}{\delta\left(\partial_\mu\psi\right)}\right]$$

Terms proportional to this expression and derivatives of it can be removed from an be removed from $\mathscr{L}^{(d)}$ without affecting observables (i.e. the S-matrix) up to $\Lambda^{-(d+1)}$ effects

Practical effect?

Lehman, Martin (2015)

Retain only the highest spin part of $\partial^n X$

$X = \phi, \psi, F$

To be specific, consider a scalar ϕ, a left Weyl fermion ψ and a field strength tensor F

$$\begin{aligned} \phi &= (0,0)\\ \psi &= \left(\tfrac{1}{2},0\right)\\ F &= (1,0)\\ \partial &= \left(\tfrac{1}{2},\tfrac{1}{2}\right) \end{aligned}$$

$$\begin{aligned} \partial^n\phi &= \left(\tfrac{n}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits}\\ \partial^n\psi &= \left(\tfrac{n+1}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits}\\ \partial^n F &= \left(\tfrac{n+2}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits} \end{aligned}$$

So the solution for the "brute force" approach is the same as for the Hilbert series approach

Derivatives

$\Rightarrow$ redundancies

Integration by parts

$$\int_M d\omega = \int_{\partial M} \omega = 0$$

means that $\partial_\mu \mathcal{O}^\mu$ terms do not affect the action S

The Hilbert series approach to getting rid of this kind of redundancies [Henning, Lu, Melia, Murayama (2017)] suggests the following strategy

#1 Calculate all operators $\mathcal{O}^{(0)}$

#2 "Subtract" from #1 all operators of the form $\mathcal{O}^{(1)} = \partial_\mu \mathcal{O}^\mu$ — redundancies

#3 We went too far. Some linear combinations of the operators $\mathcal{O}^{(1)}$ are null. "Put back" the operators of the form $\mathcal{O}^{(2)} = \partial_\mu \partial_\nu \mathcal{O}^{\mu\nu} = 0$ — redundancies of the redundancies!

#4 (...) — redundancies of the redundancies of the redundancies!

#5 (...) — redundancies of the redundancies of the redundancies of the redundancies!

(It stops here because of the space-time dimension: 4)

Getting the operators in #2 to #5: insert the total derivative ∂ as field (together with the SM ones) and treat it as a Grassmann number

(Why Grassmann numbers? $\mathcal{O}^{(n>1)} = 0$ because of a total anti-symmetry of the derivatives)

Derivatives: example

How many couplings of the form $LLHH\partial\partial$?

+

$L_iL_j\left(\partial H\right)\left(\partial H\right)$ 2 sym. contractions under permutations of L: $2\times6=12$

$L_i\left(\partial L_j\right)H\left(\partial H\right)$ 2 contractions: $2\times9=18$

$\left(\partial L_i\right)\left(\partial L_j\right)HH$ 1 sym. contraction under permutations of L: 6

-

$\partial\left[L_iL_jH\left(\partial H\right)\right]$ 2 sym. + 2 anti-sym. contractions under permutations of L: $2\times6+2\times3=18$

$\partial\left[L_i\left(\partial L_j\right)HH\right]$ 1 contraction 9

+

$\partial^2\left(L_iL_jHH\right)$ 1 anti-sym. contraction under permutations of L: 3

$$12+18+6-(18+9)+3=12$$

parameters/ couplings

Summary

1

Our understanding of SMEFT evolved slowly initially, but in recent years it has improved rapidly

2

All dim-6 operators are known (the Warsaw basis);
The Hilbert series is a powerful tool to count operators (which is a very useful information)

3

It would be useful to have the explicit SMEFT operators beyond dimension 6

4

To help achieve that aim, I suggested using a "brute-force" approach + carefully tracking the symmetries of field contractions

Thank you