Basis-invariant road to 3HDMs with symmetries

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

Workshop on Higgs and Flavour Physics, January 14–17, 2019

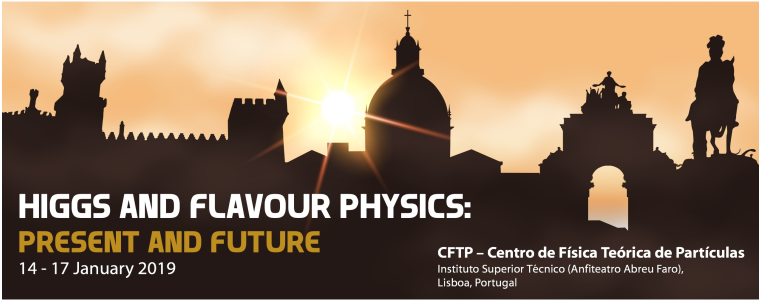
based on: I. P. Ivanov, C. Nishi, J. P. Silva, A. Trautner, arXiv:1810.13396 and work in progress

INVESTIGADOR FCT

GOVERNO DA REPÚBLICA PORTUGUESA

PROGRAMA OPERACIONAL POTENCIAL HUMANO

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL


HIGGS AND FLAVOUR PHYSICS:
PRESENT AND FUTURE
14 - 17 January 2019
CFTP – Centro de Física Teórica de Partículas
Instituto Superior Técnico (Anfiteatro Abreu Faro),
Lisboa, Portugal

Selected highlights:

- With 200 fb^{-1} analyzed, still no SUSY/exotics in sight...
- CMS or ATLAS sees a 3σ excess (local) at xxx GeV...
- LHCb Run 1+2 + Belle II data point to LFU violation at 3σ...
- stay tuned for the full Run 3 data!

Selected highlights:

- With 200 fb^{-1} analyzed, still no SUSY/exotics in sight...
- CMS or ATLAS sees a 3σ excess (local) at xxx GeV...
- LHCb Run 1+2 + Belle II data point to LFU violation at 3σ...
- stay tuned for the full Run 3 data!

Model building: business as usual

Theorists' response:

- keep proposing specific models by introducing various bSM fields;
- perform systematic scans within classes of conservative models, aiming to detect and explore everything that is possible;
- multi-Higgs models beyond 2HDM and the simplest single extension as a possible future playground.

I will focus on the scalar sector of the three-Higgs-doublet models (3HDMs) based on ϕ_a, $a = 1, 2, 3$:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,.$$

3HDM

What's new in 3HDM compared to 2HDM:

- combining nice features of 2HDM (e.g. NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009]);
- new options for *CP* violation, e.g. geometrical CPV [Branco, Gerard, Grimus, 1984],
- *CP* symmetry of order 4 (CP4) [Ivanov, Silva, 2015]:
  - mass degeneracy, *CP* eigenstates beyond *CP*-even/odd [Ivanov, Silva, 2015; Haber et al, 2018];
  - DM stabilized by CP4: [Koepke, 2018; Ivanov, Laletin, 2018];
  - quark/neutrino patterns from CP4: [Ferreira et al, 2017; Ivanov, 2018];
  - solution to strong *CP* problem: [Cherchiglia, Nishi, 2019].
- symmetries, lots of symmetries in the 3HDM scalar sector!

Symmetries in 3HDM

Particular examples of 3HDMs with symmetries begin in 1970's;
full classification only recently.

- abelian groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2\times\mathbb{Z}_2, \quad U(1), \quad U(1)\times\mathbb{Z}_2, \quad U(1)\times U(1)\,.$$

- discrete non-abelian groups: [Ivanov, Vdovin, 1210.6553]

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- symmetry breaking patterns $G \to G_v$: [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested; none works as nicely as wanted:
  - if G is large $\to$ severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small $\to$ too many free parameters, no predictive power.
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]:

  If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.

  But for large G, this is algebraically impossible.

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested; none works as nicely as wanted:
  - if G is large $\to$ severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small $\to$ too many free parameters, no predictive power.
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]:

  If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.

  But for large G, this is algebraically impossible.

Symmetries in 3HDM: flavour physics connection

- The original idea from 1970's:
  - extent G to fermion sector,
  - arrange for spontaneous violation $G \to G_v$,
  - derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested; none works as nicely as wanted:
  - if G is large $\to$ severe problems in the quark sector; A_4/S_4 illustrations in [Gonzales Felipe et al, 1302.0861, 1304.3468];
  - if G is small $\to$ too many free parameters, no predictive power.
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993; Gonzales Felipe et al, 1401.5807]:

  If the (active) Higgs sector is equipped with G, then vevs must break G completely in order to produce physical m_q's and CKM.

  But for large G, this is algebraically impossible.

Symmetries in 3HDM: flavour physics connection

Challenge

When scanning the 3HDM scalar parameter space,

one must be able to detect a G-symmetric situation

in a basis-invariant way.

We found a way to do it.

Basis-invariant methods: the traditional way

Basis invariants

Usual recipe [Botella, Silva, 1995]:

- write all couplings as tensors under basis changes,
- write down full contractions of tensors → basis invariants,
- link them to the feature you study.

It was applied, in particular, to the explicit CP-conservation in 2HDM [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$

$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$

$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

Basis invariants

Drawbacks:

- non-intuitive and highly complicated (required extensive computer-algebra searches);

  NB! [Trautner, 1812.02614] shows how to derive them!
- becomes even more complicated beyond 2HDM; conditions for CP symmetry in 3HDM via basis invariants still not established [Varzielas et al, 1603.06942];
- basis invariants are blind to the form of CP symmetry: cannot tell the usual CP from CP4.

Basis-invariant conditions with basis-covariant objects

Bilinear space formalism

[Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008].

V depends on 9 bilinears $\phi_a^\dagger \phi_b$. Arrange them as

$$r_0 = \frac{1}{\sqrt{3}} \phi_a^\dagger \phi_a \,, \quad r_i = \phi_a^\dagger (t^i)_{ab} \phi_b \,, \quad i = 1, \dots, 8 \,,$$

where $SU(3)$ generators t_i satisfy $[t_i, t_j] = i f_{ijk} t_k$ and $\{t_i, t_j\} = \delta_{ij} \mathbf{1}_3/3 + d_{ijk} t_k$.

The potential is a quadratic form of r's:

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j \,,$$

with vectors $M_i, L_i \in \mathbb{R}^8$ and an 8×8 matrix Λ_{ij}.

Basis changes: $Adj(SU(3)) \subset SO(8)$ rotations.

CP-transformations: certain reflections of $\mathbb{R}^8$.

Bilinear space formalism

[Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008].

V depends on 9 bilinears $\phi_a^\dagger \phi_b$. Arrange them as

$$r_0 = \frac{1}{\sqrt{3}} \phi_a^\dagger \phi_a \,, \quad r_i = \phi_a^\dagger (t^i)_{ab} \phi_b \,, \quad i = 1, \dots, 8 \,,$$

where $SU(3)$ generators t_i satisfy $[t_i, t_j] = i f_{ijk} t_k$ and $\{t_i, t_j\} = \delta_{ij} \mathbf{1}_3/3 + d_{ijk} t_k$.

The potential is a quadratic form of r's:

$$V = -M_0 r_0 - M_i r_i + \Lambda_{00} r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j \,,$$

with vectors $M_i, L_i \in \mathbb{R}^8$ and an 8×8 matrix Λ_{ij}.

Basis changes: $Adj(SU(3)) \subset SO(8)$ rotations.

CP-transformations: certain reflections of $\mathbb{R}^8$.

Constructions in the adjoint space

All structural properties of 3HDM are encoded in M, L, and Λ.

These are basis-covariant objects in $\mathbb{R}^8$, but their "relative orientation" is basis-invariant.

The toolbox

Suppose vectors $a, b \in \mathbb{R}^8$. Define new products:

$$F_i^{(ab)} \equiv 2f_{ijk}a_j b_k\,, \quad D_i^{(ab)} \equiv \sqrt{3}d_{ijk}a_j b_k\,, \quad D_i^{(aa)} \equiv \sqrt{3}d_{ijk}a_j a_k\,.$$

Applied to the eigenvectors of Λ, these products help detect basis-invariant structures in $\Lambda \Rightarrow$ symmetries in 3HDM.

Explicit CP-conservation in 3HDM

Example 1: usual (order-2) CP [Nishi, hep-ph/0605153]:

- There exist three eigenvectors e, e', e'', which are closed under f-product and satisfy $F^{(ee')} = e''$;
- M and L are orthogonal to e, e', e''.

Example 2: CP4 [Ivanov, Nishi, Silva, Trautner, 1810.13396]:

- there exists an eigenvector a such that $D^{(aa)} = a$;
- there exist three eigenvectors e, e', e'' which satisfy $F^{(ae)} = F^{(ae')} = F^{(ae'')} = 0$;
- M, L, $d_{ijk}\Lambda_{jk}$, and $d_{ijk}(\Lambda^2)_{jk}$ are parallel to a.

Weinberg's model

Example 3: Weinberg's model ($\mathbb{Z}_2 \times \mathbb{Z}_2$):

- There exist eigenvectors a, b such that $F^{(ab)} = 0$;
- The other six eigenvectors break into three pairs; within each pair, the eigenvectors e, e' satisfy

$$D^{(ee')} = 0, \quad D^{(ee)} = D^{(e'e')} \in \mathrm{span}(a, b)\,.$$

- $M, L \in \mathrm{span}(a, b)$.

List of such conditions for all symmetries in 3HDM in progress...

Conclusions

- Efficient parameter space scans in multi-Higgs models must be able to detect symmetries in a basis invariant way.
- We found a way how to do it in the scalar sector of 3HDM.
- Stay tuned for the list of these conditions for each symmetry group G!