Weak scale from clockwork mechanism in heterotic M-theory

Marek Olechowski

Institute of Theoretical Physics
Faculty of Physics, University of Warsaw

COST workshop:
"Higgs and Favour Physics: Present and Future"
Lisbon, 15 January 2019

based on: S.H. Im, H.P. Nilles and M.O. arXiv:1811.11838 to appear in JHEP

Outline

- **Motivation**
- **Clockwork mechanism and General Linear Dilaton**
- **Realization in minimal heterotic M-theory (Hořava-Witten model)**
- **Heterotic M-theory with vector multiplets**
- **Conclusions**

Motivation

- **Understanding the origin of large hierarchies of scales (and small couplings) is a major challenge in theoretical physics**
- **Some of proposed mechanisms based on extra dimensions**
  - large extra dimensions (LED)
  - warped extra dimensions (RS)
  - linear dilaton model (LD)
- **They may be considered as various General Linear Dilaton (GLD) models**
  - generalizations of continuous version of (recently discussed) clockwork mechanism

Motivation

- **Understanding the origin of large hierarchies of scales (and small couplings) is a major challenge in theoretical physics**

- **Some of proposed mechanisms based on extra dimensions**
  - large extra dimensions (LED)
  - warped extra dimensions (RS)
  - linear dilaton model (LD)
- **They may be considered as various General Linear Dilaton (GLD) models**
  - generalizations of continuous version of (recently discussed) clockwork mechanism

- **Which of such models have UV-completions? (containing gravity and simple and/or interesting)**
- **Which may be derived from fundamental higher-dimensional theories like string- or M-theory?**

Clockwork mechanism:

- device to obtain light degrees of freedom with (strongly) suppressed couplings
  within theory without small fundamental parameters
- generalization of aligned axion mechanism
  Kim, Nilles, Peloso, 2004
- name suggested in Kaplan, Rattazzi, 2015
- related to deconstruction
- generalization of discrete clockwork to continuous one proposed in Giudice, McCullough, 2016
- problems of such generalization
  Craig, Garcia, Sutherland, 2017
- description of General Continuous Clockwork (GCCW)
  Choi, Im, Shin, 2017

Discrete scalar clockwork action ($q > 1$)

$$\int d^4x \left[\sum_{i=0}^{N} \frac{1}{2} (\partial_\mu \phi_i)^2 + \sum_{i=0}^{N-1} \frac{1}{2} m^2 (\phi_{i+1} - q\phi_i)^2 \right]$$

Mass matrix

$$m^2 \begin{pmatrix} 1 & -q & 0 & \dots & 0 \\ -q & 1+q^2 & -q & \dots & 0 \\ 0 & -q & 1+q^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1+q^2 & -q \\ 0 & 0 & 0 & -q & q^2 \end{pmatrix}$$

has one massless eigenstate $\chi_0 = \mathcal{N} \sum_{i=0}^{N} \frac{\phi_n}{q^i}$

component at each successive site is q times smaller than at the previous site

for large N:

coupling of such massless boson at 0-th and N-th sites are very different

Sites $i = 0 \dots N$ may be interpreted as points in 5-th dimension

Continuum limit ($\Delta r \equiv \pi R/N$, $N \to \infty$):

$$\sum_i \to \frac{1}{\Delta r}\int_0^{\pi R} \mathrm{d}y \qquad m_i \to \frac{m(y)}{\Delta r} \qquad q \to 1 + k \cdot \Delta r$$

$$\phi_i(x) \to \Phi(x,y)\,\Delta r^{1/2} \qquad (\phi_{i+1} - \phi_i) \to \partial_y \Phi(x,y)\,\Delta r^{3/2}$$

with redefined 5D scalar field $\Phi \to e^{2ky}\Phi$

$$\int \mathrm{d}^5x\, e^{2ky}\left[\frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2}m^2(y)(\partial_y \Phi)^2\right]$$

Continuous clockwork

Sites $i = 0 \ldots N$ may be interpreted as points in 5-th dimension
Continuum limit ($\Delta r \equiv \pi R/N$, $N \to \infty$):

$$\sum_i \to \frac{1}{\Delta r}\int_0^{\pi R} \mathrm{d}y \qquad m_i \to \frac{m(y)}{\Delta r} \qquad q \to 1 + k \cdot \Delta r$$

$$\phi_i(x) \to \Phi(x,y)\,\Delta r^{1/2} \qquad (\phi_{i+1} - \phi_i) \to \partial_y \Phi(x,y)\,\Delta r^{3/2}$$

with redefined 5D scalar field $\Phi \to e^{2ky}\Phi$

$$\int \mathrm{d}^5 x\, e^{2ky}\left[\frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2} m^2(y)(\partial_y \Phi)^2\right]$$

can be obtained from 5D diffeomorphism invariant lagrangian

$$\int \mathrm{d}^5 x \sqrt{-g}\,\frac{1}{2}\, g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi$$

in the background

$$\mathrm{d}s^2 = e^{2k_1 y}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2k_2 y}\mathrm{d}y^2$$

General Linear Dilaton

$$\mathcal{S}_5 = M_5^3 \int \mathrm{d}^5 x \sqrt{-g} \left(\frac{1}{2} \mathcal{R}_5 - \frac{1}{2} \partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-(2\hat{c}/\sqrt{3})S} \right.$$
$$\left. - e^{-(\hat{c}/\sqrt{3})S} \left[\Lambda_0 \frac{\delta(y)}{\sqrt{g_{55}}} + \Lambda_\pi \frac{\delta(y-\pi R)}{\sqrt{g_{55}}} \right] \right)$$

4D flat background solution if: $-\Lambda_0 = \Lambda_\pi = \pm 6 \sqrt{\frac{2}{3} \left(\frac{\Lambda_b}{\hat{c}^2 - 4} \right)}$

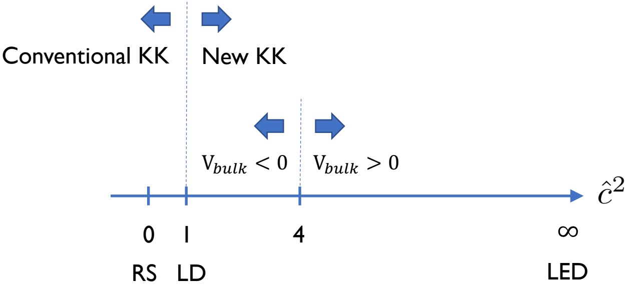
General linear dilaton background: $(\hat{c}/\sqrt{3}) S = k_2 y$

General Linear Dilaton

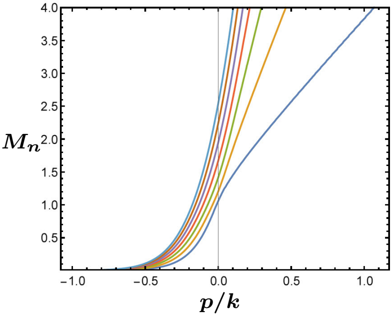
$$\mathcal{S}_5 = M_5^3 \int \mathrm{d}^5x \sqrt{-g} \left(\tfrac{1}{2}\,\mathcal{R}_5 - \tfrac{1}{2}\,\partial_\alpha S \partial^\alpha S - \Lambda_b\, e^{-(2\hat{c}/\sqrt{3})S} - e^{-(\hat{c}/\sqrt{3})S} \left[\Lambda_0 \frac{\delta(y)}{\sqrt{g_{55}}} + \Lambda_\pi \frac{\delta(y-\pi R)}{\sqrt{g_{55}}} \right] \right)$$

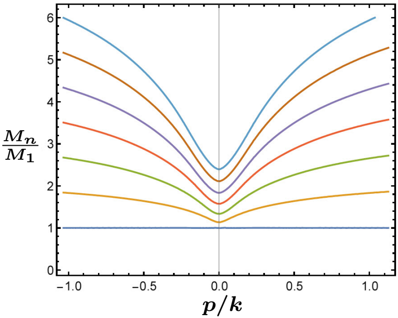
4D flat background solution if: $-\Lambda_0 = \Lambda_\pi = \pm 6 \sqrt{\frac{2}{3}\left(\frac{\Lambda_b}{\hat{c}^2 - 4}\right)}$

General linear dilaton background: $(\hat{c}/\sqrt{3})S = k_2 y$



KK spectrum in GLD models — Choi, Im, Shin, 2017





$$\frac{p}{k} = 2\,\frac{1-\hat{c}^2}{2+\hat{c}^2} \qquad\qquad \hat{c}^2 > 1 \Rightarrow p/k < 0$$

- **RS:** $\hat{c}^2 = 0$ $\quad p/k = 1$
- **LD:** $\hat{c}^2 = 1$ $\quad p/k = 0$

Strongly coupled $E_8 \times E_8$ heterotic M-theory $\rightarrow$ 11D SUGRA

$$\mathcal{S}_{11} = -\frac{1}{2\kappa^2}\int_{\mathcal{M}^{11}} \mathrm{d}^{11}x\sqrt{-g}\left(-\mathcal{R} + G\wedge\star G + 2\sqrt{2}C\wedge G\wedge G\right)$$
$$-\frac{1}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_{i=1}^{2}\int_{\mathcal{M}^{10}_{(i)}} \mathrm{d}^{10}x\sqrt{-g}\left(\mathrm{tr}F_{(i)}\wedge F_{(i)} - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)$$

Compactification on warped product $X^6 \times S^1/\mathbb{Z}_2$

supersymmetry $\rightarrow$ non-zero flux: $G_{ABCD} = -\frac{\mu}{48}\epsilon_{ABCD}{}^{EF}\omega_{EF}$

$$\mu \equiv \frac{\sqrt{2}}{\pi V_0}\left(\frac{\kappa}{4\pi}\right)^{2/3}\int_{X^6}\omega\wedge\left(\mathrm{tr}F_{(1)}\wedge F_{(1)} - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)$$

Hořava-Witten model

Strongly coupled $E_8 \times E_8$ heterotic M-theory $\to$ 11D SUGRA

$$\mathcal{S}_{11} = -\frac{1}{2\kappa^2}\int_{\mathcal{M}^{11}} \mathrm{d}^{11}x\sqrt{-g}\left(-\mathcal{R} + G\wedge\star G + 2\sqrt{2}C\wedge G\wedge G\right)$$
$$-\frac{1}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_{i=1}^{2}\int_{\mathcal{M}^{10}_{(i)}} \mathrm{d}^{10}x\sqrt{-g}\left(\mathrm{tr}F_{(i)}\wedge F_{(i)} - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)$$

Compactification on warped product $X^6 \times S^1/\mathbb{Z}_2$

supersymmetry $\to$ non-zero flux: $\quad G_{ABCD} = -\frac{\mu}{48}\epsilon_{ABCD}{}^{EF}\omega_{EF}$

$$\mu \equiv \frac{\sqrt{2}}{\pi V_0}\left(\frac{\kappa}{4\pi}\right)^{2/3}\int_{X^6}\omega\wedge\left(\mathrm{tr}F_{(1)}\wedge F_{(1)} - \frac{1}{2}\mathrm{tr}\mathcal{R}\wedge\mathcal{R}\right)$$

Standard embedding (Hořava-Witten) $\mu < 0$:

- volume V of Calabi-Yau X^6 decreases with $|x^{11}|$
- V at our brane fixed by the observable sector gauge coupling: $V_0 M_{11}^6 = (4\pi)^{2/3}\alpha_{\mathrm{GUT}}^{-1}$
- $\Rightarrow$ upper bound on length of 11-th dimension πR_{11}
- $M_W \lll (\pi R_{11})^{-1} < M_{11} < M_{\mathrm{Pl}}$

Minimal heterotic M-theory

Non-standard embedding with $\mu > 0$:

- volume V of Calabi-Yau X^6 increases with $|x^{11}|$
- length of 11-th dimension πR_{11} may be quite large
- hierarchy problem of the weak vs Planck scale may be addressed

$$M_{\rm Pl}^2 \approx 8aM_{11}^2(M_{11}\pi R_{11})^2$$

$M_{11} \equiv \kappa^{-2/9}$
$\frac{1}{16\pi^2}\int_{X^6}\omega\wedge\left({\rm tr}F^2 - \frac{1}{2}{\rm tr}\mathcal{R}^2\right) = aV_0^{1/3}$

Relation typical for $N = 2$ flat extra dimensions

$M_{11} \sim 1.3a^{-1/4}$ **TeV**, $\quad \pi R_{11} \lesssim 100\,\mu$**m**
enough to obtain the correct value of $M_{\rm Pl}$

Minimal heterotic M-theory

Compactification of 11D SUGRA on Calabi-Yau X^6
only with universal hypermultiplet and gravity multiplet ($h_{(1,1)} = 1$)
$\Rightarrow$ gravity-modulus system described by the GLD action

$$\mathcal{S}_5 = \frac{1}{\kappa_5^2} \int_{\mathcal{M}^5} \mathrm{d}^5 x \sqrt{-g} \left[\frac{1}{2} \mathcal{R}_5 - \frac{1}{2} \partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-(2\hat{c}/\sqrt{3})S} \right]$$
$$- \frac{1}{\kappa_5^2} \sum_{i=1,2} \int_{\mathcal{M}^4_{(i)}} \mathrm{d}^4 x \sqrt{-g} \, \Lambda_{(i)} e^{-(\hat{c}/\sqrt{3})S}$$

with: $\hat{c}^2 = 6$ $\Lambda_b = \frac{\mu^2}{384}$, $\Lambda_{(1)} = -\Lambda_{(2)} = \frac{\mu}{4\sqrt{2}}$

Dilaton related to the overall volume modulus from universal hypermultiplet:
$\hat{V} \equiv \exp(\sqrt{2} S)$

Minimal heterotic M-theory

Compactification of 11D SUGRA on Calabi-Yau X^6
only with universal hypermultiplet and gravity multiplet ($h_{(1,1)} = 1$)
$\Rightarrow$ gravity-modulus system described by the GLD action

$$\mathcal{S}_5 = \frac{1}{\kappa_5^2} \int_{\mathcal{M}^5} \mathrm{d}^5 x \sqrt{-g} \left[\frac{1}{2} \mathcal{R}_5 - \frac{1}{2} \partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-(2\hat{c}/\sqrt{3})S} \right] - \frac{1}{\kappa_5^2} \sum_{i=1,2} \int_{\mathcal{M}^4_{(i)}} \mathrm{d}^4 x \sqrt{-g} \, \Lambda_{(i)} e^{-(\hat{c}/\sqrt{3})S}$$

with: $\hat{c}^2 = 6$ $\quad \Lambda_b = \frac{\mu^2}{384}$, $\quad \Lambda_{(1)} = -\Lambda_{(2)} = \frac{\mu}{4\sqrt{2}}$

Dilaton related to the overall volume modulus from universal hypermultiplet:
$\hat{V} \equiv \exp(\sqrt{2} S)$

$\hat{c}^2 > 1 \Rightarrow$ non-conventional spectrum of KK states

$$M_n \approx 29 n a^{3/8} M_{11} \left(\frac{M_{11}}{M_{\mathrm{Pl}}} \right)^{5/4}$$

masses as for $N = 8/5 = 1.6$ flat extra dimensions (but non-degenerate)

Heterotic M-theory with vector multiplets (in 5D)

- **compactified on CY space with the Hodge number** $h_{(1,1)} > 1$
- $h_{(1,1)}$ **Kähler moduli** t^i $(i = 1, \ldots, h_{(1,1)})$ **defined by** $\omega = t^i \omega_i$
- **intersection numbers**

$$d_{ijk} \equiv \frac{1}{V_0} \int_{X^6} \omega_i \wedge \omega_j \wedge \omega_k$$

- $h_{(1,1)}$ **flux parameters**

$$\mu_i \equiv \frac{\sqrt{2}}{\pi V_0} \left(\frac{\kappa}{4\pi} \right)^{2/3} \int_X \omega_i \wedge \left(\mathrm{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, **only** $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ **(same as for** $h_{(1,1)} = 1$**):**
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu\text{m} \quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, **only** $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ (same as for $h_{(1,1)} = 1$):
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions
- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(10)$ TeV
  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, **only** $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ **(same as for** $h_{(1,1)} = 1$**):**
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions
- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(10)$ TeV
  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions
- $\mu_1 = 0 \qquad \mu_2 \neq 0$:
  - $\hat{c}^2 = 10$
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, **only** $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ **(same as for** $h_{(1,1)} = 1$**):**
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions
- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(10)$ TeV
  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions
- $\mu_1 = 0 \qquad \mu_2 \neq 0$:
  - $\hat{c}^2 = 10$
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

Warped product of one large (flat) and six curved extra dimensions

Heterotic M-theory with vector multiplets

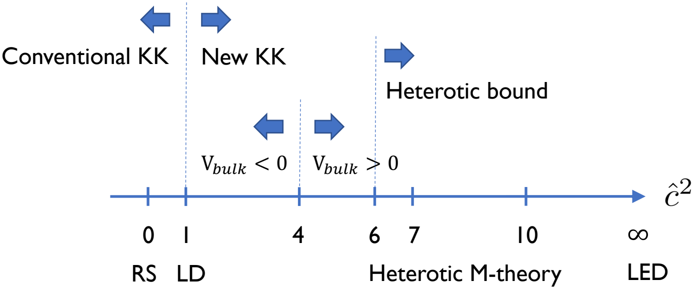
Simple example: $h_{(1,1)} = 2$, only $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0 \qquad$ (same as for $h_{(1,1)} = 1$):
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100 \mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions
- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
  - $\pi R_{11} \sim 100 \mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(10)$ TeV
  - KK spectrum similar to that of $N = 1.5$ flat extra dimensions
- $\mu_1 = 0 \qquad \mu_2 \neq 0$:
  - $\hat{c}^2 = 10$
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100 \mu m \quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

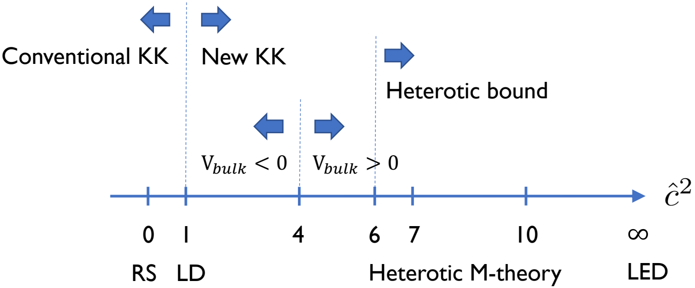
Warped product of one large (flat) and six curved extra dimensions

We have not found solutions corresponding to different values of $\hat{c}^2$ even for more complicated CY spaces

It is not difficult to see that in general heterotic M-theory $\hat{c}^2 \geq 6$



It is not difficult to see that in general heterotic M-theory $\hat{c}^2 \geq 6$



Smaller values of $\hat{c}^2 = 1, 4$ were obtained in 5D SUGRA
with decoupled universal hypermultiplet

Kehagias, Riotto, 2017; Antoniadis et al., 2017

Problematic from the point of view of higher dimensional string- or M-theory (LD $\hat{c}^2 = 1$ may be related to 6D "Little String Theory")

Conclusions

- Genaral Linear Dilaton models (5D)
  - 2-parameter class of potential solutions to the weak-scale hierarchy problem (using continuous clockwork mechanism)
  - models differ in properties of KK masses and coupling (some are quite unconventional)
  - are there consistent UV-completions of GLD models? Yes, but probably only for a very limited discrete set of parameters
- Heterotic M-theory may be such 11D UV-completion
  - non-standard embedding (of spin connection in the gauge group) is necessary
  - minimal version (modification of Hořava-Witten model) corresponds to GLD with $\hat{c}^2 = 6$
    - Planck-scale hierarchy as for 2 flat extra dimensions
    - KK spectrum as for 1.6 flat extra dimensions
    - fundamental scale not very much higher than the weak scale
  - heterotic bound: $\hat{c}^2 \geq 6$
  - models constructed only for $\hat{c}^2 = 6,\ 7,\ 10$
- Previously found 5D SUGRA models with $\hat{c}^2 = 1,\ 4$: uplift to higher dimensional string- or M-theory seems to be problematic