Spontaneous CP Violation and Scalar FCNC in a 2HDM

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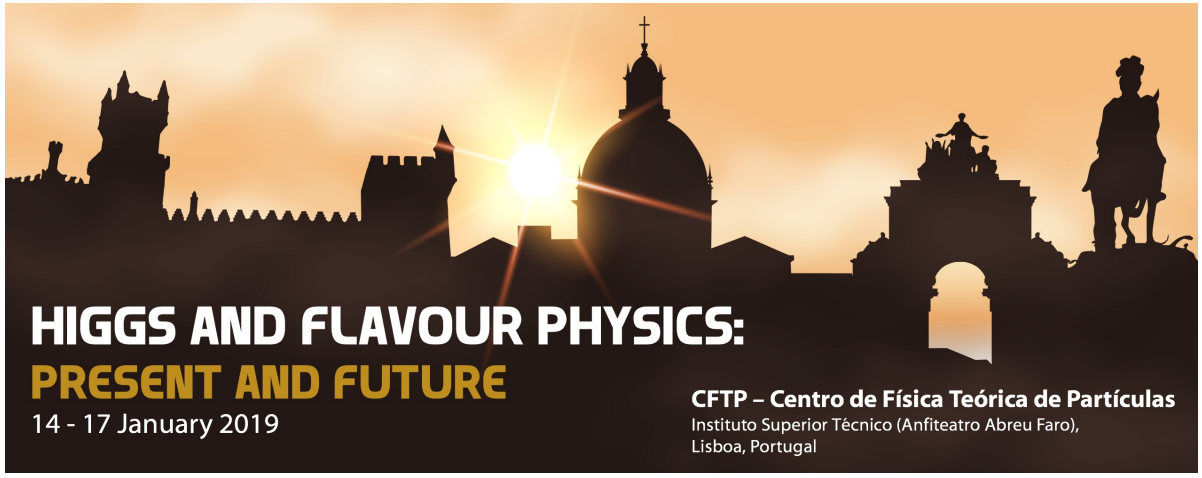

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1 Introduction
2 A 2HDM with SCPV and SFCNC
- Yukawa couplings & Symmetry
- Scalar Sector, Spontaneous CP Violation & Physical Couplings
- Phenomenology, Constraints & New Physics prospects

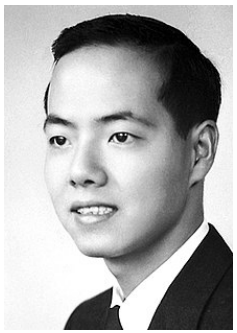
Introduction

. . . old but gold!

PHYSICAL REVIEW D VOLUME 8, NUMBER 4 15 AUGUST 1973

A Theory of Spontaneous T Violation*

T. D. Lee
Department of Physics, Columbia University, New York, New York 10027
(Received 11 April 1973)



A theory of spontaneous T violation is presented. The total Lagrangian is assumed to be invariant under the time reversal T and a gauge transformation (e.g., the hypercharge gauge), but the physical solutions are not. In addition to the spin-1 gauge field and the known matter fields, in its simplest form the theory consists of two complex spin-0 fields. Through the spontaneous symmetry-breaking mechanism of Goldstone and Higgs, the vacuum expectation values of these two spin-0 fields can be characterized by the shape of a triangle and their quantum fluctuations by its vibrational modes, just like a triangular molecule. T violations can be produced among the known particles through virtual excitations of the vibrational modes of the triangle which has a built-in T-violating phase angle. Examples of both Abelian and non-Abelian gauge groups are discussed. For renormalizable theories, all spontaneously T-violating effects are finite. It is found that at low energy, below the threshold of producing these vibrational quanta, T violation is always quite small.

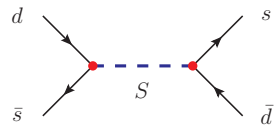
N.B.

[5]There also exist in the literature discussions of other theories of spontaneous *CP* violation, different from those considered in this paper: T. D. Lee, Phys. Rev. 137, B1621 (1965); Roger Dashen, Phys. Rev. D 3, 1879 (1971); J. Nuyts, Phys. Rev. Lett. 26, 1604 (1971); 27, 361(E) (1971); M. A. B. Bég, Phys. Rev. D 4, 3810 (1971). See also Appendix D.

Spontaneous CP Violation (SCPV)

- There is a CP transformation under which
  the Lagrangian is invariant
- . . . but there is *no* CP transformation under which
  the vacuum is invariant

- Same breaking/hiding mechanism used for gauge symmetries
- Full Poincaré group symmetry (with P $\rightarrow$ CP)
- Phenomenologically appealing, two incomplete generations known in 1973
  - Superweak ($\Delta F = 2$) CP Violation:
    $K_L \rightarrow \pi^+\pi^-$ from CPV in mixing (ϵ_K):



- N.B. no SCPV in the SM

Lee's model – 2HDM

Scalar potential, 2 scalar $SU(2)_L$ doublets Φ_i (same Y)

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2 \Phi_1^\dagger\Phi_1 + \mu_{22}^2 \Phi_2^\dagger\Phi_2 + \left(\mu_{12}^2 \Phi_1^\dagger\Phi_2 + \text{H.c.}\right)$$
$$+ \lambda_1 (\Phi_1^\dagger\Phi_1)^2 + \lambda_2 (\Phi_2^\dagger\Phi_2)^2 + 2\lambda_3 (\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4 (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+ \left(\lambda_5 (\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}\right) + \left(\lambda_6 (\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7 (\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{H.c.}\right) .$$

μ_{11}^2, μ_{22}^2, $\lambda_{i=1,\ldots,4}$ real; μ_{12}^2, $\lambda_{j=5,6,7}$ complex

- CP transformation

$$(CP)\,\Phi_j(\vec{x},t)\,(CP)^\dagger = e^{i\varphi}\,\Phi_j^*(-\vec{x},t)$$

CP invariance implies μ_{12}^2, $\lambda_{j=5,6,7}$ *real*

Lee's model – 2HDM

- Vacuum $\langle \Phi_j \rangle$

$$\langle \Phi_j \rangle = \begin{pmatrix} 0 \\ e^{i\theta_j} v_j/\sqrt{2} \end{pmatrix}$$

$$(CP) \langle \Phi_j \rangle (CP)^\dagger = e^{i\varphi} \langle \Phi_j \rangle^*$$

Spontaneous CP Violation requires *relative* vacuum phases

- The question is then to find $\mathcal{V}(\Phi_1, \Phi_2)$ such that
  - $\{\langle \Phi_1 \rangle, \langle \Phi_2 \rangle\}$ with good electroweak symmetry breaking
  - There is a *physical* relative phase among the different $\langle \Phi_j \rangle$
- CP invariance also requires *real* Yukawa couplings

$$\text{e.g} \qquad \mathscr{L}_\text{Y} = -\bar{Q}_{La} (Y_j^d)_{ab} \Phi_j d_b - \bar{d}_b (Y_j^d)_{ab} \Phi_j^\dagger Q_{La}$$

$(Y_j^d)_{ab}$ real

Scalar FCNC (SFCNC)

- Although the original motivation was CPV in K^0–$\bar{K}^0$, generic 2HDM $\Rightarrow$

  uncontrolled Scalar Flavour Changing Neutral Couplings

- Yukawa couplings

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}_L^0(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)d_R^0 - \bar{Q}_L^0(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)u_R^0 + \mathrm{H.c.}$$

- Vacuum expectation values for EWSB

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ e^{i\theta_1}v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ e^{i\theta_2}v_2/\sqrt{2} \end{pmatrix}$$

$\theta = \theta_2 - \theta_1$, $v^2 = v_1^2 + v_2^2$, $c_\beta \equiv v_1/v$, $s_\beta \equiv v_2/v$, $t_\beta \equiv \tan\beta$

- Higgs basis

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \text{ with } \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \ \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

$$\langle H_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Scalar FCNC (SFCNC)

- Yukawa couplings

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L^0(M_d^0 H_1 + N_d^0 H_2)d_R^0 - \frac{\sqrt{2}}{v}\bar{Q}_L^0(M_u^0 \tilde{H}_1 + N_u^0 \tilde{H}_2)u_R^0 + \mathrm{H.c.}$$

- Mass matrices

$$M_d^0 = \frac{v e^{i\theta_1}}{\sqrt{2}}(c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2)\,, \quad M_u^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}}(c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2)$$

- SFCNC encoded in N_d^0 and N_u^0:

$$N_d^0 = \frac{v e^{i\theta_1}}{\sqrt{2}}(-s_\beta \Gamma_1 + e^{i\theta} c_\beta \Gamma_2)\,, \quad N_u^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}}(-s_\beta \Delta_1 + e^{-i\theta} c_\beta \Delta_2)$$

uncontrolled Scalar Flavour Changing Neutral Couplings

Scalar FCNC (SFCNC)

Uncontrolled SFCNC are *problematic*
Popular scenario to eliminate them:

- Natural Flavour Conservation ($\mathbb{Z}_2$)
  only one scalar doublet couples to the fermions of a given charge
  S.L. Glashow & S. Weinberg, PRD15 (1977)
  Excessively effective!
  - in 2HDM, no scalar mediated CP Violation
  - in 2HDM, in fact, does not allow for Spontaneous CP Violation
    [N.B. soft symmetry breaking bypass]
- nevertheless, beyond 2 in nHDM,
  - Spontaneous CP Violation
  - CP Violation in $\Delta F = 2$ transitions from charged scalars

G.C. Branco, PRL44 (1980)
G.C. Branco, PRD22 (1980)

Goal: realistic 2HDM

- scalar potential giving SCPV (vacuum phase θ)
- θ generates a complex CKM matrix
- SFCNC effects under control (in agreement with experiment)

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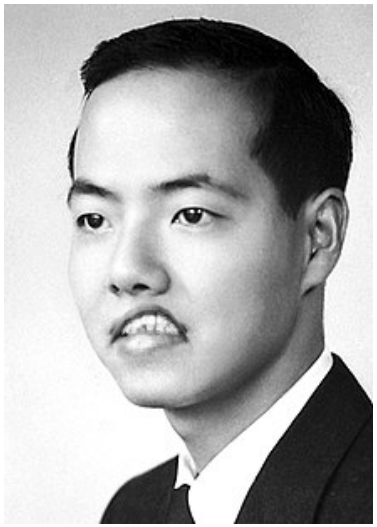

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F.J. Botella, G.C. Branco & MN, arXiv:1808.00493

A 2HDM with SCPV and SFCNC

1. Yukawa couplings & Symmetry
2. Scalar Sector, Spontaneous CP Violation & Physical Couplings
3. Phenomenology, Constraints & New Physics prospects

Yukawa couplings & Symmetry

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- Yukawa couplings

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}^0_L(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)d^0_R - \bar{Q}^0_L(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)u^0_R + \mathrm{H.c.}$$

- Symmetry under $\mathbb{Z}_2$ transformations (gBGL)

$$\Phi_1 \mapsto \Phi_1, \quad \Phi_2 \mapsto -\Phi_2, \quad d^0_{Rk} \mapsto d^0_{Rk}, \quad u^0_{Rk} \mapsto u^0_{Rk}, \quad k = 1,2,3$$
$$Q^0_{Lj} \mapsto Q^0_{Lj}, \quad j = 1,2, \quad Q^0_{L3} \mapsto -Q^0_{L3}$$

Yukawa couplings & Symmetry

- Yukawa couplings

$$\mathscr{L}_{\mathrm{Y}} = -\bar{Q}^0_L(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)d^0_R - \bar{Q}^0_L(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)u^0_R + \mathrm{H.c.}$$

- Symmetry under $\mathbb{Z}_2$ transformations (gBGL)

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$$Q^0_{Lj} \mapsto Q^0_{Lj}, \quad j = 1,2, \quad Q^0_{L3} \mapsto -Q^0_{L3}$$

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

J. Alves, F.J. Botella, G.C. Branco, F. Cornet-Gómez & MN,
EPJC77 (2017) 9, 585 [arXiv:1703.03796]

- CP invariance of $\mathscr{L}_{\mathrm{Y}}$

$$\Gamma_j^* = \Gamma_j, \qquad \Delta_j^* = \Delta_j$$

(once again)

- Vacuum expectation values for EWSB

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ e^{i\theta_1} v_1/\sqrt{2} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ e^{i\theta_2} v_2/\sqrt{2} \end{pmatrix},$$

$\theta = \theta_2 - \theta_1$, $v^2 = v_1^2 + v_2^2$, $c_\beta \equiv v_1/v$, $s_\beta \equiv v_2/v$, $t_\beta \equiv \tan\beta$

- Higgs basis

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$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Yukawas

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L^0 (M_d^0 H_1 + N_d^0 H_2) d_R^0 - \frac{\sqrt{2}}{v}\bar{Q}_L^0 (M_u^0 \tilde{H}_1 + N_u^0 \tilde{H}_2) u_R^0 + \text{H.c.}$$

Mass matrices

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Mass matrices

$$M_d^0 = \frac{v e^{i\theta_1}}{\sqrt{2}} (c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2), \quad M_u^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}} (c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2)$$

- **Important:** with $\hat{M}_d^0$ and $\hat{M}_u^0$ *real*

$$M_d^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0, \qquad M_u^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \hat{M}_u^0$$

$$M_d^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{i\theta} \mathrm{P}_3 \right] \hat{M}_d^0, \quad M_u^0 = \left[(\mathbf{1} - \mathrm{P}_3) + e^{-i\theta} \mathrm{P}_3 \right] \hat{M}_u^0$$

$$\mathrm{P}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Bidiagonalisation of M_d^0, M_u^0

$$\mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{U}_{d_R} = \mathrm{diag}(m_{d_i}), \quad \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{U}_{u_R} = \mathrm{diag}(m_{u_i})$$

- $M_d^0 M_d^{0\dagger}$

$$M_d^0 M_d^{0\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \hat{M}_d^0 \hat{M}_d^{0\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix}$$

$\hat{M}_d^0 \hat{M}_d^{0\,T}$ *real* and *symmetric*

$$\mathcal{O}_L^{d\,T} \hat{M}_d^0 \hat{M}_d^{0\,T} \mathcal{O}_L^d = \mathrm{diag}(m_{d_i}^2) \qquad \text{with real orthogonal } \mathcal{O}_L^d$$

$$\mathcal{U}_{d_L}^\dagger M_d^0 M_d^{0\dagger} \mathcal{U}_{d_L} = \mathrm{diag}(m_{d_i}^2)\,, \text{ with } \quad \mathcal{U}_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \mathcal{O}_L^d$$

- Similarly

$$\mathcal{U}_{u_L}^\dagger M_u^0 M_u^{0\dagger} \mathcal{U}_{u_L} = \mathrm{diag}(m_{u_i}^2)\,, \text{ with } \quad \mathcal{U}_{u_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \mathcal{O}_L^u$$

- Right-handed transformations

$$M_d^{0\dagger} M_d^0 = \hat{M}_d^{0\,T} \hat{M}_d^0, \qquad M_u^{0\dagger} M_u^0 = \hat{M}_u^{0\,T} \hat{M}_u^0$$

$$\mathcal{O}_R^{d\,T} M_d^{0\dagger} M_d^0 \mathcal{O}_R^d = \mathrm{diag}(m_{d_i}^2), \quad \mathcal{O}_R^{u\,T} M_u^{0\dagger} M_u^0 \mathcal{O}_R^u = \mathrm{diag}(m_{u_i}^2)$$

with real orthogonal $\mathcal{O}_R^d$ and $\mathcal{O}_R^u$

- Finally

$$M_d = \mathrm{diag}(m_{d_i}) = \mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{O}_R^d, \quad M_u = \mathrm{diag}(m_{u_i}) = \mathcal{U}_{u_L}^\dagger M_u^0 \mathcal{O}_R^u$$

- The CKM matrix $V \equiv \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$ is

$$V = \mathcal{O}_L^{u\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} \mathcal{O}_L^d$$

requires $e^{i2\theta} \neq \pm 1$ for CP violation!

Scalar FCNC

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

- Scalar Flavour Changing Neutral Couplings (SFCNC) encoded in N_d^0 and N_u^0:

$$N_d^0 = \frac{v e^{i\theta_1}}{\sqrt{2}}(-s_\beta \Gamma_1 + e^{i\theta} c_\beta \Gamma_2)\,, \quad N_u^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}}(-s_\beta \Delta_1 + e^{-i\theta} c_\beta \Delta_2)$$

- Matrix relations

$$N_d^0 = t_\beta M_d^0 - (t_\beta + t_\beta^{-1})\mathrm{P}_3 M_d^0, \quad N_u^0 = t_\beta M_u^0 - (t_\beta + t_\beta^{-1})\mathrm{P}_3 M_u^0$$

- In the mass bases

$$N_d \equiv \mathcal{U}_{d_L}^\dagger N_d^0 \mathcal{O}_R^d\,, \quad N_u \equiv \mathcal{U}_{u_L}^\dagger N_u^0 \mathcal{O}_R^u$$

Scalar FCNC: N_d

- $N_d \equiv \mathcal{U}_{d_L}^\dagger N_d^0 \mathcal{O}_R^d$ with $N_d^0 = t_\beta M_d^0 - (t_\beta + t_\beta^{-1})\mathrm{P}_3 M_d^0$

$$N_d = t_\beta \mathcal{U}_{d_L}^\dagger M_d^0 \mathcal{O}_R^d - (t_\beta + t_\beta^{-1})\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 M_d^0 \mathcal{O}_R^d =$$
$$t_\beta M_d - (t_\beta + t_\beta^{-1})\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L} M_d$$

- All *new* information encoded in $\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L}$

$$[\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L}]_{ij} = \left[\mathcal{O}_L^{d\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\theta} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\theta} \end{pmatrix} \mathcal{O}_L^d \right]_{ij}$$

- Unit vectors $\hat{r}_{[\mathrm{d}]}$ (real) and $\hat{n}_{[\mathrm{d}]}$ (complex)

$$\hat{r}_{[\mathrm{d}]j} \equiv [\mathcal{O}_L^d]_{3j}\,, \quad \hat{n}_{[\mathrm{d}]j} \equiv [\mathcal{U}_{d_L}]_{3j} = e^{i\theta}\hat{r}_{[\mathrm{d}]j}$$

$$\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L} = \mathcal{O}_L^{d\,T} \mathrm{P}_3 \mathcal{O}_L^d, \quad [\mathcal{U}_{d_L}^\dagger \mathrm{P}_3 \mathcal{U}_{d_L}]_{ij} = \hat{n}^*_{[\mathrm{d}]i}\hat{n}_{[\mathrm{d}]j} = \hat{r}_{[\mathrm{d}]i}\hat{r}_{[\mathrm{d}]j}$$

N_d and N_u

- Similarly for N_u with

$$\hat{r}_{[\mathrm{u}]j} \equiv [\mathcal{O}_L^u]_{3j} \quad \hat{n}_{[\mathrm{u}]j} \equiv [\mathcal{U}_{u_L}]_{3j} = e^{-i\theta}\hat{r}_{[\mathrm{u}]j}$$

$$\mathcal{U}_{u_L}^\dagger \mathrm{P}_3 \mathcal{U}_{u_L} = \mathcal{O}_L^{u\,T} \mathrm{P}_3 \mathcal{O}_L^u, \quad [\mathcal{U}_{u_L}^\dagger \mathrm{P}_3 \mathcal{U}_{u_L}]_{ij} = \hat{n}^*_{[\mathrm{u}]i}\hat{n}_{[\mathrm{u}]j} = \hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]j}$$

- N_d and N_u are *real* [N.B. phase convention dependent statement]

$$[N_d]_{ij} = t_\beta \delta_{ij} m_{d_i} - (t_\beta + t_\beta^{-1})\hat{n}^*_{[\mathrm{d}]i}\hat{n}_{[\mathrm{d}]j} m_{d_j}$$
$$[N_u]_{ij} = t_\beta \delta_{ij} m_{u_i} - (t_\beta + t_\beta^{-1})\hat{n}^*_{[\mathrm{u}]i}\hat{n}_{[\mathrm{u}]j} m_{u_j}$$

- $\hat{n}_{[\mathrm{d}]}$ and $\hat{n}_{[\mathrm{u}]}$ are not independent

$$\hat{n}_{[\mathrm{d}]i} = \hat{n}_{[\mathrm{u}]j} V_{ji}\,, \quad \hat{n}_{[\mathrm{u}]i} = V^*_{ij}\,\hat{n}_{[\mathrm{d}]j}$$

- SFCNC are *controlled*: quark masses and normalized $\hat{n}_{[\mathrm{d}]}$ and $\hat{n}_{[\mathrm{u}]}$

CKM and SFCNC

An important property

- CKM:

$$V = \mathcal{O}_L^{u\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} \mathcal{O}_L^d$$

- SFCNC controlled by

$$\hat{r}_{[\mathrm{d}]j} \equiv [\mathcal{O}_L^d]_{3j}\,, \quad \hat{r}_{[\mathrm{u}]j} \equiv [\mathcal{O}_L^u]_{3j}$$

- Absence of SFCNC in one sector:
  $\hat{r}_{[\mathrm{q}]}$ has two vanishing components, e.g. $\hat{r}_{[\mathrm{d}]} = (0,0,1)$
- . . . but in that case CKM *is not CP violating!*

$$V = \mathcal{O}_L^{u\,T} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2\theta} \end{pmatrix} \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The Scalar Sector, Spontaneous CP Violation and Physical Couplings

Scalar sector

- 2HDM potential
  - CP invariant (all couplings are real)
  - $\mathbb{Z}_2$ symmetry, softly broken by $\mu_{12}^2 \neq 0$

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2\Phi_1^\dagger\Phi_1 + \mu_{22}^2\Phi_2^\dagger\Phi_2 + \mu_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1)$$
$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_5[(\Phi_1^\dagger\Phi_2)^2+(\Phi_2^\dagger\Phi_1)^2]$$

- Minimization of $V(v_1, v_2, \theta) \equiv \mathcal{V}(\langle\Phi_1\rangle, \langle\Phi_2\rangle)$

$$\frac{\partial V}{\partial \theta} = -v_1 v_2 \sin\theta(\mu_{12}^2 + 2\lambda_5 v_1 v_2 \cos\theta) = 0$$
$$\frac{\partial V}{\partial v_1} = \mu_{11}^2 v_1 + \lambda_1 v_1^3 + (\lambda_3 + \lambda_4) v_1 v_2^2 + v_2(\mu_{12}^2 \cos\theta + \lambda_5 v_1 v_2 \cos 2\theta) = 0$$
$$\frac{\partial V}{\partial v_2} = \mu_{22}^2 v_2 + \lambda_2 v_2^3 + (\lambda_3 + \lambda_4) v_1^2 v_2 + v_1(\mu_{12}^2 \cos\theta + \lambda_5 v_1 v_2 \cos 2\theta) = 0$$

- For spontaneous CP violation: solution $\{v_1, v_2, \theta\}$ with $\theta \neq 0, \pm\pi/2, \pm\pi$
- Trade

$$\mu_{12}^2 = -2\lambda_5 v_1 v_2 \cos\theta$$
$$\mu_{11}^2 = -(\lambda_1 v_1^2 + (\lambda_3 + \lambda_4 - \lambda_5) v_2^2)$$
$$\mu_{22}^2 = -(\lambda_2 v_2^2 + (\lambda_3 + \lambda_4 - \lambda_5) v_1^2)$$

- Expand fields

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}$$

- Higgs basis

$$H_1 = \begin{pmatrix} G^+ \\ (v + \mathrm{h}^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (\mathrm{R}^0 + i\mathrm{I}^0)/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ \mathrm{I}^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \begin{pmatrix} \mathrm{h}^0 \\ \mathrm{R}^0 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

- Identify would-be Goldstone bosons $G^\pm$ and G^0

$$G^\pm = c_\beta \varphi_1^\pm - s_\beta \varphi_2^\pm, \quad G^0 = c_\beta \eta_1 - s_\beta \eta_2$$

- Charged scalar $H^\pm = s_\beta \varphi_1^\pm - c_\beta \varphi_2^\pm$

$$\mathcal{V}(\Phi_1, \Phi_2) \supset v^2(\lambda_5 - \lambda_4) H^+ H^- \Rightarrow m^2_{H^\pm} = v^2(\lambda_5 - \lambda_4)$$

- Neutral scalars

$$\mathcal{V}(\Phi_1, \Phi_2) \supset \frac{1}{2} \begin{pmatrix} \mathrm{h}^0 & \mathrm{R}^0 & \mathrm{I}^0 \end{pmatrix} \mathcal{M}_0^2 \begin{pmatrix} \mathrm{h}^0 \\ \mathrm{R}^0 \\ \mathrm{I}^0 \end{pmatrix} \qquad \mathcal{M}_0^2 = \mathcal{M}_0^{2\,T}$$

with

$$\begin{aligned}
[\mathcal{M}_0^2]_{11} &= 2v^2 \left\{ \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2 c_\beta^2 s_\beta^2 [\lambda_{345} + 2\lambda_5 c_\theta^2] \right\} \\
[\mathcal{M}_0^2]_{22} &= 2v^2 \left\{ c_\beta^2 s_\beta^2 (\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_5 (c_\beta^2 - s_\beta^2)^2 c_\theta^2 \right\} \\
[\mathcal{M}_0^2]_{12} &= 2v^2 s_\beta c_\beta \left\{ -\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + (c_\beta^2 - s_\beta^2)[\lambda_{345} + 2\lambda_5 c_\theta^2]) \right\} \\
[\mathcal{M}_0^2]_{13} &= -v^2 \lambda_5 s_{2\beta} s_{2\theta} \\
[\mathcal{M}_0^2]_{23} &= -v^2 \lambda_5 c_{2\beta} s_{2\theta} \\
[\mathcal{M}_0^2]_{33} &= 2v^2 \lambda_5 s_\theta^2
\end{aligned}$$

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 - \lambda_5$$

- Physical neutral scalars

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} \mathrm{h}^0 \\ \mathrm{R}^0 \\ \mathrm{I}^0 \end{pmatrix}$$

where

$$\mathcal{R}^T \, \mathcal{M}_0^2 \, \mathcal{R} = \mathrm{diag}(m_h^2, m_H^2, m_A^2) \,, \quad \mathcal{R}^{-1} = \mathcal{R}^T$$

- $\mathcal{R}$ "mixes", a priori, all three neutral scalars
- assume h is the lightest one, Higgs-like, $m_h = 125$ GeV
- Notice

$$\mathrm{Tr}[\mathcal{M}_0^2] = m_h^2 + m_H^2 + m_A^2 = 2v^2[\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \lambda_5]$$
$$\det[\mathcal{M}_0^2] = m_h^2 \, m_H^2 \, m_A^2 = 2v^6 \lambda_5 (\lambda_1 \lambda_2 - \lambda_{345}^2) \sin^2 2\beta \, \sin^2 \theta$$

No decoupling regime

Yukawa couplings

- $\mathscr{L}_{\mathrm{S\bar{q}q}}$, $S = h,\ H,\ A,\ H^{\pm}$ with $H_q \equiv \frac{N_q + N_q^{\dagger}}{2}$, $A_q \equiv \frac{N_q - N_q^{\dagger}}{2}$:

$$[H_q]_{ij} = t_\beta \delta_{ij} m_{q_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{q}]i} \hat{n}_{[\mathrm{q}]j} \frac{m_{d_i} + m_{d_j}}{2}$$

$$[A_q]_{ij} = (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{q}]i} \hat{n}_{[\mathrm{q}]j} \frac{m_{d_i} - m_{d_j}}{2}$$

- Neutral [$s = 1, 2, 3$ for $S = h, H, A$, respectively]

$$\mathscr{L}_{S\bar{\mathrm{q}}\mathrm{q}} = -\frac{S}{v} \left\{ \bar{d} \left[\mathcal{R}_{1s} M_d + \mathcal{R}_{2s} H_d + i \mathcal{R}_{3s} A_d \right] d + \bar{d} \left[\mathcal{R}_{2s} A_d + i \mathcal{R}_{3s} H_d \right] \gamma_5 d \right\}$$
$$- \frac{S}{v} \left\{ \bar{u} \left[\mathcal{R}_{1s} M_u + \mathcal{R}_{2s} H_u - i \mathcal{R}_{3s} A_u \right] u + \bar{u} \left[\mathcal{R}_{2s} A_u - i \mathcal{R}_{3s} H_u \right] \gamma_5 u \right\}$$

- Charged

$$\mathscr{L}_{H^{\pm}\bar{\mathrm{q}}\mathrm{q}} = -\frac{\sqrt{2} H^+}{v} \left[\bar{u}_L V N_d d_R - \bar{u}_R N_u^{\dagger} V d_L \right]$$
$$- \frac{\sqrt{2} H^-}{v} \left[\bar{d}_R N_d^{\dagger} V^{\dagger} u_L - \bar{d}_L V^{\dagger} N_u u_R \right]$$

Phenomenology, Constraints & New Physics prospects

Goals of the analysis

- establish that the model is viable
  (after imposing reasonable constraints)
- explore prospects for the observation
  of some definite non-SM signal
  - flavour changing decays $t \to hc, hu$ (LHC) and $h \to bs, bd$ (ILC)
  - representative low energy observable:
    time dependent CP violating asymmetry in $B_s \to J/\Psi\Phi$
    [SM prediction $A^{CP}_{J/\Psi\Phi} \simeq -0.04$, current exp. -0.030 ± 0.033]
  - not here: direct observation of new scalars

Parameters

$$\{\hat{r}_{[\mathrm{d}]}, R, \theta, v^2, m_h, t_\beta, m^2_{H^\pm}, \mathcal{R}\}$$

$$\{2+3+1+0+0+1+1+3\} = 11$$

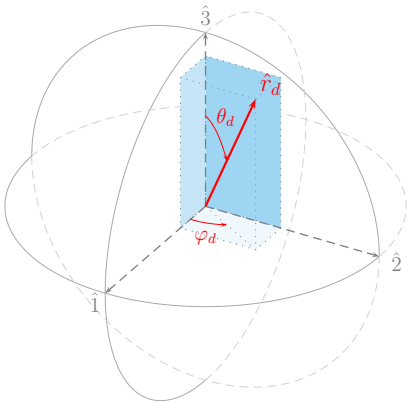
Constraints

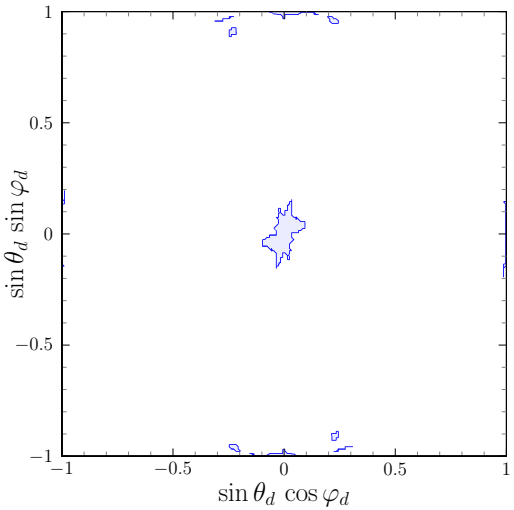
- Good CKM matrix:
  - $|V_{ij}|$ in first two rows
  - CPV phase $\gamma \equiv \arg(-V_{ud}V_{ub}^{*}V_{cb}V_{cd}^{*})$ (the only tree level one)
- Scalar sector
  - Oblique parameters S and T
  - Scalar potential bounded from below
  - Perturbative unitarity of scattering processes $(2 \to 2)$, perturbative λ_j's
  - $m_{H^{\pm}}, m_H, m_A \geq 150$ GeV

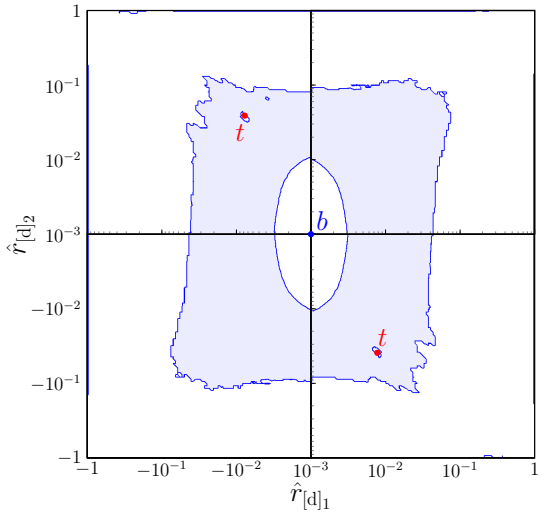
Constraints

- Scalars & Yukawas
  - Production × decay signal strengths of Higgs-like h [ATLAS+CMS from Run I + Run II]
  - Neutral meson mixings
    - B_d^0–$\bar{B}_d^0$ and B_s^0–$\bar{B}_s^0$: mass differences ΔM_{B_d}, ΔM_{B_s} and mixing × decay CP asymmetries in $B_d \to J/\Psi K_S$ and $B_s \to J/\Psi\Phi$
    - K^0–$\bar{K}^0$: scalar mediated contribution to M_{12}^K does not yield sizable contributions to ϵ_K and ΔM_K
    - D^0–$\bar{D}^0$: short distance contribution to M_{12}^D verifies $|M_{12}^D| < 3 \times 10^{-2}\,\text{ps}^{-1}$
  - $\text{Br}(B \to X_s\gamma)$ (i.e. $b \to s\gamma$)
  - Bounds on rare top decays $t \to hq$

Results

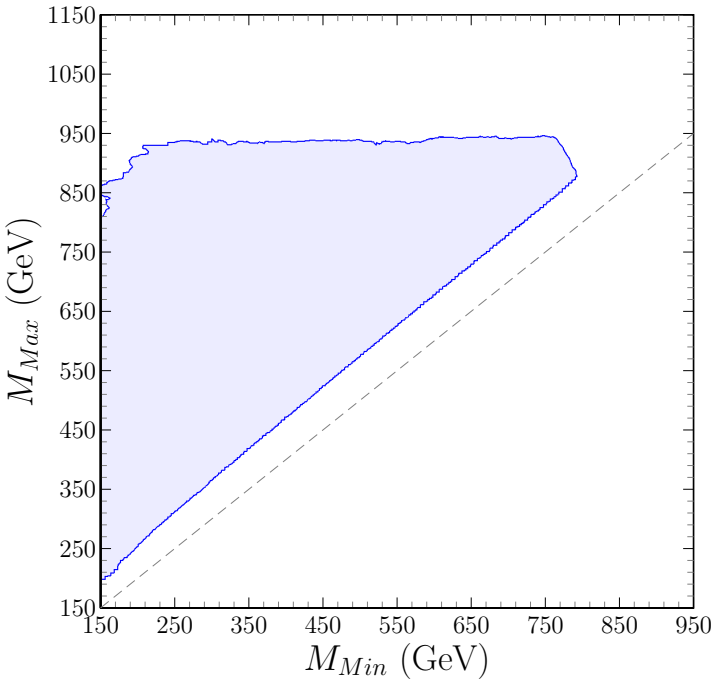


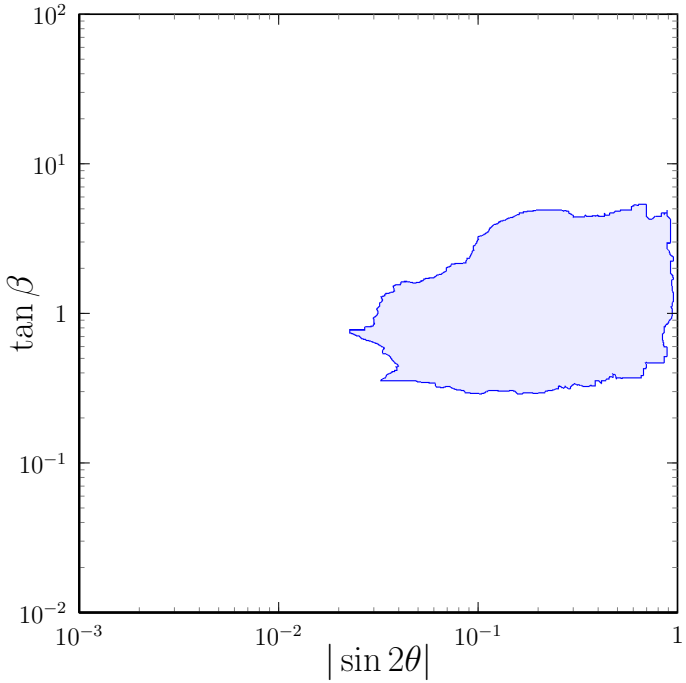




N.B. Allowed regions at 3σ in 2D-$\Delta\chi^2$

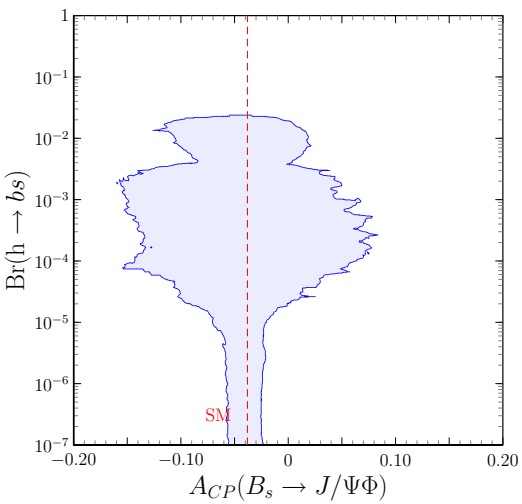
Results

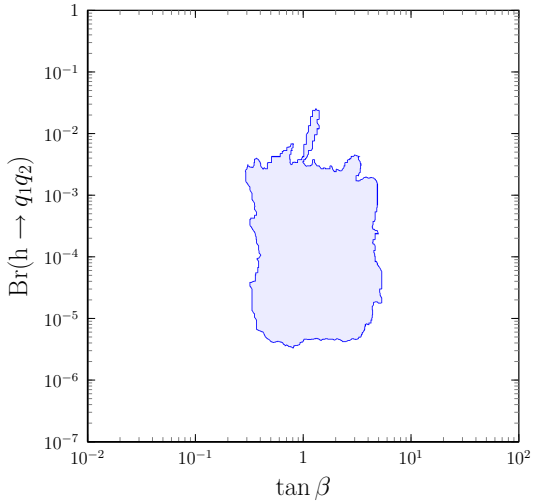


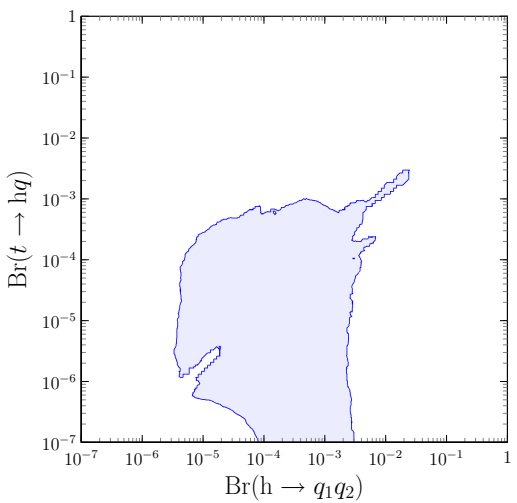


$$M_{Min} \equiv \min(m_H, m_A, m_{H^\pm}), \quad M_{Max} \equiv \max(m_H, m_A, m_{H^\pm})$$

Results







$$\mathrm{Br}(t \to hq) \equiv \mathrm{Br}(t \to hc) + \mathrm{Br}(t \to hu)$$
$$\mathrm{Br}(h \to bq) \equiv \mathrm{Br}(h \to bs) + \mathrm{Br}(h \to bd)$$
$$\mathrm{Br}(h \to q_1 q_2) \equiv \mathrm{Br}(h \to bq) + \mathrm{Br}(h \to sd) + \mathrm{Br}(h \to cu)$$

Summary and Conclusions

- Spontaneous CP Violation:
  - very appealing possibility, original motivation for 2HDMs
  - but potential problem with SFCNC
- Viable 2HDM with SCPV and controlled SFCNC
  - SCPV, complex CKM
  - All new scalar masses below 950 GeV
  - SFCNC mediated rare decays $t \to hq$ and $h \to q_1 q_2$ can be within experimental reach (LHC & ILC)
  - Additional potential effects, e.g. $A^{CP}_{J/\Psi\Phi}$
  - + correlations
- In progress
  - Lepton sector
  - Collider phenomenology
  - Strong CP

Thank you!

Backup

BGL limits

- Consider again

$$[N_d]_{ij} = t_\beta \delta_{ij} m_{d_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{d}]i} \hat{n}_{[\mathrm{d}]j} m_{d_j}$$
$$[N_u]_{ij} = t_\beta \delta_{ij} m_{u_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{u}]i} \hat{n}_{[\mathrm{u}]j} m_{u_j}$$

  with $\hat{n}_{[\mathrm{d}]i} = \hat{n}_{[\mathrm{u}]j} V_{ji}$ and $\hat{n}_{[\mathrm{u}]i} = V^*_{ij}\, \hat{n}_{[\mathrm{d}]j}$

- Choose, for example, $\hat{n}_{[\mathrm{u}]} = (0,0,1)$ (i.e. $\hat{n}_{[\mathrm{u}]j} = \delta_{3j}$):

$$\hat{n}_{[\mathrm{d}]i} = V_{3i}$$

- Then

$$[N_d]_{ij} = t_\beta \delta_{ij} m_{d_i} - (t_\beta + t_\beta^{-1}) V^*_{ti} V_{tj} m_{d_j}$$
$$[N_u]_{ij} = \mathrm{diag}(m_u t_\beta, m_c t_\beta, -m_t t_\beta^{-1})$$

- This is just the *top* BGL model

G.C. Branco, W. Grimus & L. Lavoura
PLB380 (1996) 119 [hep-ph/9601383]

BGL limits

- Consider again

$$[N_d]_{ij} = t_\beta \delta_{ij} m_{d_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{d}]i} \hat{n}_{[\mathrm{d}]j} m_{d_j}$$
$$[N_u]_{ij} = t_\beta \delta_{ij} m_{u_i} - (t_\beta + t_\beta^{-1}) \hat{n}^*_{[\mathrm{u}]i} \hat{n}_{[\mathrm{u}]j} m_{u_j}$$

  with $\hat{n}_{[\mathrm{d}]i} = \hat{n}_{[\mathrm{u}]j} V_{ji}$ and $\hat{n}_{[\mathrm{u}]i} = V^*_{ij}\, \hat{n}_{[\mathrm{d}]j}$

- Choose, for example, $\hat{n}_{[\mathrm{d}]} = (0,0,1)$ (i.e. $\hat{n}_{[\mathrm{d}]j} = \delta_{3j}$):

$$\hat{n}_{[\mathrm{u}]i} = V^*_{i3}$$

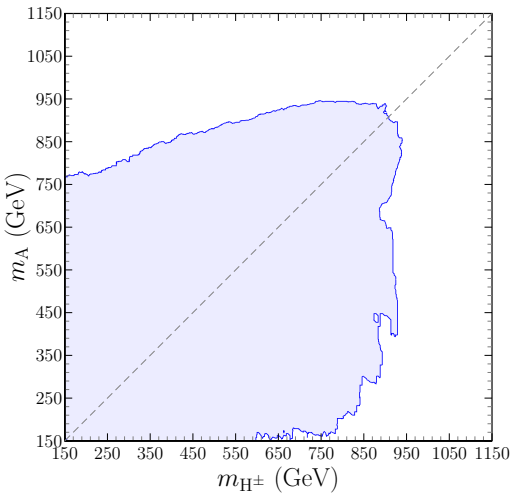
- Then

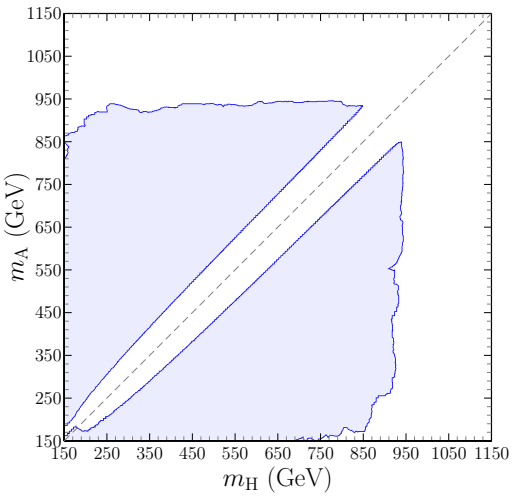
$$[N_d]_{ij} = \mathrm{diag}(m_d t_\beta, m_s t_\beta, -m_b t_\beta^{-1})$$
$$[N_u]_{ij} = t_\beta \delta_{ij} m_{u_i} - (t_\beta + t_\beta^{-1}) V_{ib} V^*_{jb} m_{u_j}$$

- This is just the *bottom* BGL model

G.C. Branco, W. Grimus & L. Lavoura
PLB380 (1996) 119 [hep-ph/9601383]

Results


1150
1050
950
850
750
650
550
450
350
250
150
m_A (GeV)
150 250 350 450 550 650 750 850 950 1050 1150
$m_{H^\pm}$ (GeV)


1150
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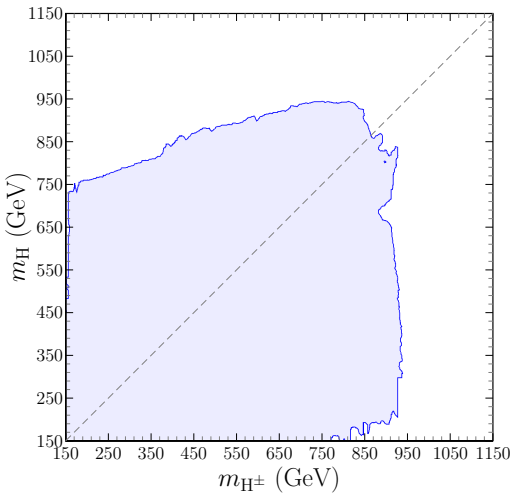
150
m_A (GeV)
150 250 350 450 550 650 750 850 950 1050 1150
m_H (GeV)


1150
1050
950
850
750
650
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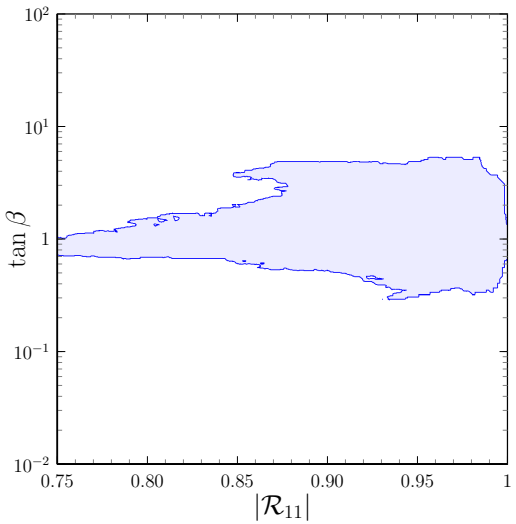
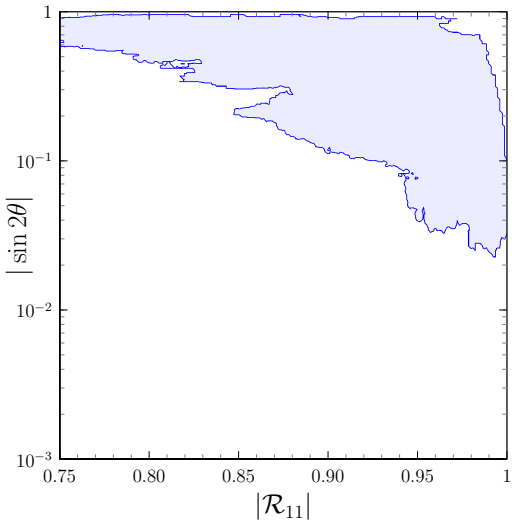
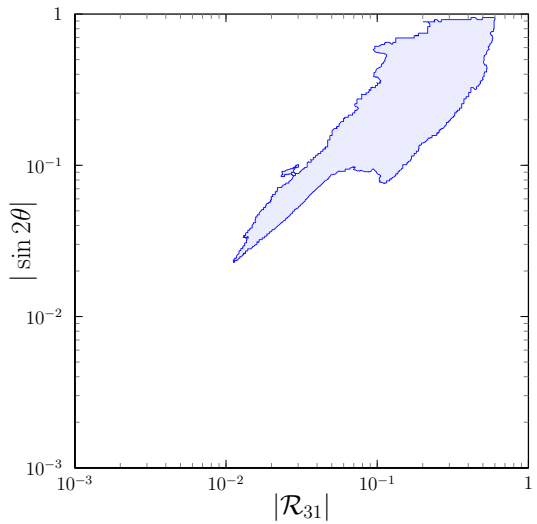
450
350
250
150
m_H (GeV)
150 250 350 450 550 650 750 850 950 1050 1150
$m_{H^\pm}$ (GeV)

Results


10^2
10^1
1
10^{-1}
10^{-2}
$\tan\beta$
0.75
0.80
0.85
0.90
0.95
1
$|\mathcal{R}_{11}|$


1
10^{-1}
10^{-2}
10^{-3}
$|\sin 2\theta|$
0.75
0.80
0.85
0.90
0.95
1
$|\mathcal{R}_{11}|$


1
10^{-1}
10^{-2}
10^{-3}
$|\sin 2\theta|$
10^{-3}
10^{-2}
10^{-1}
1
$|\mathcal{R}_{31}|$

Complex CKM matrix?

- In terms of $R \equiv \mathcal{O}_L^{u\,T}\mathcal{O}_L^d$

$$\hat{r}_{[\mathrm{d}]i} = \hat{r}_{[\mathrm{u}]j} R_{ji}\,, \quad \hat{r}_{[\mathrm{u}]i} = R_{ik}\hat{r}_{[\mathrm{d}]k}$$

- With

$$V = \mathcal{O}_L^{u\,T}[\mathbf{1} + (e^{i2\theta} - 1)\mathrm{P}_3]\mathcal{O}_L^d \Rightarrow V_{ij} = R_{ij} + (e^{i2\theta} - 1)S_{ij}$$

and

$$S_{ij} \equiv [\mathcal{O}_L^{u\,T}\mathrm{P}_3\mathcal{O}_L^d]_{ij} = \hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{d}]j} = R_{ik}\hat{r}_{[\mathrm{d}]k}\hat{r}_{[\mathrm{d}]j} = \hat{r}_{[\mathrm{u}]i}\hat{r}_{[\mathrm{u}]k}R_{kj}$$

real and imaginary parts of V_{ij} read

$$\mathrm{Re}(V_{ij}) = R_{ij} - 2s_\theta^2 S_{ij}, \quad \mathrm{Im}(V_{ij}) = s_{2\theta}S_{ij}$$

- Compute the imaginary part of
rephasing invariant quartet $V_{i_1 j_1} V_{i_1 j_2}^* V_{i_2 j_2} V_{i_2 j_1}^*$

CKM

$$\mathrm{Im}\left(V_{i_1 j_1} V^*_{i_1 j_2} V_{i_2 j_2} V^*_{i_2 j_1}\right) = \sin 2\theta \left\{ \begin{array}{l} 4 s_\theta^2 \begin{pmatrix} R_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \\ -S_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \end{pmatrix} \\ +4 s_\theta^2 \begin{pmatrix} S_{i_1 j_1} S_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \\ -S_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \\ +S_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} S_{i_1 j_2} \\ -R_{i_1 j_1} S_{i_2 j_1} S_{i_2 j_2} S_{i_1 j_2} \end{pmatrix} \\ + \begin{pmatrix} S_{i_1 j_1} R_{i_2 j_1} R_{i_2 j_2} R_{i_1 j_2} \\ -R_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} R_{i_1 j_2} \\ +R_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \\ -R_{i_1 j_1} R_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \end{pmatrix} \end{array} \right\}$$

. . . not very illuminating

- SFCNC and CP violating CKM
  - If $\hat{r}_{[\mathrm{q}]}$ had two vanishing components, *no SFCNC in that sector*
    but then *no CP violation in CKM*
  - That is having no tree level SFCNC in one quark sector
    is *incompatible* with a CP violating CKM matrix

CKM and SFCNC

$$\mathrm{Im}\left(V_{i_1 j_1} V^*_{i_1 j_2} V_{i_2 j_2} V^*_{i_2 j_1}\right) = \sin 2\theta \left\{ \begin{array}{c} 4 s_\theta^2 \begin{pmatrix} R_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \\ -S_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \end{pmatrix} \\ +4 s_\theta^2 \begin{pmatrix} S_{i_1 j_1} S_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \\ -S_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \\ +S_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} S_{i_1 j_2} \\ -R_{i_1 j_1} S_{i_2 j_1} S_{i_2 j_2} S_{i_1 j_2} \end{pmatrix} \\ + \begin{pmatrix} S_{i_1 j_1} R_{i_2 j_1} R_{i_2 j_2} R_{i_1 j_2} \\ -R_{i_1 j_1} S_{i_2 j_1} R_{i_2 j_2} R_{i_1 j_2} \\ +R_{i_1 j_1} R_{i_2 j_1} S_{i_2 j_2} R_{i_1 j_2} \\ -R_{i_1 j_1} R_{i_2 j_1} R_{i_2 j_2} S_{i_1 j_2} \end{pmatrix} \end{array} \right\}$$

- If $\hat{r}_{[\mathrm{q}]}$ has two vanishing components $\qquad [S_{ij} = \hat{r}_{[\mathrm{u}]i} \hat{r}_{[\mathrm{d}]j}]$
  - S_{ij} has only a non vanishing row (column), for which $S_{ij} = R_{ij}$
  - with $i_1 \neq i_2$ and $j_1 \neq j_2$, only two terms $\neq 0$
  - they have opposite sign!
- At the end of the day,

  6 parameters + t_β for CKM and all SFCNC