Belle II

NAGOYA UNIVERSITY

Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Global track finding algorithms

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Inverted CERN School of Computing
4-7th March 2019

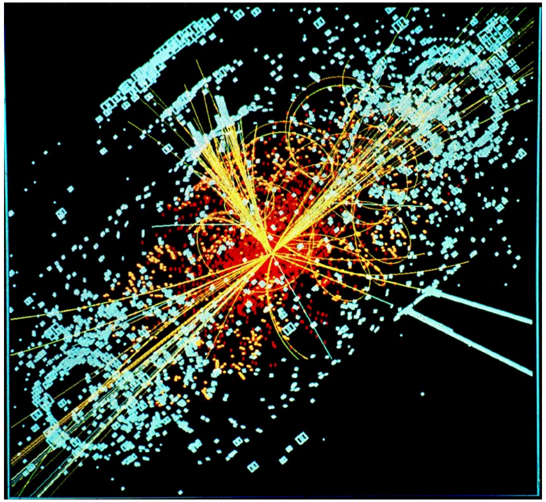
Who am I

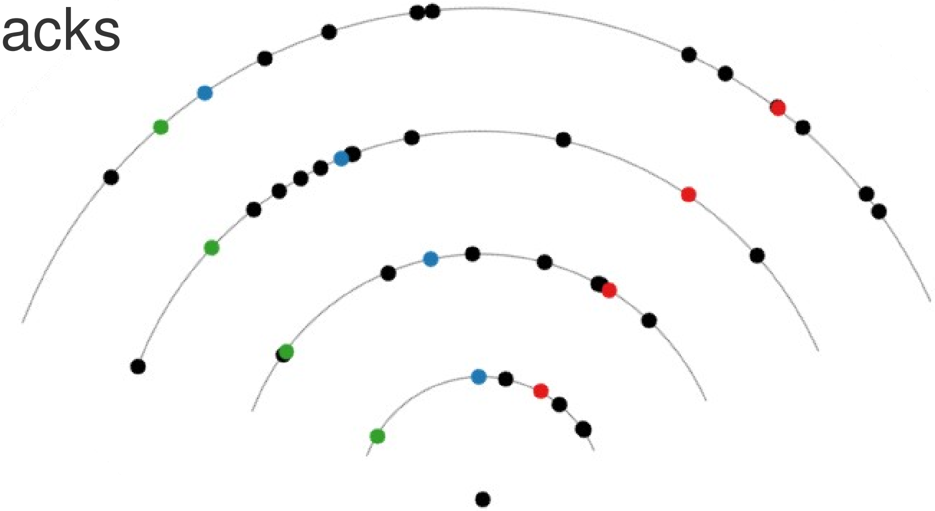
- Belle II experiment, Japan
  - e+e- collider
  - B physics
- Search for magnetic monopoles
  - Non-helical tracking
- “Physics” background

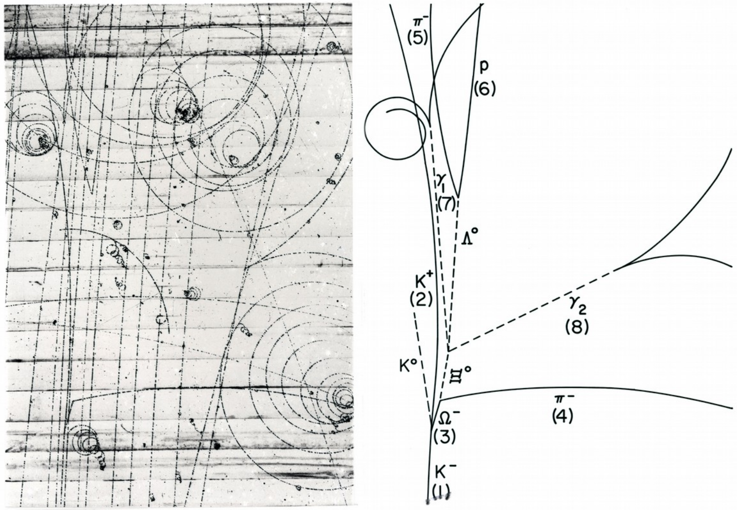
Introduction

Physics

- Fundamental laws govern collisions
- Final state particles are all we get
- Charged particles produce tracks
  - Set of points (x,y,z) in space
- Disentangle all hits into subsets
- Fit tracks







Track finding

- Challenges
  1. Track multiplicity (pileup, low pt)
  2. Fake hits
  3. Measurement errors (resolution, alignment)
  4. Material effects / inhomogeneous magnetic field

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- Local track finding
  - Hit by hit
  - Segment by segment

- Global track finding
  - All hits at once

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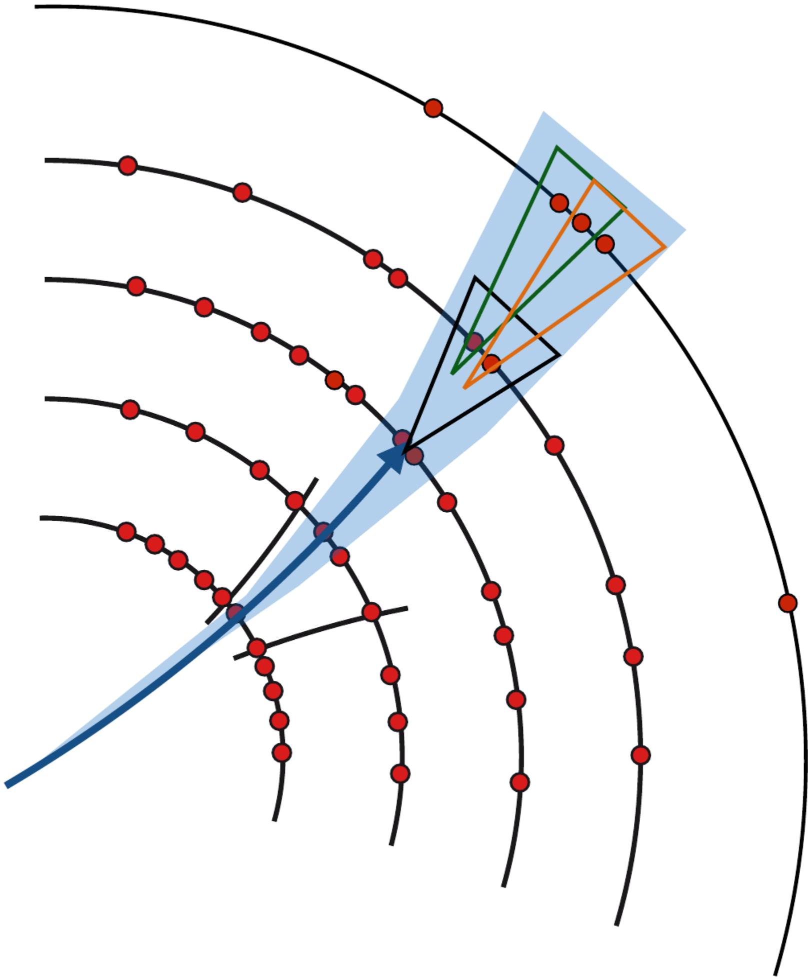
  - Hit by hit
  - Segment by segment
  - Good vs 3 4

- Global track finding
  - All hits at once
  - Good vs 1 2

Local track finding

Track seeding and following

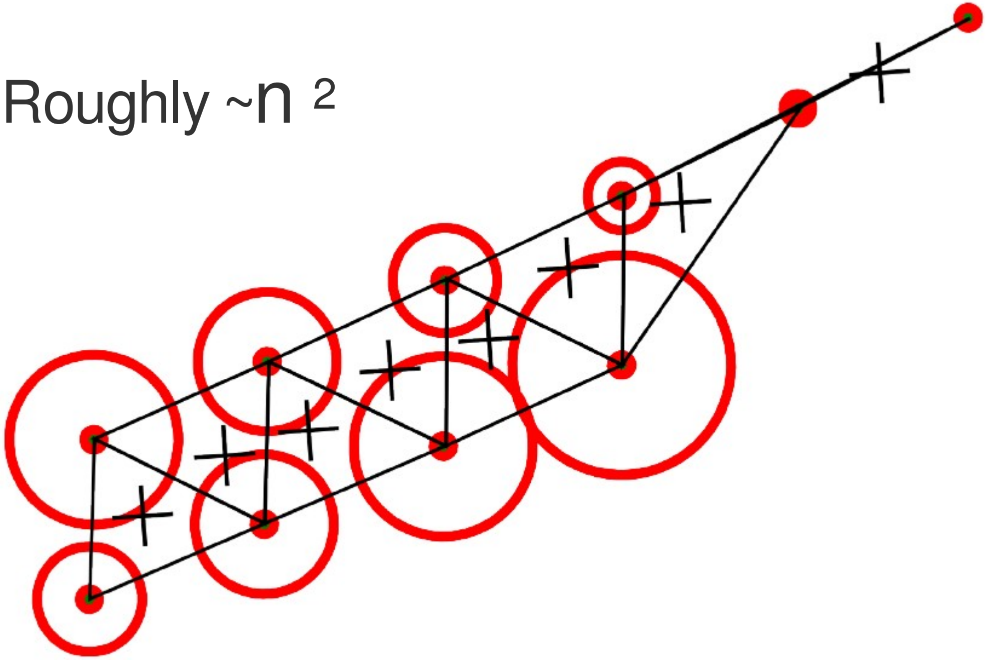
- Seed track window
- Find hits compatible with the track
- Update and advance to new layer
- Smart filters
- Roughly ~n $^{2\text{-}3}$

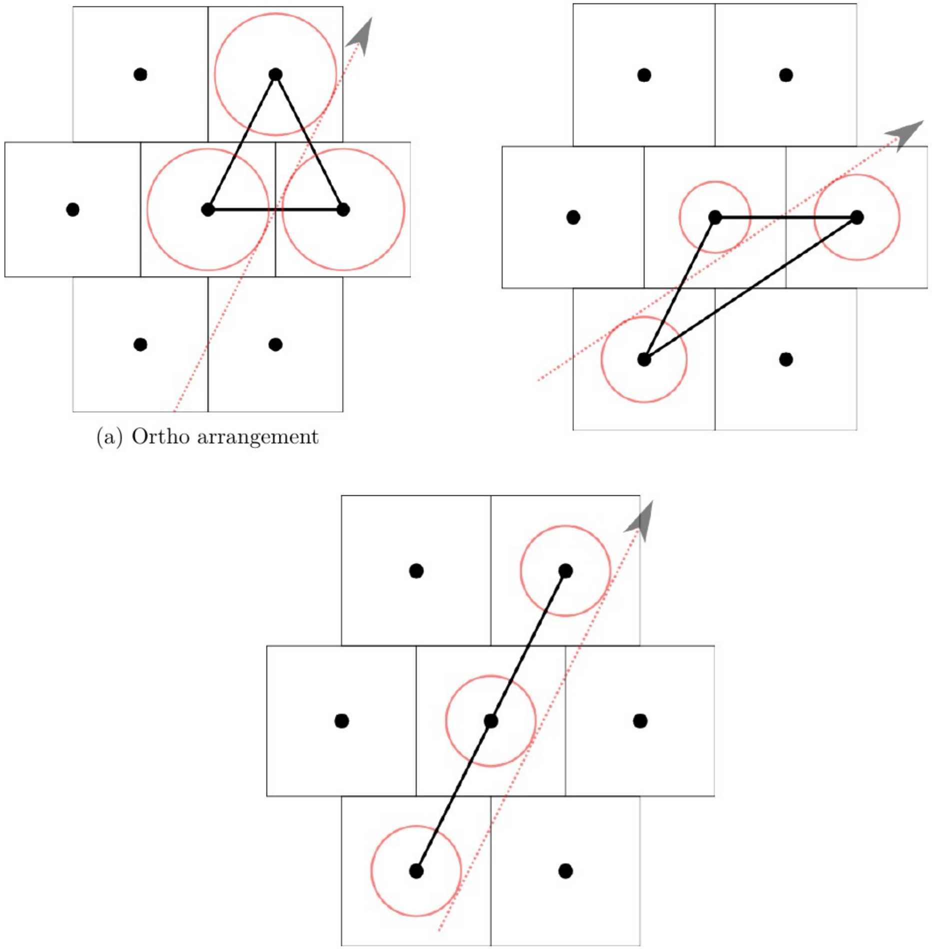


Cellular automaton

- Build clusters
- Build segments
- Build tracks

- Roughly ~n^2

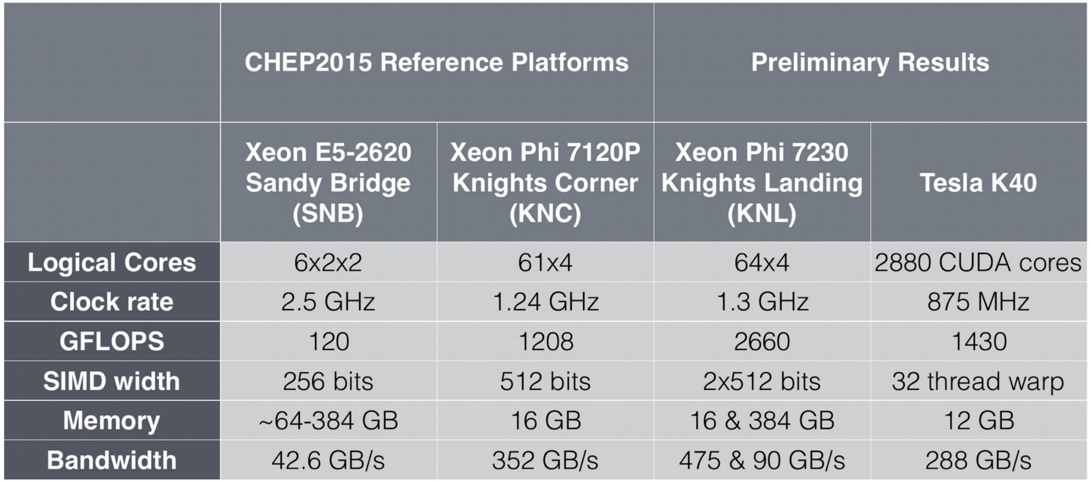




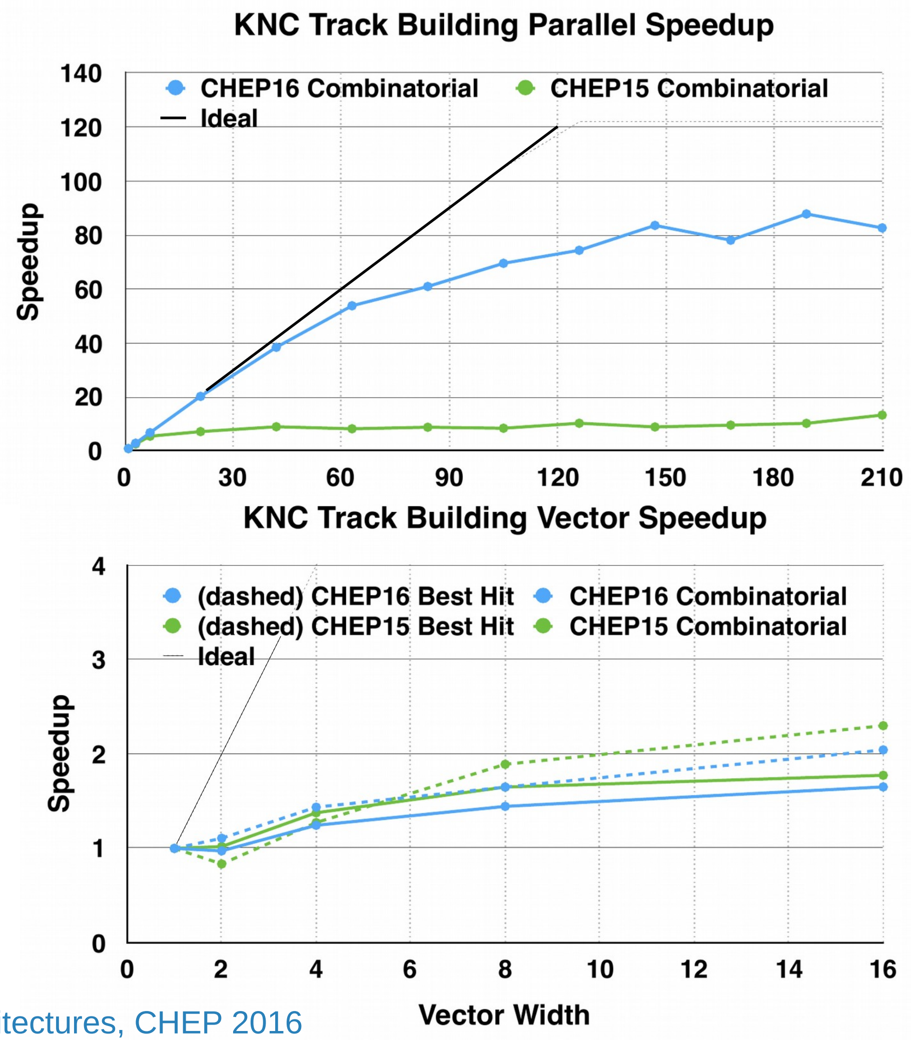
(a) Ortho arrangement

Speedup

- Can be parallelised well



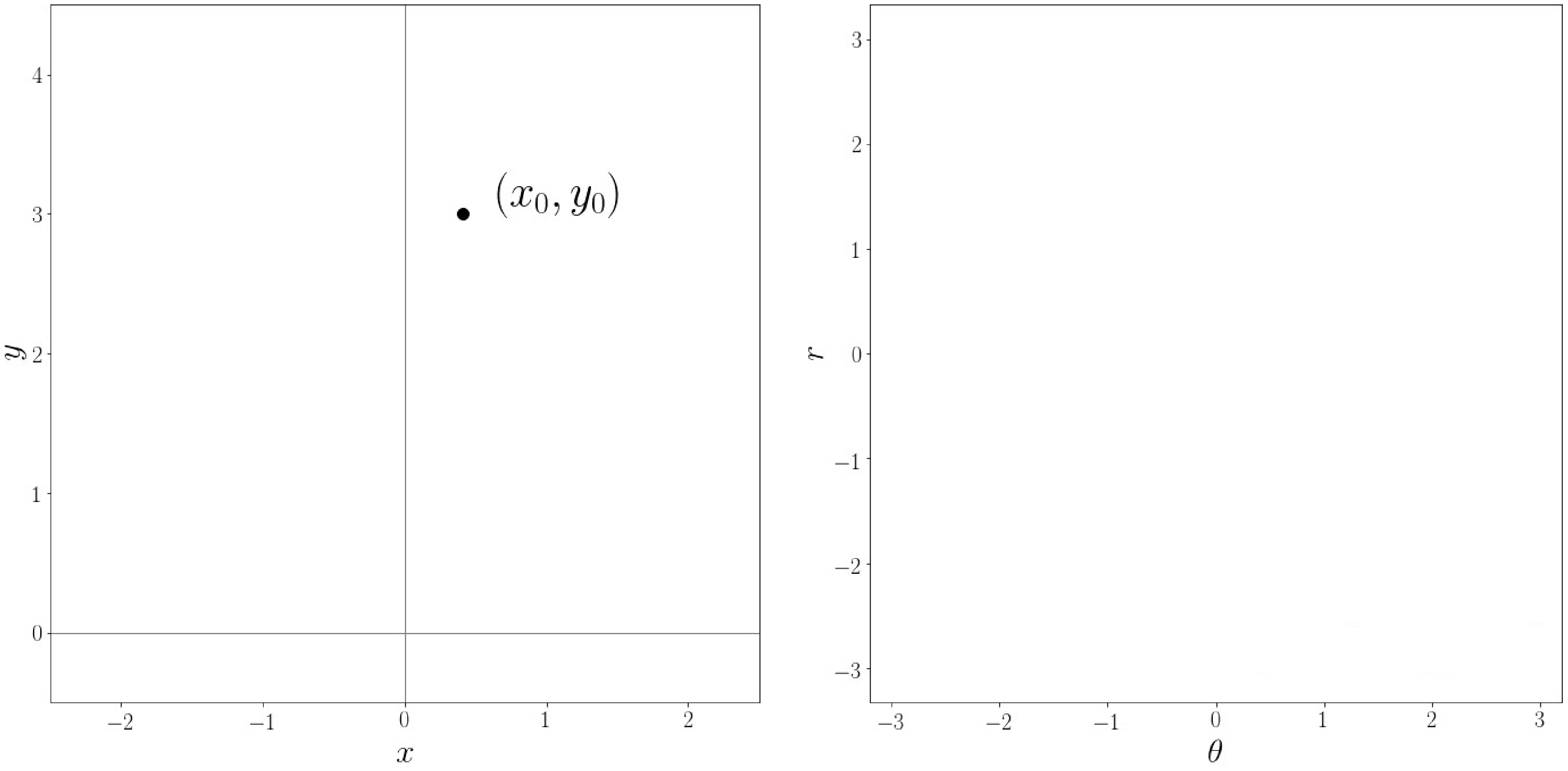
	CHEP2015 Reference Platforms		Preliminary Results	
	Xeon E5-2620 Sandy Bridge (SNB)	Xeon Phi 7120P Knights Corner (KNC)	Xeon Phi 7230 Knights Landing (KNL)	Tesla K40
Logical Cores	6x2x2	61x4	64x4	2880 CUDA cores
Clock rate	2.5 GHz	1.24 GHz	1.3 GHz	875 MHz
GFLOPS	120	1208	2660	1430
SIMD width	256 bits	512 bits	2x512 bits	32 thread warp
Memory	~64-384 GB	16 GB	16 & 384 GB	12 GB
Bandwidth	42.6 GB/s	352 GB/s	475 & 90 GB/s	288 GB/s



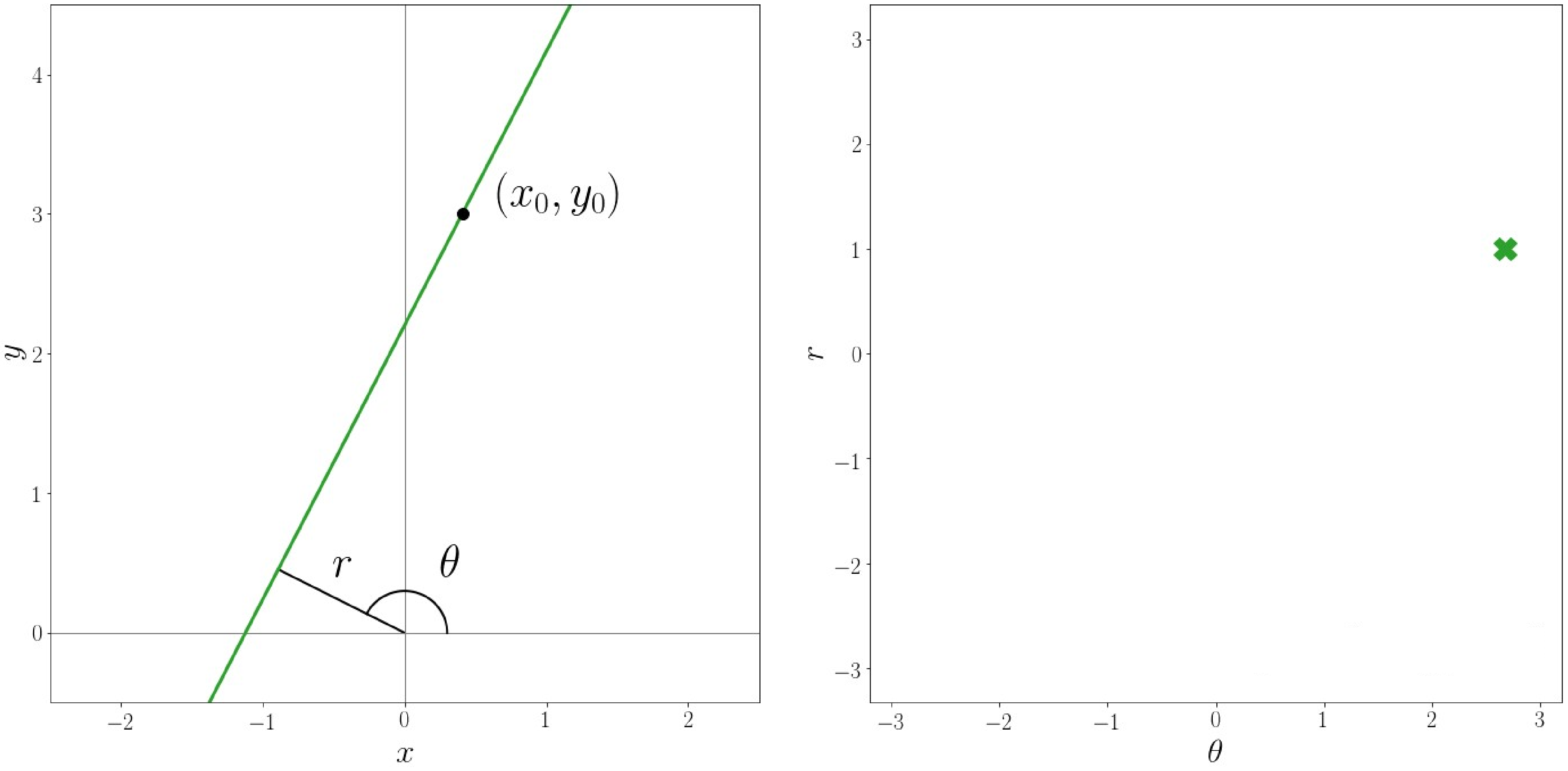
Kalman filter tracking on parallel architectures, CHEP 2016

Global track finding

Hough transform



Hough transform



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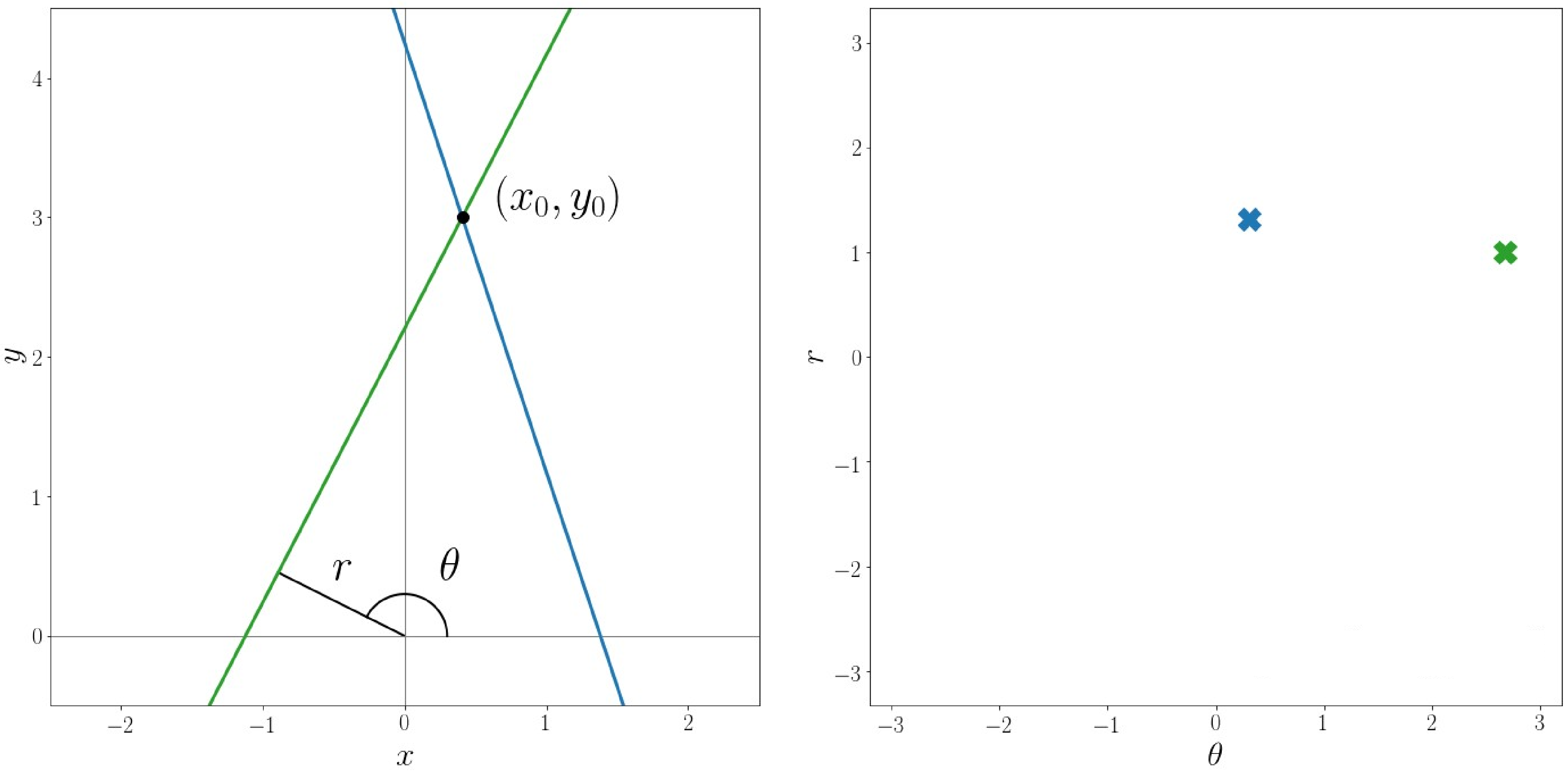

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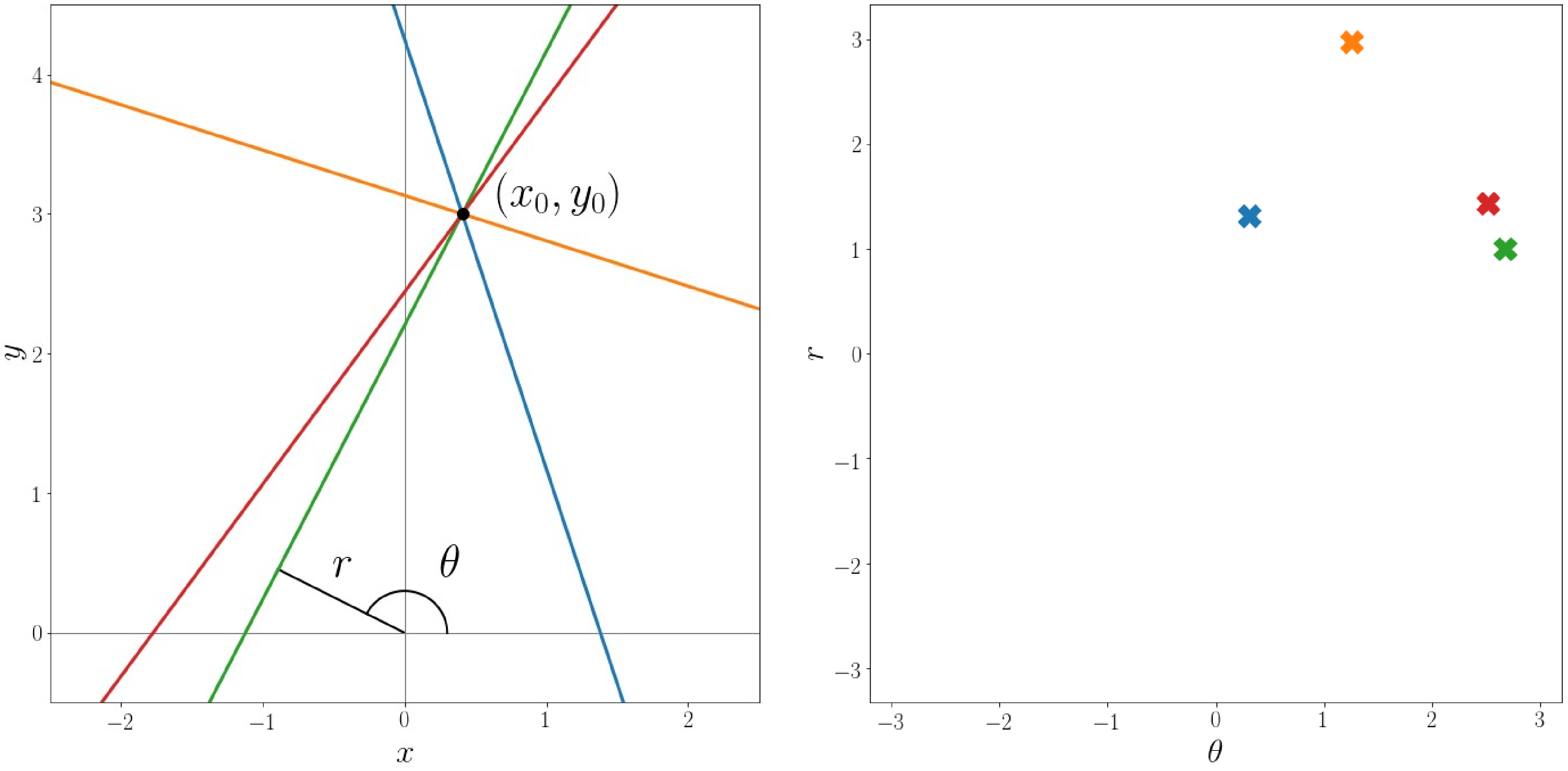

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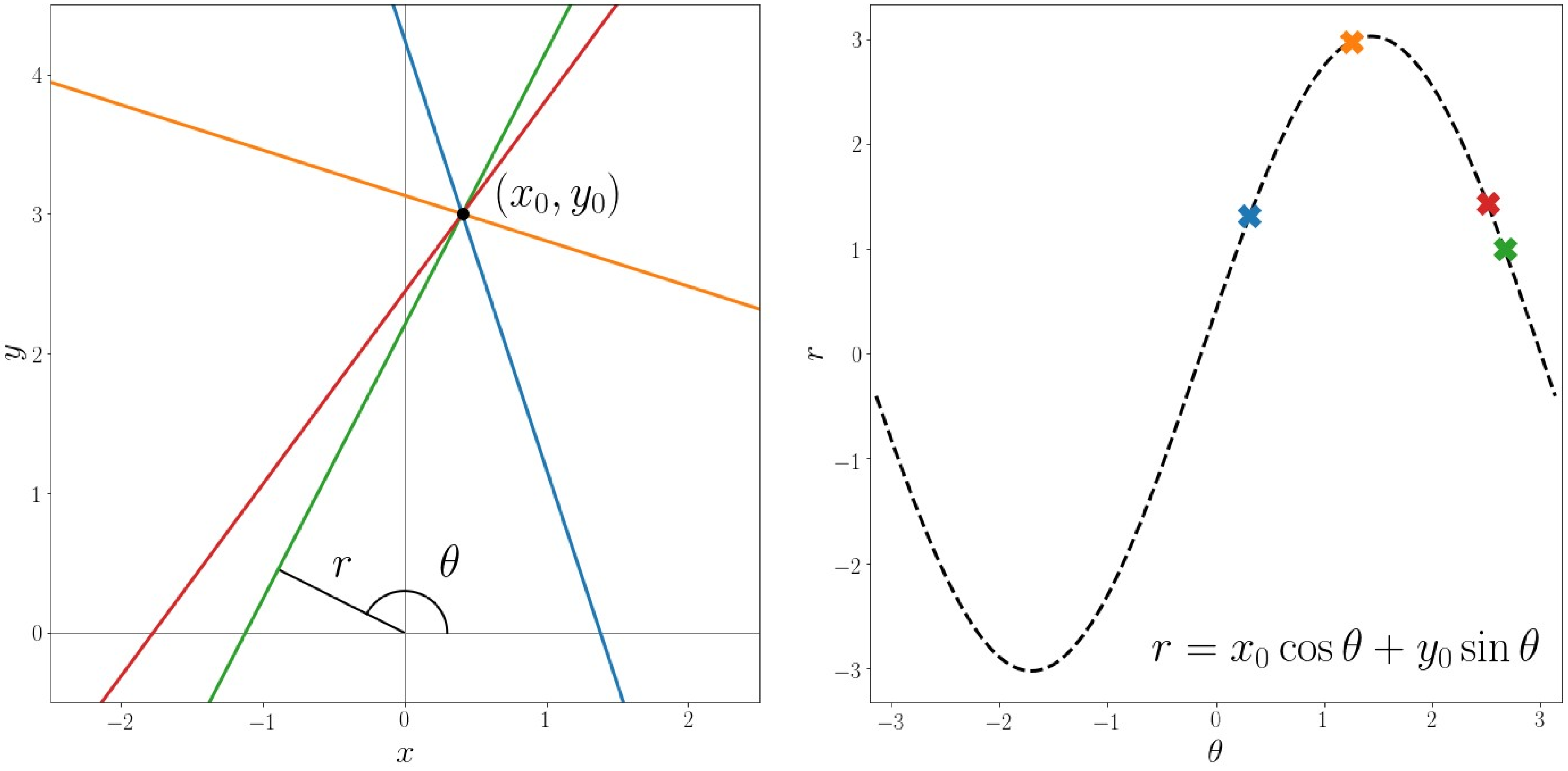
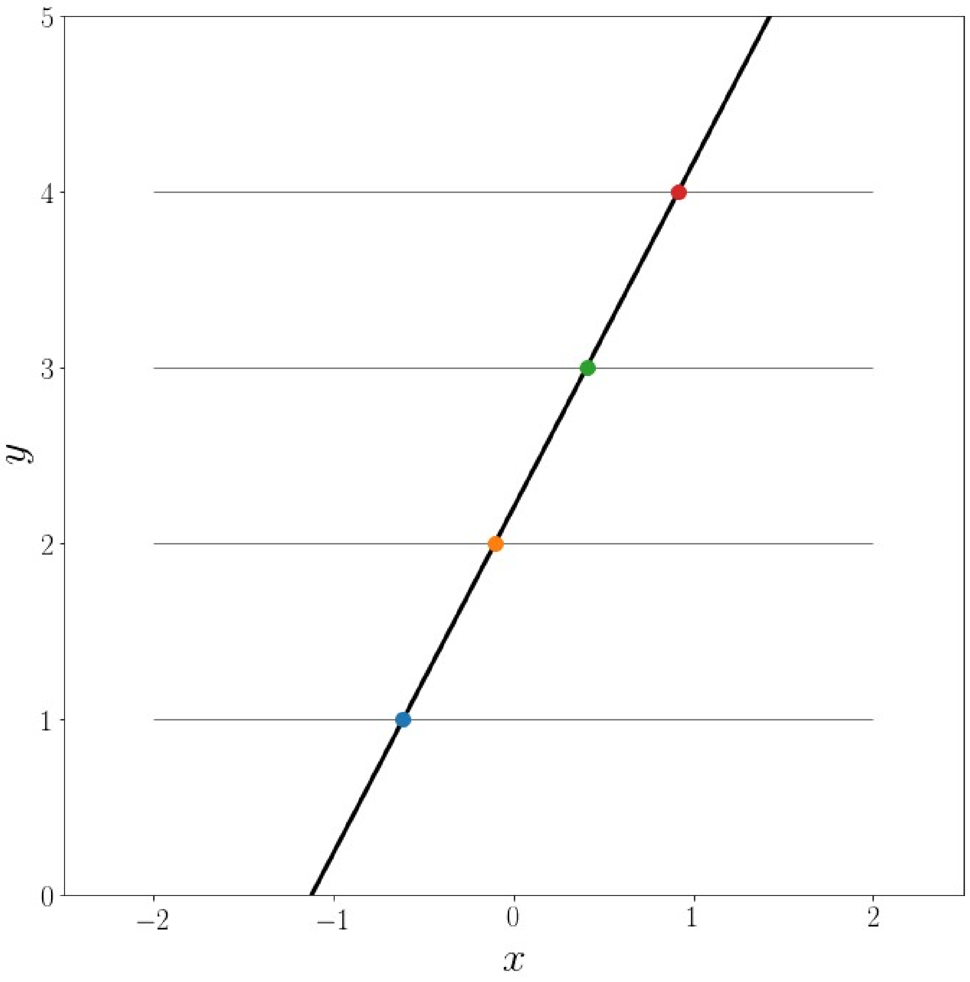
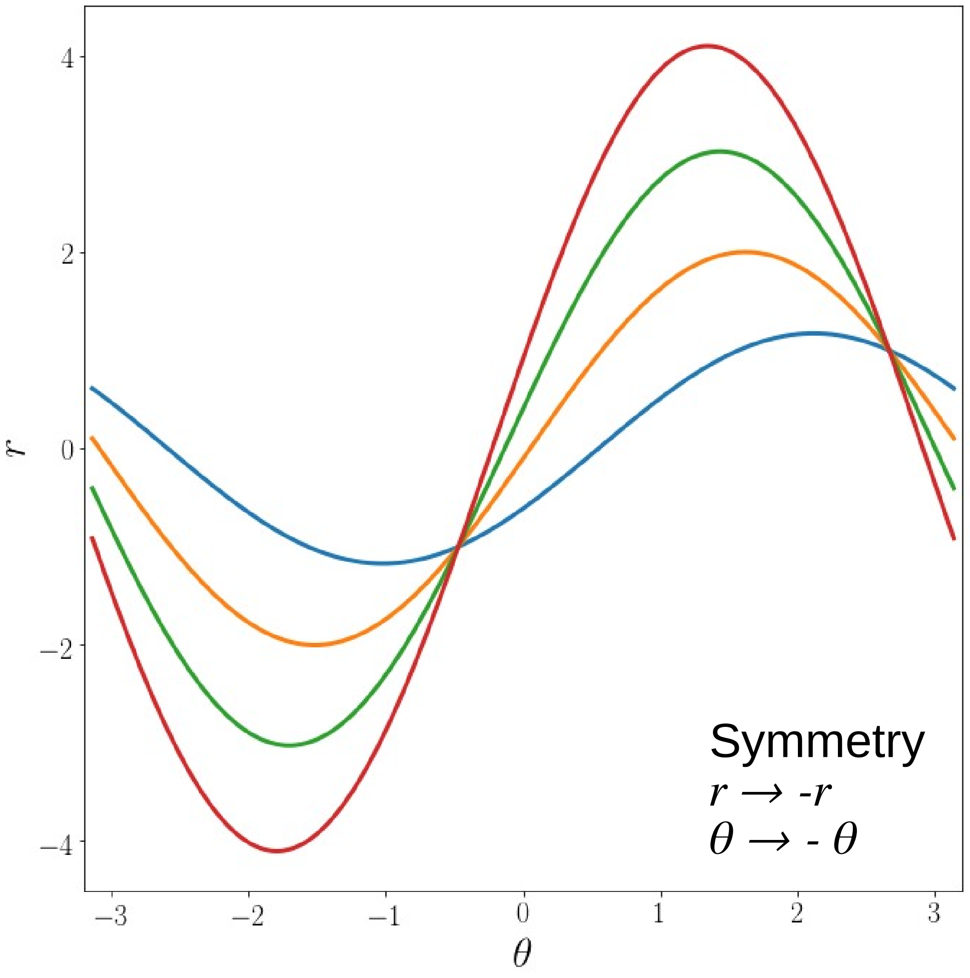


Hough transform: finding a line



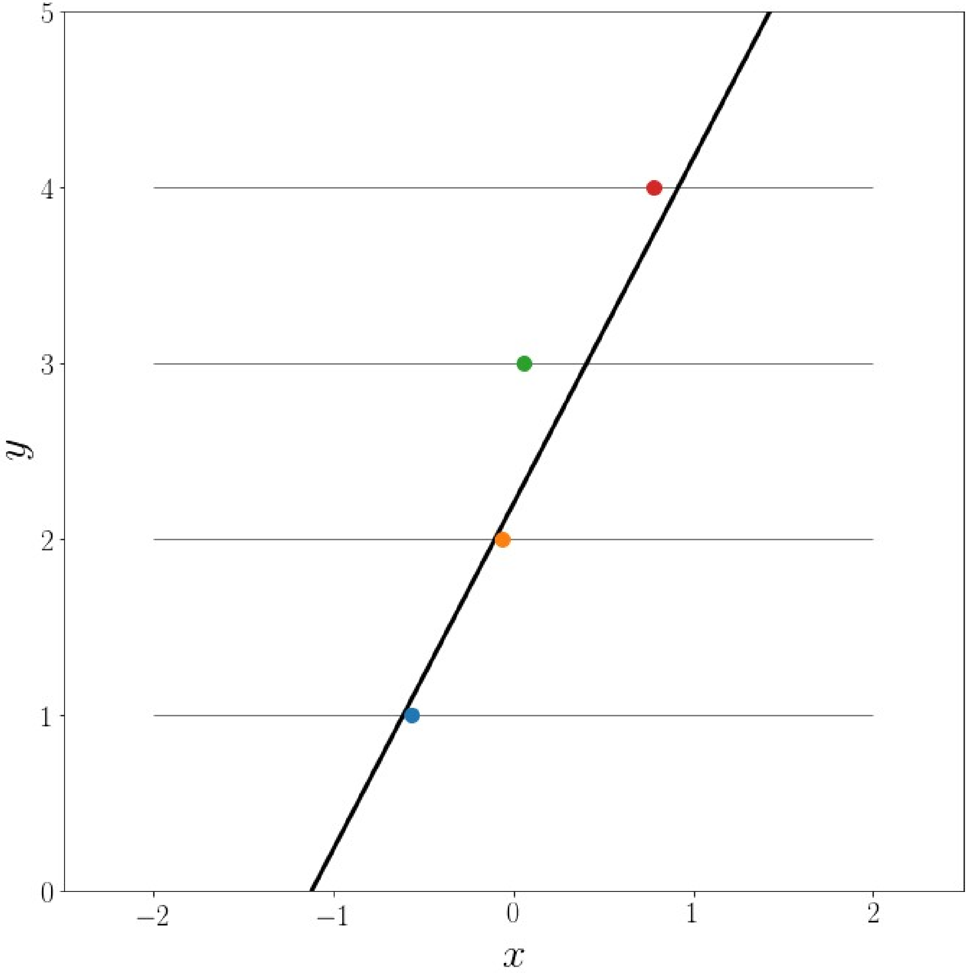


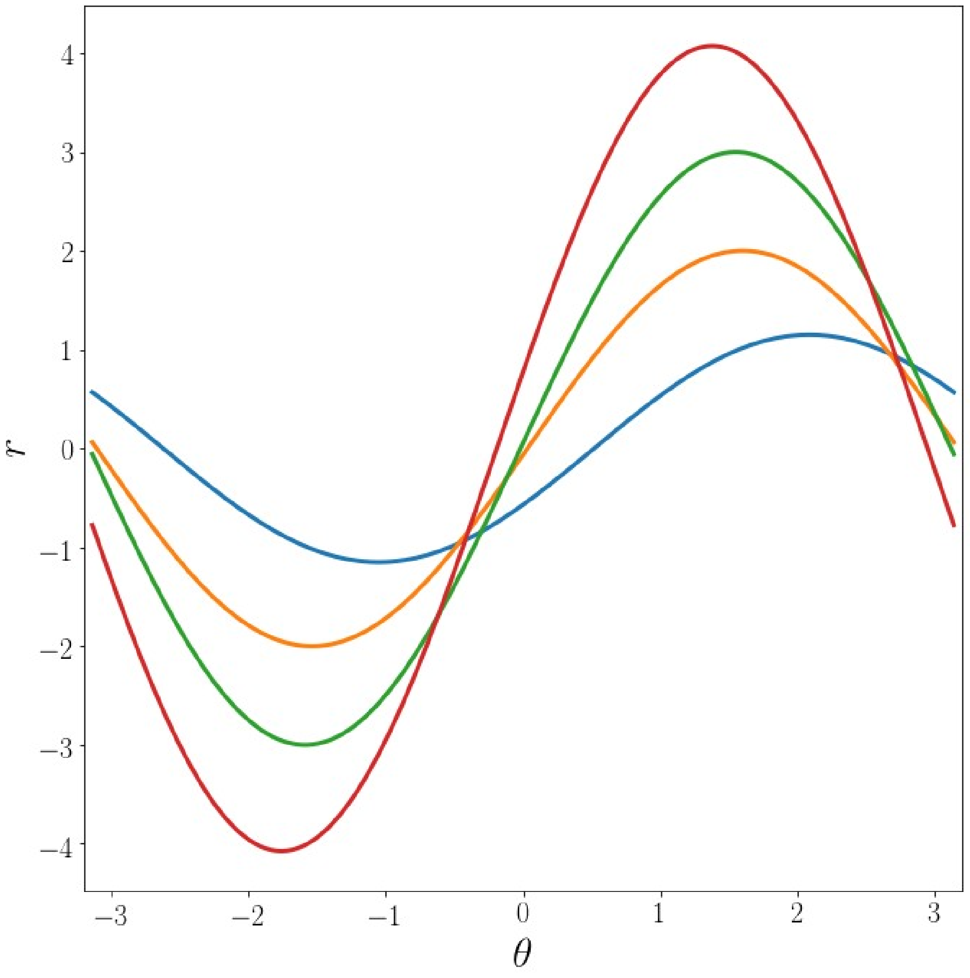
Symmetry

$r \to -r$

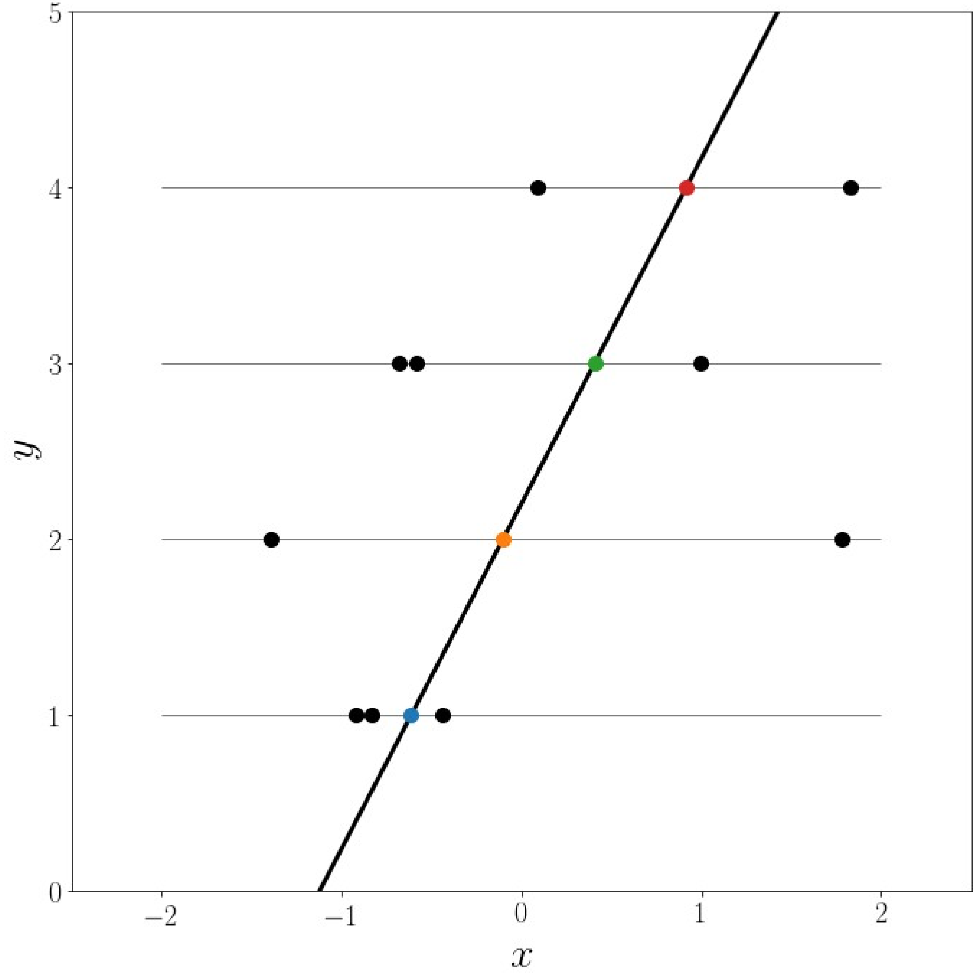
$\theta \to -\theta$

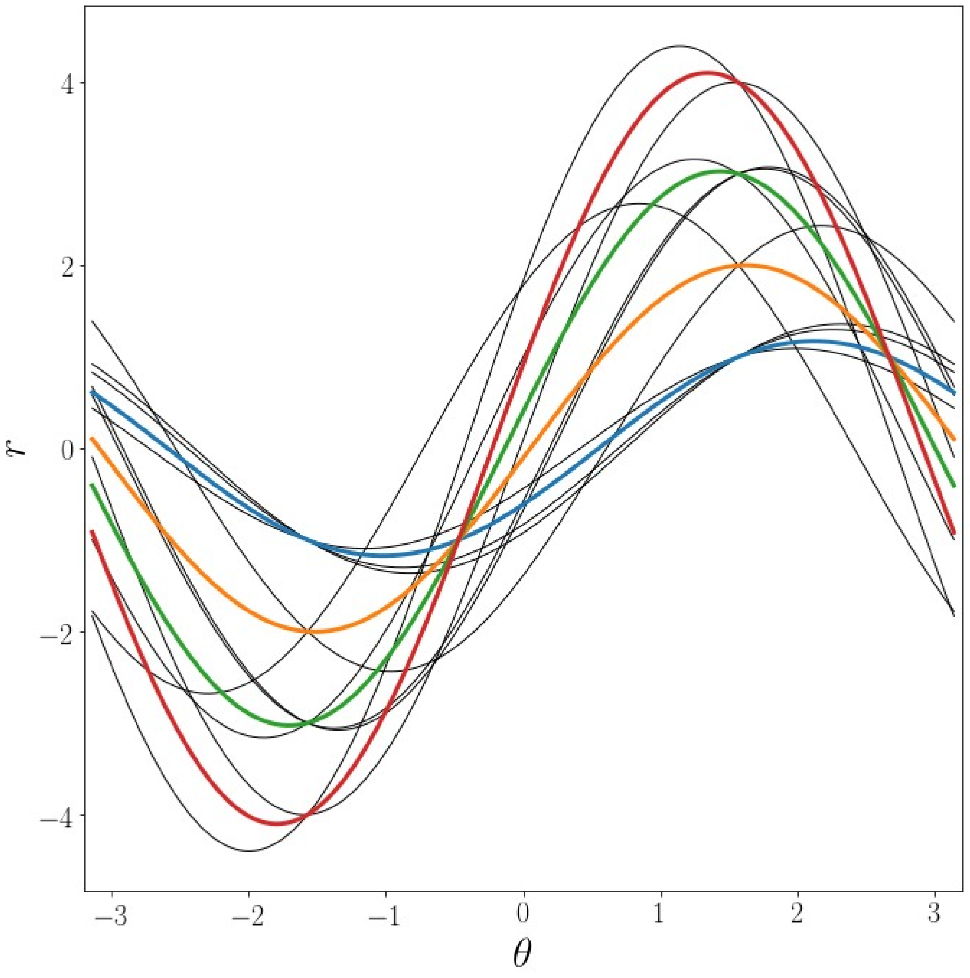
Hough transform: errors





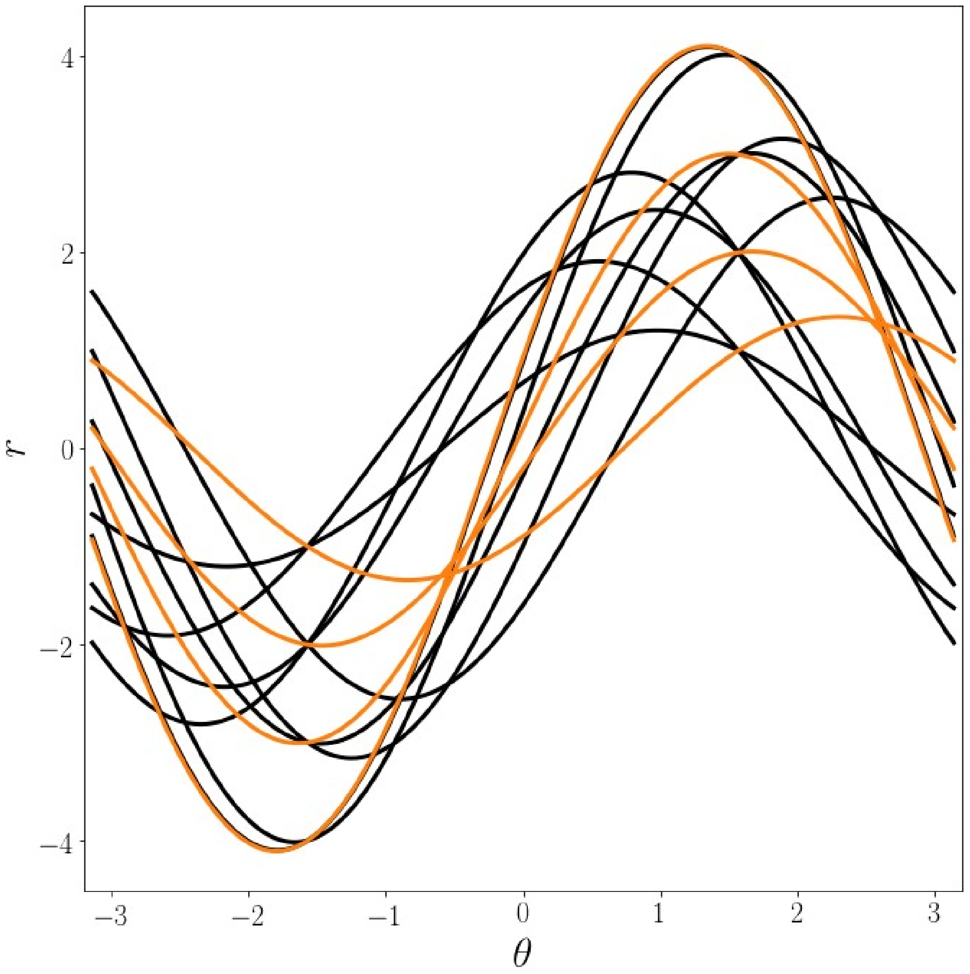
Hough transform: noise





Hough transform summary

- Hits ↔ curves in hough space
- Noise ↔ extra curves
- Errors ↔ spread intersections



Hough transform summary

- Hits ↔ curves in hough space
- Noise ↔ extra curves
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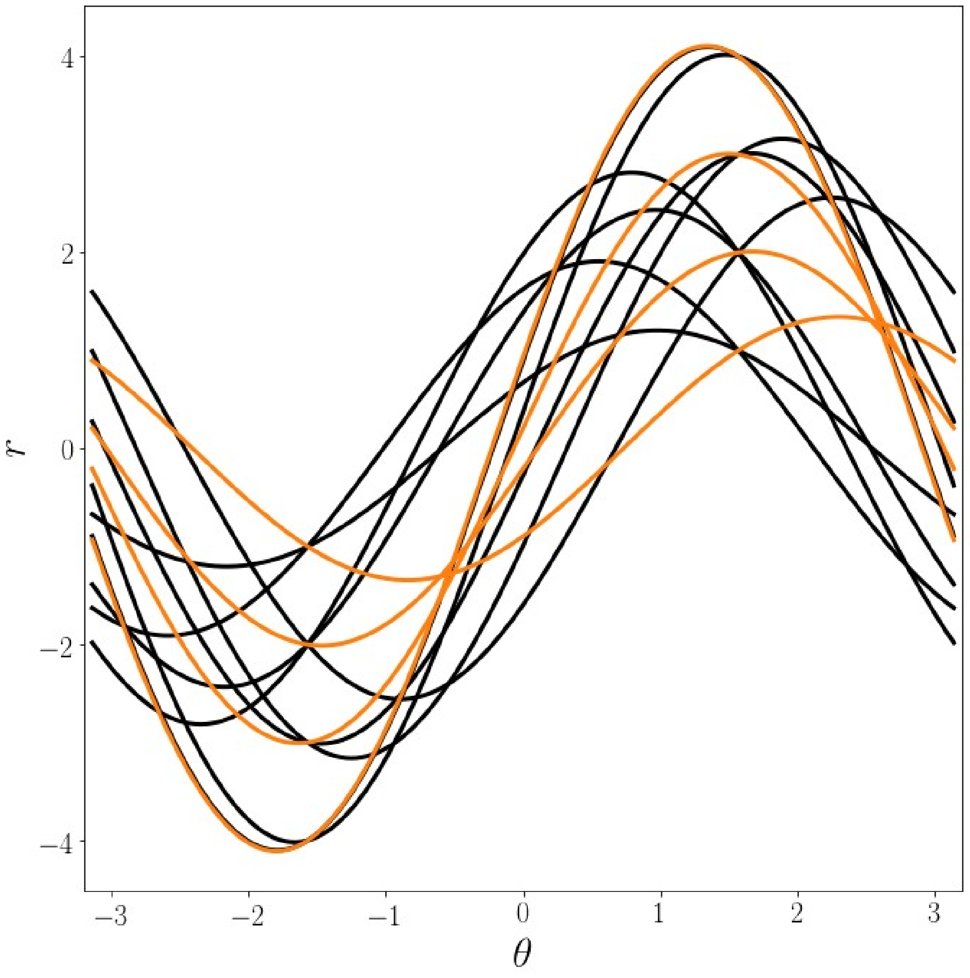
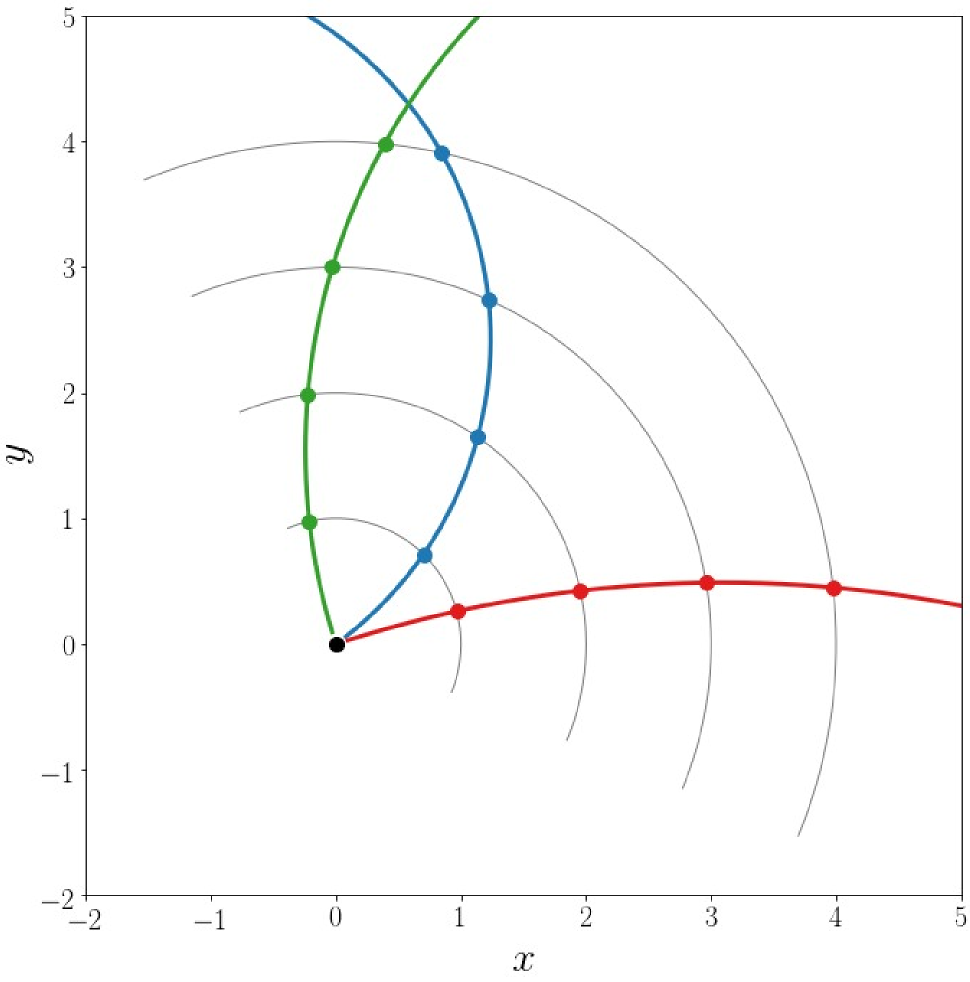
- Pattern finding translated to finding most dense regions in hough space
- Basic idea behind global track finders
- Can be extended



Conformal Hough transform

Conformal transform



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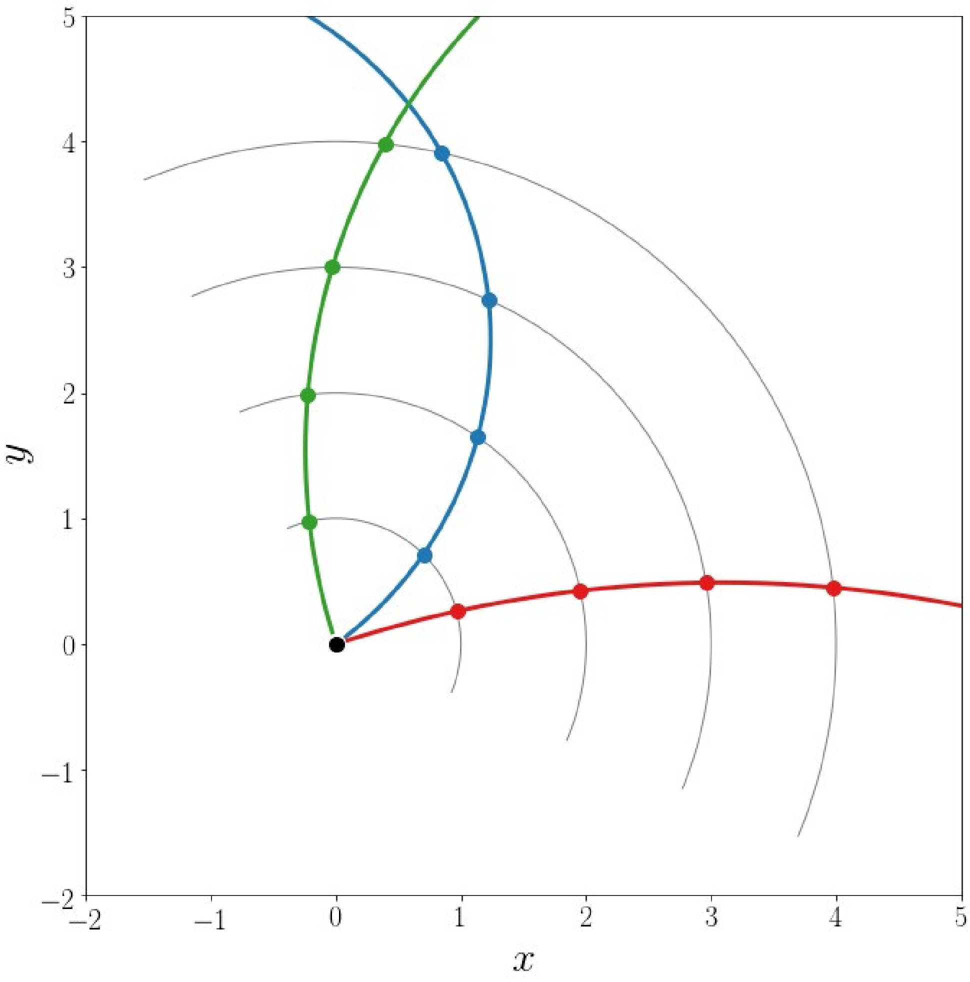
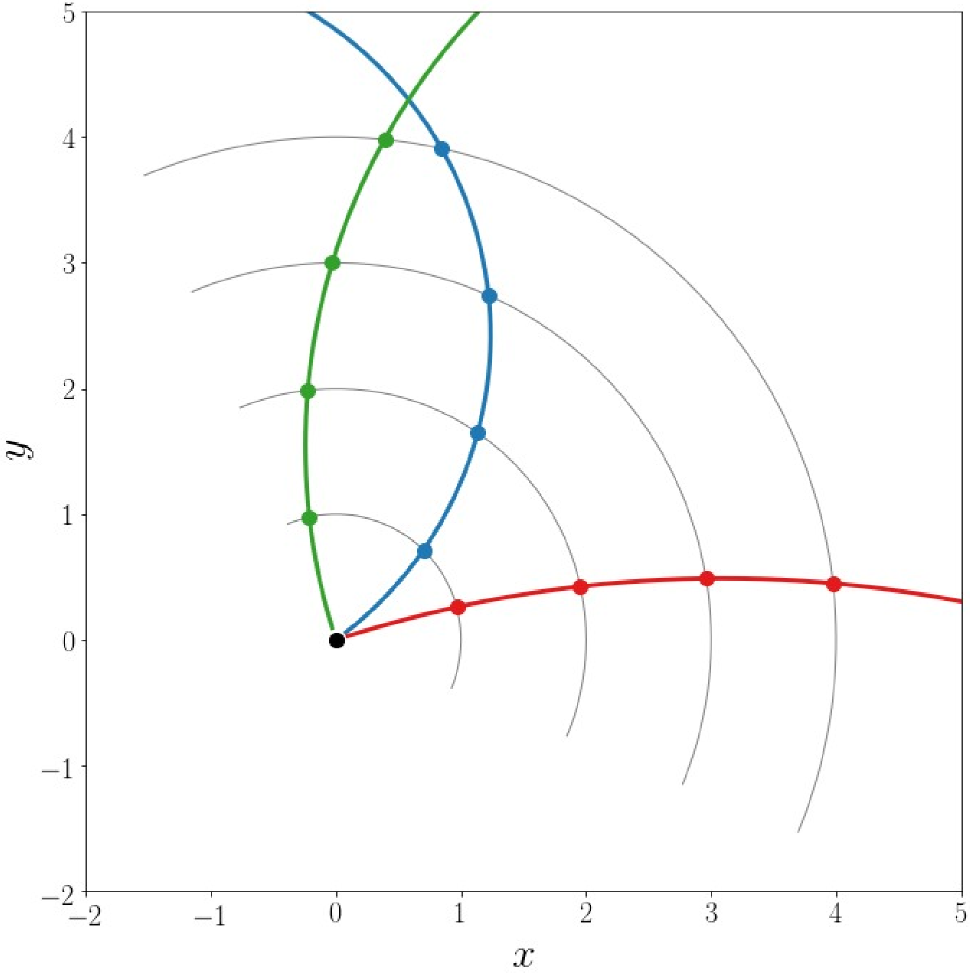
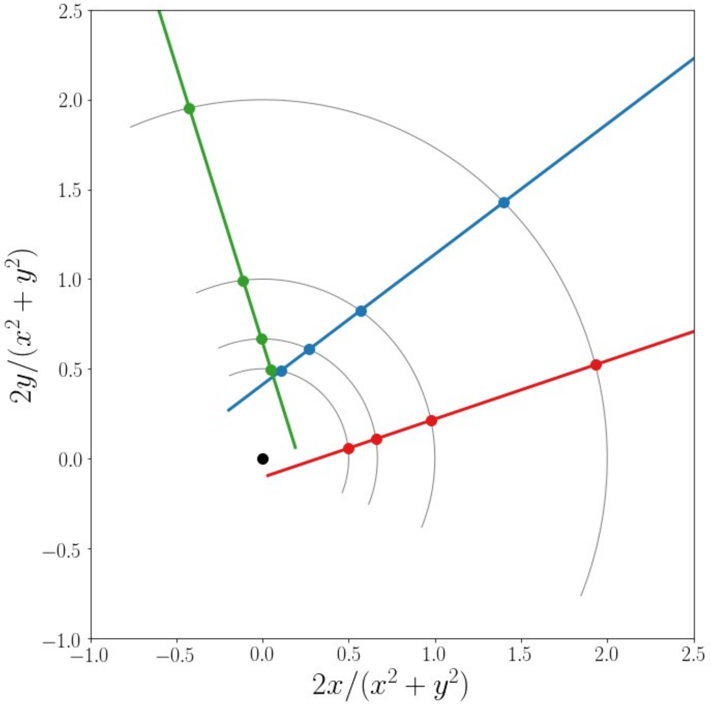


- Inversion against origin
  - $x,\ y \longrightarrow 2x/(x^2+y^2),\ 2y/(x^2+y^2)$
  - Circles ↔ circles
  - Circles through origin ↔ lines

Conformal transform



- Inversion against origin
  - *x, y ⟶ 2x/(x²+y²), 2y/(x²+y²)*
  - Circles ↔ circles
  - Circles through origin ↔ lines



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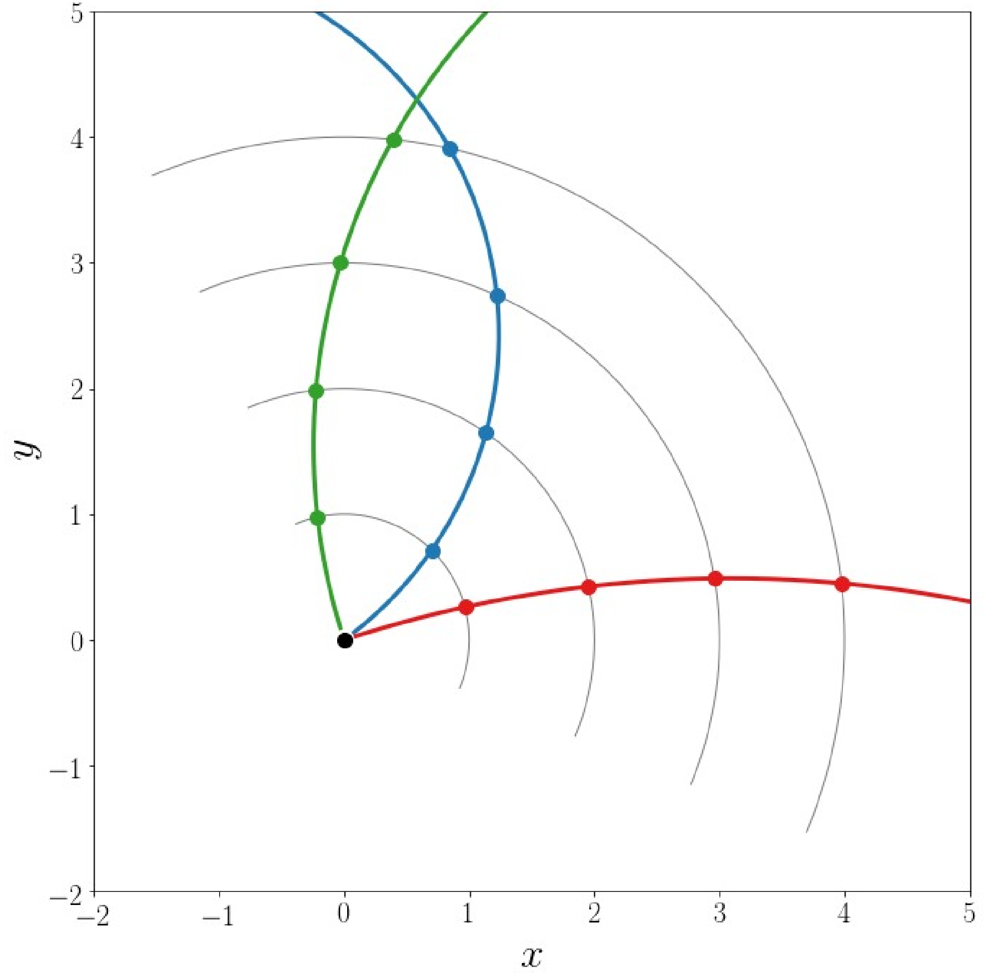
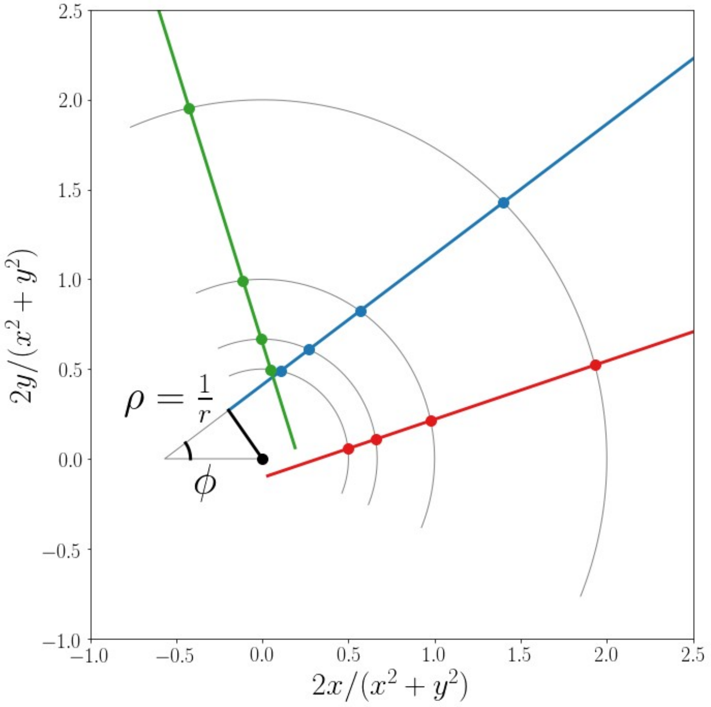


- Line parameters
  - Track curvature
  - Track direction



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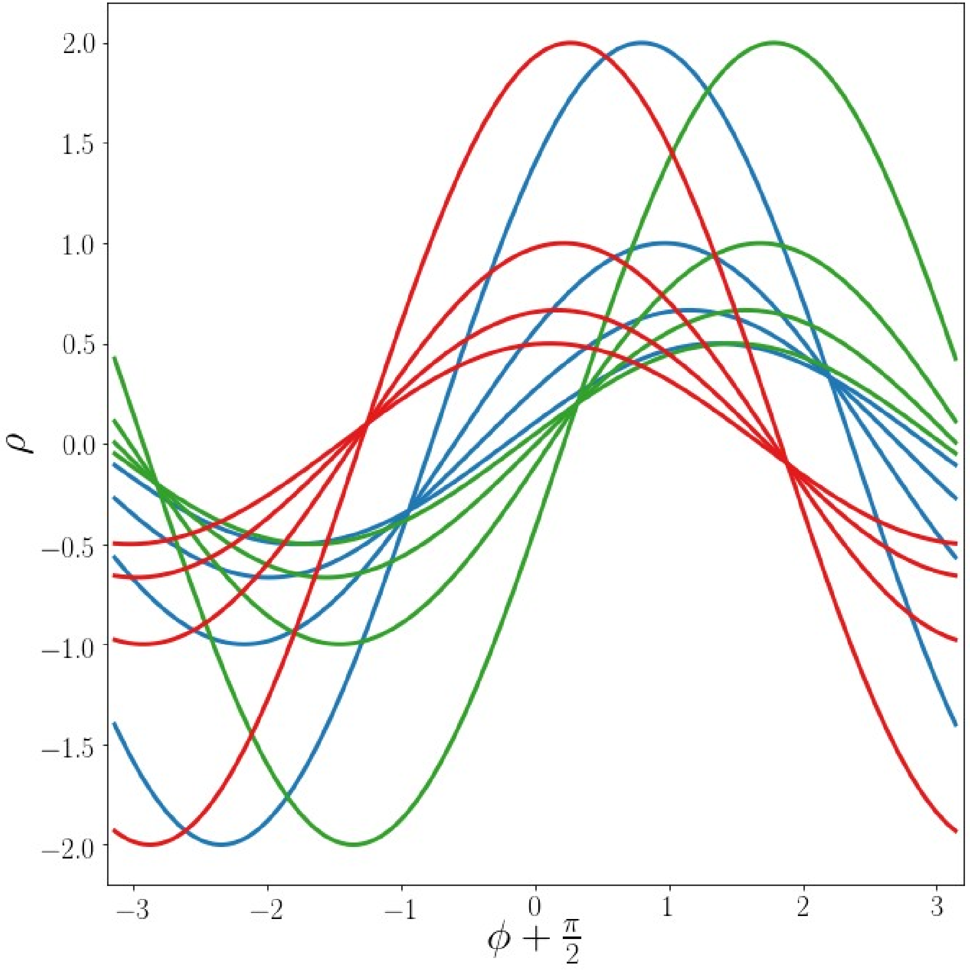

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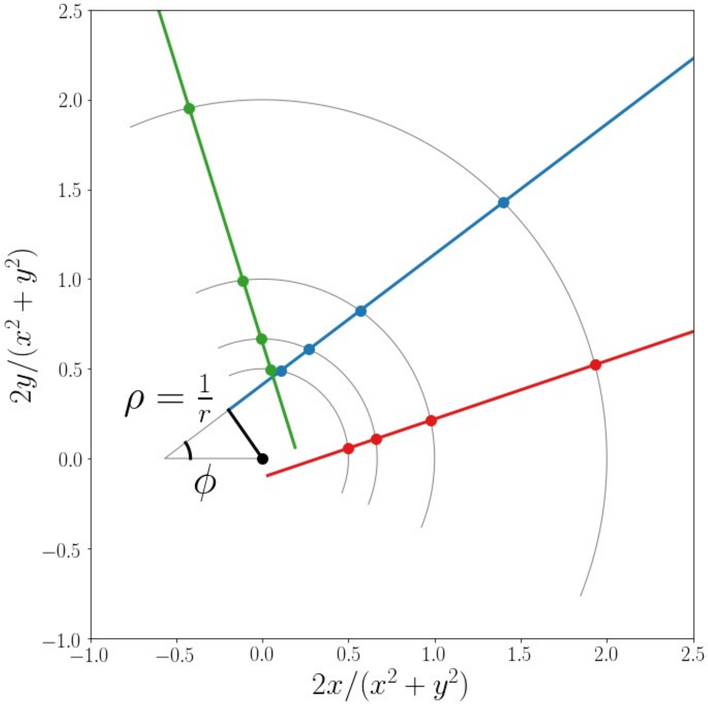

Legendre transform

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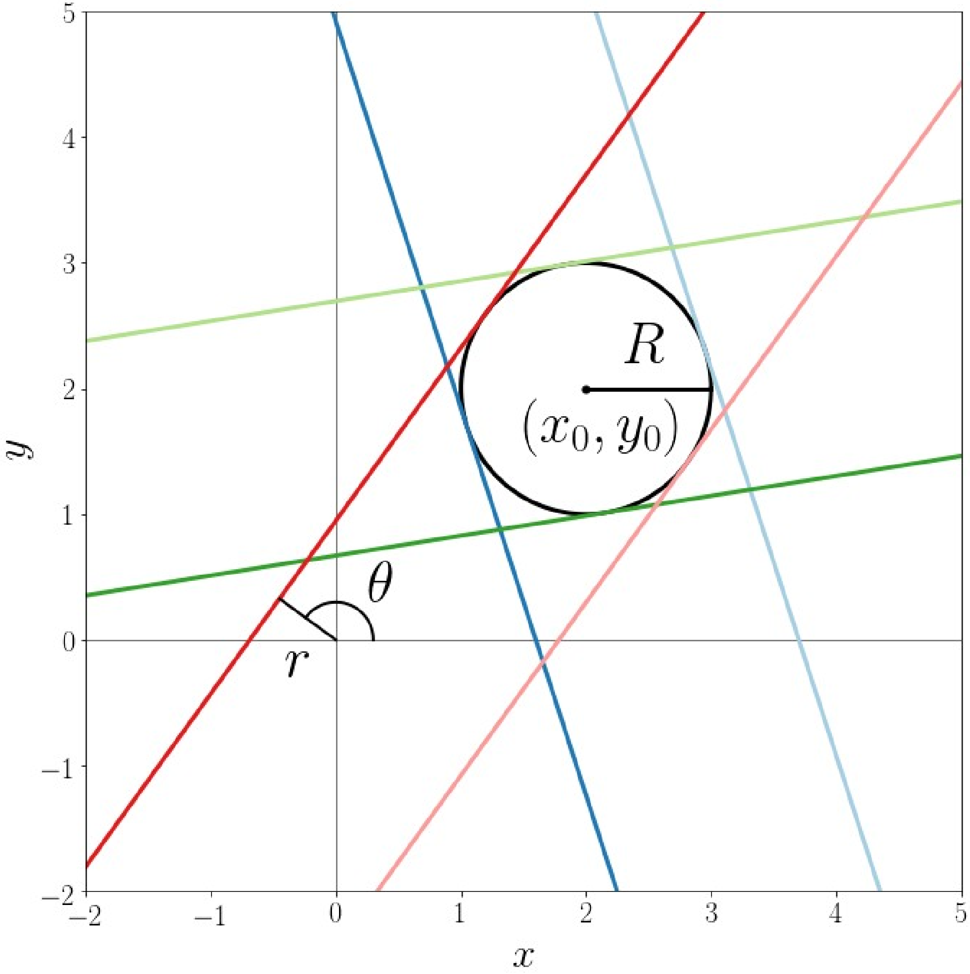
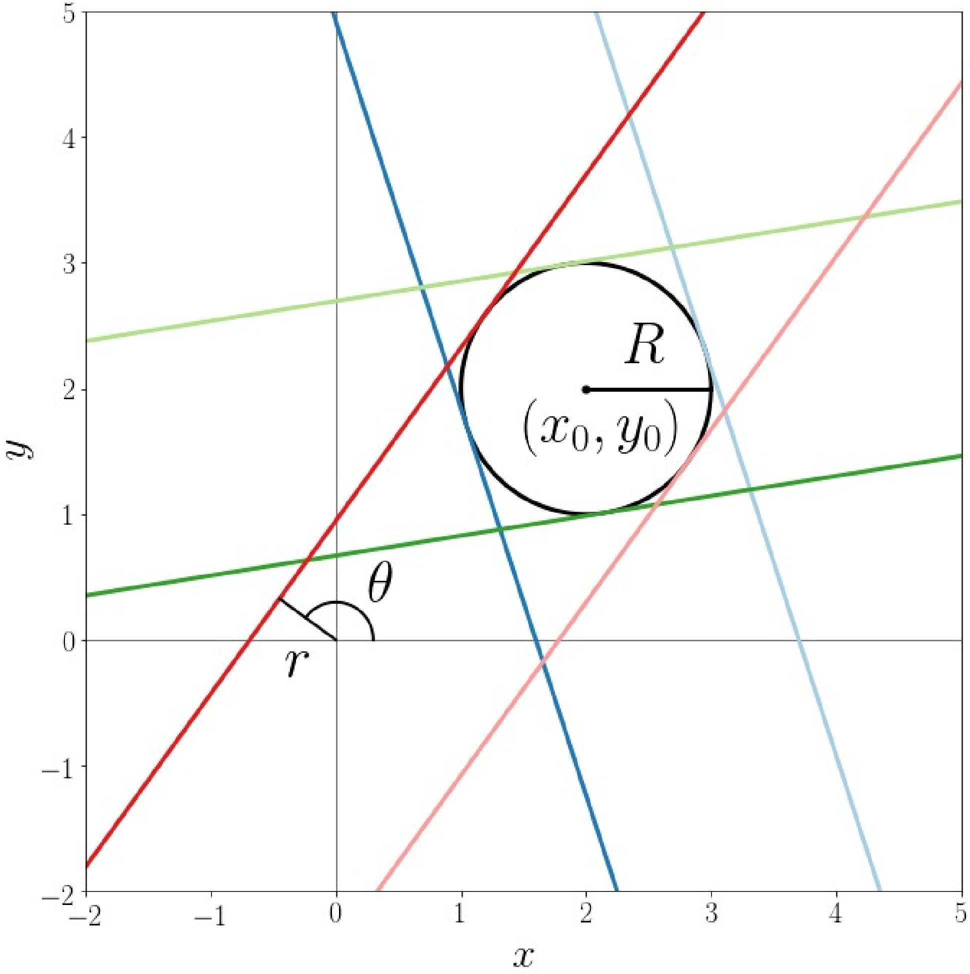


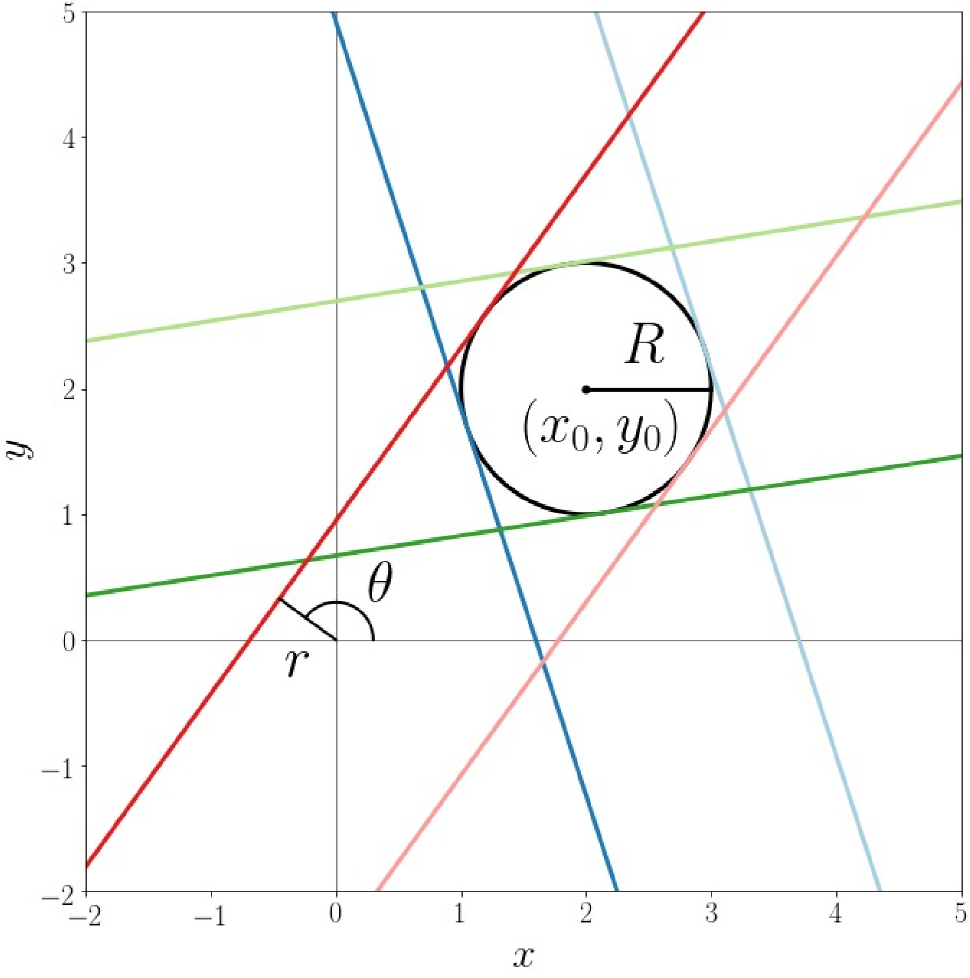
- For drift circles

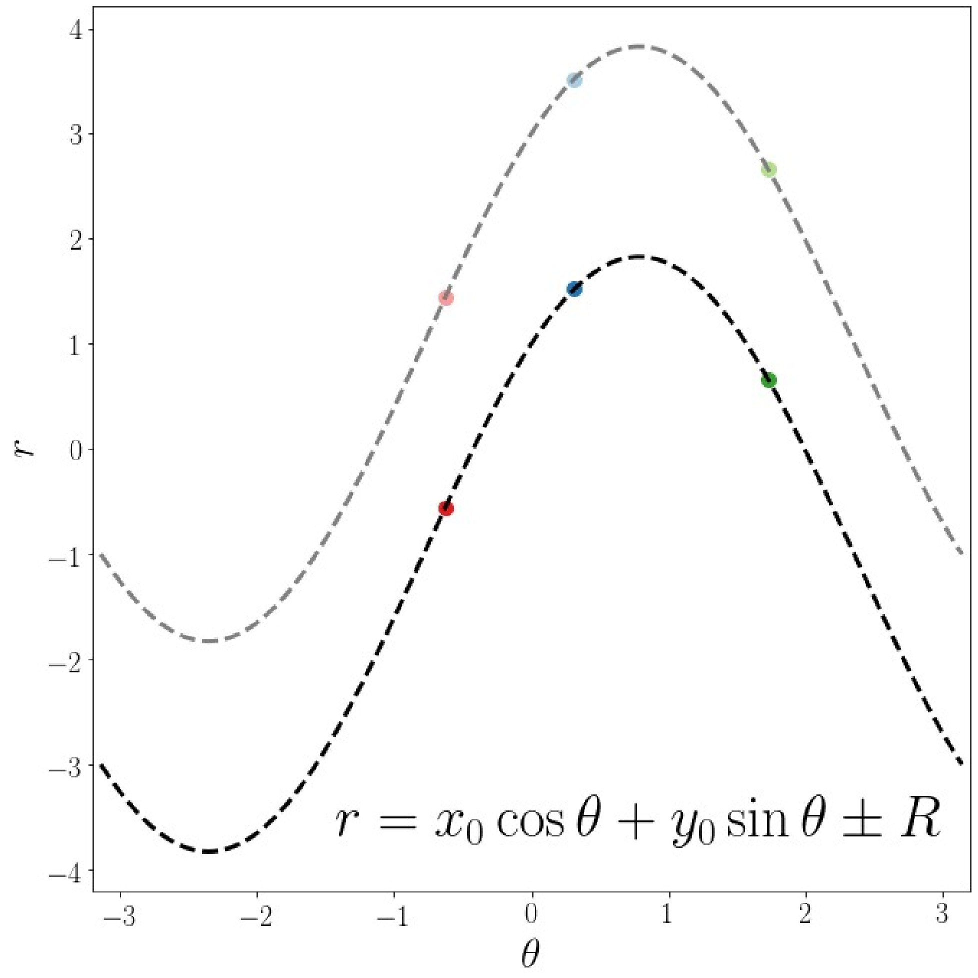
Legendre transform



- For drift circles
- Pairs of tangents
- Same idea

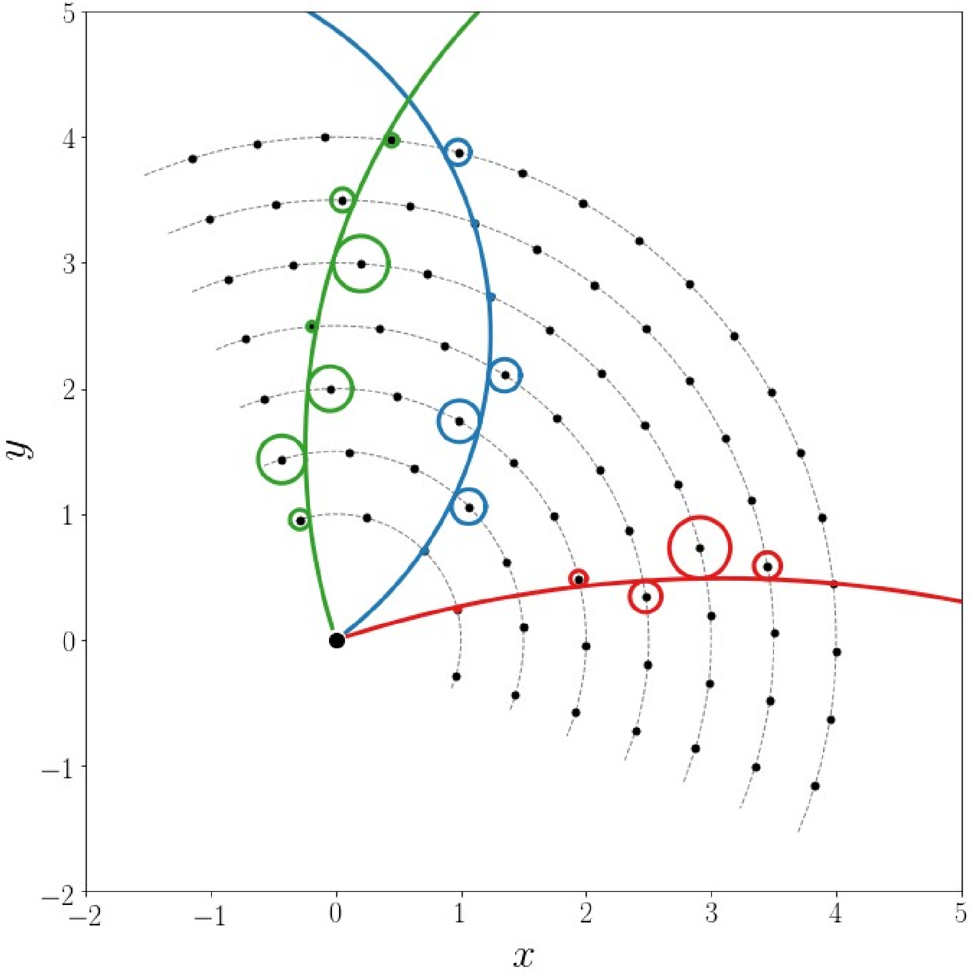
Legendre transform





$$r = x_0 \cos\theta + y_0 \sin\theta \pm R$$

Conformal Legendre transform



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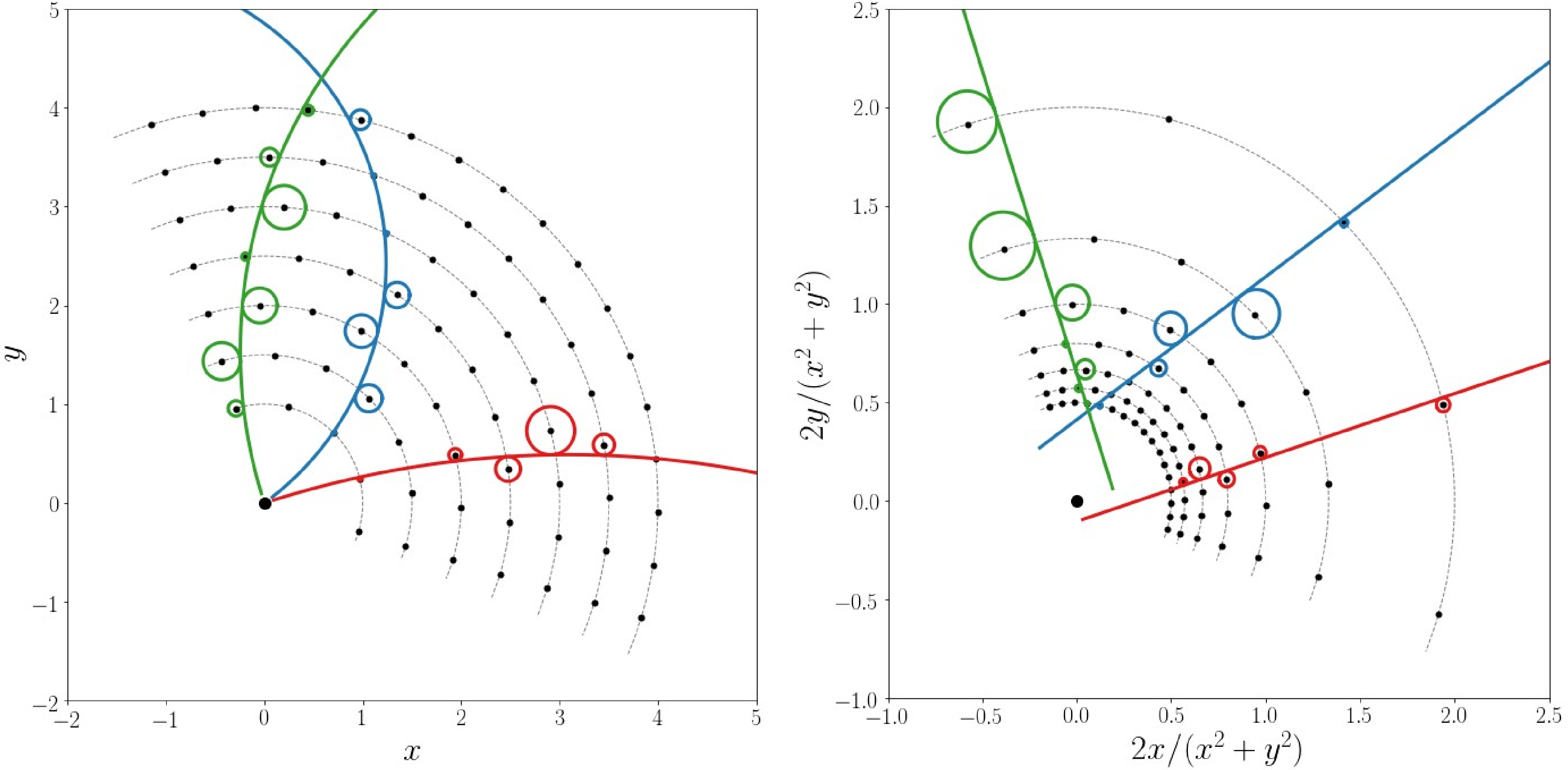

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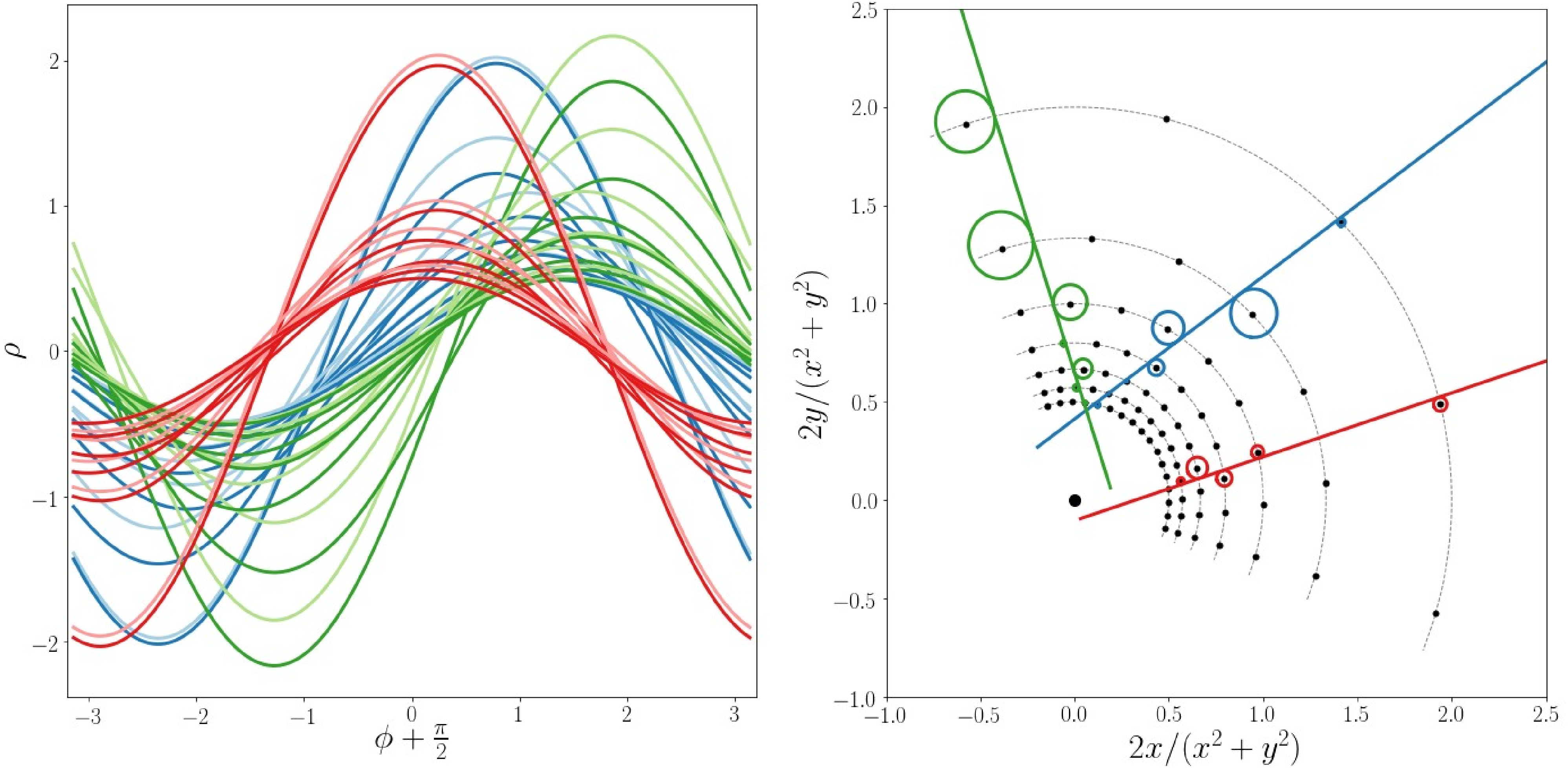

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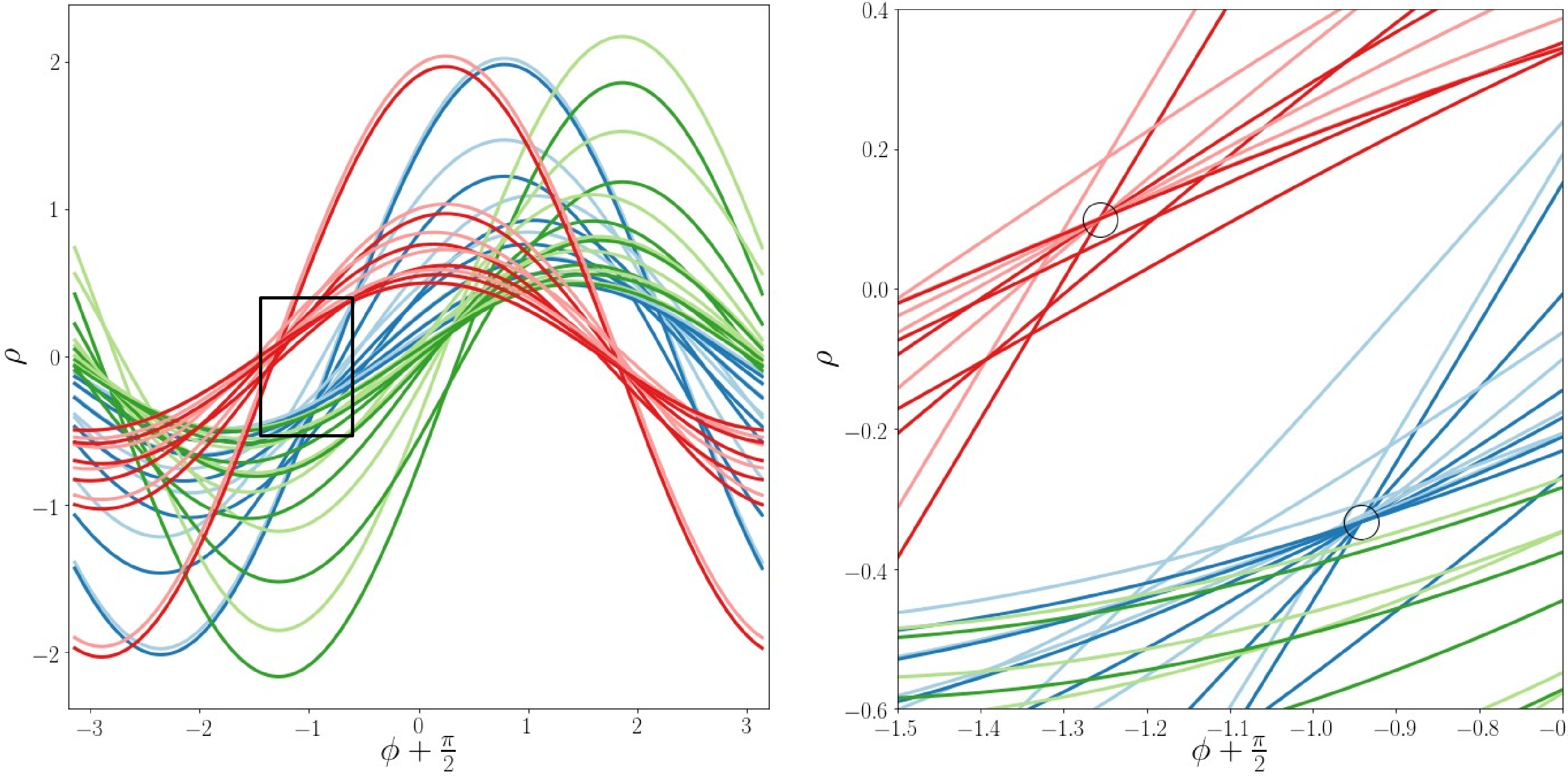
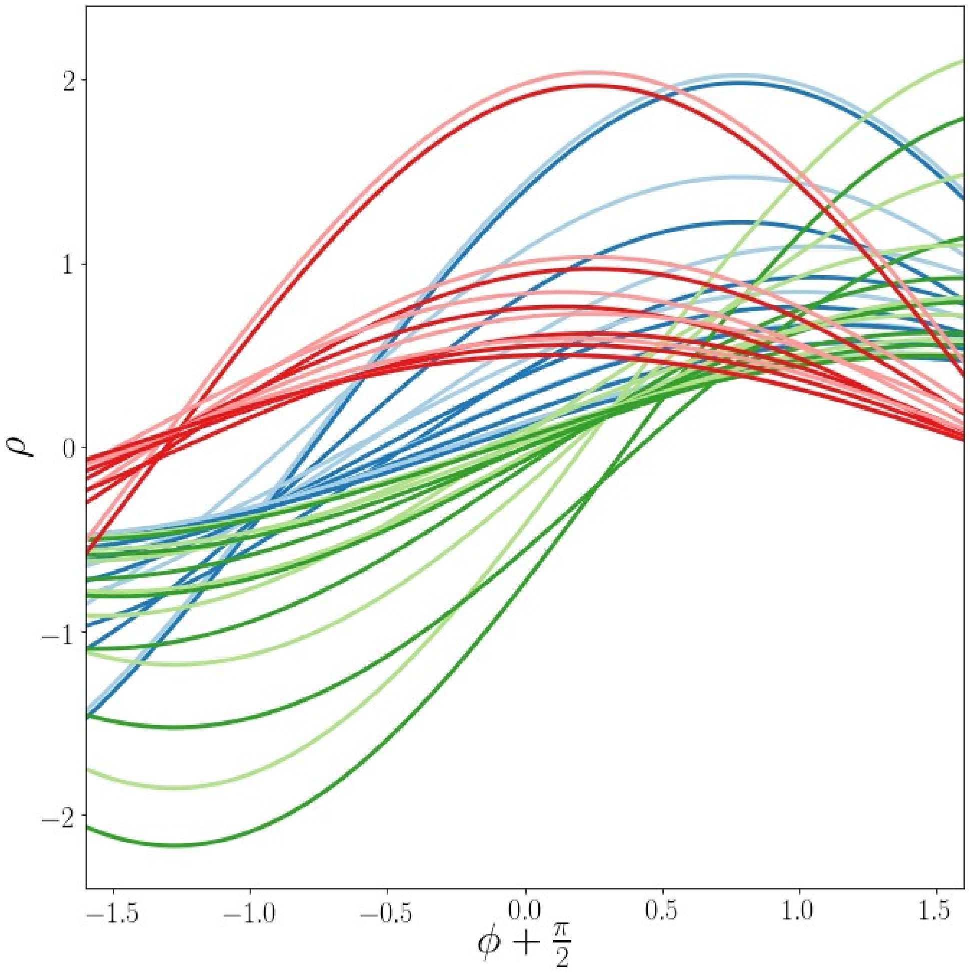
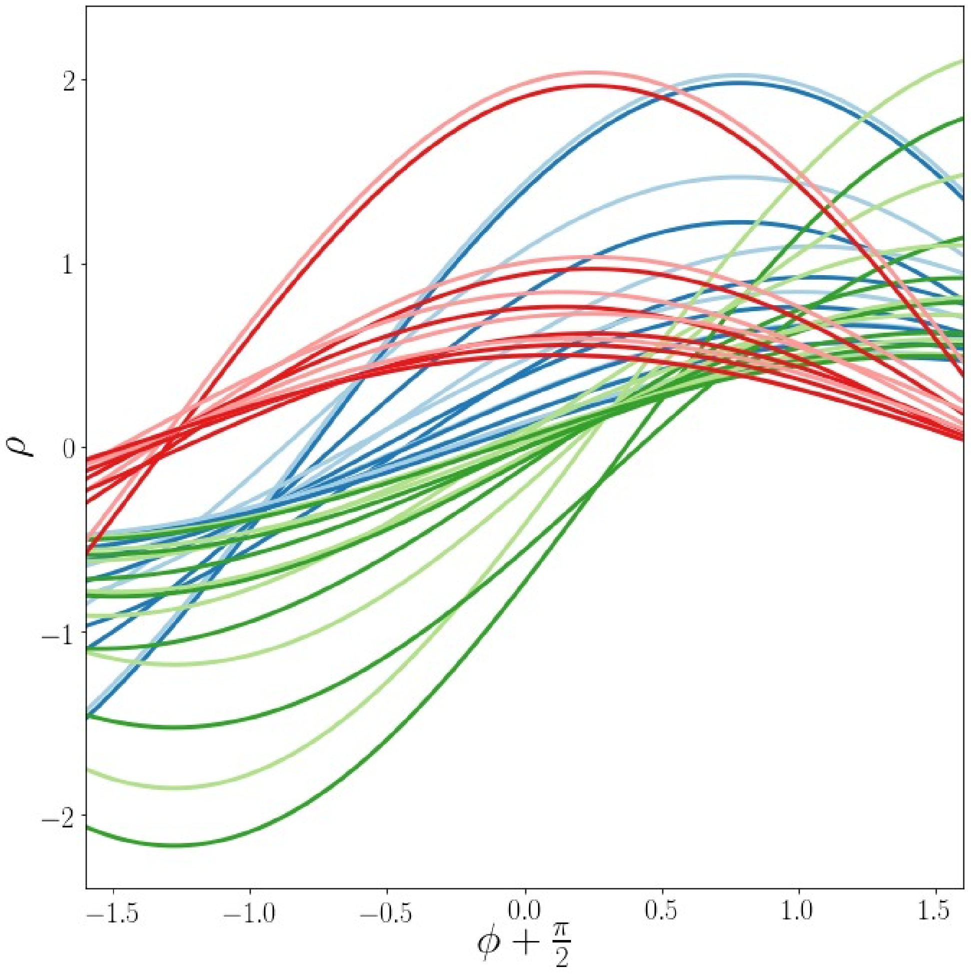


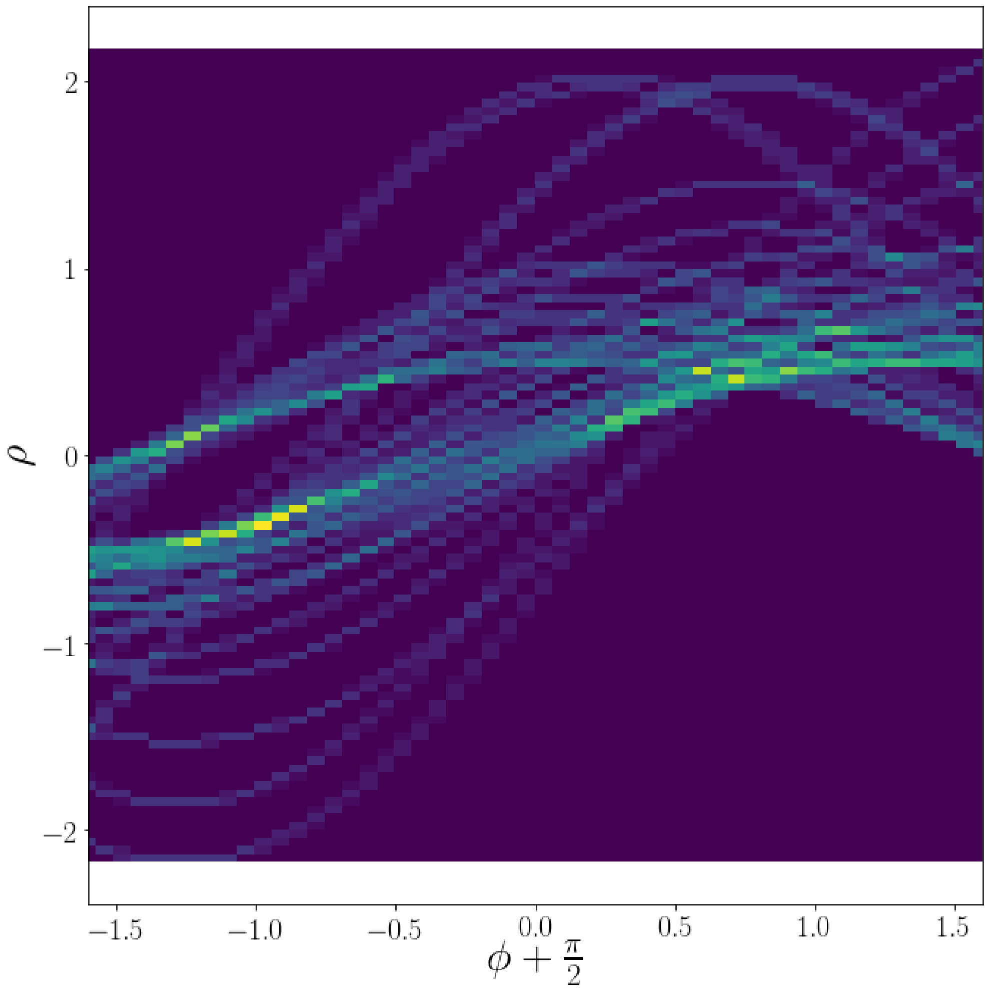
Finding intersections

Histograms



Histograms


2
1
0
−1
−2
ρ
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
$\phi + \frac{\pi}{2}$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−1.5
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0.0
0.5
1.0
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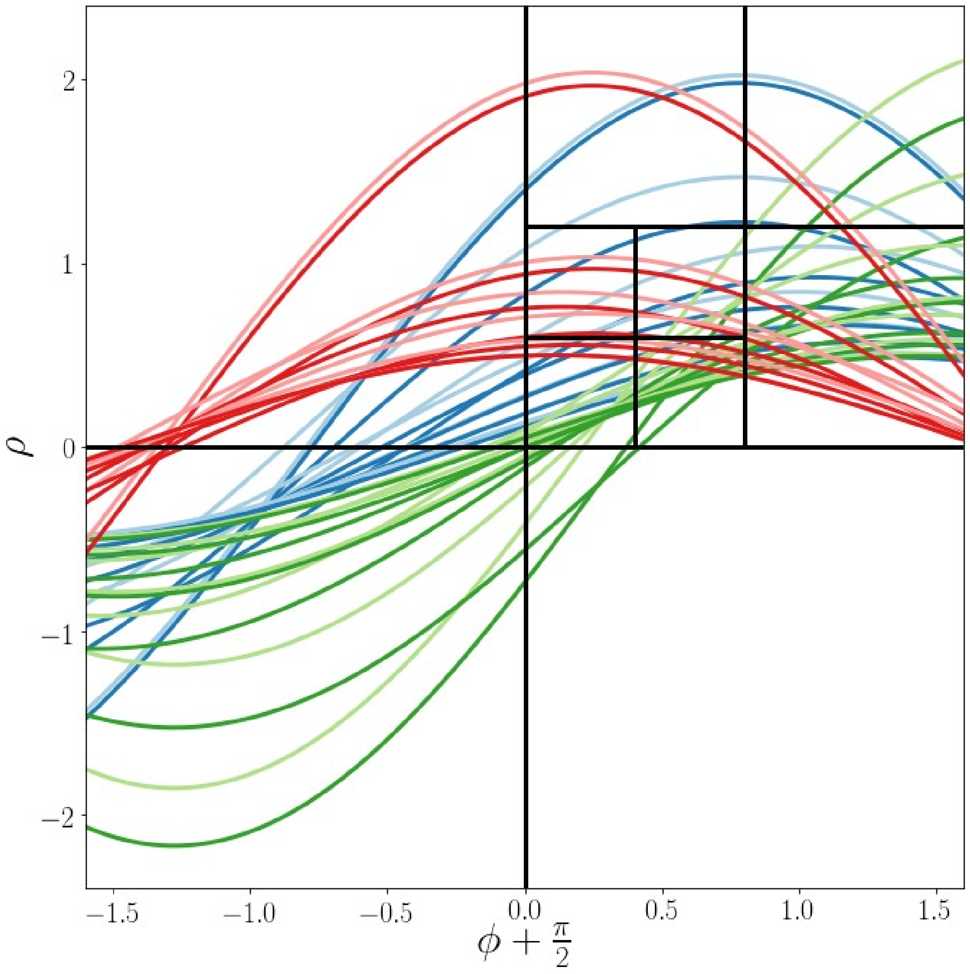
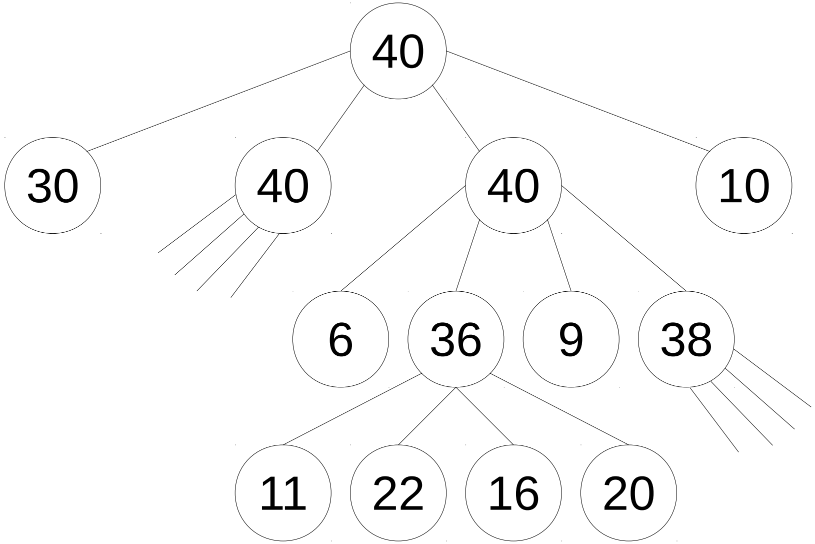
$\phi + \frac{\pi}{2}$

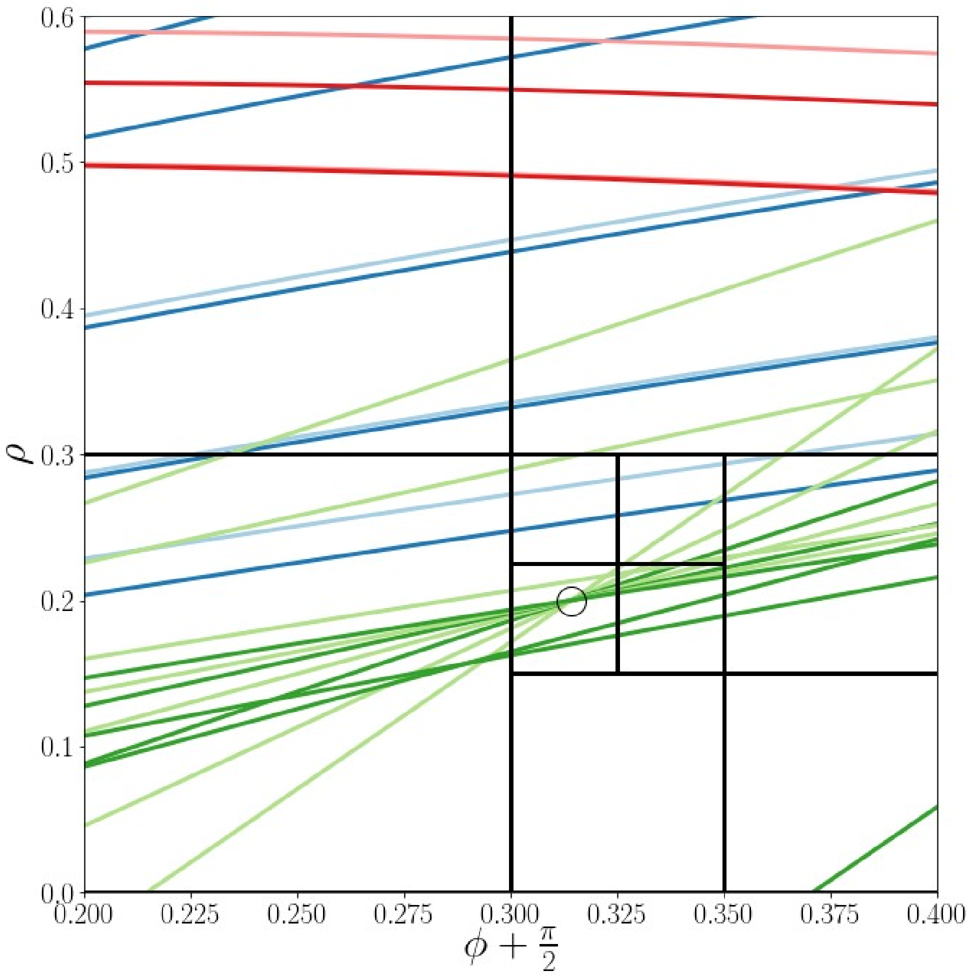
Quadtrees



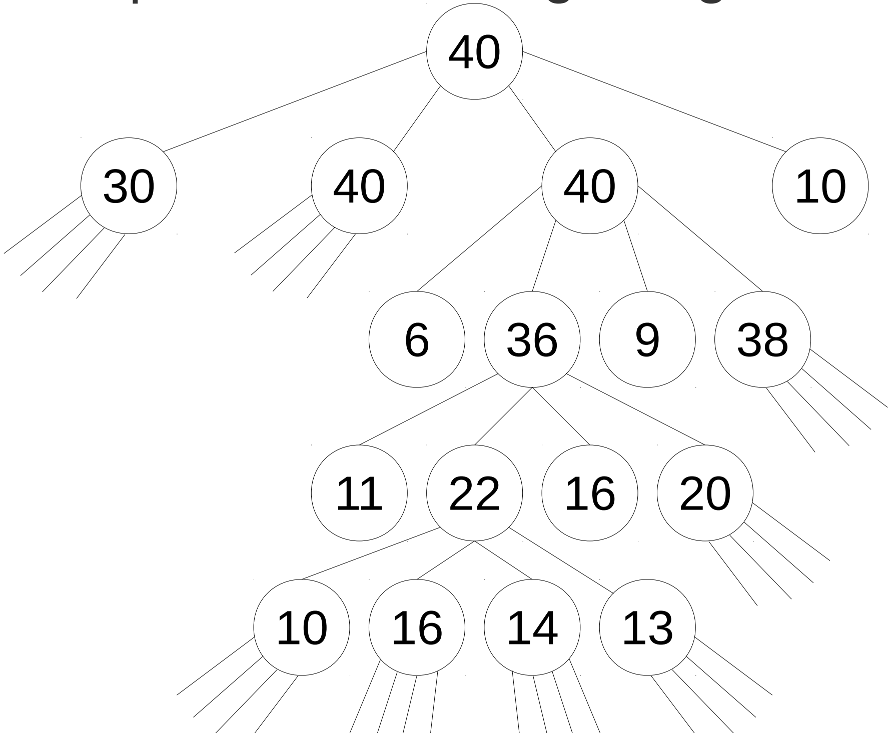
- 2D binary search
- At each step follow node with highest number of hits



Quadtrees

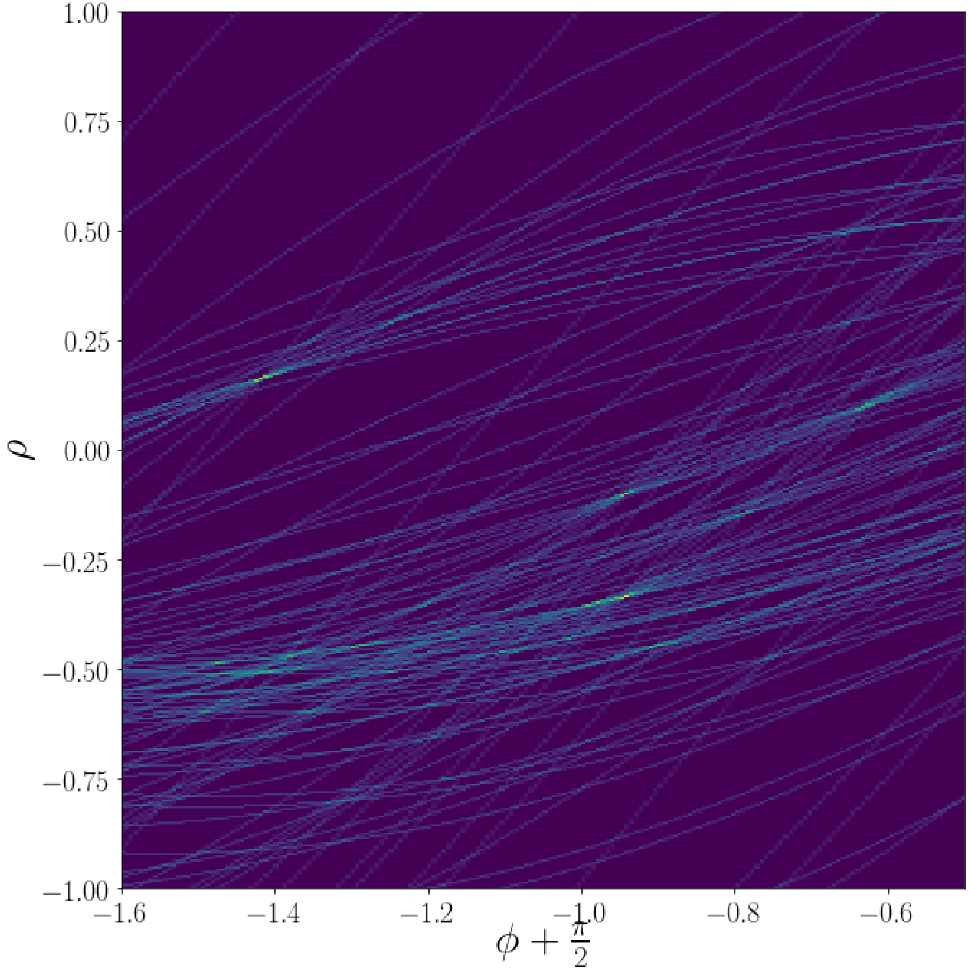


- Stop when desired precision or depth is reached
- Take found hits
- Repeat from the beginning



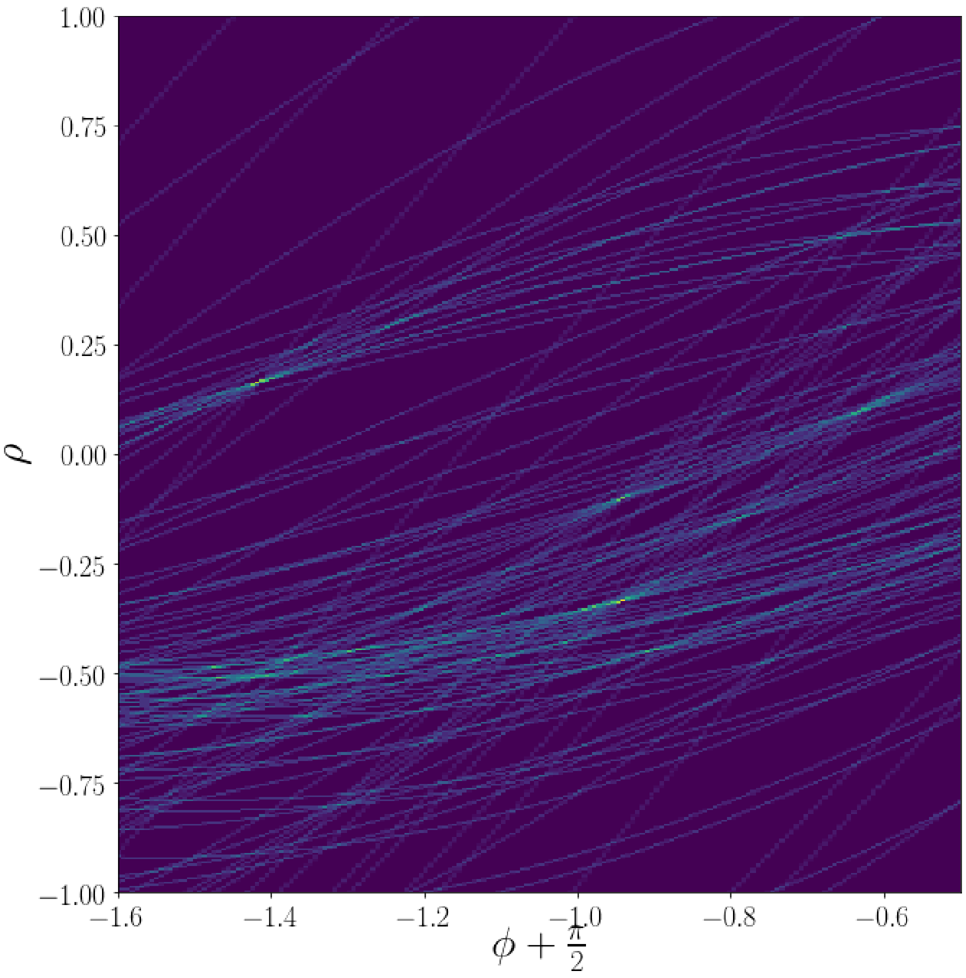
Discussion

Conformal Legendre



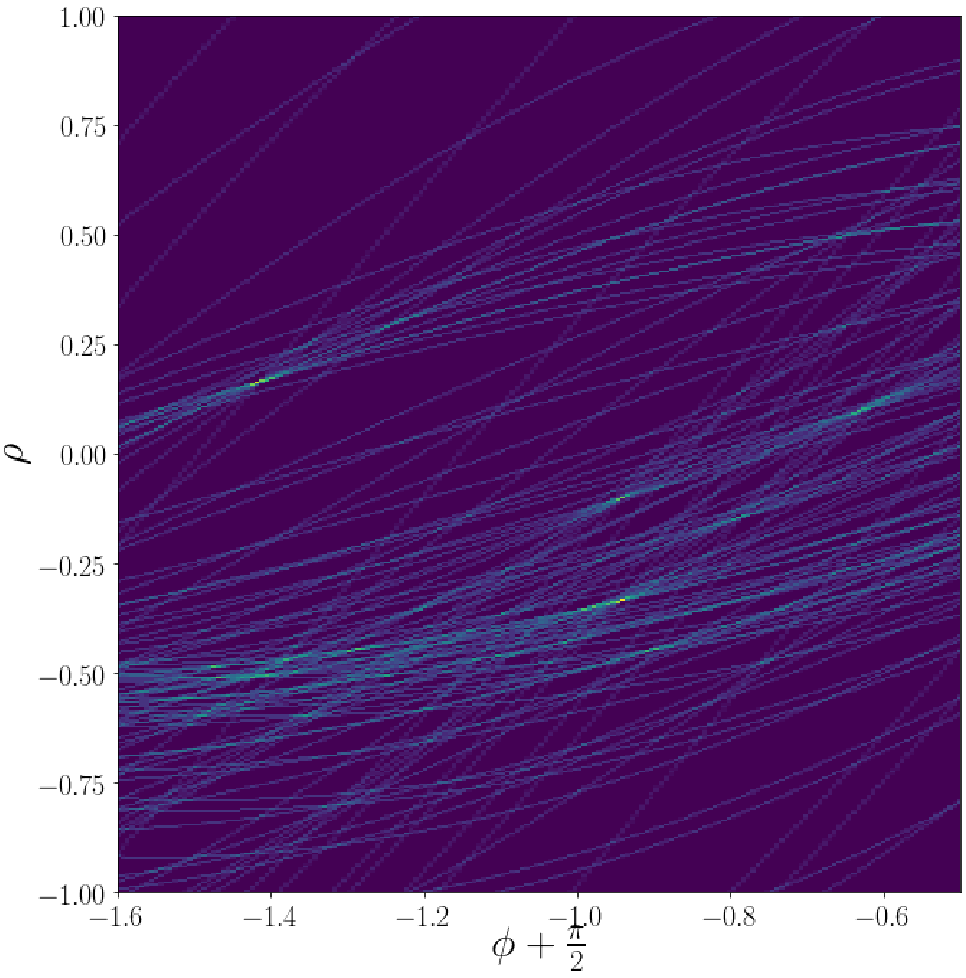
Conformal Legendre

- Complexity does not depend on curvature (pt)



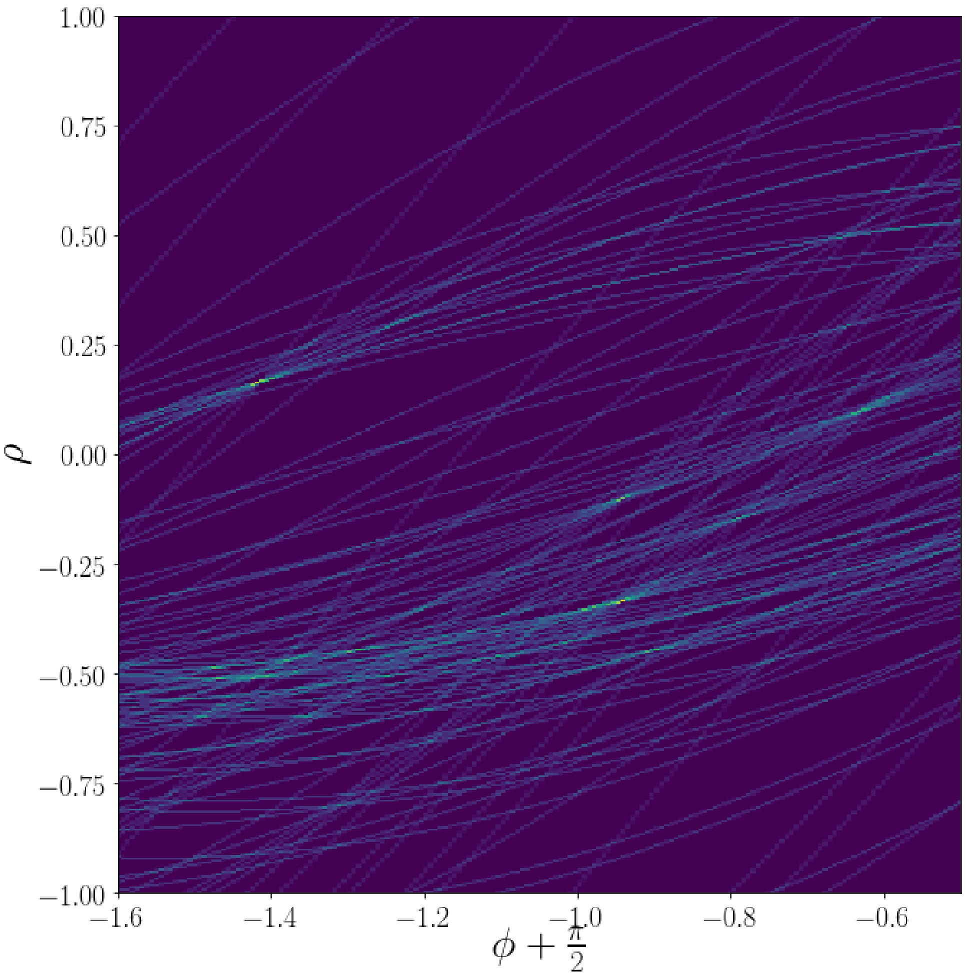
Conformal Legendre

- Complexity does not depend on curvature (pt)
- All hits have same weight



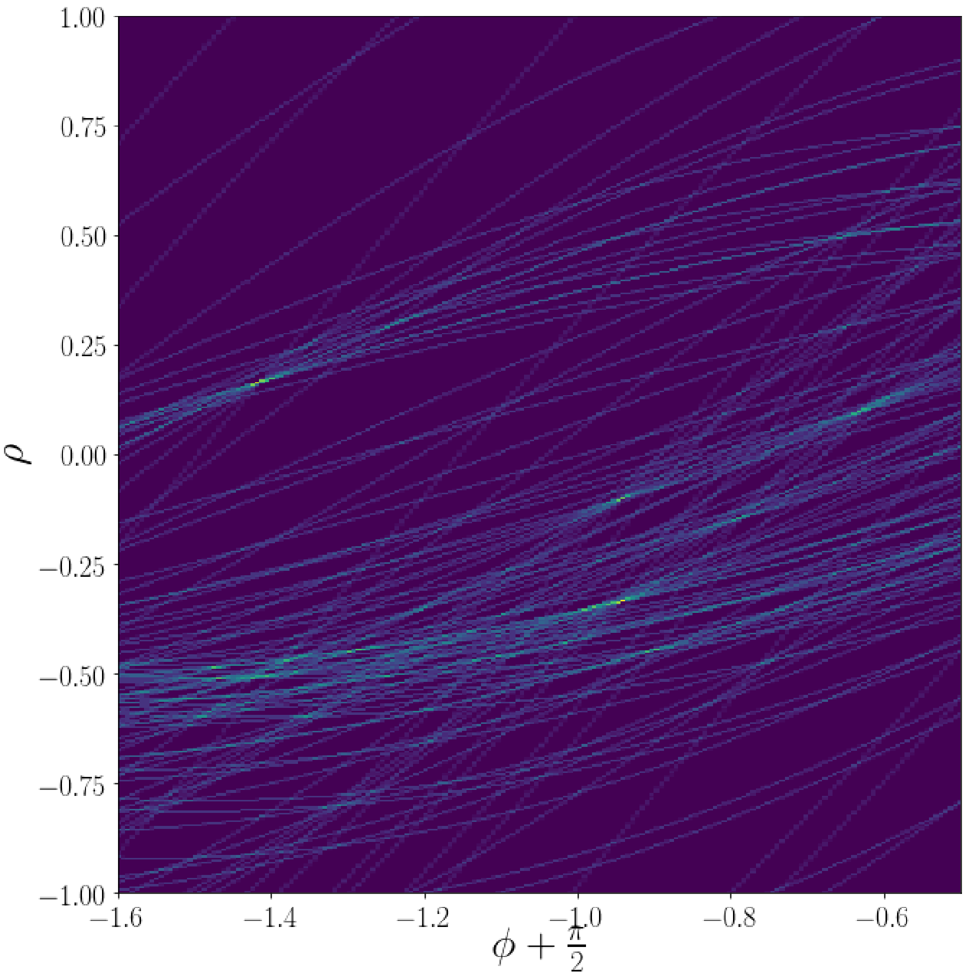
Conformal Legendre

- Complexity does not depend on curvature (pt)
- All hits have same weight
- Only circular tracks from the origin



Conformal Legendre

- Complexity does not depend on curvature (pt)
- All hits have same weight
- Only circular tracks from the origin
- Noise is tolerable as long as errors are relatively small



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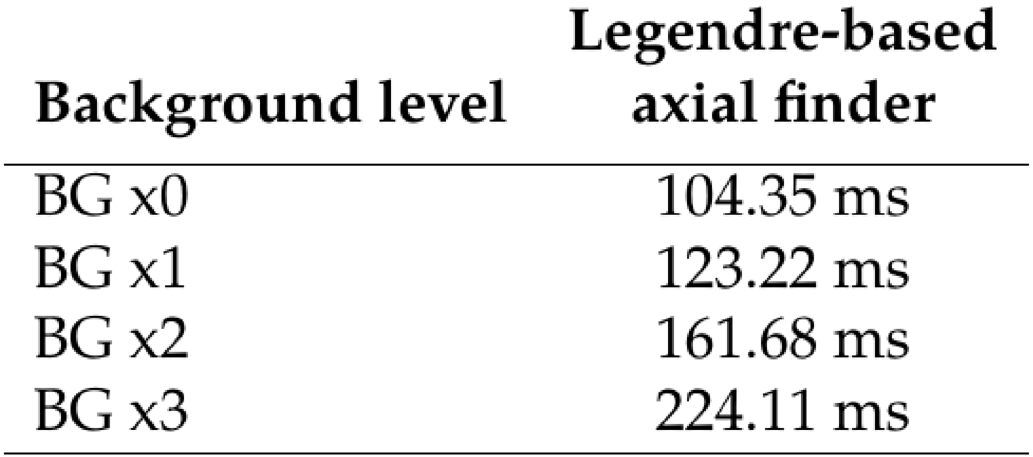
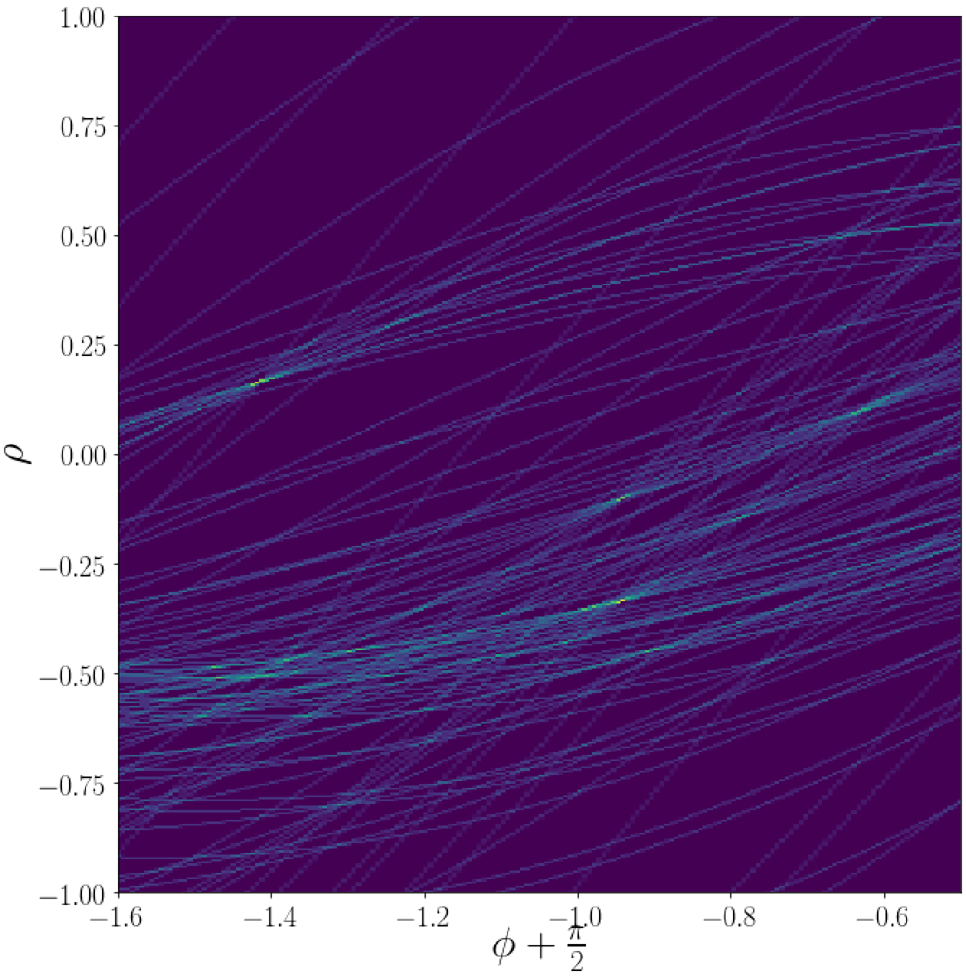
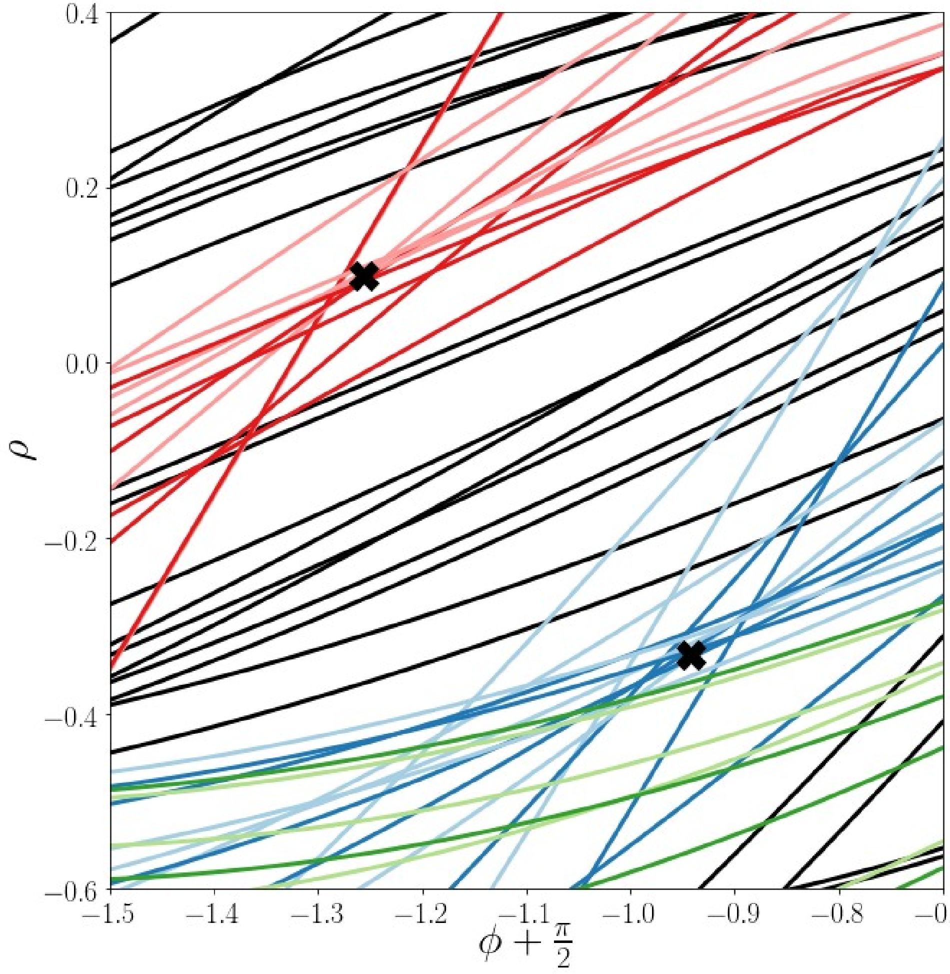


Background level	Legendre-based axial finder
BG x0	104.35 ms
BG x1	123.22 ms
BG x2	161.68 ms
BG x3	224.11 ms

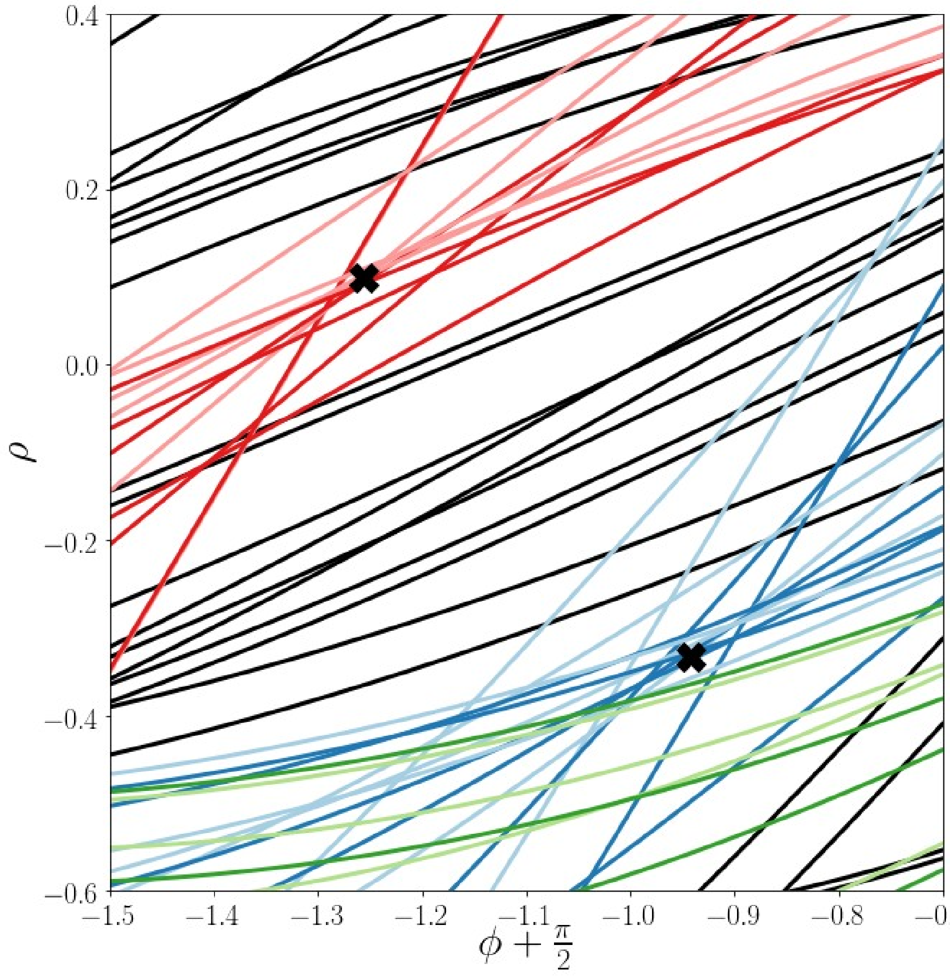


When to use



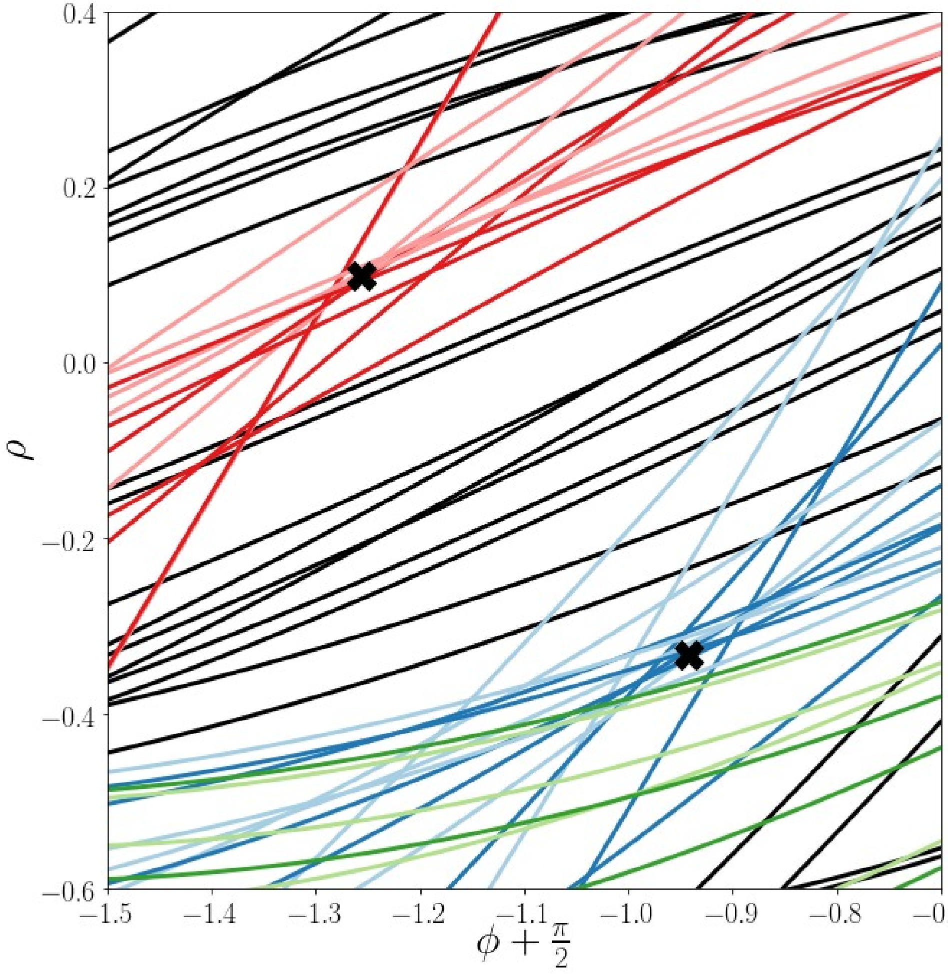
When to use

- Drift chambers (Legendre)



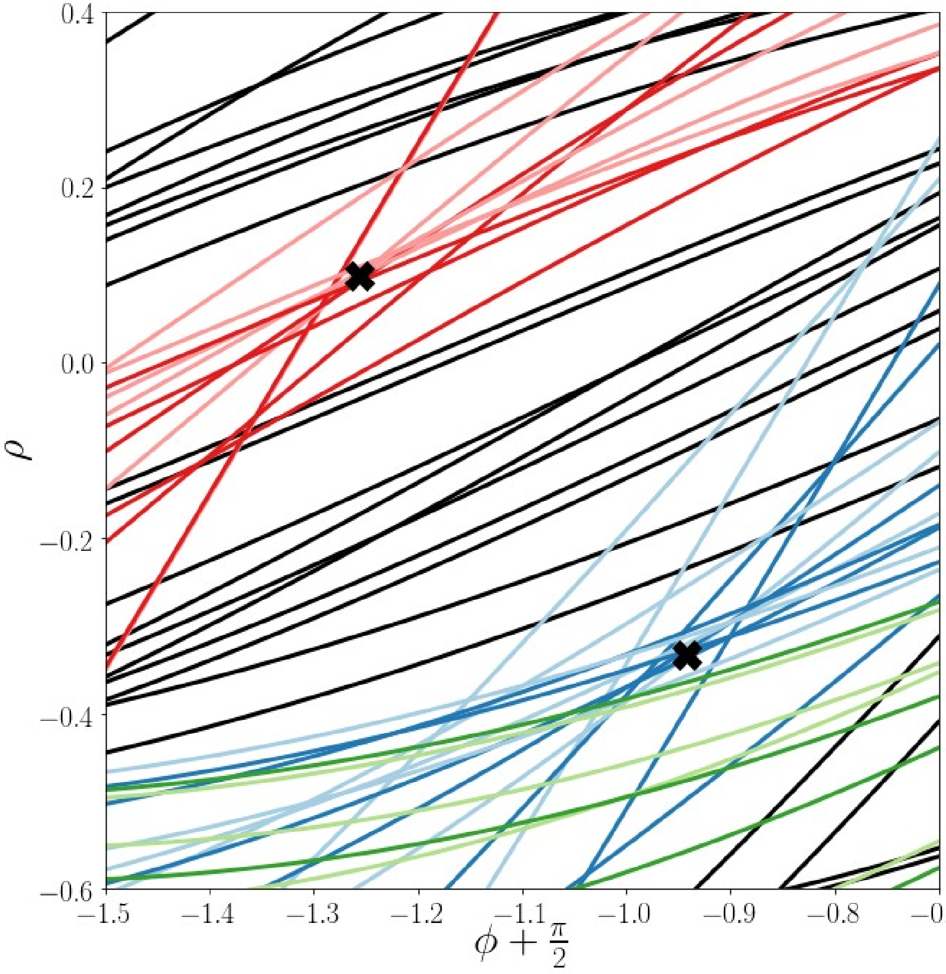
When to use

- Drift chambers (Legendre)
- Number of hits per track is high



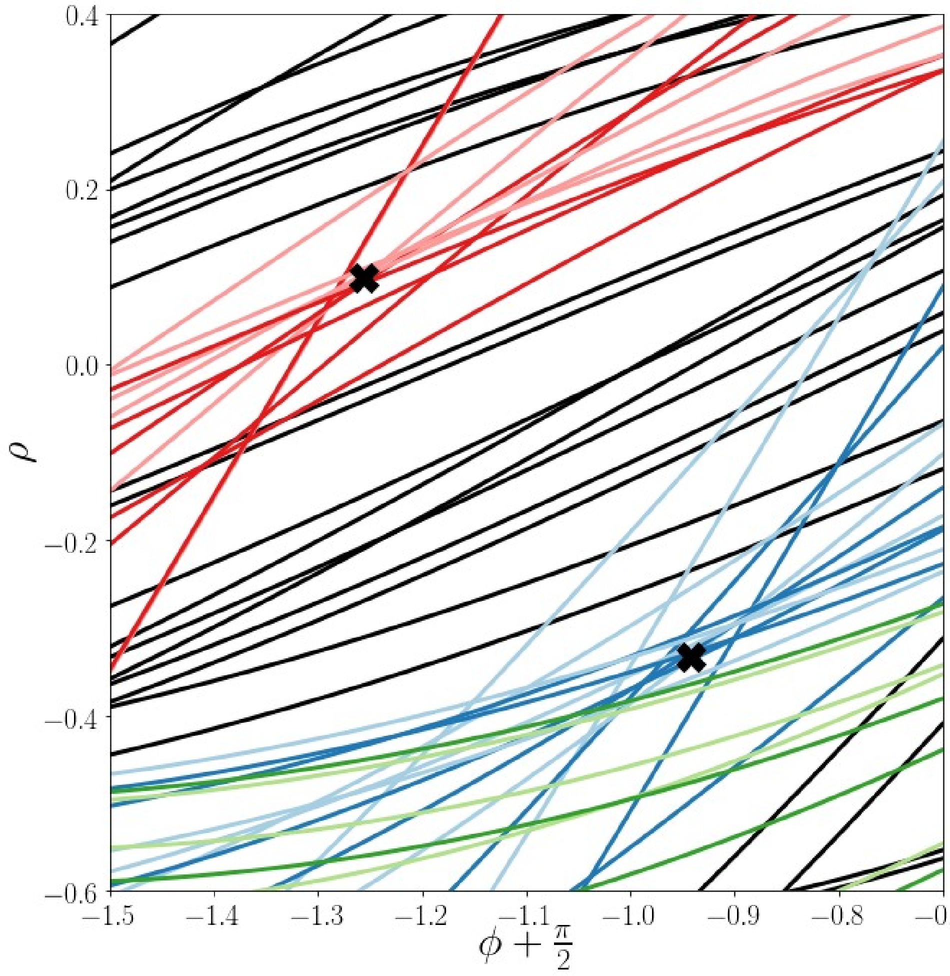
When to use

- Drift chambers (Legendre)
- Number of hits per track is high
- Tracks are strict circles from origin or lines



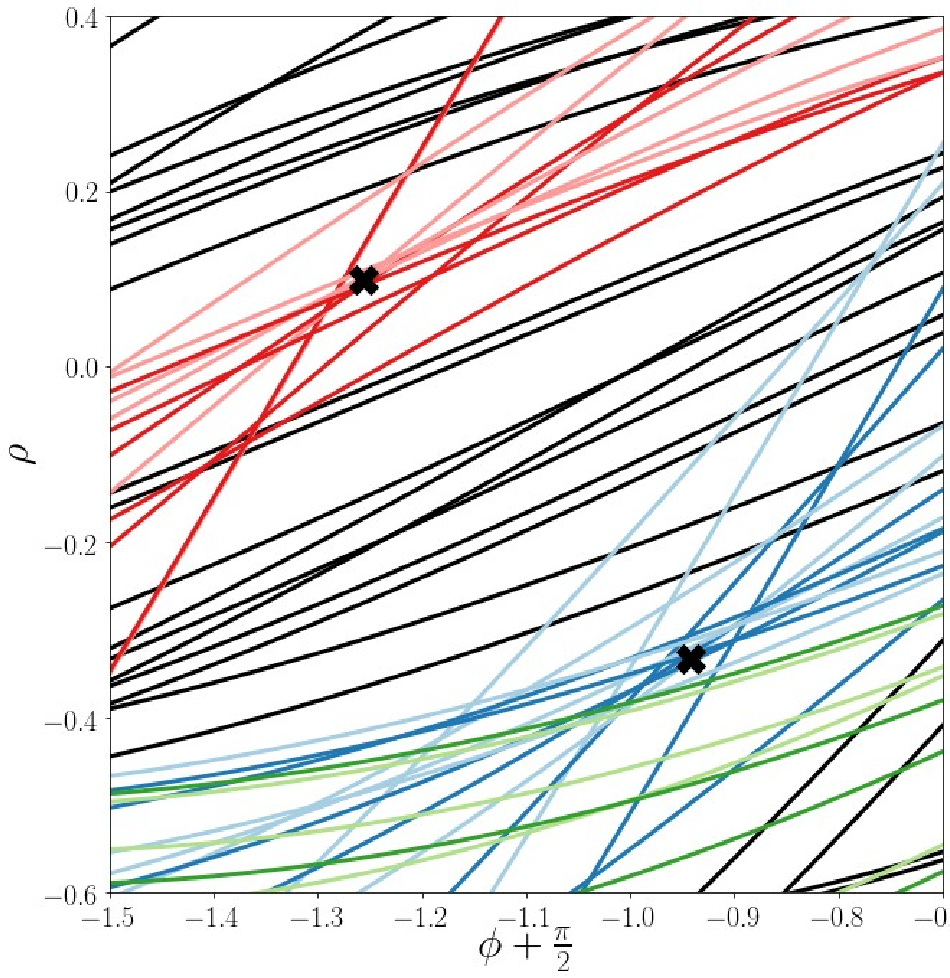
When to use

- Drift chambers (Legendre)
- Number of hits per track is high
- Tracks are strict circles from origin or lines
- High efficiency across all pt is desired



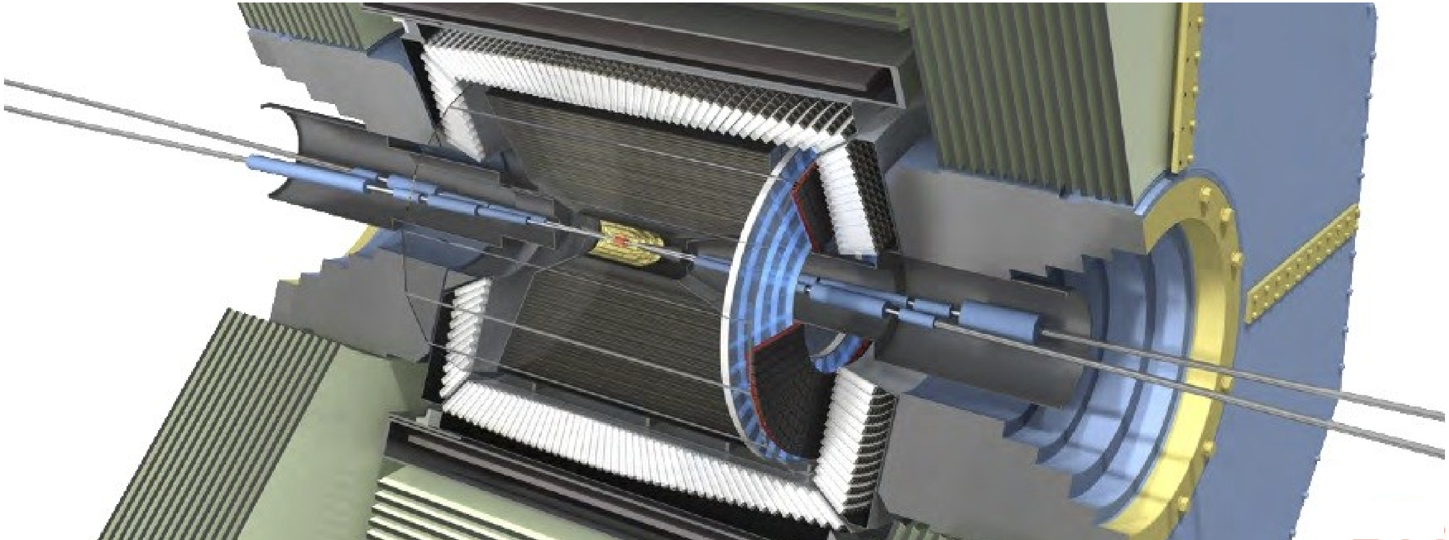
When to use

- Drift chambers (Legendre)
- Number of hits per track is high
- Tracks are strict circles from origin or lines
- High efficiency across all pt is desired
- Measurements are precise
- High noise
- High multiplicity



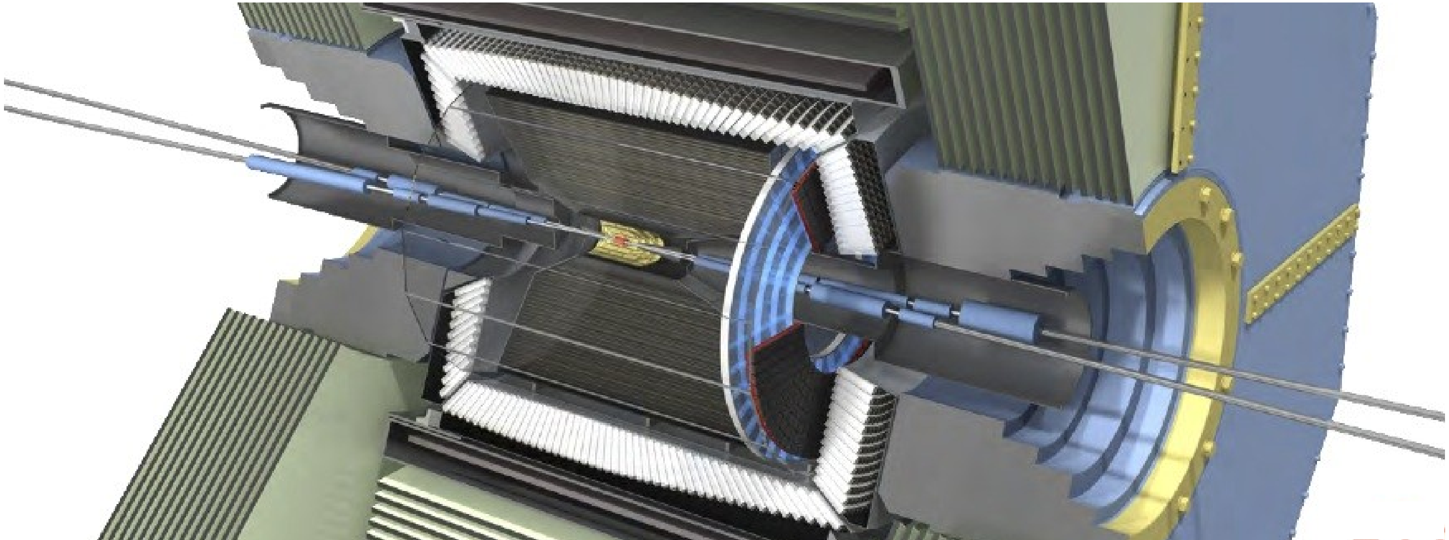
Example of Belle II

- Tracking detectors
  - Central Drift Chamber 56 layers
  - Inner Vertex Detector 6 layers



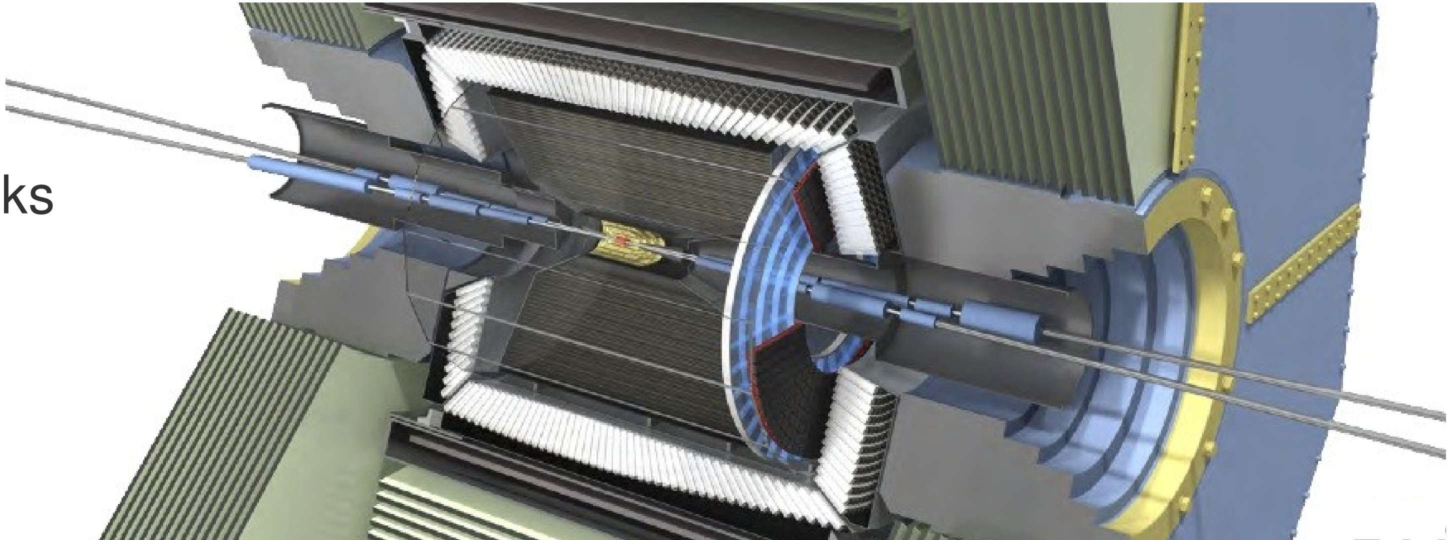
Example of Belle II

- Tracking detectors
  - Central Drift Chamber 56 layers
  - Inner Vertex Detector 6 layers
- Environment
  - ~11 tracks / event
  - High noise especially close to IP 200 – 500%



Example of Belle II

- Tracking detectors
  - Central Drift Chamber 56 layers
  - Inner Vertex Detector 6 layers
- Environment
  - ~11 tracks / event
  - High noise especially close to IP 200 – 500%
- Challenges
  - Reconstruct **all** tracks
  - Low fake rate



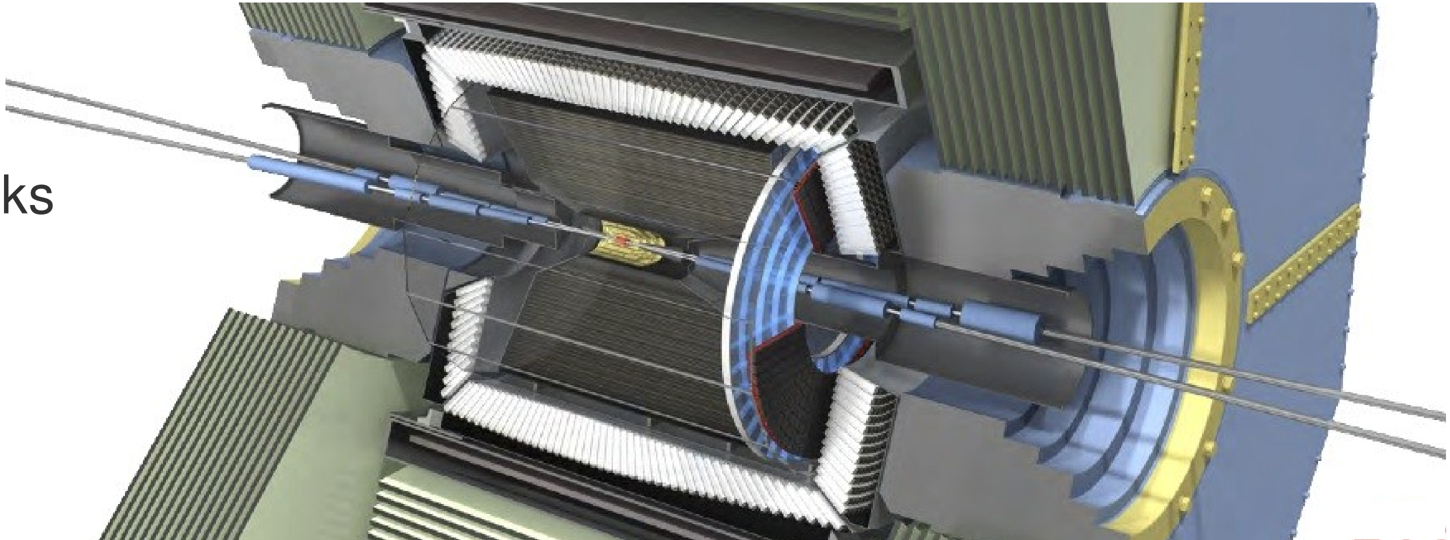
Example of Belle II

- Tracking detectors
  - Central Drift Chamber 56 layers
  - Inner Vertex Detector 6 layers
- Environment
  - ~11 tracks / event
  - High noise especially close to IP 200 – 500%
- Challenges
  - Reconstruct **all** tracks
  - Low fake rate

- Conformal Legendre (xy)
  Hough (sz)

  +

  Cellular automaton



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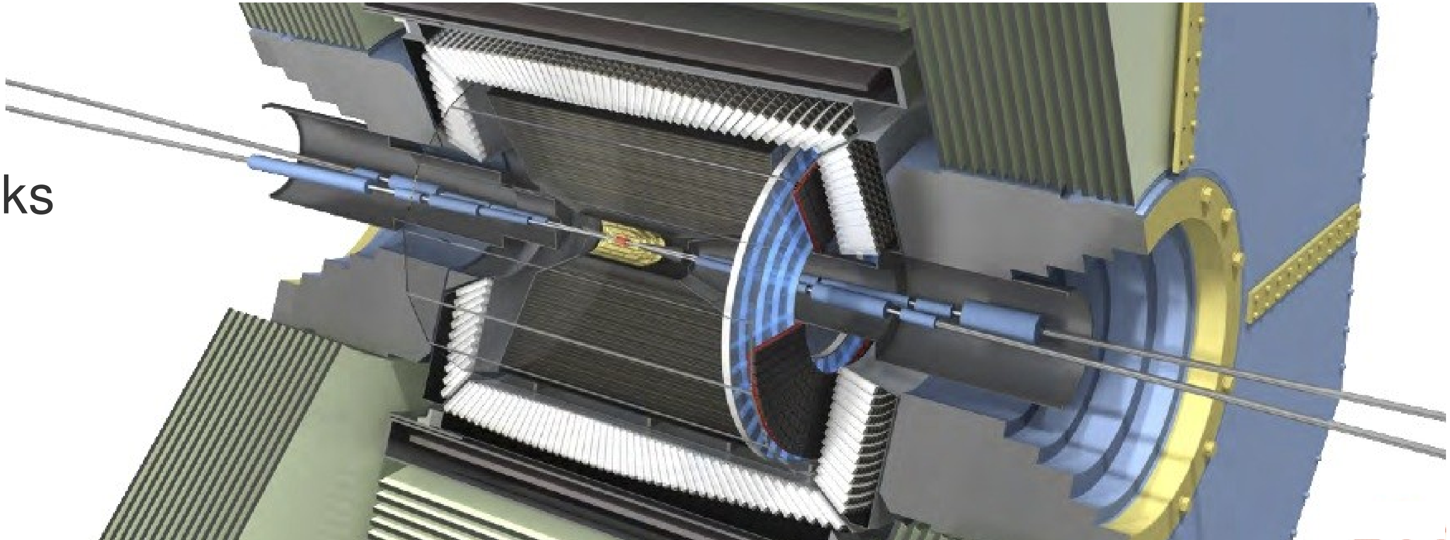
- Cellular automaton

  +

  Combinatorial Kalman filter

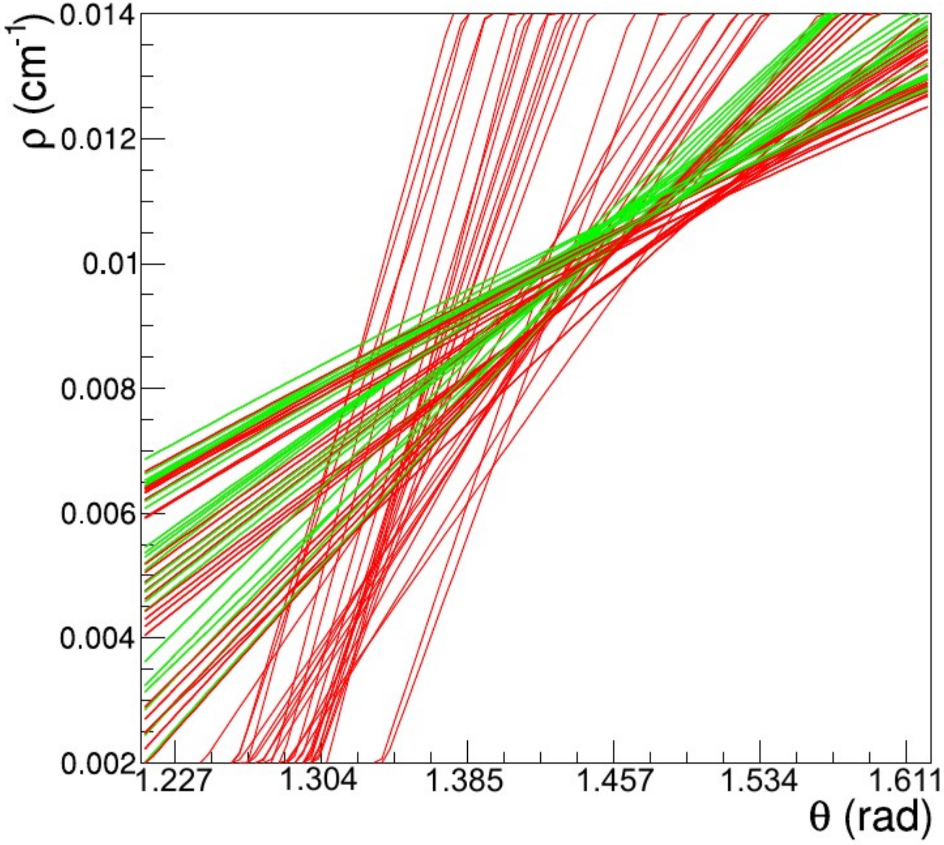


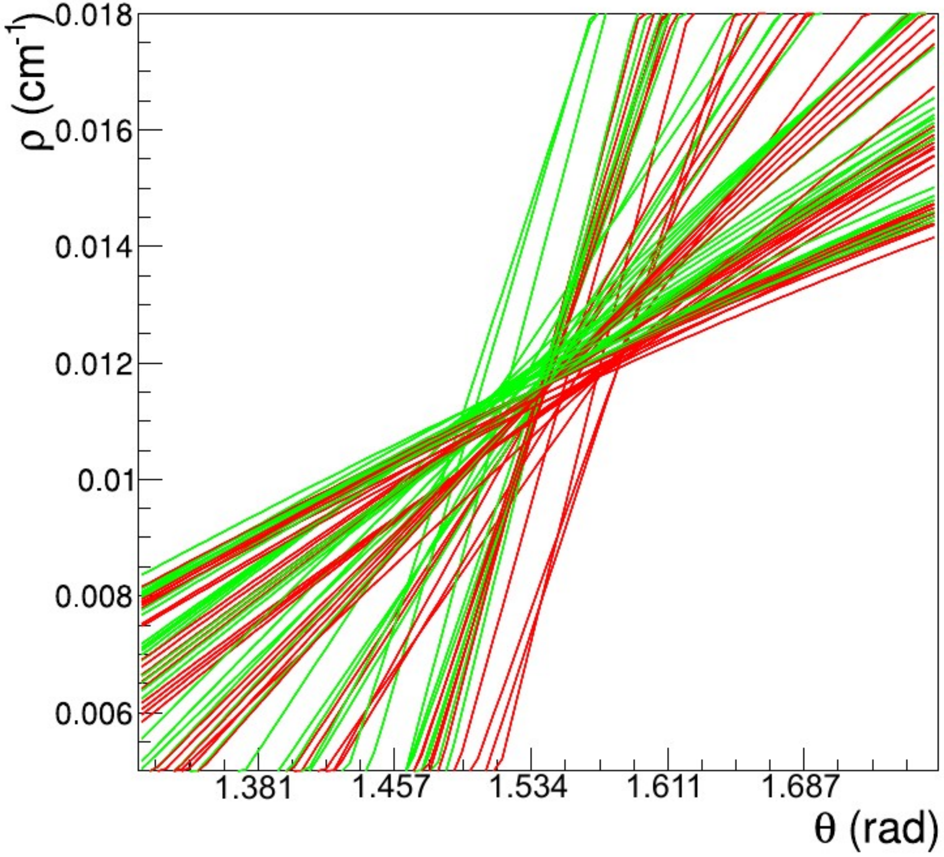
Thank you

Backup

Tracks from a slightly displaced vertex

- Repeat search wrt point of closest approach of a candidate





(b) Reference to POCA after the fitting

Legendre: Mathematical sense

- Switching to different variables
- By the simple fact of

$$d(xy) = xdy + ydx$$

- Thermodynamics
  - Internal energy ↔ Enthalpy etc.
- Theoretical mechanics
  - Lagrangian ↔ Hamiltonian

$$f(x,y) \qquad u = \frac{\partial f}{\partial x} \qquad w = \frac{\partial f}{\partial y}$$

$$df = udx + wdy$$

$$dg = udx - ydw$$

$$dh = -xdu + wdy$$

$$dk = -xdu - ydw$$

$$g(x,w) = f - wy$$

$$h(u,y) = f - ux$$

$$k(u,w) = f - ux - wy$$

Legendre: Geometrical sense

- For one-dimensional functions
  f(x) ↔ *F(p)*
- Concave case

$$F(p) = \sup_x [px - f(x)] = -\inf_x [f(x) - px]$$

- Convex case

$$F(p) = \sup_x [f(x) - px] = -\inf_x [px - f(x)]$$

- Supremum is found at a x_0 where

$$\left.\frac{df}{dx}\right|_{x=x_0} = p$$

(p, F(p)) in Legendre space corresponds to a certain tangent line to the original function *f(x)*

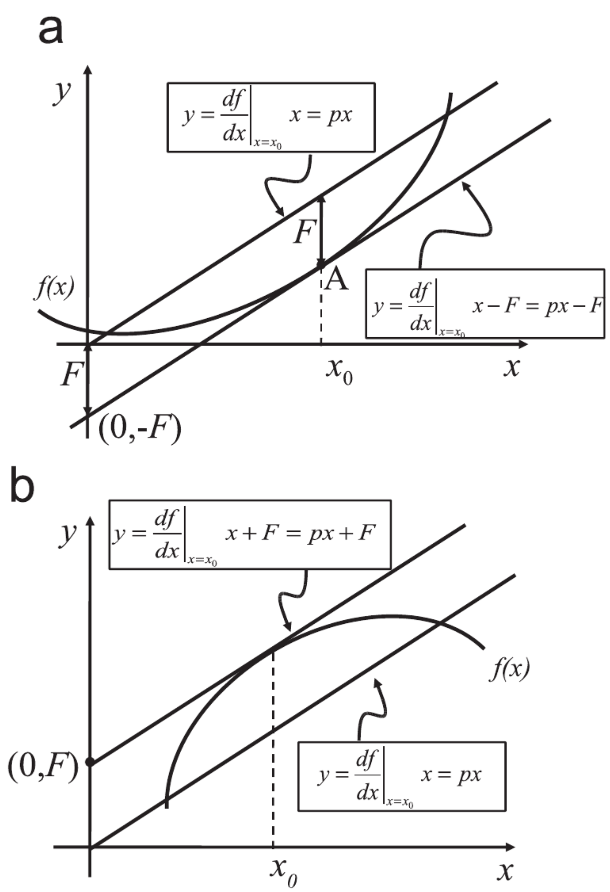

a
y
$y = \left.\frac{df}{dx}\right|_{x=x_0} x = px$
F
A
f(x)
$y = \left.\frac{df}{dx}\right|_{x=x_0} x - F = px - F$
F
x_0
x
(0,-F)
b
y
$y = \left.\frac{df}{dx}\right|_{x=x_0} x + F = px + F$
f(x)
(0,F)
$y = \left.\frac{df}{dx}\right|_{x=x_0} x = px$
x_0
x

Fig. 1. The Legendre transform corresponding (a) to a convex and (b) to a concave function.

(Drift) circles

- Derivative

$$p = -\frac{x - x_0}{\sqrt{R^2 - (x - x_0)^2}} \Rightarrow x = x_0 \frac{|p|R}{\sqrt{p^2 + 1}}$$

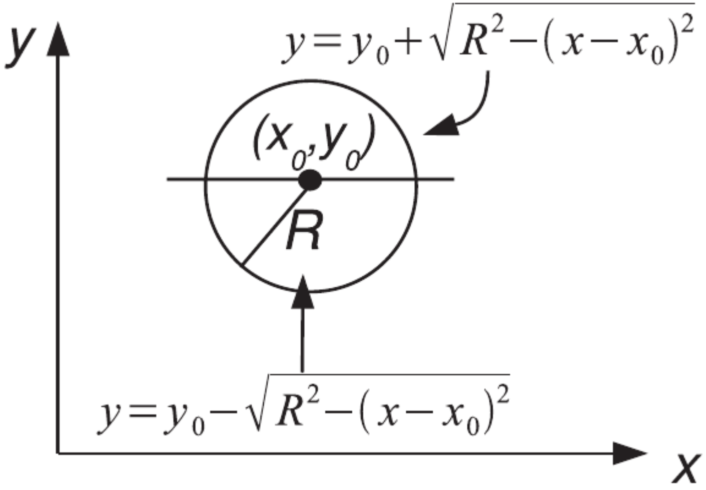
- Concave and convex parts

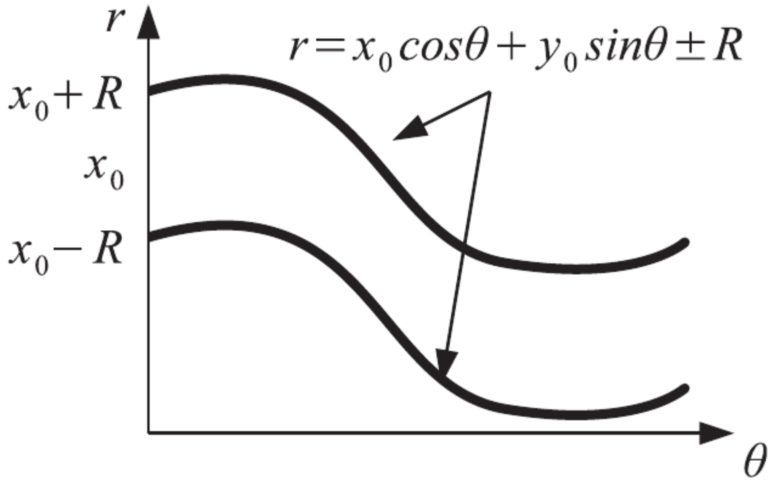
$$F_1(p) = f_1(x) - px = y_0 - x_0 p + R\sqrt{p^2 + 1}$$

$$F_2(p) = x_0 p - y_0 + R\sqrt{p^2 + 1}$$

- Switch to polar coordinates

$$r = x\cos(\theta) + y\sin(\theta)$$





All tangent lines to a circle in real space are represented as two sinograms in legendre space