Overview on Models of Neutrino Masses and Flavour Mixing

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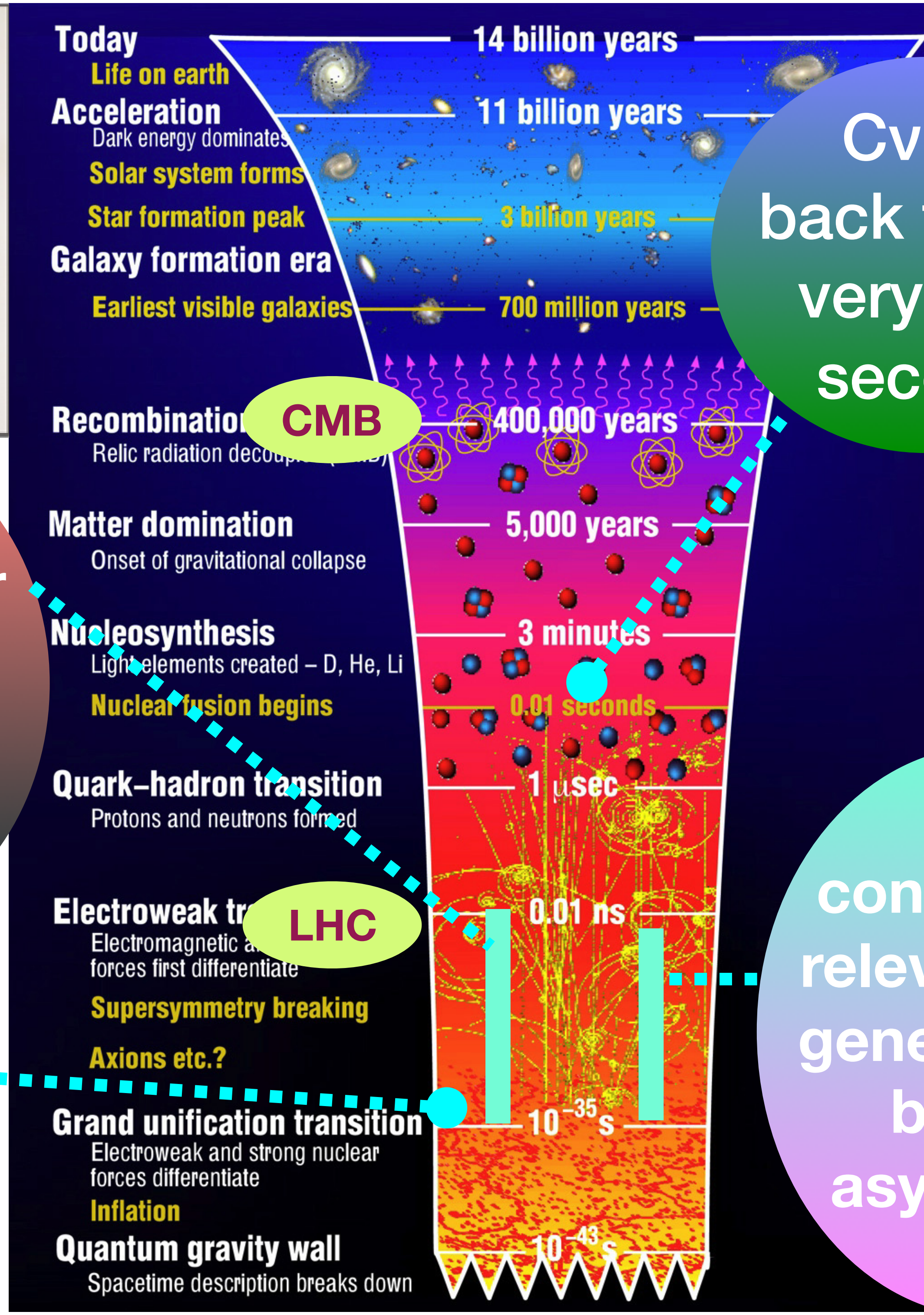
31st Rencontres de Blois on “Particle Physics and Cosmology,” Blois, France, June 6, 2019

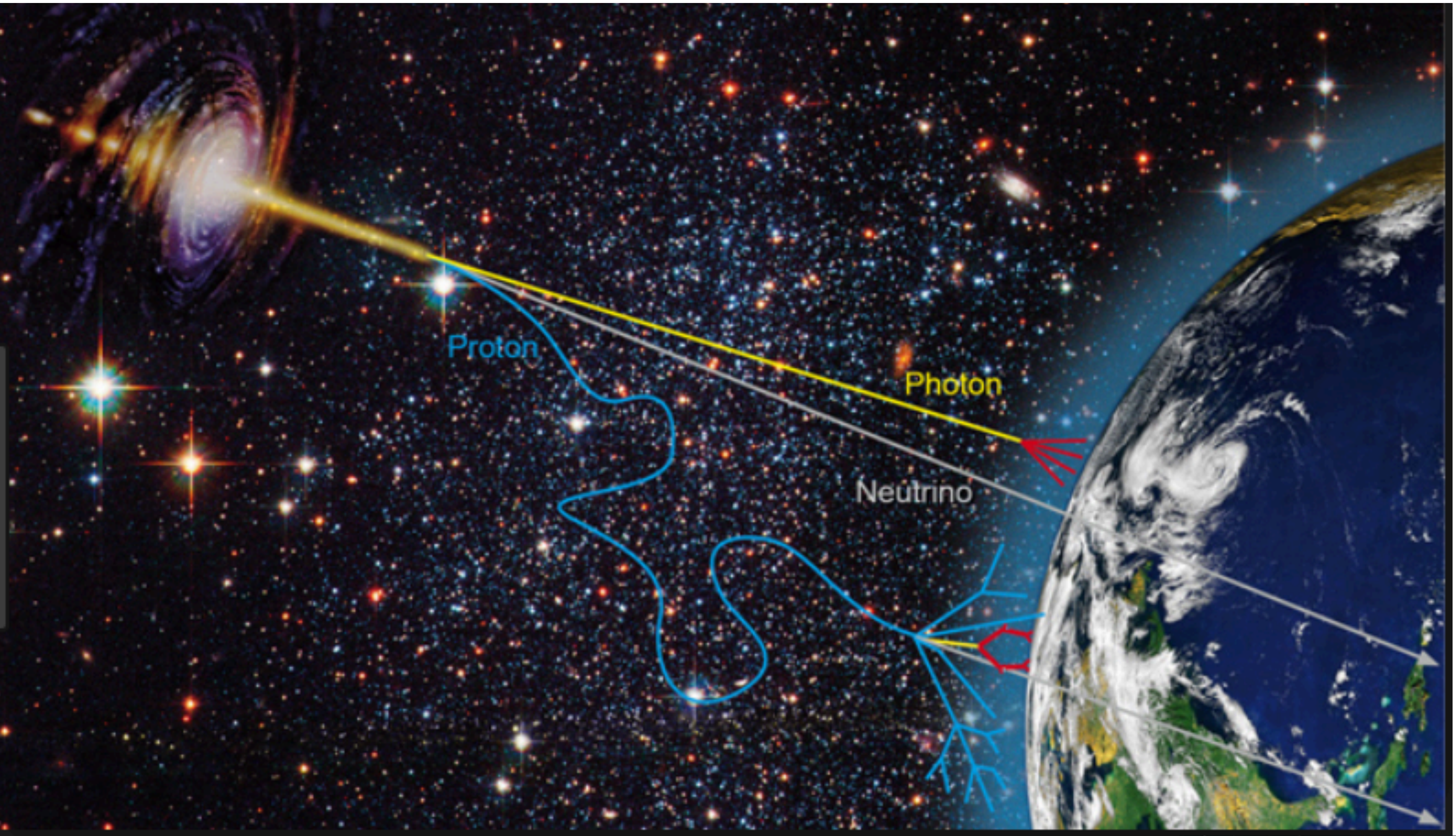


CνB - back to the very first second

operator for ν mass generation unknown

conceivable relevance for generation of baryon asymmetry

unique window into GUT scale physics



[Photo credit: Astroparticle Physics - DESY]

Neutrinos as messengers

Talks (Tu) Luigi Antonio, Fusco, (Wed) Giulia Illuminati, Juliana Stachurska, Daniel García-Fernández

Earth Tomography

Talks (Tu) Sergio Palomares-Ruiz

Where Do We Stand?

Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni, Schwetz, 1811.05487

- Latest 3 neutrino global analysis:

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$
$\sin^2\theta_{23}$	$0.582^{+0.015}_{-0.019}$	$0.428 \to 0.624$	$0.582^{+0.015}_{-0.018}$	$0.433 \to 0.623$
$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	$40.9 \to 52.2$	$49.7^{+0.9}_{-1.0}$	$41.2 \to 52.1$
$\sin^2\theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	$0.02044 \to 0.02437$	$0.02263^{+0.00065}_{-0.00066}$	$0.02067 \to 0.02461$
$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	$8.22 \to 8.98$	$8.65^{+0.12}_{-0.13}$	$8.27 \to 9.03$
$\delta_{CP}/^\circ$	217^{+40}_{-28}	$135 \to 366$	280^{+25}_{-28}	$196 \to 351$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.525^{+0.033}_{-0.031}$	$+2.431 \to +2.622$	$-2.512^{+0.034}_{-0.031}$	$-2.606 \to -2.413$

➡ hints of $\theta_{23} \neq \pi/4$

➡ expectation of Dirac CP phase δ

➡ preference for normal hierarchy

Recent T2K result ➪ $\delta \simeq -\pi/2$, consistent with global fit best fit value

Where Do We Stand?

- search for absolute mass scale:
  - end point kinematic of tritium beta decays

$$m_{\nu_e} < 2.2 \text{ eV (95\% CL)} \quad \text{Mainz}$$
$$m_{\nu_\mu} < 170 \text{ keV}$$
$$m_{\nu_\tau} < 15.5 \text{ MeV}$$

$$\text{Tritium} \to He^3 + e^- + \overline{\nu}_e$$

KATRIN: increase sensitivity ~ 0.2 eV

Talks (Tu) by Ann-Kathrin Schütz, Guido Fantini, Luca Gironi, Claudia Nones, Justo Martin-Albo

  - neutrinoless double beta decay

$$\text{current bound:} \quad |\langle m \rangle| \equiv \left| \sum_{i=1,2,3} m_i U_{ie}^2 \right|$$ < (0.061-0.165) eV (Kamland-Zen, 2016)

  - Cosmology $\sum(m_{\nu_i}) < 0.12 \text{ eV}$

Talks (Wed) Christian Reichardt

N_{eff} = 2.99 ± 0.17 [Planck 2018] ⇒ fully thermalized sterile neutrino disfavored

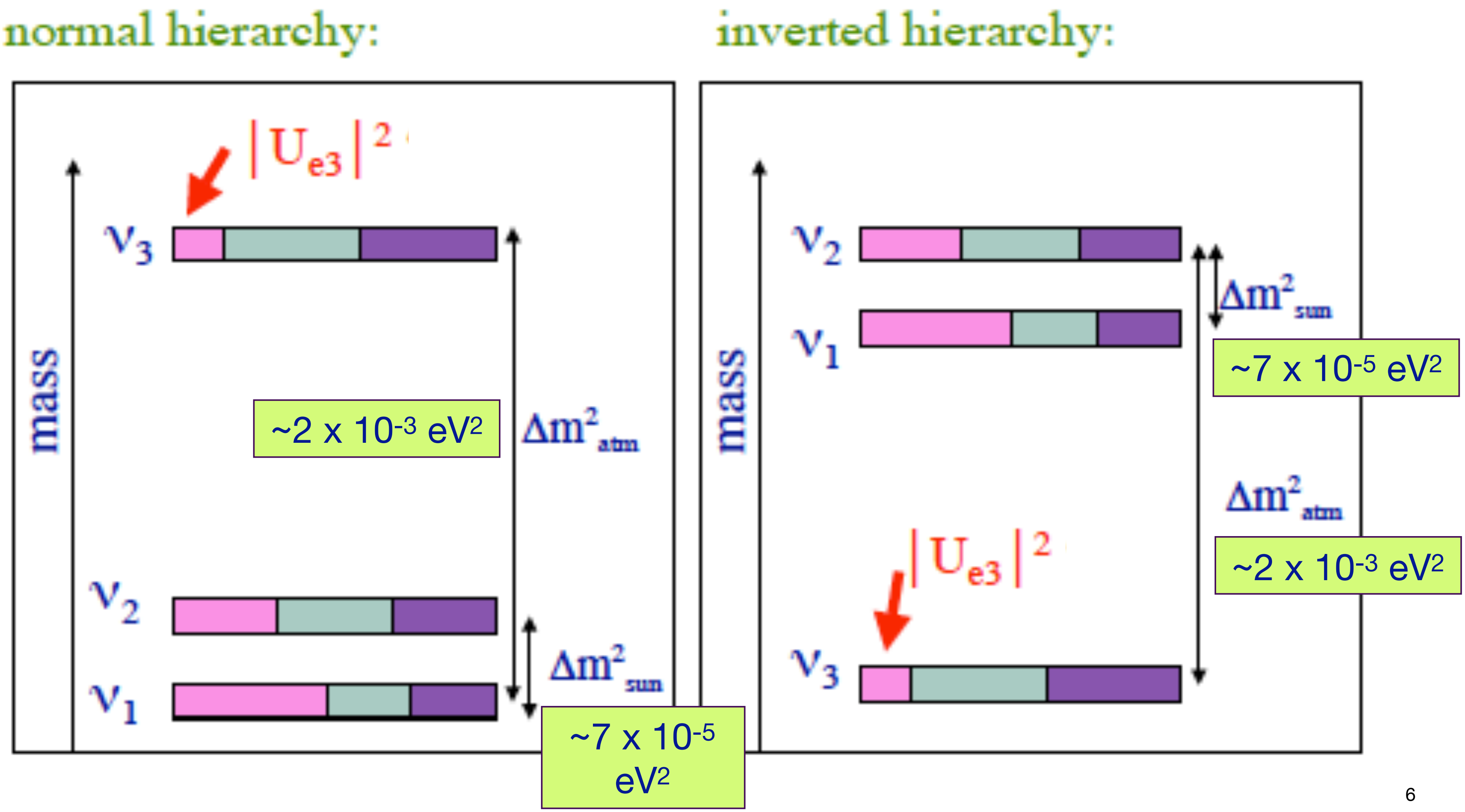
  - EM properties of Neutrinos

Talks (Tu) Alexander Studenikin

  - Astrophysical Neutrinos

Where Do We Stand?

- The known knowns:



Open Questions - Neutrino Properties

- ☞ Majorana vs Dirac?
- ☞ CP violation in lepton sector?
- ☞ Absolute mass scale of neutrinos?
- ☞ Mass ordering: sign of (Δm_{13}^2)?
- ☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?
- ☞ Sterile neutrino(s)?

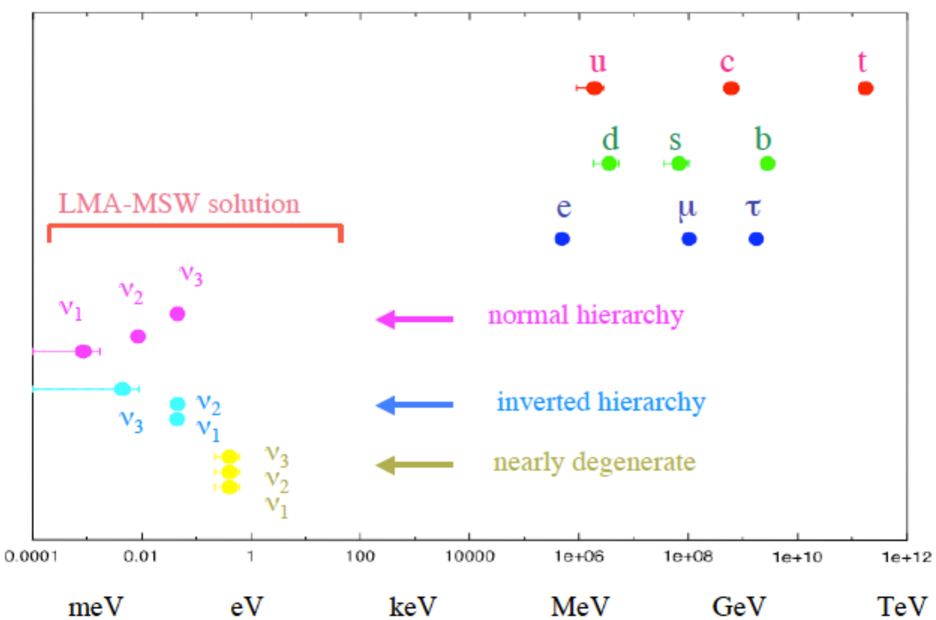
a suite of current and upcoming experiments to address these puzzles

some can only be answered by oscillation experiments

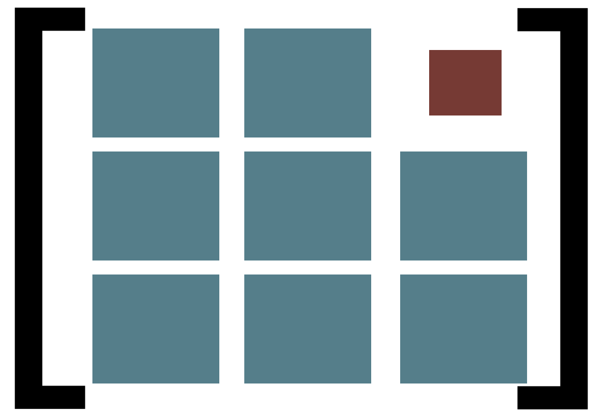
Open Questions - Theoretical

☞ Smallness of neutrino mass:

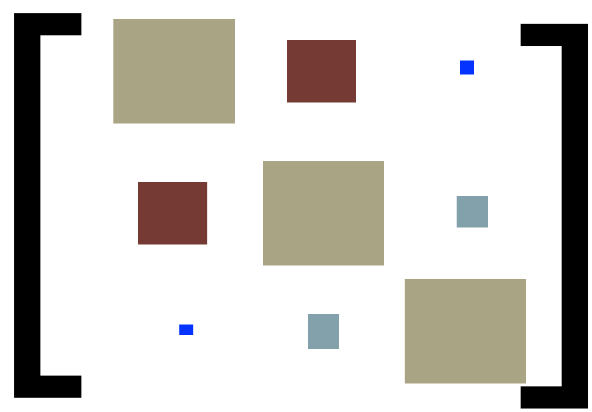
$$m_\nu \ll m_{e,\,u,\,d}$$



☞ Flavor structure:



leptonic mixing

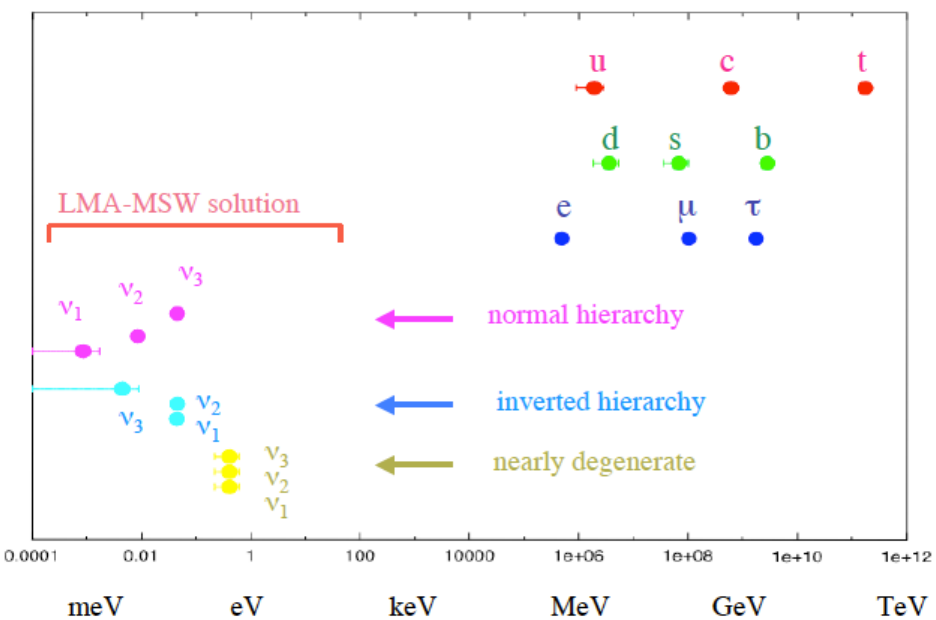


quark mixing

Open Questions - Theoretical

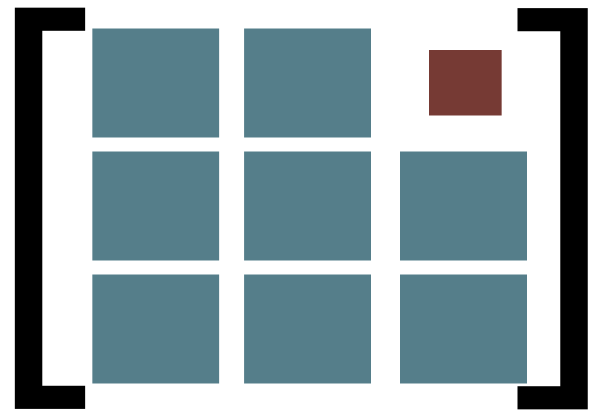
☞ Smallness of neutrino mass:

$$m_\nu \ll m_{e,\,u,\,d}$$

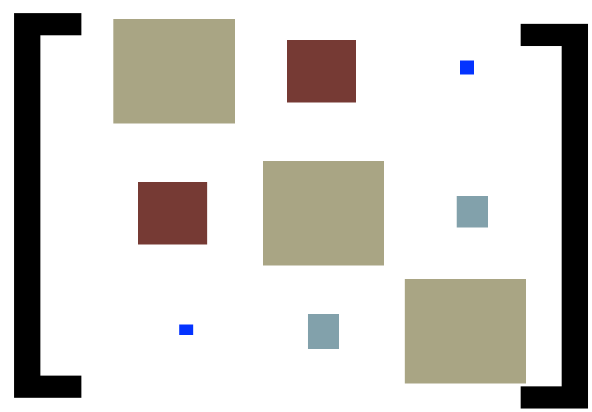


Fermion mass and hierarchy problem ➟ Many free parameters in the Yukawa sector of SM

☞ Flavor structure:



leptonic mixing



quark mixing

Smallness of neutrino masses

What is the operator for neutrino mass generation?

- Majorana vs Dirac
- scale of the operator
- suppression mechanism

Neutrino Mass beyond the SM

- SM: effective low energy theory

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \boxed{\frac{\mathcal{O}_{5D}}{M} + \frac{\mathcal{O}_{6D}}{M^2} + \ldots} \longrightarrow \boxed{\text{new physics effects}}$$

- only one dim-5 operator: most sensitive to high scale physics

Weinberg, 1979

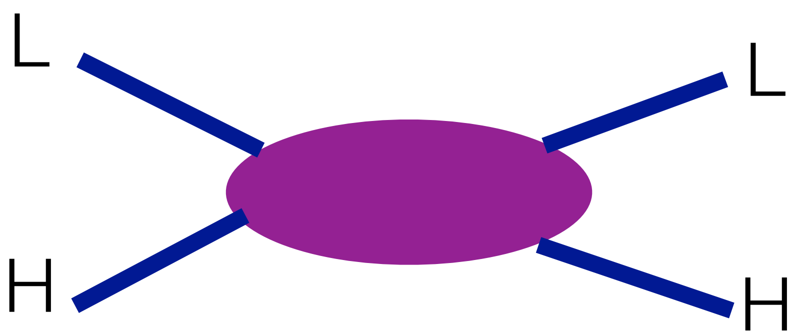
$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

- $m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.1$ eV with $v \sim 100$ GeV, $\lambda \sim O(1) \Rightarrow M \sim 10^{14}$ GeV
- Lepton number violation **ΔL = 2** ⇨ Majorana fermions

↕

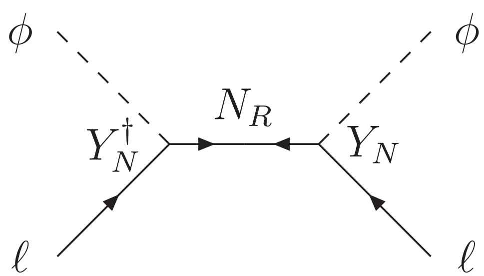
GUT scale

Neutrino Mass beyond the SM



3 possible portals

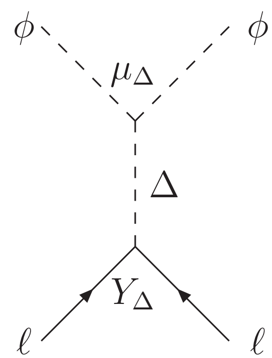
Type-I seesaw



N_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,1,0)

Minkowski, 1977; Yanagida, 1979; Glashow, 1979;
Gell-mann, Ramond, Slansky,1979;
Mohapatra, Senjanovic, 1979;

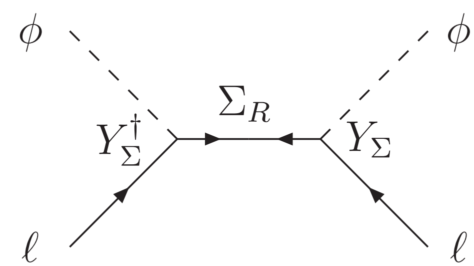
Type-II seesaw



Δ: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,3,2)

Lazarides, 1980; Mohapatra, Senjanovic, 1980

Type-III seesaw



$$\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$$

Σ_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,3,0)

Foot, Lew, He, Joshi, 1989; Ma, 1998

Why are neutrinos light? (Type-I) Seesaw Mechanism

- Adding the right-handed neutrinos:

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$$m_\nu \sim m_{light} \sim \frac{m_D^2}{M_R} << m_D$$

$$m_{heavy} \sim M_R$$

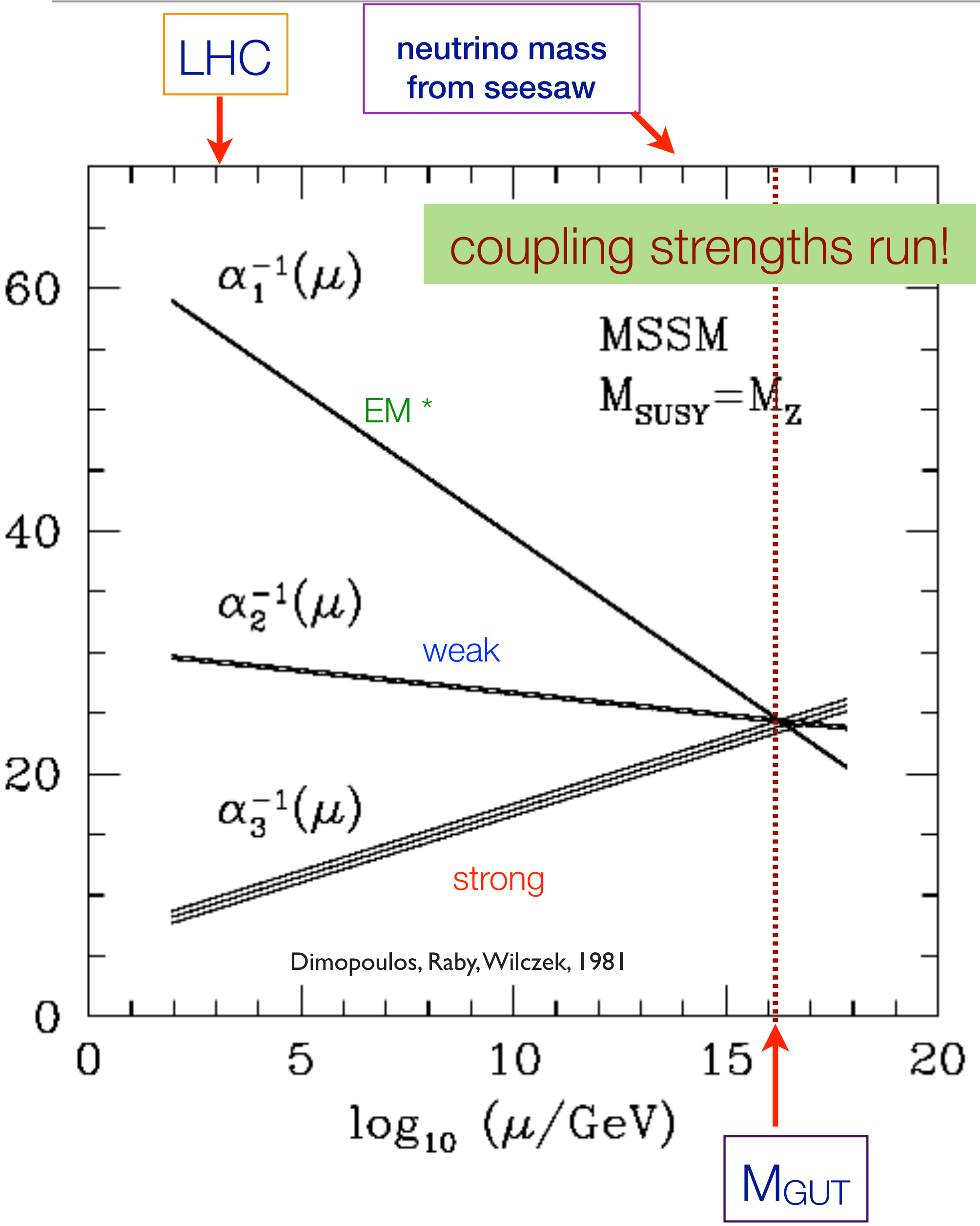
For $m_{\nu_3} \sim \sqrt{\Delta m^2_{atm}}$

If $m_D \sim m_t \sim 180 \;\; GeV$

$$\Longrightarrow \boxed{M_R \sim 10^{15}\; GeV \;\; (\text{GUT !!})}$$

Minkowski, 1977; Yanagida, 1979; Gell-Mann, Ramond, Slansky, 1979; Mohapatra, Senjanovic, 1981

Grand Unification Naturally Accommodates Seesaw



- ☞ origin of the heavy scale ⇒ $U(1)_{B-L}$
- ☞ exotic mediators ⇒ predicted in many GUT theories, e.g. SO(10)

$$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$$

$$+ (3^*, 1, -2/3) \sim (u^c \; u^c \; u^c)$$

$$+ (3^*, 1, 1/3) \sim (d^c \; d^c \; d^c)$$

$$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$$

$$+ (1, 1, 1) \sim e^c$$

$$+ (1, 1, 0) \sim \nu^c$$

Fritzsch, Minkowski, 1975

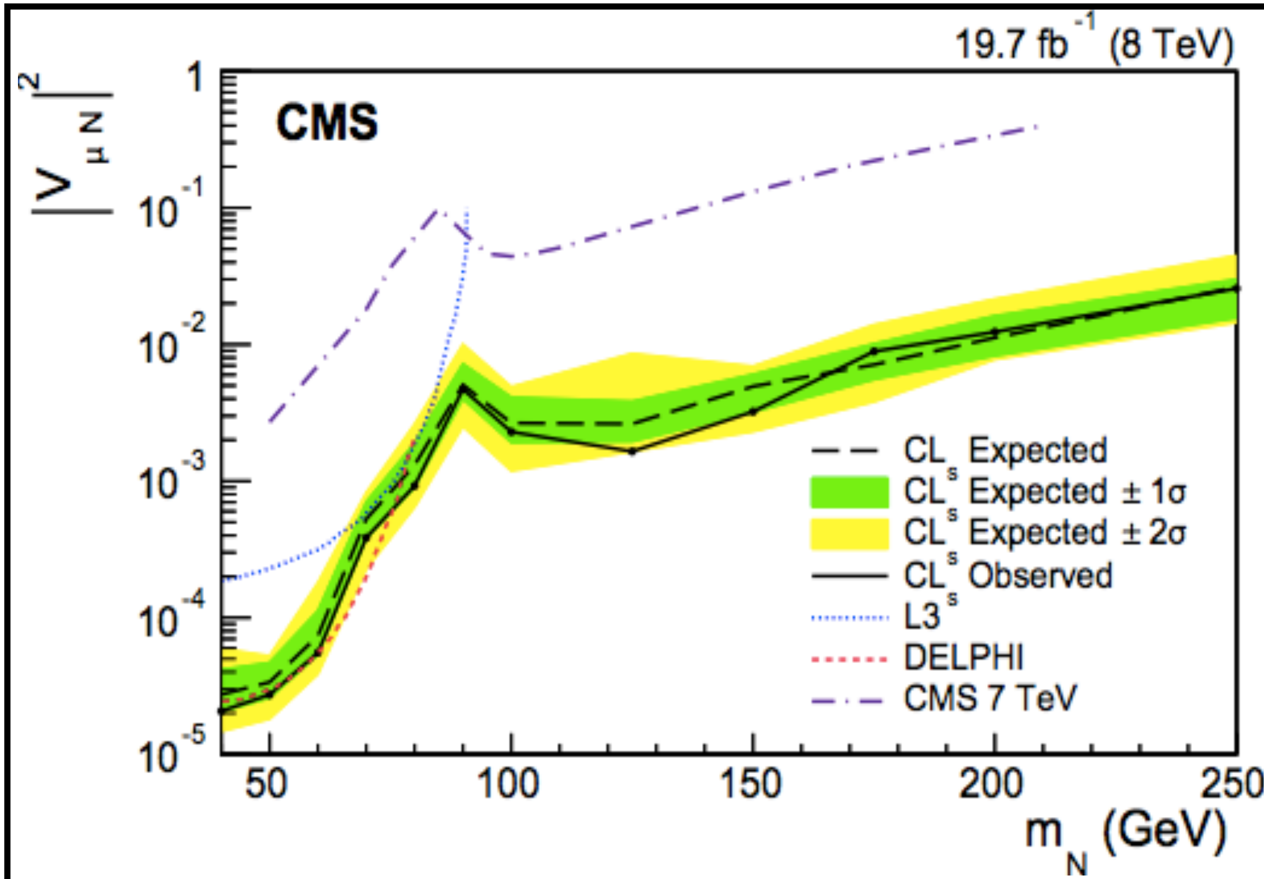
Low Scale Seesaws

[References see e.g. Review: M.-C. C., J. Huang, 1105.3188]

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.1$ eV with $v \sim 100$ GeV, $\lambda \sim 10^{-6}$
$\Rightarrow$ M ~ 10^2 GeV

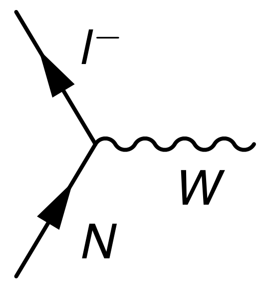
- New particles:
  - Type I seesaw: generally decouple from collider experiments
  - Type II seesaw: $\Delta^{++} \to e^+ e^+ , \; \mu^+ \mu^+ , \; \tau^+ \tau^+$
  - Type III seesaw: observable displaced vertex, dark matter candidate
  - inverse seesaw: non-unitarity effects
  - radiative mass generation: model dependent - singly/doubly charged SU(2) singlet, even colored scalars in loops, dark matter candidate
- New interactions:
  - LR symmetric model: W_R
  - R parity violation: $\tan^2 \theta_{\rm atm} \simeq \dfrac{BR(\tilde{\chi}^0_1 \to \mu^\pm W^\mp)}{BR(\tilde{\chi}^0_1 \to \tau^\pm W^\mp)}$
  - .....

Cautions!!! Is it really the ν_R in Type I seesaw?



Expanded view of the region:
40 GeV < m_N < 250 GeV

RH neutrino production thru active-sterile mixing:



$$\propto V = \frac{m_D}{M_R} \sim \frac{10^{-4}\ \text{GeV}}{100\ \text{GeV}} = 10^{-6}$$

RH neutrino relevant for ν mass generation

➪ $|V_{\mu N}|^2 = 10^{-12}$

unless extremely fine-tuned

Kersten, Smirnov (2007)

What if neutrinos are Dirac?

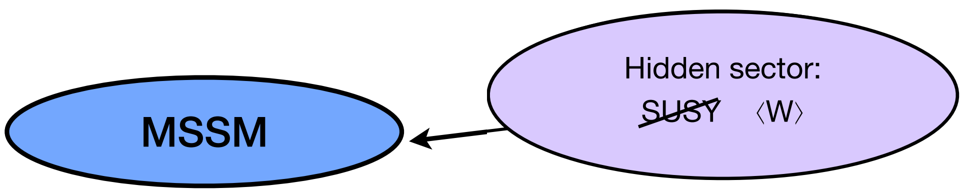
Dirac Neutrinos and SUSY Breaking

- naturally small Dirac neutrino masses?

- before SUSY breaking: absence of Dirac neutrino masses (as well as Weinberg operator)
- after SUSY breaking: realistic effective Dirac neutrino masses generated

$$\boxed{Y_\nu \sim \frac{m_{3/2}}{M_\mathrm{P}} \sim \frac{\mu}{M_\mathrm{P}}}$$

Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (2001)



- similar to the Giudice-Masiero Mechanism for the mu problem

Giudice, Masiero (1988)

$$\boxed{\mu \sim \langle \mathscr{W} \rangle / M_\mathrm{P}^2 \sim m_{3/2}}$$

- need a symmetry reason for the absence of these operators before SUSY breaking

Dirac Neutrinos and SUSY Breaking

- Symmetry realization in MSSM: discrete R symmetries, $\mathbb{Z}_M^R$

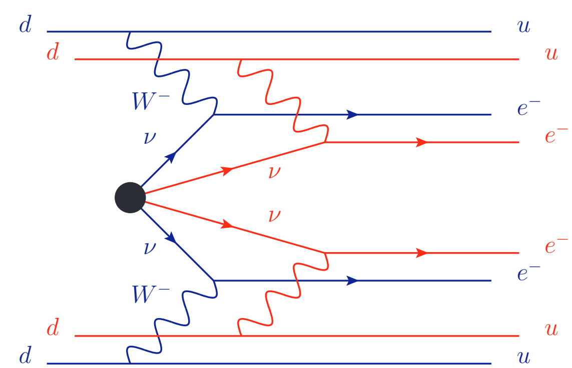
M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)

  - **Dirac neutrinos**, with naturally small masses
  - Δ L = 2 operators forbidden to all orders ⇒ no neutrinoless double beta decay
  - **New signature: lepton number violation ΔL = 4 operators, $(\nu_R)^4$, allowed ⇒ new LNV processes, e.g.**

M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)

    - neutrinoless quadruple beta decay

Heeck, Rodejohann (2013)

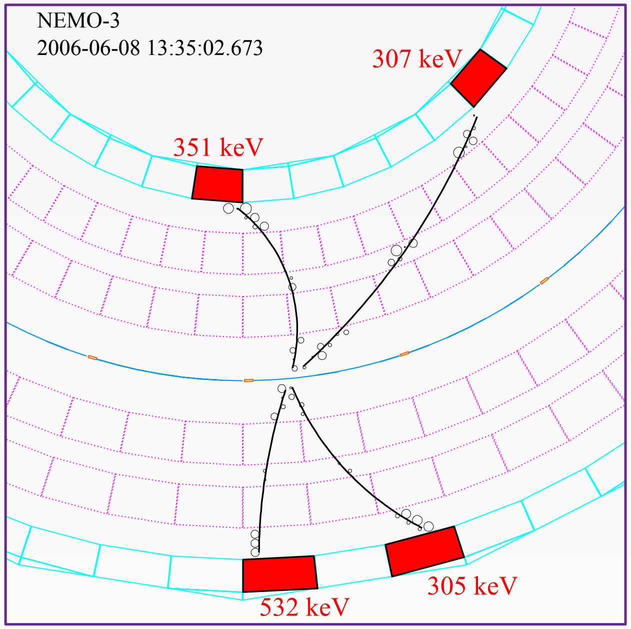


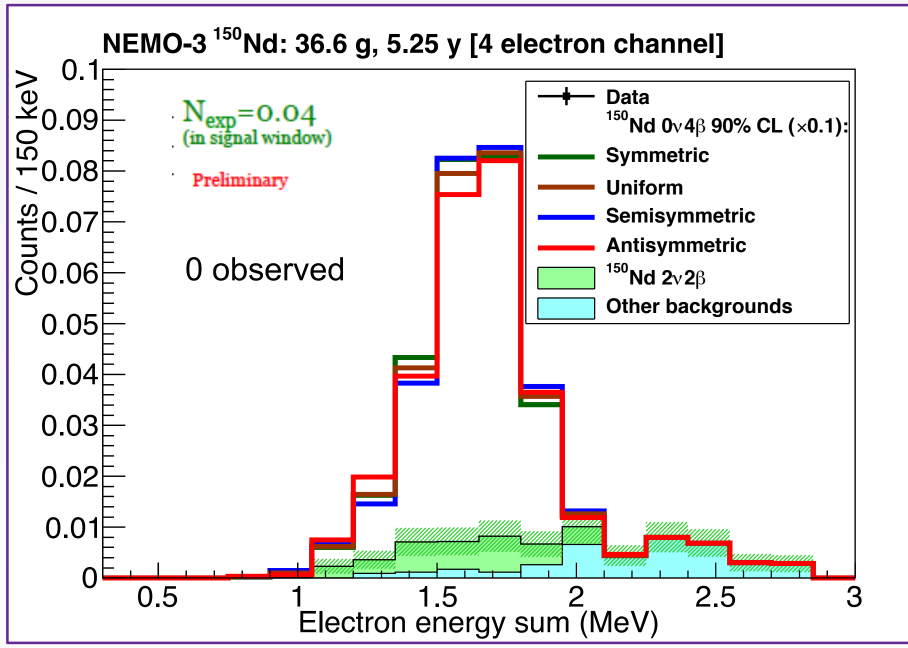
- mu term is naturally small
- dangerous proton decay operators forbidden/suppressed
- can also give dynamical generation of RPV operators with size predicted

M.-C. C., M. Ratz, V. Takhistov (2015)

Quadruple (!) beta decay — 0ν4b

ΔL = 4 BSM physics with Dirac neutrinos





Only possible with full topological reconstruction of all electrons

90%CL limit	Symmetric	Uniform	Semi-symmetric	Anti-symmetric
Observed	3.2×10^{21}y	2.6×10^{21}y	1.7×10^{21}y	1.1×10^{21}y
Sensitivity	3.7×10^{21}y	3.0×10^{21}y	2.0×10^{21}y	1.3×10^{21}y

(combined limits for 3 topologies) Preliminary

NEMO-3 (2017):

$T_{1/2} > (1.1\text{-}3.2) \times 10^{21}$ yrs

Theory expectation:

Heeck, Rodejohann (2013)

$$\frac{\tau_{1/2}^{0\nu4\beta}}{\tau_{1/2}^{2\nu2\beta}} \simeq \left(\frac{Q_{0\nu2\beta}}{Q_{0\nu4\beta}}\right)^{11}\left(\frac{\Lambda^4}{q^{12}G_F^4}\right) \simeq 10^{46}\left(\frac{\Lambda}{\mathrm{TeV}}\right)^4.$$

Flavor structure

anarchy

vs

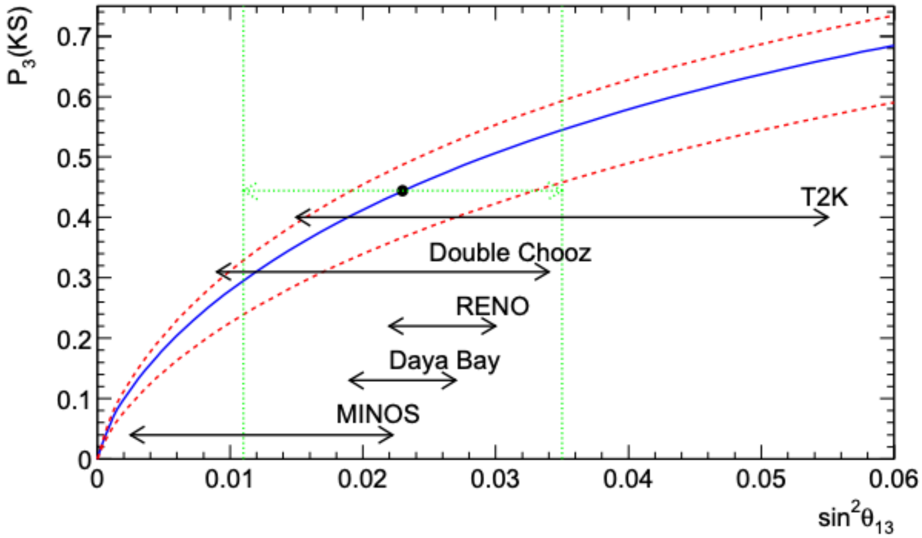
symmetry

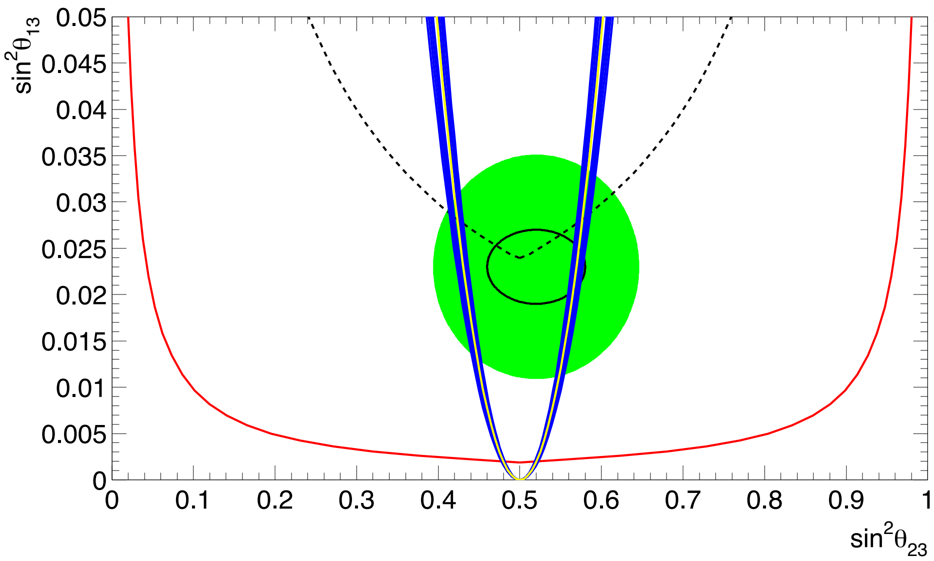
Anarchy

Hall, Murayama, Weiner (2000);
de Gouvea, Murayama (2003)

- there are no parametrically small numbers
- large mixing angle, near mass degeneracy statistically preferred

de Gouvea, Murayama (2012)



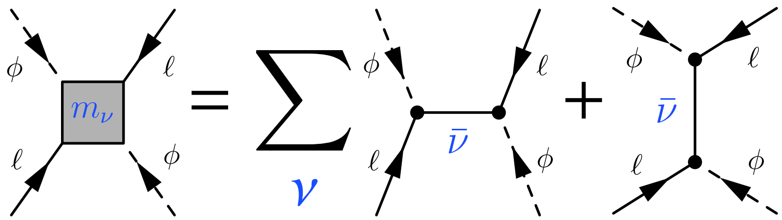


- UV theory prediction can resemble anarchy
  - warped extra dimensions
  - heterotic string theory

Expectations from Heterotic String Theories

- heterotic string models: O(100) RH neutrinos

Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez, Ratz (2007)

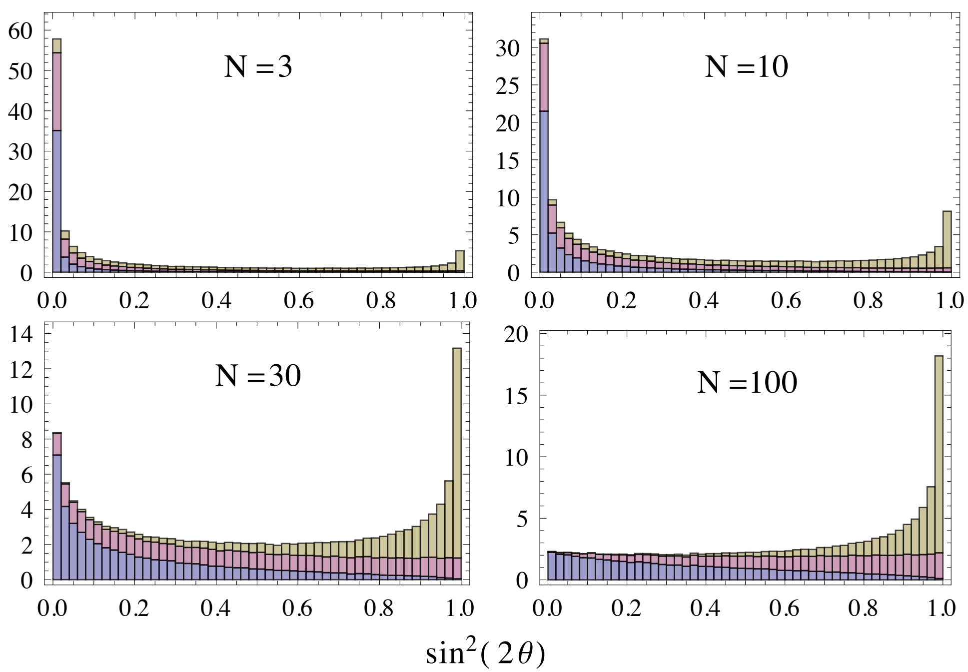


$$m_\nu \sim \frac{v^2}{M_*}$$

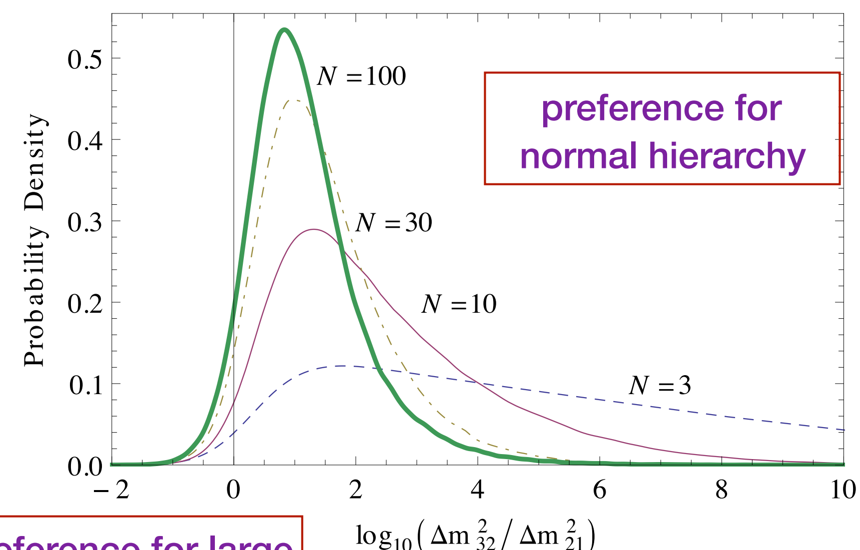
$$M_* \sim \frac{M_{\mathrm{GUT}}}{10...100}$$

- statistical expectations with large N (= # of RH neutrinos)

Feldstein, Klemm (2012)



preference for large mixing angles



preference for normal hierarchy

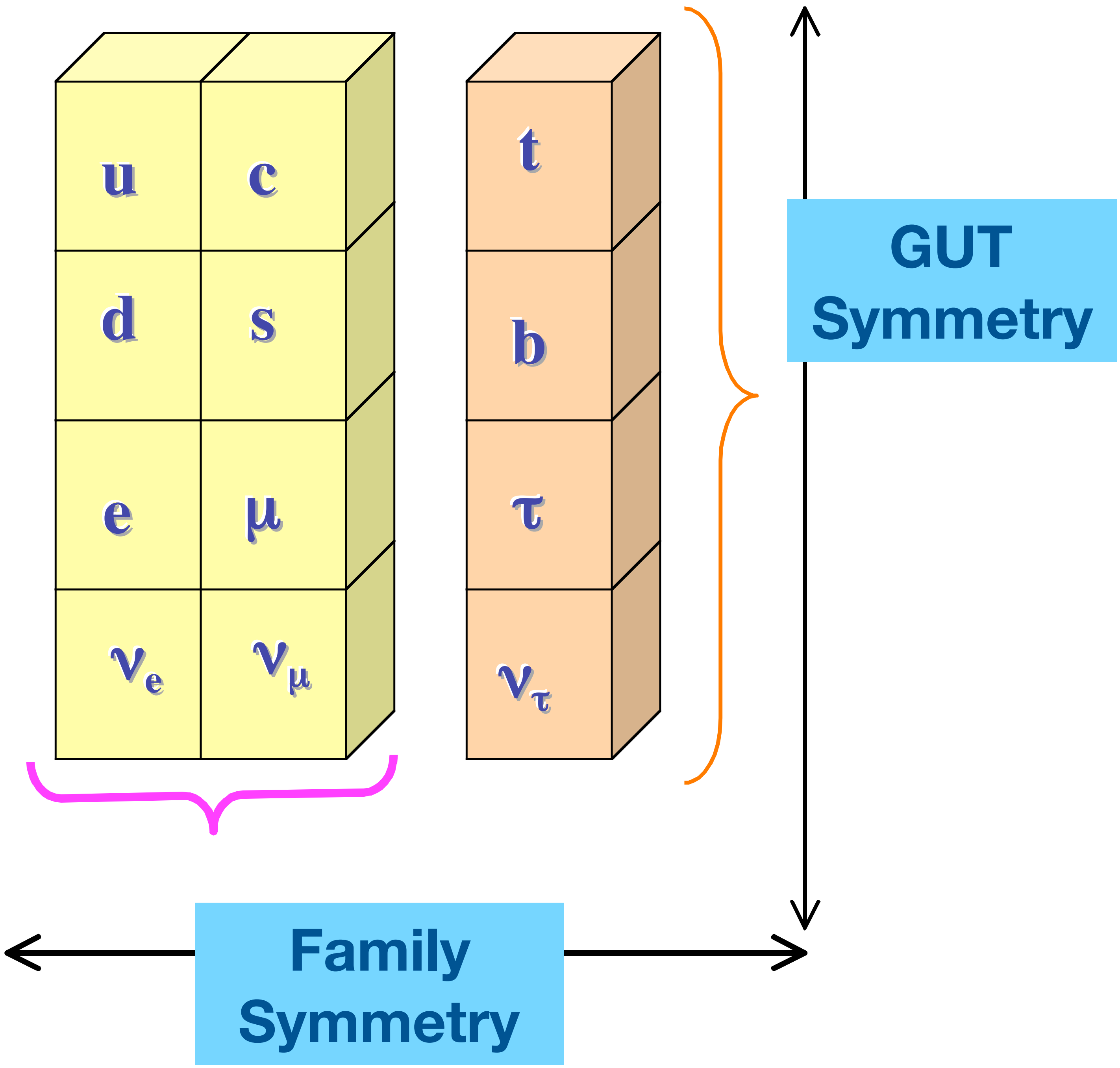
Symmetry Relations

Grand Unified Theories: GUT symmetry

Quarks ↔ Leptons

Family Symmetry:

e-family ↔ muon-family ↔ tau-family



u
c
t
d
s
b
e
μ
τ
ν_e
ν_μ
ν_τ
GUT Symmetry
Family Symmetry

Symmetry Relations

Symmetry ⇒ relations among parameters

⇒ reduction in number of fundamental parameters

Symmetry Relations

Symmetry ⇒ relations among parameters ⇒ reduction in number of fundamental parameters

Symmetry ⇒ experimentally testable correlations among physical observables

Symmetry Relations

Symmetry ⇒ experimentally testable correlations among physical observables

mass hierarchy

CP phase

$0\nu\beta\beta$

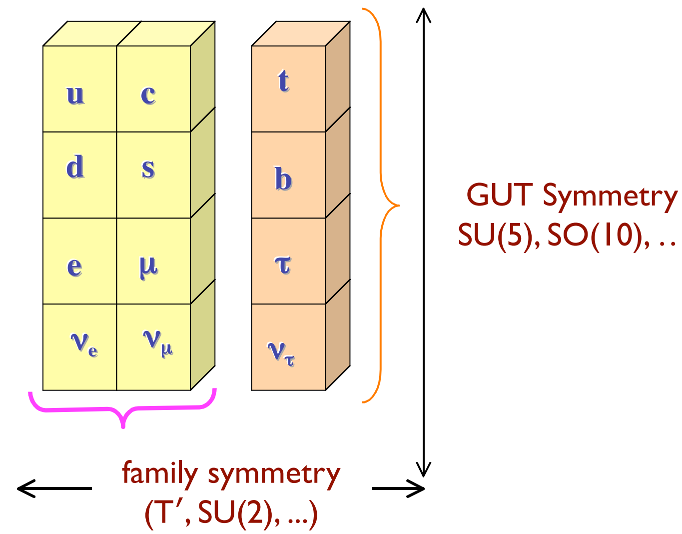
cLFV

mixing angles

Testing correlations ⇒ Precision

Origin of Flavor Mixing and Mass Hierarchies

- several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T' (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
- Extra dimensional origin
- Modular symmetry



Tri-bimaximal Neutrino Mixing

- Latest Global Fit (3σ)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ $[\theta^{lep}_{23} \sim 49.7°]$

Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni, Schwetz, 1811.05487

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ $[\theta^{lep}_{12} \sim 33.8°]$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ $[\theta^{lep}_{13} \sim 8.61°]$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\text{TBM}} = 1/3$$

$$\sin\theta_{13,\text{TBM}} = 0.$$

- Leading Order: TBM (from symmetry) + higher order corrections/contributions
- More importantly, corrections to the kinetic terms

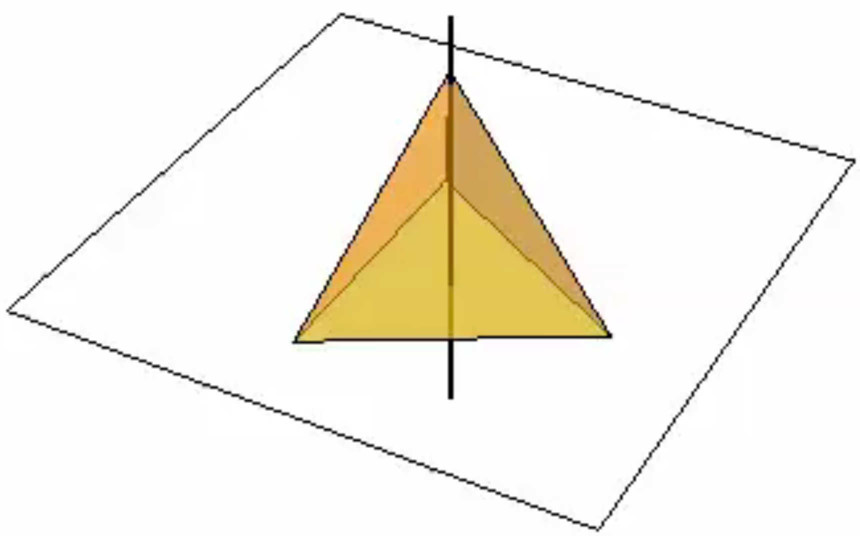
Leurer, Nir, Seiberg ('93);
Dudas, Pokorski, Savoy ('95)

  - small for quarks
  - sizable in discrete symmetry models for leptons M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)

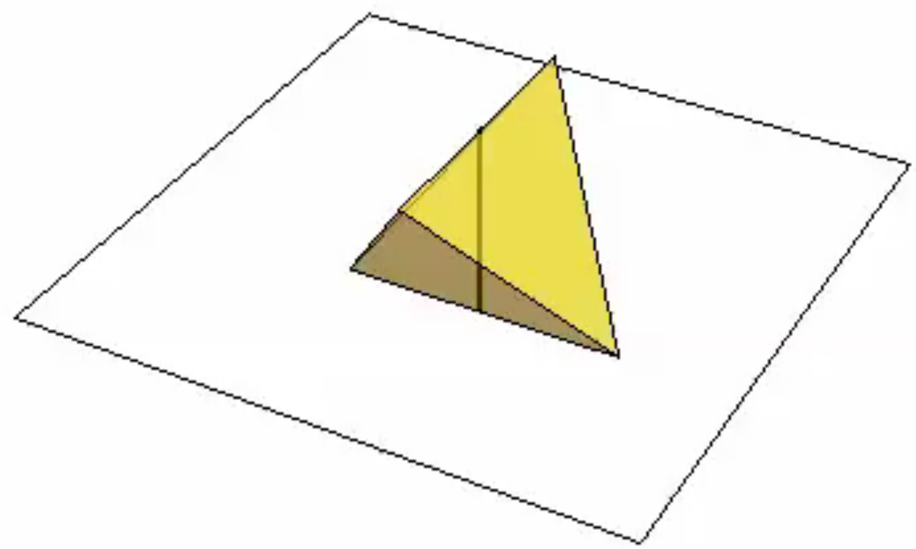
Example: Tetrahedral Group A_4

- Smallest group giving rise to tri-bimaximal neutrino mixing: **tetrahedral group A_4**

T: (1234) → (2314)



S: (1234) →(4321)



Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

2 free parameters

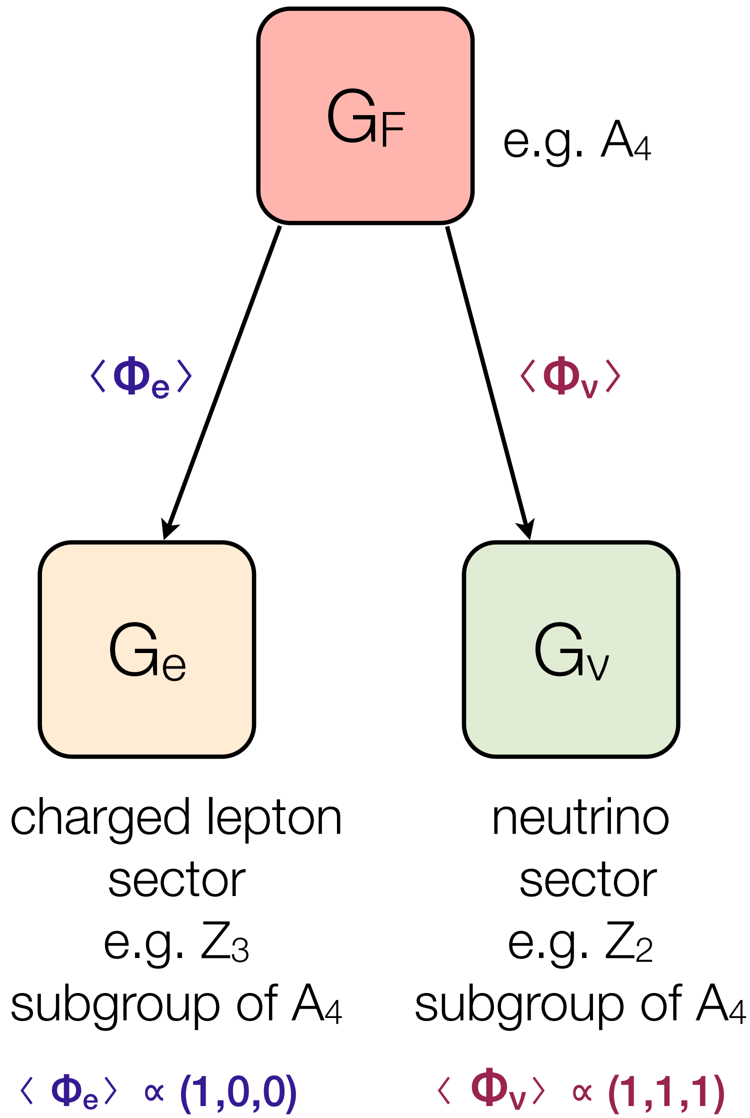
relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

- 2 independent parameters for 3 masses ⇒ 1 relation

General Structure



Example: SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T´: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- Symmetries ⇒ **correlations among quark and lepton mixing parameters**

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2}}$$

CG's of SU(5) & T´

no free parameters!

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

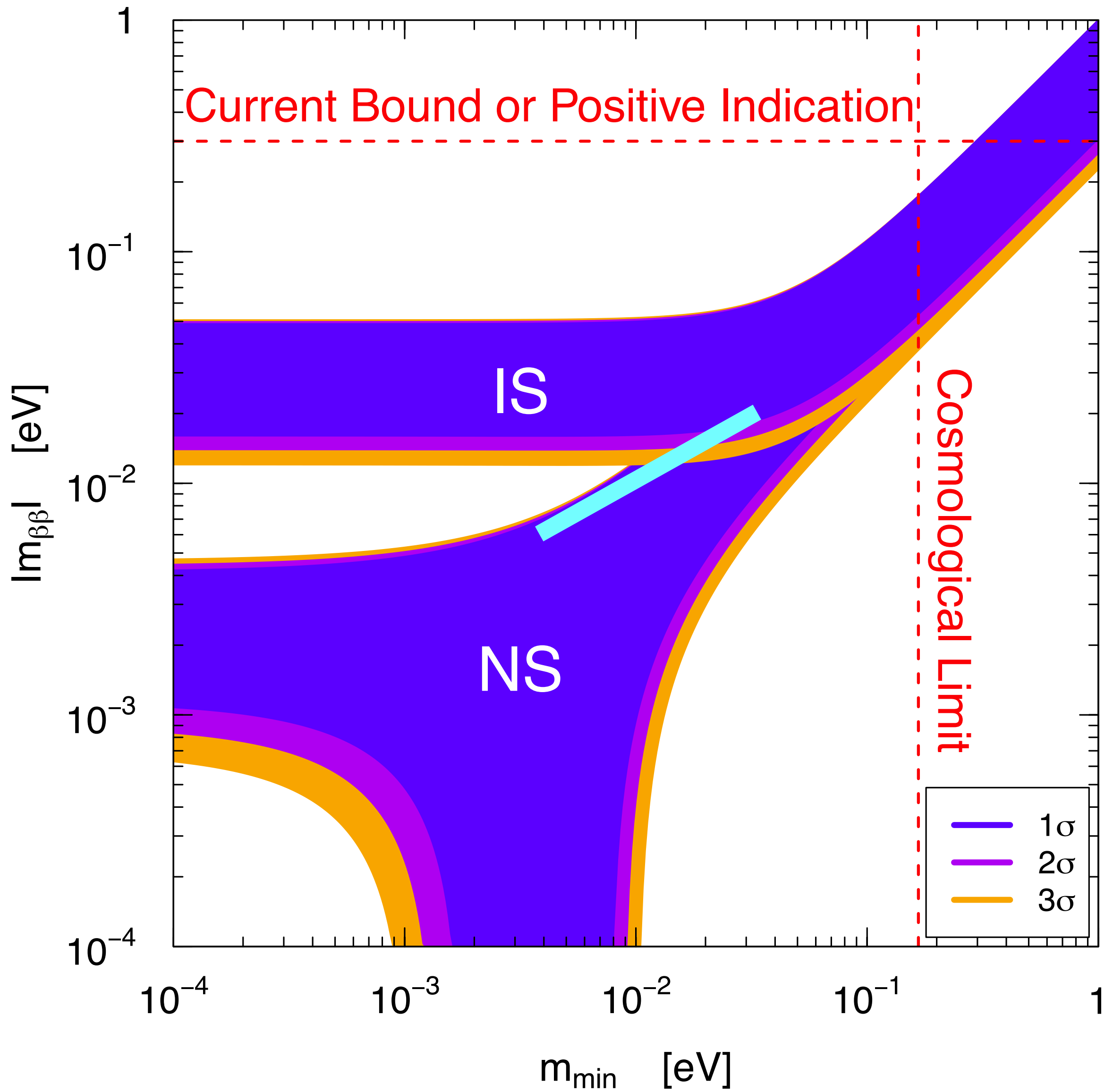
neutrino solar mixing

1/2

quark Cabibbo mixing

leptonic CP phase

Neutrinoless Double Beta Decay



our model prediction

sum rule among masses
⇒ small predicted region

[Plot taken from C. Giunti, LIONeutrino2012]

Symmetry Relations

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	49.7°	40.9° - 52.2°
θ^e_{12}	33.82°	31.61° - 36.27°
θ^e_{13}	8.61°	8.22° -8.98°

- QLC-I $\theta_c + \theta_{sol} \cong 45^\circ$ Raidal, '04; Smirnov, Minakata, '04

  (BM) $\boxed{\theta^q_{23} + \theta^e_{23} \cong 45^\circ}$ ☜ **slight inconsistent**

- QLC-II $\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_c / 2) * \cos\delta_e$ Ferrandis, Pakvasa; Dutta, Mimura; M.-C.C., Mahanthappa

  (TBM) $\boxed{\theta^e_{13} \cong \theta_c / 3\sqrt{2}}$ ☜ **Too small**

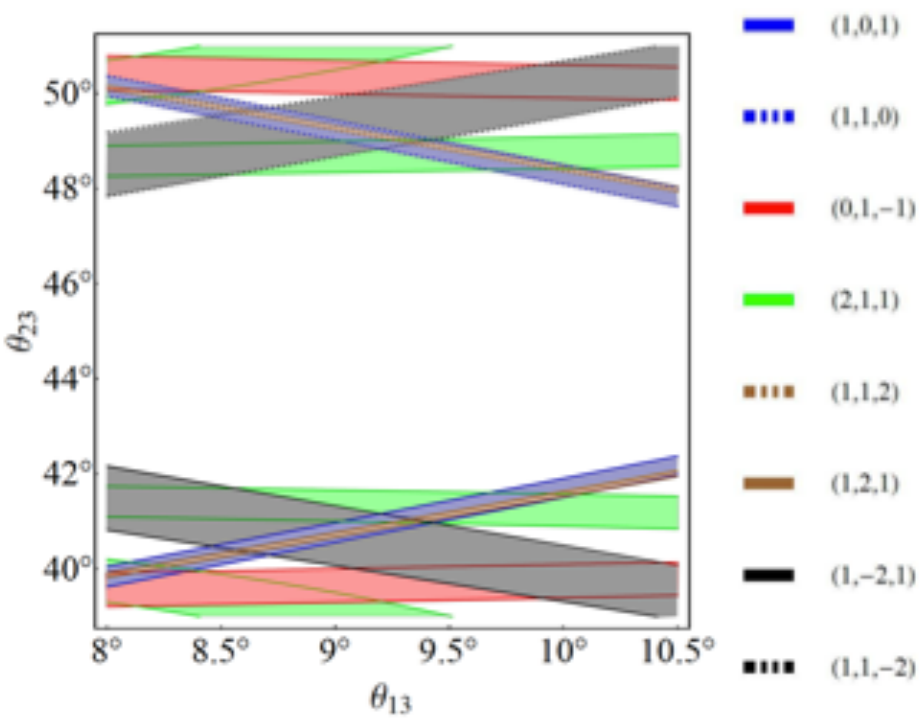
- testing symmetry relations: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector

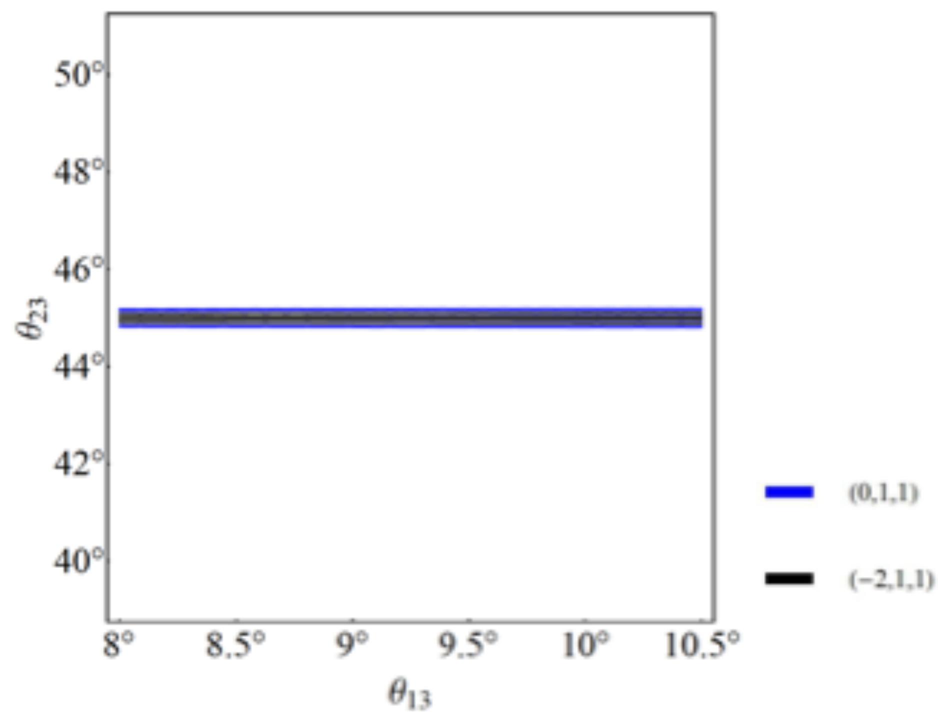
“Large” Deviations from TBM in A_4

M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)

- Different A4 breaking patterns:



normal



inverted

deviations correlated

non-maximal θ_{23} ⇨ normal hierarchy

mass ordering ⇨ symmetry breaking patterns

Another Example: A_5

P. Ballett, S. Pascoli, J. Turner (2015)

- Correlations among different mixing parameters

G_e	θ_{12}	θ_{23}	$\sin\alpha_{ji}$	δ
$\mathbb{Z}_3$	$35.27^\circ + 10.13^\circ\, r^2$	45°	0	90°
				270°
$\mathbb{Z}_5$	$31.72^\circ + 8.85^\circ\, r^2$	$45^\circ \pm 25.04^\circ\, r$	0	0°
				180°
		45°	0	90°
				270°
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$36.00^\circ - 34.78^\circ\, r^2$	$31.72^\circ + 55.76^\circ\, r$	0	0°
				180°
		$58.28^\circ - 55.76^\circ\, r$	0	0°
				180°

TABLE I. Numerical predictions for the correlations found in this paper. The dimensionless parameter $r \equiv \sqrt{2}\sin\theta_{13}$ is constrained by global data to lie in the interval $0.19 \lesssim r \lesssim 0.22$ at 3σ. The predictions for θ_{12} and θ_{23} shown here ne-

CP Violation

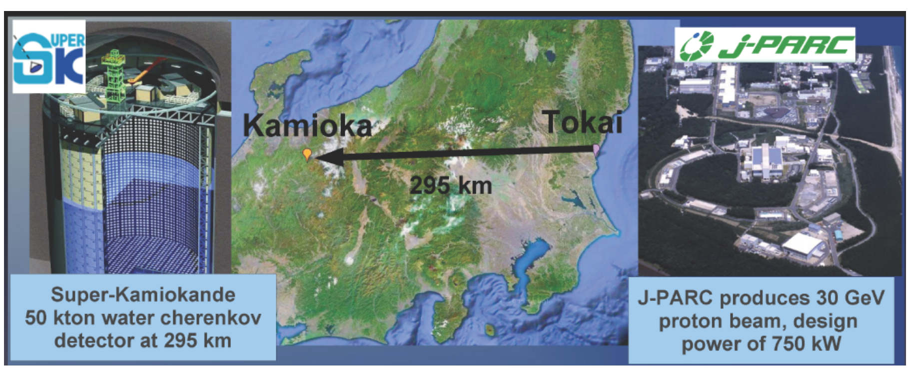
CP Violation in Neutrino Oscillation

- With leptonic Dirac CP phase $\delta \neq 0$ ➜ leptonic CP violation
- Predict different transition probabilities for neutrinos and antineutrinos

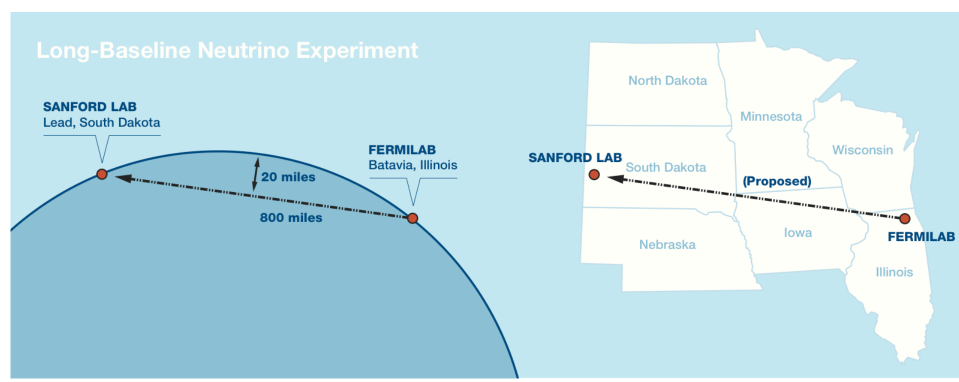
$$P\,(\nu_\alpha \rightarrow \nu_\beta) \neq P\,(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta)$$

- One of the major scientific goals at current and planned neutrino experiments

T2K



DUNE/LBNF

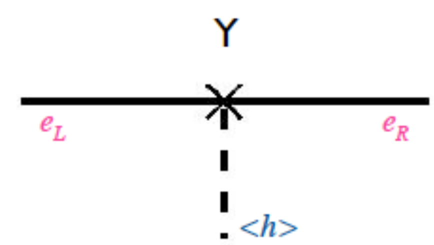


Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

M.-C.C., K.T. Mahanthappa, Phys. Lett. B681, 444 (2009)

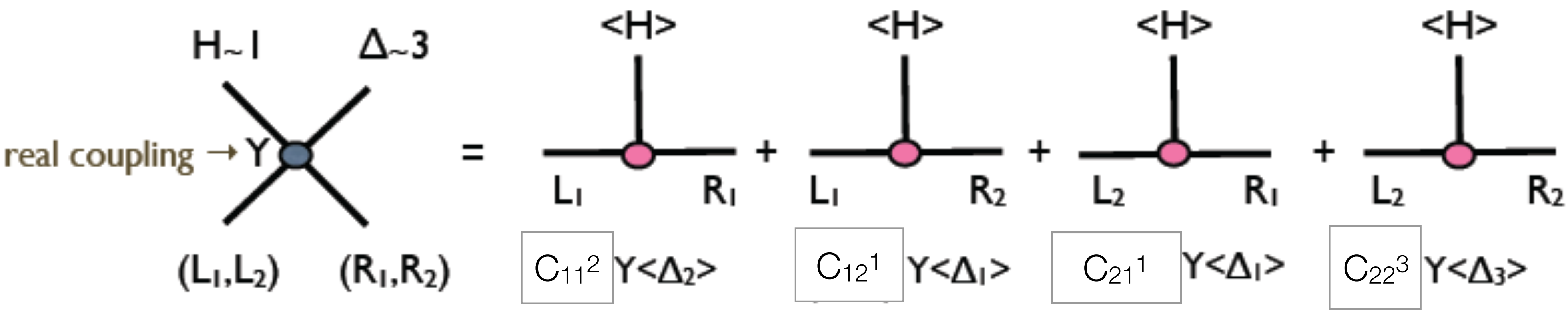
CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass hierarchy)

Group Theoretical Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

Basic idea

Discrete symmetry ***G***



$C_{ij}{}^{k}$: complex CG coefficients of ***G***

- if Z_3 symmetric $\Rightarrow \langle\Delta_1\rangle = \langle\Delta_2\rangle = \langle\Delta_3\rangle \equiv \langle\Delta\rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$$\begin{array}{cc} & (\; L_1 \quad\quad L_2 \;) \\ M = & \begin{pmatrix} C_{11}{}^{2} & C_{21}{}^{1} \\ C_{12}{}^{1} & C_{22}{}^{3} \end{pmatrix} Y\langle\Delta\rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \end{array}$$

CP Transformation

- Canonical CP transformation

$$\phi(x) \overset{C\mathcal{P}}{\longmapsto} \eta_{C\mathcal{P}} \ \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

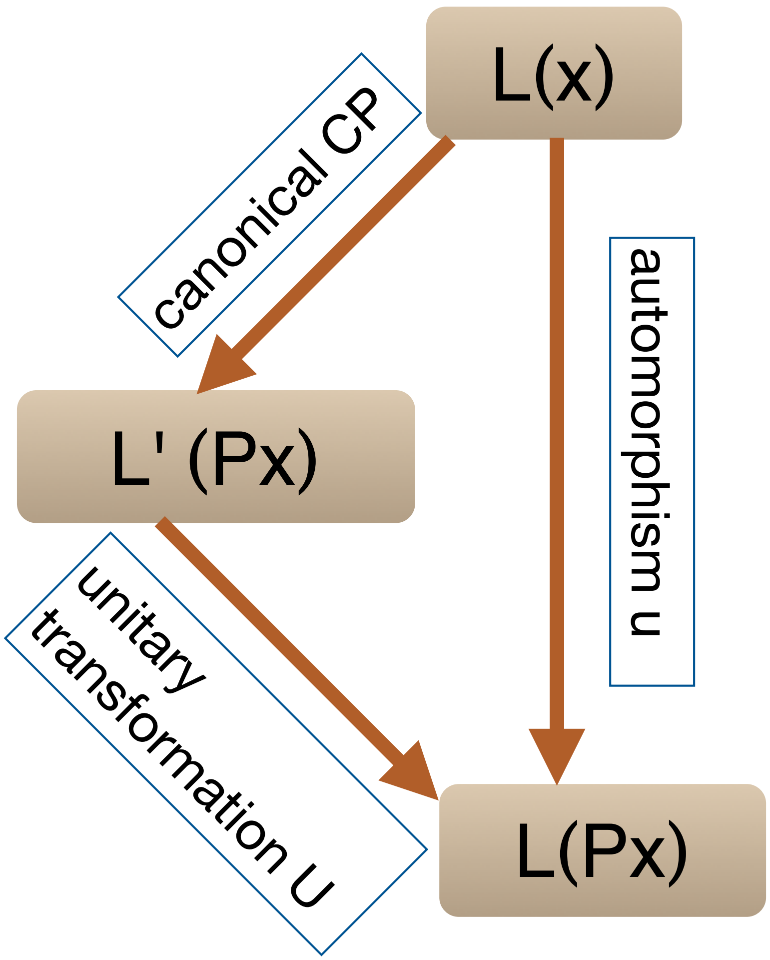
$$\Phi(x) \overset{\widetilde{C\mathcal{P}}}{\longmapsto} U_{\mathrm{CP}} \, \Phi^*(\mathcal{P} \ x)$$

unitary matrix

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ➪ G and physical CP transformations do not commute



$$\Phi(x) \xmapsto{\widetilde{CP}} U_{\mathrm{CP}}\,\Phi^*(\,\mathcal{P}\;x)$$

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a **class-inverting, involutory** automorphism of G

➪ non-existence of such automorphism in certain groups

➪ calculable physical CP violation in generic setting

examples: T_7, $\Delta(27)$, ...

Novel Origin of CP (Time Reversal) Violation

M.-C.C, M. Fallbacher,
K.T. Mahanthappa, M. Ratz,
A. Trautner, NPB (2014)

complex CGs ⇨ CP symmetry cannot be defined for certain groups

CP Violation from Group Theory!

Sterile Neutrinos

- All previous discussions applicable to sterile neutrinos also
- Tension with standard cosmology: sterile neutrinos as test of standard cosmology
- Tension with non-unitarity
- Reversed spectrum for neutrino less double beta decay

Talks (Tu) by Stefan Schoppmann, Carlos Arguelles

MaVaNs

R. Fardon, A. Nelson, N. Weiner (2003)

- Exotic scalar field A (acceleron) with *logarithmic*, *temperature*-dependent potential
    - Dark Energy density: $\Lambda^4 \sim (10^{-2.5}\ \mathrm{eV})^4 \sim (\Delta m^2)^2$
- A-dependent “heavy” Majorana neutrino masses

$$m_N(A) = m_0 + \kappa A$$

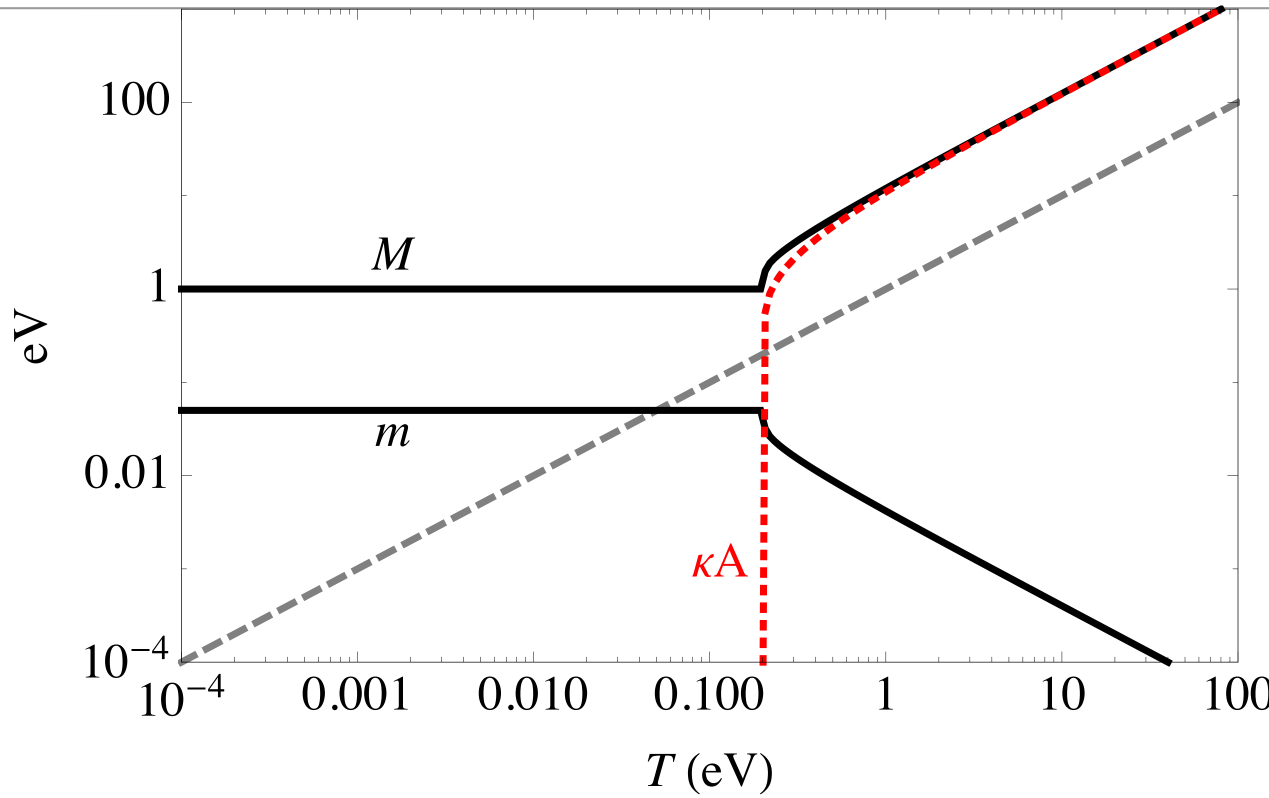
$$m_\nu(A) = m_D^2 / (m_0 + \kappa A)$$

T> 0.1 eV: A $\propto$ T
T < 0.1 eV: A → 0

- Active-Sterile mixing ~ $(m_{active} / M_{sterile})^{1/2}$

MaVaNs

A. Ghalsasi, D. McKeen, A. Nelson (2016)



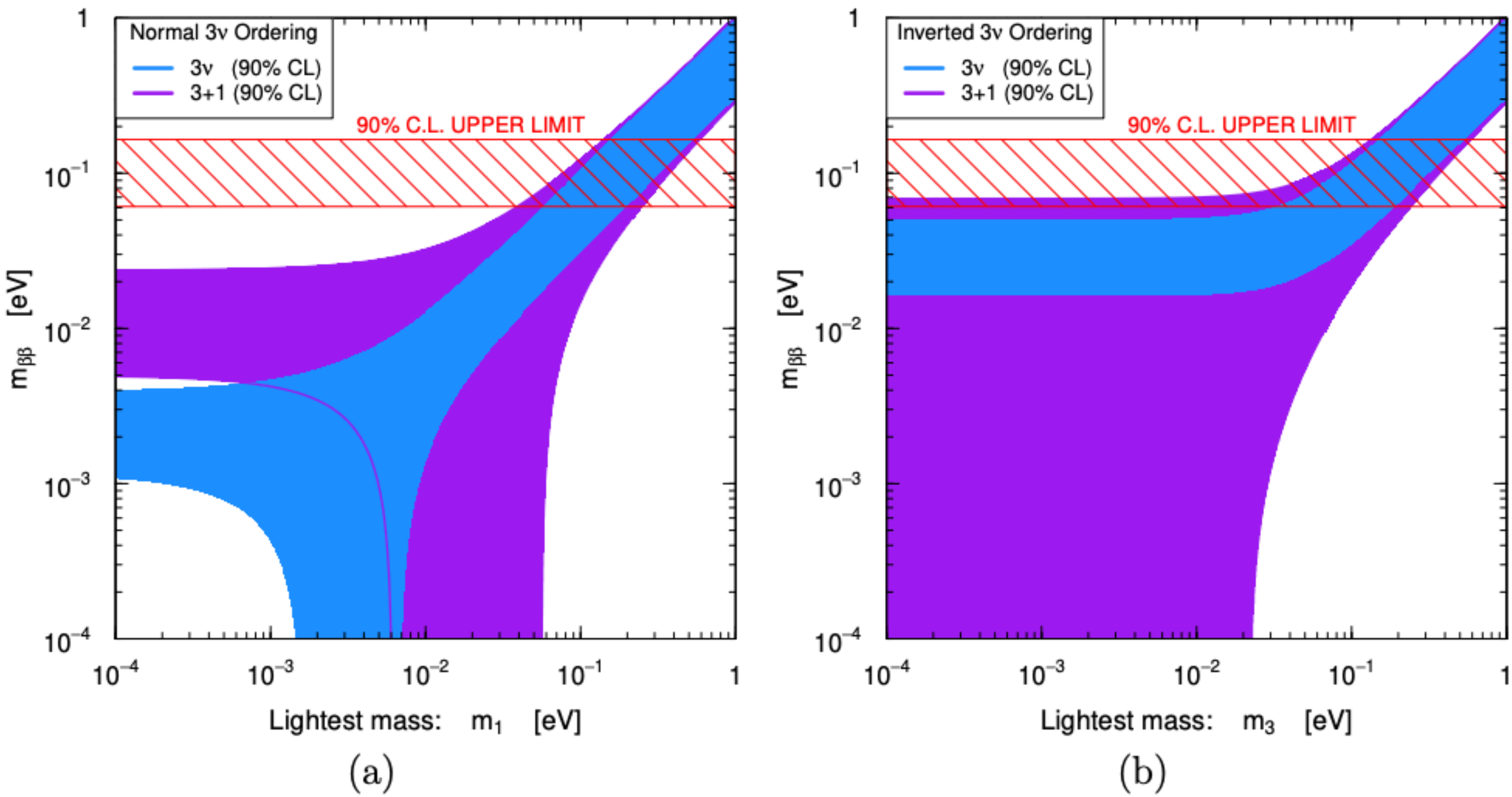
Terrestrial Experiments:
sizable active-sterile mixing

Early Universe (T>0.1 eV):
small active-sterile mixing

Consistent with Cosmology; Bonus: DE

Neutrinoless Double Beta Decay

$$|m_{\beta\beta}| = \left| \sum_{k=1}^{4} U_{ek}^2 \, m_k \right|$$



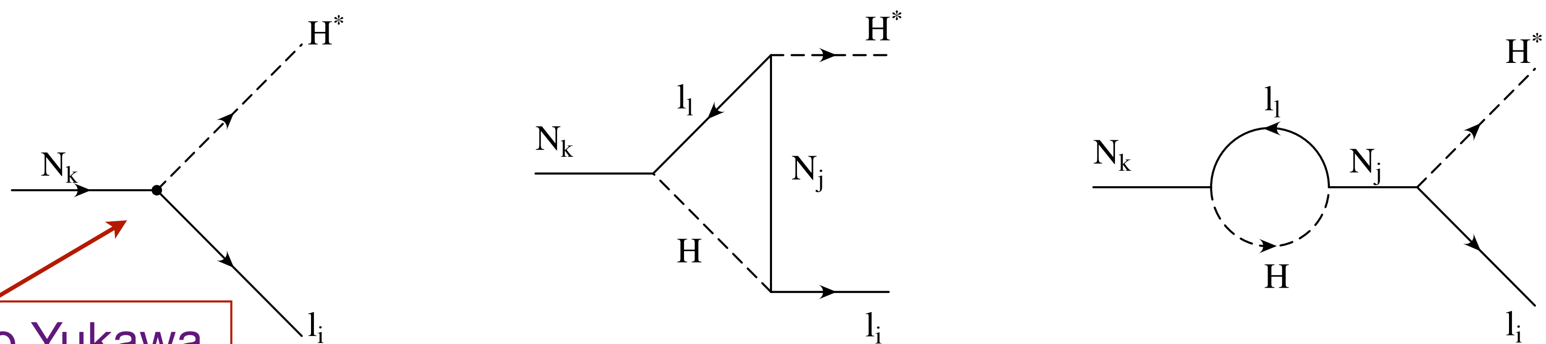
C. Giunti, T. Lasserre (2019)

Cosmological Connections

Standard Leptogenesis

Fukugita, Yanagida, 1986

- RH heavy neutrino decay:
  - quantum interference of tree-level & one-loop diagrams ⇒ primordial lepton number asymmetry **ΔL**



Neutrino Yukawa coupling

leptons antileptons

$$\epsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}$$

Leptonic CP violation ⇒ **ΔL** ∝ $\left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]$ ≠ 0

Dirac Leptogenesis

K. Dick, M. Lindner, M. Ratz, D. Wright, 2000;
H. Murayama, A. Pierce, 2002

- Leptogenesis possible even when neutrinos are Dirac particles **(no ΔL = 2 violation)**

- Characteristics of Sphaleron effects:
  - only left-handed fields couple to sphalerons
  - sphalerons change (B+L) but not (B-L)
  - sphaleron effects in equilibrium for T > Tew

late time LR equilibration of neutrinos making Dirac leptogenesis possible with primordial **ΔL = 0**

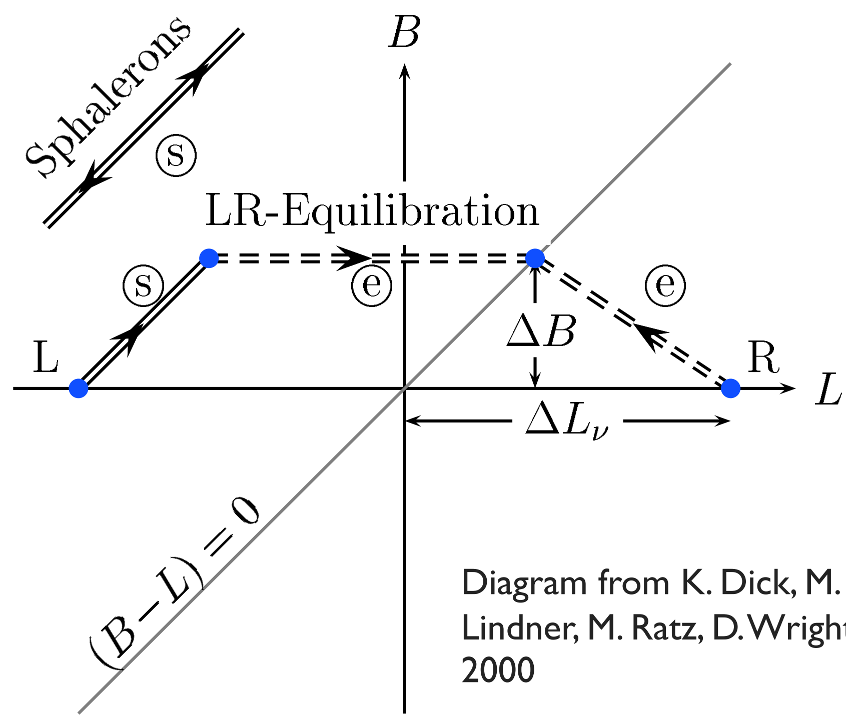


Diagram from K. Dick, M. Lindner, M. Ratz, D. Wright, 2000

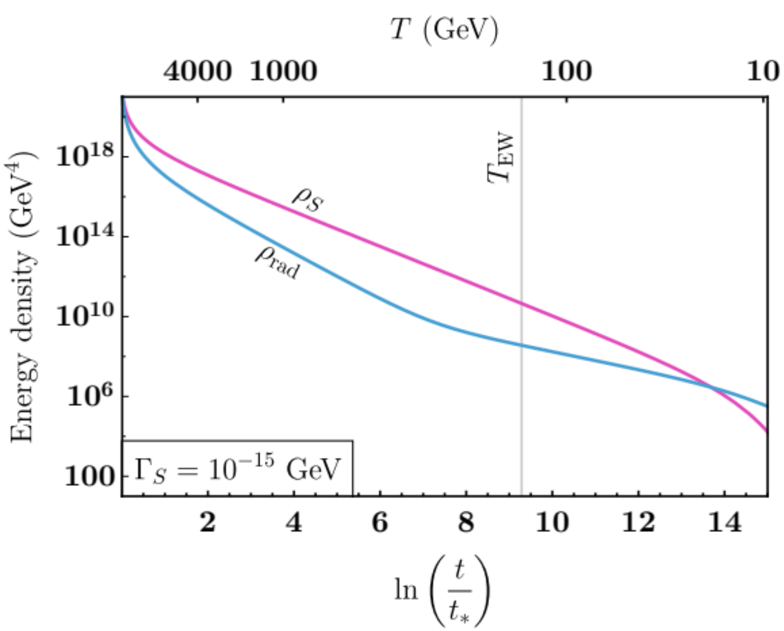
Baryogenesis through Flavon Decay

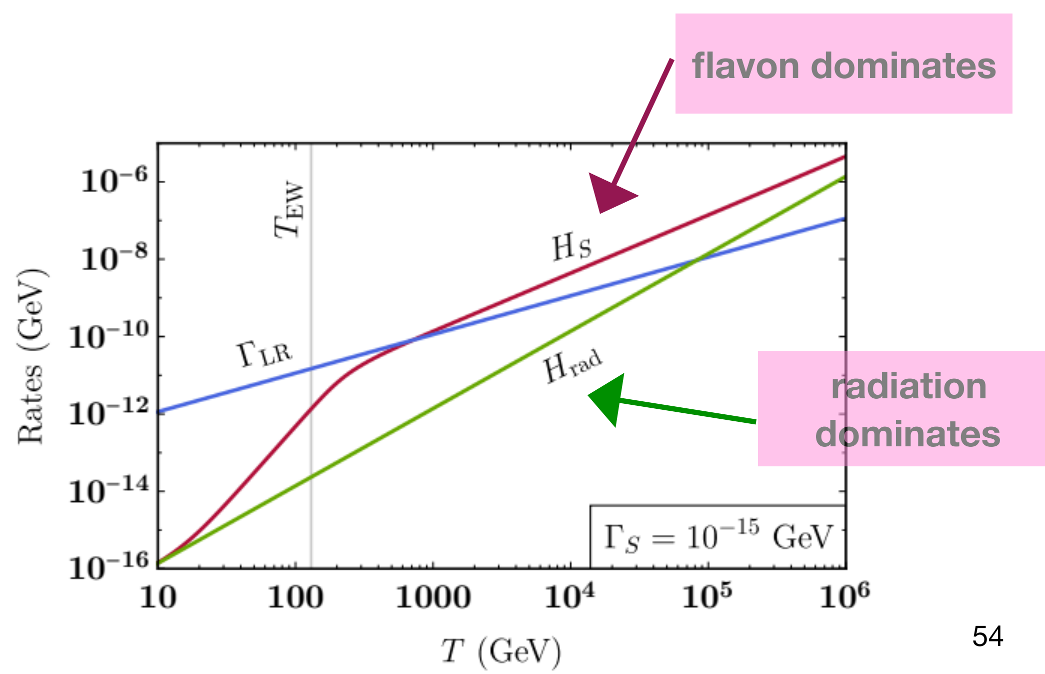
M.-C.C, S. Ipek, M. Ratz (2019)

- **Radiation dominates:** LR equilibration for electrons @ T~10^5 GeV
- Froggatt-Nielsen Models for flavor structure and mass hierarchy ⇒ Flavon
- Asymmetry due to flavon decay (Δ L = 0)

$$S \rightarrow \bar{\ell}_{\mathrm{L}} + \phi + e_{\mathrm{R}} \qquad\qquad S^* \rightarrow \ell_{\mathrm{L}} + \phi^* + \bar{e}_{\mathrm{R}}$$

- **Flavon dominates**: Hubble increases so that RH electrons do not equilibrate before EWPT





Outlook

Summary

- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM

- Symmetries:
  - can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry $\Rightarrow$ predictive power
  - Symmetries $\Rightarrow$ Correlations, Correlations, Correlations!!!

- **Dirac vs Majorana?** - should remain open minded!
  - naturally light Dirac neutrinos from discrete R-symmetry
  - suppressed nucleon decays and naturally small mu term

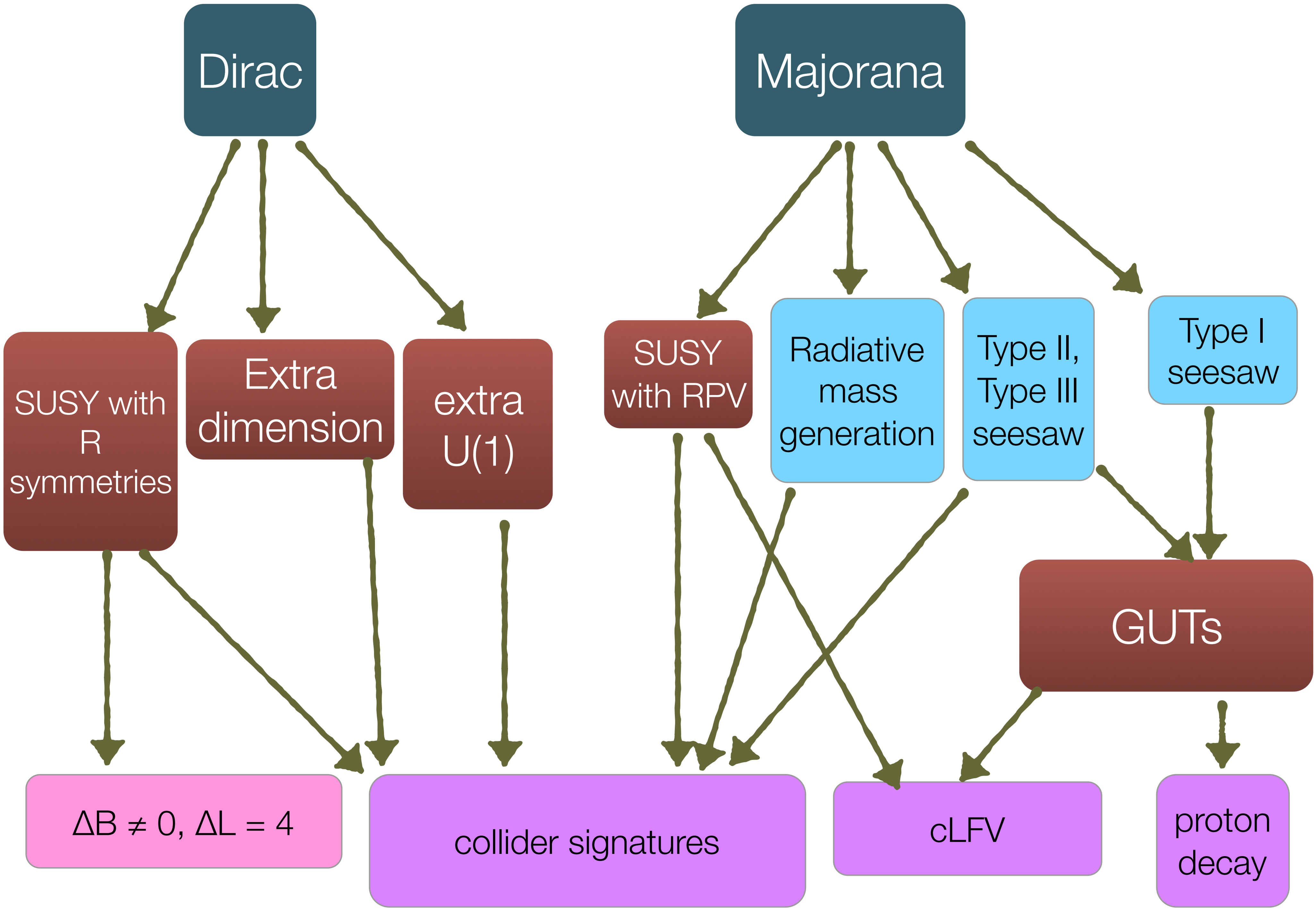
Summary

- **Discrete Groups (of Type I) affords a Novel origin of CP violation:**
  - **Complex CGs ⇒ Group Theoretical Origin of CP Violation**
- **NOT all outer automorphisms correspond to physical CP transformations**
- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \qquad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

class inverting, involutory automorphisms $\Leftrightarrow$ **physical CP transformations**


Dirac
Majorana
SUSY with R symmetries
Extra dimension
extra U(1)
SUSY with RPV
Radiative mass generation
Type II, Type III seesaw
Type I seesaw
GUTs
ΔB ≠ 0, ΔL = 4
collider signatures
cLFV
proton decay

Discussions

1. question 1
2. question 2
3. question 3

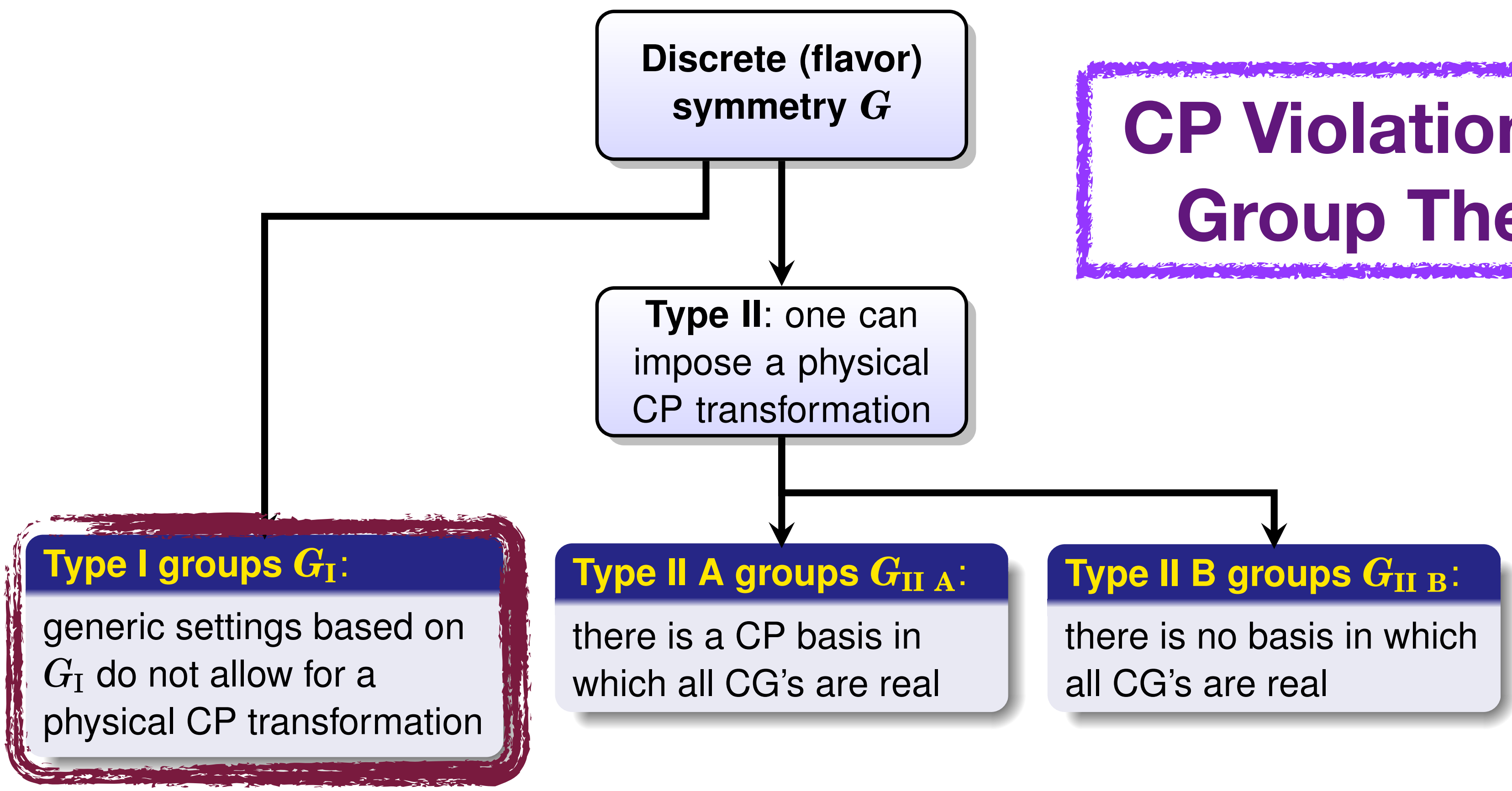
Backup Slides

Group Theoretical Origin of CP Violation: a toy model

Novel Origin of CP (Time Reversal) Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**



CP Violation from Group Theory!

Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\boxed{\Delta(27)}$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	$\boxed{(27,3)}$	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	$\boxed{\mathrm{T}'}$	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	$\boxed{(24,3)}$	(24,12)	(60,5)

- Type IIB

group	$\boxed{\Sigma(72)}$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	$\boxed{(72,41)}$	(144,120)

Example for a type I group:

$$\mathbf{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H^{ij}_\Psi\, Y\, \overline{\Psi}_i \Psi_j + H^{ij}_\Sigma\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\, \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3}$

“flavor” structures determined by (complex) CG coefficients

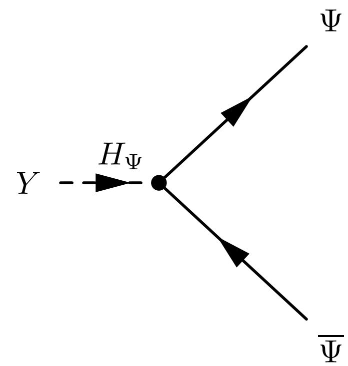
arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

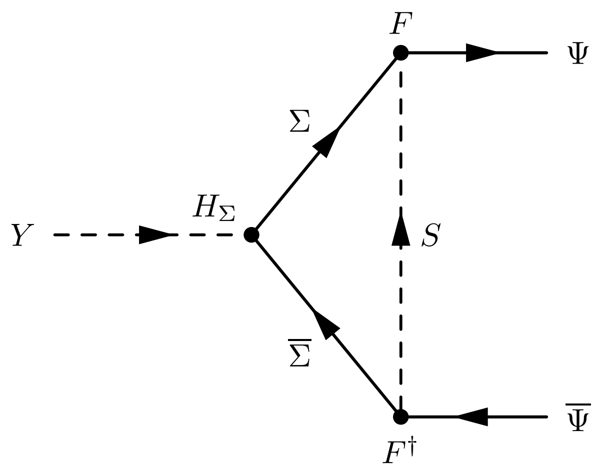
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

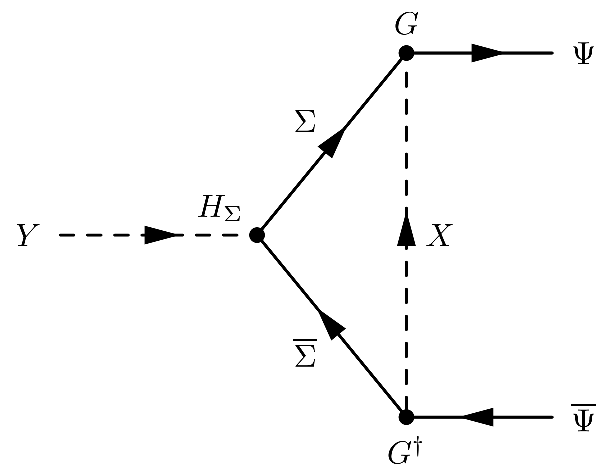
- Particle decay $Y \longrightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y\to\overline{\Psi}\Psi)-\Gamma(Y^*\to\overline{\Psi}\Psi)}{\Gamma(Y\to\overline{\Psi}\Psi)+\Gamma(Y^*\to\overline{\Psi}\Psi)}$$

$$\propto \quad \mathrm{Im}\,[I_S]\;\mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\,H_\Psi\,F\,H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\;\mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\,H_\Psi\,G\,H_\Sigma^\dagger\right)\right]$$

$$= \quad |f|^2\;\mathrm{Im}\,[I_S]\;\mathrm{Im}\,[h_\Psi\,h_\Sigma^*] + |g|^2\;\mathrm{Im}\,[I_X]\;\mathrm{Im}\,[\omega\,h_\Psi\,h_\Sigma^*]\ .$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2\ \mathrm{Im}\left[I_S\right]\ \mathrm{Im}\left[h_\Psi\, h_\Sigma^*\right] + |g|^2\ \mathrm{Im}\left[I_X\right]\ \mathrm{Im}\left[\omega\, h_\Psi\, h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi\, h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by an outer automorphism (i.e. GCP) of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$$\Delta(27) \subset \mathrm{SG}(54,5): \begin{cases} (X, Z) & : \text{ doublet} \\ (\Psi, \Sigma^C) & : \text{ hexaplet} \\ \phi & : \text{ non–trivial 1–dim. representation} \end{cases}$$

- non–trivial $\langle\phi\rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$

  Type IIA → Type I

- allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\mathrm{toy}} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}} \langle\phi\rangle \left(|X|^2 - |Z|^2\right) + \text{h.c.}\right]$

  CG coefficient of $\mathrm{SG}(54,5)$

- CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2 \, |h_\Psi|^2 \, \mathrm{Im}\left[\,\omega\,\right] \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

CP Transformation

- Canonical CP transformation

$$\phi(x) \xrightarrow{\mathcal{CP}} \eta_{\mathcal{CP}} \; \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \xrightarrow{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}} \, \Phi^*(\mathcal{P} \; x)$$

unitary matrix

Generalized CP Transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987)

- setting w/ discrete symmetry G **G and CP transformations do not commute**
- generalized CP transformation Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)
- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \xmapsto{\widetilde{\mathcal{CP}}} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \xmapsto{\widetilde{\mathcal{CP}}} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

The Bickerstaff-Damhus automorphism (BDA)

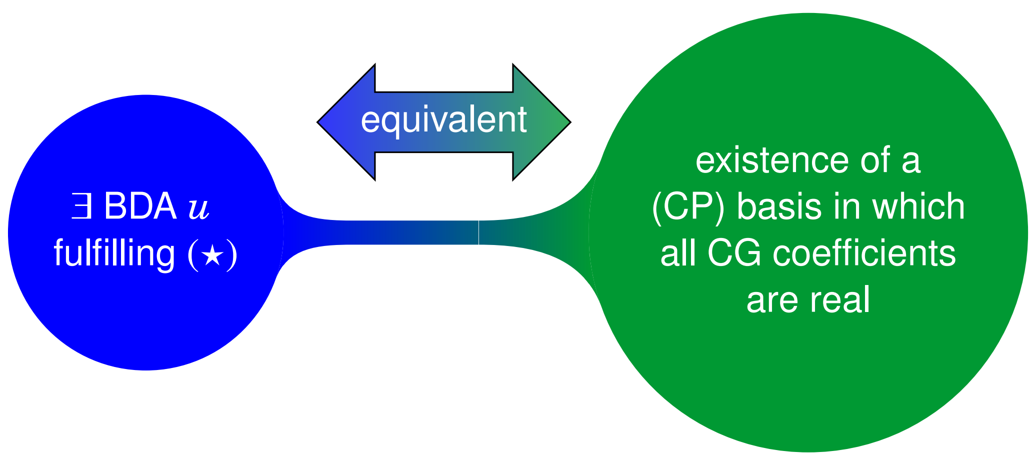
- Bickerstaff-Damhus automorphism (BDA) $\boldsymbol{u}$

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(u(g)) \;=\; U_{\boldsymbol{r}_i}\;\rho_{\boldsymbol{r}_i}(g)^*\,U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i \qquad (\,\star\,)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|}\sum_{g\in G}\mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

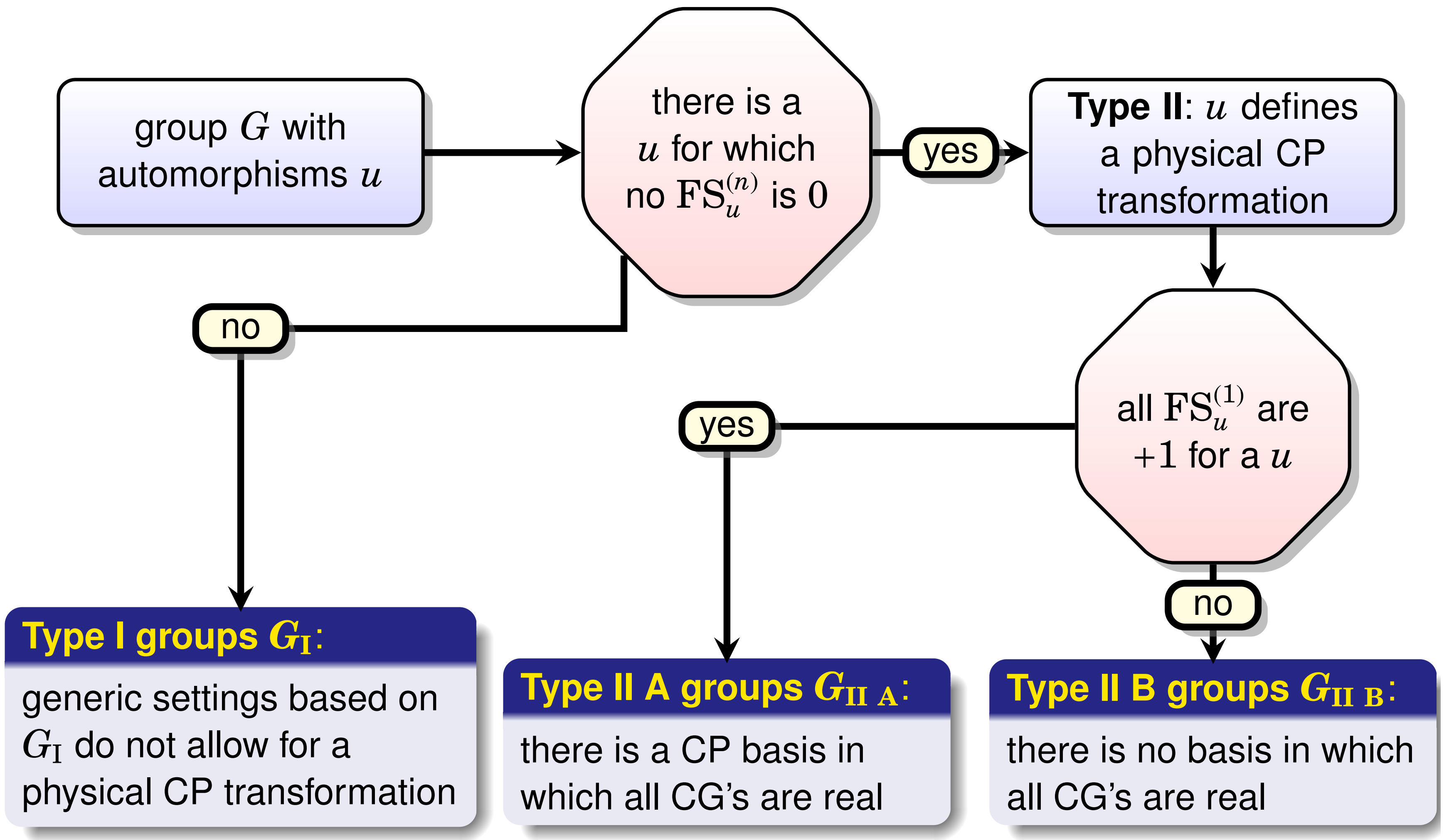
Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|}\sum_{g\in G}\left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta}\left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)



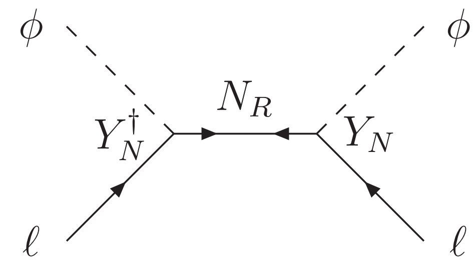
Low Scale Seesaw Scenarios

- New particles:
  - Type I seesaw: generally decouple from collider experiments
  - Type II seesaw: $\Delta^{++} \to e^+ e^+ \, , \, \mu^+ \mu^+ \, , \, \tau^+ \tau^+$
  - Type III seesaw: observable displaced vertex   Franceschino, Hambye, Strumia,2008
  - Inverse seesaw: non-unitarity effects
  - Radiative mass generation: model dependent - singly/doubly charged SU(2) singlet, even colored scalars in loops
- New interactions:
  - LR symmetric model: W_R
  - R parity violation: $\tan^2 \theta_{\mathrm{atm}} \simeq \frac{BR(\tilde{\chi}_1^0 \to \mu^\pm W^\mp)}{BR(\tilde{\chi}_1^0 \to \tau^\pm W^\mp)}$ Mukhopadhyaya, Roy, Vissani, 1998
  - .....

TeV Scale Seesaw Models

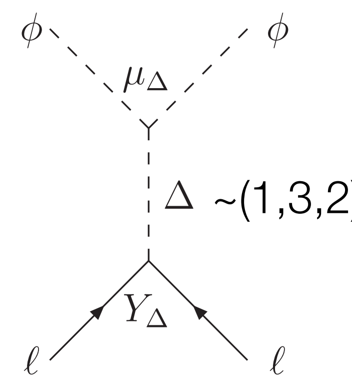
- With new particles:
  - type-I seesaw
    - generally decouple from collider physics

Kersten, Smirnov, 2007



  - type-II seesaw

Lazarides, 1980; Mohapatra, Senjanovic, 1980



    - TeV scale doubly charged Higgs ⇔ small couplings
    - unique signatures:

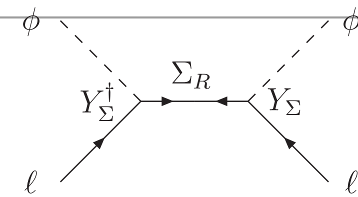
$$\Delta^{++} \to e^+ e^+ , \ \mu^+ \mu^+ , \ \tau^+ \tau^+$$

    - decay BR ↔ mass ordering

Perez, Han, Huang, Li, Wang, ‘08; Han, Mukhopadhyaya, Si, Wang, ‘07; Akeroyd, Aoki, Sugiyama, ‘08; ...

-

TeV Scale Seesaw Models



Σ_R: ~(1,3,0)

- With new particles:
  - type-III seesaw — Foot, Lew, He, Joshi, 1989; Ma, 1998
    - TeV scale triplet decay : observable displaced vertex

$$\tau \leq 1\ \mathrm{mm} \times \left(\frac{0.05\ \mathrm{eV}}{\sum_i m_i}\right)\left(\frac{100\ \mathrm{GeV}}{\Lambda}\right)^2$$

Franceschino, Hambye, Strumia,2008

    - neutral component Σ^0 can be dark matter candidate

E. J. Chun, 2009

  - Radiative Seesaw
    - Zee-Babu model (neutrino mass at 2 loop)
      - singly+doubly charged SU(2) singlet scalars

Zee 1986; Babu, 1989

    - neutrino mass at higher loops: TeV scale RH neutrinos
    - loop particles can also have color charges

Krauss, Nasri, Trodden, 2003; E. Ma, 2006; Aoki, Kanemura, Seto, 2009

      - enhanced production cross section
    -

Perez, Han, Spinner, Trenkel, 2011

TeV Scale Seesaw Models

- With new interactions:
  - SUSY LR Model:
    - tested via searches for W_R

Azuleos et al 06; del Aguila et al 07, Han et al 07; Chao, Luo, Xing, Zhou, '08; ...

- More Naturally: inverse seesaw or higher dimensional operators or Extra Dim
  - inverse seesaw

Mohapatra,1986; Mohapatra, Valle, 1986; Gonzalez-Garcia, Valle, 1989

    - non-unitarity effects
    - enhanced LFV (both SUSY and non-SUSY cases)
    - correlation

Hirsch, Kernreiter, Romao, del Moral, 2010

$$\frac{\mathrm{BR}(\tilde{\chi}_1^{\pm} \to \tilde{N}_{1+2} + \mu^{\pm})}{\mathrm{BR}(\tilde{\chi}_1^{\pm} \to \tilde{N}_{1+2} + \tau^{\pm})} \propto \frac{\mathrm{BR}(\mu \to e + \gamma)}{\mathrm{BR}(\tau \to e + \gamma)}$$

A Novel Origin of CP Violation

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)