Defining and Measuring the Top Quark Mass
(Theory Aspects of Top MC Mass)

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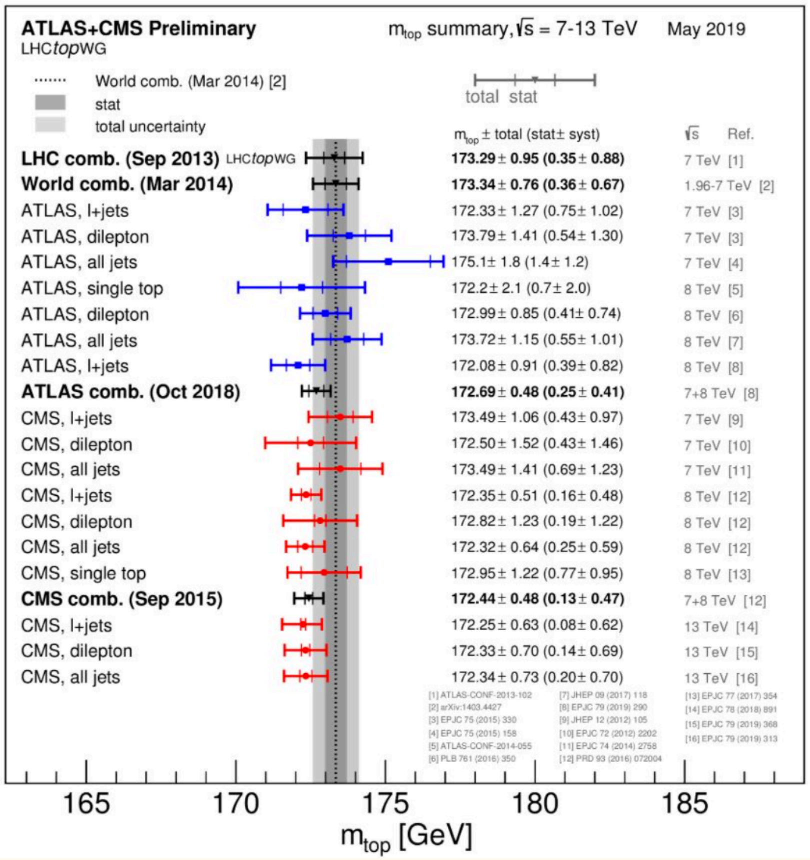
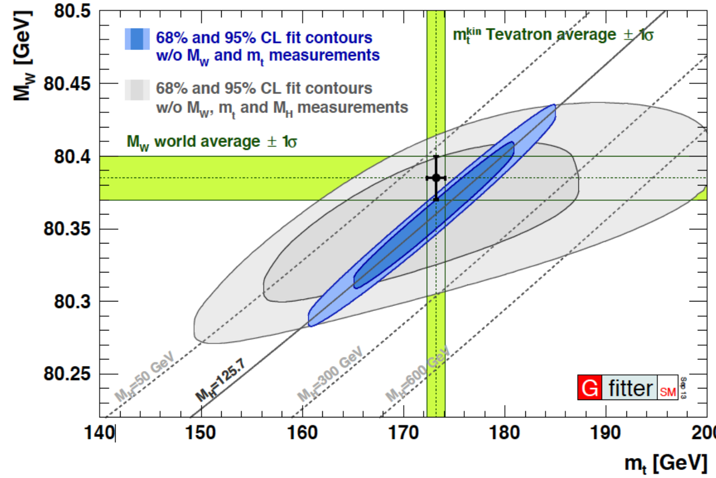
Content

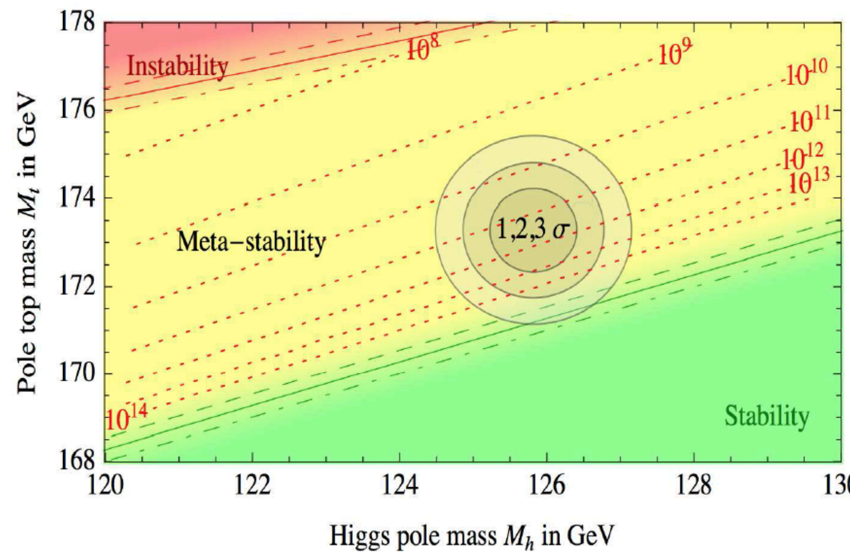
- Concepts of most precise experimental top mass determination based on direct reconstruction
- Why top mass measurements are highly non-trivial and require extended studies and many new developments.
- Recent theoretical studies
- Analysis of shower cut Q_0 in angular ordered parton showers
- Conclusions and outlook

→ Marco Vanadia: Top mass measurements at ATLAS and CMS
→ Thomas Jezo : Top quark modelling

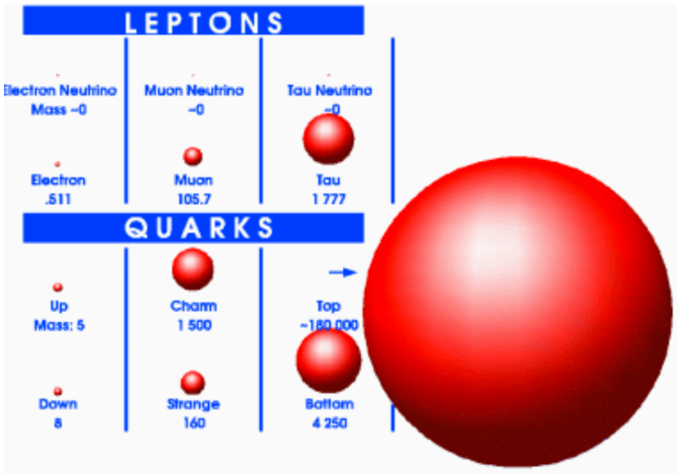
Motivation for Top Mass Measurements







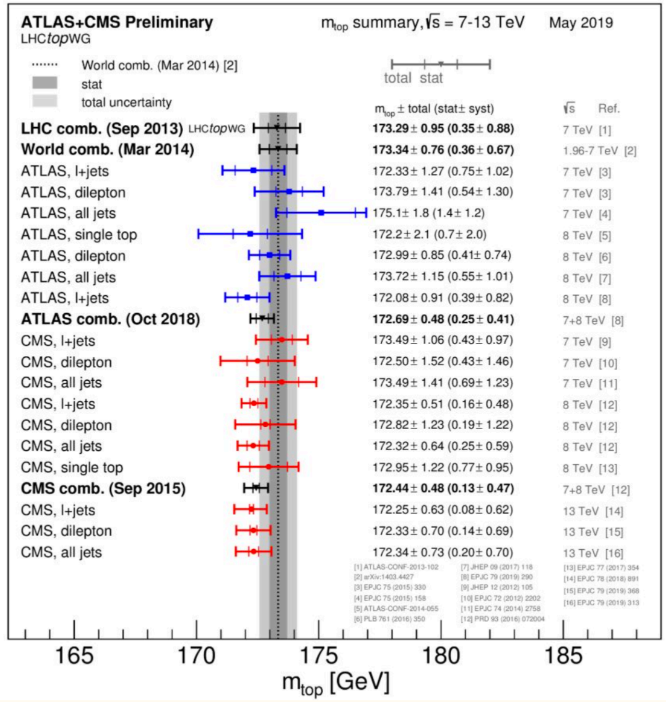
Why is the Top a Hard Theory Problem?



1) Color charge: $\Lambda_{QCD} \sim 0.5$ GeV
2) Large decay width: $\Gamma_t \sim 1.4$ GeV
3) Large mass: $m_t \sim 172$ GeV

- Top quark not observable: color neutralization modifies observable momenta by at least some non-perturbative contributions.
- Resonant and non-resonant processes contribute.
- Large logarithms of ratios: Λ_{QCD} / m_t , $\Gamma_t / m_t \rightarrow \delta m_t \sim \Gamma \alpha_s(\Gamma) \sum_i \alpha_s^i \log^i (m_t/\Gamma)$
- Reminder: Top mass is not a physical mass, but a scheme-dependent parameter.
- Excellent theoretical knowledge on a number of highly non-trivial aspects are required to avoid being dependent on “good modelling”.

Top Mass Measurements at the LHC



Aims:

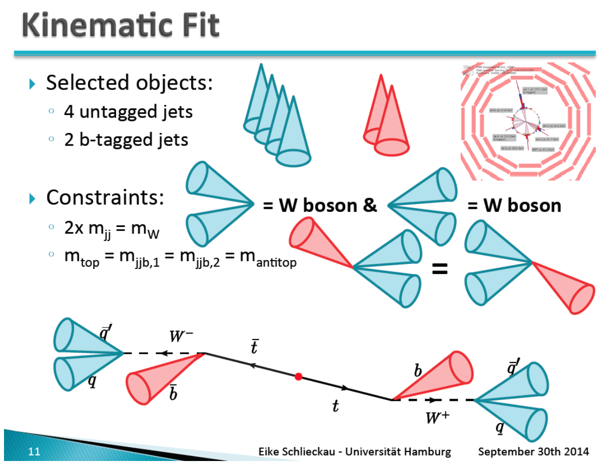
- Small experimental errors
  - High statistics ✔
  - Systematics ("MC errors") !
- Small theoretical errors
  - Many loops / legs !
  - Large log resummations !
  - Non-perturbative corrections !

Interconnected at high precision

→ Precise control of mass scheme

Measurement Methods

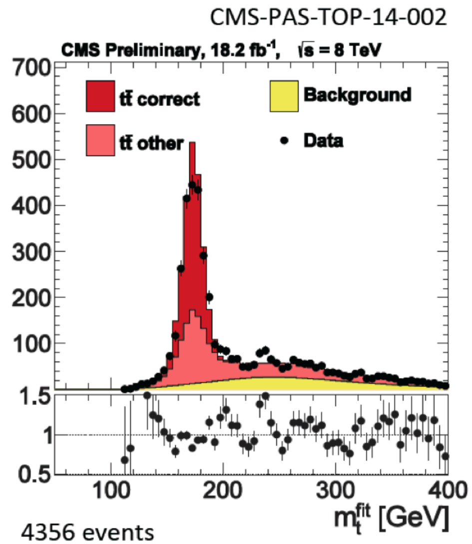
Direct kinematic methods:

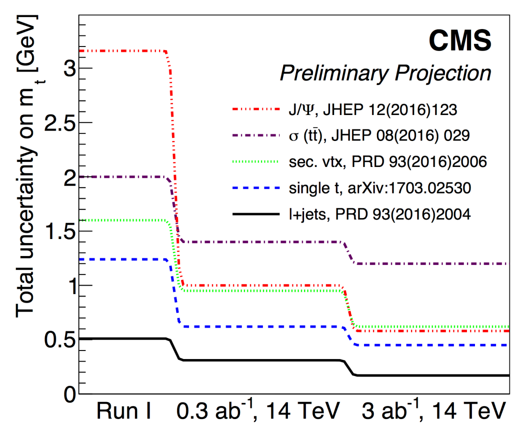


$$m_t^{\mathrm{MC}} = 174.34 \pm 0.64 \quad \text{(Tevatron final, 2014)}$$
$$m_t^{\mathrm{MC}} = 172.44 \pm 0.49 \quad \text{(CMS Run-1 final, 2015)}$$
$$m_t^{\mathrm{MC}} = 172.69 \pm 0.48 \quad \text{(ATLAS Run-1 final, 2018)}$$

Most precise method.
I will focus on it!

e.g. template fits





- ⊕ High leading order top mass sensitivity
- ⊕ Insensitive to norm uncertainties (pdf, ..)

- ⊖ Resummation and hadronization dominated
- ⊖ Purely based on MC
- ⊖ MC uncertainties ?
- ⊖ Meaning of m_t^{MC} ?

Relevant already today because of very high leading order sensitivity.

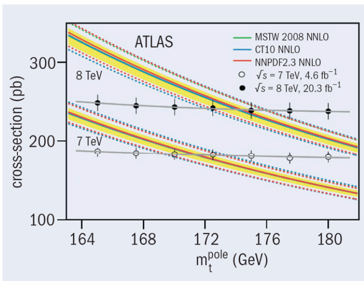
$\Delta^{ex}\, m_t \sim 200$ MeV
(HL-LHC projection)

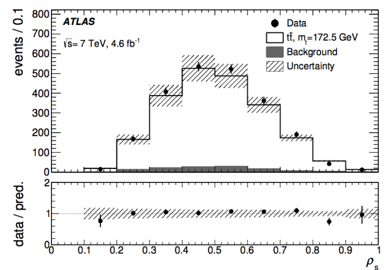
For an ultimate precision of 200 MeV all methods are going to have the same level of complication.

Top Mass Measurement Methods

Indirect global methods:

- Other measurements based on NLO and NNLO pQCD calculations of $\boldsymbol{\sigma}(tt, ttj)$:
  → "pole mass measurements", but uncertainties larger than for direct method



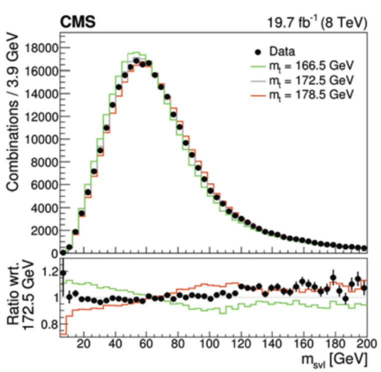


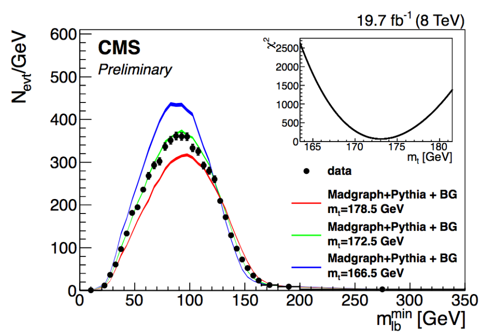
- ⊕ pQCD-FO calculations dominate
- ⊕ Control of mass scheme
- ⊖ Lower top mass sensitivity
- ⊖ High sensitivity to norm uncertainties (pdf, α_S, ..)

arXiv:1904.05237 : m_t^{pole} = 170. 5 ± 0.8 GeV from $d\boldsymbol{\sigma}(tt)/dX$, $X=N_{jet}$, M_{tt} , y_{tt} + NLO/PS

arXiv:1905.02302 : m_t^{pole} = 171. 1 ± 1.1 GeV from $\boldsymbol{\sigma}(tt+jet)$ + NLO-QCD

- Hadron/lepton methods (M_{Bl}, M_{bl}, E_B, E_l, T_2, ..):

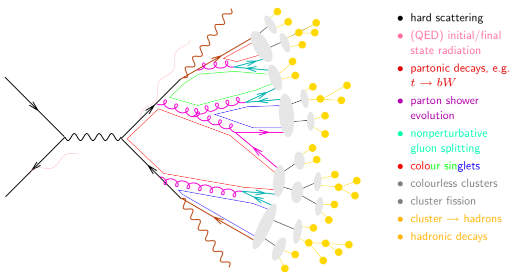




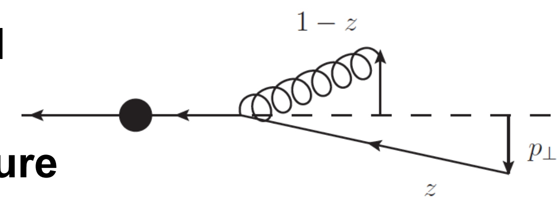
- ⊕ Experimentally clean
- ⊕ Partly based on pQCD
- ⊖ lower top mass sensitivity
- ⊖ strong dependence on MC simulations
- ⊖ Significant hadronization effects

What is the issue with the MC mass?

Which scheme is used by Monte-Carlo event generators?



1) Hard matrix elements
2) Parton shower
3) Hadronization Model



Better statement: Scheme is (un)determined by the structure <u>and</u> the theoretical (in)precision of the parton shower.

<u>Quark mass scheme encodes the amount of unresolved radiation</u> surrounding the heavy quark in our calculations. Pole mass assumes all real radiation is resolved.

Naïve view: Parton shower describes all radiation explicitly: m_t^{MC} must be close to m_t^{pole} !

But:

a) Parton showers are working in the approximation of a stable top quark. (Narrow width approximation)

b) Parton showers are not uniformly precise for all observables. (They are not fully next-to-leading order precise for all observables even with NLO matching.)

c) The shower cutoff Q_0 treats all radiation below Q_0 as unresolved.

Where do we stand today?

Without any further systematic study of what the quantum structure of parton showers and MC event generators is, the only conservative (= absolutely save and undisputable) statement one can make is that the MC top mass parameter agrees with the pole mass within an theoretical uncertainty of size of the top quark width:

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} \pm \max(\Gamma_t, Q_0)$$

$$\Gamma_t \approx 1.4\ \mathrm{GeV}$$
$$Q_0 = (0.5 - 1.25)\ \mathrm{GeV}$$

To this precision one can consider the top quark a well defined "top particle" and its "physical mass" is m_t^{MC}.

This is the inherent precision of ALL current top quark mass measurements.

But we want and need much more precision than that !

To get a better understanding of m_t^{MC} we have to better understand all aspects of MC event generators related to observables relevant to top mass measurements.

1) Analyze parton showers
2) Understand hadronization models conceptually

} Scrutinize MC event generators

3) Eventually: new theory developments (NLL-MC, theory calculations, …) are mandatory

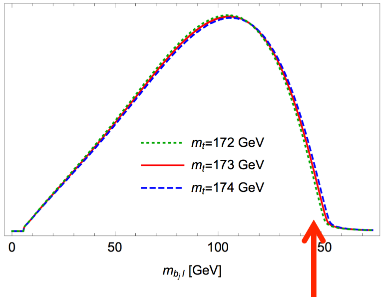
How serious is the issue? Recent Work

Ravasio, Jezo, Nason, Oleari, arXiv: 1801.03944 (m_t^{MC} from direct reconstruction)

- POWHEG study: NLO corrections in various approximations (production, decay, full off-shell) leads to small numerical differences ($hva,\ t\bar{t}dec,\ b\bar{b}4\ell$)
- Numerical effects on the observed end point (e.g. peak position of reconstr. inv. Mass) MC dependent (Pythia **(<200 MeV**) compared to Herwig (**up to 1 GeV**))

Corcella, Franceschini, Kim, arXiv: 1712.05801 (m_t^{MC} from alternative methods)

- Dependence of m_t^{MC} determination from kinematic decay distributions on fragmentation parameters in Pythia 8 and Herwig 6
- **Hadronization model parameters cannot be determined precise enough** such that alternative fragmentation based methods (exclusive observables, m_{Bl}, E_B) can compete with direct mass measurements.
- Endpoints not sensitive to hadronization model variations (fragmentation)



Heinrich, Maier, Nisius, Schlenk, Schulze, Winter: 1709.08615 (alternative methods)

- Effects of off-shell top production compared to narrow width approximation ($M_{j_b\ell}$)
- Effects related to off-shell effects as large as **0.5 − 1 GeV** for m_t^{MC} determination

Monte-Carlo Top Quark Mass Parameter

To start the systematic considerations we should set up a notation so that we can discuss the different issues in a systematic way.

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + \Delta_m^{\mathrm{pert}} + \Delta_m^{\mathrm{non-pert}} + \Delta_m^{\mathrm{MC}}$$

pQCD contribution:

- Perturbative correction
- Depends on MC parton shower setup
- (Affected by finite width effects?)

Non-perturbative contribution:

- Effects of hadronization model
- May depend on parton shower setup

Monte Carlo shift:

- Contribution arising from systematic MC uncertainties
- E.g. color reconnection, b-jet modeling, (finite width), ...
- Should be covered by 'MC uncertainty' or better negligible

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- Contribution arising from systematic MC uncertainties
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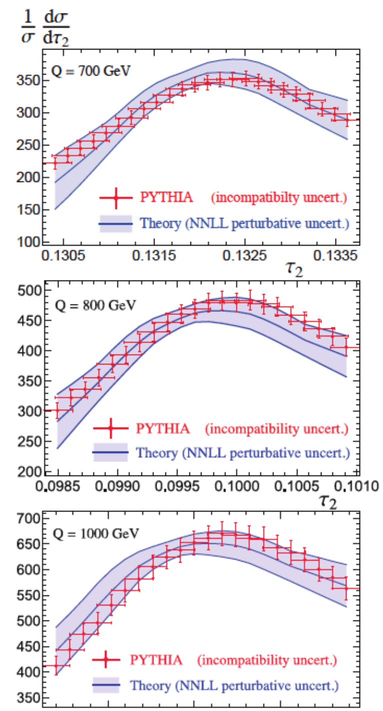
- Scrutinize theoretical content of MC event generators, so that we can write an equality in the first place.

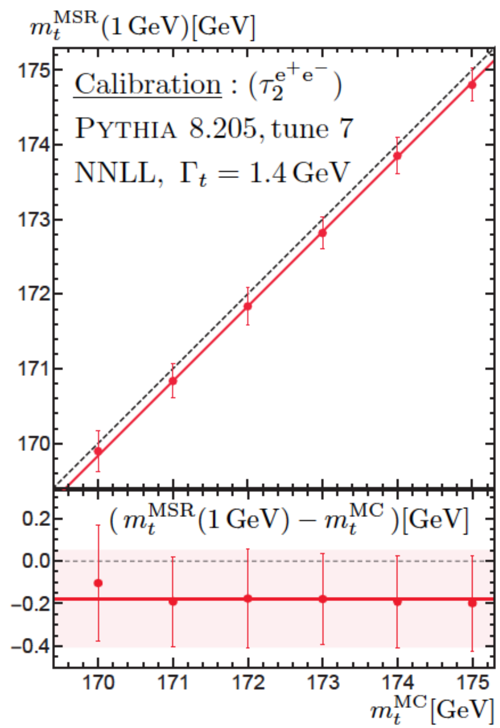
Lebel of systematics of MC decides whether the equality of to be understood phenomenologically or field theoretically

Monte-Carlo Top Quark Mass Parameter

Quantitative examinations of m_t^{MC}

- Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart (2017), arxiv:1608.01318
  - numerical relation between Pythia MC top mass and MSR mass using 2-jettiness in e^+e^- in the resonance region from calibration fits
  - $m_t^{\rm MC} = m_t^{\rm MSR}(1\,{\rm GeV}) + (0.18 \pm 0.22)\,{\rm GeV}$ $\qquad m_t^{\rm MC} = m_t^{\rm pole} + (0.57 \pm 0.28)\,{\rm GeV}$
  - universality conjectured but not proven



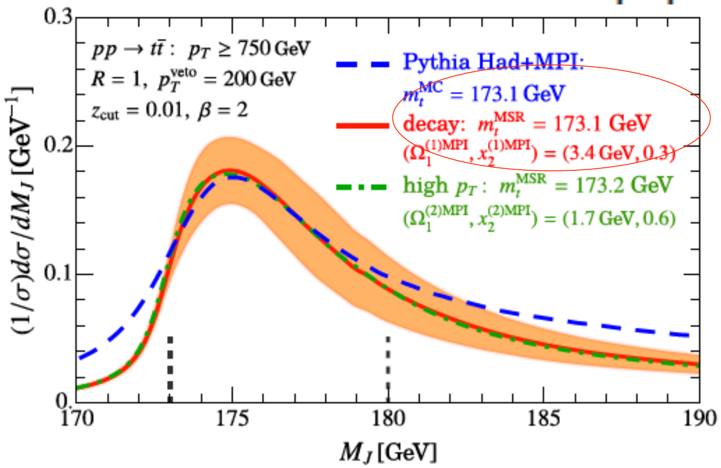


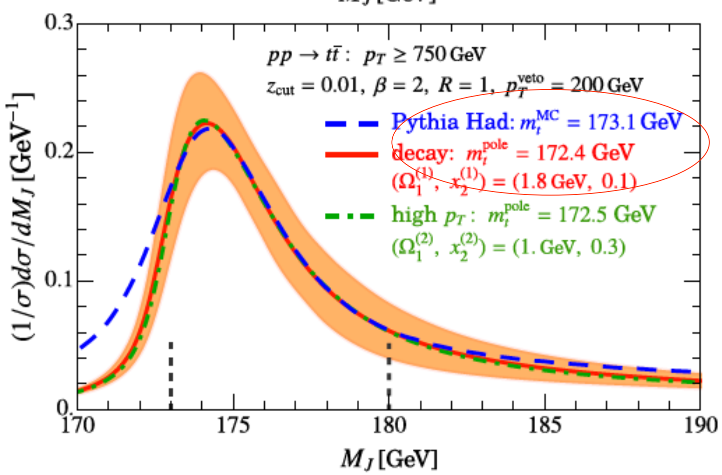
- Fits of NNLL+NLO+had.corr. theory predictions with Pythia 8.205
- Good agreement between Pythia and analytic calculation
- **"MC top mass calibration"**

Monte-Carlo Top Quark Mass Parameter

Quantitative examinations of m_t^{MC}

- Hoang, Mantry, Pathak, Stewart (2017), arxiv:1708.02586
  - extension of the framework to groomed jet masses at the LHC for boosted top quarks
  - results consistent with e^+e^- calibration
  - full calibration analysis still to be done





- Comparison of NLL+had.corr. theory predictions with Pythia 8.205
- Good description of Pythia output.

e^+e^- Calibration Result: Top Width Dependence

Plätzer, Preisser, Samitz, AHH, w.i.p.

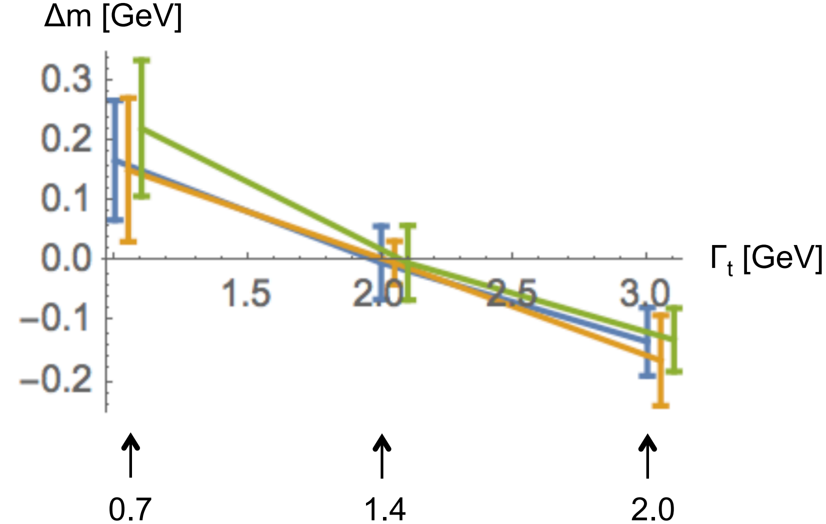
Top width dependence

$$\Delta m = m_t^{MSR}[\Gamma_t] - m_t^{MSR}[\Gamma_t\!=\!1.4]$$

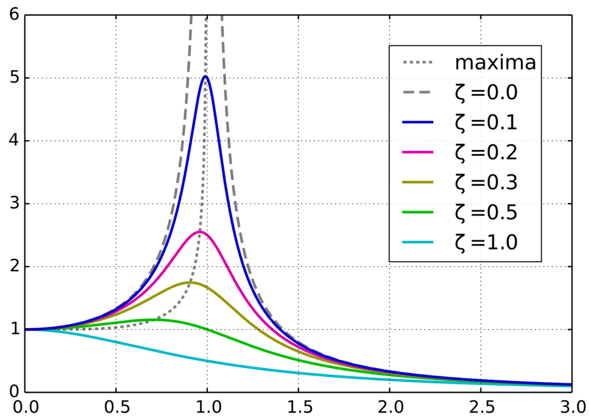
- Sensitivity to top width value.
- Pythia resonance peak position does not depend on value of Γ_t (inadequate modelling!)
- Theory resonance peak position increases correctly with Γ_t

$\alpha_S(M_Z)$=0.118

m_t^{MC}=173



- Three colors: tunes 1, 3, 7
- Error bars: standard deviation of best mass value distribution in 500 profile function fits

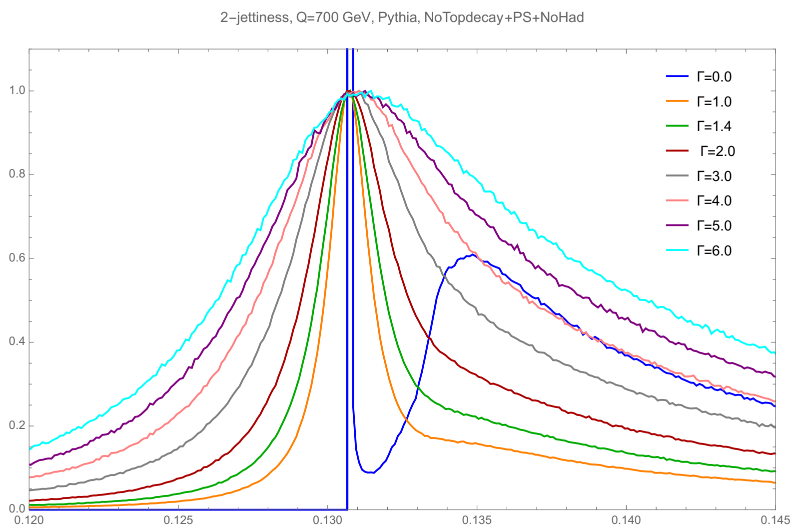


"driven harmonic oscillator"

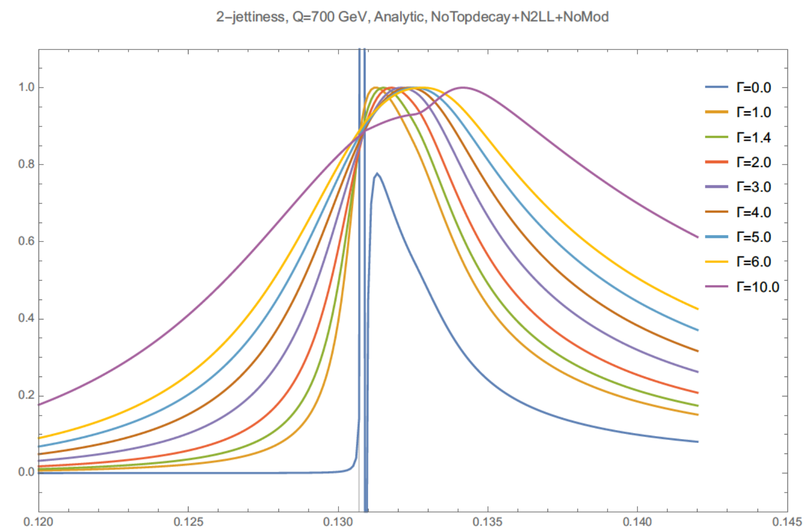
Top Resonance: factorization vs. Pythia

Plätzer, Preisser, Samitz, AHH, w.i.p.

Pythia



QCD Factorization



- **Pythia does not describe the top width dependence in a way compatible with theory.**
- MC generators themselves need to be scrutinized and understood thoroughly in order to fix the relation of the MC top quark mass to field theory masses.

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{pole}} + \Delta_m^{\mathrm{pert}} + \Delta_m^{\mathrm{non-pert}} + \Delta_m^{\mathrm{MC}}$$

Numerical calibration useful tool, but cannot distinguish the three contributions

Cutoff in Angular Ordered Parton Showers

Plätzer, Samitz, AHH; JHEP 1810 (2018) 200

The first step of a systematic theoretical examination:

Δ_m^{pert} can be examined at $\mathcal{O}(\alpha_S)$ for $\boldsymbol{\tau}_2$ (2-jettiness) in the resonance region for e^+e^- collisions:

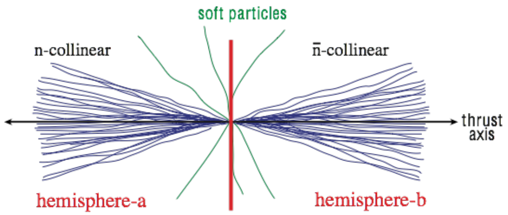
Restrictions:

- Parton level (with common shape function for non-pert. Effects)
- Boosted top quarks (factorization and shower algorithm reliable)
- Narrow width approximation
- Examination of radiation in top production
- Dijet limit (NLL precision for angular ordered PS)

2-Jettiness $\boldsymbol{\tau}_2$ distribution In the peak region (for e^+e^- and boosted tops) can be discribed using QCD dijet-factorization at NLL+NLO and coherent branching (CB) at NLL.

Questions to be addressed:

- Are current MCs precise enough so the m_t^{MC} is well-defined at $\mathcal{O}(\alpha_S)$?
- Which role does the shower cut Q_0 play quantitatively?

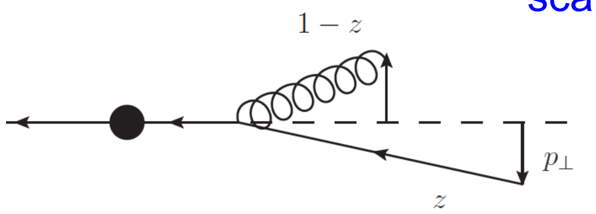


Cutoff in Angular Ordered Parton Showers

Catani, Marchesini, Webber 1991
Gieseke, Stephens, Webber, 2003

→ Coherent branching: (basis of the Herwig parton shower)

scale in α_S: $\mu^2 = p_\perp^2 + (1-z)^2 m^2$ cutoff: $p_\perp^2 > Q_0^2$



Usually not present in analytic QCD !

→ QCD factorization (SCET+bHQET):

Fleming, Mantri, Stewart, AHH, 2007

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Ultra-collinear radiation

Large-angle soft radiation

- Correspondences can be cross checked by explicit computations.

Cutoff in Angular Ordered Parton Showers

Plätzer, Samitz, AHH; JHEP 1810 (2018) 200

Q_0=0 :

→ Computational scheme of resummed pQCD calculations

→ Coherent branching algorithm can be solved analytically in the same way

Outcome:

- Equivalence of CB and SCET at NLL order for Q_0=0 (massive quark case new!)
- NLL precision sufficient to specify the mass scheme at $\mathcal{O}(\alpha_S)$
- **Generator mass m_t is the pole mass m_t^{pole} for Q_0=0 !**

But for MC event generation parton showers require $Q_0 \gtrsim 1$ GeV, so it is mandatory to consider a finite shower cut !

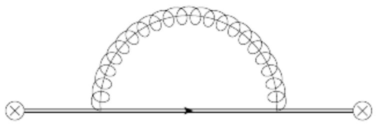
Cutoff in Angular Ordered Parton Showers

Plätzer, Samitz, AHH; JHEP 1810 (2018) 200

$Q_0>0$:

- Pole of the top quark propagator = $m_t^{CB}(Q_0) \neq m_t^{pole}$ (**coherent branching mass**)

$$m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \frac{2}{3}Q_0\alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$



- In the presence of the shower cut the **ultra-collinear radiation** generated by CB produces exactly the mass scheme change correction that is required so that the generator mass is exactly the coherent branching mass $m_t^{CB}(Q_0)$.

$$\sigma(m_1, Q, \ldots) = \sigma(m_2, Q, \ldots) + \delta m \times \frac{\rm d}{{\rm d}m}\sigma(m, Q, \ldots)\Big|_{m=m_1} + \ldots$$

$$\delta m = m_2 - m_1$$

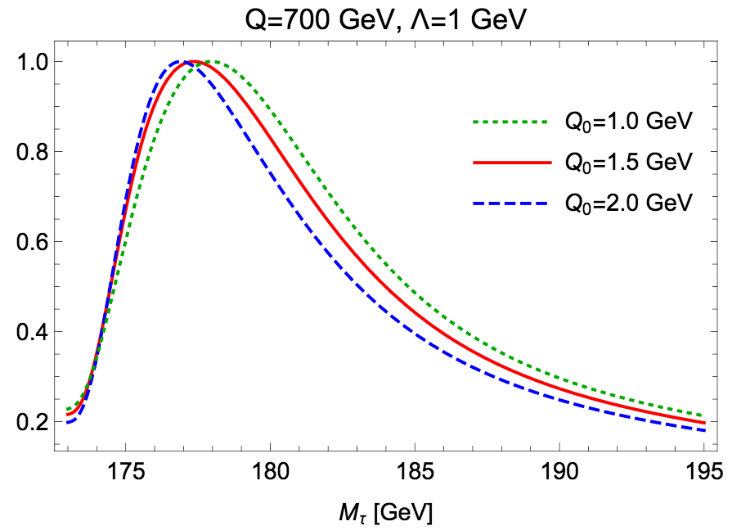
Scheme change correction

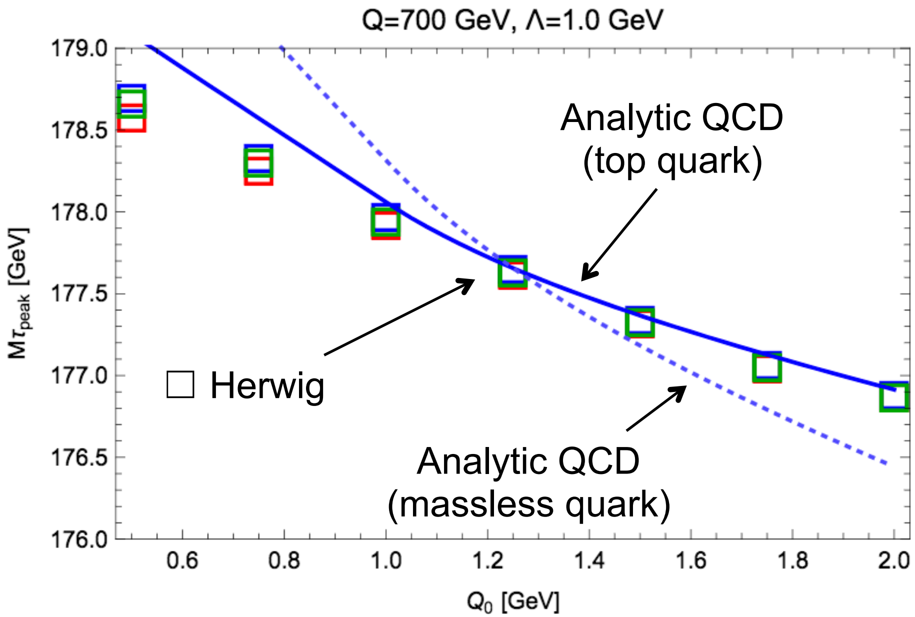
- The shower cut also affects **large-angle soft radiation. T**he corresponding effects are directly tied to the amount of hadronization effects that are fixed by tuning (effects are the same for massless quarks)

Q_0 Dependence: Herwig vs analytic QCD

Peak position of $M_\tau = \dfrac{Q^2\tau_2}{2m_t} \quad (Q = E_{\rm cm})$

- Depends on value of Q_0 (while keeping hadronization effects unchanged)
- Relative Q_0 dependence of hadronization and the top mass depends on Q





- Herwig simulations in full agreement with analytic calculation for CB algorithm
- M_{bl} endpoint and M_{bW} resonance position show compatible Q_0 behavior.

Plätzer, Samitz, AHH; JHEP 1810 (2018) 200

Relation of $m_t^{CB}(Q_0)$ to other Masses

Herwig 7: Q_0 = 1.25 GeV → $m_t^{Herwig} = m_t^{CB}(1.25\ \text{GeV})$

Be aware of the restrictions!

MSR Mass

$$m_t^{\rm MC} = m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) = m_t^{\rm pole} - 0.67\, Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$m_t^{\rm MSR}(Q_0) = m_t^{\rm pole} - \frac{4}{3\pi} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2) = m_t^{\rm pole} - 0.42\, Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\to\ m_t^{\rm MSR}(Q_0) - m_t^{\rm CB}(Q_0) = 0.24\, Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2)$$

$$\boxed{m_t^{\rm MSR}(Q_0) - m_t^{\rm CB}(Q_0) = (0.190 \pm 0.070)\ \text{GeV}}$$

$$\alpha_S^{\overline{\rm MS}}(M_Z) = 0.118$$

- CB and MSR masses do not suffer from the $O(\Lambda_{QCD})$ renormalon → good convergence
- Uncertainty estimated from difference between α_s in $\overline{\rm MS}$ and MC schemes
- MSR mass can be related to $\overline{m}_t(\overline{m}_t)$ with uncertainty of 15 MeV.

- Precision sufficient for all possible applications at the LHC
- Two-loop corrections to $m_t^{CB}(Q_0)$-m_t^{pole} needed for ILC top quark physics

Relation of $m_t^{CB}(Q_0)$ to other Masses

Herwig 7: Q_0 = 1.25 GeV → $m_t^{Herwig} = m_t^{CB}(1.25\ \text{GeV})$

Pole Mass

$$m_t^{\text{MSR}}(Q_0) - m_t^{\text{CB}}(Q_0) = (0.190 \pm 0.070)\ \text{GeV}$$

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(Q_0) = (0.350 \pm 0.250)\ \text{GeV}$$

Lepenik, Preisser, AHH 2017
[±110 MeV: Beneke, Marquard, Nason, Steinhauser 2017]

$$\rightarrow \boxed{m_t^{\text{pole}} - m_t^{\text{CB}}(Q_0) = (0.540 \pm 0.260)\ \text{GeV}}$$

All order relation!

- Pole mass suffers from the $O(\Lambda_{QCD})$ renormalon → irreducible ambiguity 250 MeV
- **Pole mass around 0.5 GeV larger than Herwig top generator mass.**
  Shift as large as current experimental uncertainty from direct methods.

Conclusions / Status / Outlook

- NLL accurate parton showers are needed to consistently control the MC top mass parameter at the field theoretic level (i.e. we can define its relation to other mass schemes, such as pole or MSbar, at O'(α_S).

- For dijet observables one can determine the **perturbative** contributions to the relation between m_t^{MC} and the pole mass (Δ_m^{pert})

$$m_t^{\rm CB}(Q_0) = m_t^{\rm pole} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

- Statement valid for: (current restrictions)

  - Boosted top quarks
  - Narrow width approximation
  - Top production sensitive observables
  - Herwig angular-ordered PC

  - e^+e^- annihilation
  - Only perturbative contribution
  - 2-jet observable

Systematic work on MC top quark mass just started!

Conclusions / Status / Outlook

- Future work:

  - Include top decay: $M_{b\text{-jet-lepton}}$ (NW-factorization) (straightforward)
  - Non-perturbative effects / retuning analyses $\rightarrow \Delta_m^{nonpert}$ (straightforward)
  - pp collisions (straightforward / medium)

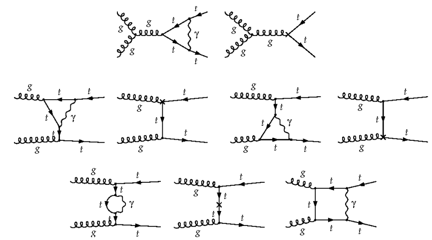
  - Analytic treatment of dipole shower [Pythia, Sherpa] (medium)
  - Intermediate p_T top quarks (medium / hard)
  - Current technology parton shower for unstable top quark (medium / hard)

  - NLL accurate parton shower for stable top quark (hard)
  - Include top decay (non-resonant) → beyond resonance aware (hard)
  - NLL accurate parton shower for unstable top quark (very hard)

Backup Slights

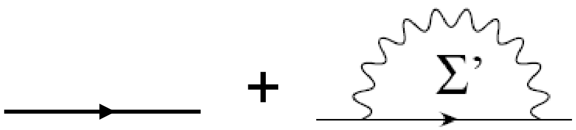
Heavy Quark Mass Schemes

Experimental precision is so high that theoretical calculations must include higher order quantum corrections.



Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

$$m^0 \, \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$$

One has to absorb the UV divergence into the mass parameter, which causes the mass parameter to be scheme-dependent.

All mass schemes are related through a perturbative series.

$$m^{\text{schemeA}} - m^{\text{schemeB}} = \#\,\alpha_s + \#\,\alpha_s^2 + \#\,\alpha_s^3 + \ldots$$

A good scheme choice is one that gives systematically (not accidentally) good convergence. But there are almost always class of schemes one can use.

Heavy Quark Mass Schemes

Common Mass Renormalization Schemes:

Pole mass: mass = closest to concept of a classic rest mass

$$m^0 = m^{\rm pole} + \delta m^{\rm pole} \qquad \delta m^{\rm pole} = \Sigma(m, m)$$

But very infrared sensitive: has ambiguity of 250 MeV !

Problem: Pretends that virtual and real radiation can be distinguished to zero momentum.

$\overline{\text{MS}}$ mass: $$m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi}\frac{1}{\epsilon}$$

Input for studies of SM vacuum stability. No ambiguity.

Makes no explicit assumption about resolution scale of real radiation, but technicallly the resolution scale is μ ≳ m.

MSR mass: $$m^{\rm MSR}(R) = m^{\rm pole} - \Sigma^{\rm fin}(R, R, \mu) \qquad \text{for } R < m$$

Interpolates between MSbar and pole mass. No ambibuity for R > Λ_{QCD}

$$m_t^{\rm MSR}(R=0) = m^{\rm pole}$$

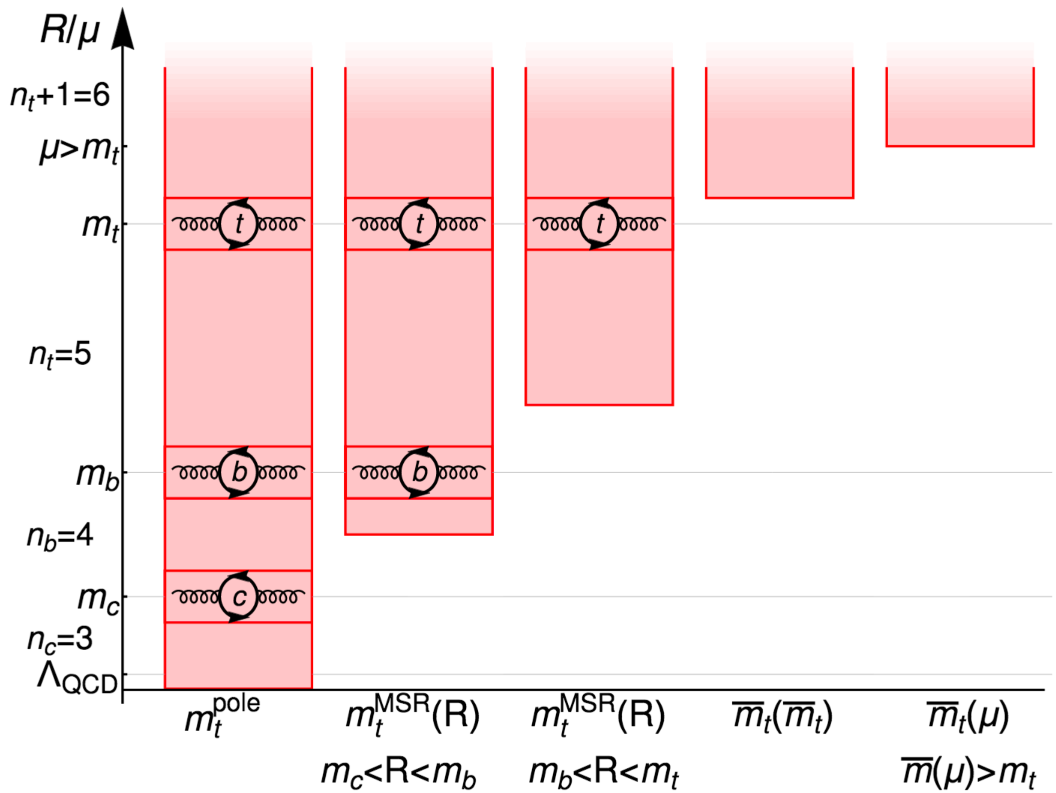
$$m_t^{\rm MSR}(R=\overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

(Formal only, because R < Λ_{QCD} impossible due to Landau pole)

Heavy Quark Mass Schemes

Perturbative self-energy corrections absorbed into the mass parameter

Origin of renormalon ambiguity



Pole mass contains contributions down to zero momentum.

→ Ambiguity

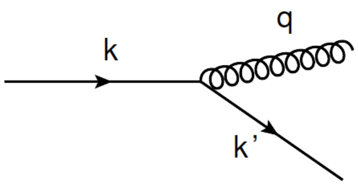
Short-distance masses contains contributions down to momentum $R>\Lambda_{QCD}$

→ No ambiguity

Coherent Branching

Dokshitzer, Fadin, Khoze (1982)
Bassetto, Ciafaloni, Marchesini (1983)
Catani, Marchesini, Webber (1991)
Gieseke, Stephens, Webber (2003)

→ default shower in Herwig



momentum conservation

$$k^2 - m^2 = \frac{k'^2 - m^2}{z} + \frac{q^2}{1-z} + \frac{q_\perp^2 + m^2(1-z)^2}{z(1-z)}$$

ordering variable

$$\tilde{q}^2 = \frac{q_\perp^2 + m^2(1-z)^2}{z^2(1-z)^2}$$

angular ordering (color coherence)

$$z_i^2 \tilde{q}_i^2 > \tilde{q}_{i+1}^2$$

→ jet mass distribution (inv. mass generated from CB from one hard quark)

$$J(k^2, Q^2, m^2) = \delta(k^2 - m^2) + \int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_{\frac{m}{\tilde{q}}}^{1} \mathrm{d}z\, P_{qq}\left[\alpha_s(z(1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right]$$
$$\times \left[zJ(zk^2 - (1-z)(z^2\tilde{q}^2 - m^2), z^2\tilde{q}^2, m^2) - J(k^2, \tilde{q}^2, m^2)\right]$$

→ parton level $\boldsymbol{\tau}$ distribution

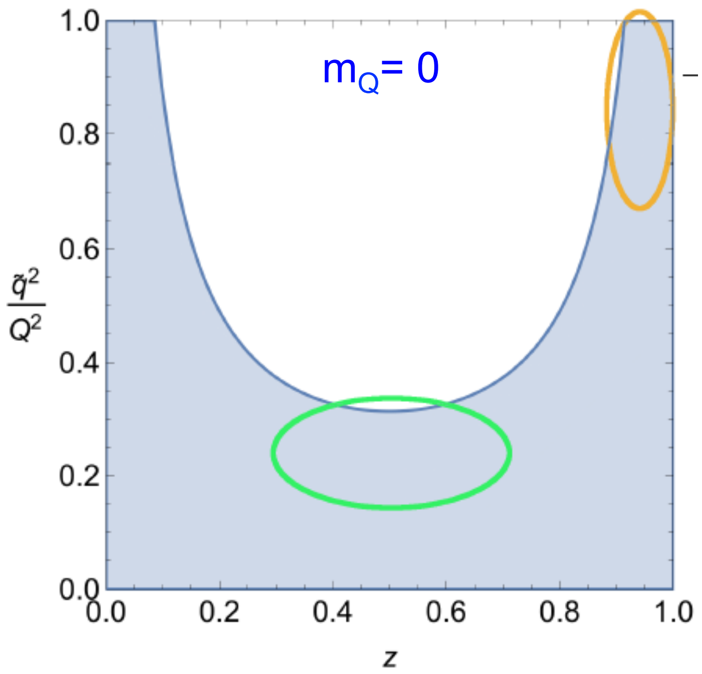
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \int \mathrm{d}k_1^2 \mathrm{d}k_2^2\, \delta\left(\tau - \frac{k_1^2 + k_2^2}{Q^2}\right) J(k_1^2, Q^2, m^2) J(k_2^2, Q^2, m^2)$$

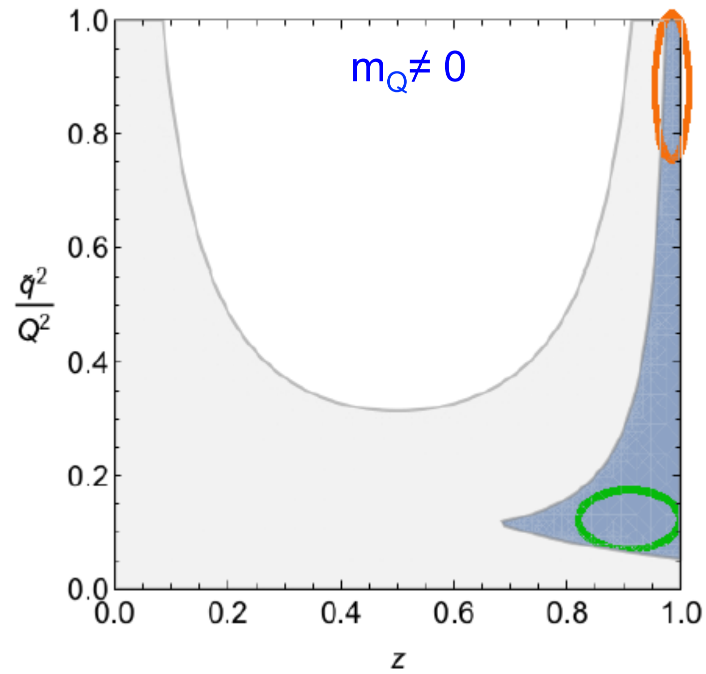
→ exponentiation in Lapace space

Catani, Trentadue, Turnock, Webber (1993)

$$\tilde{J}(\nu, Q^2, m^2) = \int_{m^2}^{\infty} \mathrm{d}k^2\, \mathrm{e}^{-\frac{\nu}{Q^2}(k^2 - m^2)} J(k^2, Q^2, m^2)$$
$$\approx \exp\left[\int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_{\frac{m}{\tilde{q}}}^{1} \mathrm{d}z\, P_{qq}\left[\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right]\left(\mathrm{e}^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}} - 1\right)\right]$$

Phase Space and Power Counting (Q_0=0)





phase space regions for $\tau_{\text{peak}} \sim \frac{\Lambda}{Q} \ll 1$, $m=0$		
	coherent branching	QCD factorization
n-coll.	$z \sim (1-z) \sim 1$ $\tilde{q} \sim (Q\Lambda)^{\frac{1}{2}}$ $q_\perp \sim (Q\Lambda)^{\frac{1}{2}}$	$q^\mu \sim (\Lambda, Q, (Q\Lambda)^{\frac{1}{2}})$
soft	$1-z \sim \frac{\Lambda}{Q}$, $z \sim 1$ $\tilde{q} \sim Q$ $q_\perp \sim \Lambda$	$q^\mu \sim (\Lambda, \Lambda, \Lambda)$

phase space regions for $\tau_{\text{peak}} - \tau_{\text{min}} \sim \frac{\Lambda}{Q} \ll 1$, $m \neq 0$		
	coherent branching	QCD factorization
u. coll.	$1-z \sim \frac{Q\Lambda}{m^2}$, $z \sim 1$ $\tilde{q} \sim m$ $q_\perp \sim \frac{Q}{m}\Lambda$	$q^\mu \sim (\Lambda, \frac{Q^2}{m^2}\Lambda, \frac{Q}{m}\Lambda)$
soft	$1-z \sim \frac{\Lambda}{Q}$, $z \sim 1$ $\tilde{q} \sim Q$ $q_\perp \sim \Lambda$	$q^\mu \sim (\Lambda, \Lambda, \Lambda)$

Cutoff in Angular Ordered Parton Showers

Plätzer, Samitz, AHH; arXiv:1807.06617

→ Is NLO information in the PS mandatory to specify the mass scheme?

Consider partonic cross section at NLO for massless quarks:

$$\frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi}\Big\{ \underbrace{-8\left[\frac{\ln\tau}{\tau}\right]_+}_{\mathrm{LL}} \underbrace{-6\left[\frac{1}{\tau}\right]_+}_{\mathrm{NLL}} + \underbrace{\delta(\tau)\Big(\frac{2\pi^2}{3}-2\Big)}_{\mathrm{N^2LL}}\Big\} + \mathcal{O}(\alpha_s^2)$$

(NLL PS corresponds to NLL' in SCET)

Use that NNLL@NLO part is proportional to LO $\quad f(\tau) = \frac{d\hat{\sigma}}{d\tau} \otimes S_{\mathrm{mod}}$

$$f_{\mathrm{NLO}}(\tau) = \tilde{f}_{\mathrm{NLO}}(\tau) + \alpha f_{\mathrm{LO}}(\tau)$$

$$f(\tau) = f_{\mathrm{LO}}(\tau) + f_{\mathrm{NLO}}(\tau) = (1+\alpha) f_{\mathrm{LO}}(\tau) + \tilde{f}_{\mathrm{NLO}}(\tau) + \mathcal{O}(\alpha_s^2)$$

LO peak position

$$f'_{\mathrm{LO}}(\tau_0) = 0$$

NLO peak position $\quad \delta\tau \sim \alpha_s$

$$f'(\tau_0 + \delta\tau) \stackrel{!}{=} 0 + \mathcal{O}(\alpha_s^2)$$

$$= (1+\alpha) f'_{\mathrm{LO}}(\tau_0+\delta\tau) + \tilde{f}'_{\mathrm{NLO}}(\tau_0+\delta\tau) + \mathcal{O}(\alpha_s^2)$$

$$= (1+\alpha)\underbrace{f'_{\mathrm{LO}}(\tau_0)}_{=0} + \delta\tau f''_{\mathrm{LO}}(\tau_0) + \tilde{f}'_{\mathrm{NLO}}(\tau_0) + \mathcal{O}(\alpha_s^2)$$

Same holds for massive quarks !

$$\rightarrow \quad \delta\tau = \frac{-\tilde{f}'_{\mathrm{NLO}}(\tau_0)}{f''_{\mathrm{LO}}(\tau_0)} + \mathcal{O}(\alpha_s^2)$$

→ NLL resummation has full NLO information on the peak position

CB: Impact of Shower Cut Q_0

→ recall NLL resummed thrust distribution in Laplace space

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q, m) = \exp\left[2\int_{m^2}^{Q^2}\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\int_{\frac{m}{\tilde{q}}}^{1}\mathrm{d}z\, P_{qq}\left[\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right]\left(\mathrm{e}^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}} - 1\right)\right]$$

→ introduce cutoff: $\theta\left(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\right) = 1 - \theta\left(\frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2} - \tilde{q}^2\right)$

=unreleased radiation
(radiation prevented by shower cut)

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q, m, Q_0) = \mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q, m) \times \mathrm{e}^{\mathcal{I}(\nu, Q, Q_0, m)}$$

→ expand $\mathcal{I}$ up to terms $\mathcal{O}\left(\frac{\nu^2 Q_0^2}{Q^2}, \frac{Q_0^2}{m^2}, \frac{m^2}{Q^2}\right)$

$$\mathcal{I} \approx \frac{2\nu}{Q^2}\int_{m^2}^{Q^2}\mathrm{d}\tilde{q}^2\int_{\frac{m}{\tilde{q}}}^{1}\mathrm{d}z\,(1-z)P_{qq}\left[\alpha_s((1-z)\tilde{q}), z, \frac{m^2}{\tilde{q}^2}\right]\theta\left(\frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2} - \tilde{q}^2\right) + \ldots$$

$$\approx -\nu\frac{\alpha_s(Q_0)}{4\pi}\left(16 C_F\frac{Q_0}{Q} - 8\pi C_F\frac{Q_0 m}{Q^2}\right) + \ldots$$

mass-independent shift
→ change to hadronization !

mass-dependent shift
→ change to mass !

→ term linear in ν gives shift in **τ**-space

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q, m, Q_0) = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau}\left(\tau + \frac{\alpha_s(Q_0)}{4\pi}\left[16 C_F\frac{Q_0}{Q} - 8\pi C_F\frac{Q_0 m}{Q^2}\right], Q, m, Q_0 = 0\right)$$