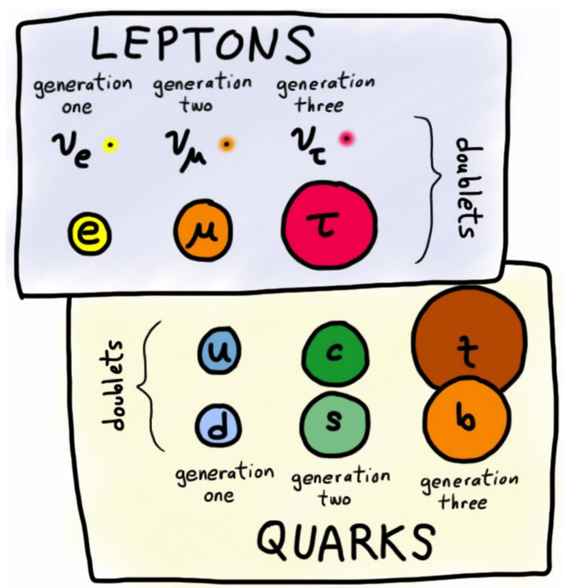
Exploring Beyond-the-SM physics at Low Energy

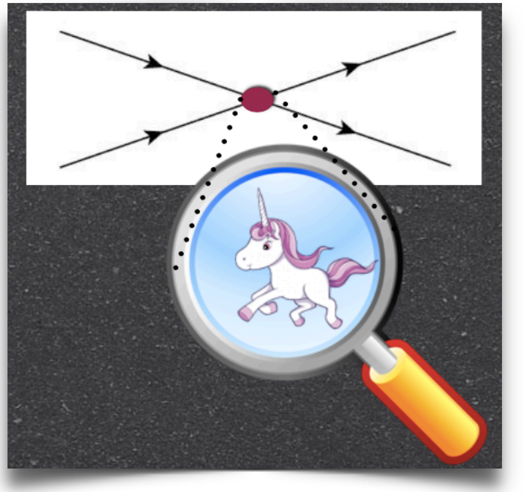
Blois 2019 June 2019



Martín González-Alonso
CERN-TH

Low-Energy Probes of New Physics

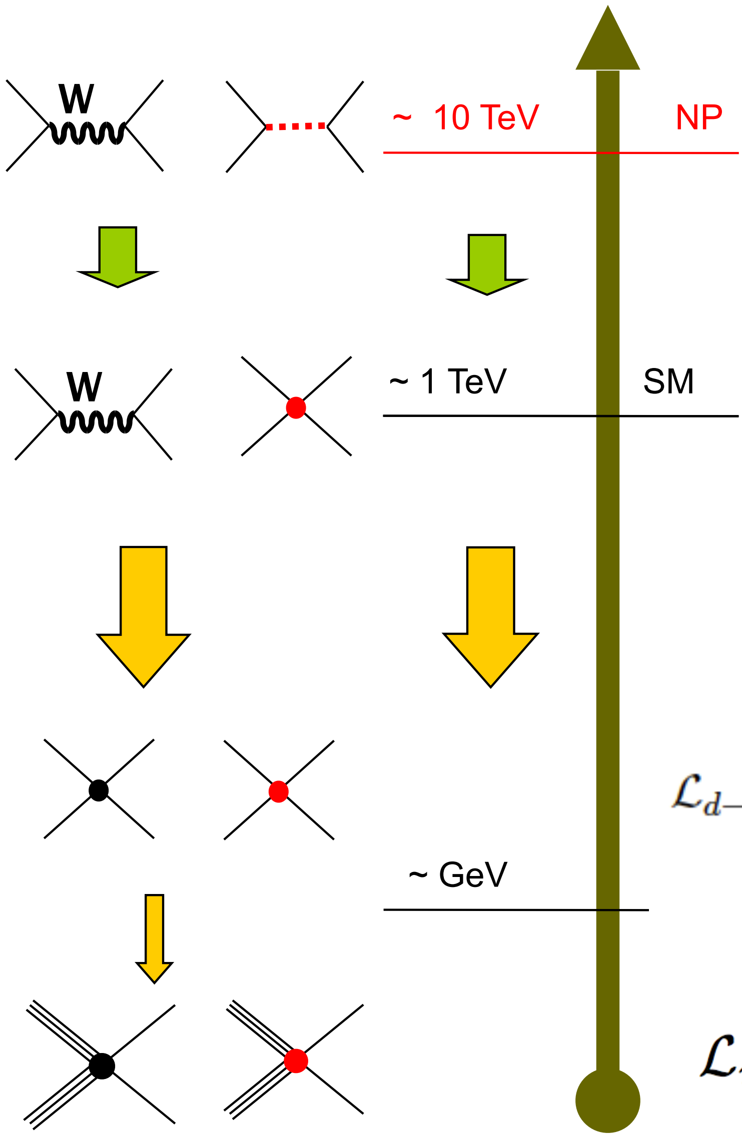
- I'll focus on precision measurements in non-forbidden processes:
  - Both exp & theory (lattice!) precision needed
  - Precision ~ 10^{-2} - 10^{-3} → Λ ~ O(1) TeV
  - Much higher scales if SM is suppressed ($\pi \to e\nu$, CPV, CKM, ...)



- Still a very wide subject:
  - Leptonic processes, flavor (kaons, B's, LFU, ...), ...
  - Nuclear decays, atomic PV, neutrino, ...
  - Z/W data (LEP & LHC), LEP2, top, Higgs, ... → low-energy?

- I'll assume "heavy NP" → Effective Field Theory

EFT 101



$$\mathcal{L}(x) = \mathcal{L}\,(\text{SM fields, bSM fields})$$

$$\mathcal{L}_{eff.} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2}\sum \alpha_i \mathcal{O}_i$$

Standard Model EFT

[Buchmuller & Wyler'86,
Leung et al.'86,
Grzadkowksi et al., 10,
Jenkins et al'13,
Alonso et al.'13, ...]

$$\mathcal{L}_{d\to u\ell^-\bar{\nu}_\ell} = -\frac{4G_F V_{ij}}{\sqrt{2}}\left[\bar{\ell}_L\gamma_\mu\nu\cdot\bar{u}\gamma^\mu d_L + \sum_{\rho\delta\Gamma}\epsilon^{\Gamma}_{\rho\delta}\,\bar{\ell}_\rho\Gamma\nu\cdot\bar{u}\Gamma d_\delta\right]$$

Low-energy EFT

[Cirigliano et al'09,
Aebischer al.'15,
Jenkins et al'18, ...]

$$\mathcal{L}_{\pi,N,\ldots} = \ldots$$

EFT: motivation

Take your favorite precision experiment:

→ Implications for NP model M?

$$O_{i,exp} - O_{i,SM} = f_i\,(g', M')$$

Nontrivial:

- Atomic/nuclear/hadronic/PDF TH;
- Correlations;
- Cuts, SM assumed?
- Large logs resummation

EFT: motivation

Take your favorite precision experiment:

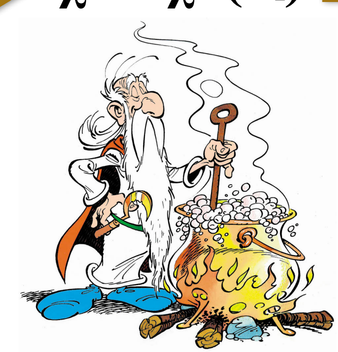
→ Implications for NP model M?

$$O_{i,exp} - O_{i,SM} = f_i\,(g', M')$$

Nontrivial:

- Atomic/nuclear/hadronic/PDF TH;
- Correlations;
- Cuts, SM assumed?
- Large logs resummation

$$O_{i,exp} - O_{i,SM} = \delta O\,(\alpha_1, \alpha_2, \ldots, \alpha_{80})$$ → $$\chi^2 = \chi^2(\alpha_i)$$ → Specific NP model $$\frac{\alpha_6^{(i)}}{\Lambda^2} = f_i\,(g_{NP}, M_{NP})$$



EFT as a model-independent framework to interpret, combine & compare low-E experiments (& a bridge to models)

EFT as a model-independent framework to interpret, combine & compare low-E experiments (& a bridge to models)

(Sort of) well known in many cases

Example: Electroweak Precision Data

EFT as a model-independent framework to interpret, combine & compare low-E experiments (& a bridge to models)

(Sort of) well known in many cases

Example: Electroweak Precision Data → Flavor-general (!!) SMEFT fit

[Efrati, Falkowski & Soreq, JHEP'15;
Falkowski & Mimouni, JHEP'16;
Falkowski, MGA & Mimouni, JHEP'17]

EWPO fit in the flavorful SMEFT

[Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
  - Z- & W-pole data
  - $e^+e^- \to l^+l^-$, qq
  - Low-energy processes:
    - Nuclear and hadron decays ($d \to ul\nu$)
    - Neutrino scattering
    - PV in atoms and in scattering
    - Leptonic tau decays

Observable	Experimental value	Ref.	SM prediction	Definition
Γ_Z [GeV]	2.4952 ± 0.0023	[47]	2.4950	$\sum_f \Gamma(Z \to ff)$
$\sigma_{\rm had}$ [nb]	41.541 ± 0.037	[47]	41.484	$\frac{12\pi}{m_Z^2}\frac{\Gamma(Z\to e^+e^-)\Gamma(Z\to q\bar{q})}{\Gamma_Z^2}$
R_e	20.804 ± 0.050	[47]	20.743	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to e^+e^-)}$
R_μ	20.785 ± 0.033	[47]	20.743	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to \mu^+\mu^-)}$
R_τ	20.764 ± 0.045	[47]	20.743	$\frac{\sum_q \Gamma(Z\to q\bar{q})}{\Gamma(Z\to \tau^+\tau^-)}$
$A_{\rm FB}^{0,e}$	0.0145 ± 0.0025	[47]	0.0163	$\frac{3}{4}A_e^2$
$A_{\rm FB}^{0,\mu}$	0.0169 ± 0.0013	[47]	0.0163	$\frac{3}{4}A_e A_\mu$
$A_{\rm FB}^{0,\tau}$	0.0188 ± 0.0017	[47]	0.0163	$\frac{3}{4}A_e A_\tau$
R_b	0.21629 ± 0.00066	[47]	0.21578	$\frac{\Gamma(Z\to bb)}{\sum_q \Gamma(Z\to q\bar{q})}$
R_c	0.1721 ± 0.0030	[47]	0.17226	$\frac{\Gamma(Z\to c\bar{c})}{\sum_q \Gamma(Z\to q\bar{q})}$
$A_b^{\rm FB}$	0.0992 ± 0.0016	[47]	0.1032	$\frac{3}{4}A_e A_b$
$A_c^{\rm FB}$	0.0707 ± 0.0035	[47]	0.0738	$\frac{3}{4}A_e A_c$
A_e	0.1516 ± 0.0021	[47]	0.1472	$\frac{\Gamma(Z\to e_L^+e_L^-)-\Gamma(Z\to e_R^+e_R^-)}{\Gamma(Z\to e^+e^-)}$
A_μ	0.142 ± 0.015	[47]	0.1472	$\frac{\Gamma(Z\to \mu_L^+\mu_L^-)-\Gamma(Z\to \mu_R^+\mu_R^-)}{\Gamma(Z\to \mu^+\mu^-)}$
A_τ	0.136 ± 0.015	[47]	0.1472	$\frac{\Gamma(Z\to \tau_L^+\tau_L^-)-\Gamma(Z\to \tau_R^+\tau_R^-)}{\Gamma(Z\to \tau^+\tau^-)}$
A_e	0.1498 ± 0.0049	[47]	0.1472	$\frac{\Gamma(Z\to e_L^+e_L^-)-\Gamma(Z\to e_R^+e_R^-)}{\Gamma(Z\to \tau^+\tau^-)}$
A_τ	0.1439 ± 0.0043	[47]	0.1472	$\frac{\Gamma(Z\to \tau_L^+\tau_L^-)-\Gamma(Z\to \tau_R^+\tau_R^-)}{\Gamma(Z\to \tau^+\tau^-)}$
A_b	0.923 ± 0.020	[47]	0.935	$\frac{\Gamma(Z\to b_L b_L)-\Gamma(Z\to b_R b_R)}{\Gamma(Z\to b\bar{b})}$
A_c	0.670 ± 0.027	[47]	0.668	$\frac{\Gamma(Z\to c_L\bar{c}_L)-\Gamma(Z\to c_R\bar{c}_R)}{\Gamma(Z\to c\bar{c})}$
A_s	0.895 ± 0.091	[48]	0.935	$\frac{\Gamma(Z\to s_L\bar{s}_L)-\Gamma(Z\to s_R\bar{s}_R)}{\Gamma(Z\to s\bar{s})}$
R_{uc}	0.166 ± 0.009	[45]	0.1724	$\frac{\Gamma(Z\to u\bar{u})+\Gamma(Z\to c\bar{c})}{2\sum_q \Gamma(Z\to q\bar{q})}$

Class	Observable	Exp. value
$\nu_e\nu_e qq$	$R_{\nu_e\bar{\nu}_e}$	0.41(14)
$\nu_\mu\nu_\mu qq$	$(g_L^{\nu_\mu})^2$	0.3005(28)
	$(g_R^{\nu_\mu})^2$	0.0329(30)
	$\theta_L^{\nu_\mu}$	2.500(35)
	$\theta_R^{\nu_\mu}$	$4.56^{+0.42}_{-0.27}$
PV low-E $eeqq$	$g_{AV}^{eu} + 2g_{AV}^{ed}$	0.489(5)
	$2g_{AV}^{eu} - g_{AV}^{ed}$	−0.708(16)
	$2g_{VA}^{eu} - g_{VA}^{ed}$	−0.144(68)
	$g_{VA}^{eu} - g_{VA}^{ed}$	−0.042(57) −0.120(74)
PV low-E $\mu\mu qq$	$b_{\rm SPS}(\lambda = 0.81)$	$-1.47(42) \cdot 10^{-4}$
	$b_{\rm SPS}(\lambda = 0.66)$	$-1.74(81) \cdot 10^{-4}$
$d(s) \to u\ell\nu$	$\epsilon_i^{d_j\ell}$	eq. (3.17)
$e^+e^- \to q\bar{q}$	$\sigma(q\bar{q})$	$f(\sqrt{s})$
	σ_c, σ_b	
	A_{FB}^{cc}, A_{FB}^{bb}	

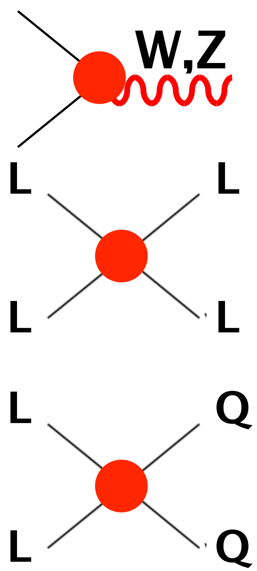
Class	Observable	Exp. value
$\nu_\mu\nu_\mu ee$	$g_{LV}^{\nu_\mu e}$	−0.040(15)
	$g_{LA}^{\nu_\mu e}$	−0.507(14)
$e^-e^- \to e^-e^-$	g_{AV}^{ee}	0.0190(27)
$\nu_\mu\gamma^* \to \nu_\mu\mu^+\mu^-$	$\frac{\sigma}{\sigma_{\rm SM}}$	1.58(57) 0.82(28)
$\tau \to \ell\nu\nu$	$G_{\tau e}^2/G_F^2$	1.0029(46)
	$G_{\tau\mu}^2/G_F^2$	0.981(18)
$e^+e^- \to \ell^+\ell^-$	$\frac{d\sigma(ee)}{d\cos\theta}$	$f(\sqrt{s})$
	$\sigma_\mu, \sigma_\tau, \mathcal{P}_\tau$	
	A_{FB}^μ, A_{FB}^τ	

Observable	Experimental value	Ref.	SM prediction	Definition
m_W [GeV]	80.385 ± 0.015	[50]	80.364	$\frac{g_L v}{2}(1+\delta m)$
Γ_W [GeV]	2.085 ± 0.042	[45]	2.091	$\sum_f \Gamma(W \to ff')$
$\mathrm{Br}(W \to e\nu)$	0.1071 ± 0.0016	[51]	0.1083	$\frac{\Gamma(W\to e\nu)}{\sum_f \Gamma(W\to ff')}$
$\mathrm{Br}(W \to \mu\nu)$	0.1063 ± 0.0015	[51]	0.1083	$\frac{\Gamma(W\to \mu\nu)}{\sum_f \Gamma(W\to ff')}$
$\mathrm{Br}(W \to \tau\nu)$	0.1138 ± 0.0021	[51]	0.1083	$\frac{\Gamma(W\to \tau\nu)}{\sum_f \Gamma(W\to ff')}$
R_{Wc}	0.49 ± 0.04	[45]	0.50	$\frac{\Gamma(W\to cs)}{\Gamma(W\to ud)+\Gamma(W\to cs)}$
R_σ	0.998 ± 0.041	[52]	1.000	$g_L^{Wq_3}/g_{L,\rm SM}^{Wq_3}$

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)$$

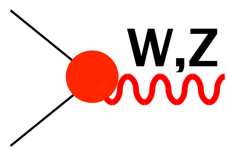


Results given at the EW scale
(QEDxQCD running included in precise low-E observables)
[MGA, M. Camalich & Mimouni, PLB'17]

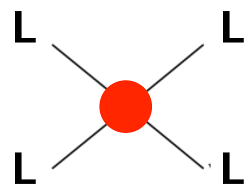
EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

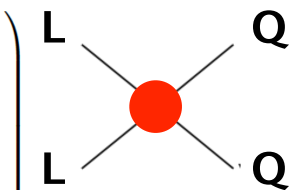
$$\mathbf{O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \longrightarrow \chi^2 = \chi^2\,(c_i)}$$



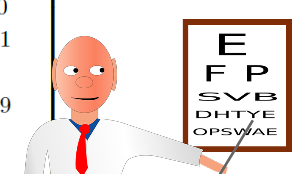
$$\begin{pmatrix} \delta g_L^{We} \\ \delta g_L^{W\mu} \\ \delta g_L^{W\tau} \\ \delta g_L^{Ze} \\ \delta g_L^{Z\mu} \\ \delta g_L^{Z\tau} \\ \delta g_R^{Ze} \\ \delta g_R^{Z\mu} \\ \delta g_R^{Z\tau} \\ \delta g_L^{Zu} \\ \delta g_L^{Zc} \\ \delta g_L^{Zt} \\ \delta g_R^{Zu} \\ \delta g_R^{Zc} \\ \delta g_L^{Zd} \\ \delta g_L^{Zs} \\ \delta g_L^{Zb} \\ \delta g_R^{Zd} \\ \delta g_R^{Zs} \\ \delta g_R^{Zb} \\ \delta g_R^{Wq_1} \end{pmatrix} = \begin{pmatrix} -1.00 \pm 0.64 \\ -1.36 \pm 0.59 \\ 1.95 \pm 0.79 \\ -0.023 \pm 0.028 \\ 0.01 \pm 0.12 \\ 0.018 \pm 0.059 \\ -0.033 \pm 0.027 \\ 0.00 \pm 0.14 \\ 0.042 \pm 0.062 \\ -0.8 \pm 3.1 \\ -0.15 \pm 0.36 \\ -0.3 \pm 3.8 \\ 1.4 \pm 5.1 \\ -0.35 \pm 0.53 \\ -0.9 \pm 4.4 \\ 0.9 \pm 2.8 \\ 0.33 \pm 0.17 \\ 3 \pm 16 \\ 3.4 \pm 4.9 \\ 2.30 \pm 0.88 \\ -1.3 \pm 1.7 \end{pmatrix} \times 10^{-2}.$$



$$\begin{pmatrix} [c_{\ell\ell}]_{1111} \\ [c_{\ell e}]_{1111} \\ [c_{ee}]_{1111} \\ [c_{\ell\ell}]_{1221} \\ [c_{\ell\ell}]_{1122} \\ [c_{\ell e}]_{1122} \\ [c_{\ell e}]_{2211} \\ [c_{ee}]_{1122} \\ [c_{\ell\ell}]_{1331} \\ [c_{\ell\ell}]_{1133} \\ [c_{\ell e}]_{1133} \\ [c_{\ell e}]_{3311} \\ [c_{ee}]_{1133} \\ [\hat{c}_{\ell\ell}]_{2222} \\ [c_{\ell\ell}]_{2332} \end{pmatrix} = \begin{pmatrix} 1.01 \pm 0.38 \\ -0.22 \pm 0.22 \\ 0.20 \pm 0.38 \\ -4.8 \pm 1.6 \\ 1.5 \pm 2.1 \\ 1.5 \pm 2.2 \\ -1.4 \pm 2.2 \\ 3.4 \pm 2.6 \\ 1.5 \pm 1.3 \\ 0 \pm 11 \\ -2.3 \pm 7.2 \\ 1.7 \pm 7.2 \\ -1 \pm 12 \\ -2 \pm 21 \\ 3.0 \pm 2.3 \end{pmatrix} \times 10^{-2}.$$



$$\begin{pmatrix} [c_{\ell q}^{(3)}]_{1111} \\ [\hat{c}_{eq}]_{1111} \\ [\hat{c}_{\ell u}]_{1111} \\ [\hat{c}_{\ell d}]_{1111} \\ [\hat{c}_{eu}]_{1111} \\ [\hat{c}_{ed}]_{1111} \\ [\hat{c}_{\ell q}^{(3)}]_{1122} \\ [c_{\ell u}]_{1122} \\ [\hat{c}_{\ell d}]_{1122} \\ [c_{eq}]_{1122} \\ [c_{eu}]_{1122} \\ [\hat{c}_{ed}]_{1122} \\ [\hat{c}_{\ell q}^{(3)}]_{1133} \\ [c_{\ell d}]_{1133} \\ [c_{eq}]_{1133} \\ [c_{ed}]_{1133} \\ [c_{\ell q}^{(3)}]_{2211} \\ [c_{\ell q}]_{2211} \\ [c_{\ell u}]_{2211} \\ [c_{\ell d}]_{2211} \\ [\hat{c}_{eq}]_{22\ldots} \\ [c_{\ell equ}]_{1\ldots} \\ [c_{\ell edq}]_{1\ldots} \\ [c_{\ell equ}^{(3)}]_{11\ldots} \\ \epsilon_P^{d\mu}(2\ \mathrm{GeV}) \end{pmatrix} = \begin{pmatrix} -2.2 \pm 3.2 \\ 100 \pm 180 \\ -5 \pm 11 \\ -5 \pm 23 \\ -1 \pm 12 \\ -4 \pm 21 \\ -61 \pm 32 \\ 2.4 \pm 8.0 \\ -310 \pm 130 \\ -21 \pm 28 \\ -87 \pm 46 \\ 270 \pm 140 \\ -8.6 \pm 8.0 \\ -1.4 \pm 10 \\ -3.2 \pm 5.1 \\ 18 \pm 20 \\ -1.2 \pm 3.9 \\ 1.3 \pm 7.6 \\ 15 \pm 12 \\ \\ \\ \\ \\ \\ \\ \end{pmatrix} \times 10^{-2}.$$



Bounds: 10^{-4} - O(1)

[c= 10^{-2} ⟶ Λ=2.5 TeV]

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

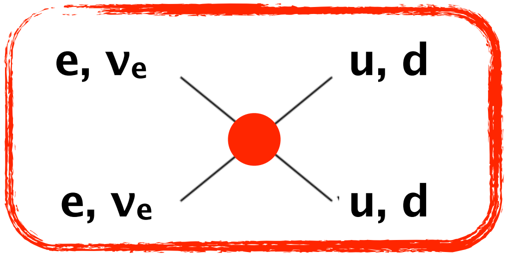
- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$\mathbf{O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)}$$

- Public likelihood: $\chi^2 = \chi^2\,(c_i)$
  www.dropbox.com/s/26nh71oebm4o12k/SMEFTlikelihood.nb?dl=0

- It allows us to study the interplay of experiments in a more general setup

  → eeqq: best bounds come from APV or CKM-unitarity!
  [competitive with LHC]



$$(\bar{\ell}_1 \gamma_\mu \ell_1)(\bar{q}_1 \gamma^\mu q_1)$$

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

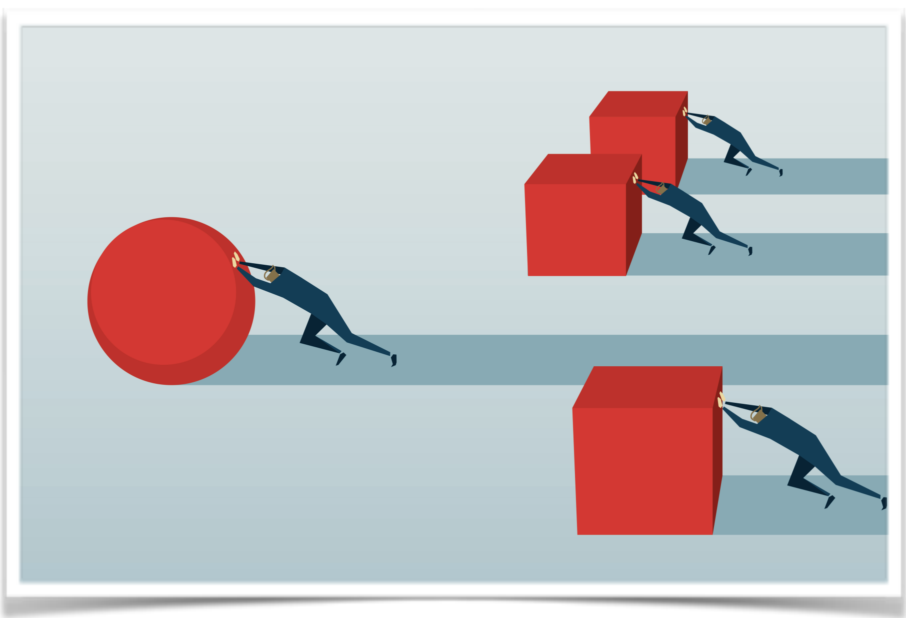
- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)$$

RGE!

Specific NP model

$$\frac{\alpha_6^{(i)}}{\Lambda^2} = f_i\,(g_{NP}, M_{NP})$$



Extra-dims [Megías et., 1703.06019],
Z' flavor gauge bosons [Cline & Camalich, 1706.08510],
Minimal Z' models [Alioli et al., 1712.02347],
...

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)$$

- **Can't we do the same with flavor data?**
  - Practical complication: hadronic FF

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)$$

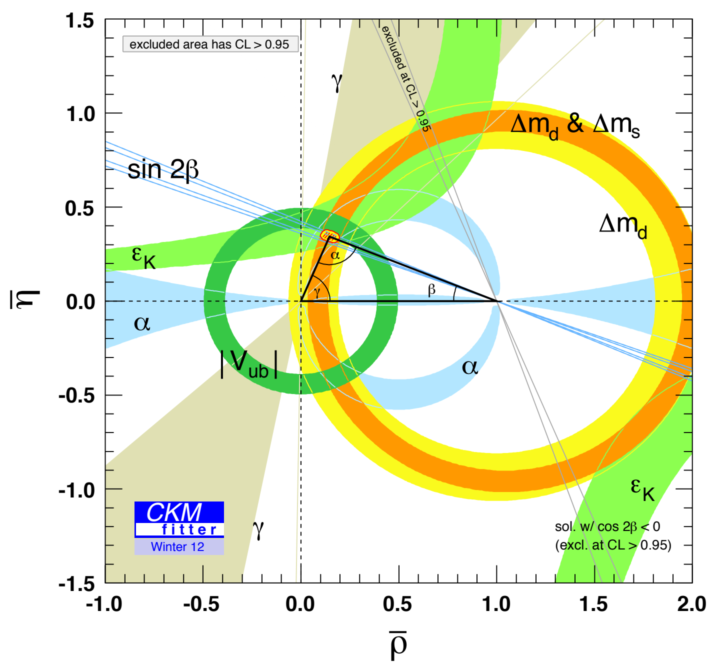
- **Can't we do the same with flavor data?**
  - Practical complication: hadronic FF
  - LFU ratios: intense activity recently ("B anomalies")
    [Aebischer et al'19, Algueró et al'19, Ciuchini et al.'19, Arbey et al.'19, ...]

EWPO fit in the flavorful SMEFT [Falkowski, MGA & Mimouni, 2017]

- 258 experimental input
- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$O = O_{SM} + O\,(c_1, c_2, \ldots, c_{80}) \rightarrow \chi^2 = \chi^2\,(c_i)$$

- **Can't we do the same with flavor data?**
  - Practical complication: hadronic FF
  - LFU ratios: intense activity recently ("B anomalies")
    [Aebischer et al'19, Algueró et al'19, Ciuchini et al.'19, Arbey et al.'19, ...]
  - UV meaning of the famous CKM-triangle plot?
    Still to be done in the general SMEFT
    The presence of CKM factors requires some care
    [Descotes-Genon, Falkowski, Fedele, MGA, & Virto, '19]



EFT as a model-independent framework
to interpret, combine & compare
low-E experiments
(& a bridge to models)

(Sort of) well known in many cases
Example: Electroweak Precision Data

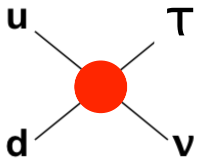
EFT as a model-independent framework to interpret, combine & compare low-E experiments (& a bridge to models)

(Sort of) well known in many cases

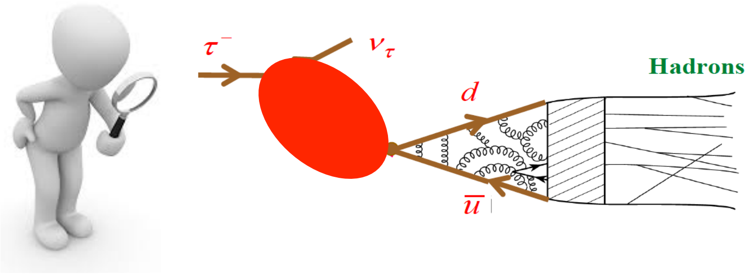
Example: Electroweak Precision Data

Not so much in others

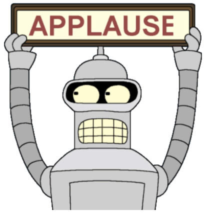
Ex. #1: Hadronic Tau decays



Hadronic tau decays as NP probes

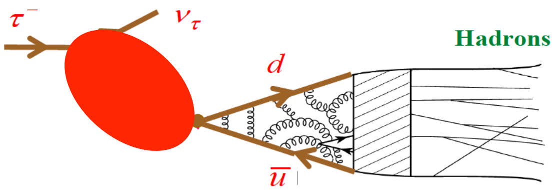


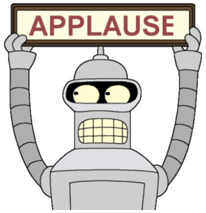
- Great EXP & TH precision used in the past to extract SM quantities: α_s, V_{us}, $(g\text{-}2)_{\mu,had}$, …



- UV meaning?

Hadronic tau decays as NP probes



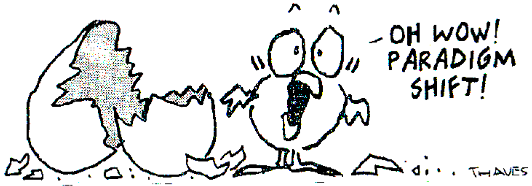


- Great EXP & TH precision used in the past to extract SM quantities: α_s, V_{us}, $(g\text{-}2)_{\mu,\text{had}}$, …

- UV meaning?

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} \;=\; & -\frac{G_F V_{ud}}{\sqrt{2}} \Bigg[\left(1+\epsilon_L^{d\tau}\right) \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma_5)d \\
& +\epsilon_R^{d\tau}\; \bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau\; \bar{u}\gamma^\mu(1+\gamma_5)d \\
& +\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}\left[\epsilon_S^{d\tau} - \epsilon_P^{d\tau}\gamma_5\right] d \\
& +\epsilon_T^{d\tau}\; \bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5)d \Bigg] + \text{h.c.}
\end{aligned}
$$

Cirigliano et al. '10



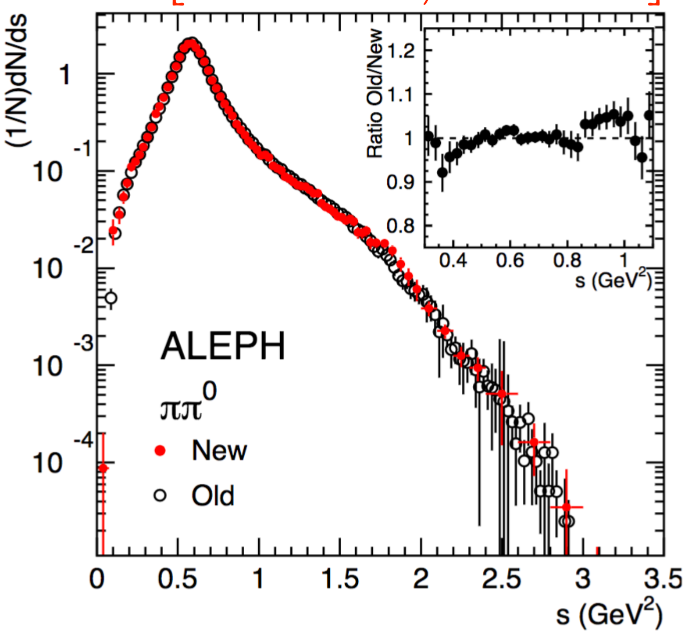
[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19 (in press)]

$\tau \longrightarrow \pi\pi\nu$ as a NP probe

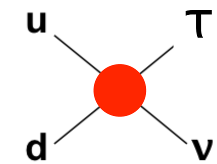
[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

[Davier et al., 1312.1501]



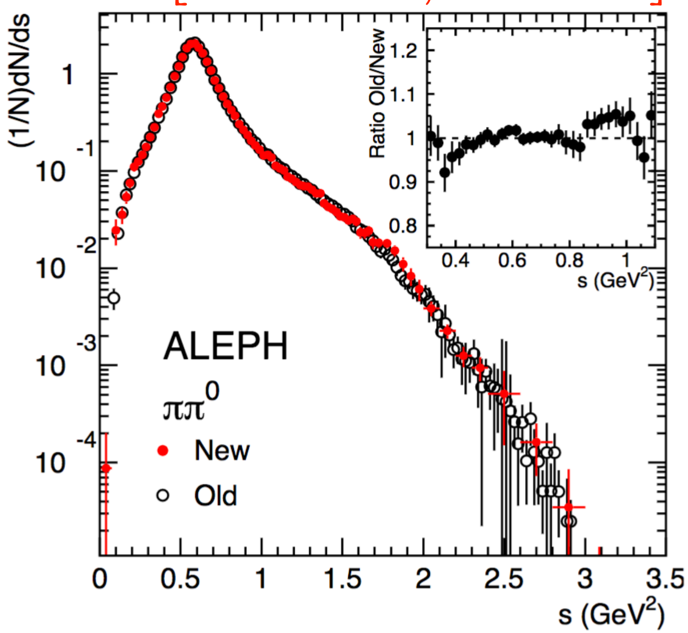
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



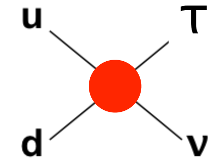
[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

[Davier et al., 1312.1501]



... But the QCD description is more involved
→ Hadronic physics probe;

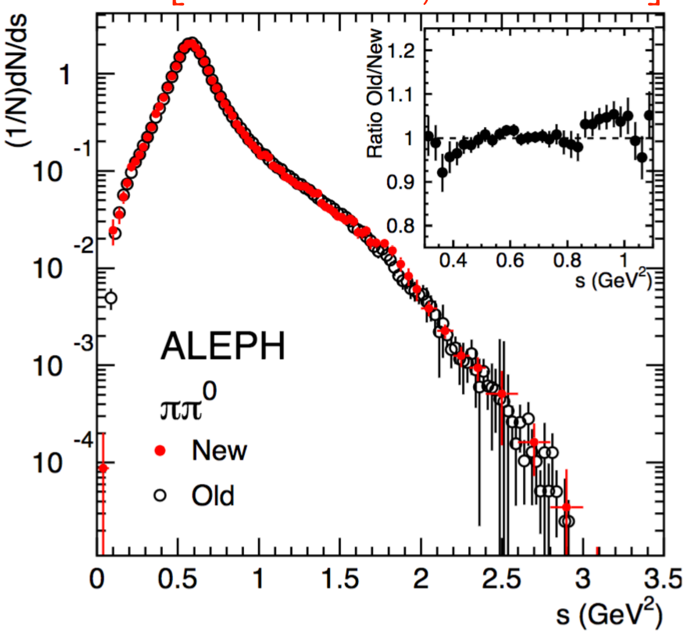


$\tau \to \pi\pi\nu$ as a NP probe

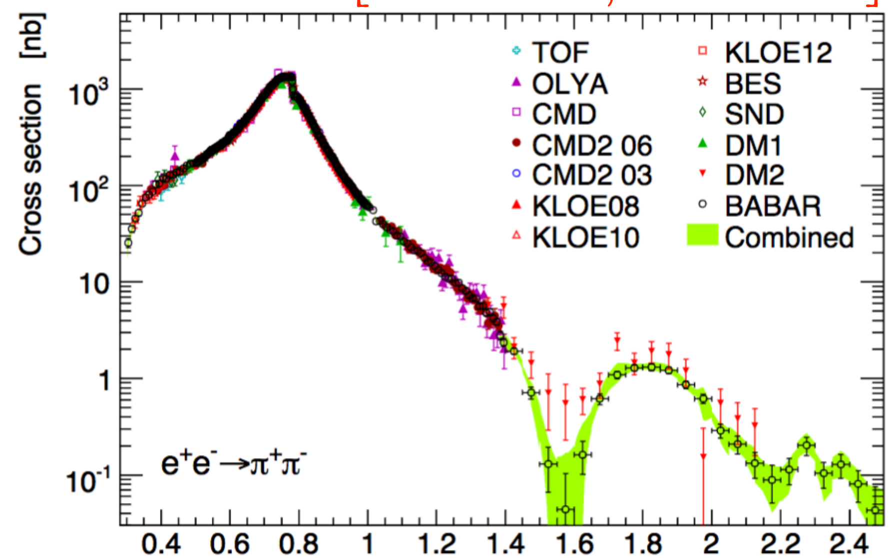
[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

[Davier et al., 1312.1501]



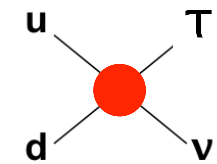
[Davier et al., 1706.09436]



[+ SU(2) breaking corrections]

~~... But the QCD description is more involved~~
~~→ Hadronic physics probe;~~

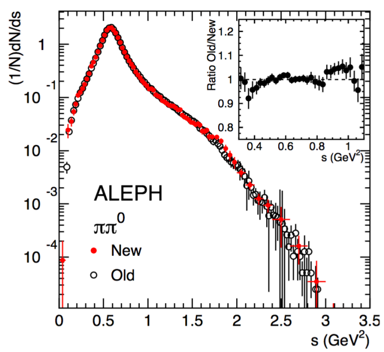
Way out:
To extract the SM value from $e^+e^- \to \pi\pi$
(which is free of heavy NP)!



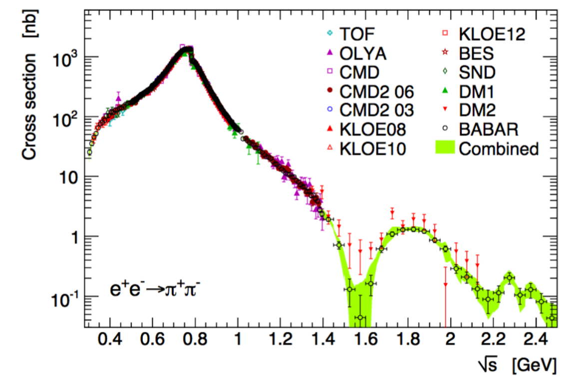
$\tau \longrightarrow \pi\pi\nu$ as a NP probe

[V. Cirigliano, A. Falkowski, MGA, &
A. Rodríguez-Sánchez, PRL'19]

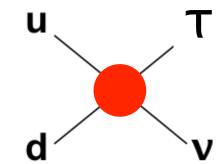
- Precise data;



vs.

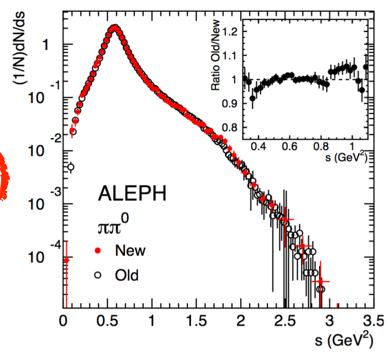


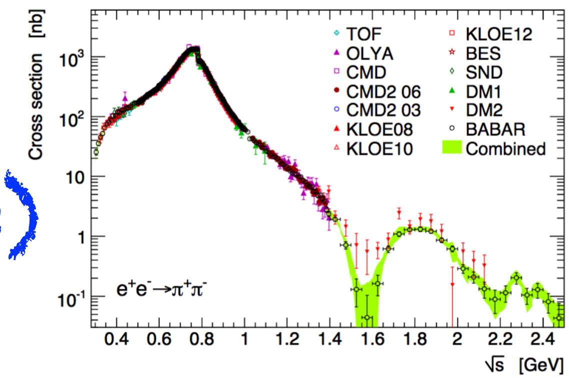
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

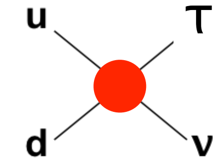
- Precise data;

$\int$ K(s) 

ds vs. $\int$ K(s) 

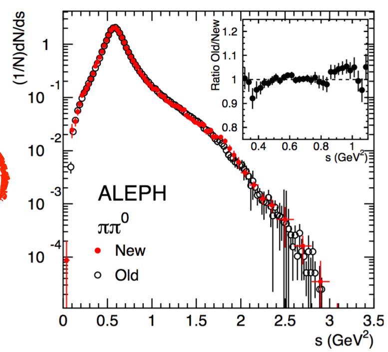
ds

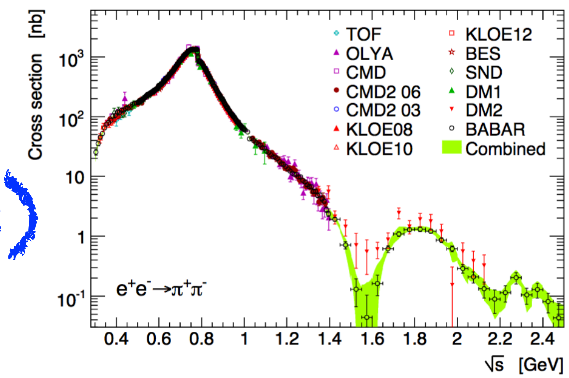
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

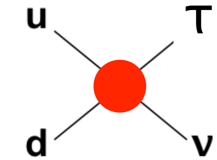
$\int K(s)$


ds vs. $\int K(s)$


ds

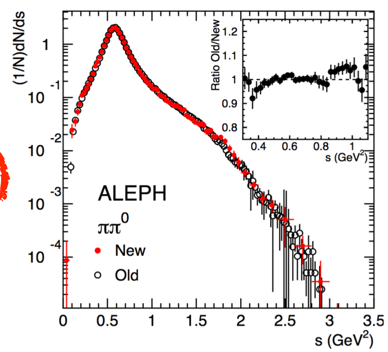
$$a_\mu^{had,\,LO}[\pi\pi]_{\tau\text{-data}} \qquad a_\mu^{had,\,LO}[\pi\pi]_{e^+e^-\text{-data}}$$

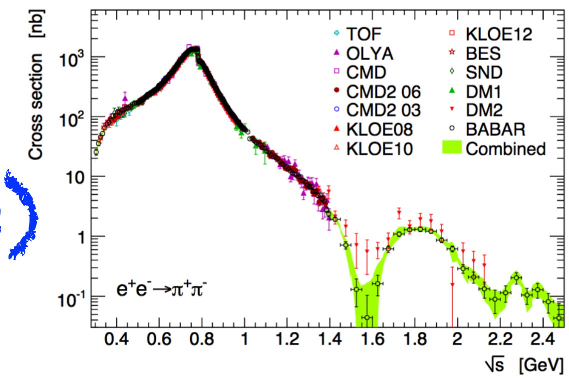
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

$\int K(s)$ 

ds vs. $\int K(s)$ 

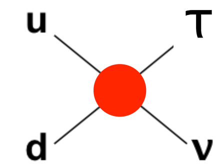
ds

$a_\mu^{had,\, LO}[\pi\pi]_{\tau\text{-data}}$ $\qquad$ $a_\mu^{had,\, LO}[\pi\pi]_{e^+e^-\text{-data}}$

Using [Davier et al., 1706.09436]:

$$\frac{a_\mu^\tau - a_\mu^{ee}}{2\, a_\mu^{ee}} = \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e + 1.7\, \epsilon_T^\tau = (8.9 \pm 4.4)\cdot 10^{-3}$$

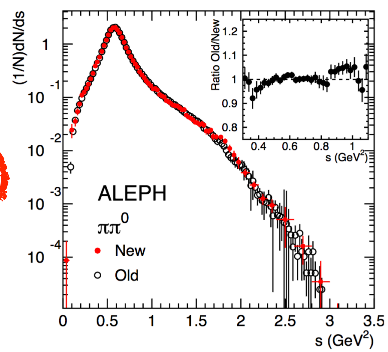
Main error: EXP !

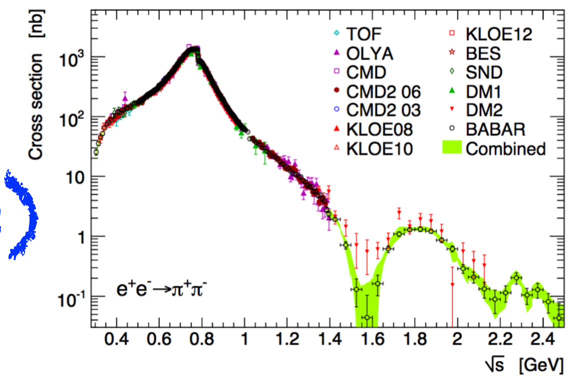


$\tau \longrightarrow \pi\pi\nu$ as a NP probe

[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

- Precise data;

$\int K(s)$ 

ds vs. $\int K(s)$ 

ds

$a_\mu^{\text{had, LO}}[\pi\pi]_{\tau\text{-data}}$ $\qquad$ $a_\mu^{\text{had, LO}}[\pi\pi]_{e^+e^-\text{-data}}$

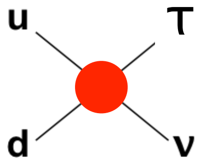
Using [Davier et al., 1706.09436]:

$$\frac{a_\mu^\tau - a_\mu^{ee}}{2\, a_\mu^{ee}} = \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e + 1.7\, \epsilon_T^\tau = (8.9 \pm 4.4) \cdot 10^{-3}$$

Main error: EXP !

- More data coming (& better agreement...);
- Lattice input too [M. Bruno et al., 1811.00508]
- Full spectrum available

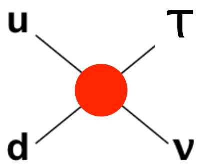
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, &
A. Rodríguez-Sánchez, PRL'19]

$$\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e + 1.7\,\epsilon_T^\tau = (8.9 \pm 4.4) \cdot 10^{-3},$$

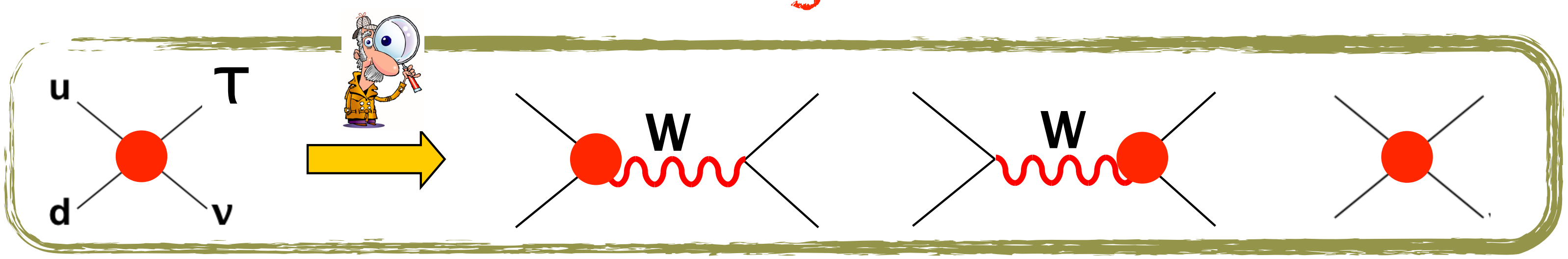
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

$$\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e + 1.7\,\epsilon_T^\tau = (8.9 \pm 4.4) \cdot 10^{-3},$$

SMEFT matching



+ RGE running

[MGA, M. Camalich & Mimouni, PLB'17]

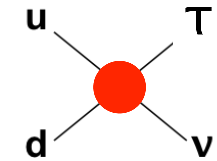
$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee11}$$

$$\epsilon_R^\tau = \delta g_R^{Wq_1},$$

$$\epsilon_{S,P}^\tau = -\frac{1}{2}[c_{lequ} \pm c_{ledq}]^*_{\tau\tau 11} ,$$

$$\epsilon_T^\tau = -\frac{1}{2}[c_{lequ}^{(3)}]^*_{\tau\tau 11} .$$

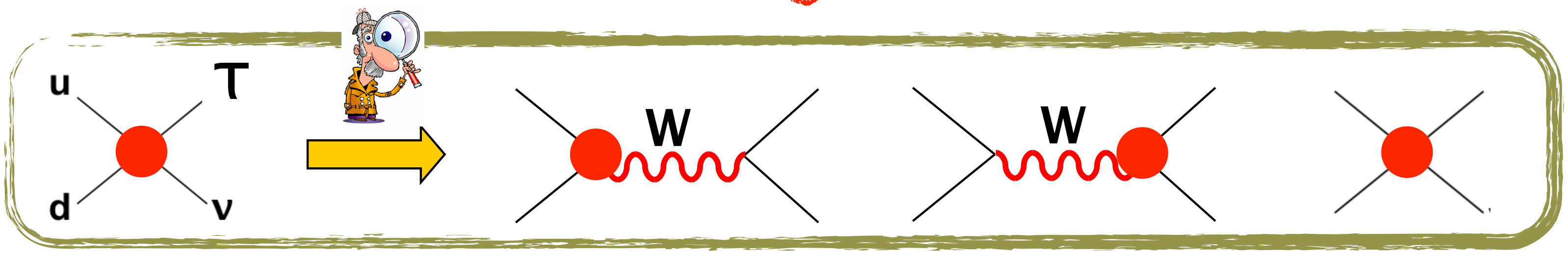
$\tau \longrightarrow \pi\pi\nu$ as a NP probe



[V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]

$$\epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e + 1.7\,\epsilon_T^\tau = (8.9 \pm 4.4) \cdot 10^{-3}$$

SMEFT matching



$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee11}$$

$$\epsilon_R^\tau = \delta g_R^{Wq_1},$$

$$\epsilon_{S,P}^\tau = -\frac{1}{2}[c_{lequ} \pm c_{ledq}]^*_{\tau\tau 11}\ ,$$

$$\epsilon_T^\tau = -\frac{1}{2}[c_{lequ}^{(3)}]^*_{\tau\tau 11}\ .$$

+ RGE running

[MGA, M. Camalich & Mimouni, PLB'17]

+ comparison with other EWPO, LHC, models, ...

EFT as a model-independent framework to interpret, combine & compare low-E experiments (& a bridge to models)

(Sort of) well known in many cases

Example: Electroweak Precision Data

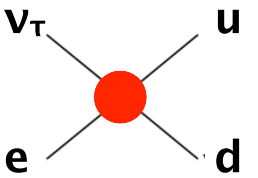
Not so much in others

Ex. #1: Hadronic Tau decays (no access to τqq in the previous EWPO fit)

Ex. #2: Reactor neutrino oscillations

[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

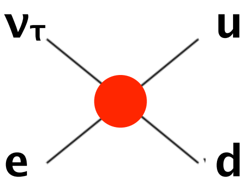
NP bounds from Neutrino Oscillation data



[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

- Similar to flavor physics: $O = O\,(\theta_i,\, \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i,\, \Delta m^2,\, \varepsilon_j)$

NP bounds from Neutrino Oscillation data

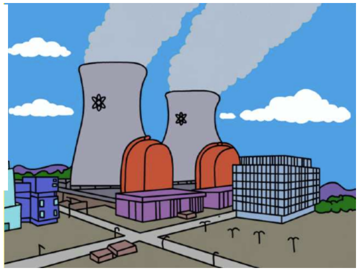


[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

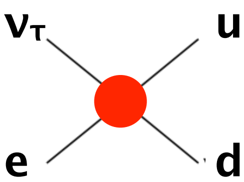
- Similar to flavor physics: $O = O\,(\theta_i,\ \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i,\ \Delta m^2,\ \epsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) = 1 - \sin^2\left(\frac{\Delta m^2_{31} L}{4 E_\nu}\right) \sin^2\left(2\theta_{13}\right)$$

[PS: no anomaly in far/near ratios]



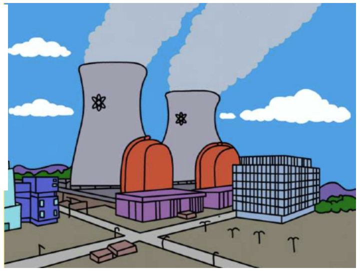
NP bounds from Neutrino Oscillation data



[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

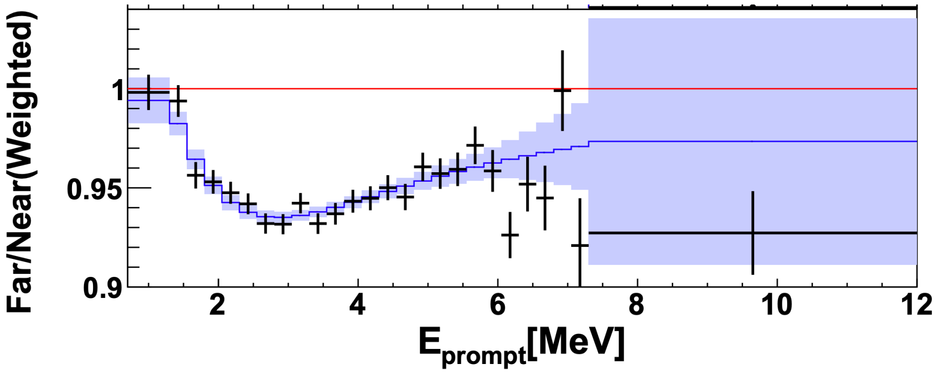
- Similar to flavor physics: $O = O\ (\theta_i, \Delta m^2)$
- NP constrained by the observed consistency: $O = O\ (\theta_i, \Delta m^2, \varepsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) \;=\; 1 - \sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right)\sin^2\left(2\theta_{13}\right)$$

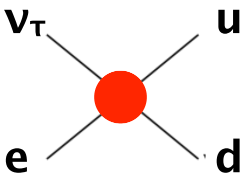


[PS: no anomaly in far/near ratios]

- Precision: $\theta_{13} = 0.0856(29)$
  [DayaBay'18, ~4M neutrino events!]



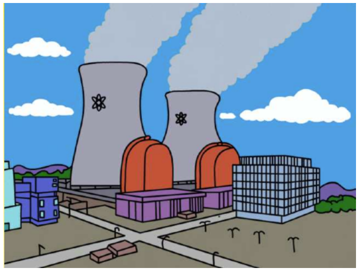
NP bounds from Neutrino Oscillation data



[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

- Similar to flavor physics: $O = O\,(\theta_i,\ \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i,\ \Delta m^2,\ \varepsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

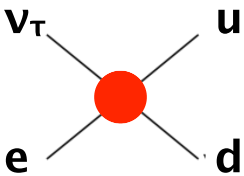
$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) \;=\; 1 - \sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right) \sin^2\left(2\theta_{13}\right)$$



  - Precision: $\theta_{13} = 0.0856(29)$
    [DayaBay'18, ~4M neutrino events!]

  - Again: UV-meaning of the good agreement with the SM?

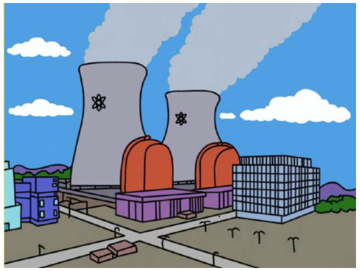
NP bounds from Neutrino Oscillation data



[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

- Similar to flavor physics: $O = O\,(\theta_i,\ \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i,\ \Delta m^2,\ \epsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

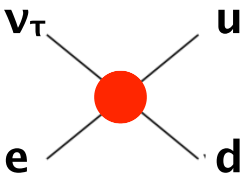
$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) = 1 - \sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right) \sin^2\left(2\theta_{13}\right)$$



  - Precision: $\theta_{13} = 0.0856(29)$
    [DayaBay'18, ~4M neutrino events!]

  - Again: UV-meaning of the good agreement with the SM?
    - No access to NC NSI (negligible matter effects): ννqq

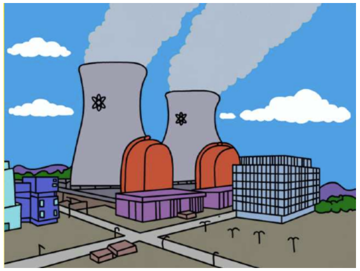
NP bounds from Neutrino Oscillation data



[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

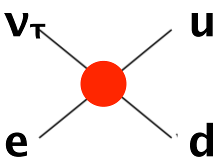
- Similar to flavor physics: $O = O\,(\theta_i, \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i, \Delta m^2, \epsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

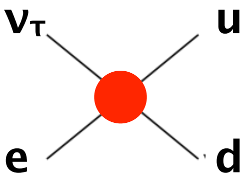
$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) \quad = \quad 1 - \sin^2\left(\frac{\Delta m^2_{31} L}{4E_\nu}\right)\sin^2\left(2\theta_{13}\right)$$



- Precision: $\theta_{13} = 0.0856(29)$
  [DayaBay'18, ~4M neutrino events!]

- Again: UV-meaning of the good agreement with the SM?
  - No access to NC NSI (negligible matter effects): ννqq
  - Non-standard V-A ($e_L \gamma_\mu \nu_\tau \; u_L \gamma^\mu d_L$) gets hidden: $\theta_{13} \to \theta'_{13}$ [Ohlsson-Zhang'09]



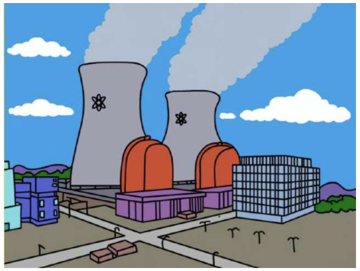


NP bounds from Neutrino Oscillation data

[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

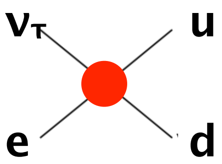
- Similar to flavor physics: $O = O\,(\theta_i,\ \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i,\ \Delta m^2,\ \epsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

$$\begin{aligned} P_{\bar\nu_e\to\bar\nu_e}(L,E_\nu) &= 1-\sin^2\left(\frac{\Delta m^2_{31}L}{4E_\nu}\right)\sin^2\left(2\theta_{13}-\alpha_D\frac{m_e}{E_\nu-\Delta}-\alpha_P\frac{m_e}{f_T(E_\nu)}\right) \\ &+ \sin\left(\frac{\Delta m^2_{31}L}{2E_\nu}\right)\sin(2\theta_{13})\left(\beta_D\frac{m_e}{E_\nu-\Delta}-\beta_P\frac{m_e}{f_T(E_\nu)}\right)+\mathcal{O}(\epsilon_X^2) \end{aligned}$$

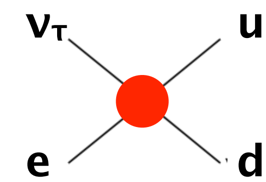


  - Precision: $\theta_{13} = 0.0856(29)$
    [DayaBay'18, ~4M neutrino events!]

  - Again: UV-meaning of the good agreement with the SM?
    - No access to NC NSI (negligible matter effects): ννqq
    - Non-standard V-A ($e_L\gamma_\mu\nu_\tau\ u_L\gamma^\mu d_L$) gets hidden: $\theta_{13} \rightarrow \theta'_{13}$ [Ohlsson-Zhang'09]
    - S, T and Im(V+A) can be probed



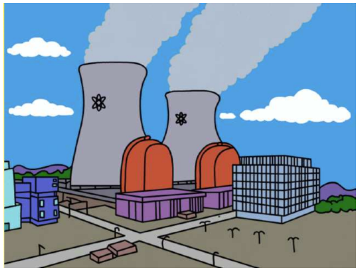
NP bounds from Neutrino Oscillation data

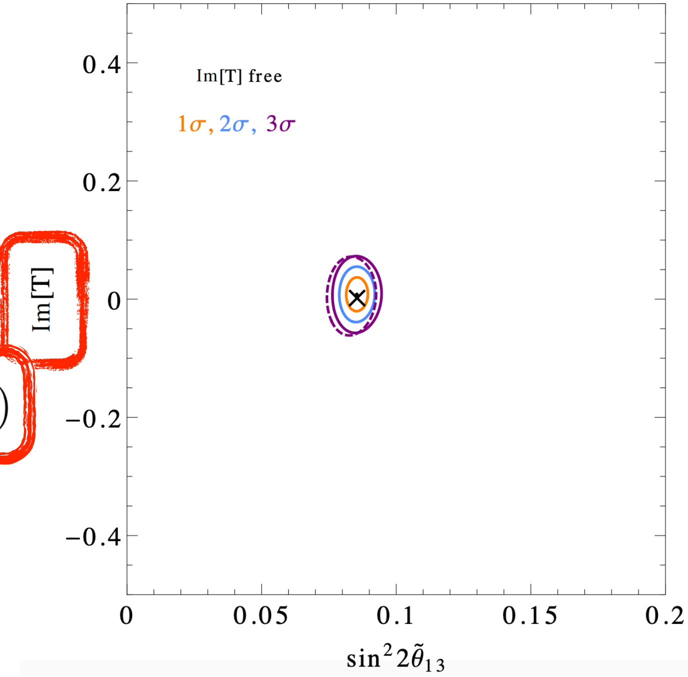


[A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]

- Similar to flavor physics: $O = O\,(\theta_i, \Delta m^2)$
- NP constrained by the observed consistency: $O = O\,(\theta_i, \Delta m^2, \epsilon_j)$
- Concrete example:
  short-baseline reactor neutrino experiments

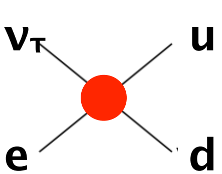
$$
\begin{aligned}
P_{\bar\nu_e\to\bar\nu_e}(L,E_\nu) &= 1-\sin^2\left(\frac{\Delta m^2_{31}L}{4E_\nu}\right)\sin^2\left(2\theta_{13}-\alpha_D\frac{m_e}{E_\nu-\Delta}-\alpha_P\frac{m_e}{f_T(E_\nu)}\right)\\
&+\sin\left(\frac{\Delta m^2_{31}L}{2E_\nu}\right)\sin(2\theta_{13})\left(\beta_D\frac{m_e}{E_\nu-\Delta}-\beta_P\frac{m_e}{f_T(E_\nu)}\right)+\mathcal{O}(\epsilon_X^2)
\end{aligned}
$$



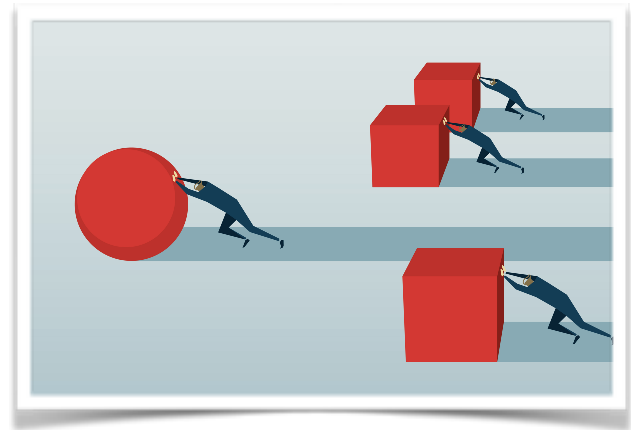


% level bounds (TeV scale)

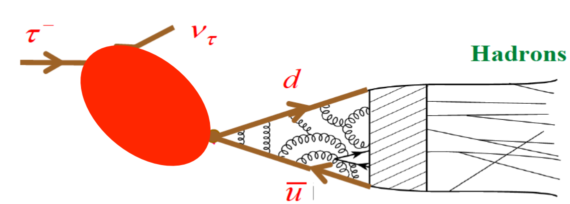
$$[Q^{(3)}_{\ell equ}]_{\alpha 111} = (\bar\ell^m_\alpha\sigma_{\mu\nu}e_1)\epsilon_{mn}(\bar q^n_1\sigma^{\mu\nu}u_1)$$



Summary

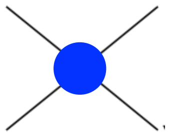


- Many precious low-E precision measurements
- The (SM)EFT is an ***efficient*** framework to combine / compare / interpret precision low-E experiments
- Intense activity in recent years:
EFT basis, RGEs, **global fits**, BSM matching, ...
- The UV information of many precision measurements has not been explored:
    - $\tau \longrightarrow \pi\pi\nu$
    [V. Cirigliano, A. Falkowski, MGA, & A. Rodríguez-Sánchez, PRL'19]
    - Reactor neutrinos
    [A. Falkowski, MGA, & Z. Tabrizi, JHEP'19]



Backups

eeqq interactions



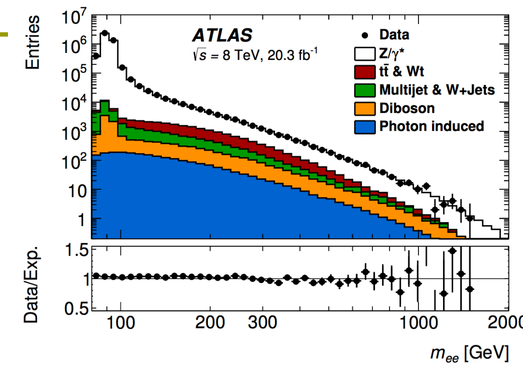
$$\bar{\ell}_1 \gamma_\mu \ell_1 \cdot \bar{q}_1 \gamma^\mu q_1$$

$c_{lq} \times 10^3$

APV	1.6 ± 1.1
QWEAK	-2.3 ± 4.0
PVDIS	24 ± 35
LEP-2	-42 ± 28

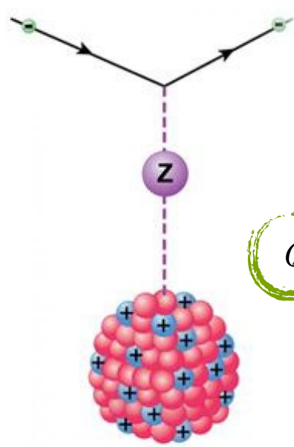
LHC	$2.5^{+1.9}_{-2.5}$

pp→e$^+$e$^-$



Less precision compensated by higher E: $A_{4f} \sim s/\Lambda^2$

LHC run 2 & HL-LHC
→ ~10^{-4} level bounds
[Greljo-Marzocca, 2017]



$$Q_W^{\text{Cs}} = -72.62 \pm 0.43$$

[Wood et al., Science, 1997]

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Letter of Intent to the ISOLDE and Neutron Time-of-Flight Committee

Laser Cooling of Ra ions for Atomic Parity Violation

May 31, 2017

L. Willmann[1], K. Jungmann[1], N. Severijns[2], K. Wendt[3]

"The ion Ra+ renders the possibility for a 5x improvement in the accuracy of $\sin^2\theta_w$ within 1 week of measurement time"

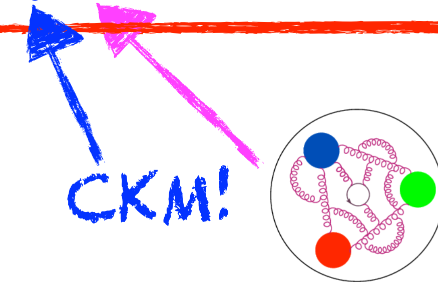
[Falkowski, MGA & Mimouni, 2017]

Global EFT fit of flavor data?

- Difficulties (flavor vs EWPO):
  - Nonperturbative QCD input (form factors);
  - CKM parameters (no hierarchy of observables)

$$\mathbf{O} = \mathbf{O}_{SM}\,(V_{ij};\theta_k) + \delta\mathbf{O}\,(V_{ij};\theta_k;\varepsilon_i)$$

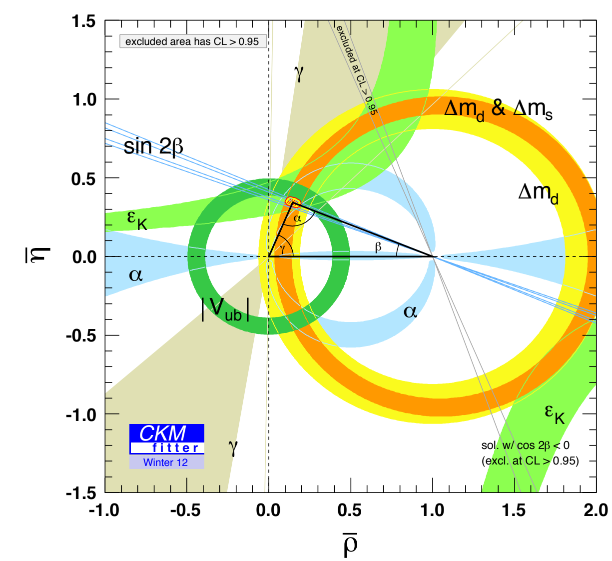
$$\rightarrow \chi^2 = \chi^2\,(\tilde{V}_{ij};\theta_k;\varepsilon_i)$$



- Traditional approach:
  no NP in tree-level extraction of CKM from CC processes

  → Makes sense (Λ_{NP} >> TeV in other processes), but...
  - It's unnecessary
  - Inconsistent with the EFT counting / philosophy;
  - BSM ~ SM-like?
  - Hints in R(D), R(D*) [only 0.3 "suppression"]
  - Tree-level CC processes can be very suppressed (CKM, chiral suppression, ...)

- *UV-meaning of the consistency of the whole CKM paradigm???*



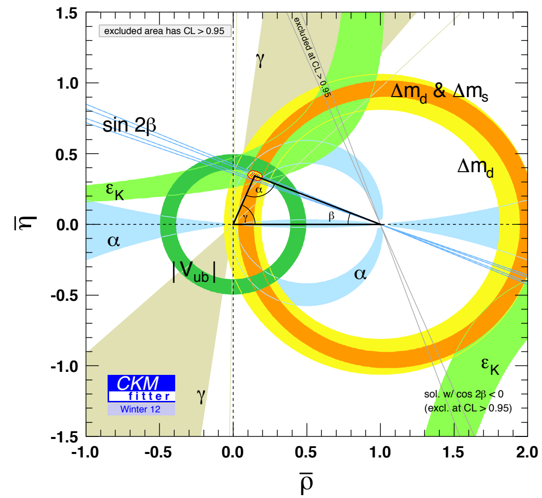
CKM parameters in the SMEFT

[Descotes-Genon, Falkowski, Fedele, MGA, & Virto, 1812.08163]

O = O

$$O = O_{SM}(W_i;\theta_k) + O(W_i;\theta_k;c_i)$$
$$\rightarrow \chi^2 = \chi^2(\widetilde{W}_i;\theta_k;c_i)$$
$$\rightarrow \chi^2 = \chi^2(c_i)$$

$W_i=(\lambda, A, \rho, \eta)$



- Four "optimal" observables;

$$\boxed{\Gamma(K\to\mu\nu_\mu)/\Gamma(\pi\to\mu\nu_\mu),\quad \Gamma(B\to\tau\nu_\tau),\quad \Delta M_d,\quad \Delta M_s.}$$

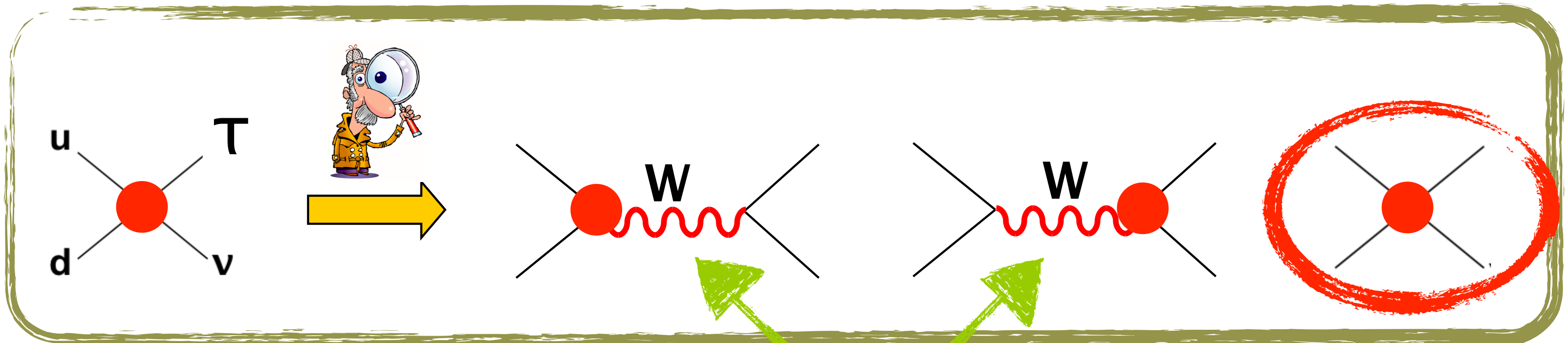
→ Four tilde Wolfenstein parameters;
→ NP effects in them known (not neglected);

$$\begin{pmatrix} \tilde\lambda = \lambda + \delta\lambda \\ \tilde A = A + \delta A \\ \tilde{\bar\rho} = \bar\rho + \delta\bar\rho \\ \tilde{\bar\eta} = \bar\eta + \delta\bar\eta \end{pmatrix} = \begin{pmatrix} 0.22537 \pm 0.00046 \\ 0.828 \pm 0.021 \\ 0.194 \pm 0.024 \\ 0.391 \pm 0.048 \end{pmatrix}, \quad \rho = \begin{pmatrix} 1 & -0.16 & 0.05 & -0.03 \\ \cdot & 1 & -0.25 & -0.24 \\ \cdot & \cdot & 1 & 0.83 \\ \cdot & \cdot & \cdot & 1 \end{pmatrix}$$

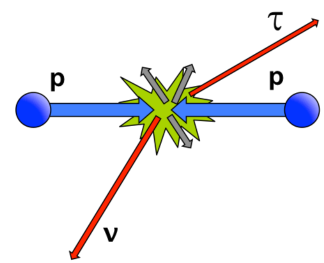
- Any other flavor observable becomes a NP probe:

$$O_\alpha = O_{\alpha,\mathrm{SM}}(W_j) + \delta O^{\mathrm{direct}}_{\alpha,\mathrm{NP}} = O_{\alpha,\mathrm{SM}}(\widetilde{W}_j) + \delta O^{\mathrm{indirect}}_{\alpha,\mathrm{NP}} + \delta O^{\mathrm{direct}}_{\alpha,\mathrm{NP}}$$

Tau, EWPO & LHC searches

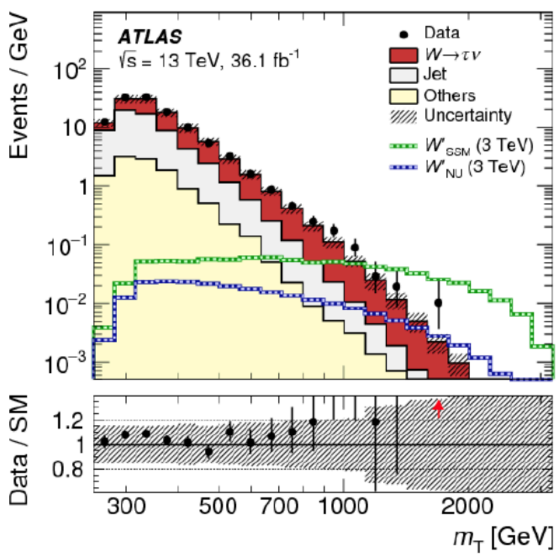


Other EWPO



Less precision compensated by higher E:

$A_{4f} \sim s/\Lambda^2$

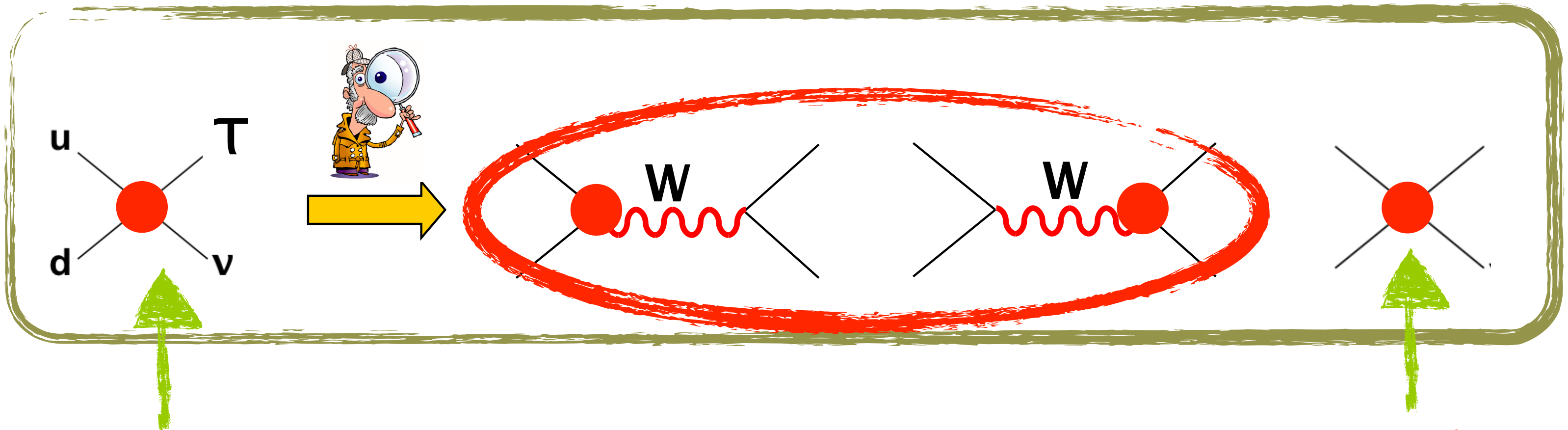


Unique low-E probes

Coefficient	ATLAS $\tau\nu$	τ and π decays
$[c_{\ell q}^{(3)}]_{\tau\tau 11}$	$[0.0, 1.6]$	$[-7.6, 2.1]$
$[c_{\ell equ}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-5.6, 2.3]$
$[c_{\ell edq}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-2.1, 5.8]$
$[c_{\ell equ}^{(3)}]_{\tau\tau 11}$	$[-3.3, 3.3]$	$[-8.6, 0.7]$

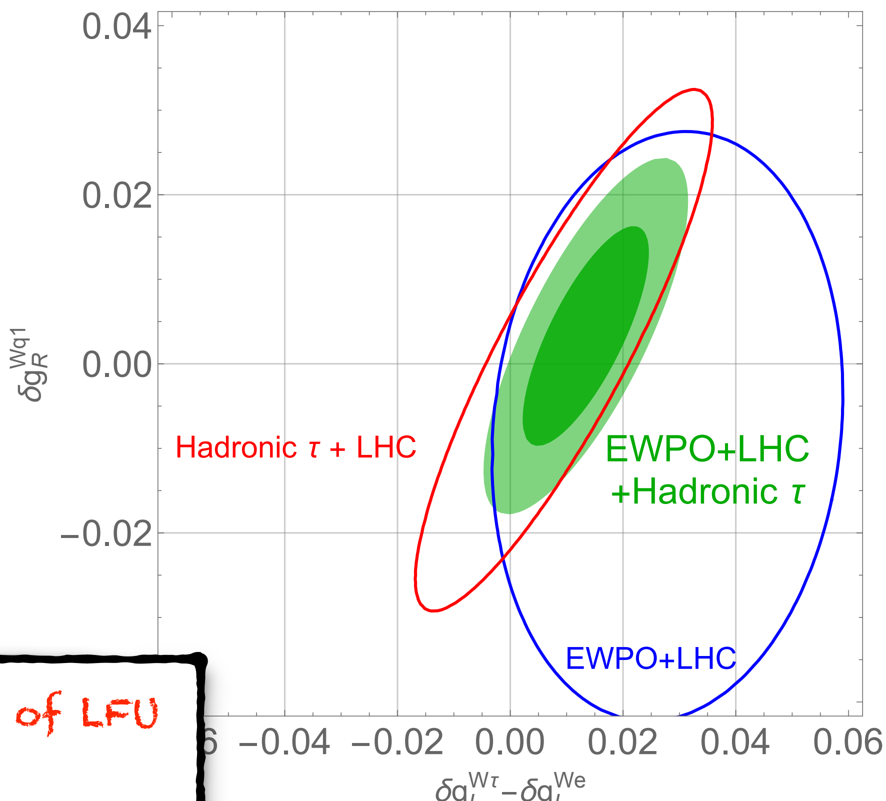
95% CL intervals (in 10^{-3} units) at $\mu = 1$ TeV

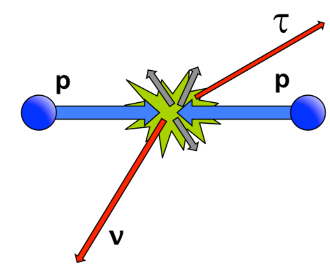
Tau, EWPO & LHC searches



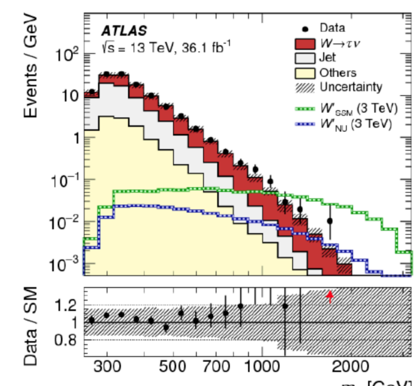
Hadronic Tau Decays

LHC!



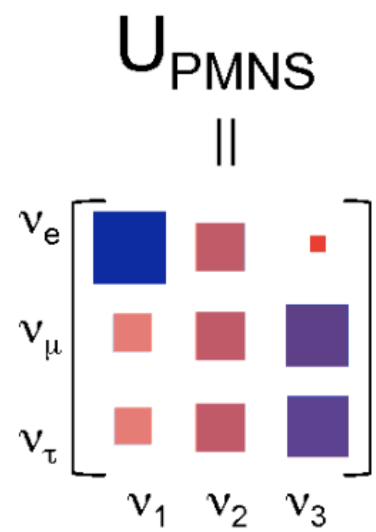


Less precision compensated by higher E: $A_{4f} \sim s/\Lambda^2$



%-level probe of LFU in the vertex!

Oscillations in EFT



Oscillation in the SM:

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

Oscillation in EFT:

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}(L, E_\nu) = \sum_{JK} C^\alpha_{JK} \exp\left(-i \frac{\Delta m^2_{JK} L}{2E_\nu}\right), \qquad C^\alpha_{JK} \equiv \frac{(\int A^P_{\alpha J} A^{P*}_{\alpha K})(\int A^D_{J\alpha} A^{D*}_{K\alpha})}{\left(\sum_I \int |A^P_{\alpha I}|^2\right)\left(\sum_{I'} \int |A^D_{I'\alpha}|^2\right)}$$

Production and Detection amplitudes

$$A^P_{\alpha J} \equiv \mathcal{M}(X^P \to \ell^-_\alpha \bar{\nu}_J Y^P), \qquad A^D_{J\alpha} \equiv \mathcal{M}(\bar{\nu}_J X^D \to \ell^+_\alpha Y^D)$$

$$A^P_{\alpha J} = U_{\alpha J} M^P_L + \sum_{X=L,R,S,P,T} [\epsilon_X U]_{\alpha J} M^P_X, \qquad A^D_{J\alpha} = U^\dagger_{J\alpha} M^D_L + \sum_{X=L,R,S,P,T} [U^\dagger \epsilon^\dagger_X]_{J\alpha} M^D_X$$

EFT in reactor experiments

The survival probability in the SM+V-A+detection+production:

$$P_{\bar{\nu}_e \to \bar{\nu}_e}(L, E_\nu) = 1 - \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) \sin^2\left(2\tilde{\theta}_{13} - \alpha_D \frac{m_e}{E_\nu - \Delta} - \alpha_P \frac{m_e}{f_T(E_\nu)}\right)$$
$$+ \sin\left(\frac{\Delta m_{31}^2 L}{2E_\nu}\right) \sin(2\tilde{\theta}_{13}) \left(\beta_D \frac{m_e}{E_\nu - \Delta} - \beta_P \frac{m_e}{f_T(E_\nu)}\right) + \mathcal{O}(\epsilon_X^2)$$

$$\tilde{\theta}_{13} = \theta_{13} + \mathrm{Re}\,[L]$$

$$\alpha_D = \frac{g_S}{3g_A^2+1}\mathrm{Re}\,[S] - \frac{3g_A g_T}{3g_A^2+1}\mathrm{Re}\,[T], \qquad \alpha_P = \frac{g_T}{g_A}\mathrm{Re}\,[T]$$
$$\beta_D = \frac{g_S}{3g_A^2+1}\mathrm{Im}\,[S] - \frac{3g_A g_T}{3g_A^2+1}\mathrm{Im}\,[T], \qquad \beta_P = \frac{g_T}{g_A}\mathrm{Im}\,[T]$$

Survival probability at the leading order depends only on off-diagonal Wilson coefficients ε_X!!!

$$[L] \equiv e^{i\delta_{\mathrm{CP}}}\left(s_{23}[\epsilon_L]_{e\mu} + c_{23}[\epsilon_L]_{e\tau}\right)$$
$$[S] \equiv e^{i\delta_{\mathrm{CP}}}\left(s_{23}[\epsilon_S]_{e\mu} + c_{23}[\epsilon_S]_{e\tau}\right)$$
$$[T] \equiv e^{i\delta_{\mathrm{CP}}}\left(s_{23}[\hat{\epsilon}_T]_{e\mu} + c_{23}[\hat{\epsilon}_T]_{e\tau}\right)$$