# Theoretical uncertainties in the $W$ boson mass determination: EW and mixed EW-QCD effects

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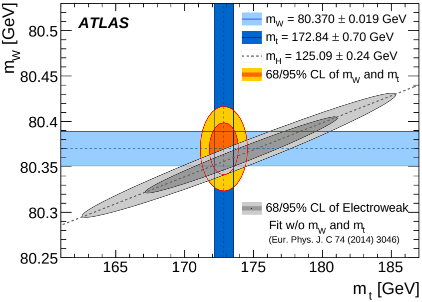
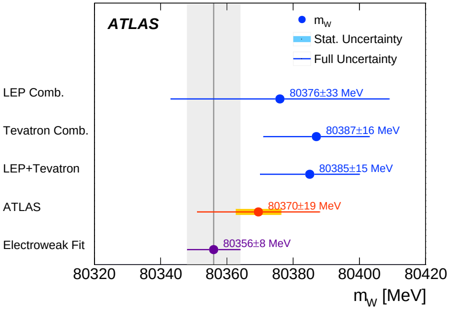
Blois, June 4, 2019

based on Phys.Rev. D96 (2017)

in collaboration with Carlo Michel Carloni Calame, Homero Martinez, Guido Montagna, Oreste Nicrosini, Fulvio Piccinini and Alessandro Vicini



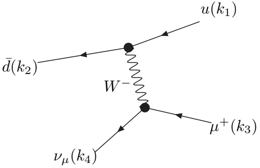

from EPJC78 (2018) no.2, 110

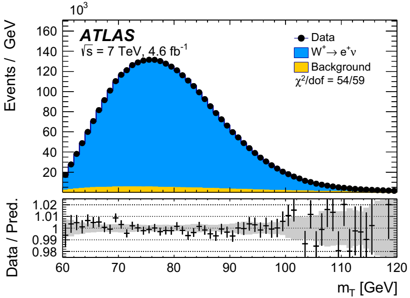
- precision test of the SM/indirect search for NP

- $M_W$ often used as input parameter for H.O. EW calculations at the LHC ($G_\mu, M_Z, M_W$ scheme and related ones)

# $M_W$ at hadron colliders (1)

Measured from charged DY




$M(l\nu)$ not measurable

$\Downarrow$

template fit to $P_{\mathrm{T}}^{l}$, $M_T^{l\nu}$

$$M_{\mathrm{T}}^{W} = \sqrt{2 p_{\mathrm{T},l} p_{\mathrm{T},\nu} (1 - \cos\Delta\phi_{l\nu})}$$

# $M_W$ at hadron colliders (2)

Template fit procedure:

1. measure a relevant distribution ($P_{\rm T}^l$, $M_T^{l\nu}$)
2. generate several Monte Carlo samples with different $M_W$ (templates)
3. the measured $M_W$ corresponds to the sample that best fits to the data

Monte Carlo simulations are affected by perturbative (radiative corrections) and non-perturbative (i.e. PDFs, $p_{\rm T}^W$ modelling) theoretical uncertainties

$$\Downarrow$$

these theory uncertainties propagate to $M_W$ measurement

# Theoretical uncertainties in $M_W$ measurement: strategy

1. pseudodata
   - Monte Carlo samples with a given theoretical accuracy
   - play the role of experimental data
2. templates
   - MC samples at NLO QCD+QCD-PS (or LO) generated for different values of $M_W$
   - will be fitted to the pseudodata
3. $\Delta M_W = M_W(\text{pseudodata}) - M_W(\text{fit output})$

# Theoretical uncertainties in $M_W$ measurement: tools (1)

- HORACE (Carloni Calame et al. hep-ph/0303102, hep-ph/050626)
  - MC event generator for DY
  - can generate events at NLO EW+QED-PS, and NLO EW+QED-PS+unresolved $l^+l^-$ radiation

- POWHEG-BOX-V2/W_ew-BMNNP (Barze et al. arXiv:1202.0465)
  - MC event generator for charged DY
  - can generate events at NLO QCD+QCD-PS and NLO (QCD+EW)+(QCD+QED)-PS
  - relies on external shower MC programs (i.e. PYTHIA, PYTHIA+PHOTOS)

# Theoretical uncertainties in $M_W$ measurement: tools (2)

- PYTHIA (Sjostrand et al. hep-ph/0603175; arXiv:0710.3820)
  - general purpose shower MC generator
  - can generate multiple QCD and QED radiation

- PHOTOS (Barberio et al. CPC 66 (1991), CPC 79 (1994), Golonka et al. hep-ph/0506026)
  - general purpose shower MC generator
  - can generate multiple QED radiation off fermions (from $W$ decay)

# Theoretical uncertainties in $M_W$ measurement: notation

- NLO EW corrections: $d\sigma = d\sigma_0 \left[1 + \delta_\alpha\right]$

- QED-PS: all order $\gamma$ radiation in leading log approx.

  $$d\sigma = d\sigma_0 \left[1 + \sum_{n=1}^{\infty} \delta'_{\alpha^n}\right]$$

- NLO EW+QED-PS: $d\sigma = d\sigma_0 \left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$

  matching replaces first PS radiation with NLO real radiation

- HORACE NLO EW+QED-PS: $d\sigma = d\sigma_0 \left[1 + \delta_\alpha + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$

- POWHEG NLO (QCD+EW)+(QCD+QED)-PS:

$$d\sigma = d\sigma_0 \left[1 \; + \; \delta_{\alpha_s} + \delta_\alpha + \sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}\right]$$

$pp \to \mu^+\nu_\mu$, fit to $M_{\mathrm{T}}(\mu^+\nu_\mu)$

| | Templates | Pseudodata | $M_W$ shifts (MeV) |
|---|---|---|---|
| 1 | LO | POWHEG(QCD) NLO | 56.0 $\pm$ 1.0 |
| 2 | LO | POWHEG(QCD)+PYTHIA(QCD) | 74.4 $\pm$ 2.0 |
| 3 | LO | HORACE(EW) NLO | -94.0 $\pm$ 1.0 |
| 4 | LO | HORACE (EW,QEDPS) | -88.0 $\pm$ 1.0 |
| 5 | LO | POWHEG(QCD,EW) NLO | -14.0 $\pm$ 1.0 |
| 6 | LO | POWHEG(QCD,EW) `two-rad`+PYTHIA(QCD)+PHOTOS | -5.6 $\pm$ 1.0 |

| | samples | $M_W$ shift (MeV) |
|---|---|---|
| $\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n} + \sum_{m=2}^{\infty} \delta'_{\alpha_s^m} + \sum_{n=2}^{\infty} \delta'_{\alpha^n}$ | [6]-[5] | 8.4 $\pm 1.4$ MeV |
| $\sum_{m=2}^{\infty} \delta'_{\alpha_s^m}$ | [2]-[1] | 18.4 $\pm 2.2$ MeV |
| $\sum_{n=2}^{\infty} \delta'_{\alpha^n}$ | [4]-[3] | 6.0 $\pm 1.4$ MeV |

$\sum_{m=1,n=1}^{\infty} \delta'_{\alpha_s^m \alpha^n}$=([6]-[5])-([2]-[1])-([4]-[3])=-16.0$\pm$3.0 MeV

in agreement with the results of Dittmaier et al. 1511.08016 for the full $\mathcal{O}(\alpha\alpha_{\mathcal{S}})$ corrections in pole approx. (-14 MeV)

| $pp \to W^+$, $\sqrt{s} = 14$ TeV | | | $M_W$ shifts (MeV) | | | |
|---|---|---|---|---|---|---|
| Templates accuracy: NLO-QCD+QCD$_{\rm PS}$ | | | $W^+ \to \mu^+ \nu$ | | $W^+ \to e^+ \nu$(dres) | |
| | Pseudodata accuracy | QED FSR | $M_T$ | $p_T^\ell$ | $M_T$ | $p_T^\ell$ |
| 1 | NLO-QCD+(QCD+QED)$_{\rm PS}$ | Pythia | -95.2±0.6 | -400±3 | -38.0±0.6 | -149±2 |
| 2 | NLO-QCD+(QCD+QED)$_{\rm PS}$ | Photos | -88.0±0.6 | -368±2 | -38.4±0.6 | -150±3 |
| 3 | NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad` | Pythia | -89.0±0.6 | -371±3 | -38.8±0.6 | -157±3 |
| 4 | NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$`two-rad` | Photos | -88.6±0.6 | -370±3 | -39.2±0.6 | -159±2 |

- dressed $e$: recombine $\gamma$ with $e$ if $\Delta R(\gamma e) < 0.1$
- bare $\mu$: corrections enhanced by logs $\alpha \log(\frac{m_\mu^2}{Q^2})$

| $pp \to W^+$, $\sqrt{s} = 14$ TeV | | | $M_W$ shifts (MeV) | | | |
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| 1 | NLO-QCD+(QCD+QED)$_{\mathrm{PS}}$ | PYTHIA | -95.2±0.6 | -400±3 | -38.0±0.6 | -149±2 |
| 2 | NLO-QCD+(QCD+QED)$_{\mathrm{PS}}$ | PHOTOS | -88.0±0.6 | -368±2 | -38.4±0.6 | -150±3 |
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- difference between QED-PS in PHOTOS and PYTHIA at $\mathcal{O}(\alpha)$
- PHOTOS $\propto \frac{1}{1-\beta\cos\theta_{l\gamma}}$
- PYTHIA -QED $\propto \frac{1}{p_{\mathrm{T}}^{\gamma}}$
- 32 MeV ($p_{\mathrm{T}}$)/ 7 MeV ($M_{\mathrm{T}}$) effect for bare $\mu$

| | $pp \to W^+$, $\sqrt{s} = 14$ TeV<br>Templates accuracy: NLO-QCD+$\mathrm{QCD_{PS}}$<br>Pseudodata accuracy | QED FSR | $M_W$ shifts (MeV)<br>$W^+ \to \mu^+\nu$<br>$M_T$ | <br><br>$p_T^\ell$ | <br>$W^+ \to e^+\nu$(dres)<br>$M_T$ | <br><br>$p_T^\ell$ |
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| 1 | NLO-QCD+$\mathrm{(QCD+QED)_{PS}}$ | Pythia | -95.2±0.6 | -400±3 | -38.0±0.6 | -149±2 |
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- first QED radiation generated by POWHEG
- difference between QED-PS in PHOTOS and PYTHIA at $\mathcal{O}(\alpha^2)$

| | $pp \to W^+$, $\sqrt{s} = 14$ TeV | | $M_W$ shifts (MeV) | | | |
|---|---|---|---|---|---|---|
| | Templates accuracy: NLO-QCD+QCD$_{\rm PS}$ | | $W^+ \to \mu^+\nu$ | | $W^+ \to e^+\nu$(dres) | |
| | Pseudodata accuracy | QED FSR | $M_T$ | $p_T^\ell$ | $M_T$ | $p_T^\ell$ |
| 1 | NLO-QCD+(QCD+QED)$_{\rm PS}$ | Pythia | -95.2±0.6 | -400±3 | -38.0±0.6 | -149±2 |
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| 4 | NLO-(QCD+EW)+(QCD+QED)$_{\rm PS}$two-rad | Photos | -88.6±0.6 | -370±3 | -39.2±0.6 | -159±2 |

- impact of non-log QED, weak and mixed EW-QCD contributions
- different effects for Photos or Pythia (different non-log QED terms)
- more stable results for $M_{\rm T}$ (less sensitive to mixed EW-QCD corrections)

| $pp \to W^+$, $\sqrt{s} = 14$ TeV | | | $M_W$ shifts (MeV) | | | |
|---|---|---|---|---|---|---|
| Templates accuracy: NLO-QCD+QCD$_{\rm PS}$ | | | $W^+ \to \mu^+\nu$ | | $W^+ \to e^+\nu$(dres) | |
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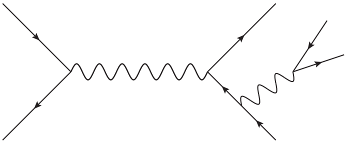
Experimental collaboration at LHC use as templates

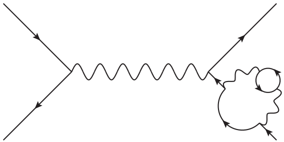
- ATLAS: Powheg-QCD+Pythia-QCD+Photos
- CMS: Powheg-QCD+Pythia-(QCD+QED)

uncertainties form
non-log QED,
weak,
mixed QCD-EW corr.

| | | $\Delta M_W$ (MeV) bare muons | |
|---|---|---|---|
| | QED FSR model | $M_T$ | $p_T^\ell$ |
| LHC | Pythia | +6.2 ± 0.8 | +29 ± 4 |
| | Photos | -0.6 ± 0.8 | -2 ± 4 |

# Higher order effects: pair radiation





same order as 2 $\gamma$ radiation

# Higher order effects: pair radiation

Unresolved pair radiation can be included in the Sudakov through the running [1] of $\alpha$

$$\alpha \Longrightarrow \alpha(s) = \begin{cases} \alpha / \left(1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2}\right) & \text{electrons only} \\ \alpha / \left(1 - \frac{\alpha}{3\pi} \ln \frac{s}{m_e^2} - \theta(s - m_\mu^2) \frac{\alpha}{3\pi} \ln \frac{s}{m_\mu^2}\right) & \text{electrons + muons} \end{cases}$$

| $pp \to W^+$, $\sqrt{s} = 14$ TeV<br>Templates accuracy: LO<br>Pseudo–data accuracy | | $M_W$ shifts (MeV)<br>$W^+ \to \mu^+\nu$<br>$M_T$ | <br><br>$p_T^\ell$ | <br>$W^+ \to e^+\nu$<br>$M_T$ | <br><br>$p_T^\ell$ |
|---|---|---|---|---|---|
| 1 | HORACE FSR-LL | -89±1 | -97±1 | -179±1 | -195±1 |
| 2 | HORACE FSR-LL + Pairs | -94±1 | -102±1 | -182±2 | -199±1 |

$\Delta M_W(\mu^+\nu) \sim 5 \pm 1$ MeV (from $\mu$) and $\sim 3 \pm 2$ MeV (from $e$)

[1] alternative implementation: N. Davidson, T. Przedzinski and Z. Was, CPC 199 (2016) 86, arXiv:1011.0937

# NNLO uncertainty: input parameter scheme

pert. EW calculations require a consistent set of input param. in the gauge sector, e.g.

- $\alpha(0)$, $M_W$ and $M_Z$
- $G_\mu$, $M_W$ and $M_Z$ to be preferred in the CC DY
- we can define

$$\begin{aligned}\alpha_\mu^{tree} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2\vartheta \\ \alpha_\mu^{1l} &\equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \sin^2\vartheta \,(1-\Delta r)\end{aligned}$$

The expressions for the cross section differ at $\mathcal{O}(\alpha^2)$

$$\begin{aligned}\alpha_0 \quad &: \qquad \sigma = \alpha_0^2 \sigma_0 + \alpha_0^3 (\sigma_{SV} + \sigma_H)\,, \\ G_\mu \; I \quad &: \qquad \sigma = (\alpha_\mu^{tree})^2 \sigma_0 + (\alpha_\mu^{tree})^2 \alpha_0 (\sigma_{SV} + \sigma_H) - 2\Delta r (\alpha_\mu^{tree})^2 \sigma_0\,, \\ G_\mu \; II \quad &: \qquad \sigma = (\alpha_\mu^{1l})^2 \sigma_0 + (\alpha_\mu^{1l})^2 \alpha_0 (\sigma_{SV} + \sigma_H)\end{aligned}$$

- potentially effects on $M_W$ because of the different sharing among different photon multiplicities

| | $p\bar{p} \to W^+$, $\sqrt{s} = 1.96$ TeV<br>Templates accuracy: LO<br>Pseudodata accuracy | Input scheme | $M_W$ shifts (MeV)<br>$W^+ \to \mu^+\nu$<br>$M_T$ | <br><br>$p_T^\ell$ |
|---|---|---|---|---|
| 1 | HORACE NLO-EW | $\alpha_0$ | -101±1 | -117±2 |
| 2 | | $G_\mu - I$ | -112±1 | -130±1 |
| 3 | | $G_\mu - II$ | -101±1 | -117±1 |
| 4 | HORACE NLO-EW+QED-PS | $\alpha_0$ | -70±1 | -81±1 |
| 5 | | $G_\mu - I$ | -72±2 | -83±1 |
| 6 | | $G_\mu - II$ | -72±1 | -82±2 |

- differences present at NLO, after matching with higher orders, become much smaller

$$\Delta M_W \sim 2 \text{ MeV} \pm 1 - 2 \text{ MeV}$$

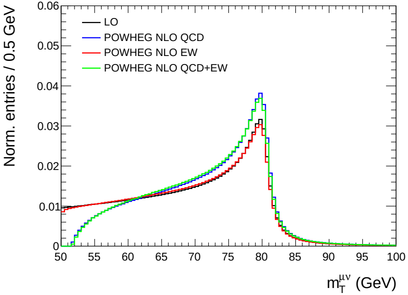
# Conclusions

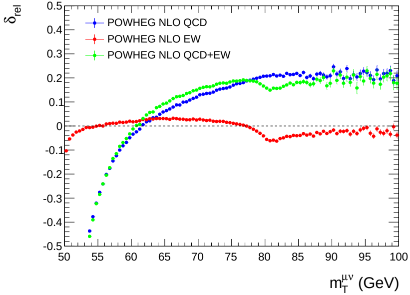
- we studied the impact of EW, QCD and mixed EW+QCD corrections on the determination of $M_W$:
  - larger shifts when $M_W$ is extracted from $p_{\rm T}^l$
  - larger shifts for bare muons
- our approx. estimate of mixed EW+QCD correction in agreement with the full $\mathcal{O}(\alpha\alpha_S)$ calculation in PA
- compared to predictions at NLO QCD matched with QCD-PS + QED-PS: missing EW and mixed EW+QCD corrections are

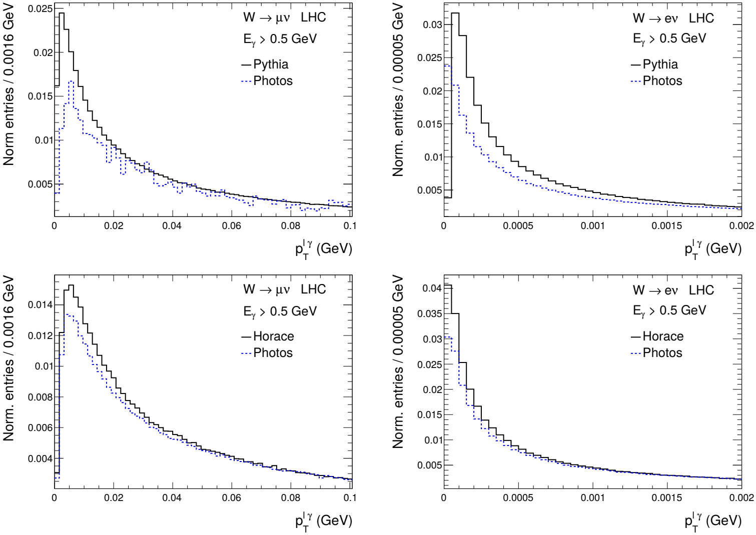
| | | $\Delta M_W\,(\mathrm{MeV})$ bare muons | |
|---|---|---|---|
| | QED FSR model | $M_T$ | $p_T^\ell$ |
| LHC | PYTHIA | +6.2 ± 0.8 | +29 ± 4 |
| | PHOTOS | -0.6 ± 0.8 | -2 ± 4 |

- the effect of pair corrections is $\Delta M_W(\mu^+\nu) \sim 5 \pm 1$ MeV (from $\mu$) and $\sim 3 \pm 2$ MeV (from $e$)

# Backup Slides


Norm. entries / 0.5 GeV
LO
POWHEG NLO QCD
POWHEG NLO EW
POWHEG NLO QCD+EW
$m_T^{\mu\nu}$ (GeV)


$\delta_{rel}$
POWHEG NLO QCD
POWHEG NLO EW
POWHEG NLO QCD+EW
$m_T^{\mu\nu}$ (GeV)


Norm entries / 0.0016 GeV
W → μν LHC
E_γ > 0.5 GeV
Pythia
Photos
p_T^{lγ} (GeV)
Norm. entries / 0.00005 GeV
W → eν LHC
E_γ > 0.5 GeV
Pythia
Photos
p_T^{lγ} (GeV)
Norm. entries / 0.0016 GeV
W → μν LHC
E_γ > 0.5 GeV
Horace
Photos
p_T^{lγ} (GeV)
Norm. entries / 0.00005 GeV
W → eν LHC
E_γ > 0.5 GeV
Horace
Photos
p_T^{lγ} (GeV)

HORACE

$$d\sigma^{\infty} = F_{SV}\ \Pi(Q^2,\varepsilon)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)|\mathcal{M}_{n,LL}|^2 d\Phi_n$$

POWHEG

$$d\sigma = \sum_{f_b}\bar{B}^{f_b}(\mathbf{\Phi}_n)\,d\mathbf{\Phi}_n\Bigg\{\Delta^{f_b}(\mathbf{\Phi}_n, p_T^{min})$$

$$+\sum_{\alpha_r\in\{\alpha_r|f_b\}}\frac{\left[d\Phi_{rad}\,\theta(k_T - p_T^{min})\,\Delta^{f_b}(\mathbf{\Phi}_n, k_T)\,R(\mathbf{\Phi}_{n+1})\right]_{\alpha_r}^{\bar{\mathbf{\Phi}}_n^{\alpha_r}=\mathbf{\Phi}_n}}{B^{f_b}(\mathbf{\Phi}_n)}\Bigg\}$$

taken from 1701.07240

| $W$-boson charge | $W^+$ | | $W^-$ | | Combined | |
|---|---|---|---|---|---|---|
| Kinematic distribution | $p_T^\ell$ | $m_T$ | $p_T^\ell$ | $m_T$ | $p_T^\ell$ | $m_T$ |
| $\delta m_W$ [MeV] | | | | | | |
| Fixed-order PDF uncertainty | 13.1 | 14.9 | 12.0 | 14.2 | 8.0 | 8.7 |
| AZ tune | 3.0 | 3.4 | 3.0 | 3.4 | 3.0 | 3.4 |
| Charm-quark mass | 1.2 | 1.5 | 1.2 | 1.5 | 1.2 | 1.5 |
| Parton shower $\mu_F$ with heavy-flavour decorrelation | 5.0 | 6.9 | 5.0 | 6.9 | 5.0 | 6.9 |
| Parton shower PDF uncertainty | 3.6 | 4.0 | 2.6 | 2.4 | 1.0 | 1.6 |
| Angular coefficients | 5.8 | 5.3 | 5.8 | 5.3 | 5.8 | 5.3 |
| Total | 15.9 | 18.1 | 14.8 | 17.2 | 11.6 | 12.9 |

Table 3: Systematic uncertainties in the $m_W$ measurement due to QCD modelling, for the different kinematic distributions and $W$-boson charges. Except for the case of PDFs, the same uncertainties apply to $W^+$ and $W^-$. The fixed-order PDF uncertainty given for the separate $W^+$ and $W^-$ final states corresponds to the quadrature sum of the CT10nnlo uncertainty variations; the charge-combined uncertainty also contains a 3.8 MeV contribution from comparing CT10nnlo to CT14 and MMHT2014.