Latest results on rare kaon decays from the NA48/2 experiment at CERN

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on behalf of the NA48/2 collaboration
Outline

• NA48/2 experiment
• Study of the $K^+ \rightarrow \pi^+\pi^0e^+e^-$ decay
• $K_{l3}$ form factors precision measurement
• Conclusion
NA48/2 kaon beam

2003+2004 ~ 6 months, ~ 2 $10^{11}$ K decays
Flux ratio: $K^+/K^- \approx 1.8$

Simultaneous $K^+$ and $K^-$ beams: large charge symmetrization of experimental conditions

Front-end achromat
Quadrupole quadruplet

• Momentum selection
• Focusing
• $\mu$ sweeping
Main detector components:

• Magnetic spectrometer (4 DCHs): 4 views/DCH inside a He tank $\Delta p/p = 1.02\% \oplus 0.044\% \times p$ [p in GeV/c].

• Hodoscope fast trigger; precise time measurement (150ps).

• Liquid Krypton EM calorimeter (LKr) High granularity, quasi-homogenous $\sigma_E/E = 3.2\%/E^{1/2} \oplus 9\%/E \oplus 0.42\%$ $\sigma_x = \sigma_y = 0.42/E^{1/2} \oplus 0.06\text{cm}$ [E in GeV]. (0.15cm@10GeV).

• Hadron calorimeter, muon veto counters, photon vetoes.
Motivation:

• Never observed so far: confirmation of BR magnitude ChPT predictions:

• If a detailed analysis possible (G. D'Ambrosio):
  Sign of interference term (IB,E),
  Magnetic term from (IB,M) interference
  Charge asymmetry — direct CPV
  ChPT predicted bump in $M(e^+e^-)$ spectrum

\[ K^\pm \to \pi^\pm\pi^0 e^+e^- \text{ decay} \]
Events selection

Signal: $\pi^\pm (\pi^0 \rightarrow \gamma\gamma)e^+e^-$

Normalization: $\pi^\pm (\pi^0_D \rightarrow \gamma e^+e^-)$

3 tracks in both cases, but one photon less in the normalization channel.

Cut for trigger efficiency: no 3 track in one HOD quadrant.

- No PID from LKr => no LKr acceptance cuts for tracks;
- Assign electron mass to the track with a charge opposite to kaon charge;
- For both other tracks using both $M(e)$ and $M(\pi^\pm)$ reconstruct $M(\pi^0)$ and $M(K^\pm)$;
- $|M(\pi^0) − M_{PDG}| < 15 \text{ MeV/c}^2$;
- $|M(K^\pm) − M_{PDG}| < 45 \text{ MeV/c}^2$;
- $|M(\pi^0) − 0.42 M(K^\pm) + 72.3 \text{ MeV/c}^2| < 6 \text{ MeV/c}^2$
Selection results

The mass resolutions (Gaussian rms) are measured from data and agree with MC

Signal: $\pi^\pm (\pi^0 \rightarrow \gamma \gamma) e^+ e^-$

- $\sigma(\pi^0 \gamma\gamma)$ ≈ 2.7 MeV/c²
- $\sigma(\pi^0 \pi^0 ee)$ ≈ 6.1 MeV/c²
- $\sigma(\pi^0 \pi^0 \pi^\pm)$ ≈ 4.2 MeV/c²
- $\sigma(\pi^\pm \pi^0)$ ≈ 1.7 MeV/c²

Normalization: $\pi^\pm (\pi^0_D \rightarrow \gamma e^+ e^-)$
Selection results

Candidates $N_N$ 16.3 x $10^6$

Background $N_{BN}$ 17288 ± 159

Acceptance $A_N$ 3.981%

L1 eff. $\varepsilon_{L1S}$ (99.767±0.003)%

L2 eff. $\varepsilon_{L2S}$ (98.495±0.006)%

Candidates $N_S$ 4919

Background $N_{BS}$ 241 ± 21

Acceptance $A_S$ 0.662%

L1 eff. $\varepsilon_{L1S}$ (99.729±0.009)%

L2 eff. $\varepsilon_{L2S}$ (98.604±0.021)%
## Background

All background contributions are estimated from simulation.

<table>
<thead>
<tr>
<th>Decay</th>
<th>N</th>
<th>$R, %$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{\mu3D}$</td>
<td>10437 ± 119</td>
<td></td>
</tr>
<tr>
<td>$K_{e3D}$</td>
<td>6851 ± 106</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.106</td>
</tr>
<tr>
<td><strong>Signal</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{3\pi D}$</td>
<td>132 ± 8</td>
<td></td>
</tr>
<tr>
<td>$K_{2\pi D\gamma}$</td>
<td>102 ± 19</td>
<td></td>
</tr>
<tr>
<td>$K_{e3D}$</td>
<td>7 ± 3</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>4.9 ± 0.4</td>
</tr>
</tbody>
</table>

$R$ — background to signal ratio
Branching ratio measurement

\[
BR = BR(\pi^{\pm}\pi^{0}) \times \Gamma(\pi^{0}_{D})/\Gamma(\pi^{0}_{\gamma\gamma}) \\
\times (N_{S}-N_{BS})/(N_{N}-N_{BN}) \\
\times (A_{N} \times \epsilon_{N})/(A_{S} \times \epsilon_{S})
\]

\[
\Gamma(\pi^{0}_{D})/\Gamma(\pi^{0}_{\gamma\gamma}) = (1.188 \pm 0.035)\%
\]

**FINAL RESULT:**

\[
BR = (4.237 \pm 0.063_{\text{stat}} \pm 0.033_{\text{syst}} \pm 0.126_{\text{ext}}) \times 10^{-6}
\]

Error is dominated by external error of \(\Gamma(\pi^{0}_{D})/\Gamma(\pi^{0}_{\gamma\gamma})\)

In perfect agreement with ChPT [EPJ C72 (2012) 1872, EPJ C78 (2018) 265]:

For IB only

\[
BR(\text{IB}) = 4.183 \times 10^{-6};
\]

When including all DE and INT terms

\[
BR(\text{IB}) = 4.229 \times 10^{-6}.
\]
**Kinematic space study**

- Predictions for IB and full BR differ only by 1%. The BR measurement is not precise enough to extract information on the contributions. But some information can be extracted from the kinematic space distribution.


- The population of 3d-boxes in the kinematic space \((q^2, T^*_\pi, E^*_\gamma)\) is used to determine the relative fraction of each component. The 3d-space is split into \(N_1\) slices along \(q^2\), \(N_2\) slices along \(T^*_\pi\) and then into \(N_3\) \(E^*_\gamma\) slices, all boxes with equal populations.

- To obtain the fractions \((M)/IB\) and \((IB-E)/IB\) reproducing the data, a \(\chi^2\) estimator is minimized:

\[
\chi^2 = \sum_{i=1}^{N_1 \times N_2 \times N_3} \frac{(N_i - M_i)^2}{\delta N_i^2 + \delta M_i^2}
\]

where \(N_i (\delta N_i)\) - data population (error), \(M_i (\delta M_i)\) - expected population (error) in box i.

The expected number of events in box I is

\[M_i = N \times (N_{iIB} + a \cdot N_{iM} + b \cdot N_{iIB-E}) + N_{iBkg}\]

where \(N\) is normalization factor. The relative contributions \((M)/IB\) and \((IB-E)/IB\) are defined as

\[
(M)/IB = (a \pm \delta a)/\rho_M, \quad (IB-E)/IB = (b \pm \delta b)/\rho_{IB-E}
\]

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M/IB)</td>
<td>0.0114 ± 0.0043_stat</td>
<td>1/71 = 0.0141 ± 0.0014_ext</td>
</tr>
<tr>
<td>((IB-E)/IB)</td>
<td>-0.0014 ± 0.0036_stat</td>
<td>-1/253 = -0.0039 ± 0.0028_ext</td>
</tr>
<tr>
<td>(\chi^2/ndf)</td>
<td>98.2/87</td>
<td></td>
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<tr>
<td>probability</td>
<td>19%</td>
<td></td>
</tr>
<tr>
<td>Correlation C(a, b)</td>
<td>0.06</td>
<td></td>
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</tbody>
</table>
Asymmetries

The simplest CP-violating asymmetry is the charge asymmetry between $K^+$ and $K^-$ partial rates integrated over the whole phase space:

$$A_{CP} = \frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0 e^+ e^-) - \Gamma(K^- \rightarrow \pi^- \pi^0 e^+ e^-)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0 e^+ e^-) + \Gamma(K^- \rightarrow \pi^- \pi^0 e^+ e^-)}$$

$$\text{BR}(K^+) = (4.151 \pm 0.078_{\text{stat}}) \times 10^{-6},\quad \text{BR}(K^-) = (4.394 \pm 0.108_{\text{stat}}) \times 10^{-6}$$

$$A_{CP} = -0.0284 \pm 0.0155_{\text{stat}},$$

the systematic and external errors cancel in the ratio. This value is translated to a single-sided limit: $|A_{CP}| < 4.82 \times 10^{-2}$ at 90% CL.

The $A_{\Phi^*_{CP}}$ and $A_{\phi_{CP}}$ angular asymmetries are defined in [Eur. Phys. J. C 72 (2012) 1872]:

$$A_{\Phi^*_{CP}} = 0.0119 \pm 0.0150_{\text{stat}}$$
$$A_{\phi_{CP}} = 0.0058 \pm 0.0150_{\text{stat}}$$

Both asymmetries are consistent with zero. The single-sided upper limits:

$$|A_{\Phi^*_{CP}}| < 3.11 \times 10^{-2}, \quad |A_{\phi_{CP}}| < 2.50 \times 10^{-2}$$ at 90% CL


$$A_{(L)P}(K^+) = 0.0059 \pm 0.0180_{\text{stat}}$$
$$A_{(L)P}(K^-) = -0.0166 \pm 0.0237_{\text{stat}}$$

The combined value is

$$A_{(L)P}(K^\pm) = -0.0023 \pm 0.0144_{\text{stat}}$$

The single-sided upper limit:

$$A_{(L)P} < 2.07 \times 10^{-2}$$ at 90% CL.
**K^± → π^0 l^± ν** (K_{l3}) form factors

experimental input for \(|V_{us}|\) extraction (in addition to \(\Gamma(K_{l3})\))

**Without radiative effects:**

\[
\rho_0 = d^2N/(dE_l\,dE_\pi) \sim A \left| f_+(t) \right|^2 + B f_+(t) f_-(t) + C \left| f_-(t) \right|^2,
\]

where \(t = (P_K - P_\pi)^2 = M_K^2 + M_\pi^2 - 2 M_K E_\pi\)

and \(f_+(t) = (f_+(t) - f_0(t))(M_K^2 - m_\pi^2)/t\).  
(just another formulation, \(f_0\) is «scalar» and \(f_+\) is «vector» FF), \(E_\pi\) is charged lepton energy, \(E_\pi\) is \(\pi^0\) energy (both in the kaon rest frame).

\[
A = M_K^2(2 E_\nu E_\pi - M_K(E_\pi^{max} - E_\pi)) + M_\nu^2((E_\pi^{max} - E_\pi)/4 - E_\nu)
\]

\[
B = M_\nu^2(E_\nu - (E_\pi^{max} - E_\pi)/2) \quad \text{negligible for Ke3}
\]

\[
C = M_\nu^2(E_\pi^{max} - E_\pi)/4 \quad \text{negligible for Ke3}
\]

\[
E_\pi^{max} = (M_K^2 + M_\pi^2 - M_\nu^2)/(2 M_K)
\]

<table>
<thead>
<tr>
<th>FF Parameterisation</th>
<th>(f_+(t, parameters))</th>
<th>(f_0(t, parameters))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Taylor expansion</strong></td>
<td>1 + (\lambda') (t/m^2_{\pi^+} + \frac{1}{2} \lambda'') ((t/m^2_{\pi^+})^2)</td>
<td>1 + (\lambda'<em>0) (t/m^2</em>{\pi^+})</td>
</tr>
<tr>
<td><strong>Pole</strong></td>
<td>(M_\nu^2 / (M_\nu^2 - t))</td>
<td>(M_s^2 / (M_s^2 - t))</td>
</tr>
<tr>
<td><strong>Dispersive</strong></td>
<td>exp( ((\Lambda_+ + H(t)) t/m^2_{\pi^+}) )</td>
<td>exp( ((\ln[C] - G(t)) t/(m_K^2 - m_\pi^0)) )</td>
</tr>
</tbody>
</table>

**Data:** 3 days from the NA48/2 data taken in 2004 (low intensity)

**Trigger:** 1 charged track (2 hodoscope hits) and $E_{LKr} > 10$ GeV

**Registered:**

- **1 track** (> 0 candidates): $P_e \geq 5$ GeV, $P_\mu \geq 10$ GeV, $R_{MUV} > 30$ cm, $|X_{MUV}, Y_{MUV}| < 115$ cm.

- **2 LKr clusters** (> 1 candidates): $E > 3$ GeV, distance to closest track > 15 cm.

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**Kaon momentum reconstruction**

**Neutrino is missing, beam geometry and average momentum $P_b$ are measured from $K_{3\pi^\pm}$**

Two solutions of the quadratic equation for $P_K$

- **Assumptions:**
  - $P_T(v) = -P_t$
  - $M(v) = 0$

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- **Best** $P_K$ solution = closest $P_{1,2}$ to the average beam momentum $P_b$ (measured from $3\pi^\pm$ decays for each run).

- **Select:** $-7.5$ GeV/c < $(P_K - P_b) < 7.5$ GeV/c

- For each event, separately for $K_{e3}$ and $K_{\mu3}$ selections, the combination with a minimum $\Delta P = |P_K - P_b|$ is the best candidate.
Selection:

\( \pi^0 \):
- A pair of clusters in-time (within 5 ns) without any in-time extra clusters (to suppress BG)
- Distance between the clusters in a pair > 20 cm
- \( E(\pi^0) > 15 \) GeV (for the trigger efficiency)
- Z of decay: from 2\( \gamma \) assuming \( \pi^0 \) mass («neutral Z»); Z > 200 cm downstream the last collimator
- DCH1 inner flunge cut for the both \( \gamma \)

Track selection and identification
- A good track in-time with the \( \pi^0 \) within 10 ns.
- No extra good track within 8 ns (against showers).
- If \( 2.0 > E_{\text{LKr}} / P_{\text{DCH}} > 0.9 \), it is an electron of \( K_{e3} \).
- If \( E_{\text{LKr}} / P_{\text{DCH}} < 0.9 \) (for true muons it cuts nothing) and there is a MUV muon associated, it is a \( K_{\mu3} \) muon.

Loose \( E_{\text{LKr}} / P_{\text{DCH}} \) cuts \(<=>\) negligible related systematics.

Neutral vertex:

\( Z_{\text{decay}} = Z (\pi^0) \); \( X_{\text{decay}}, Y_{\text{decay}} \) = impact point of reconstructed charged track on \( Z_{\text{decay}} \) plane
Specific $K_{\mu 3}$ selection cuts

Against $\pi^+\pi^0\pi^0$:

- $|P_2 - P_1| < 60$ GeV
  $\iff$ $D < 900$ GeV$^2$

[D is large when one pion is missing]

Against $K^\pm \rightarrow \pi^\pm\pi^0$

- $\pi^\pm$ misidentification as $\mu$
  $m(\pi^+\pi^0) < 0.475$ GeV/c$^2$
- $\pi^\pm \rightarrow \mu^\pm\nu$
  $m(\pi^+\pi^0) < 0.6 - P_1(\pi^0)/c$
  $m(\mu^\pm\nu) > 0.16$ GeV/c$^2$
  (to exclude $\pi^+$ mass region)
Specific $K_{e3}$ selection cut

- Transversal momentum with respect to beam axis $P_t >= 0.03$ GeV

against $K^\pm \rightarrow \pi^\pm \pi^0$ with $\pi^\pm$ misidentified as $e$ (when $E/P > 0.9$);

Common cuts:

- Beam (transverse elliptic) variable $B < 11$.  

- $P_L(\nu)^2 = (E\nu)^2/c^2 - (P_t\nu)^2 > 0.0014$ GeV$^2/c^2$

negative and zero regions are difficult to simulate exactly: sensitive to beam shape.

- $Z > -1600$ cm

(Final collimator is at $Z = -1800$ cm)
## Background

<table>
<thead>
<tr>
<th>Decay</th>
<th>$r_e, 10^{-3}$</th>
<th>$r_\mu, 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pm \to \pi^\pm (\pi^0 \to 2\gamma)$</td>
<td>0.272</td>
<td>0.392</td>
</tr>
<tr>
<td>$K^\pm \to \pi^\pm 2 (\pi^0 \to 2\gamma)$</td>
<td>0.287</td>
<td>2.192</td>
</tr>
<tr>
<td>$K^\pm \to \pi^\pm (\pi^0 \to e^+ e^- \gamma)$</td>
<td>0.049</td>
<td>0.000</td>
</tr>
<tr>
<td>$K^\pm \to \pi^\pm \gamma (\pi^0 \to 2\gamma)$</td>
<td>0.004</td>
<td>0.044</td>
</tr>
<tr>
<td>$K^\pm \to \pi^0 \mu^\pm \nu (\mu \to e\nu)$</td>
<td>0.004</td>
<td>0.000</td>
</tr>
</tbody>
</table>

$r_e$ — background to signal ratio in $K_{e^3}$ data

$r_\mu$ — background to signal ratio in $K_{\mu^3}$ data

BG contamination from $2\pi$ and $3\pi$: very small, $O(10^{-4} - 10^{-3})$

BG contamination from other channels: negligible
In order to extract form factors, the background-corrected Dalitz plots were fitted by reweighting of each MC event using variable form factor parameters.
Reconstructed lepton energy and pion energy distributions for data after background subtraction and simulated samples according to the fit results using the Taylor expansion model (other parameterisations look very similar).

The fit results and systematic uncertainties obtained (see JHEP 1810 (2018) 150):

- For $K_{e3}$
- For $K_{\mu3}$
- For the combined $K_{l3}$ result:
  Joint fits minimizing $\chi^2(K_{e3}) + \chi^2(K_{\mu3})$ with a common set of fit parameters.
## Results of the joint $K_{l3}$ ($K_{e3} + K_{\mu3}$) analysis

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_+^*$</th>
<th>$\lambda_+''$</th>
<th>$\lambda_0$</th>
<th>$m_V$</th>
<th>$m_S$</th>
<th>$\Lambda_+$</th>
<th>$\ln C$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Central values</strong></td>
<td>24.24</td>
<td>1.67</td>
<td>14.47</td>
<td>884.4</td>
<td>1208.3</td>
<td>24.99</td>
<td>183.65</td>
</tr>
<tr>
<td><strong>Statistical error</strong></td>
<td>0.75</td>
<td>0.29</td>
<td>0.63</td>
<td>3.1</td>
<td>21.2</td>
<td>0.20</td>
<td>5.92</td>
</tr>
<tr>
<td>Diverging beam component</td>
<td>0.97</td>
<td>0.35</td>
<td>0.55</td>
<td>1.1</td>
<td>32.2</td>
<td>0.08</td>
<td>9.43</td>
</tr>
<tr>
<td>Kaon momentum spectrum</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.1</td>
<td>0.7</td>
<td>0.00</td>
<td>0.19</td>
</tr>
<tr>
<td>Kaon mean momentum</td>
<td>0.04</td>
<td>0.01</td>
<td>0.04</td>
<td>0.2</td>
<td>1.7</td>
<td>0.01</td>
<td>0.47</td>
</tr>
<tr>
<td>LKr energy scale</td>
<td>0.66</td>
<td>0.12</td>
<td>0.61</td>
<td>4.9</td>
<td>17.4</td>
<td>0.32</td>
<td>5.16</td>
</tr>
<tr>
<td>LKr non-linearity</td>
<td>0.20</td>
<td>0.01</td>
<td>0.55</td>
<td>3.1</td>
<td>19.6</td>
<td>0.20</td>
<td>5.77</td>
</tr>
<tr>
<td>Residual background</td>
<td>0.08</td>
<td>0.03</td>
<td>0.04</td>
<td>0.1</td>
<td>0.7</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td>Electron identification</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.2</td>
<td>0.2</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Event pileup</td>
<td>0.23</td>
<td>0.08</td>
<td>0.08</td>
<td>0.4</td>
<td>0.2</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.23</td>
<td>0.07</td>
<td>0.03</td>
<td>0.7</td>
<td>4.3</td>
<td>0.05</td>
<td>1.11</td>
</tr>
<tr>
<td>Neutrino momentum resolution</td>
<td>0.16</td>
<td>0.04</td>
<td>0.04</td>
<td>0.9</td>
<td>3.3</td>
<td>0.06</td>
<td>0.88</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>0.29</td>
<td>0.13</td>
<td>0.20</td>
<td>1.1</td>
<td>9.9</td>
<td>0.07</td>
<td>2.82</td>
</tr>
<tr>
<td>Dalitz plot binning</td>
<td>0.05</td>
<td>0.04</td>
<td>0.06</td>
<td>0.9</td>
<td>1.1</td>
<td>0.06</td>
<td>0.29</td>
</tr>
<tr>
<td>Dalitz plot resolution</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.0</td>
<td>1.3</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>Radiative corrections</td>
<td>0.17</td>
<td>0.01</td>
<td>0.57</td>
<td>2.5</td>
<td>20.1</td>
<td>0.16</td>
<td>5.92</td>
</tr>
<tr>
<td><strong>Systematic error</strong></td>
<td>1.30</td>
<td>0.41</td>
<td>1.17</td>
<td>6.7</td>
<td>47.5</td>
<td>0.62</td>
<td>14.25</td>
</tr>
<tr>
<td><strong>Total error</strong></td>
<td>1.50</td>
<td>0.50</td>
<td>1.32</td>
<td>7.4</td>
<td>52.1</td>
<td>0.65</td>
<td>15.43</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>$-0.934$ ($\lambda_+''/\lambda_+'$)</td>
<td>0.374</td>
<td>0.354</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$0.118$ ($\lambda_+''/\lambda_0$)</td>
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<tr>
<td></td>
<td>$0.091$ ($\lambda_+''/\lambda_0$)</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$/NDF</td>
<td>979.6/1070</td>
<td>979.3/1071</td>
<td>979.7/1071</td>
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</tbody>
</table>
Joint $K_{l3}$ results comparison for quadratic parameterization

$1\sigma$ ellipses (39.4% CL) rather than 68% for better visibility
Conclusion

- The 4919 $K^\pm \rightarrow \pi^\pm \pi^0 e^+ e^-$ rare decay candidates, with a 5% background contamination, are first observed. Branching ratio is measured to be $(4.24 \pm 0.15) \times 10^{-6}$, that is in good agreement with ChPT-based theoretical predictions. The relative contributions of $(M)/IB$ and $(IB-E)/IB$ are also in agreement with the theory. Several CP-violating asymmetries and a long-distance P-violating asymmetry have been evaluated and found to be consistent with zero.


- $K_{l3}$ form factors measurement is performed by NA48/2 on the basis of 2004 run selected $4.4 \cdot 10^6 (K_{e3})$ and $2.3 \cdot 10^6 (K_{\mu3})$ events. Result is competitive with the other ones in $K_{\mu3}$ mode, and a smallest error in $K_{e3}$ has been reached, that gives us also the most precise combined $K_{l3}$ result.

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SPARES
Angular asymmetries definition


Defining sectors of the $\phi$ space between 0 and 2$\pi$ as $\Phi_1 (0, \pi/2)$, $\Phi_2 (\pi/2, \pi)$, $\Phi_3 (\pi, 3\pi/2)$, $\Phi_4 (3\pi/2, 2\pi)$, and combining them as statistically independent sector sums ($\Phi_{13} = \Phi_1 + \Phi_3$, $\Phi_{24} = \Phi_2 + \Phi_4$), one can define the long-distance asymmetry as:

$$A_{CP}^{(L)} = \frac{\int_0^{2\pi} \frac{d\Gamma}{d\phi} d\phi^*}{\int_0^{2\pi} \frac{d\Gamma}{d\phi} d\phi} = \frac{\Gamma(\Phi_{13}) - \Gamma(\Phi_{24})}{\Gamma}$$
$K_{l3}$ common cuts

$P_L(v)^2 = (E^\gamma)^2/c^2 - (P_1^\gamma)^2 > 0.0014 \text{ GeV}^2/c^2$

Negative tail and zero region are difficult to simulate exactly: sensitive to beam shape.

$Z > -1600 \text{ cm}$

Final collimator is at $Z = -1800$
Kaon momentum reconstruction

Two solutions of the quadratic equation for $P_K$:

$$P_{1,2} = \left(\phi P_{zb} \pm \sqrt{D}\right) / (E^2 - P_{zb}^2),$$

where

$\phi = 0.5 \left( M_K^2 + E^2 - P_t^2 - P_{zb}^2 \right)$,

$D = \left( \phi^2 P_{zb}^2 - (E^2 - P_{zb}^2)(M_K^2 E^2 - \phi^2) \right)$

When $D<0$, we assume $D=0$.

Assumptions:

- $P_T(v) = -P_t$
- $M(v) = 0$

Best $P_K$ solution = closest $P_{1,2}$ to the average beam momentum $P_b$ (measured from $3\pi^{\pm}$ decays for each run).

Select: $-7.5 \text{ GeV/c} < (P_K - P_b) < 7.5 \text{ GeV/c}$

For each event, separately for $K_{e3}$ and $K_{\mu3}$ selections, the combination with a minimum $\Delta P = |P_K - P_b|$ is the best candidate.
Data $3\pi^\pm$ decay: Kaon impact points X,Y at the focus plane (4500 cm)

Focused scattering is simulated in MC: 3% of beam kaons are additionally directed into a series of rings at the focus plane with a different radius > 2.2 cm.
Background processes, branching ratios $\mathcal{B}$ generated MC statistics in the fiducial volume 112 m long, $N_{\text{gen}}$, and the estimated contributions $f_{K\ell 3}$ and $f_{K\mu 3}$ in the selected $K_{e3}$ and $K_{\mu 3}$ samples. The upper limit for one of the values is shown at 95% CL.

<table>
<thead>
<tr>
<th>Process</th>
<th>$\mathcal{B}$ [%]</th>
<th>$N_{\text{gen}}$ [$10^6$]</th>
<th>$f_{K\ell 3}$ [$10^{-3}$]</th>
<th>$f_{K\mu 3}$ [$10^{-3}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0 \ (\pi^0 \rightarrow \gamma\gamma, \pi^0 \rightarrow \gamma\gamma)$</td>
<td>1.72(2)</td>
<td>62.5</td>
<td>0.286(6)</td>
<td>2.192(32)</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \pi^0 \ (\pi^0 \rightarrow \gamma\gamma)$</td>
<td>20.43(8)</td>
<td>393.2</td>
<td>0.271(6)</td>
<td>0.392(10)</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \pi_D^0 \ (\pi_D^0 \rightarrow e^+ e^- \gamma)$</td>
<td>0.243(7)</td>
<td>1.5</td>
<td>0.049(5)</td>
<td>0.0008(8)</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^0 \mu^+ \nu \ (\pi^0 \rightarrow \gamma\gamma) [\text{via } \mu \rightarrow e\bar{\nu}\nu]$</td>
<td>0.033(3)</td>
<td>174.3</td>
<td>0.044(5)</td>
<td>---</td>
</tr>
<tr>
<td>$K^\pm \rightarrow e^\pm \nu \pi^0 \pi^0 \ (\pi^0 \rightarrow \gamma\gamma, \pi^0 \rightarrow \gamma\gamma)$</td>
<td>0.0022(4)</td>
<td>5.0</td>
<td>0.019(3)</td>
<td>$&lt;4 \times 10^{-6}$</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \pi^0 \gamma \ (T_{\pi^+} = 55 - 90 \text{MeV}, \pi^0 \rightarrow \gamma\gamma)$</td>
<td>0.027(2)</td>
<td>35.3</td>
<td>0.0044(3)</td>
<td>0.071(4)</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0 \ (\pi^0 \rightarrow \gamma\gamma, \pi^0 \rightarrow e^+ e^- \gamma)$</td>
<td>0.0204(7)</td>
<td>9.9</td>
<td>0.0028(2)</td>
<td>0.0130(5)</td>
</tr>
<tr>
<td>$K^\pm \rightarrow \mu^+ \nu \pi^0 \pi^0 \ (\pi^0 \rightarrow \gamma\gamma, \pi^0 \rightarrow \gamma\gamma)$</td>
<td>0.0004(2)</td>
<td>5.0</td>
<td>0.19(11) $\times 10^{-5}$</td>
<td>0.004(2)</td>
</tr>
</tbody>
</table>
**K_{l3} events-weighting fit procedure**

We use MC radiative decay generator of C.Gatti [Eur.Phys.J. C45 (2006) 417–420] provided by KLOE collaboration. It includes $f_0 = f_0 = 1 + \lambda' t/m^2_{\pi}$. Generator for $K_{e3}$ is corrected by weighting to conform to the «universal» part of radiative effects ($K_{\mu3}$ is OK) [Cirigliano et al. Eur.Phys. J. C 23 (2002) 121]. So we measure «effective» FF-s that absorb interplay between QED and low-energy QCD.

- For each fit iteration, the model Dalitz plot is filled in with an MC simulated reconstructed center-of-mass pion and lepton energies. Each event is weighted by

  \[ w = \rho_0(E_{\pi}^{true}, E_{l}^{true}, FF_{\text{fit}}) / \rho_0(E_{\pi}^{true}, E_{l}^{true}, FF_{\text{MC generator}}), \]

  where $\rho_0$ is the non-radiative Dalitz density formula (for $K_{e3}$ an additional factor is applied to conform the universal correction).

- MINUIT package is searching for the FF parameters minimizing the standard $\chi^2$ value:

  \[ \chi^2 = \sum_{i,j} \frac{(D_{i,j} - MC_{i,j})^2}{(\delta D_{i,j})^2 + (\delta MC_{i,j})^2}, \]

  where $i,j$ means the Dalitz plot cell indices, $D_{i,j}$ is the background-corrected experimental data content of the cell, $MC_{i,j}$ is the weighted MC bin content, and $\delta D_{i,j}$, $\delta MC_{i,j}$ are the corresponding statistical errors. **Background correction contribution also has some dependence on FF due to the signal acceptance sensitivity.**

At least 20 data events per cell are required in the fit area, so $\chi^2$ works well.