Higgs boson pair and H+jet production

Matthias Kerner (UZH)

31st Rencontres de Blois

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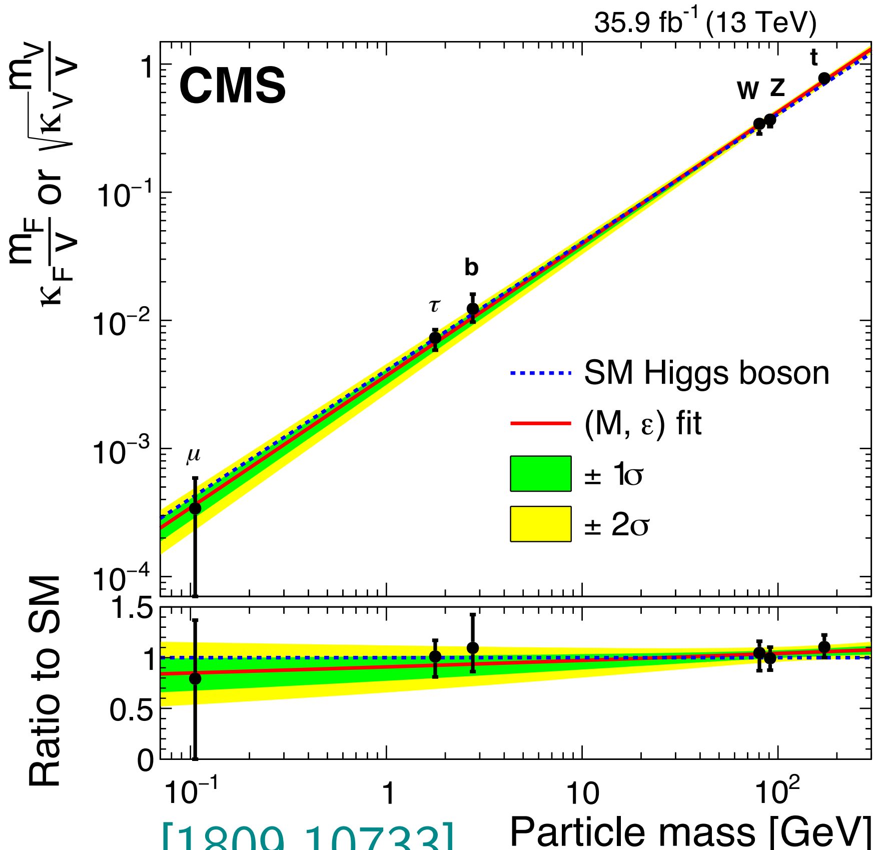
based on:

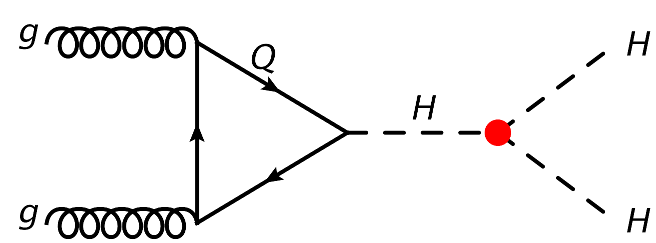
- Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke
  Phys.Rev.Lett. 117 (2016) no.1, 012001 [arXiv:1604.06447], JHEP 1610 (2016) 107 [arXiv:1608.04798]
- Jones, MK, Luisoni
  Phys.Rev.Lett. 120 (2018) no.16, 162001 [arXiv:1802.00349]

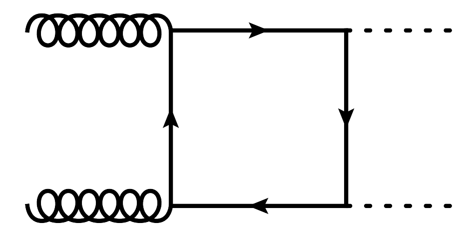
Introduction



[1809.10733]

Higgs self-interaction not yet established





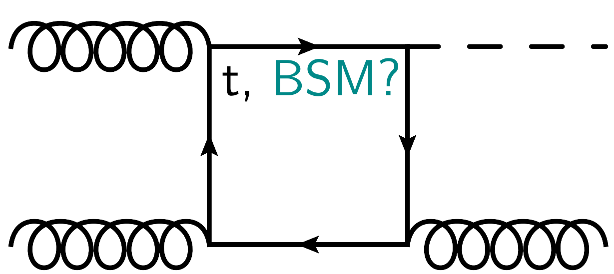
Measurements of Higgs boson pair production

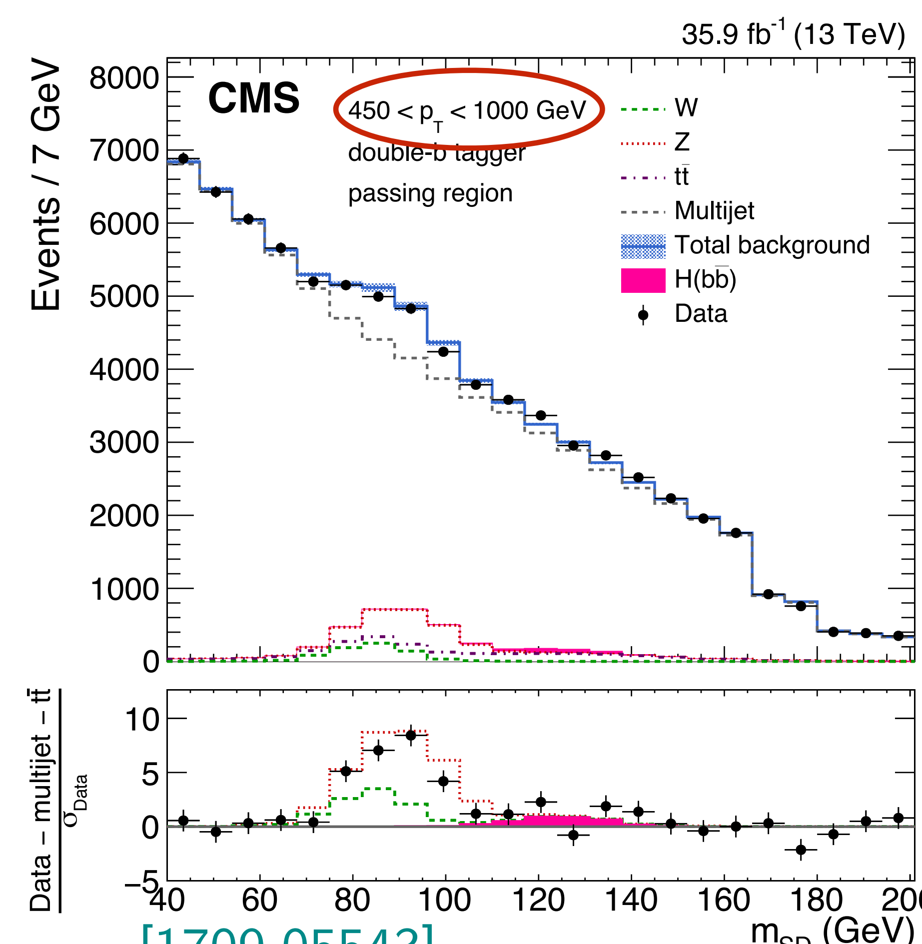
→ direct relation to Higgs potential

→ test mechanism of EW symmetry breaking

Higgs+Jet production in gluon fusion

boosted production sensitive to particle in loop

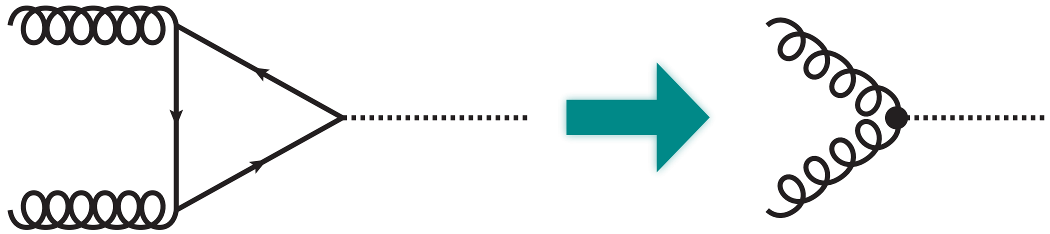




[1709.05543]

HEFT vs. full theory

Many calculations in Higgs physics done in the $m_t \to \infty$ limit (Higgs EFT)



$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4} H G_a^{\mu\nu} G_{a,\mu\nu}$$

with $\lambda = -\frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$

HEFT is only good approximation if $\mathrm{m_T}$ largest scale of process!

valid for inclusive gg→H production, but poor approximation for HH and HJ production

Higgs pair production:

HEFT valid for $\sqrt{s} \ll 2\, m_T$

but $2\, m_H < \sqrt{s}$

→ only small contributions to xsec from regions where HEFT valid

HJ production at large $\mathrm{p_T}$:

$2\, m_T < 2\, p_T < m_{HJ}$

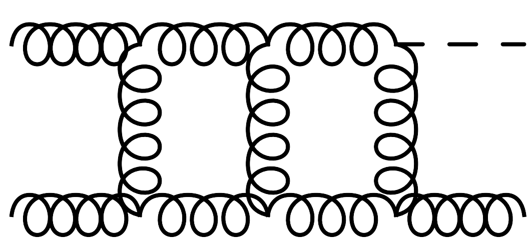
HEFT expected to break down at large pt

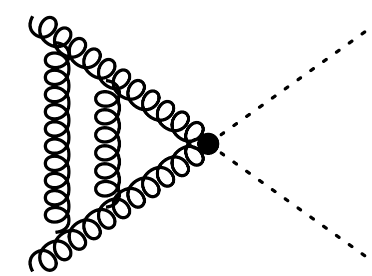
→ predictions of these processes with full top mass dependence required

Fixed Order Results

HJ and HH production are known to NNLO in heavy top approximation

	HJ	HH
NLO:	de Florian, Grazzini, Kunszt 99 Ravindran, Smith, van Neerven 02 Glosser, Schmidt 02	Dawson, Dittmaier, Spira 98
NNLO:	Boughezal, Caola, Melnikov, Petriello, Schulze 13, 14 Chen, (Martinez,) Gehrmann, Glover, Jaquier 14, 16 Boughezal, Focke, Giele, Liu, Petriello 15	de Florian, Mazzitelli 13 Grigo, Melnikov, Steinhauser 14 de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16





Results with full m_T dependence:

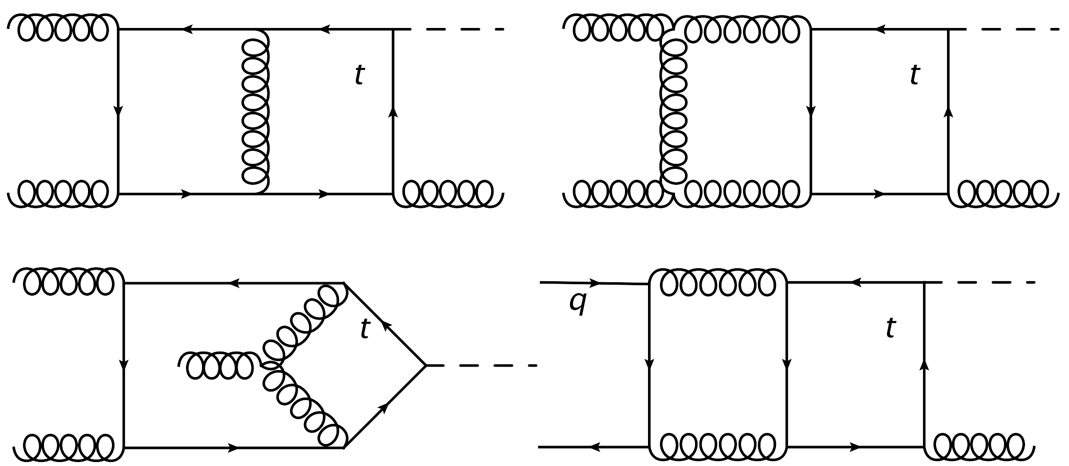
	HJ	HH
LO:	Ellis, Hinchliffe, Soldate, van der Bij 87 Baur, Glover 89	Glover, van der Bij 88
NLO:	Jones, MK, Luisoni 18 this talk	Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16 Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18

Virtual Corrections

Virtual Corrections

- computation very challenging
- 4 mass scales $s,\ t,\ m_T,\ m_H$

2-loop 4-point diagrams with internal mass



Strategies for Computation

1) fully analytic calculation [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov 16]
   - involves elliptic integrals
   - so far only planar diagrams
   - in Euclidean region ($s = m^2_{Hj} < 0$)

2) expansions:
   - in $1/m_T$: HH: [Grigo, Hoff, Melnikov, Steinhauser 13,15] [Degrassi, Giardino, Gröber 16]
     HJ: [Neumann, Wiesemann, (Harlander, Ozeren) 12,14]
   - in m_T HH: [Davies, Mishima, Steinhauser 18] → talk by Chris Wever
     HJ: [Melnikov, Tancredi, Wever 16, 17], [Kudashkin, Melnikov, Wever 17]
   - further expansion for HH: [Gröber, Maier, Rauh 17], [Bonciani, Degrassi, Giardino, Gröber 18], [Xu, Yang 18]

3) we compute all 2-loop integrals numerically

NLO Calculation — Overview

1. Form factor decomposition of amplitude
2. Integration-by-parts reduction using Reduze von Manteuffel, Studerus`12

   → exploit linear relations to express integrals with minimal set of master integrals
   use basis of finite integrals von Manteuffel, Panzer, Schabinger `15

3. Numerical evaluation of 2-loop integrals using SecDec Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke
   → applies Sector Decomposition to factorize and subtract UV/IR poles
   → integrals can be computed numerically
   - using Quasi-Monte-Carlo integration with $\mathcal{O}(n^{-1})$ scaling
     Li, Wang, Yan, Zhao `15; Review: Dick, Kuo, Sloan
   - dynamically set number of sampling points for each integral
   - parallelization on GPU

4. Use unweighted events (based on LO) for optimized phase space sampling of virtuals

5. Real radiation amplitudes generated with GoSam Cullen et.al.

Integral Reduction

IBP reduction obtained using Reduze 2 [von Manteuffel, Studerus 12]

but reduction with 4 independent scales (s, t, m_t^2, m_H^2) challenging

$\rightarrow$ modifications to Reduze code:

- specify list of required integrals
  $\rightarrow$ consider only equations containing these integrals
- change order of solving the system of equations, sorting the equations by number of unreduced integrals

useful additional simplification: fix m_t and m_H

HH reduction with fixed masses:
$m_t = 173\,\mathrm{GeV}$, $m_H = 125\,\mathrm{GeV}$
but we did not manage to obtain reduction of non-planar integrals!
$\rightarrow$ rewrite inverse propagators as scalar products to reduce rank
$\rightarrow$ directly calculate them numerically

full HJ reduction obtained twice:

- with $m_H^2/m_t^2 = 12/23$
- with full m_t and m_H dependence
  $\rightarrow$ will allow to study
  - bottom quark contributions
  - mass scheme dependence

Numerical Integration

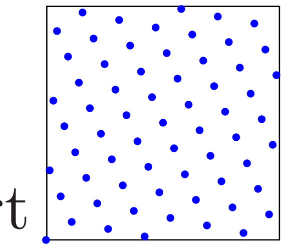
Virtual amplitude — evaluation of loop integrals

- all integrals evaluated using Quasi-Monte-Carlo integration
  - generating vector
    - constructed component-by-component [Nuyens 07]
    - minimizing worst-case error
    - for fixed lattice sizes
  - $\mathcal{O}(n^{-1})$ scaling of integration error
- parallelization on gpu
- many further optimizations ...

QMC rank-1 lattice rule

$$I = \int \mathrm{d}\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n}\sum_{i=1}^{n} f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$



$\{\ldots\} =$ fractional part

$\vec{g} =$ generating vector

$\vec{\Delta}_k =$ randomized shift

m different estimates $I_1 \ldots I_m$

$\rightarrow$ error estimate

[Li, Wang, Yan, Zhao 16]

Review: [Dick, Kuo, Sloan]

QMC integrator available Borowka, Heinrich, Jahn, Jones, MK, Schlenk 18

- as standalone single-header c++ library github.com/mppmu/qmc
- within pySecDec github.com/mppmu/secdec

H+Jet Production

HJ Results — Total Cross Section

Theory	LO [pb]	NLO [pb]
HEFT:	$\sigma_{\rm LO} = 8.22^{+3.17}_{-2.15}$	$\sigma_{\rm NLO} = 14.63^{+3.30}_{-2.54}$
$\mathrm{FT}_{\rm approx}$:	$\sigma_{\rm LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\rm NLO} = 15.07^{+2.89}_{-2.54}$
Full:	$\sigma_{\rm LO} = 8.57^{+3.31}_{-2.24}$	$\sigma_{\rm NLO} = 16.01^{+1.59}_{-3.73}$

- LHC @ 13 TeV
- $p_{T,j} > 30\,\mathrm{GeV},\ R = 0.4$, anti-$k_T$
- scale: $\frac{H_T}{2} = \frac{1}{2}\left(\sqrt{m_{\rm H}^2 + p_{t,\rm H}^2} + \sum_i |p_{t,i}|\right)$
- PDF4LHC15
- $m_H = 125\,\mathrm{GeV}$
  $m_t = \sqrt{23/12}\, m_H \approx 173.05\,\mathrm{GeV}$

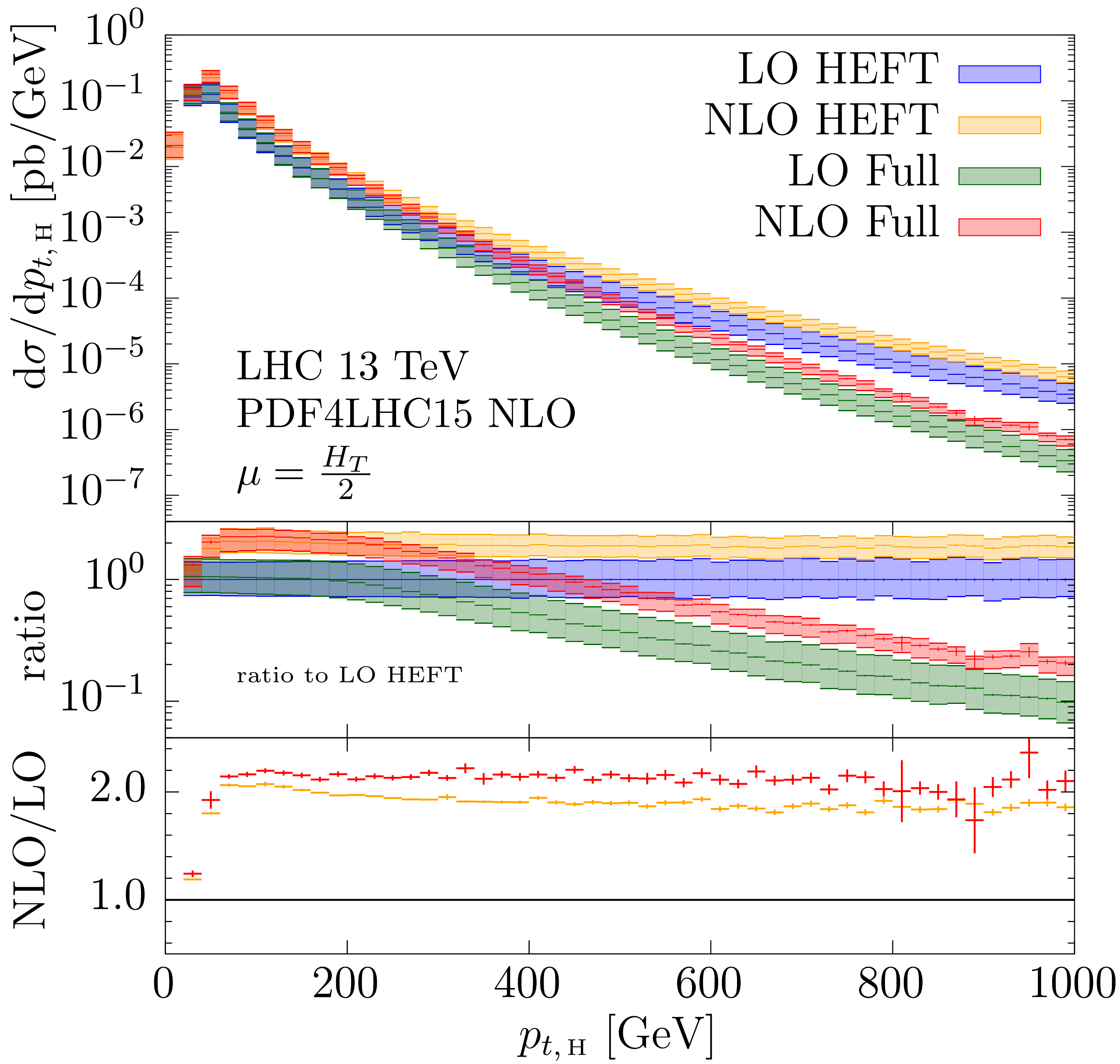
$\mathrm{FT}_{\rm approx}$:

$$\mathrm{d}\sigma_{\rm NLO}^{\rm FT_{approx}} = \int \mathrm{d}PS_2 \left(\mathrm{d}\sigma_{\rm B}^{\rm Full} + \frac{\mathrm{d}\sigma_{\rm B}^{\rm Full}}{\mathrm{d}\sigma_{\rm B}^{\rm HEFT}}\, \mathrm{d}\sigma_{\rm V}^{\rm HEFT}\right) + \int \mathrm{d}PS_3\, \mathrm{d}\sigma_{\rm R}^{\rm Full}$$

top-quark mass effects: +4.3% at LO
+9% at NLO (+6% compared to $\mathrm{FT}_{\rm approx}$)

HJ Results — p_T of Higgs boson

mass effects compared to HEFT



HEFT and full theory predict different scaling of $\mathrm{d}\sigma/\mathrm{d}p_T^2$

$\sim p_T^{-2}$ in HEFT

$\sim p_T^{-4}$ in full theory

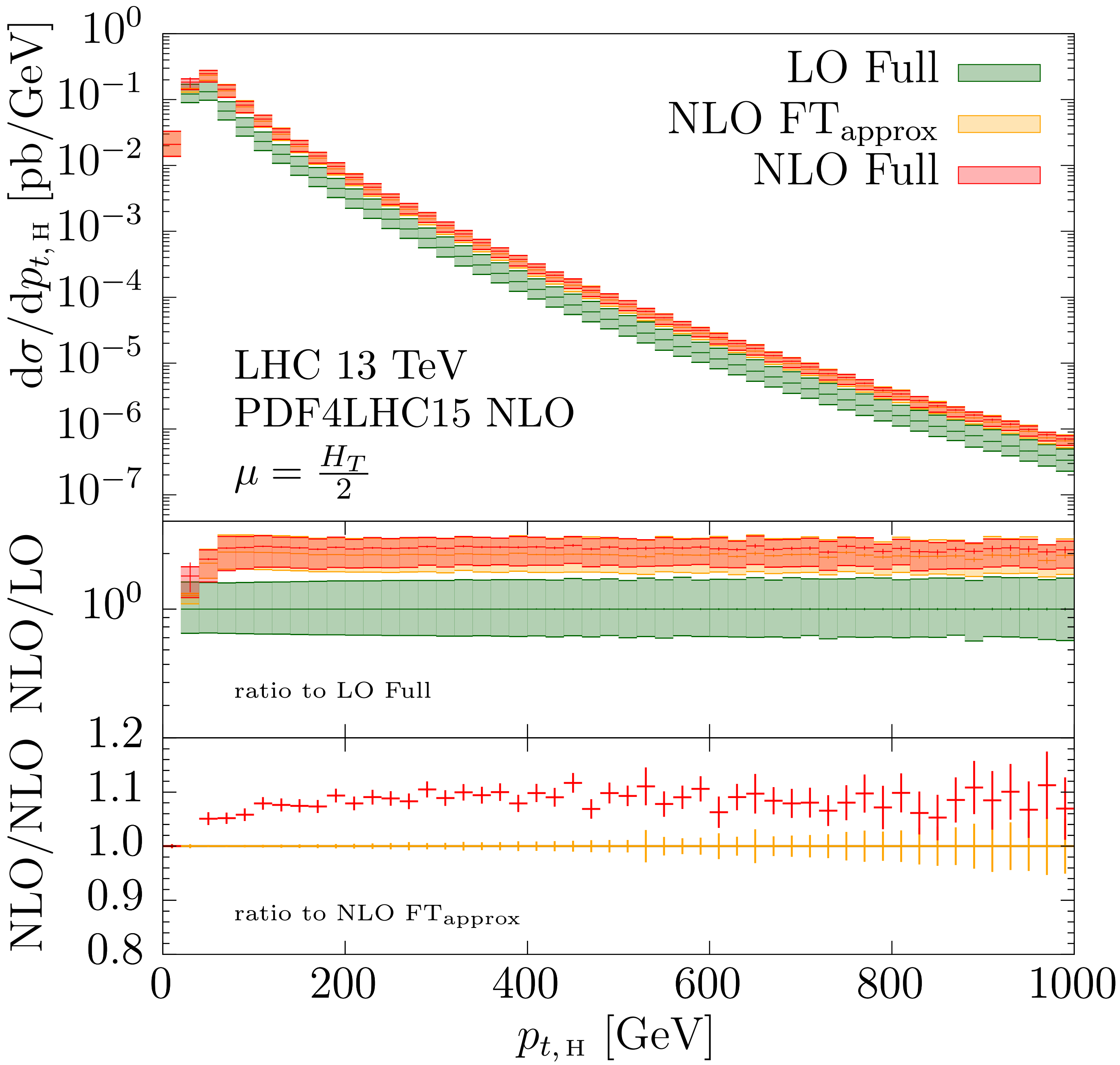
[Caola, Forte, Marzani, Muselli, Vita, 15,16]

confirmed at NLO

nearly constant K-factor in full theory

HJ Results — p_T of Higgs boson

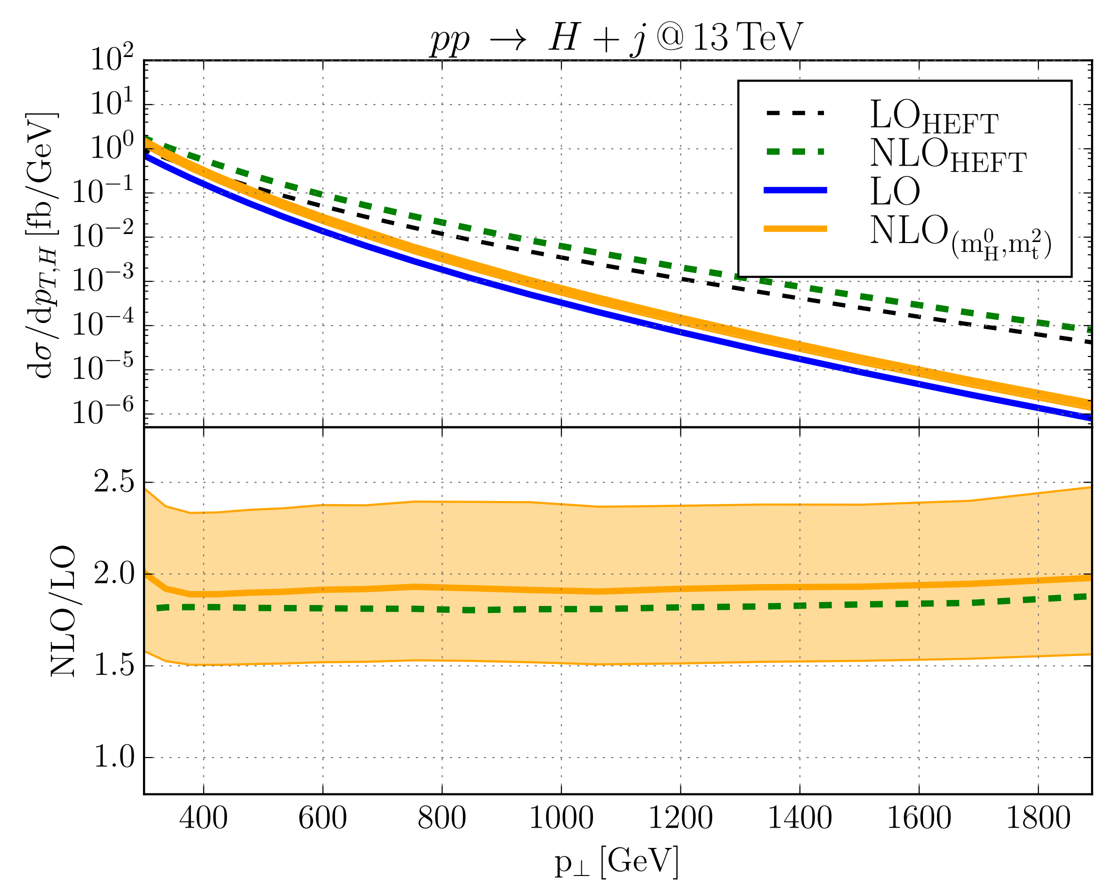
mass effects compared to FT_{approx}



FT_{approx} and full theory predict same shape of p_T distribution

nearly constant increase of ~8% due to top mass in virtual contribution

good agreement with results by Lindert, Kudashkin, Melnikov, Wever 18



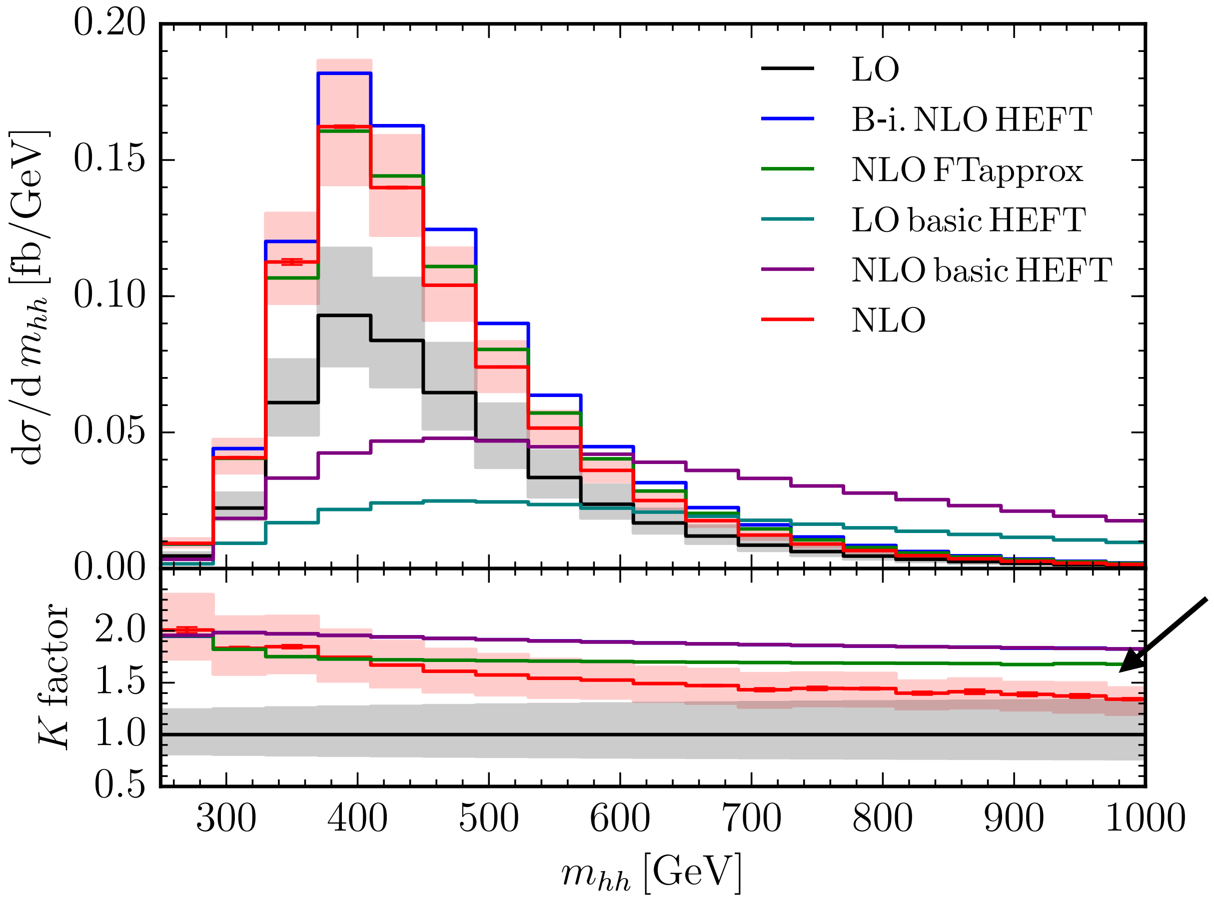
Higgs Pair Production

HH Results — fixed order NLO

$\sqrt{s}$	LO	B-i. NLO HEFT	NLO FT$_{approx}$	NLO
14 TeV	$19.85^{+27.6\%}_{-20.5\%}$	$38.32^{+18.1\%}_{-14.9\%}$	$34.26^{+14.7\%}_{-13.2\%}$	$32.91^{+13.6\%}_{-12.6\%}$
100 TeV	$731.3^{+20.9\%}_{-15.9\%}$	$1511^{+16.0\%}_{-13.0\%}$	$1220^{+11.9\%}_{-10.7\%}$	$1149^{+10.8\%}_{-10.0\%}$

-14% wrt. NLO HEFT

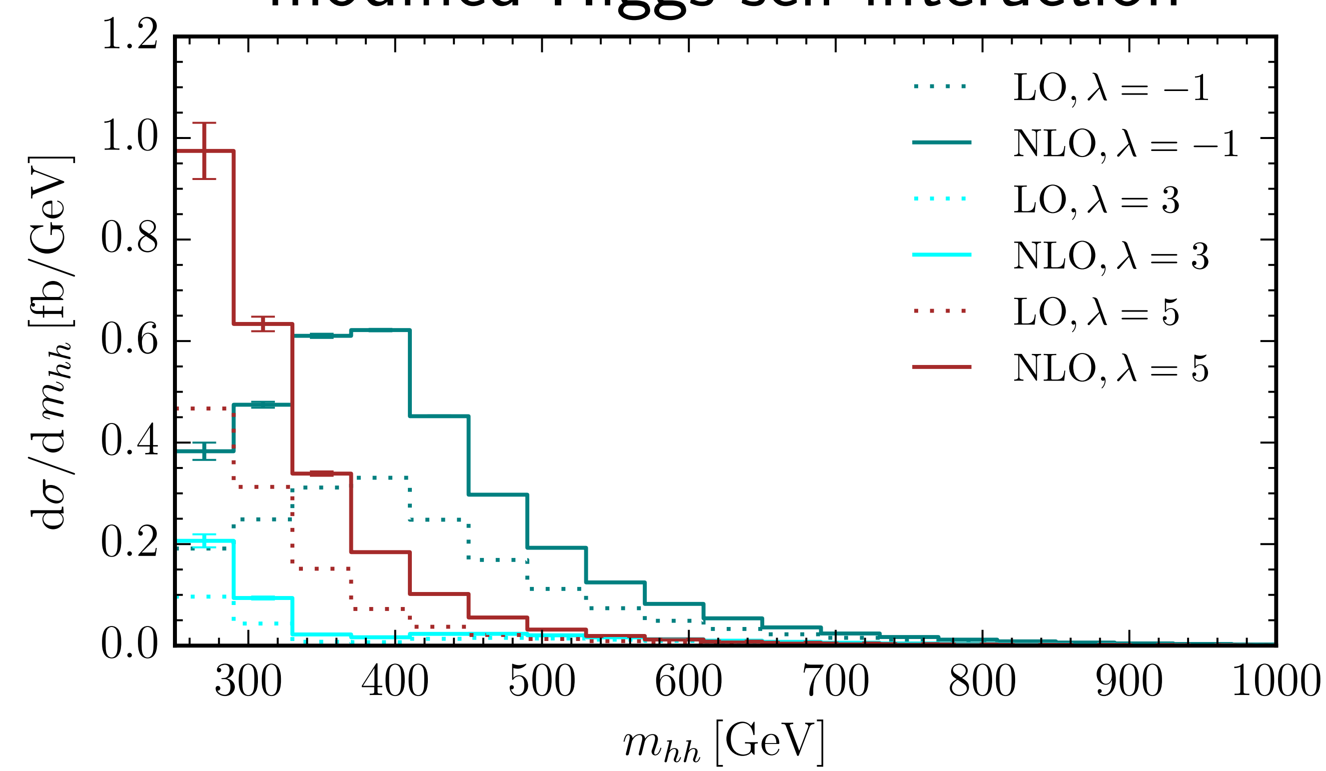
-4% wrt. NLO FT$_{\text{approx}}$



large dependence of K-factor on m_{hh}

large top mass effects in high m_{hh} region

modified Higgs self-interaction



BSM effects within Electroweak Chiral Lagrangian framework

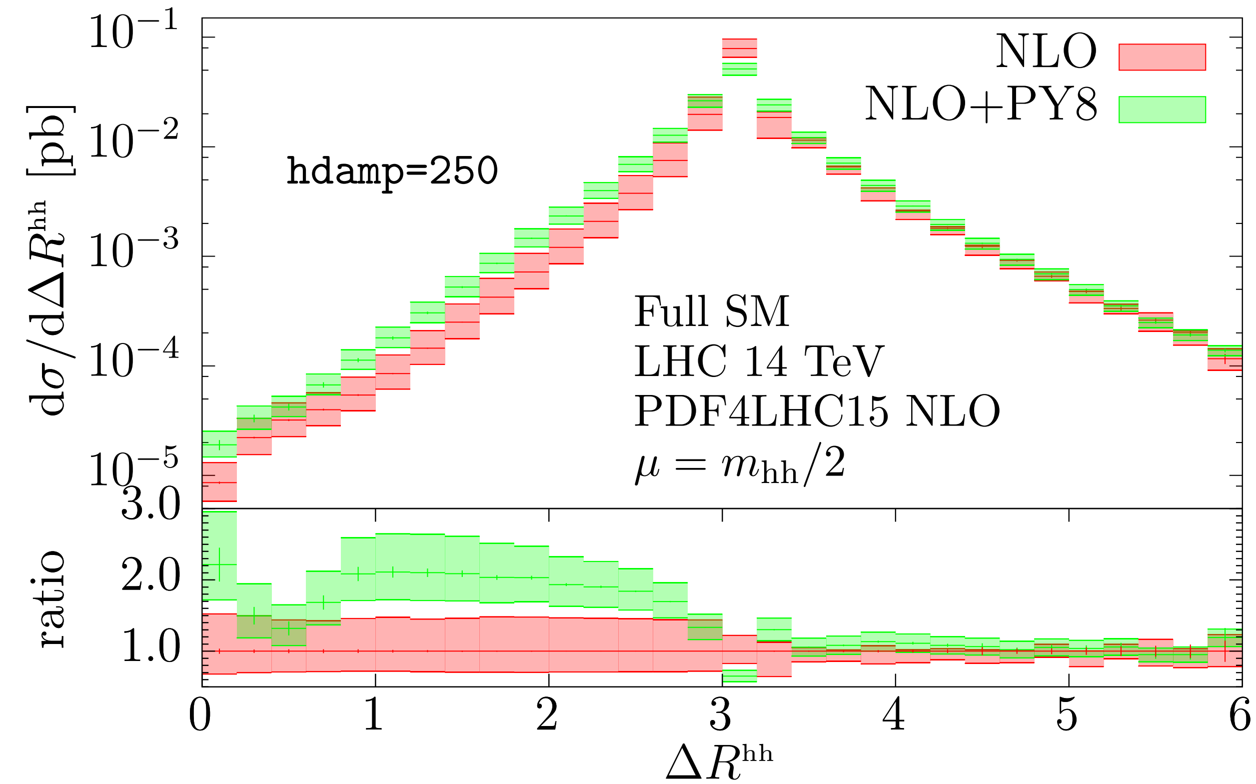
Buchalla, Capozi, Celis, Heinrich, Scyboz 18

HH Results — Parton Shower

combination with parton shower
using grid-interpolation of virtual amplitude
$\rightarrow$ publicly available in PowhegBox-V2

Heinrich, Jones, MK, Luisoni, Vryonidou 17

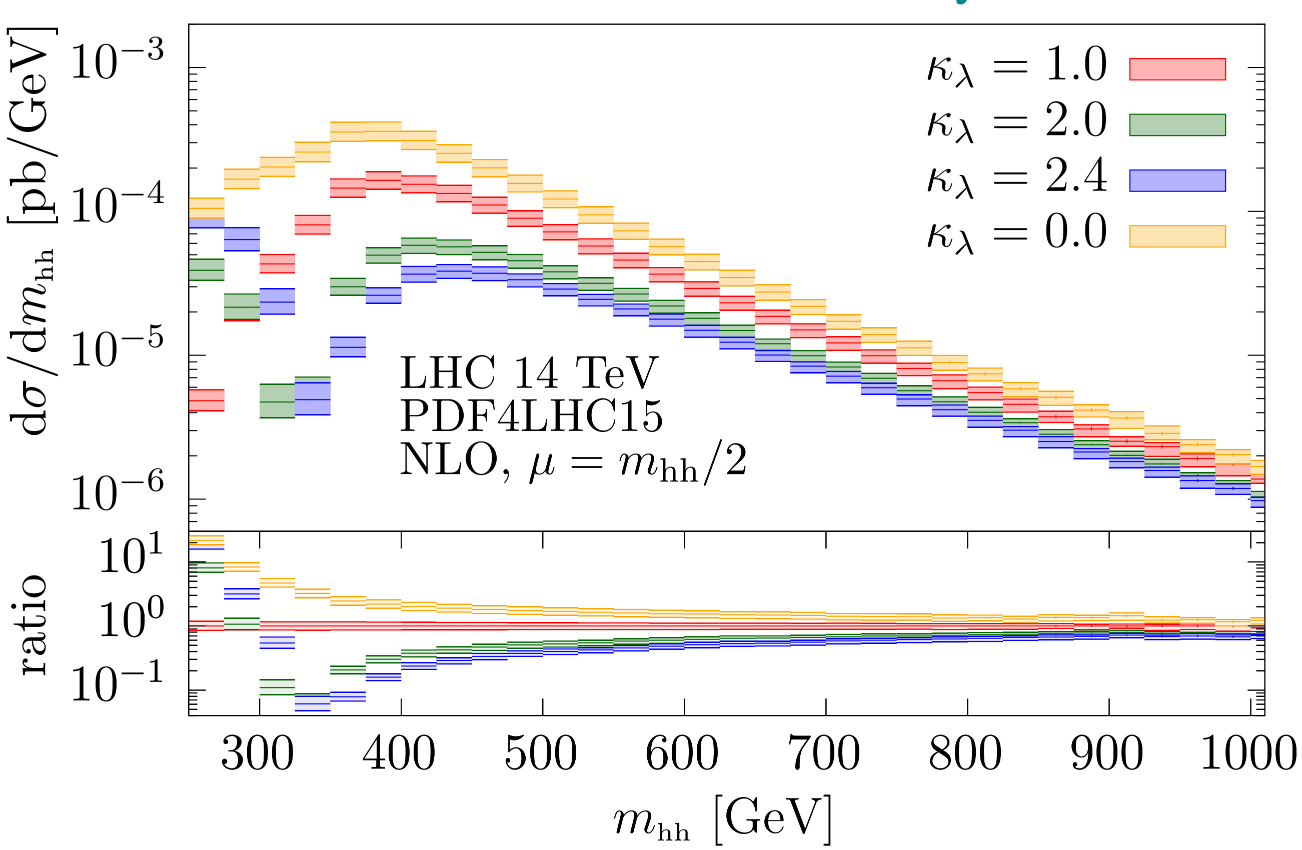
see also Jones, Kuttimalai 17

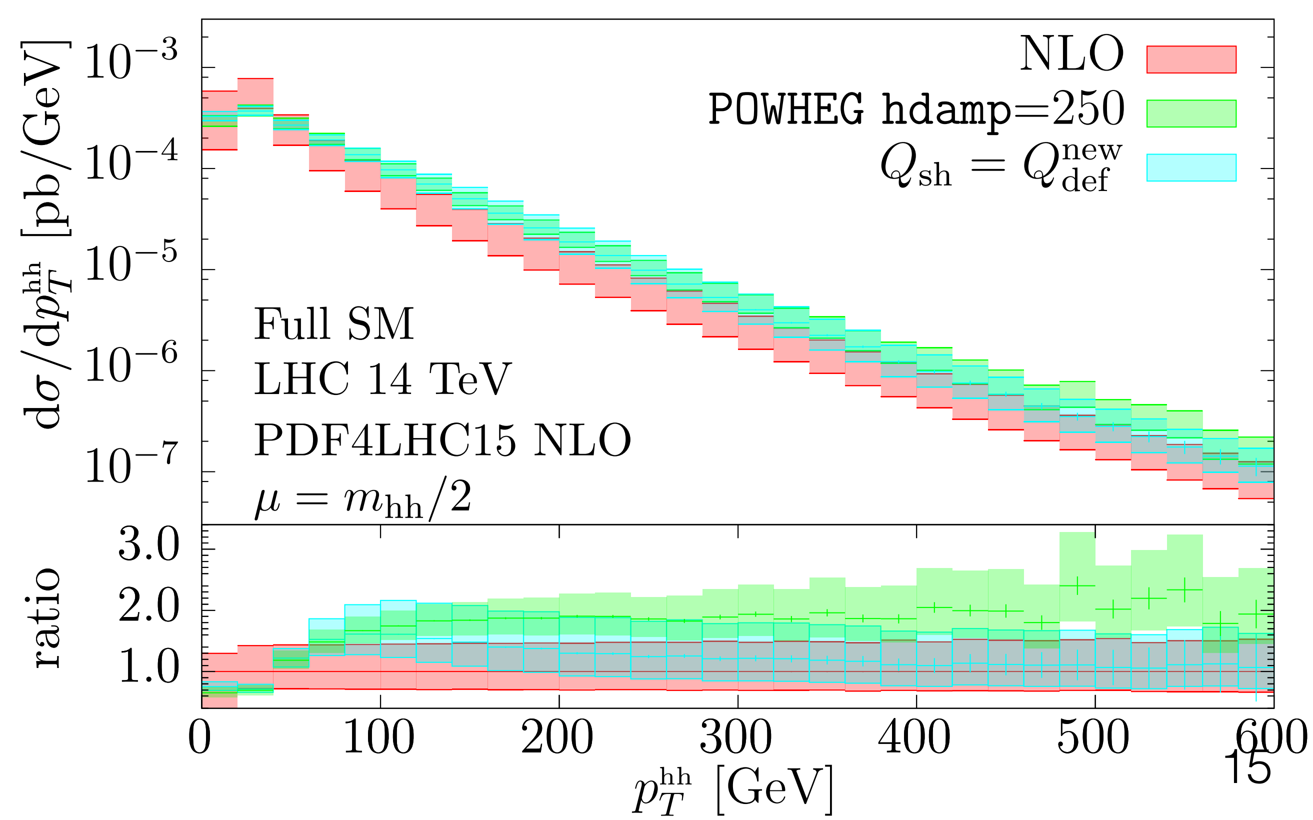


new: Variation of Higgs self-coupling κ_λ within Powheg-Box

Heinrich, Jones, MK, Luisoni, Skyboz 19

large parton shower effects/uncertainties in LO accurate phase-space regions





HH Results — Approximated NNLO

Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli 18

combination with NNLO $(m_t \to \infty)$

$\to$ approx. m_t dependence at NNLO

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	$27.78^{+13.8\%}_{-12.8\%}$	$32.88^{+13.5\%}_{-12.5\%}$	$127.7^{+11.5\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$
$\mathrm{NLO_{FTapprox}}$ [fb]	$28.91^{+15.0\%}_{-13.4\%}$	$34.25^{+14.7\%}_{-13.2\%}$	$134.1^{+12.7\%}_{-11.1\%}$	$1220^{+11.9\%}_{-10.6\%}$
$\mathrm{NNLO_{NLO-i}}$ [fb]	$32.69^{+5.3\%}_{-7.7\%}$	$38.66^{+5.3\%}_{-7.7\%}$	$149.3^{+4.8\%}_{-6.7\%}$	$1337^{+4.1\%}_{-5.4\%}$
$\mathrm{NNLO_{B-proj}}$ [fb]	$33.42^{+1.5\%}_{-4.8\%}$	$39.58^{+1.4\%}_{-4.7\%}$	$154.2^{+0.7\%}_{-3.8\%}$	$1406^{+0.5\%}_{-2.8\%}$
$\mathrm{NNLO_{FTapprox}}$ [fb]	$31.05^{+2.2\%}_{-5.0\%}$	$36.69^{+2.1\%}_{-4.9\%}$	$139.9^{+1.3\%}_{-3.9\%}$	$1224^{+0.9\%}_{-3.2\%}$
M_t unc. $\mathrm{NNLO_{FTapprox}}$	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
$\mathrm{NNLO_{FTapprox}/NLO}$	1.118	1.116	1.096	1.067

3 different methods:

1) $\mathrm{NNLO_{NLO\text{-}i}}$

rescale NLO by $K_{\mathrm{NNLO}} = \mathrm{NNLO_{HEFT}/NLO_{HEFT}}$

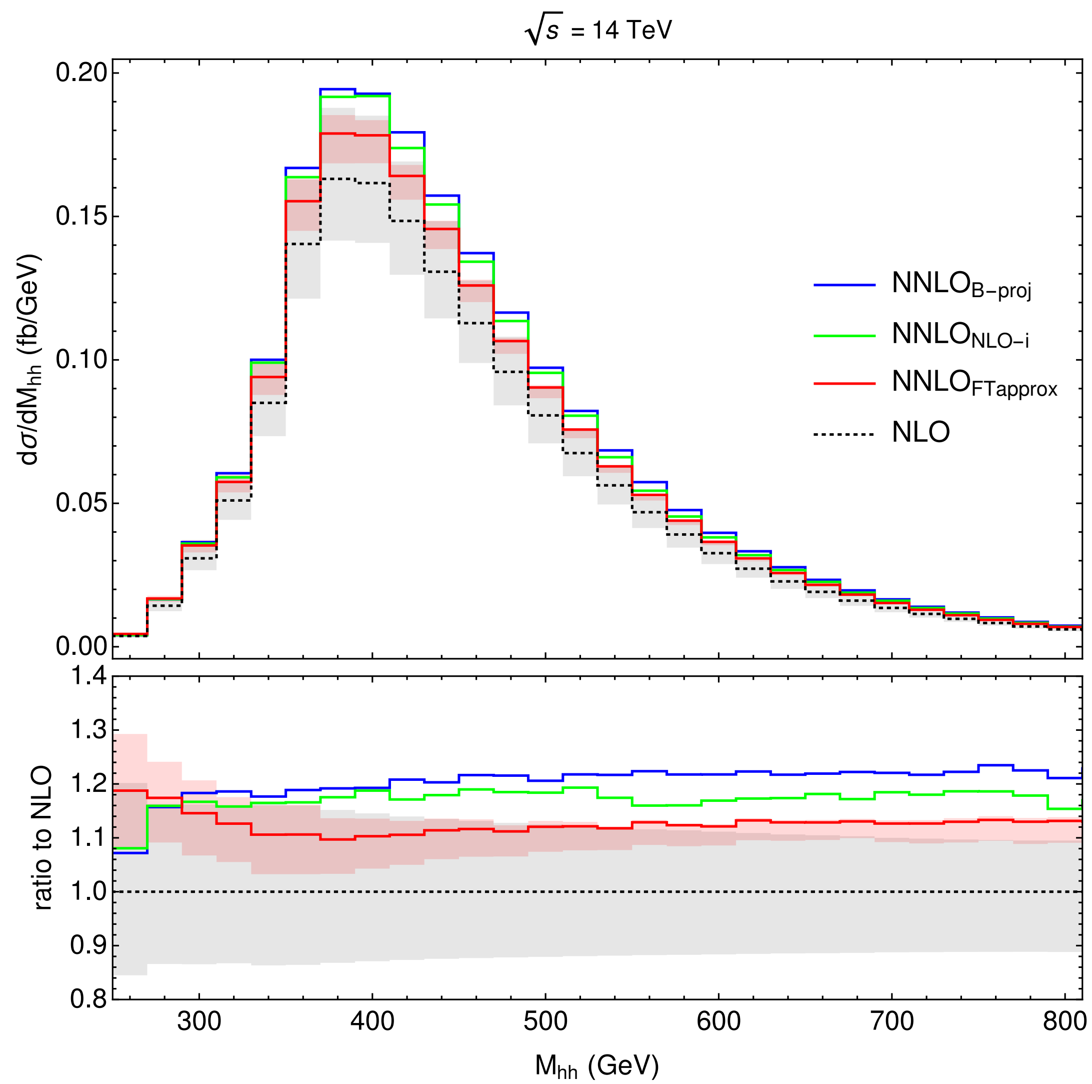
2) $\mathrm{NNLO_{B\text{-}proj}}$

project all real radiation contributions to Born configuration, rescale by $\mathrm{LO/LO_{HEFT}}$

3) $\mathrm{NNLO_{FTapprox}}$

calculate $\mathrm{NNLO_{HEFT}}$ and for each multiplicity rescale by

$$\mathcal{R}(ij \to HH + X) = \frac{\mathcal{A}^{\mathrm{Born}}_{\mathrm{Full}}(ij \to HH + X)}{\mathcal{A}^{(0)}_{\mathrm{HEFT}}(ij \to HH + X)}$$



Conclusion

HH and HJ production at NLO QCD with full m_t-dependence

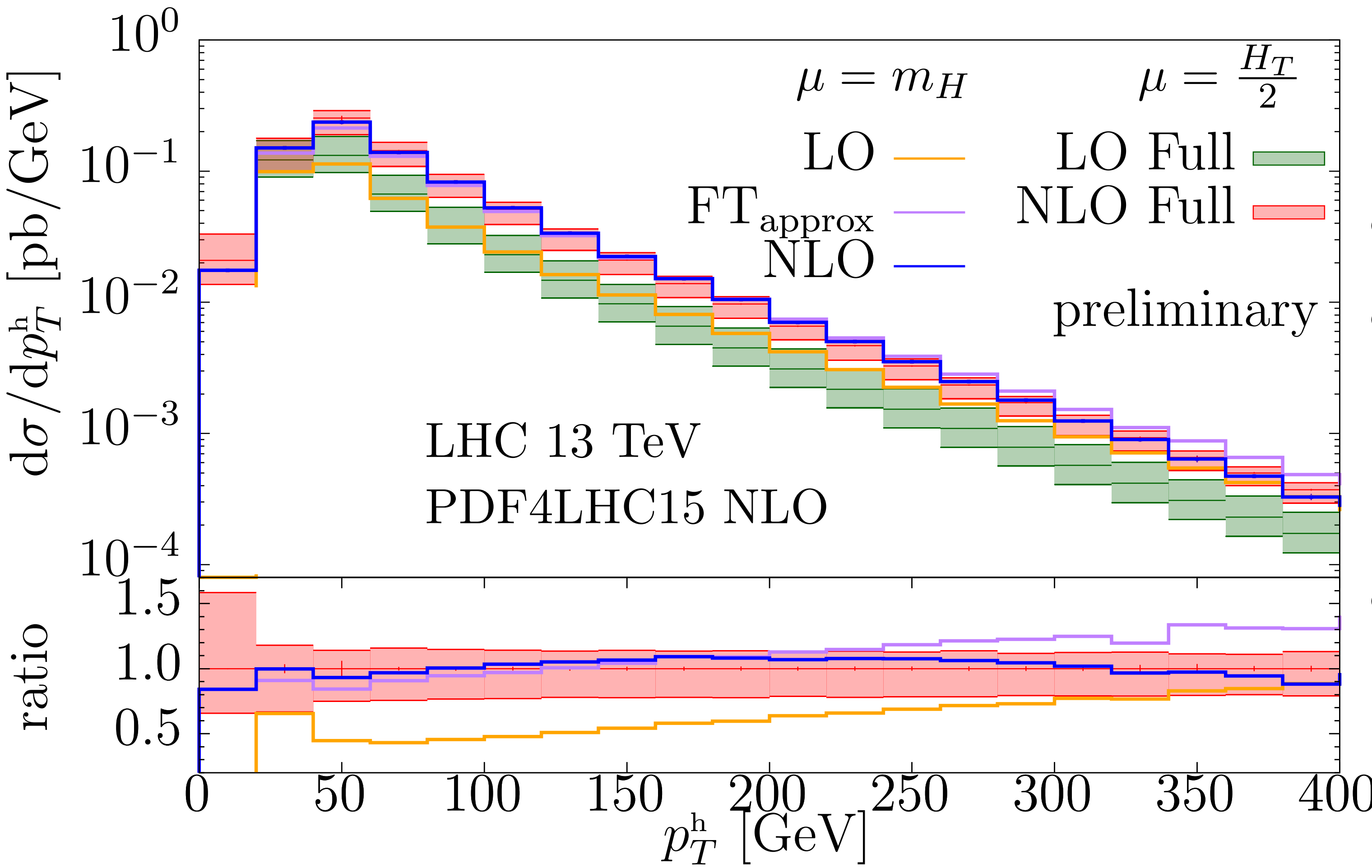
- calculation using numerical approach
  - based on sector decomposition
  - viable alternative, if analytic results not available

- HH production
  - top mass effects decrease cross section by ~4% compared to FT_{approx}
  - size of corrections increase for large m_{HH}
  - results beyond fixed order NLO available:
    - interface to parton shower
    - combination with $NNLO_{HEFT}$
- HJ production
  - top mass effects increase cross section by ~6% compared to FT_{approx}
  - only small dependence of K-Factor on p_T

Backup

HJ Results — Different scale choices

comparison of central scales $H_T/2$ and m_H



choosing $\mu_R, \mu_F = m_H$ leads to

- different shape of LO distribution
- FT$_{\text{approx}}$
  - good agreement at low p_T
  - overestimates the tail
- full result in very good agreement with results with $\mu_R, \mu_F = H_T/2$

$\rightarrow$ top-quark mass effects only small for $\mu_R, \mu_F = H_T/2$!

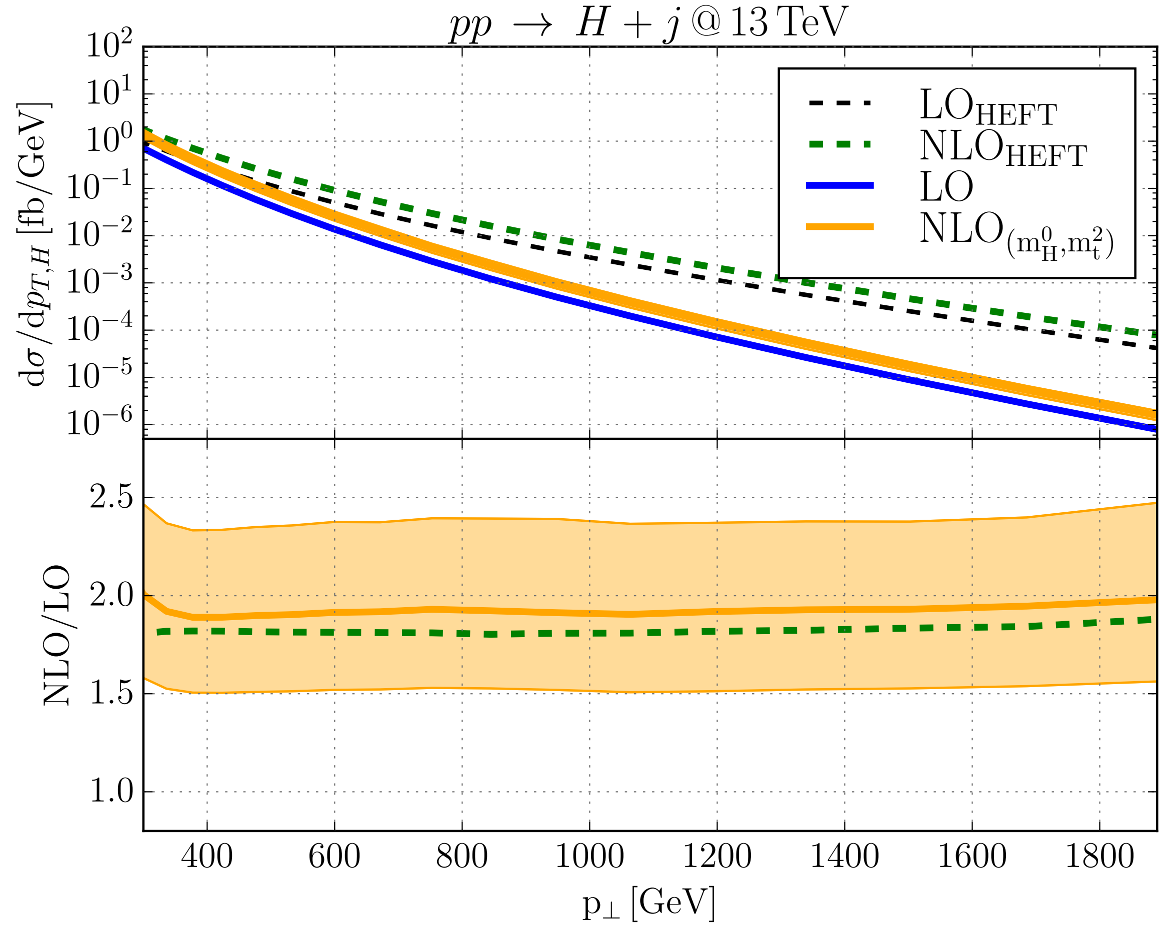
HJ Approximated Results

small m_t expansion up to $\mathcal{O}(m_t^2/p_T^2)$ [Kudashkin, Melnikov, Wever 17]

[Lindert, Kudashkin, Melnikov, Wever 18]

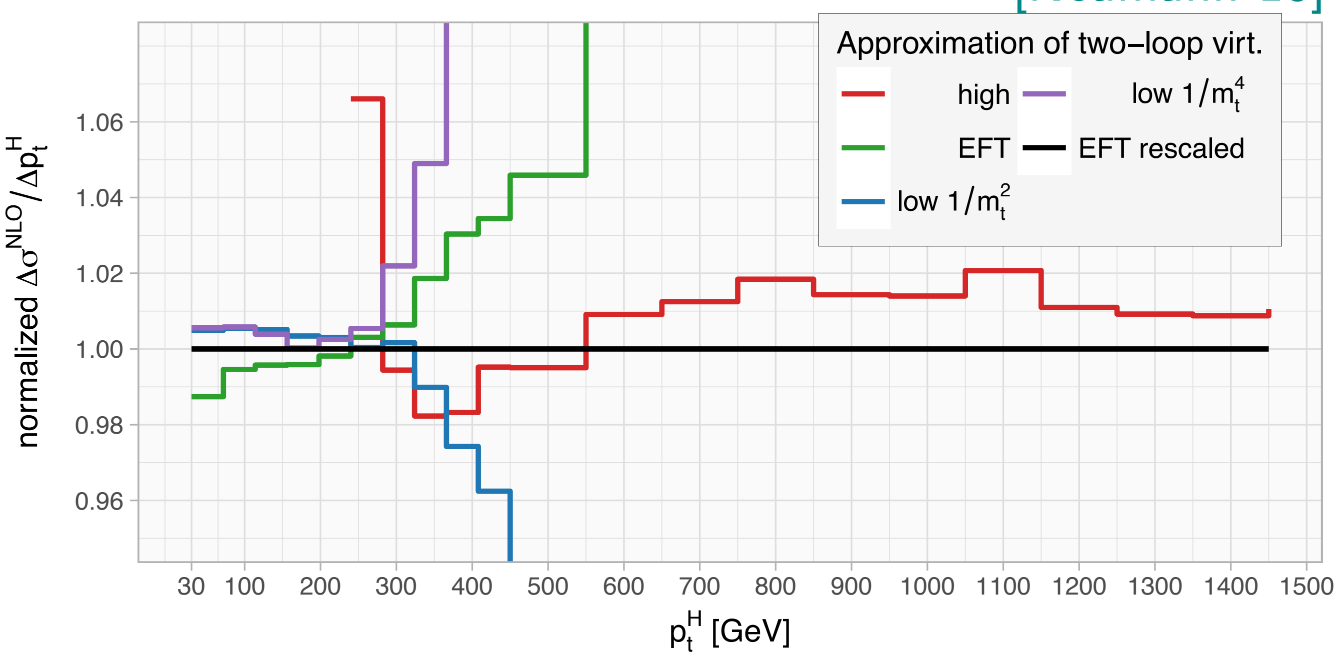
	$\mathrm{LO_{HEFT}}$ [fb]	$\mathrm{NLO_{HEFT}}$ [fb]	K	LO [fb]	NLO [fb]	K
$p_\perp > 400$ GeV	$33.8^{+44\%}_{-29\%}$	$61.4^{+20\%}_{-19\%}$	1.82	$12.4^{+44\%}_{-29\%}$	$23.6^{+24\%}_{-21\%}$	1.90
$p_\perp > 450$ GeV	$22.0^{+45\%}_{-29\%}$	$39.9^{+20\%}_{-19\%}$	1.81	$6.75^{+45\%}_{-29\%}$	$12.9^{+24\%}_{-21\%}$	1.91
$p_\perp > 500$ GeV	$14.7^{+44\%}_{-28\%}$	$26.7^{+20\%}_{-19\%}$	1.81	$3.80^{+45\%}_{-29\%}$	$7.28^{+24\%}_{-21\%}$	1.91
$p_\perp > 1000$ GeV	$0.628^{+46\%}_{-30\%}$	$1.14^{+21\%}_{-19\%}$	1.81	$0.0417^{+47\%}_{-30\%}$	$0.0797^{+24\%}_{-21\%}$	1.91

$$\frac{K^{SM}}{K^{HEFT}} = 1.04 \, .. \, 1.06$$



Approximated results predict top mass effects at the few percent level

[Neumann 18]



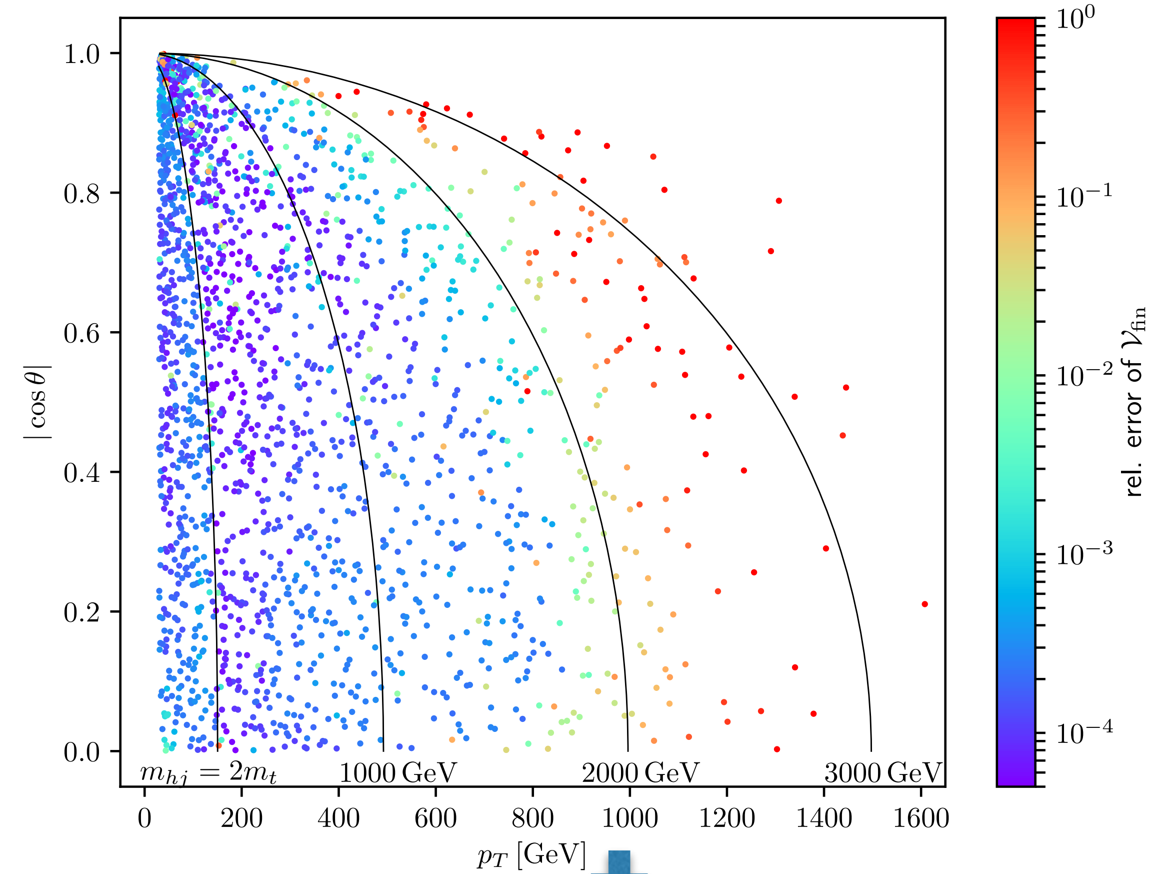
HJ Numerical Stability & Run Time

numerical evaluation of virtual amplitude:

- precision goal: 0.5% for each form factor
- wall-clock limit: 2d GPU-time (Tesla K20X GPUs)

accuracy reached for $|\mathcal{M}|^2$:

- better than per-mill
  for most points below $m_{hj} = 1.5\,\mathrm{TeV}$
- region $m_{hj} \gtrsim 2\,\mathrm{TeV}$ numerically challenging



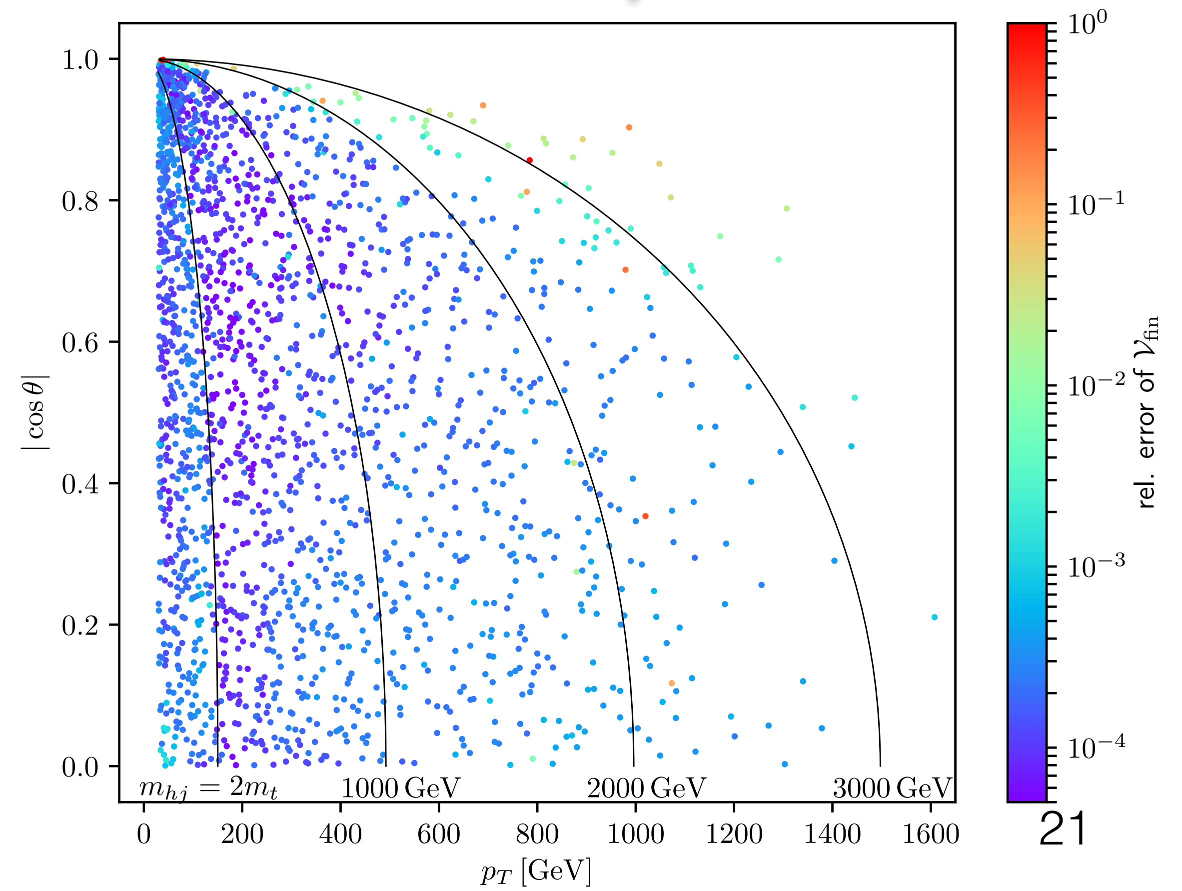
improved basis choice

- use finite integrals with $\mathrm{exponent}(\mathcal{F}) = -1$
  $\rightarrow$ possibly better convergence
- avoid poles in sectors with large #prop
- prefer basis with simple, factorizing denom.

➡ reduced median runtime 15h $\rightarrow$ <2h

➡ reduced size of code for coefficients

➡ avoid spurious poles & cancellations



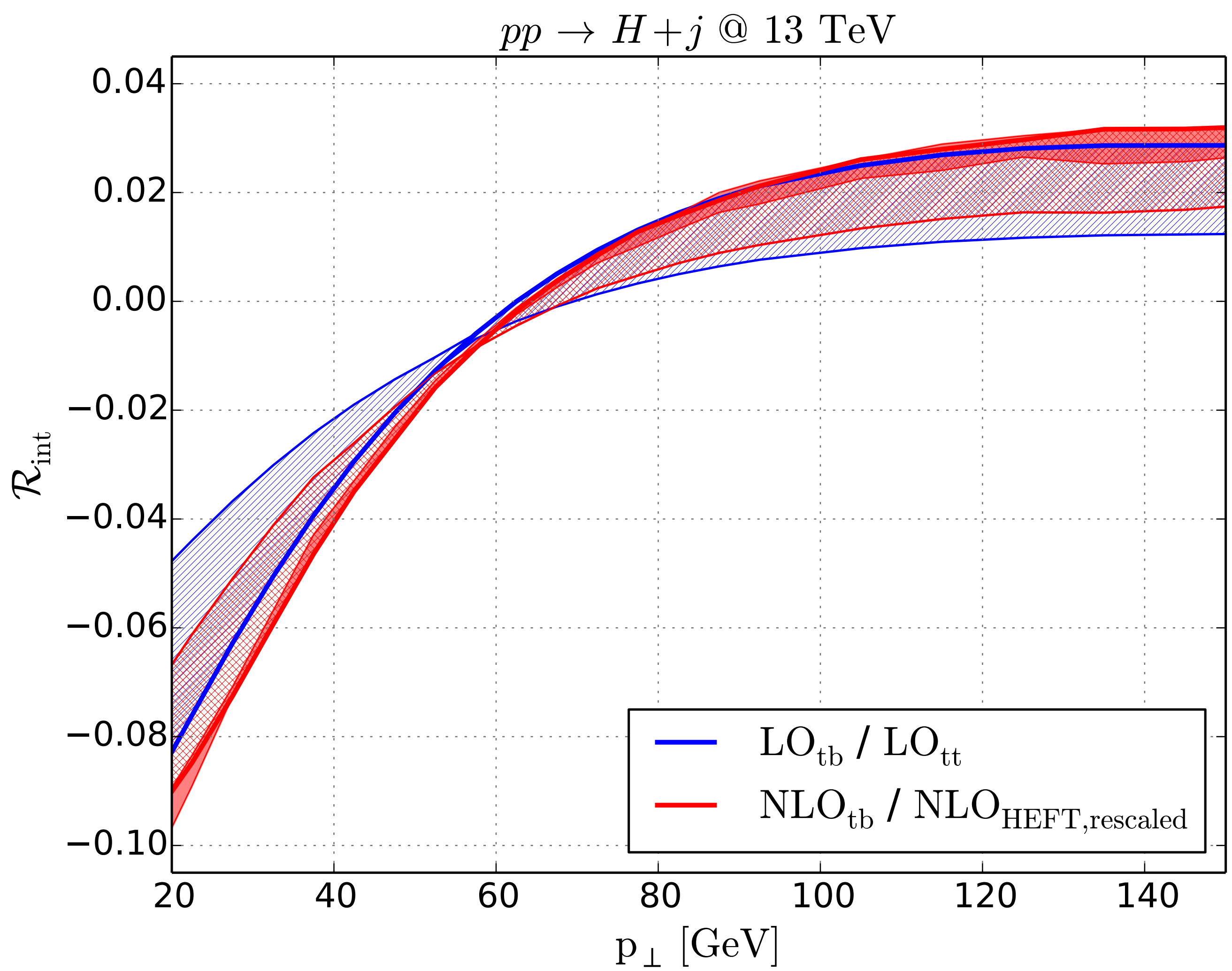
Interference of top and bottom quark

interference of top and bottom quark contributions relevant at low p_T

obtained in the limit $m_t \to \infty, \; m_b \to 0$

[Lindert, Melnikov, Tancredi, Wever 17]

$$d\sigma_{tb}^{\text{virt}} \sim \text{Re}\left[A_t^{\text{LO}} A_b^{\text{LO}*} + \frac{\alpha_s}{2\pi}\left(A_t^{\text{NLO}} A_b^{\text{LO}*} + A_t^{\text{LO}} A_b^{\text{NLO}*}\right)\right]$$



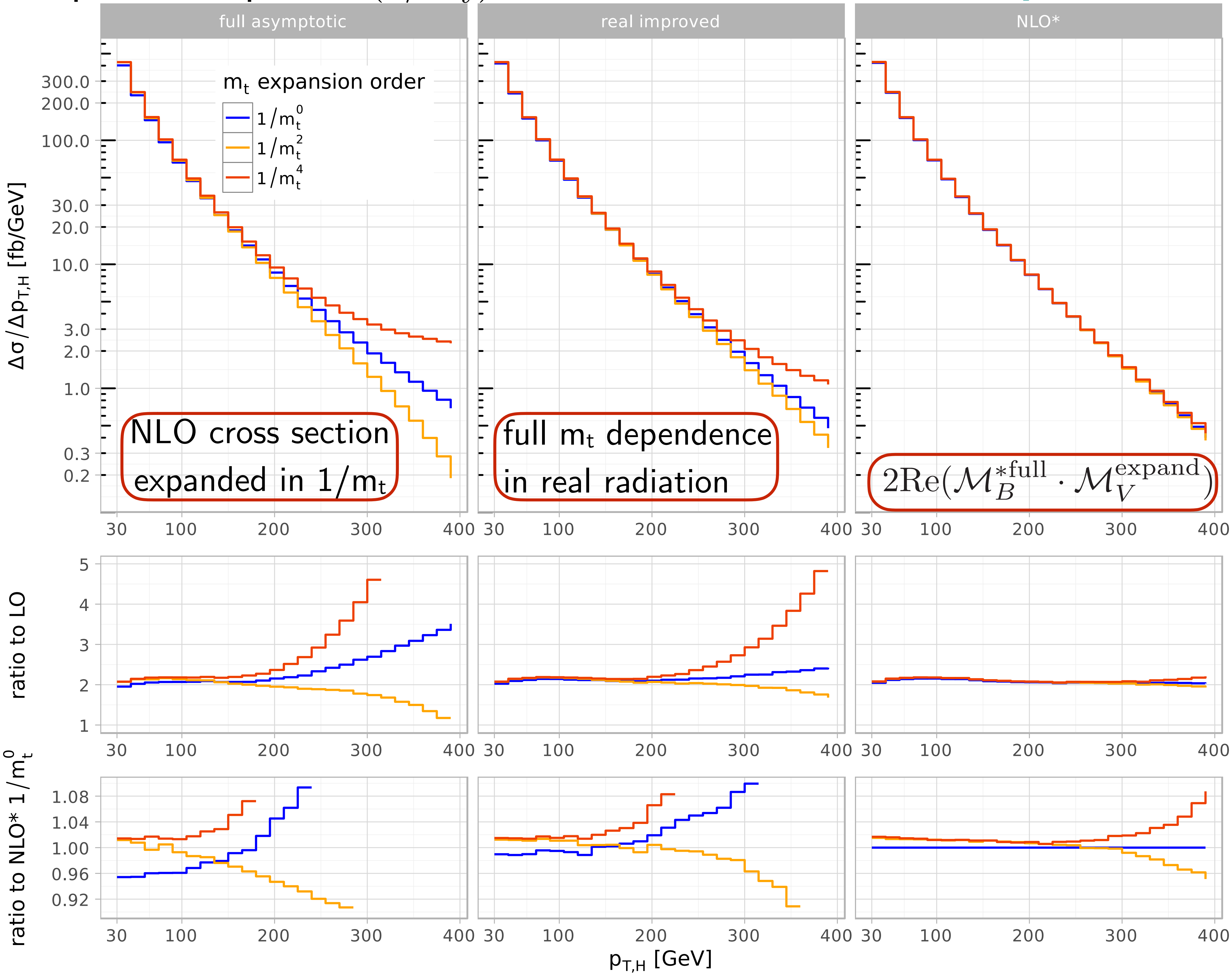
-8% at $p_T = 20\,\text{GeV}$

$+2\%$ at $p_T = 100\,\text{GeV}$

size of interference effects nearly identical at LO and NLO

Approximated Results

large m_t expansion up to $\mathcal{O}(1/m_t^4)$ [Neumann, Williams 16]



Approximated Results

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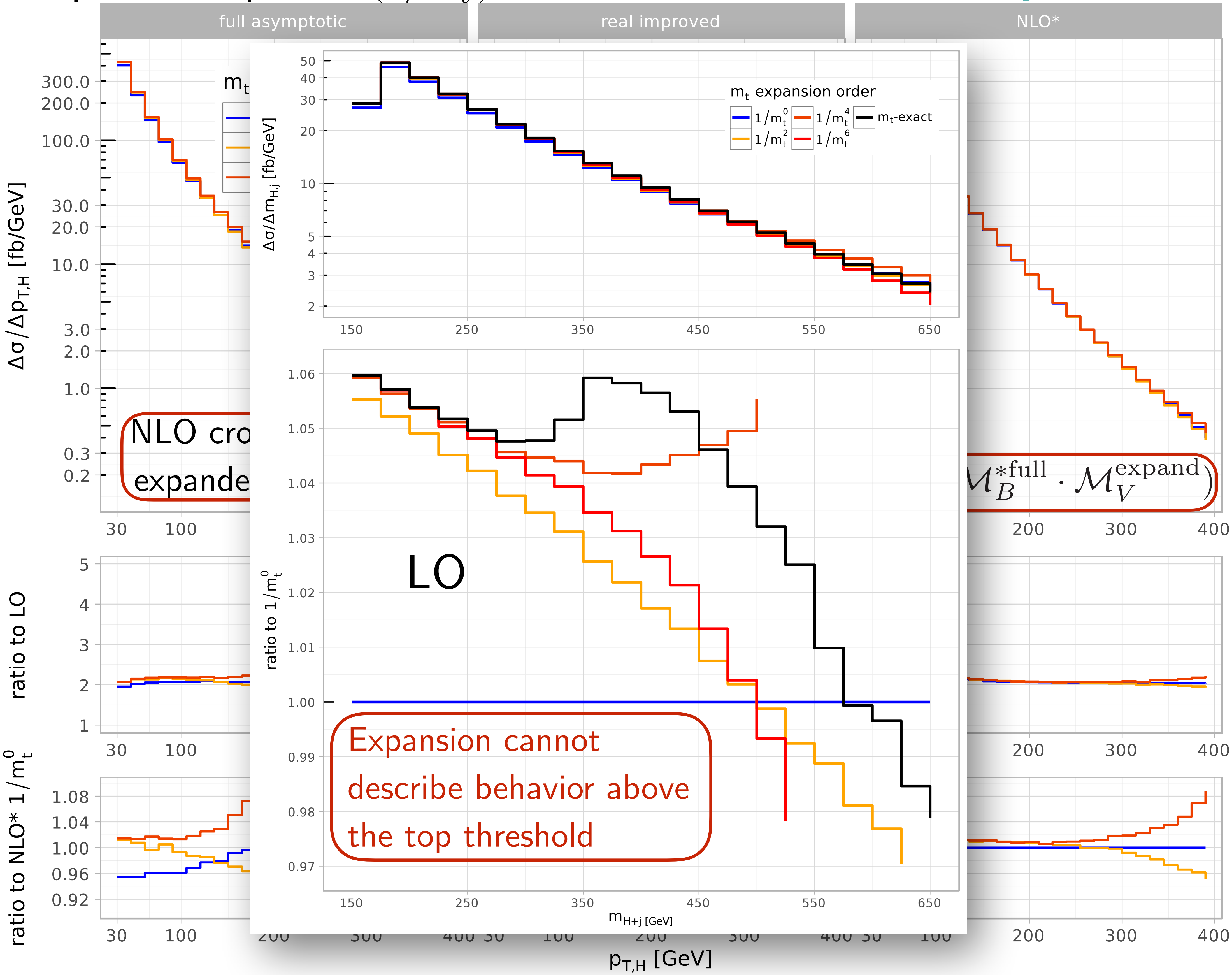


Quasi-Monte Carlo in weighted function space

assign weights $\gamma_{\mathfrak{u}}$ to each subset of dimensions $\mathfrak{u} \subseteq \{1, \ldots, d\}$ Review: Dick, Kuo, Sloan `13

weighted function spaces, typically considered in literature:

Sobolev space

norm $$||f||_\gamma^2 = \sum_{\mathfrak{u}\subseteq\{1,\ldots,d\}} \frac{1}{\gamma_{\mathfrak{u}}} \int_{[0,1]^{|\mathfrak{u}|}} \left(\int_{[0,1]^{d-|\mathfrak{u}|}} \frac{\partial^{|\mathfrak{u}|} f(\mathbf{x})}{\partial \mathbf{x}_{\mathfrak{u}}} \mathrm{d}\mathbf{x}_{-\mathfrak{u}} \right)^2 \mathrm{d}\mathbf{x}_{\mathfrak{u}}$$

Korobov space

$$||f||_\gamma^2 = \sum_{\mathbf{h}\in\mathbb{Z}^d} \frac{\prod_{j\in\mathfrak{u}(\mathbf{h})} |h_j|^{2\alpha}}{\gamma_{\mathfrak{u}(\mathbf{h})}} |\hat{f}(\mathbf{h})|^2$$

Fourier coefficient

use rank-1 lattice rule $$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \mathbf{\Delta}_k \right\}$$

$\{\ldots\} =$ fractional part ($\rightarrow x \in [0;1[$)

$\mathbf{\Delta}_k =$ randomized shifts

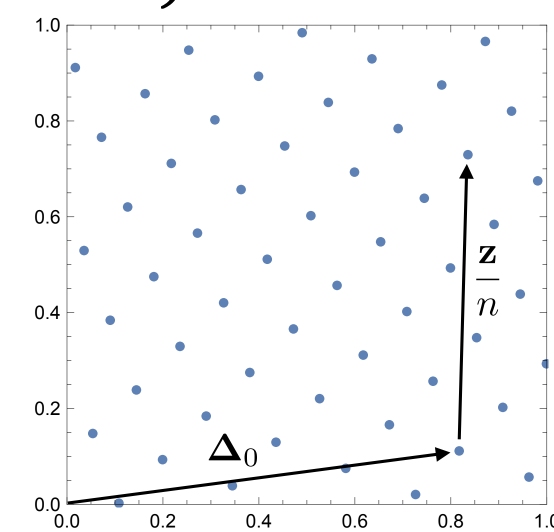
$\rightarrow m$ different estimates of Integral: $I_1, \ldots, I_m$

$\rightarrow$ error estimate of result

$\mathbf{z} =$ generating vector

constructed component-by-component Nuyens `07

minimizing worst-case error ϵ_γ



worst-case error

$$\epsilon_\gamma^2 \leq \left(\frac{1}{\psi(N)} \sum_{\emptyset\neq\mathfrak{u}\subseteq\{1,\ldots,d\}} \gamma_{\mathfrak{u}}^\lambda \left(\frac{2\zeta(2\lambda)}{(2\pi^2)^\lambda} \right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in]1/2, 1]$

$\epsilon_\gamma = \mathcal{O}\left(N^{-1}\right)$

$$\epsilon_\gamma^2 \leq \left(\frac{1}{\psi(N)} \sum_{\emptyset\neq\mathfrak{u}\subseteq\{1,\ldots,d\}} \gamma_{\mathfrak{u}}^\lambda \left(2\zeta(2\alpha\lambda)\right)^{|\mathfrak{u}|} \right)^{\frac{1}{\lambda}}$$

$\lambda \in]1/(2\alpha), 1]$

$\epsilon_\gamma = \mathcal{O}\left(N^{-\alpha}\right)$ smoothness α

first application to sector-decomposed loop integrals: Li, Wang, Yan, Zhao 15

implementation in public library coming soon! arXiv:1811.????? Borowka, Heinrich, Jahn, Jones, MK, Schlenk

can be used on CPUs and GPUs

Evaluation of amplitude

after sector decomposition and expansion in ε:

$\rightarrow$ amplitude written in terms of $\mathcal{O}(10\,\mathrm{k})$ finite integrals

Some optimizations required to reduce run time:

- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral}\, i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$$\sigma_i = \text{error estimate (including coefficients in amplitude)}$$
$$\lambda = \text{Lagrange multiplier} \qquad \sigma = \text{precision goal}$$

- parallelization on gpu
- avoid reevaluation of integrals for different orders in ε and form factors

compute only once

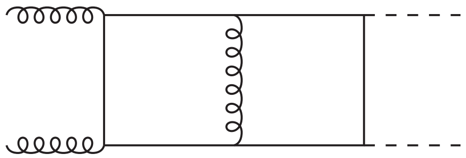
$$F^a = \sum_i \left[\left(\sum_j C^a_{i,j} \varepsilon^j \right) \cdot \left(\sum_k I_{i,k} \varepsilon^k \right) \right] = \frac{C^a_{1,-2} I_{1,0} + C^a_{1,-1} I_{1,-1} + \ldots}{\varepsilon^2} + \frac{C^a_{1,-1} I_{1,0} + \ldots}{\varepsilon^1} + \ldots$$

HH Amplitude Evaluation — Example

$\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$

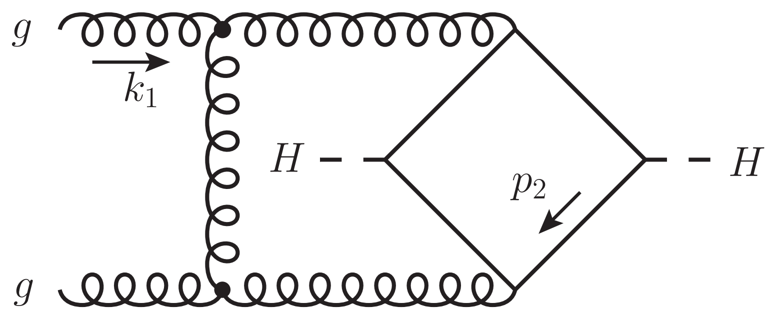
contributing integrals:

integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342



≈ 700 integrals

$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon}\Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2}+\frac{A_{-1}}{\epsilon^1}+A_0+\mathcal{O}(\epsilon)\right)$$



sector decomposition

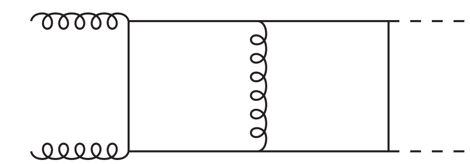
sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

slide:
MK, L&L 2016

HH Amplitude Evaluation — Example

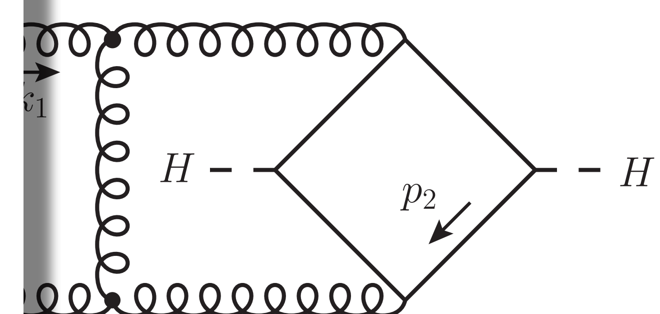
$$\sqrt{s} = 327.25\,\mathrm{GeV},\ \sqrt{-t} = 170.05\,\mathrm{GeV},\ M^2 = s/4$$

contributing integrals:

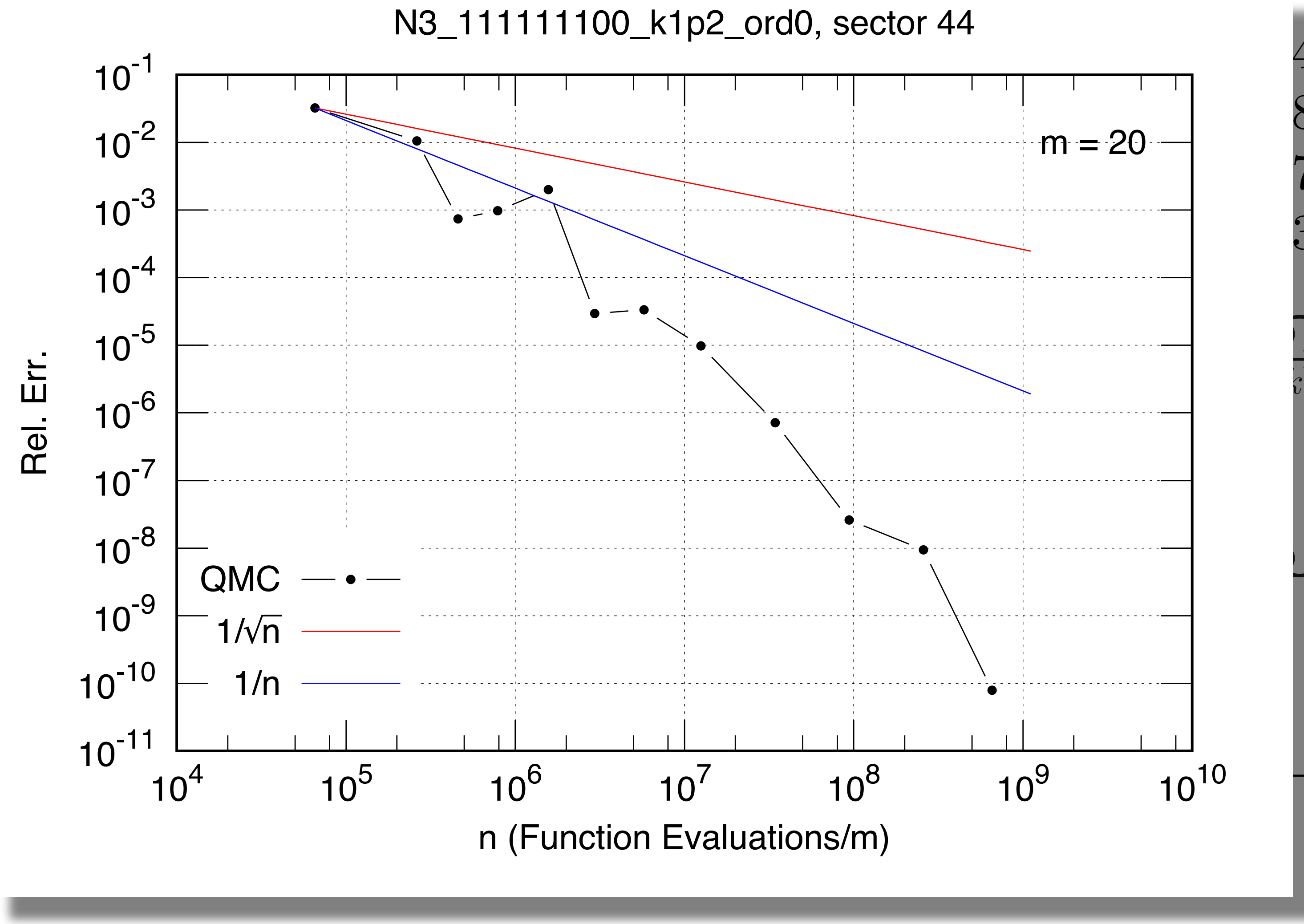


integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2			412
N3_111111100_1_or			896
N3_111111100_k1p2			794
N3_111111100_k1p2			342

≈ 700 integrals



$I(s,t,m_t^2,m_h^2) = -$ (



sector	in			
5	(-1.34e-0			
6	(-1.58e-03			
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
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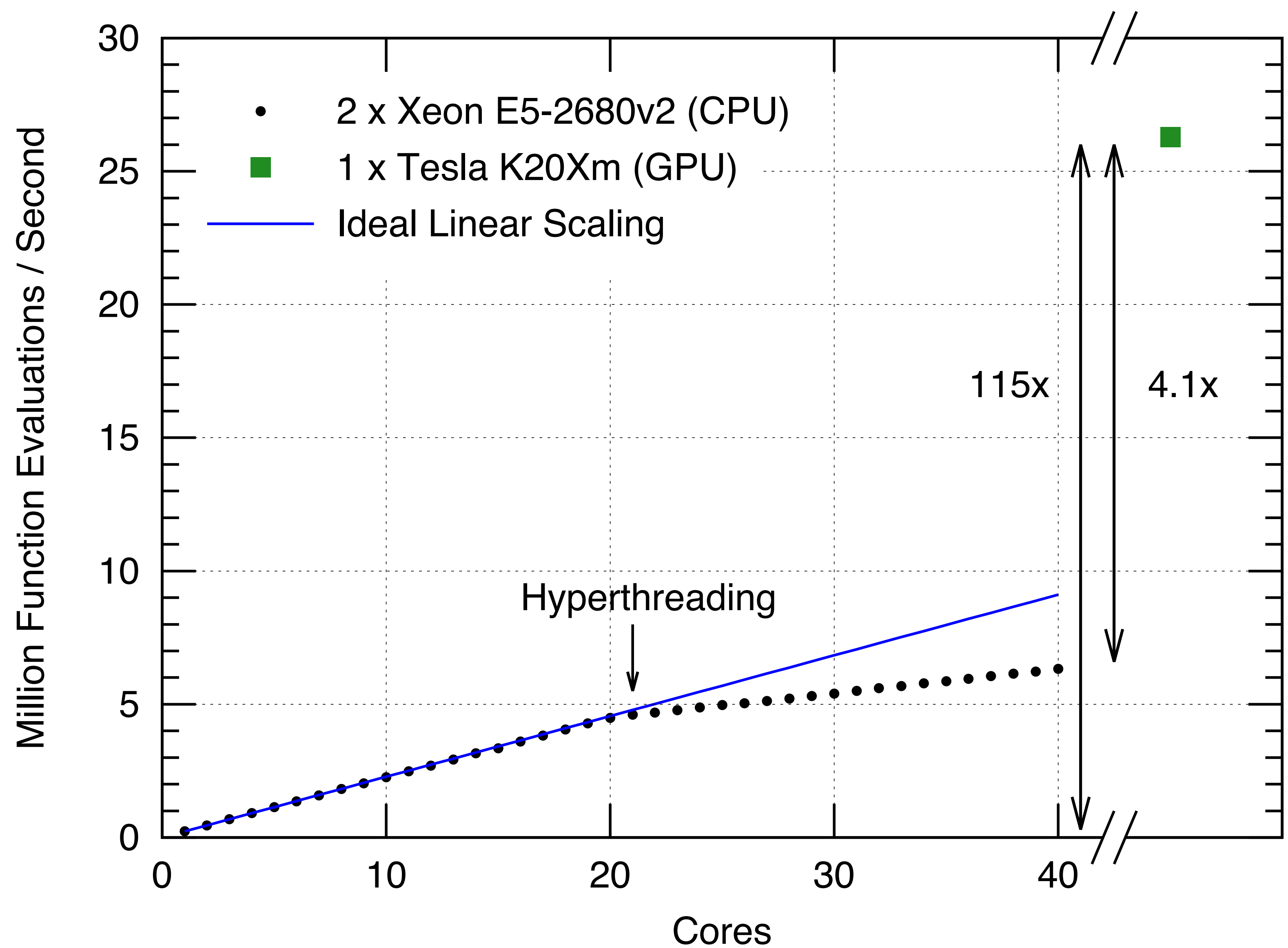
slide:
MK, L&L 2016

GPU performance



plot:
Stephen Jones

Parton Shower Interface

2-loop amplitude too slow (median 2h on gpu) for direct interface to PS
$\rightarrow$ construct grid for interpolation of virtual amplitude

- included additional points in large m_{HH} region (total of 3741 2-loop results used)
- input parameters ($\hat{s}$, $\hat{t}$) transformed to

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left| \frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})} \right|, \quad \beta = \left(1 - \frac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$$

$\rightarrow$ nearly uniform distribution of phase space points in $(x, c_\theta) \in [0,1]^2$ if $f(\beta)$ chosen according to cumulative distribution of points in original calculation

- interpolation done in 2 steps:
  1. choose equidistant grid points, estimate result at each grid point with linear interpolation of amplitude results in vicinity
  2. Clough-Tocher interpolation (as implemented in SciPy) to estimate amplitude at arbitrary sampling points

  $\rightarrow$ reduces sensitivity to uncertainties of input-data points
- available at github.com/mppmu/hhgrid

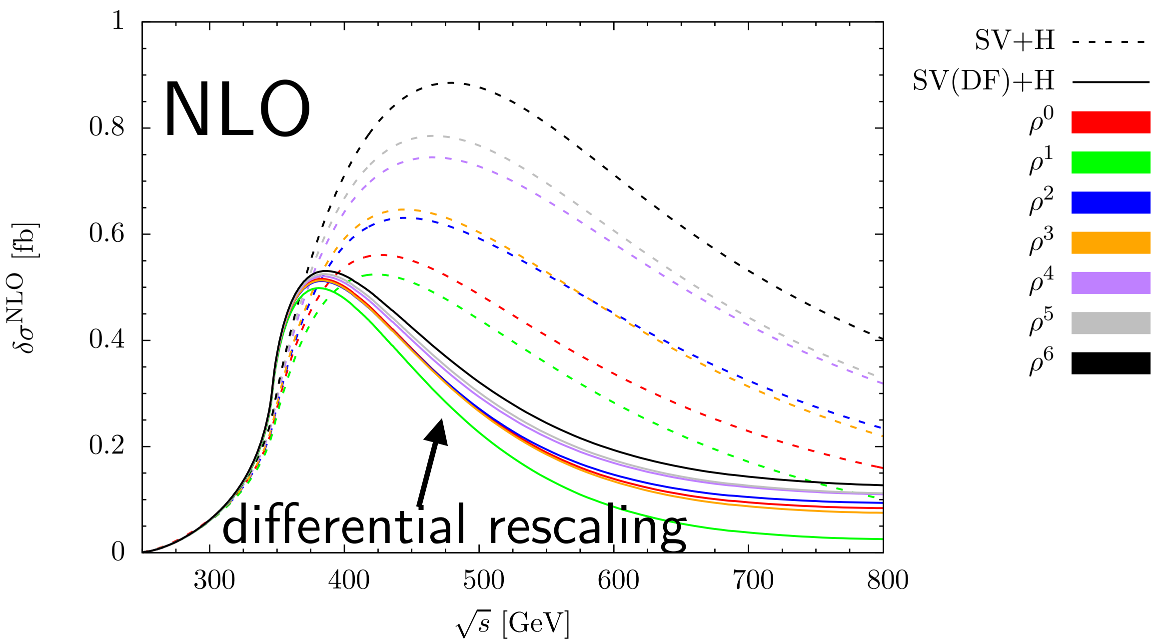
Large mass expansion

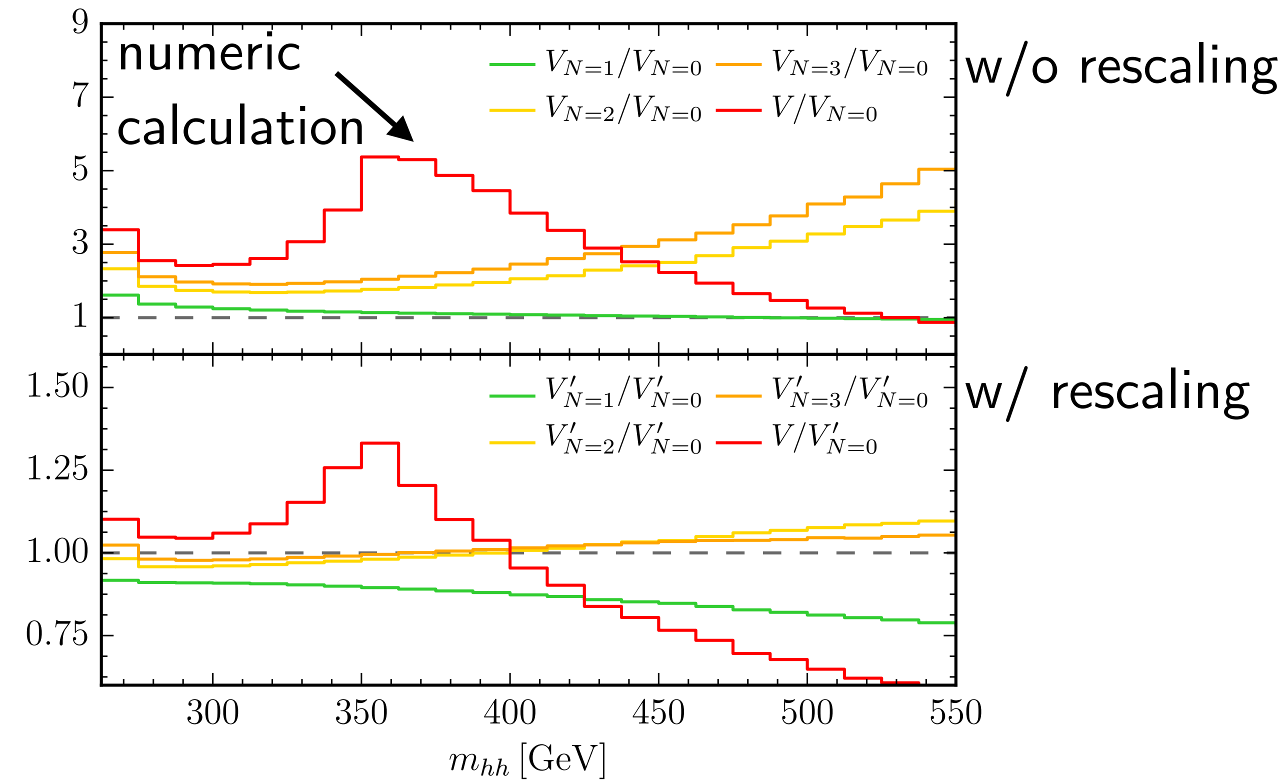
HH expansion in $1/m_t$

[Grigo, Hoff, Steinhauser 15]

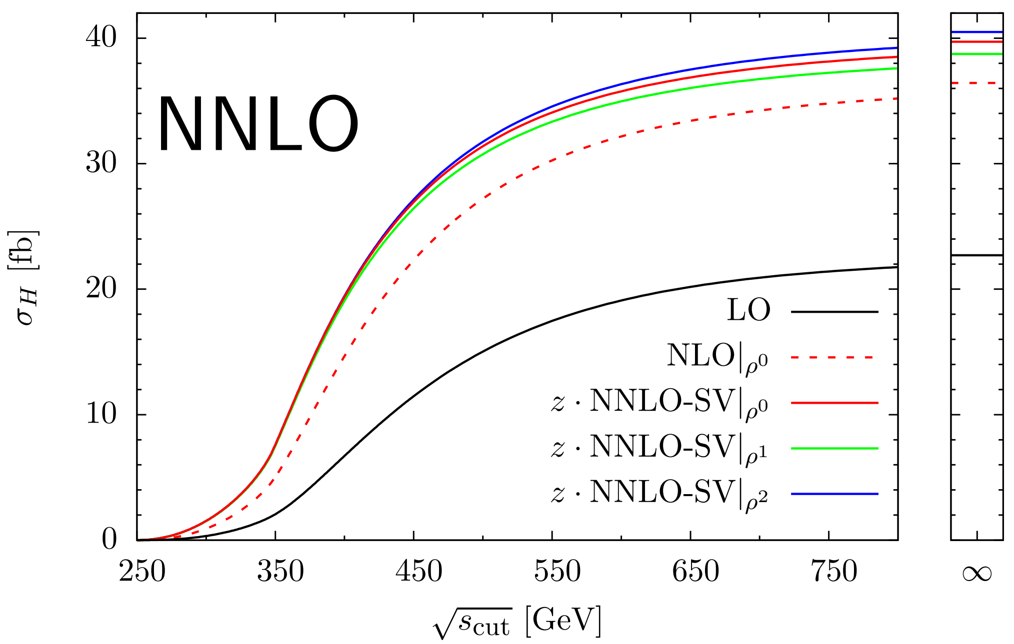
rescaling can improve convergence:

$$\mathrm{d}\sigma(m_t) \approx \mathrm{d}\sigma(m_t \to \infty)\frac{\mathrm{d}\sigma_{LO}(m_t)}{\sigma_{LO}(m_t \to \infty)}$$





[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]



expansion improves convergence only for $m_{HH} < 2m_t$

HJ: [Harlander, Neumann, Ozeren, Wiesemann 12][Neumann, Wiesemann 14] [Frederix, Frixione, Vryonidou, Wiesemann 16] [Neumann, Williams 16]

ZZ: [Campbell, Ellis, Czakon, Kirchner 16] [Caola, Dowling, Melnikov, Röntsch, Tancredi 16]

ZH: [Hasselhuhn, Luthe, Steinhauser, 16]

Low mass expansion

can describe

→ b-quark contributions

→ t-quark contributions at large s,t

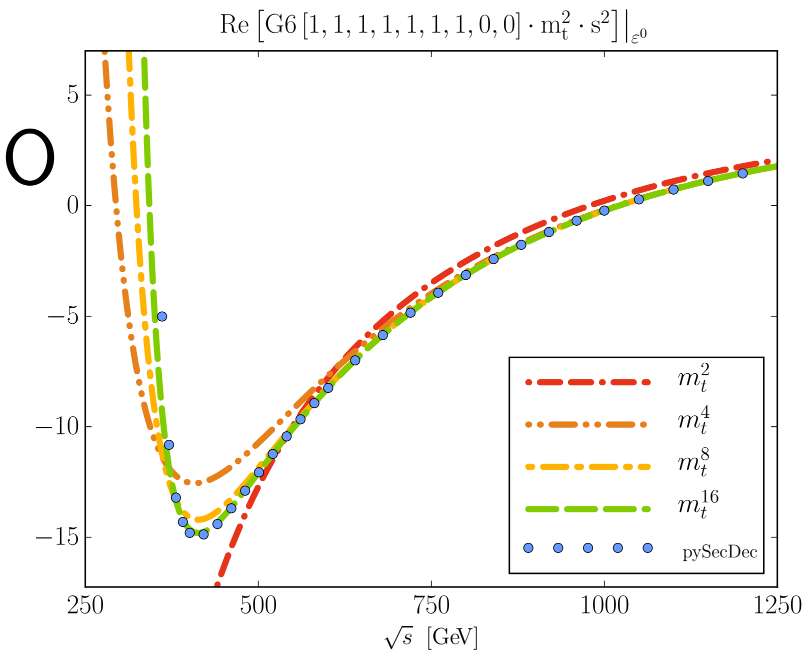
HJ production → Chris

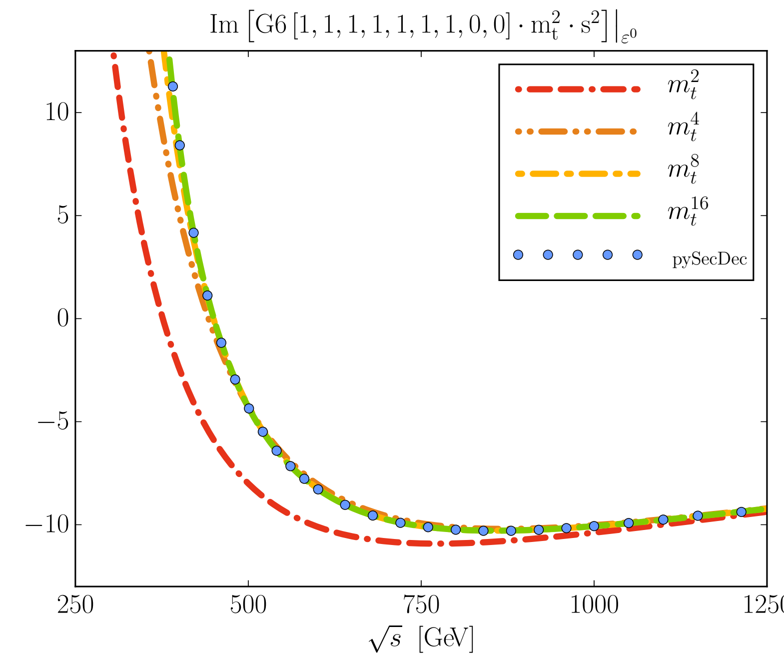
HH production

[Davies, Mishima, Steinhauser, Wellmann 18]

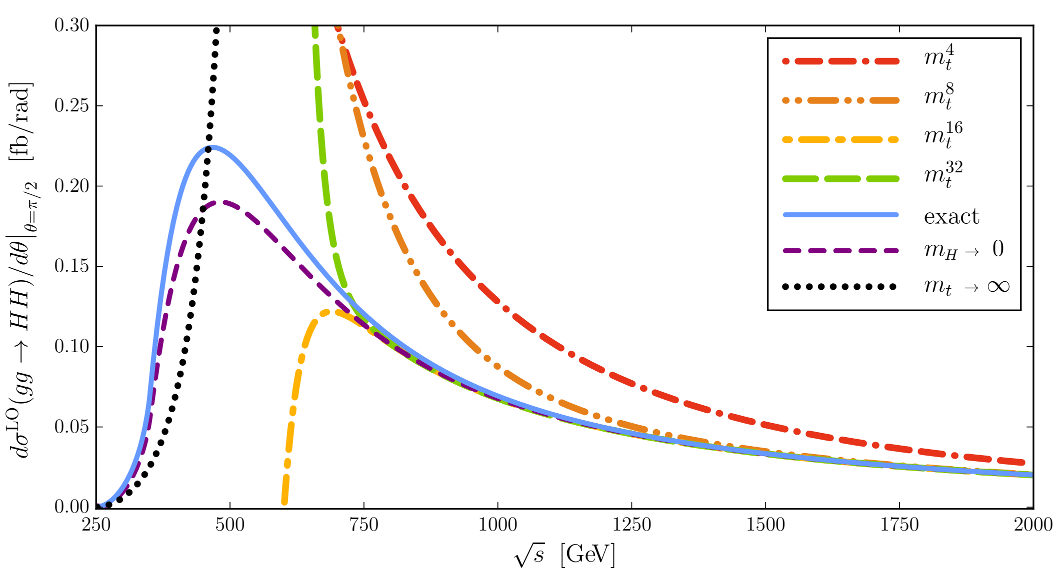
many expansion terms required

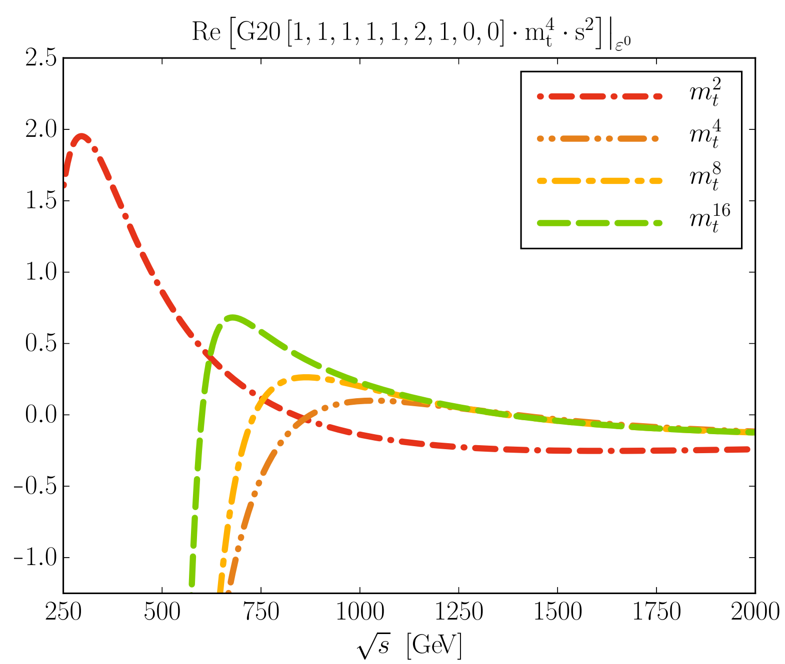
NLO

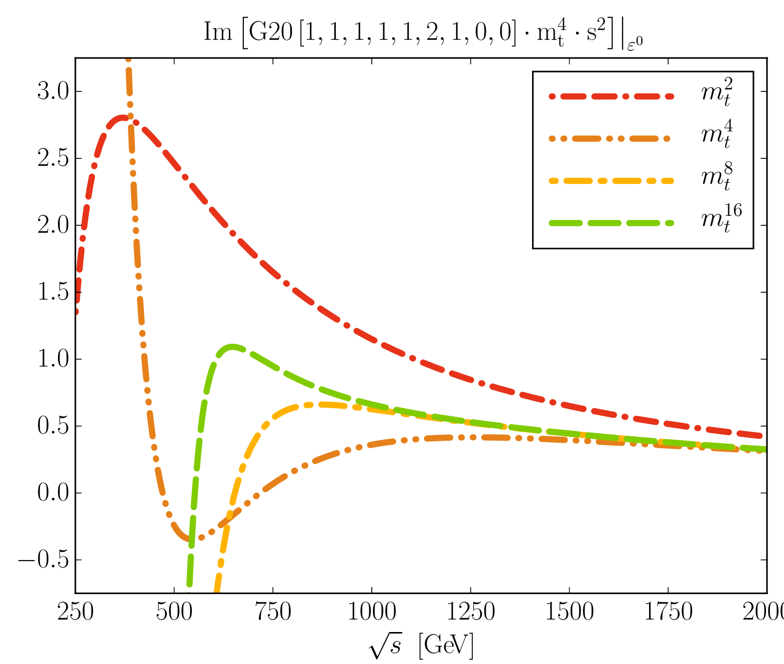




LO







Padé approximation & threshold behavior

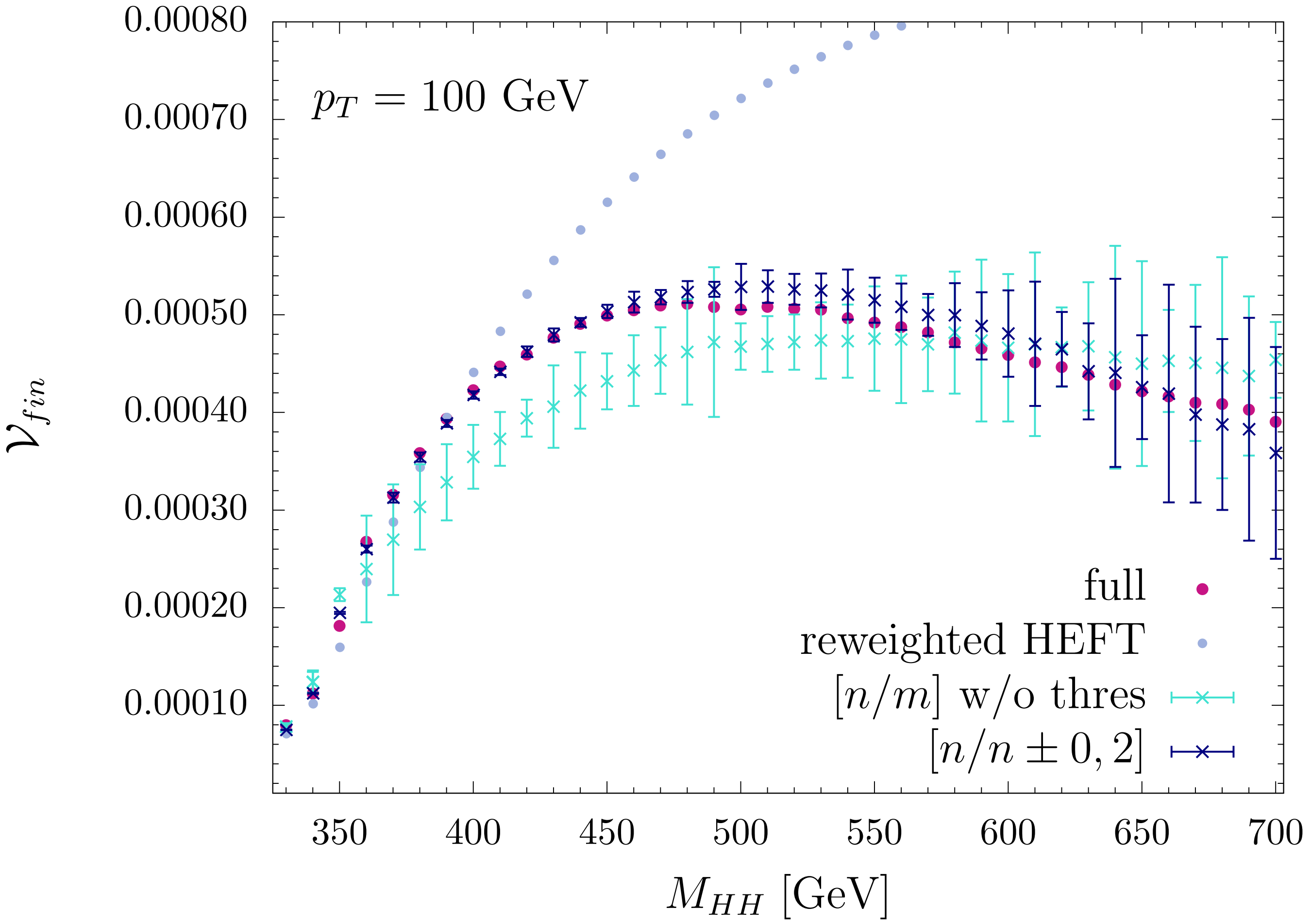
HH production [Gröber, Maier, Rauh 17]

Conformal mapping

$$z = \frac{4\omega}{(1+\omega)^2} \qquad z = \frac{s + i0}{4m_t^2}$$

Padé approximations

$$[n/m](\omega) = \frac{\sum_{i=0}^{n} a_i \omega^i}{1 + \sum_{j=1}^{m} b_j \omega^j}$$



ZZ production: Padé approximation
[Campbell, Ellis, Czakon, Kirchner 16]

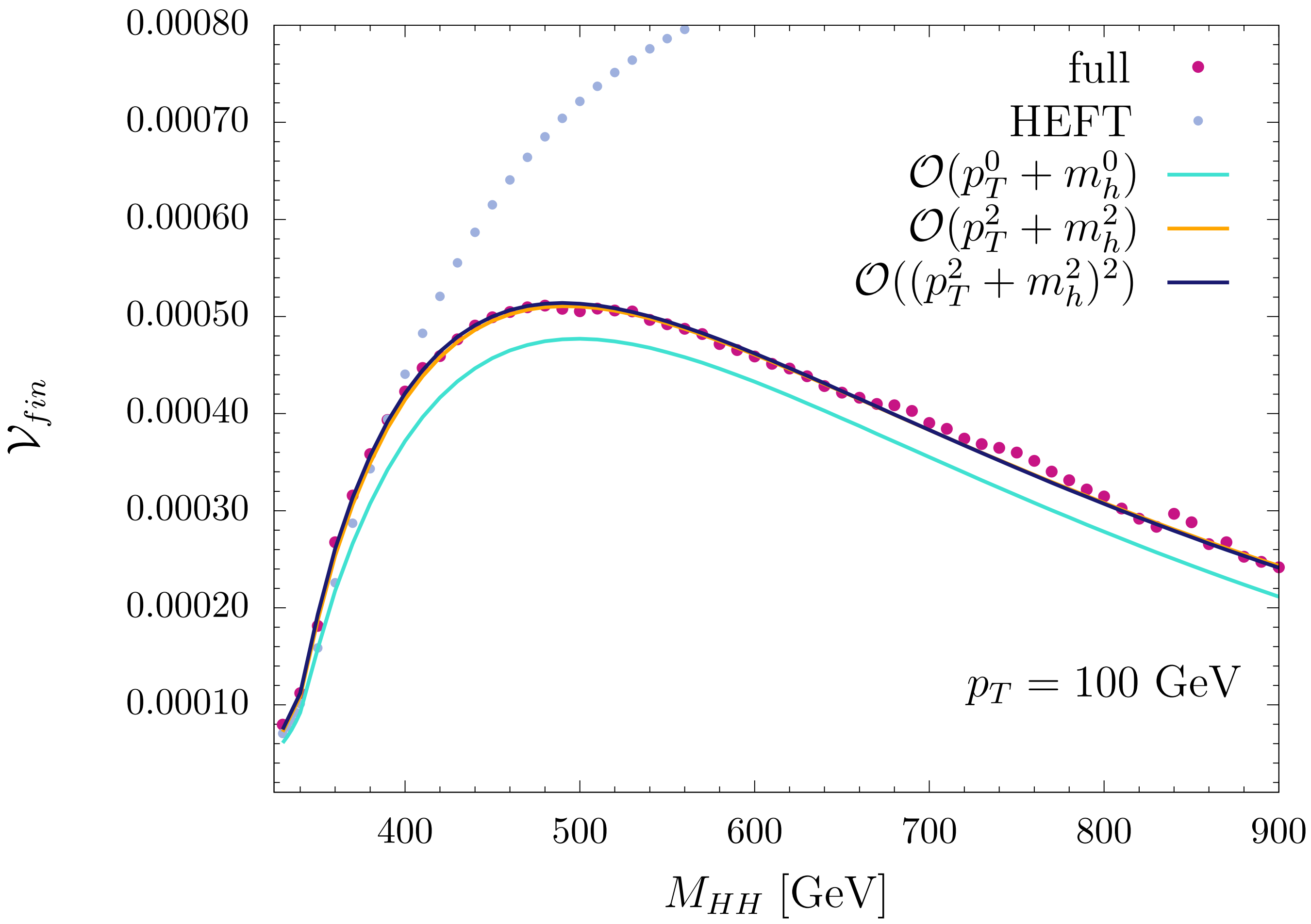
Expansion in E_T

HH production [Bonciani, Degrassi, Giardino, Gröber 18]

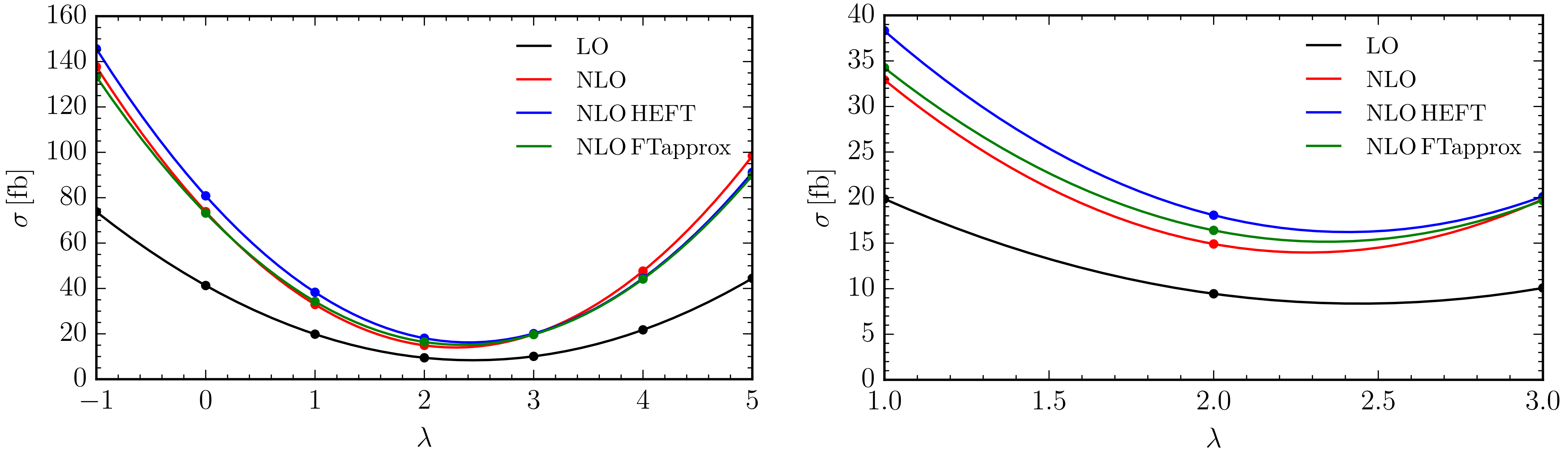
$$p_T^2 + m_H^2 \leq \frac{\hat{s}}{4}$$

→ expand in $\sqrt{p_T^2 + m_H^2}$

→ solve remaining dependence on $\hat{s}$ and m_t

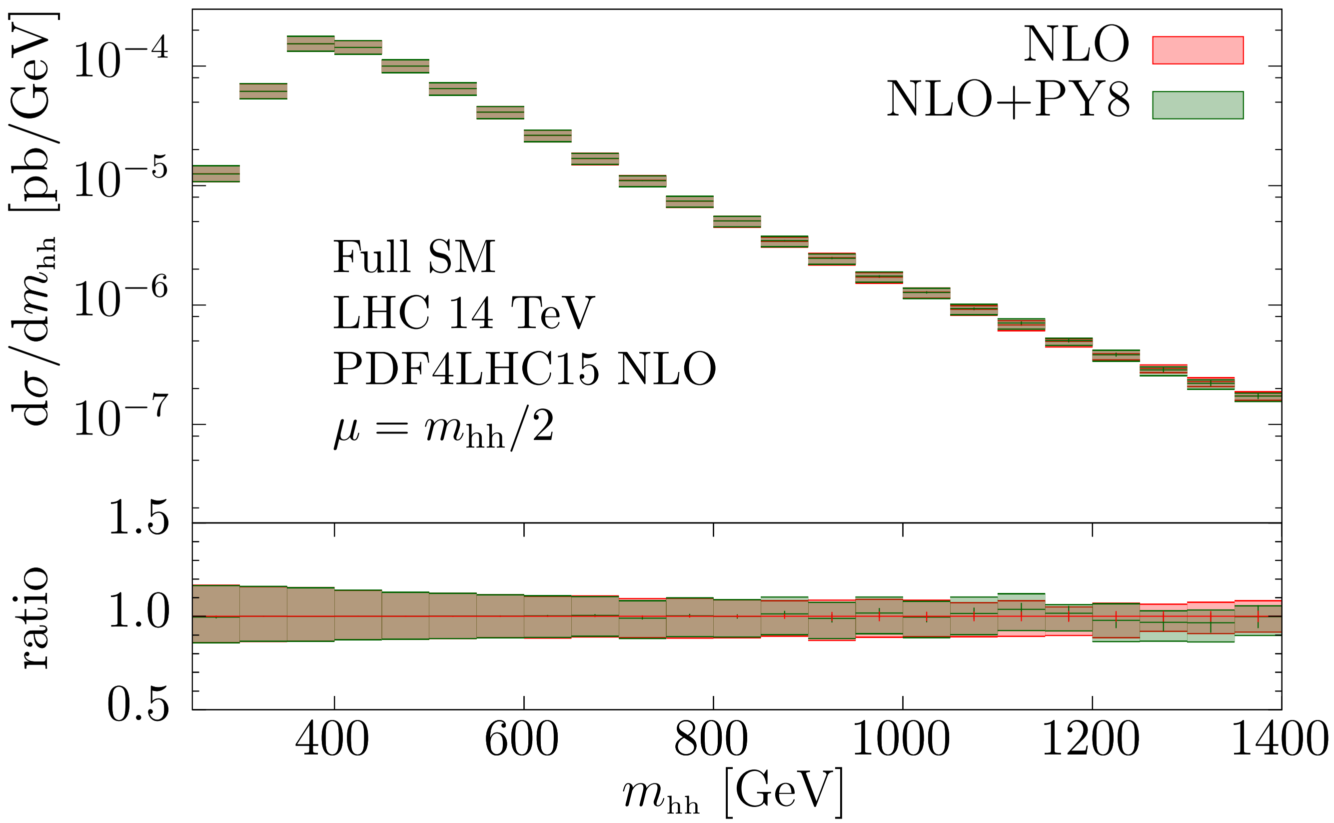


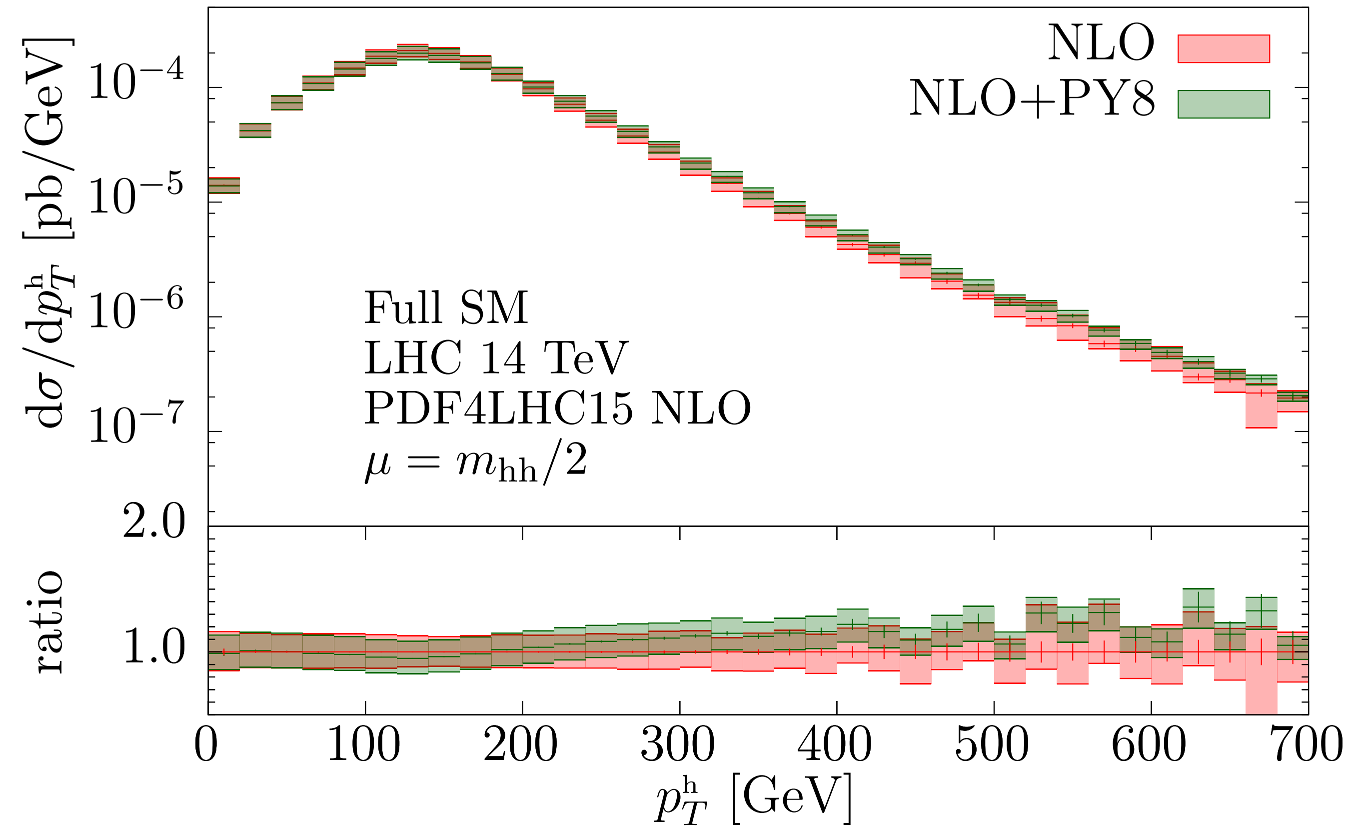
modified Higgs self-interactions



HH Results including Parton Shower

Powheg + Pythia8





no effect on invariant mass

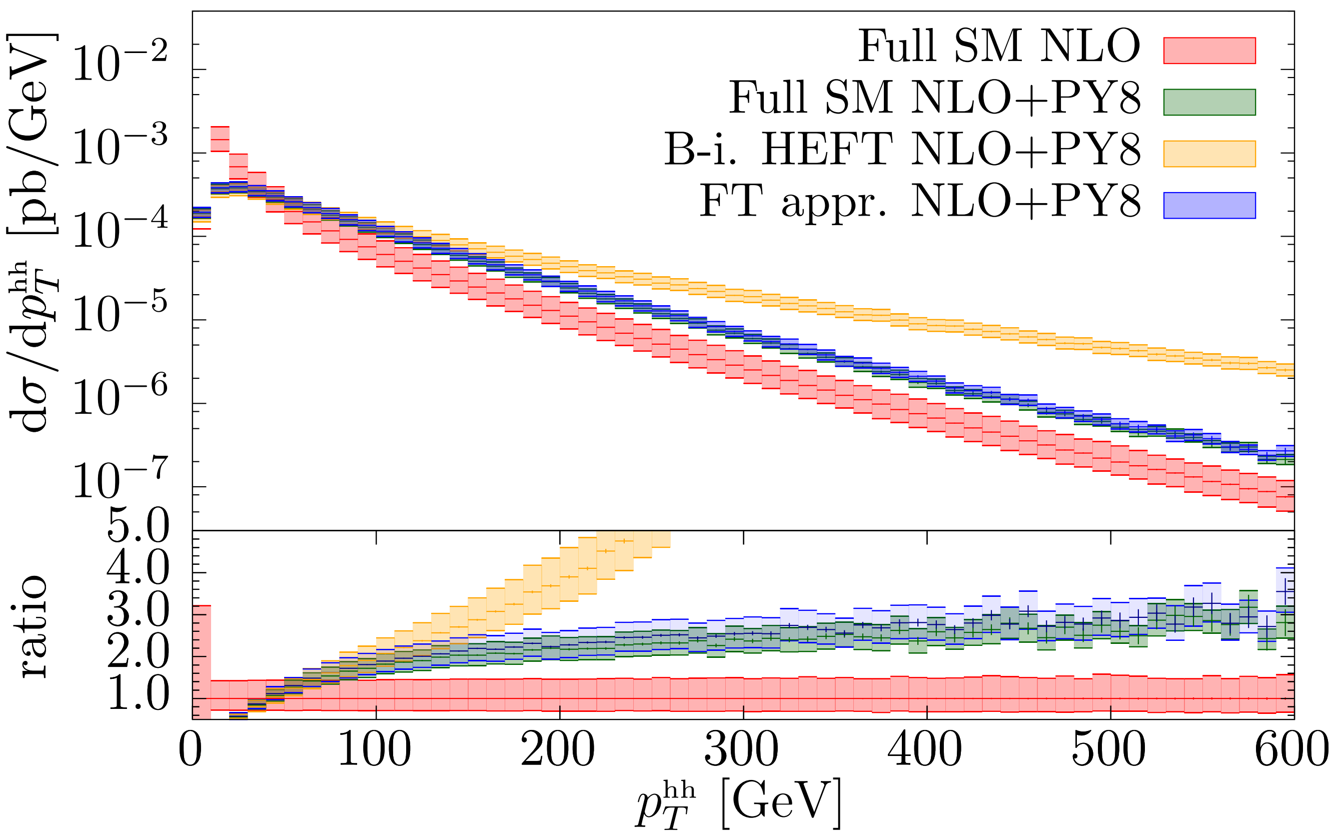
parton shower enhances tails of p_T distributions

only small parton shower effects on NLO accurate observables

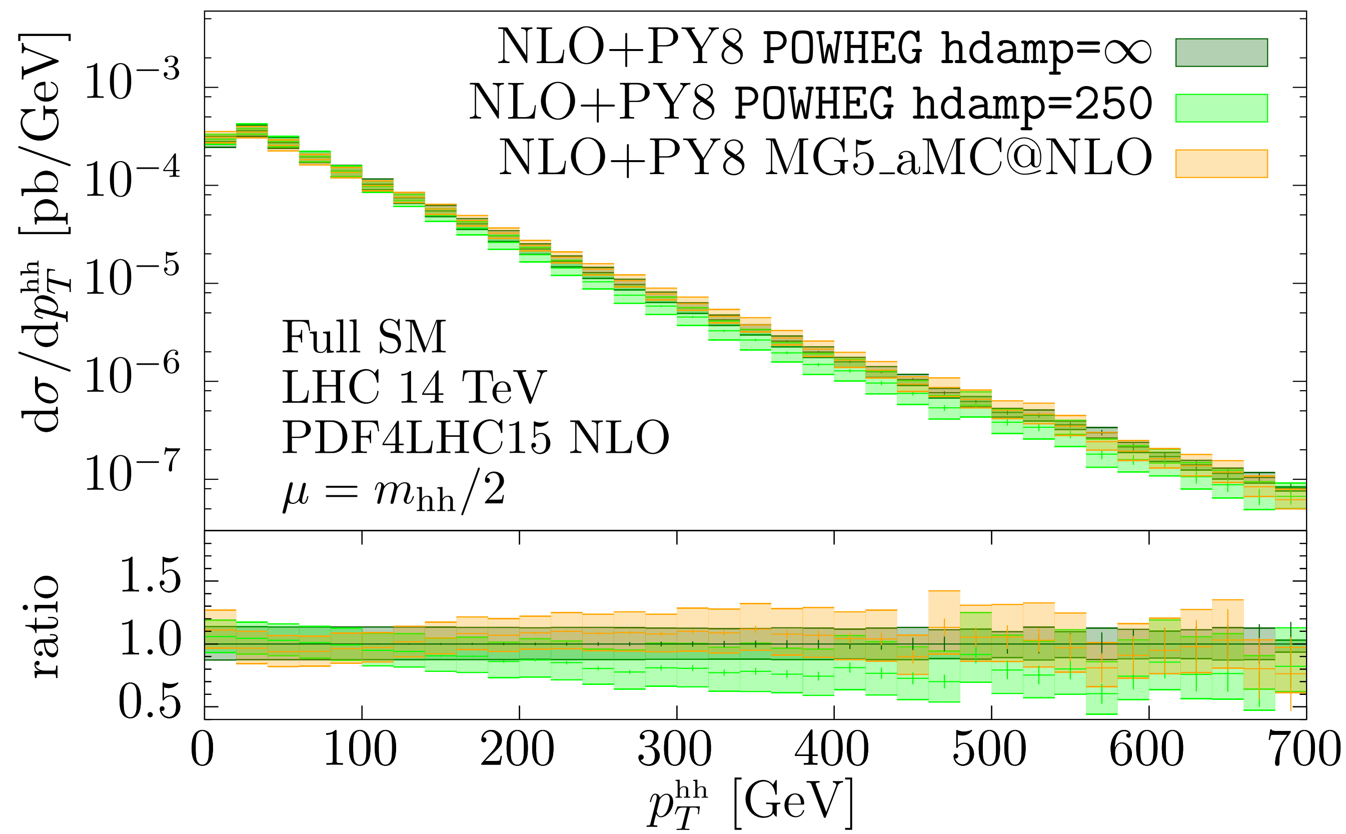
HH Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. p_T^{hh}

Powheg



MadGraph5_aMC@NLO

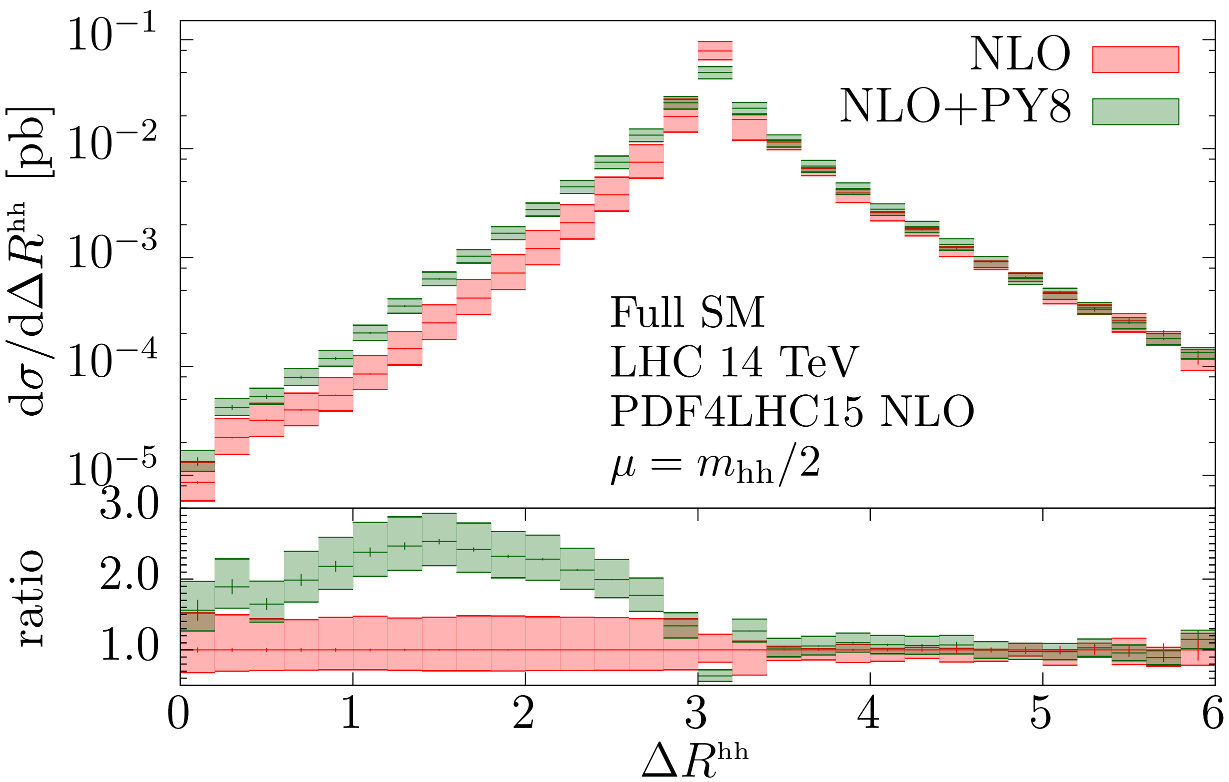


- parton shower enhances tail p_T^{hh} distribution by factor of ~2
- difference of matching schemes of ~20%
- small difference between full NLO and FT approx. result

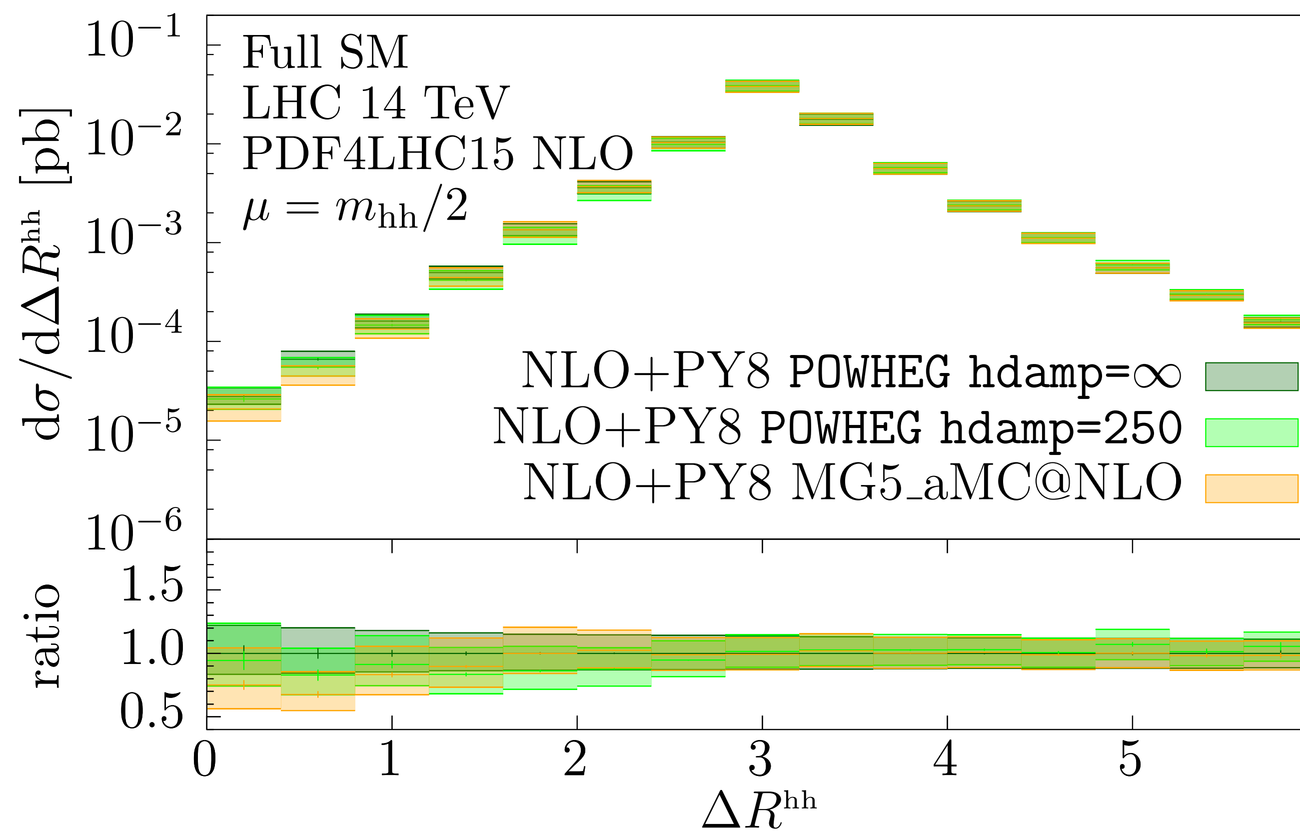
HH Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. ΔR^{hh}

Powheg



MadGraph5_aMC@NLO



$\Delta R^{hh} < \pi$

- filled by real radiation
  only LO accurate
- parton shower corrections
  up to factor of ~2.5
- differences due to matching
  method visible

$\Delta R^{hh} > \pi$

- NLO accurate
- small dependence on
  parton shower / matching

HH top mass scheme dependence

Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV},$$
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=400\text{ GeV}} = 0.1609(4)^{+7\%}_{-7\%}\,\text{fb/GeV},$$
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=600\text{ GeV}} = 0.03204(9)^{+0\%}_{-26\%}\,\text{fb/GeV},$$
$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV},$$