# Subtraction Methods at Next-to-next-leading order

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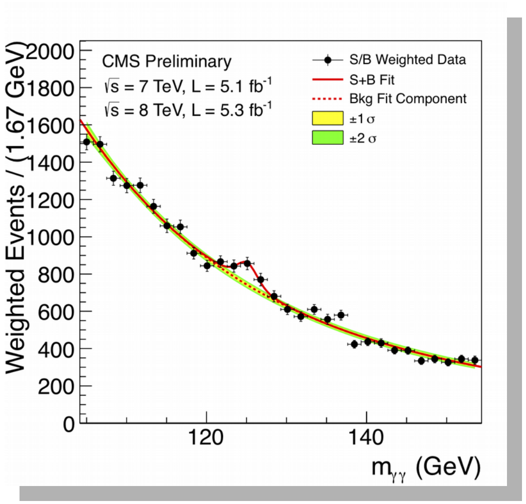
# Disclaimer

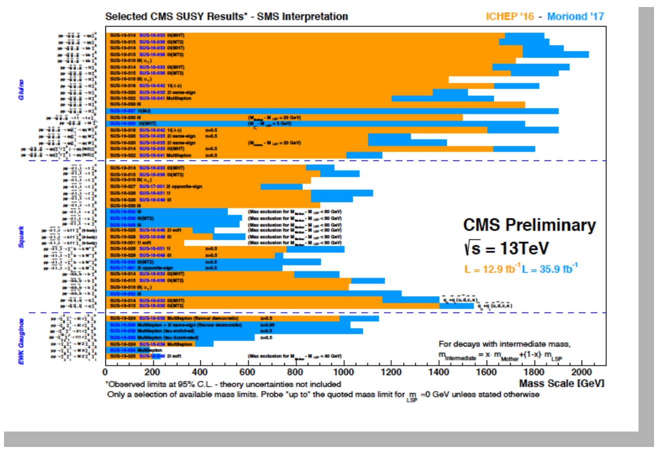
- Substantial progress on NNLO subtraction schemes over the last ~ 15 years, as a result of the hard work of many people.
- Not possible to give a summary of this work.
- Instead, I will try to give an overview of the current status of NNLO subtractions:
  - What has been done?
  - What can we hope to do in the near future?

# Precision physics at the LHC

Discovery of Higgs boson...




+ absence of enduring evidence for new physics...



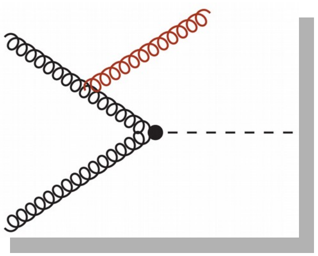

➞ **Precision physics programme** at LHC:

- Extensive studies of Higgs boson: fully understand the nature of EWSB.
- Search for BSM physics through subtle deviations from SM background.
- Determine fundamental parameters of nature.

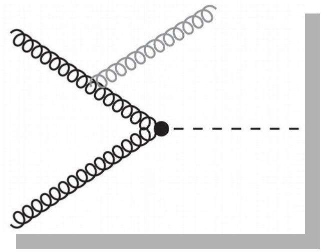
- Percent-level predictions require next-to-next-to-leading order (NNLO) accuracy.
  - Two-loop amplitudes.
  - Subtraction methods.

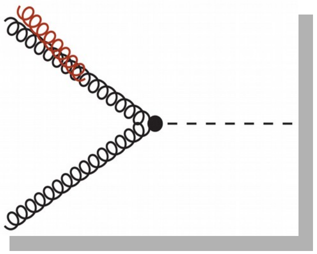
# Need for Subtractions

*"Subtractions?! But you knew how to do subtractions when you were six years old!"* -- My mother.



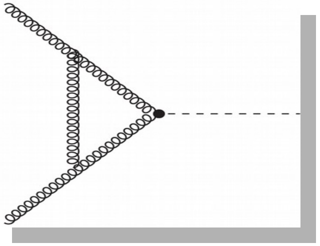
- Beyond LO in perturbative QCD : need real radiative corrections.
- Integrate over phase space: $\int |\mathcal{M}|^2 F_J d\phi_4$ diverges!
- Unphysical infrared divergence: due to soft and/or collinear radiation.





- Combine with virtual corrections: guaranteed to cancel with IR poles in loop amplitude (KLN theorem).

$\mathcal{M}_{1-\text{loop}} \sim 1/\epsilon^2$



- Cancellation only manifest after integrating over full phase space of emitted parton:

  ⇨ lose kinematic information, not fully differential.

# Subtractions at NLO and NNLO

**Subtraction scheme:**

Extract singularities without integrating over full phase space of radiated parton:

- Singularities manifest as poles in $1/\epsilon$ cancel against poles in virtual correction.

[Catani, Seymour '96; Frixione, Kunszt, Signer '96-'97]

- Subtractions at NLO fully solved.
  - Essential precursor to automation of NLO computations.
- Much more complicated at NNLO!
  - Singularity structure much more complicated – singularities overlap!
- This was ***the*** major obstacle to computing $2 \to 2$ processes at NNLO.
- Substantial progress over last decade – several approaches applied with great success:
  - Slicing methods.
  - Subtraction methods.
- (Almost) all $2 \to 2$ processes known at NNLO...
- ... but problem not completely solved, as at NLO.

# Slicing methods at NNLO

- Basic idea:
  - ➢ identify an observable that is sensitive to IR radiation;
  - ➢ use it to slice up the phase space into an unresolved part and a (partially) resolved part.

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int_0^\delta \left[|\mathcal{M}|^2 F_J d\phi_d\right]_{\text{s.c.}} + \int_\delta^1 |\mathcal{M}_J|^2 F_J d\phi_4 + \mathcal{O}(\delta)$$

Divergent — Born-like; Soft-collinear approximation — NLO+jet

- Observables:
  - ➢ qT [Catani, Grazzini '07]
  - ➢ N-jettiness [Gaunt *et al* '15; Boughezal *et al* '15]

- Pros:
  - ✔ Exploits vast experience in NLO calculations.
  - ✔ Simpler than subtraction schemes – phenomenological results quicker.

- Cons:
  - ✗ Non-local – potential issues of numerical stability.
  - ✗ Cutoff scale ambiguous.
  - ✗ Power corrections can be large.

# Slicing Methods at NNLO

- Public codes for fully differential NNLO results:
  - ➢ MATRIX (based on qT) [Kallweit, Grazzini, Wiesemann, '17]
  - ➢ MCFM (based on N-jettiness) [Boughezal *et al.*, '17]

- Power corrections in cutoff:
  - ✔ Reduce dependence on cutoff parameter.
  - ✔ Allow more stable, efficient computations.
- Next-to-leading power corrections for NNLO computations

$$\tau \frac{d\sigma^{2,2}}{d\tau} = \alpha_s^2 \tau \left( \underline{C_3^{2,2} \log^3 \tau} + C_2^{2,2} \log^2 + C_1^{2,2} \log \tau + C_0 \right)$$

- Leading log corrections known for color singlet production
  - ➢ SCET [Moult *et al.*, '16-'17; Ebert *et al.*, hep-ph/1812.08189]
  - ➢ QCD [Boughezal, Isgro, Liu, Petriello, '17-'18]
- Looking forward to corrections for jet final states.

# Subtraction Methods at NNLO

- Basic idea: identify a function *S* which:
  - reproduces the matrix elements in the unresolved limits;
  - is (relatively) simple and can be integrated over the unresolved phase space.
- Subtract and add back:

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

Divergent

Finite;
integrate in 4-dim.

Counterterm;
Explicit singularities

- Pros:
  - ✔ Local – better numerical stability.
  - ✔ No issues of cutoff or power corrections.
  - ✔ Historically, subtraction outperformed slicing at NLO.
- Cons:
  - ✗ Difficult to identify good subtraction function.
  - ✗ Highly non-trivial to integrate counterterm – singularities overlap.

# Subtraction Methods at NNLO

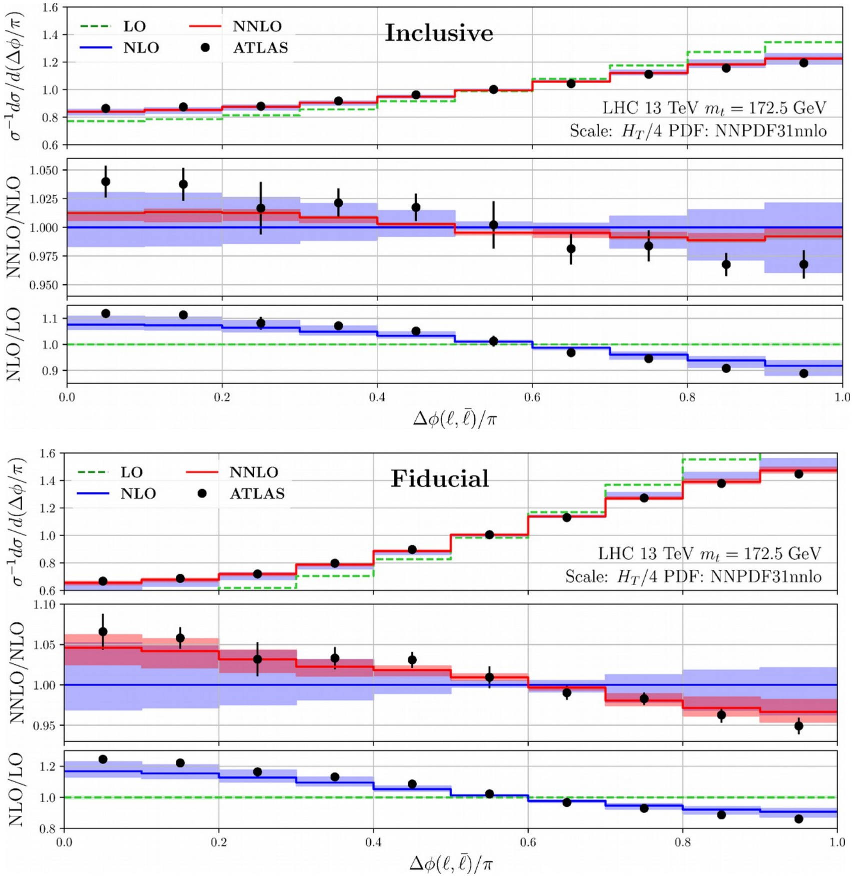
- Mature, all-purpose subtractions for LHC:
  - Antenna [Gehrmann-de Ridder, Gehrmann, Glover ‘05, …]
  - STRIPPER [Czakon ‘10, ‘11]
- Specialized subtraction schemes:
  - Projection-to-Born [Cacciari *et al* ‘15]
    (applicable to very few processes BUT can be extended to N3LO)
  - CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca ‘05, …]
    (partons in final state only).
- Next-generation subtractions, under construction:
  - Nested soft-collinear [Caola, Melnikov, R.R. ‘17, ‘19, ...]
  - Geometric [Herzog ‘18]
  - Local analytic sector [Magnea *et al* ‘18]

# Phenomenological Results

Many recent results rely on effective subtraction methods:

[Behring, Czakon, Mitov, Papanastasiou, Poncelet, hep-ph/1905.05407]

- STRIPPER subtraction.
- Spin correlations in top pair production.
- NNLO corrections to top pair production and top decay.
- Complete NNLO corrections to complicated production and decay process!

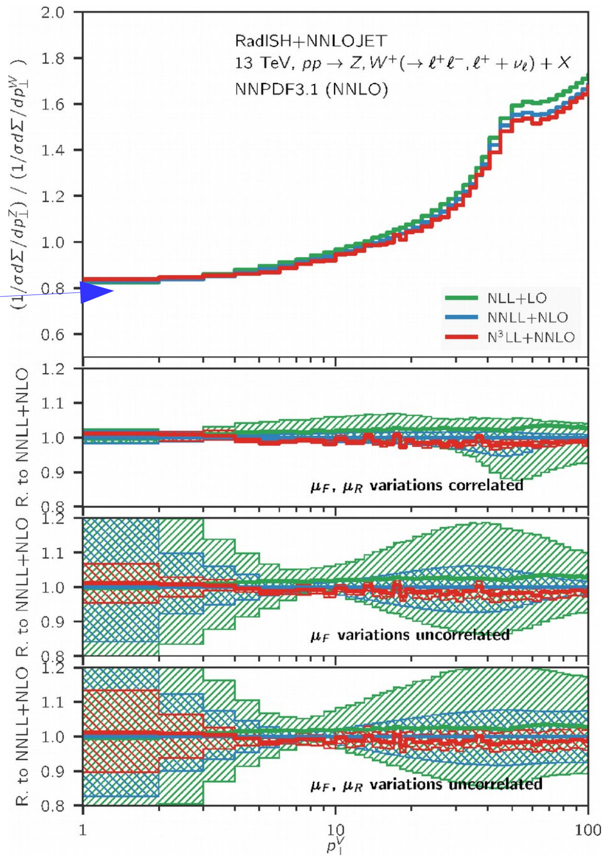


# Phenomenological Results

- Many recent results rely on effective subtraction methods:

[Bizon *et al.*, hep-ph/1905.05171]

- Antenna subtraction.
- Transverse momentum distribution of $W$ and $Z$ production at NNLO+N3LL.
- Very low values of pT!
- Help understand correlation of theoretical uncertainties in W and Z production – $W$ mass determination!




[Verbytskyi *et al.*, hep-ph/1902.08158]

- CoLoRFul subtraction.
- Precision extraction of strong coupling from 2- and 3-jet rates in e+e- collisions.
- $\alpha_s(M_Z) = 0.11881 \pm 0.00063\ (\text{exp.}) \pm 0.00101\ (\text{hadr.}) \pm 0.00045\ (\text{ren.}) \pm 0.00034(\text{res.}).$

# Next-generation subtractions

- Going to 2 → 3 processes [e.g. trijet, H+2j, V+2j] challenging for subtraction schemes.
- No NNLO subtraction method is ideal:
  - Local;
  - Analytic;
  - Flexible;
  - General.
- Motivates new attempts at constructing subtraction schemes:
  - Nested soft-collinear [Caola, Melnikov, R.R. '17, '19, ...]
  - Geometric [Herzog '18]
  - Local analytic sector [Magnea *et al.* '18]

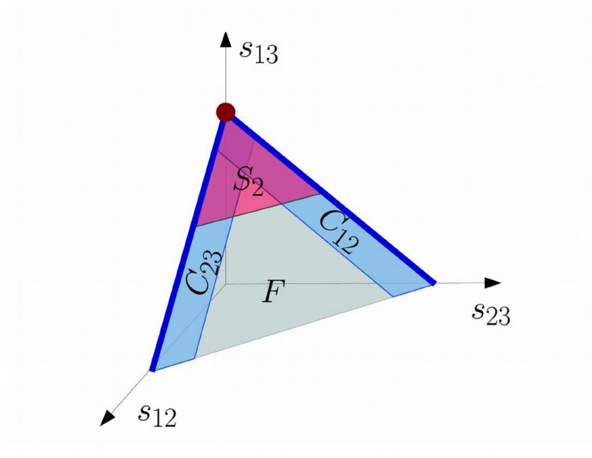
# Local Analytic Subtractions

[Magnea *et al.*, '18]

- Combine advantages of two NLO methods: FKS & Catani-Seymour.
  - Sector decomposition to separate overlapping singularities.
  - Counterterms written as sum over dipoles.
  - Different phase space parametrization in each term.
- Local & analytic.
- Leads to extremely simple counterterms.
- Proof-of-concept: $e^+e^- \to q\bar{q}$ (nf terms)
- Extensions for hadronic collisions: work in progress.

# Geometric Subtraction

[Herzog '18]




- Identify singular regions in $s_{ij}$ space.
- Construct slicing scheme:
  - Slicing parameter depends on Feynman diagrams!
- Promote to local subtraction scheme.
- Explicit ordering of limits to remove overlapping singularities.
- Proof-of-concept: pole structure for H→gg (nf=0).

# Nested soft-collinear subtractions

[Caola, Melnikov, RR, ‘17]

- Use color coherence to separate soft and collinear singularities.
- Use sector decomposition (cf. STRIPPER) to separate overlapping collinear singularities.
- Nested subtraction of singularities:
  - Clear physical origin of singularities.
- Local & analytic.
  [Caola, Delto, Frellesvig, Melnikov, hep-ph/1807.05835;
  Delto, Melnikov, hep-ph/1901.05213].
- Results for color singlet production and decay.
  [Caola, Melnikov, RR, hep-ph/1902.02081;
  Caola, Delto, Melnikov, RR, hep-ph/1906.xxxxx]
- Phenomenological application: $WH(\rightarrow b\bar{b})$ [Caola, Luisoni, Melnikov, RR, ‘17]
- Extension for final state jet at LHC straightforward (work in progress).

# Towards N3LO

- Fully differential N3LO calculations: even more complicated singularities.
- Possible for simplest ($1\to2$ and $2\to1$) processes, using
  - qT
  - Projection-to-Born.
- Requires extremely good control of NNLO singularities in presence of additional hard jet.

- Higgs production:
  - qT slicing + Antenna subtraction for NNLO H+jet

    [Chieri, Chen, Gehrmann, Glover, Huss, hep-ph/1807.11501]
- Deep inelastic scattering
  - Projection-to-Born + antenna for NNLO DIS+jet

    [Currie *et al.*, '18;

    Gehrmann *et al.*, hep-ph/1812.06104]
- VBF
  - Projection-to-Born + Projection-to-Born for VBF+jet

    [Dreyer, Karlberg, hep-ph/1811.07906]
- $H \to b\bar{b}$
  - Projection-to-Born + N-Jettiness for $H\to b\bar{b}$+jet

    [Mondini, Schiavi, Williams, hep-ph/1904.08960]

# Conclusions

- Infrared singularities are a major obstacle to fully differential predictions at NNLO.
- Significant progress over last decade on treating these singularities.
- A number of subtraction and slicing methods are fully developed: $2 \rightarrow 2$ processes are all known at NNLO.
- Power corrections for slicing:
  - more stable, remove ambiguities related to cutoff scale.
- New approaches to subtraction schemes:
  - Quest for local, analytic, general subtraction scheme for $2 \rightarrow 3$ processes.

THANK YOU FOR YOUR ATTENTION!

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