Bound state formation in colored coannihilation scenarios of dark matter

Julia Harz

in collaboration with

Kalliopi Petraki

based on

JHEP 1904 (2019) 130, [arXiv:1901.10030]
JHEP 1902 (2019) 186, [arXiv:1811.05478]
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Technische Universität München

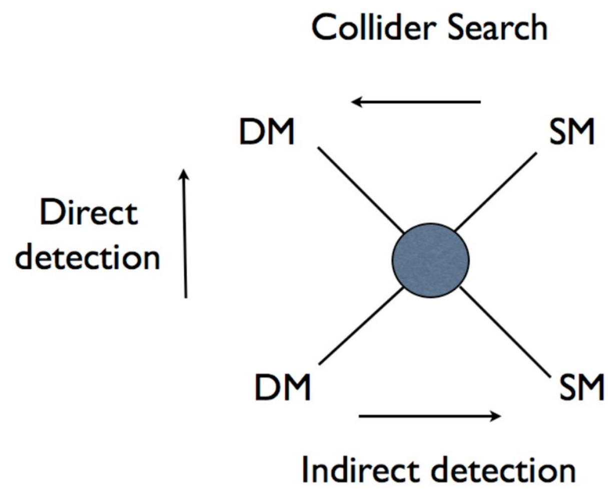
Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

WIMP miracle?

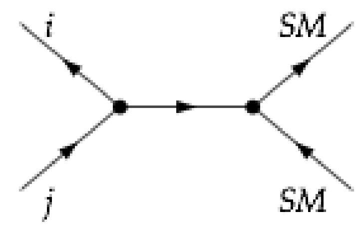
$$\Omega h^2 \sim \frac{10^{-10}\mathrm{GeV}^{-2}}{\langle \sigma v \rangle} \sim 0.1$$

$$\Rightarrow \quad \langle \sigma v \rangle \sim \frac{g^4}{m_\chi^2} \sim 10^{-9}\mathrm{GeV}^{-2}$$



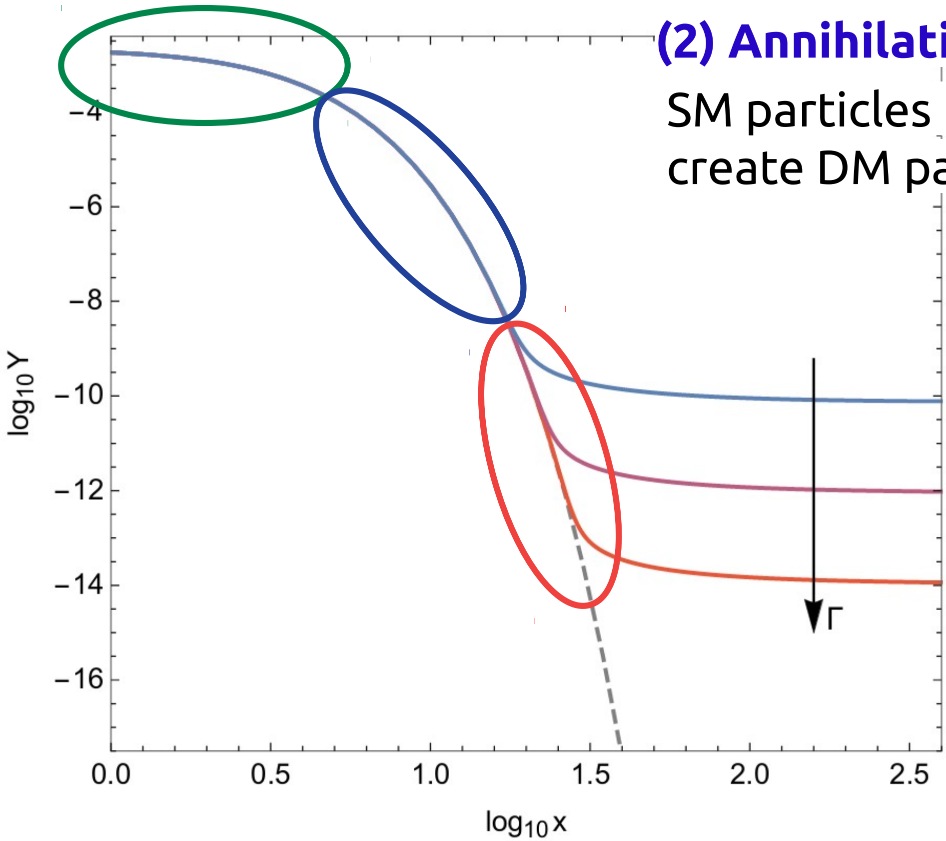
- no observations at the LHC, direct or indirect detection so far that supports the *minimal* WIMP model
- reason could be the realisation of a more complex WIMP model, *e.g. higher DM mass, complex dark sector, co-annihilation scenarios etc.*

Freeze-out with co-annihilation



(1) Thermal equilibrium regime (T >> m)

annihilation and production of DM
in thermal equilibrium $Y \approx \text{const.}$



$x = m_\chi / T$

cooling down

(2) Annihilation regime (T ~ m/10)

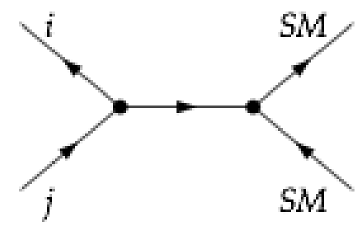
SM particles not energetic enough to
create DM particles $Y \approx \exp(-m_{DM}/T)$

(3) Freeze-out (T ~ m/30)

Annihilation rate falls behind expansion rate
→ DM abundance

$$\boxed{\begin{gathered}\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \\ \Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\text{crit}}} \propto \frac{1}{< \sigma_{\text{eff}} v >}\end{gathered}}$$

Freeze-out with co-annihilation



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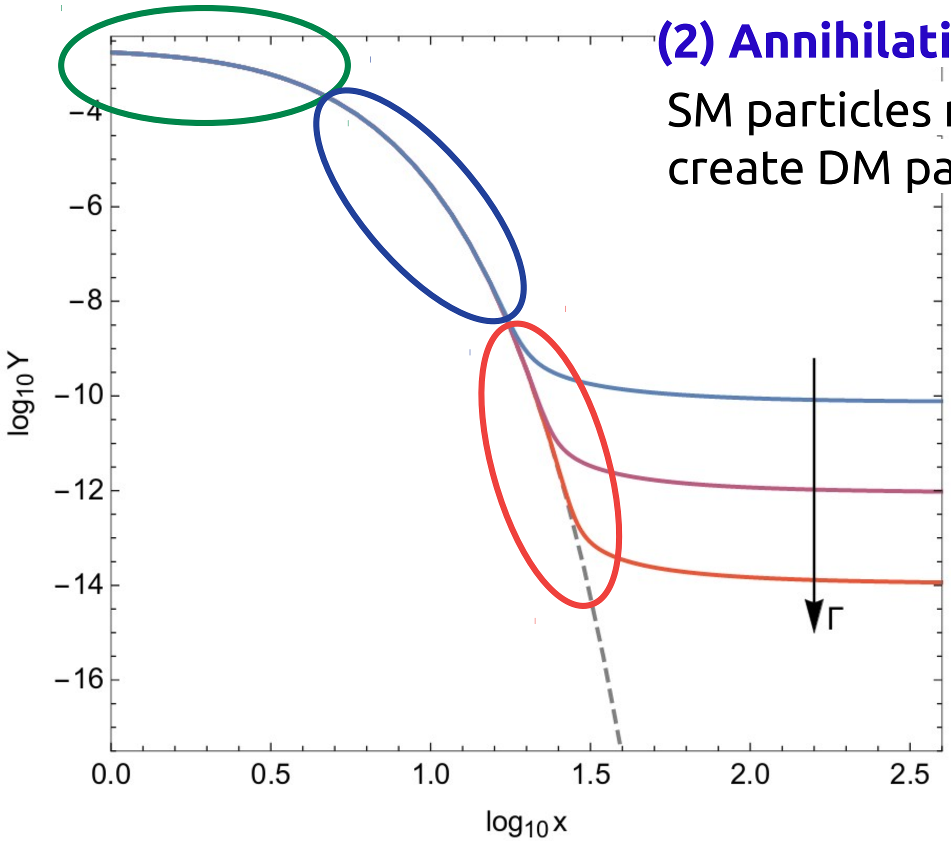


$$x = m_\chi / T$$

cooling down →

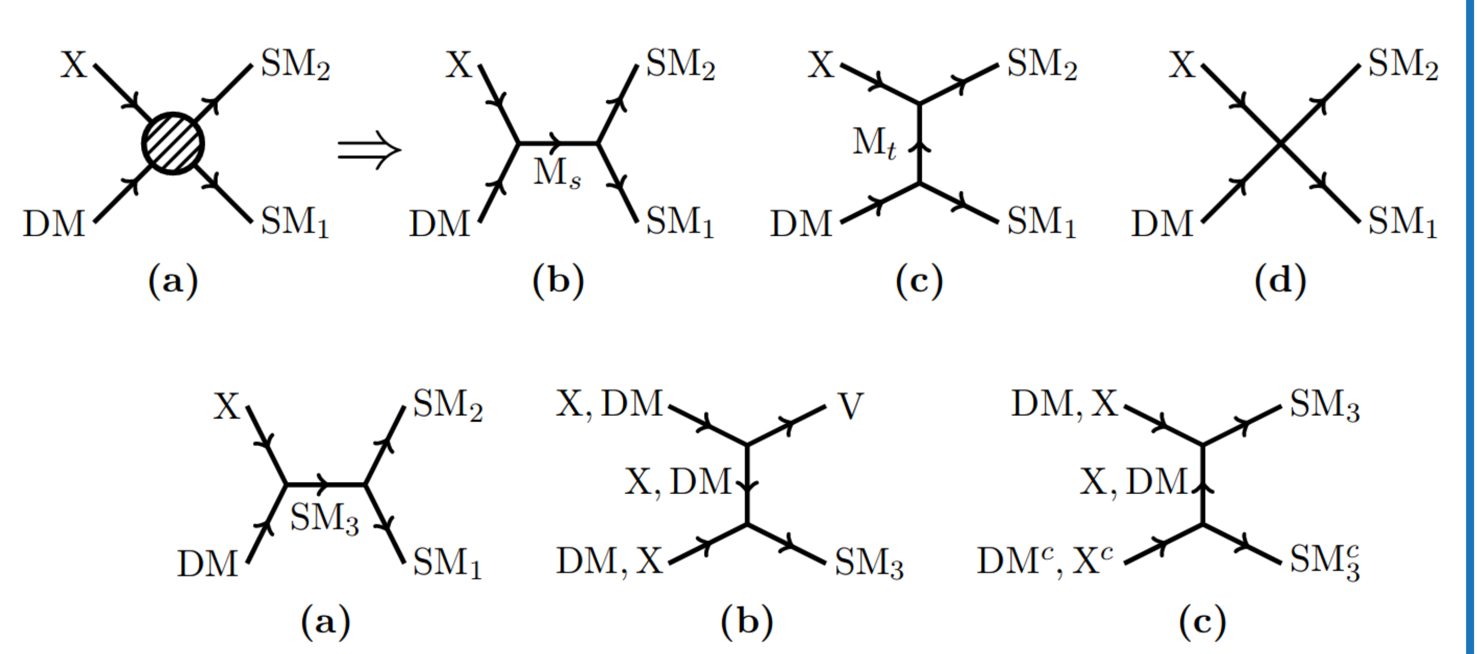
$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

$$\Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\text{crit}}} \propto \frac{1}{< \sigma_{\text{eff}} v >}$$

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$

co-annihilation $\quad \frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}}$

Recent specific focus on (colored) Coannihilation



Coloured coannihilations: Dark matter phenomenology meets non-relativistic EFTs, Biondini et al (2018)

Cornering Colored Coannihilation, El Hedri et al (2018)

Stop Coannihilation in the CMSSM and SubGUT Models, Ellis et al (2018)

Simplified Phenomenology for Colored Dark Sector, El Hedri et al (2017)

The Coannihilation Codex, Baker et al (2016)

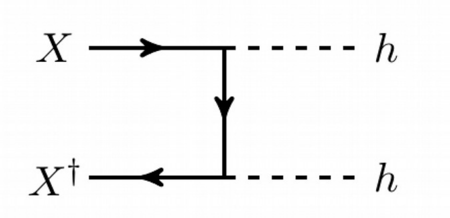
Anatomy of Coannihilation with a Scalar Top Partner, Ibarra et al (2015)

To name a few examples…

Effects impacting the relic abundance

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle}$$

Born level annihilation



$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm tree} v_{\rm rel}$$

usual DM codes include *only* born level calculation

Effects impacting the relic abundance

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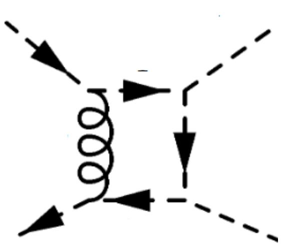
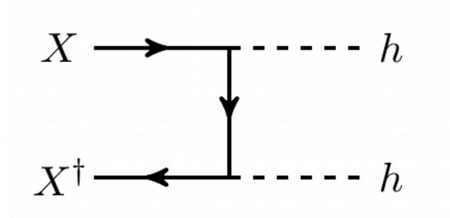
Higher order corrections



$$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{NLO}} v_{\text{rel}}$$

can lead to corrections of around 20% to the DM abundance

Born level annihilation



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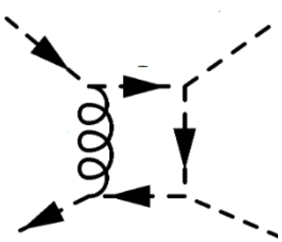

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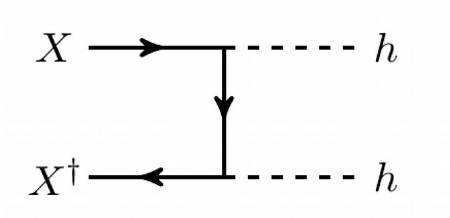

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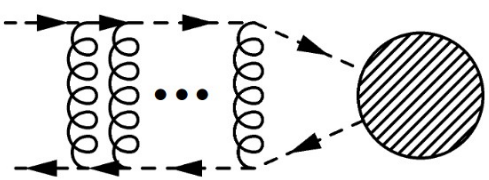
Sommerfeld enhancement



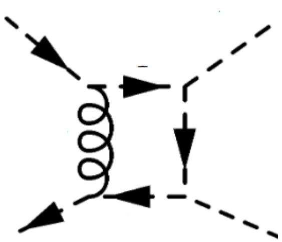
$$\left(\frac{\alpha}{v_{\rm rel}}\right)^n \sim 1$$

$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm tree} v_{\rm rel} \times S_0$$

Effects impacting the relic abundance

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle}$$

Higher order corrections



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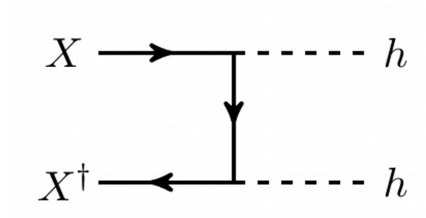
Born level annihilation



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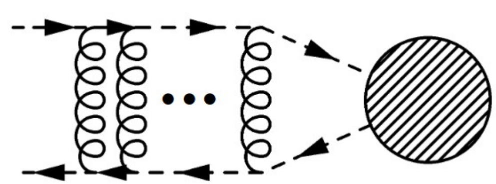
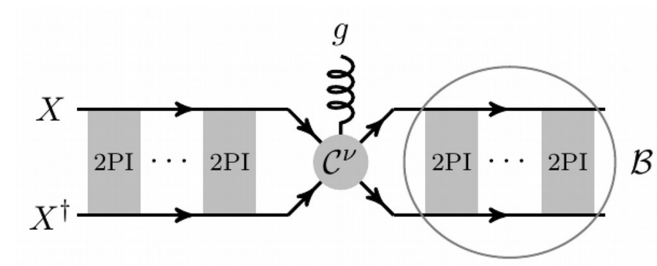


$$\left(\frac{\alpha}{v_{\rm rel}}\right)^n \sim 1$$

$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm tree} v_{\rm rel} \times S_0$$

Bound state formation



$$\langle \sigma_{\rm eff} v_{\rm rel} \rangle = \langle \sigma_{\rm ann} v_{\rm rel} \rangle + \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff}$$

bound state formation and subsequent decay open up a new effective DM annihilation channel

What had been done before?

non-exhaustive list!

Higher order corrections

- **Squark-pair annihilation into quarks at next-to-leading order.** S. Schmiemann, JH, B. Herrmann, M. Klasen, K. Kovařík, Phys. Rev. D99 095015 (2019)
- **Theoretical uncertainty of the supersymmetric dark matter relic density from scheme and scale variations,** JH, Herrmann, Klasen, Kovarik, Steppeler (2016)
- **SUSY-QCD corrections to stop annihilation into electroweak final states including Coulomb enhancement effects,** JH, Herrmann, Klasen, Kovařík, Meinecke (2015)
- **One-loop corrections to neutralino-stop coannihilation revisited,** JH, Herrmann, Klasen, Kovarik (2015)
- **One-loop corrections to gaugino (co)annihilation into quarks in the MSSM,** Herrmann, Klasen, Kovarik, Meinecke, Steppeler (2014)
- **Neutralino-stop coannihilation into electroweak gauge and Higgs bosons at one loop,** JH, Herrmann, Klasen, Kovarik, Le Boulc'h (2013)

Sommerfeld enhancement

- **A Sommerfeld Toolbox for Colored Dark Sectors,** El Hedri, Kaminska, Vries (2017)
- **Self-consistent Calculation of the Sommerfeld Enhancement,** Blum, Sato, Slatyer (2016)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos III. Computation of the Sommerfeld enhancements**, Beneke, Hellmann, Ruiz-Femenia (2015)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos II. P-wave and next-to-next-to-leading order S-wave coefficients**, Hellmann, Ruiz-Femenía (2013)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos I. General framework and S-wave annihilation,** Beneke, Hellmann, Ruiz-Femenia (2013)

What had been done before?

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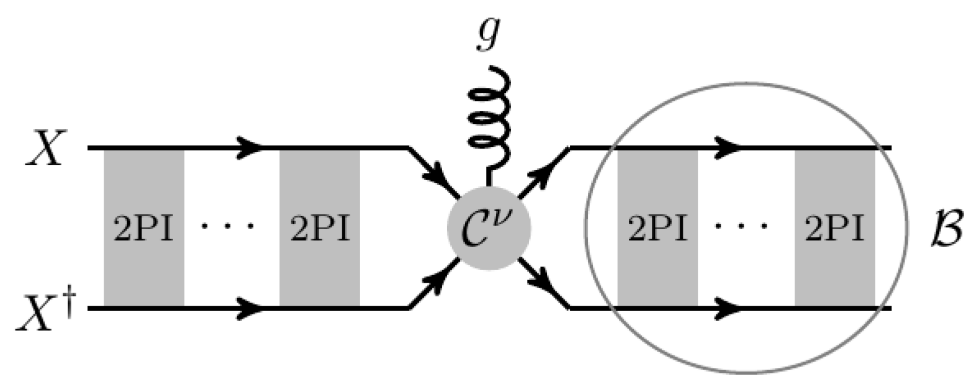
Bound state formation

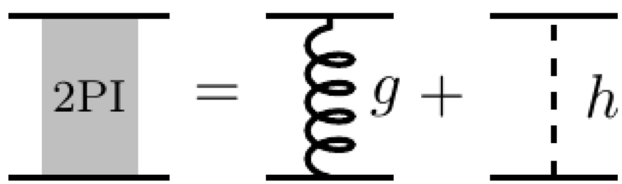
- **Dark Matter Sommerfeld-enhanced annihilation and Bound-state decay at finite temperature,** Binder, Covi, Mukaida (2018)
- **Thermal dark matter co-annihilating with a strongly interacting scalar,** Biondini, Vogl (2018)
- **Bound-state effects for dark matter with Higgs-like mediators,** Biondini (2018)
- **Reappraisal of dark matter co-annihilating with a top or bottom partner,** Keung, Low, Zhang (2017)
- **Cosmological Implications of Dark Matter Bound States,** Mitridate, Redi, Smirnov, Strumia (2017)
- **Effects of QCD bound states on dark matter relic abundance,** Liew, Luo (2017)
- **Simplified Phenomenology for Colored Dark Sectors,** El Hedri, Kaminska, de Vries, Zurita (2017)
- **Capture and Decay of Electroweak WIMPonium,** Asadi, Baumgart, Fitzpatrick, Krupczak, Slatyer (2016)

What was missing?

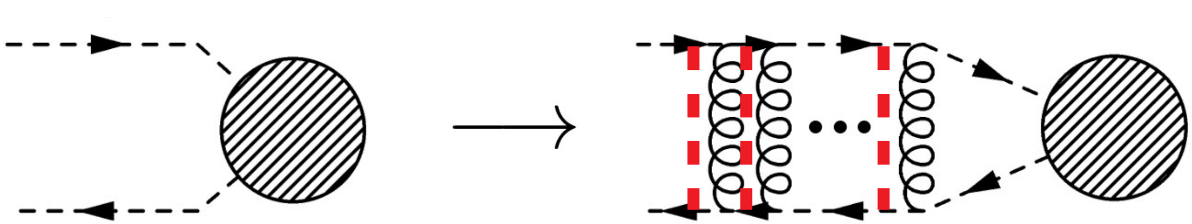
colored coannihilation

accurate treatment of unstable **bound state formation** in unbroken non-abelian theories





including the effect of the **Higgs boson**



effect of **the Higgs boson** in the context of Sommerfeld enhancement for **annihilation process**

Simplified model

- **Lagrangian**

DM Majorana fermion χ; co-annihilating with complex scalar X charged under SU(3)$_c$

$$\delta\mathcal{L} = (D_{\mu,ij}X_j)^\dagger (D^\mu_{ij'}X_{j'}) - m_X^2 X_j^\dagger X_j$$
$$+ \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 - g_h m_X \, h X_j^\dagger X_j$$

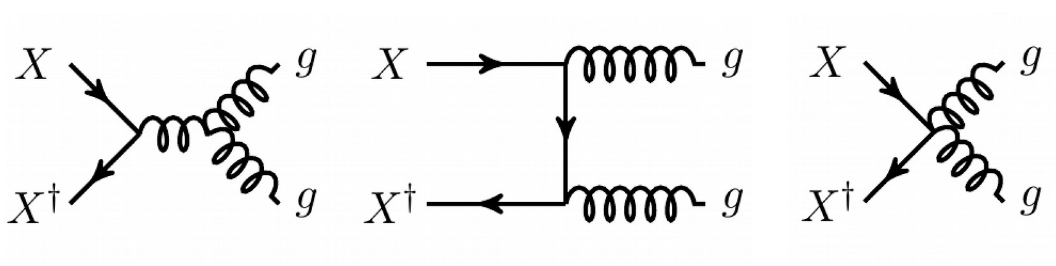
Simplified model

- **Lagrangian**

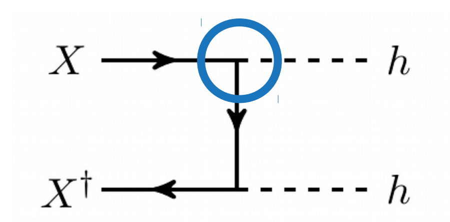
DM Majorana fermion χ; co-annihilating with complex scalar X charged under SU(3)$_c$

$$\delta\mathcal{L} = (D_{\mu,ij}X_j)^\dagger\,(D^{\mu}_{ij'}X_{j'}) - m_X^2\,X_j^\dagger X_j + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 - g_h m_X\;h\,X_j^\dagger X_j$$

- **Annihilation processes:**



$$(\sigma v_{\rm rel})^{\rm pert}_{XX^\dagger\to gg} = \frac{14}{27}\,\frac{\pi\alpha_s^2}{m_X^2}$$

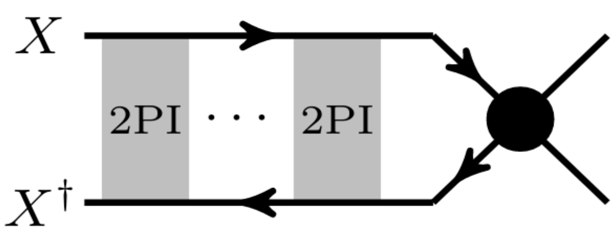


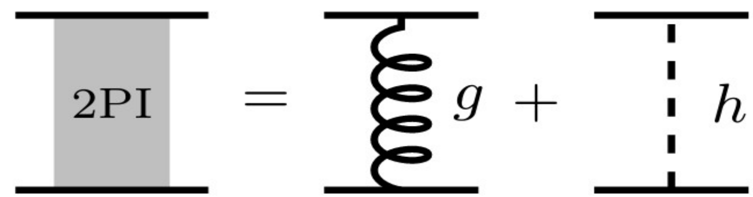
$$\alpha_h = \frac{g_h^2}{16\pi}$$

$$(\sigma v_{\rm rel})^{\rm pert}_{XX^\dagger\to hh} = \frac{4\pi\alpha_h^2}{3m_X^2}\,\frac{(1-m_h^2/m_X^2)^{1/2}}{[1-m_h^2/(2m_X^2)]^2}$$

we neglect p-wave suppressed contributions $\quad X\bar{X}\to q\bar{q},\, X\bar{X}\to gh$

Higgs as mediator of long-range interactions





$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{scatt}}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r}) \quad \text{with} \quad \mathrm{V}_{\text{scatt}}(r) = -\frac{\alpha_g^S}{r} - \frac{\alpha_h}{r}\,e^{-m_h r}$$

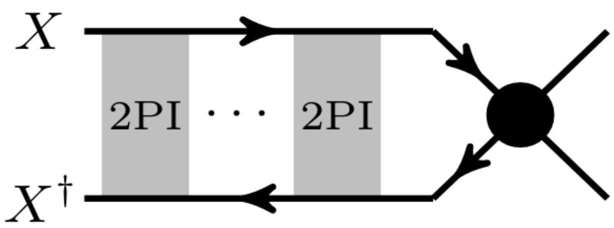
Characteristic parameters:

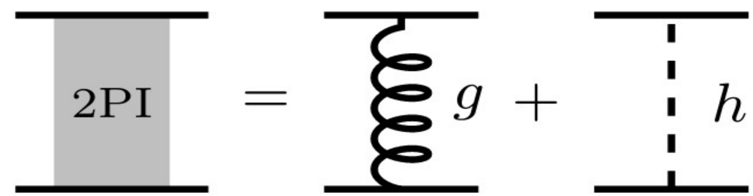
$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\text{rel}}} = \frac{\alpha_{g,h}}{v_{\text{rel}}} \qquad \frac{\text{Bohr momentum}}{\text{momentum exchange of unbound particles}} > 1$$

$$\mathrm{d}_h \equiv \frac{\mu\alpha_h}{m_h} \qquad \frac{\text{Interaction range}}{\text{Bohr radius}} > 1$$

$$\left\{\nabla_{\mathbf{z}}^2 + 1 + \frac{2}{z}\left[\zeta_g + \zeta_h\,\exp\left(-\frac{\zeta_h z}{d_h}\right)\right]\right\}\phi_{\mathbf{k}} = 0 \quad \Longrightarrow \quad \boxed{S_0(\zeta_g, \zeta_h, d_h) \equiv |\phi_{\mathbf{k}}(0)|^2}$$

Higgs as mediator of long-range interactions





$$\left[-\frac{\nabla^2}{2\mu} + V_{\mathrm{scatt}}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r}) \quad \text{with} \quad \mathrm{V}_{\mathrm{scatt}}(r) = -\frac{\alpha_g^S}{r} - \frac{\alpha_h}{r}\,e^{-m_h r}$$

Color decomposition: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

$$\alpha_g^S \equiv \alpha_s^S \times \begin{cases} C_{\mathbf{1}} = C_F = 4/3 \\ C_{\mathbf{8}} = C_F - C_A/2 = -1/6 \end{cases}$$

attractive singlet

repulsive octet

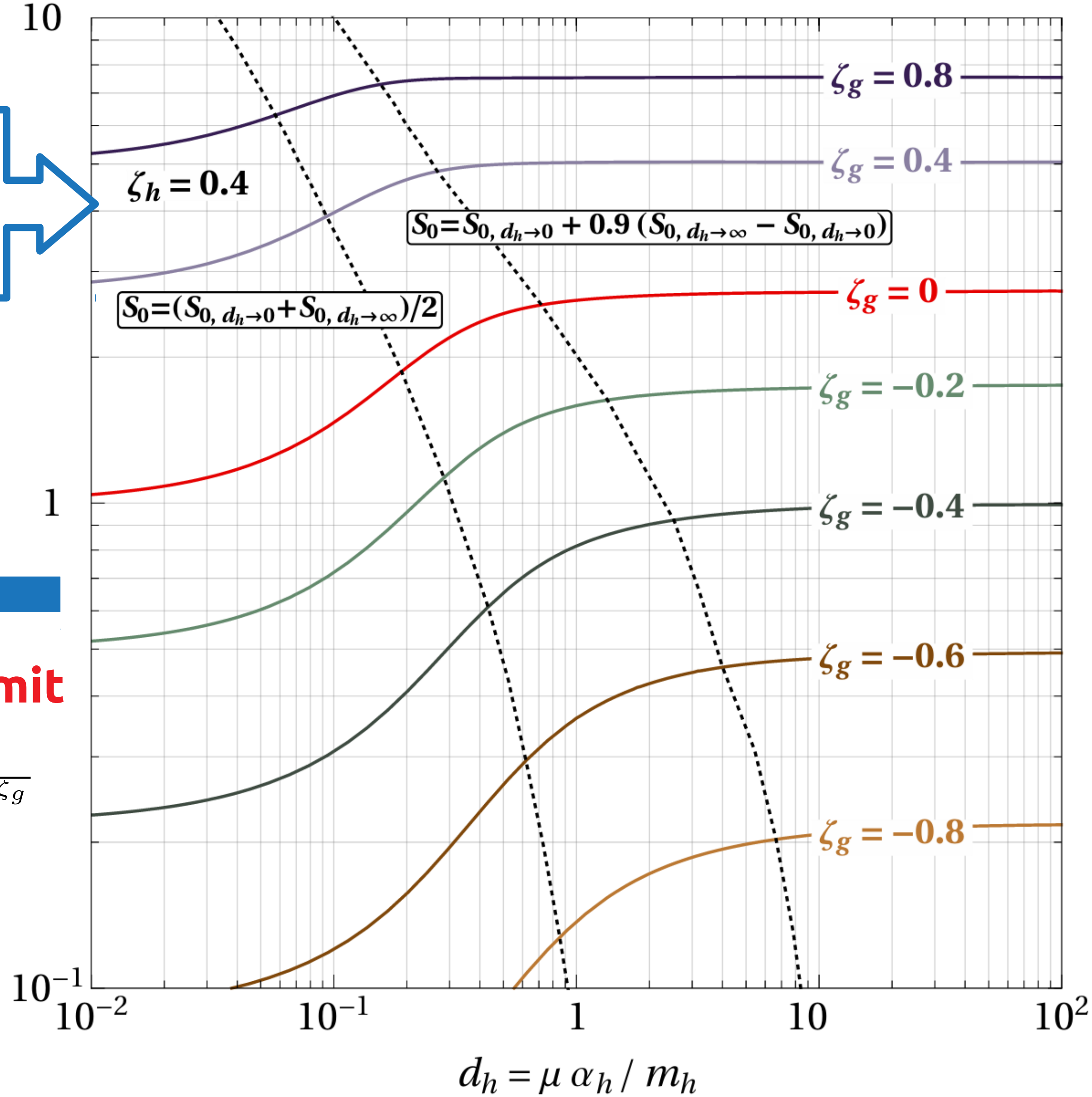
$$(\sigma v_{\mathrm{rel}})_{XX^\dagger\to gg} = (\sigma v_{\mathrm{rel}})^{\mathrm{pert}}_{XX^\dagger\to gg} \times \left(\frac{2}{7}S_0^{[\mathbf{1}]} + \frac{5}{7}S_0^{[\mathbf{8}]}\right) \qquad S_0^{[\mathbf{1}]} = S_0[\zeta_g^{[\mathbf{1}]},\,\zeta_h,\,d_h]$$

$$(\sigma v_{\mathrm{rel}})_{XX^\dagger\to hh} = (\sigma v_{\mathrm{rel}})^{\mathrm{pert}}_{XX^\dagger\to hh} \times S_0^{[\mathbf{1}]} \qquad S_0^{[\mathbf{8}]} = S_0[\zeta_g^{[\mathbf{8}]},\,\zeta_h,\,d_h]$$

Higgs enhancement

$v_{\rm rel} \approx 0.2$

$\alpha_h \approx 0.08$



$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}}$$

$$d_h \equiv \frac{\mu\alpha_h}{m_h}$$

Coulomb limit

$$S_0 \simeq \frac{2\pi(\zeta_g+\zeta_h)}{1-e^{-2\pi(\zeta_g+\zeta_h)}}$$

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JH, Petraki, (2018)

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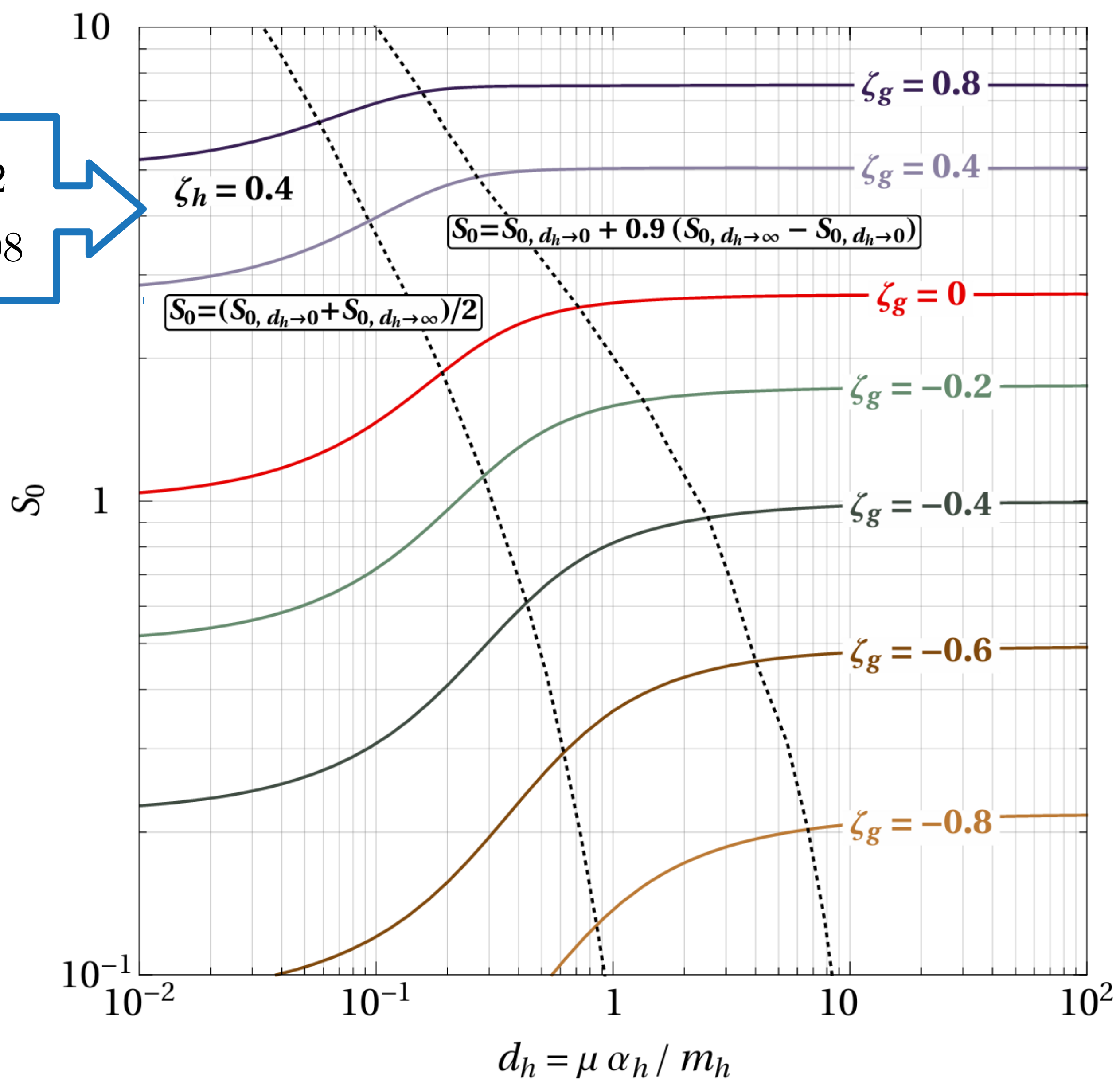

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Higgs enhances the attraction of the singlet state

JH, Petraki, (2018)

Higgs enhancement

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$$d_h \equiv \frac{\mu\alpha_h}{m_h}$$

$v_{\rm rel} \approx 0.2$

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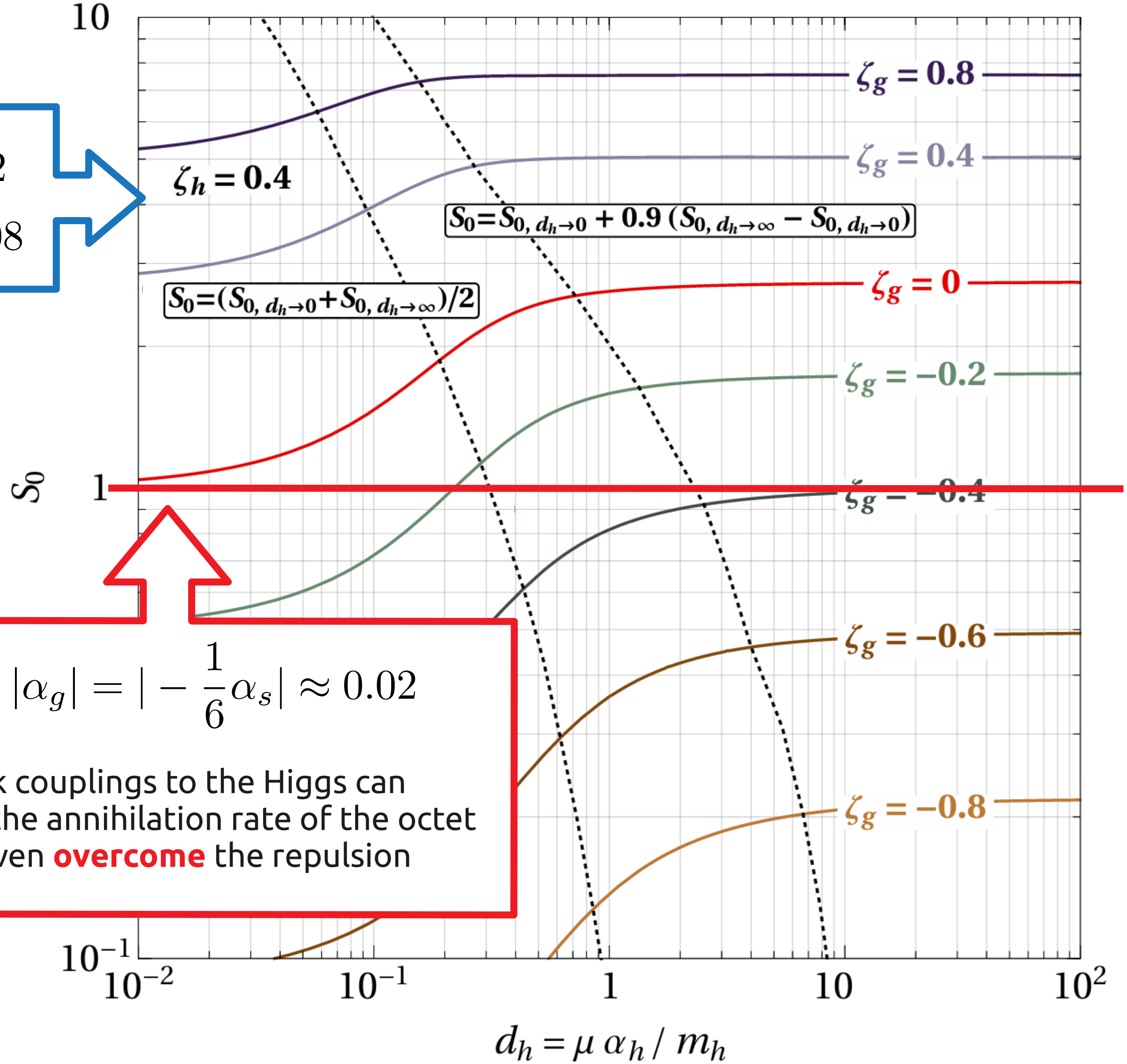


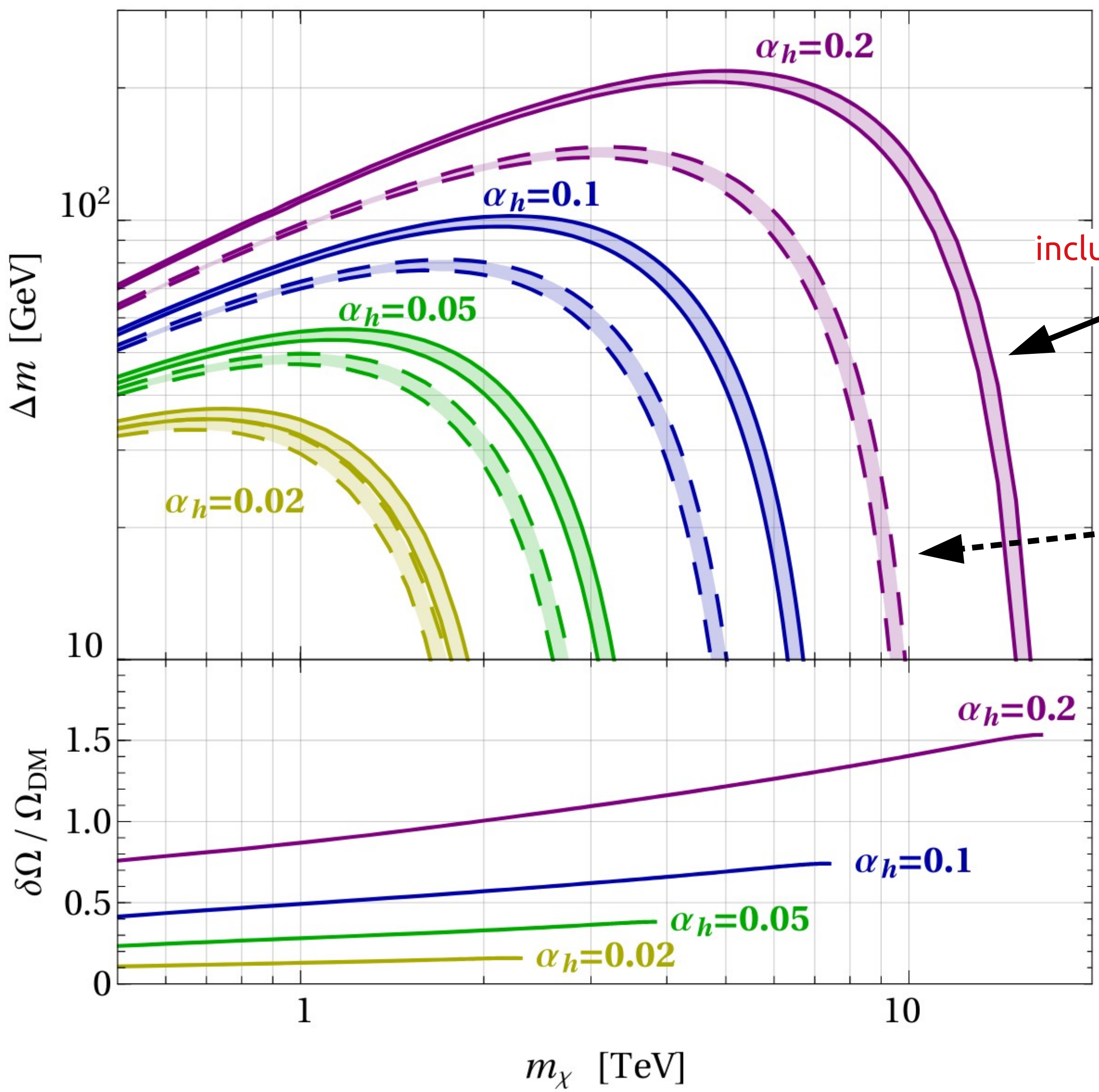
$$\alpha_h \approx |\alpha_g| = |-\frac{1}{6}\alpha_s| \approx 0.02$$

Even weak couplings to the Higgs can **enhance** the annihilation rate of the octet state or even **overcome** the repulsion

Higgs reduces the repulsion of the octet state

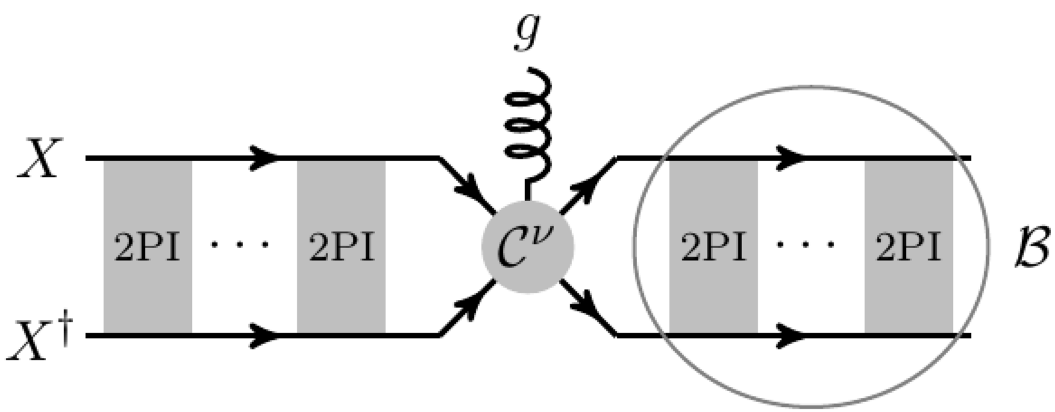
JH, Petraki, (2018)

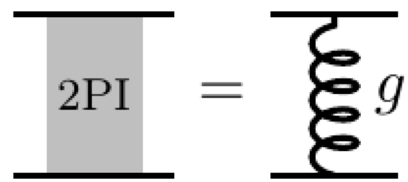
Impact of Higgs enhancement on the relic density



JH, Petraki, (2018)

Bound states with gluon exchange





$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]}$ **bound state formation**

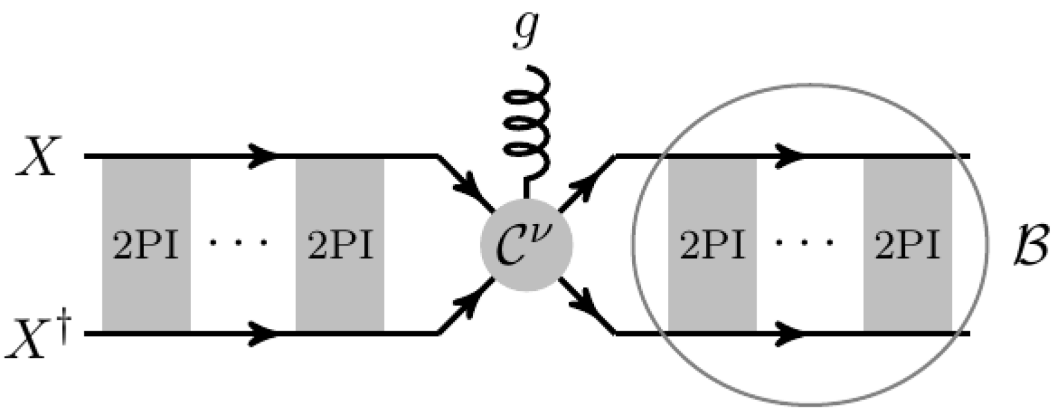
$(\mathrm{XX}^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]} \to (X + X^\dagger)_{[\mathbf{8}]}$ **bound state ionisation**

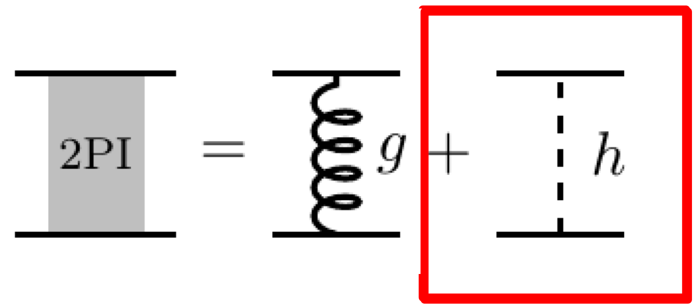
$\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} \to g_{[\mathbf{8}]}\ g_{[\mathbf{8}]}$ **bound state decay**

$$\boxed{\langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle_{\mathrm{eff}} = \langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \times \left(\frac{\Gamma_{\mathrm{dec}}}{\Gamma_{\mathrm{dec}} + \Gamma_{\mathrm{ion}}} \right)}$$

→ additional "annihilation" channel alters the relic density prediction

Bound states with gluon and Higgs exchange





$$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]}$$

$$(X + X^\dagger)_{[\mathbf{1}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g_{[\mathbf{8}]}\}_{\mathbf{1_S}}$$

$$(X + X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g_{[\mathbf{8}]}\}_{\mathbf{8_S}\text{ or }\mathbf{8_A}}$$

bound state formation

Higgs may allow

(1) to form tighter bound states

(2) to form color octet bound states

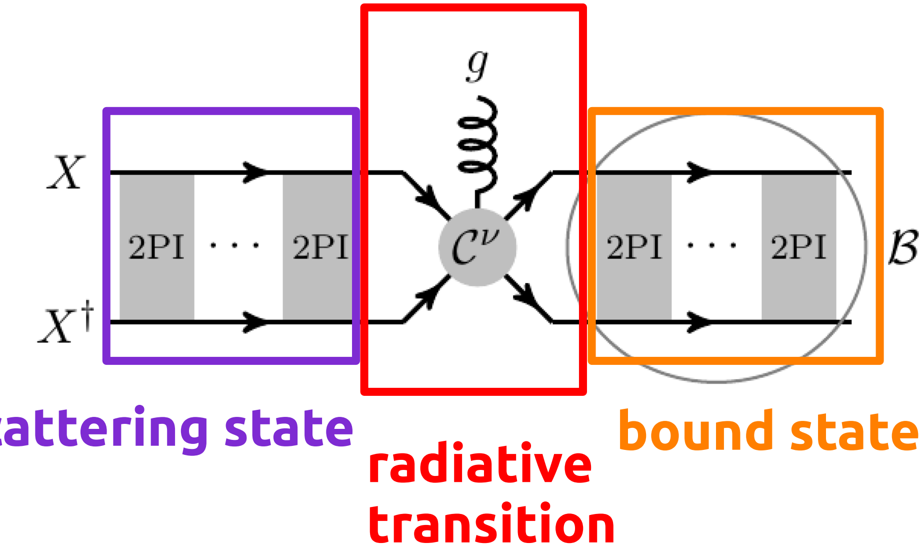
$$\boxed{\langle \sigma_{\rm BSF} v_{\rm rel}\rangle_{\rm eff} = \langle \sigma_{\rm BSF} v_{\rm rel}\rangle \times \left(\frac{\Gamma_{\rm dec}}{\Gamma_{\rm dec} + \Gamma_{\rm ion}}\right)}$$

→ additional "annihilation" channel alters the relic density prediction

Bound state formation

$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{scatt}}(\mathbf{r})\right] \phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\, \phi_{\mathbf{k}}(\mathbf{r})$$

$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{bound}}(\mathbf{r})\right] \psi_{n\ell m}(\mathbf{r}) = \mathcal{E}_{n\ell}\, \psi_{n\ell m}(\mathbf{r})$$



Bound state formation

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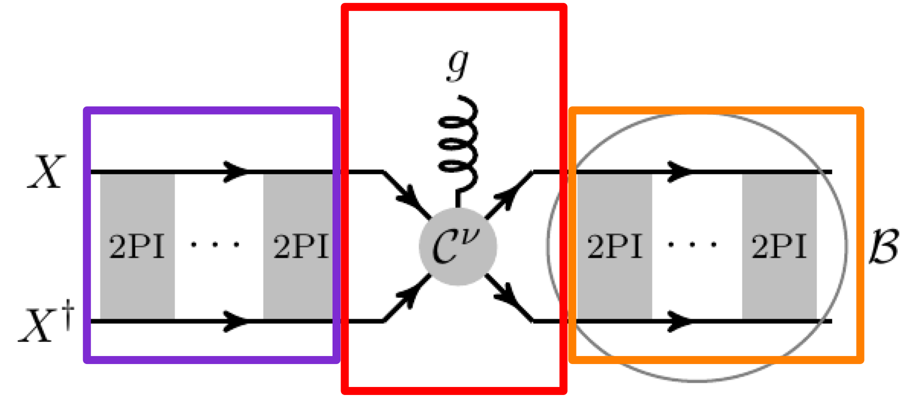
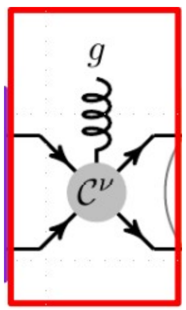


scattering state

radiative transition

bound state



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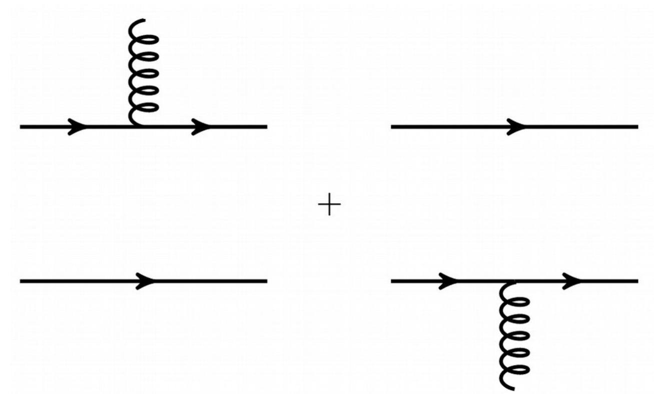

Effects of QCD bound states on dark matter relic abundance, Liew, Luo (2017)
Simplified Phenomenology for Colored Dark Sectors, El Hedri, Kaminska, de Vries, Zurita (2017)

transition amplitude

Bound state formation

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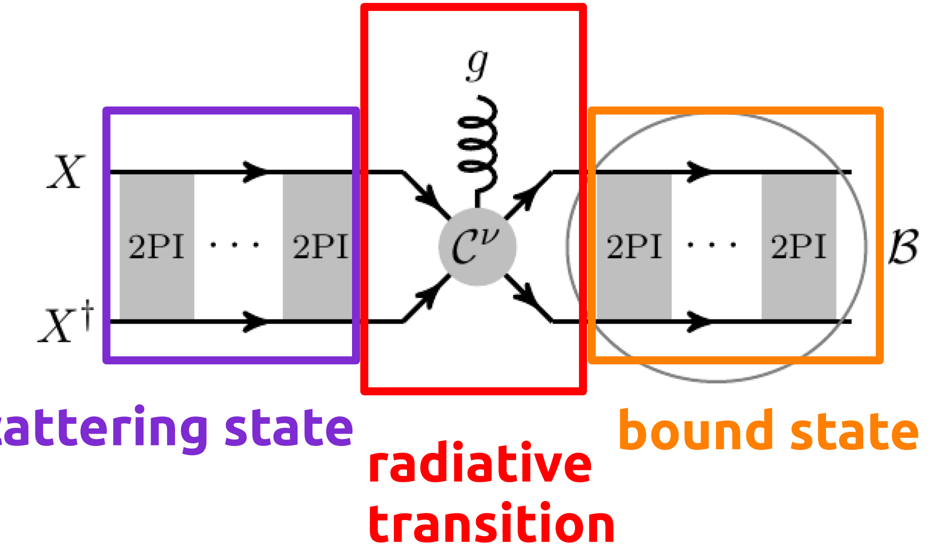

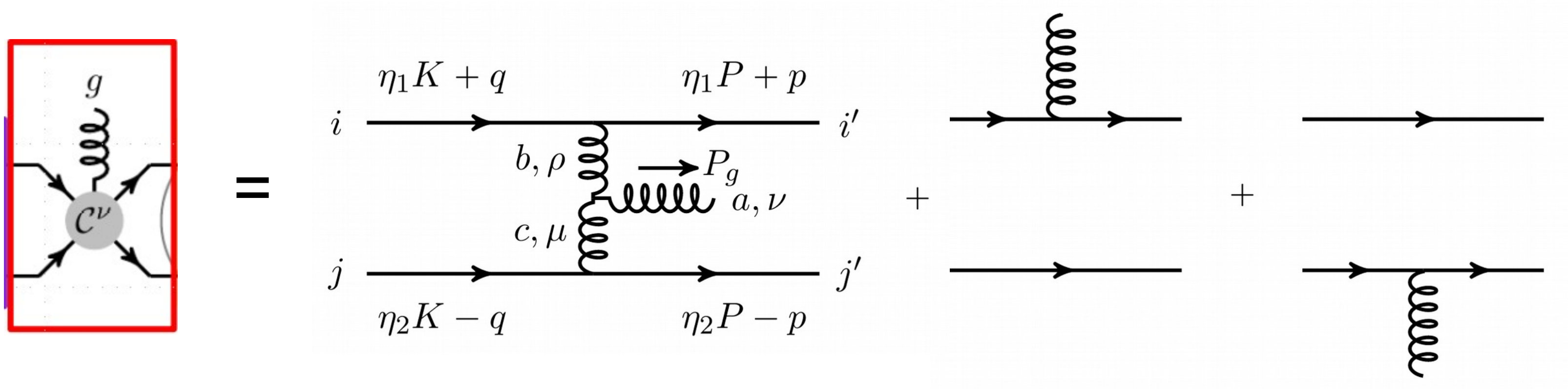

Reappraisal of dark matter co-annihilating with a top or bottom partner, Keung, Low, Zhang (2017)
Cosmological Implications of Dark Matter Bound States, Mitridate, Redi, Smirnov, Strumia (2017)
Capture and Decay of Electroweak WIMPonium, Asadi, Baumgart, Fitzpatrick, Krupczak, Slatyer (2016)

transition amplitude

Bound state formation

$$\left[-\frac{\nabla^2}{2\mu} + V_{\rm scatt}(\mathbf{r})\right] \phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\, \phi_{\mathbf{k}}(\mathbf{r})$$

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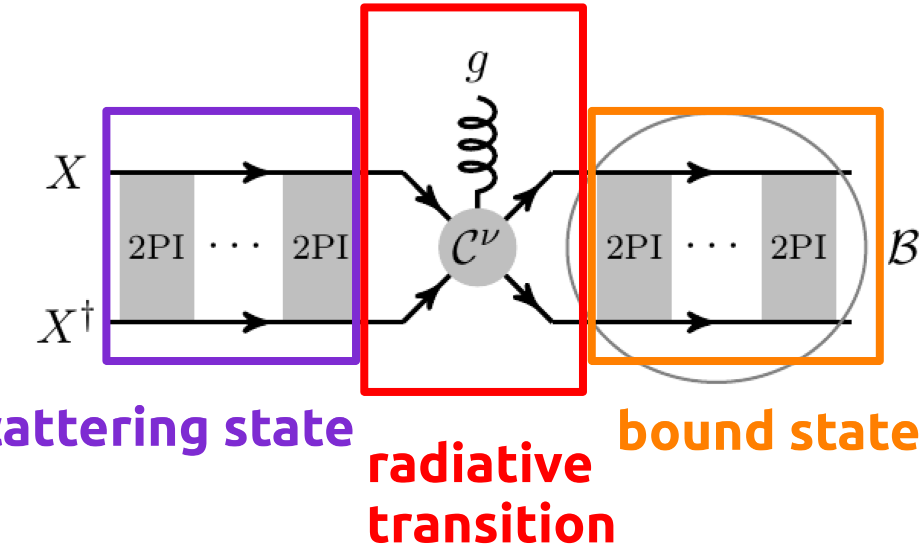
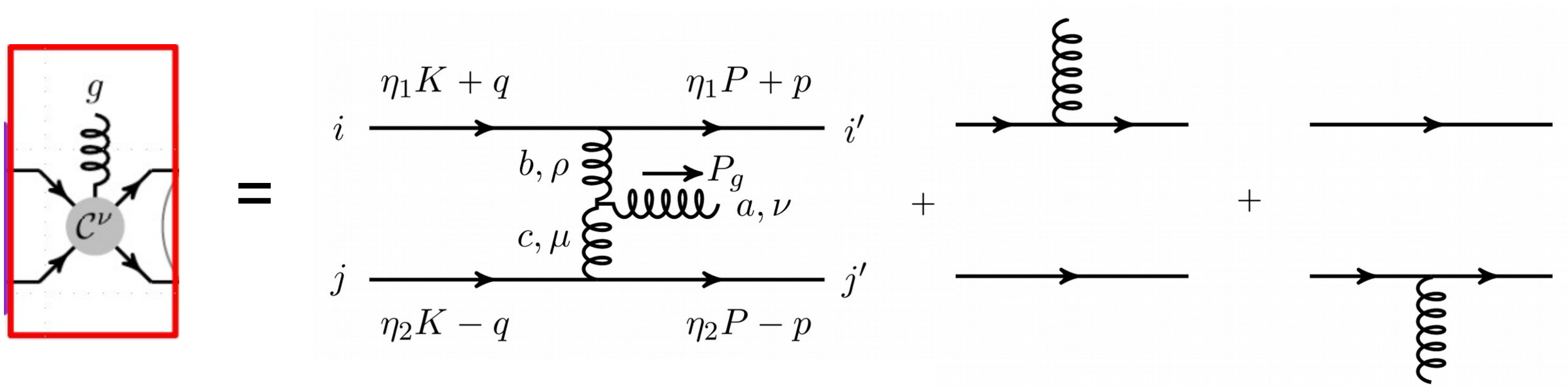




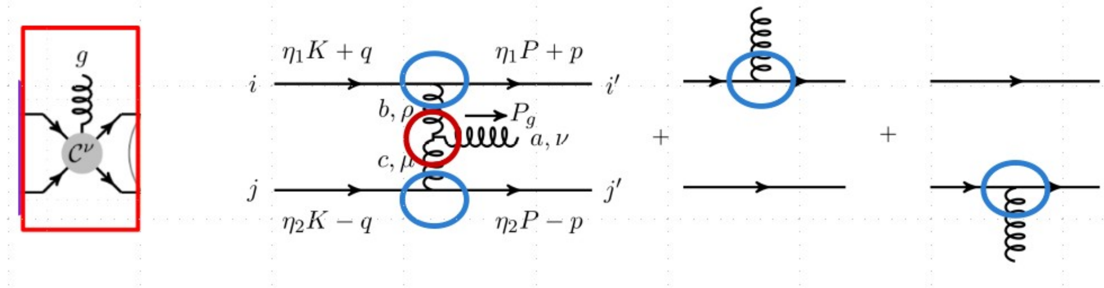
Derivation from Feynman diagrammatic approach, see
DM bound states from Feynman diagrams, Petraki et al. JHEP 1506 (2015) 128

$$[\mathcal{M}^{\nu}_{\mathbf{k}\to\{n\ell m\}}]^{a}_{ii',jj'} = \frac{1}{\sqrt{2\mu}} \int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\; \tilde{\psi}^*_{n\ell m}(\mathbf{p})\; \tilde{\phi}_{\mathbf{k}}(\mathbf{q})\; [\mathcal{M}^{\nu}_{\rm trans}(\mathbf{q},\mathbf{p})]^{a}_{ii',jj'}$$

transition amplitude

Bound state formation

transition amplitude



with: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

$$\frac{1}{9} \left| \mathcal{M}_{\mathbf{k}\to 100}^{[\mathbf{8}]\to[\mathbf{1}]} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{4}{27} \left[1 + \frac{3}{2} \left(\frac{\alpha_s^B}{\alpha_h + \alpha_g^B} \right) \right]^2 \left| \mathcal{J}_{\mathbf{k},100}^{[\mathbf{8},\mathbf{1}]} \right|^2$$

$$\text{for} \quad \alpha_h \to 0 : \qquad \to \left[1 + \frac{9}{8} \right]^2$$

expected to have significant effect!

Comparison with Quarkonium literature:

Perturbative heavy quark - anti-quark systems, M. Beneke, hep-ph/9911490

Running of the heavy quark production current and 1 / v potential in QCD, A. V. Manohar and I. W. Stewart, Phys. Rev. D63 (2001) 054004

*Renormalization group analysis of the QCD quark potential to order v**2,* A. V. Manohar and I. W. Stewart, Phys. Rev. D62 (2000) 014033

Thermal width and gluo-dissociation of quarkonium in pNRQCD, N. Brambilla, et al, JHEP 12 (2011) 116

Contributions to the effective BSF cross section

Remember:

$$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]}$$ **bound state formation**

$$(\mathrm{XX}^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]} \to (X + X^\dagger)_{[\mathbf{8}]}$$ **bound state ionisation**

$$\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} \to g_{[\mathbf{8}]}\ g_{[\mathbf{8}]}$$ **bound state decay**

$$\langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle_{\mathrm{eff}} = \langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \times \left(\frac{\mathit{\Gamma}_{\mathrm{dec}}}{\mathit{\Gamma}_{\mathrm{dec}} + \mathit{\Gamma}_{\mathrm{ion}}} \right)$$

Contributions to the effective BSF cross section

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bound state ionisation

$$(\mathrm{XX}^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]} \to (X+X^\dagger)_{[\mathbf{8}]}$$

$$\mathit{\Gamma}_{\mathrm{ion}} = g_g \int_{\omega_{\mathrm{min}}}^{\infty} \frac{d\omega}{2\pi^2} \frac{\omega^2}{e^{\omega/T}-1}\, \sigma_{ion}$$

$$\sigma_{ion} = \frac{g_X^2}{g_g g_{\mathcal{B}}} \frac{\mu^2 v_{rel}^2}{\omega^2}\, \sigma_{\mathrm{BSF}}$$ Milne relation

bound state decay

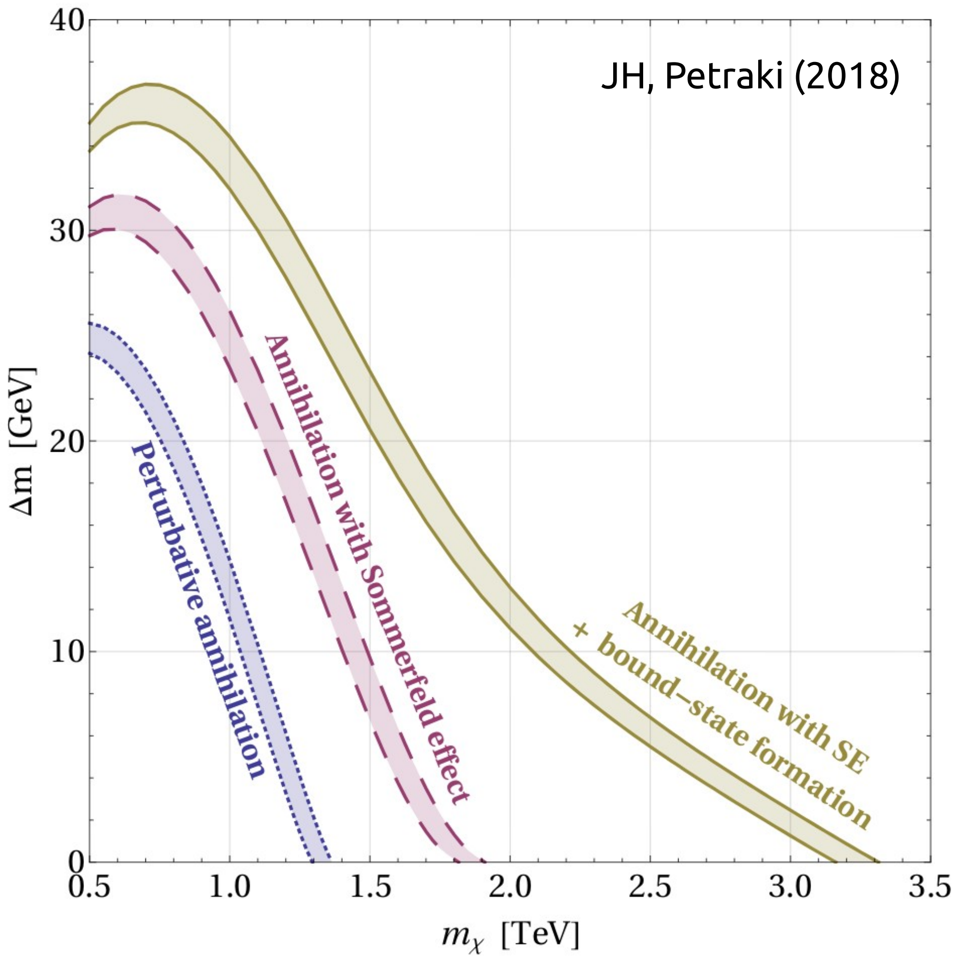
$$\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} \to g_{[\mathbf{8}]}\ g_{[\mathbf{8}]}$$

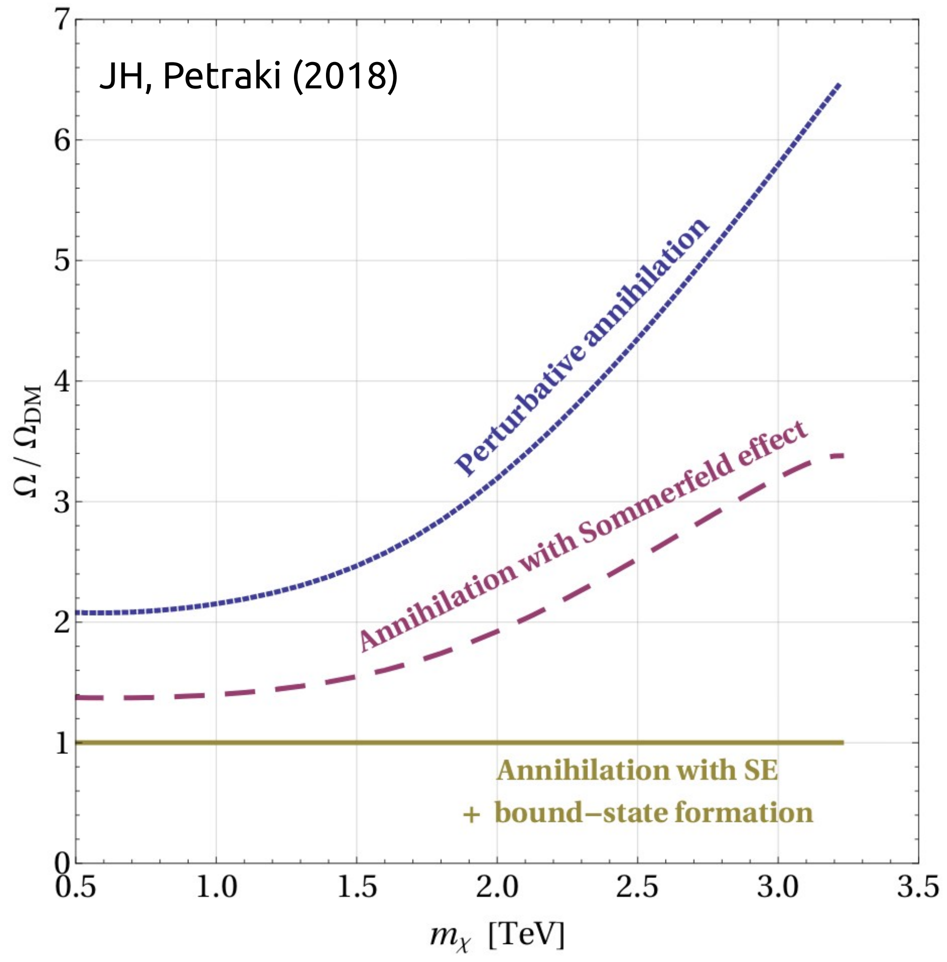
$$\mathit{\Gamma}_{\mathrm{dec}} = (\sigma_{\mathrm{ann},[\mathbf{1},\mathbf{8}]}^{s-\mathrm{wave}} v_{\mathrm{rel}})\ |\psi_{n\ell m}^{[\mathbf{1},\mathbf{8}]}(0)|^2$$

$$|\psi_{1,0,0}^{[\mathbf{1},\mathbf{8}]}(0)|^2 = \frac{\mu^3(\alpha_h + \alpha_{g\ ,[\mathbf{1},\mathbf{8}]}^B)^3}{\pi}$$

Impact on the relic density

gluon exchange only

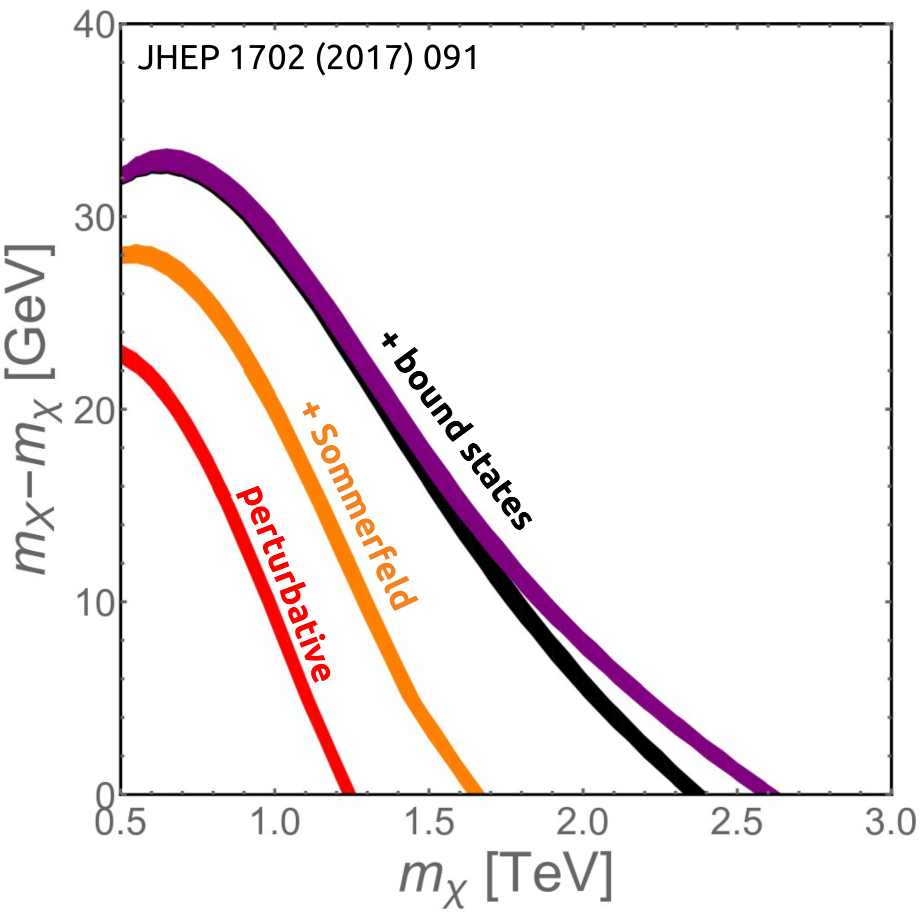


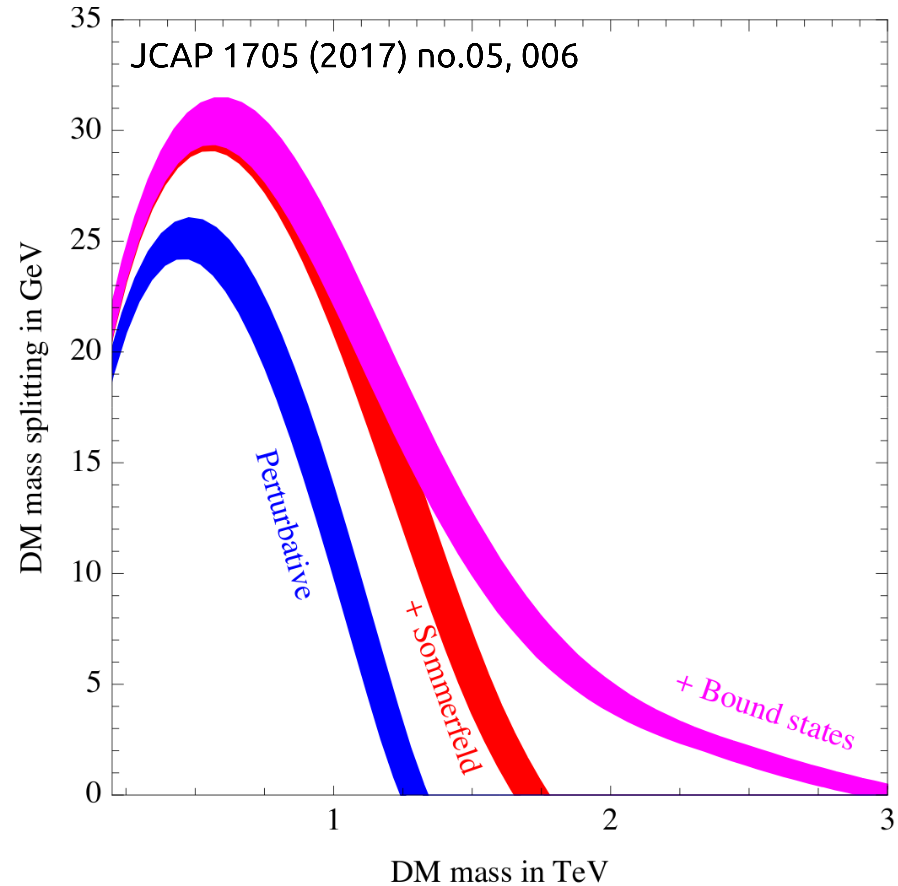


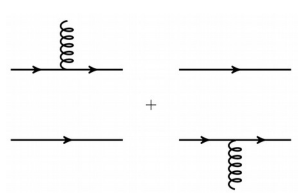
→ neglecting BSF and Sommerfeld enhancement would lead to a wrong relic density prediction by a factor 2 to 7

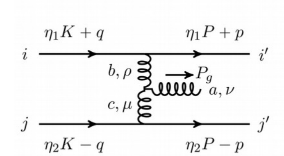
Comparison with previous results

gluon exchange only

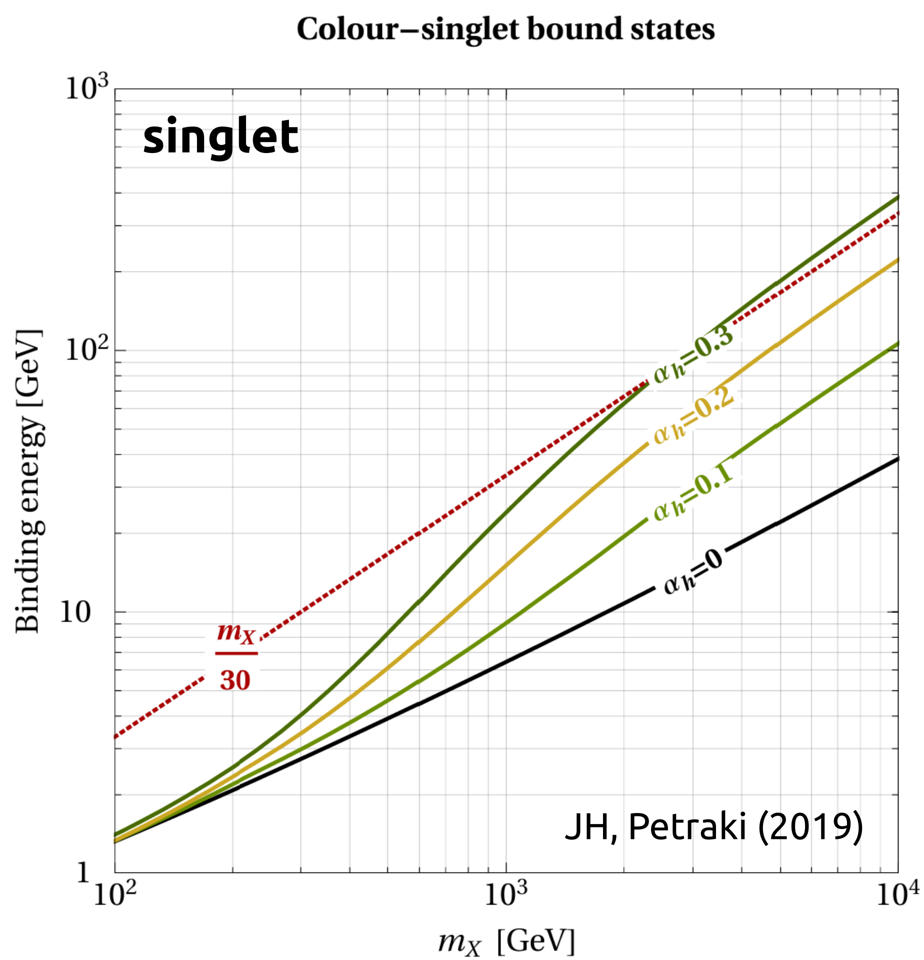


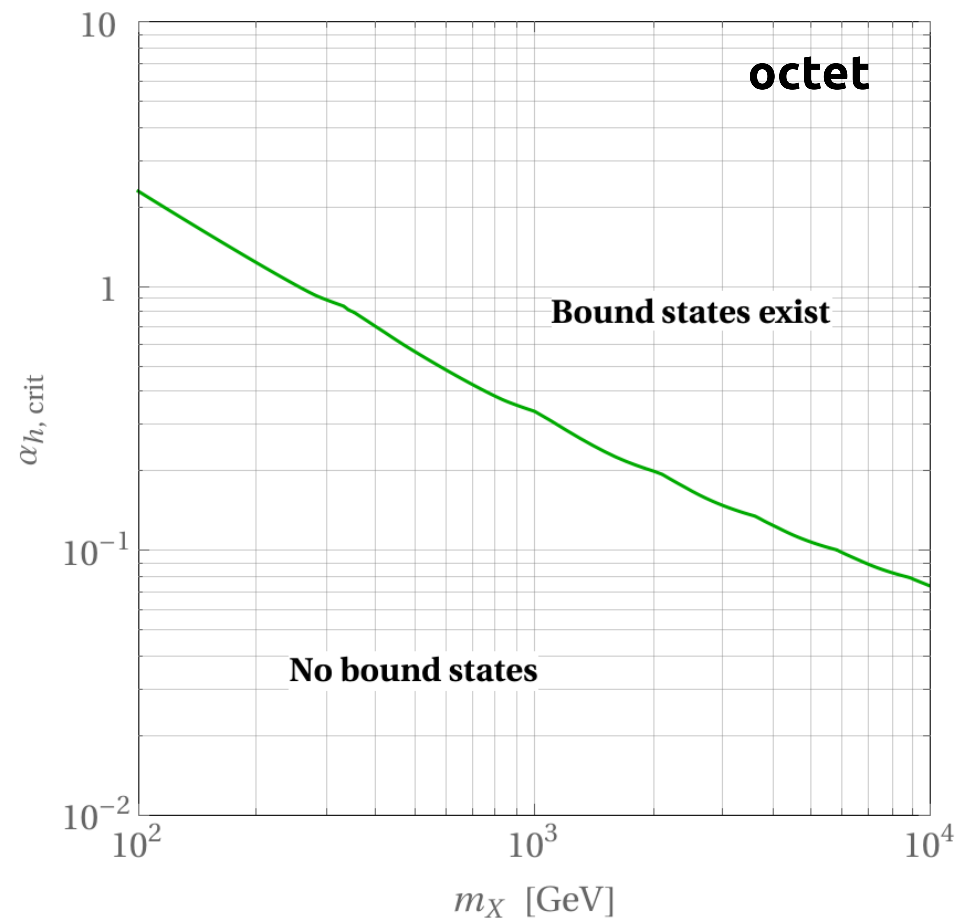






Impact of the Higgs on the formation of bound states

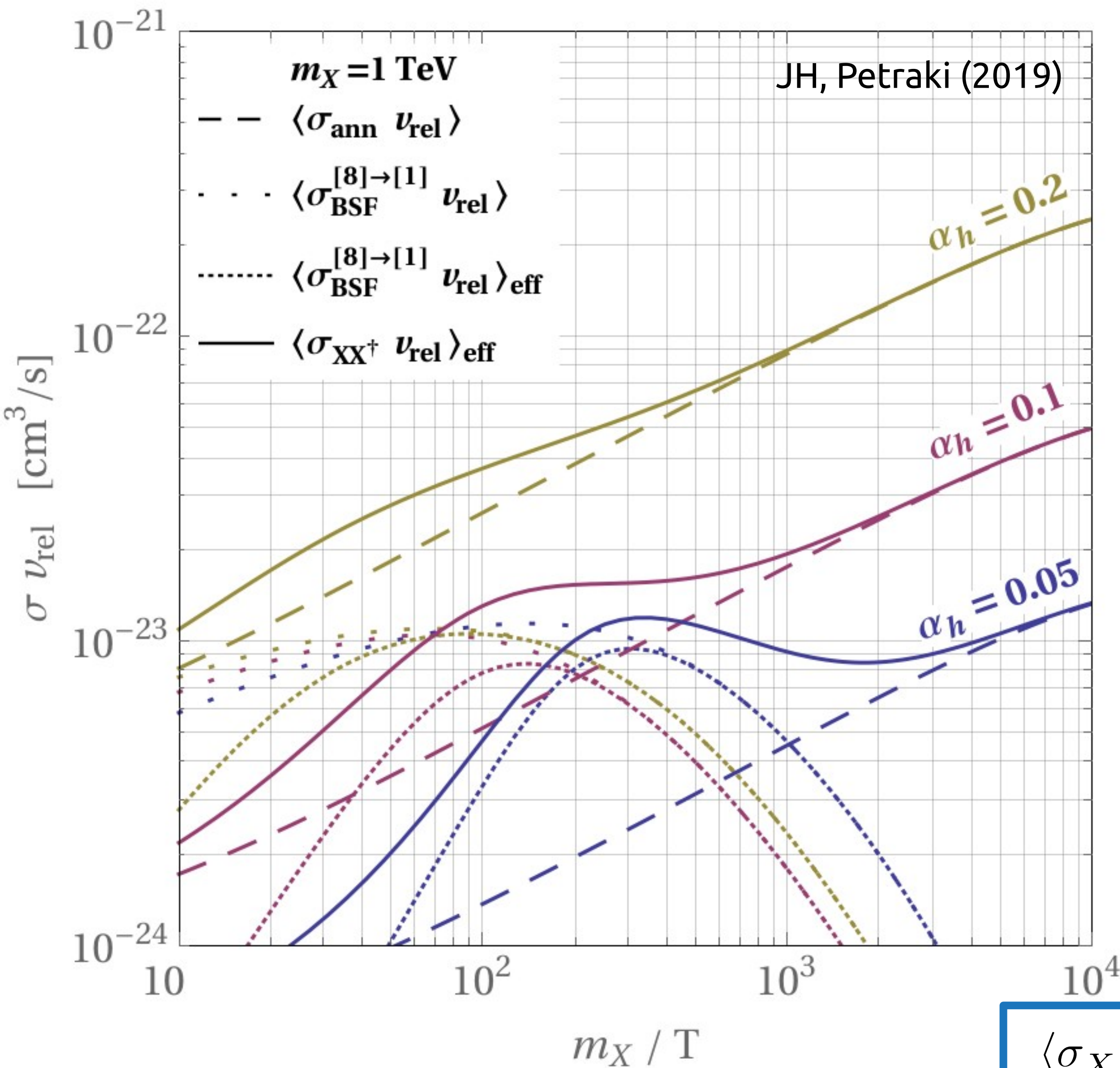




tighter bound states

additional bound states

Impact of the Higgs on the effective BSF cross section



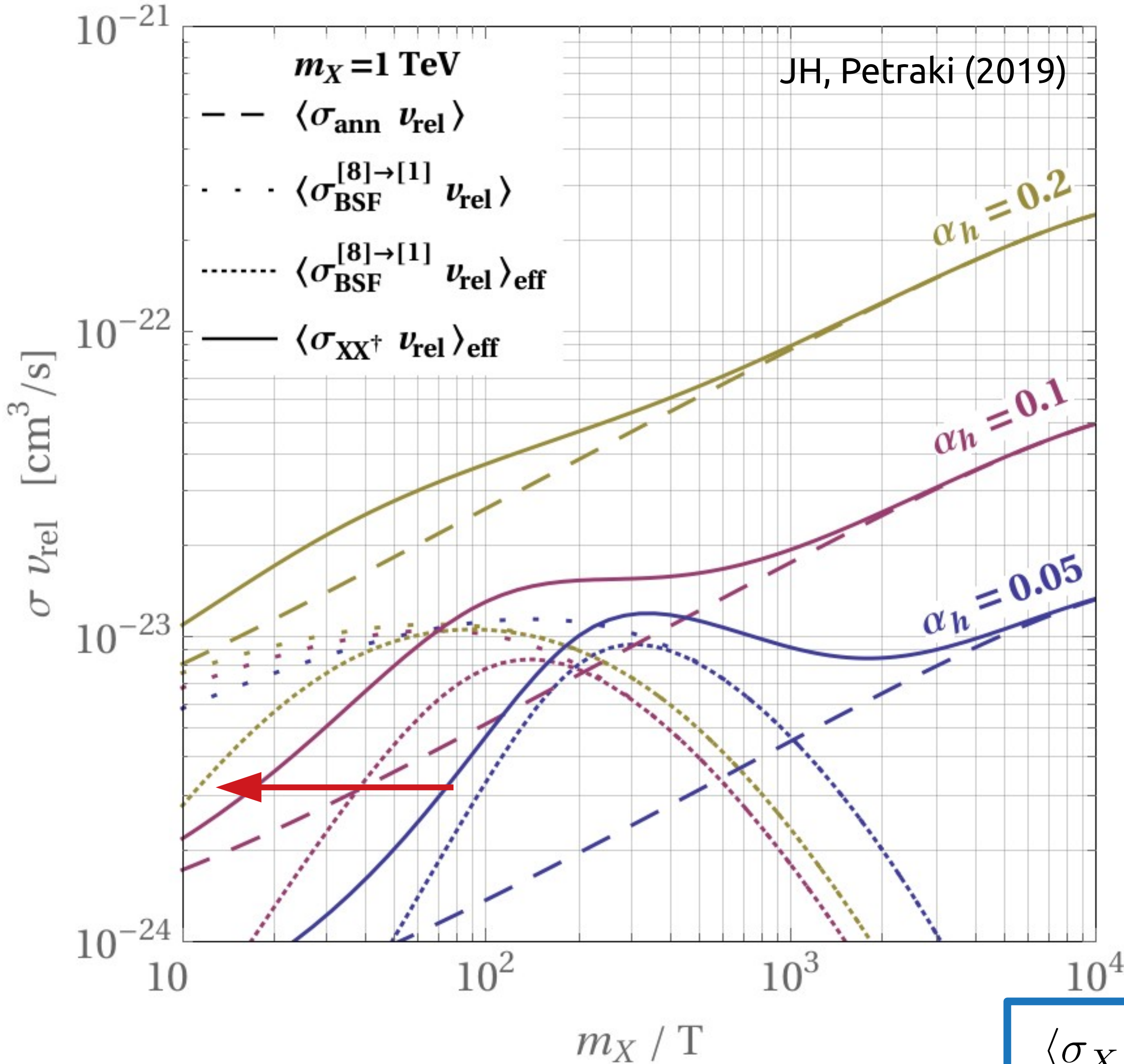
with Higgs exchange

- **Higgs coupling increases the binding energy**
- **a larger binding energy renders bound-state dissociation inefficient earlier, when the DM density is larger**
- **this enhances the efficiency to deplete DM**

$$\langle \sigma_{XX^\dagger} v_{\rm rel} \rangle = \langle \sigma_{\rm ann}\, v_{\rm rel} \rangle + \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff}$$

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} = \langle \sigma_{\rm BSF} v_{\rm rel} \rangle \times \left(\frac{\Gamma_{\rm dec}}{\Gamma_{\rm dec} + \Gamma_{\rm ion}} \right)$$

Impact of the Higgs on the effective BSF cross section



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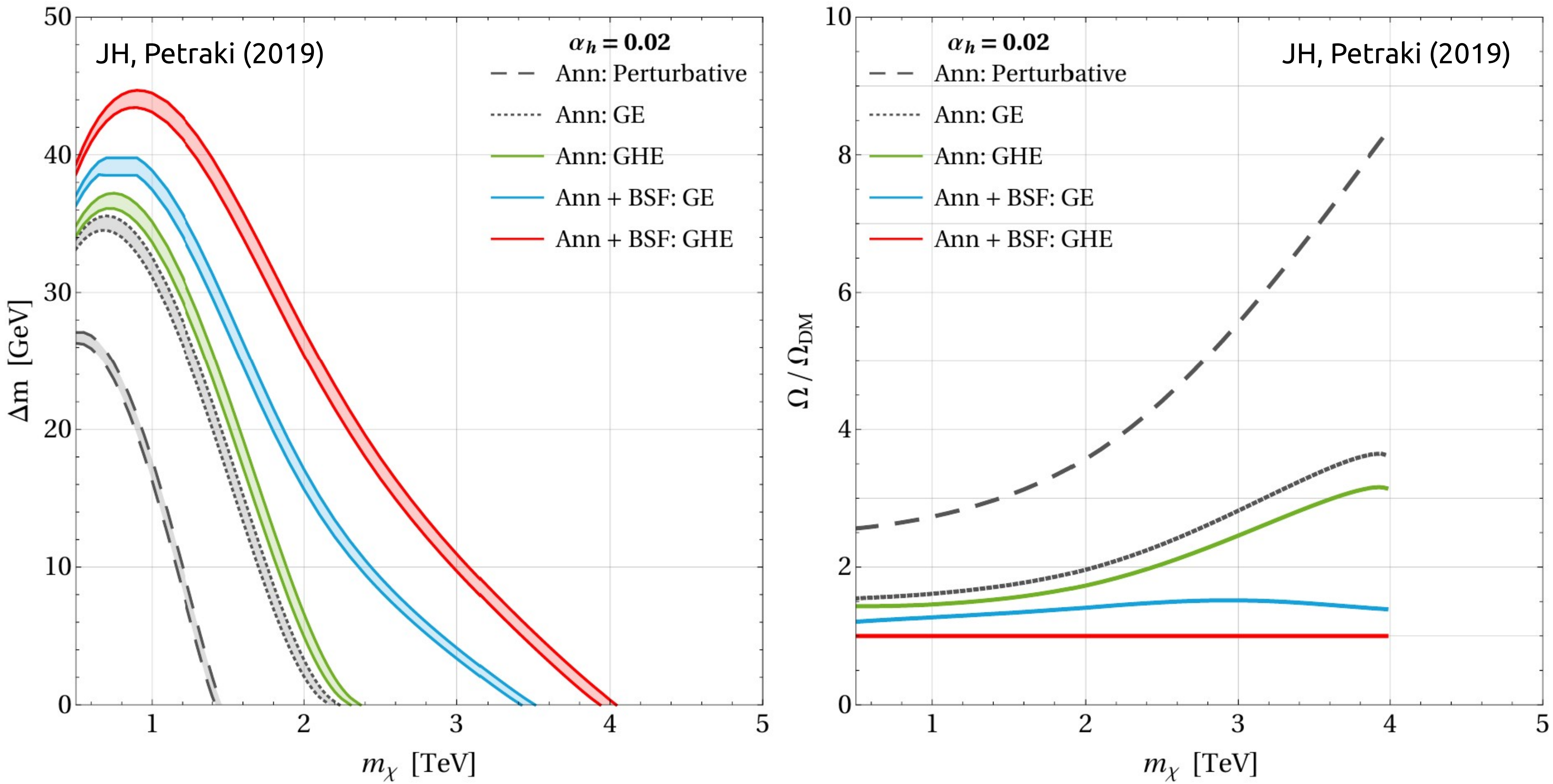
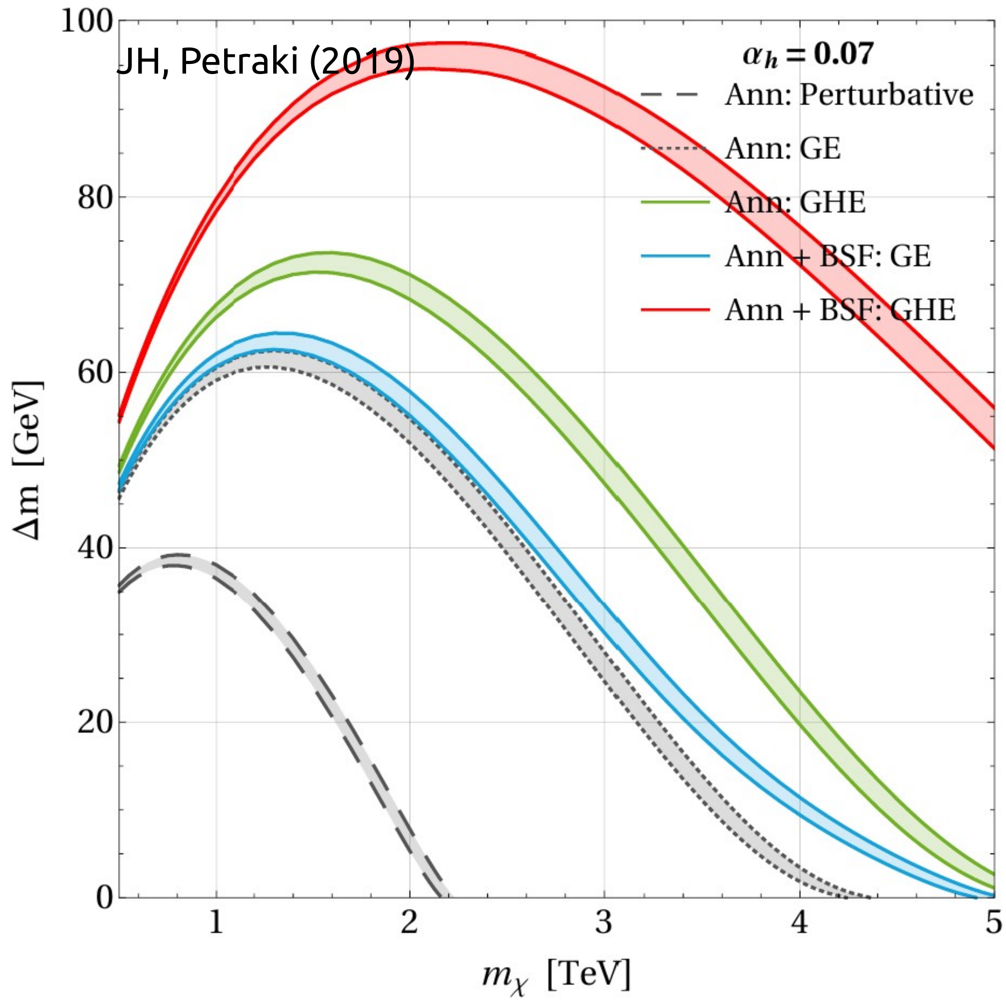
$$\langle \sigma_{XX^\dagger} v_{rel} \rangle = \langle \sigma_{ann} \, v_{rel} \rangle + \langle \sigma_{BSF} v_{rel} \rangle_{eff}$$

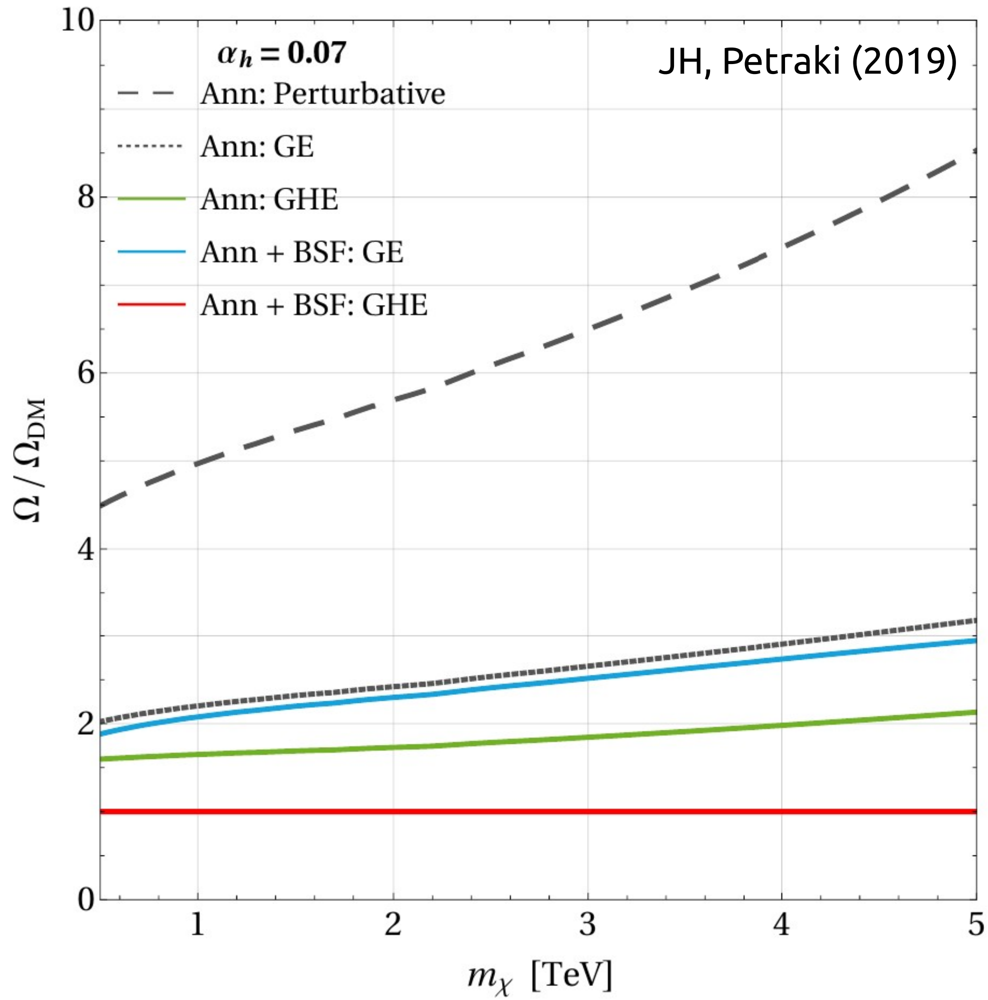
$$\langle \sigma_{BSF} v_{rel} \rangle_{eff} = \langle \sigma_{BSF} v_{rel} \rangle \times \left(\frac{\Gamma_{dec}}{\Gamma_{dec} + \Gamma_{ion}} \right)$$

Impact on the relic density (with Higgs exchange)

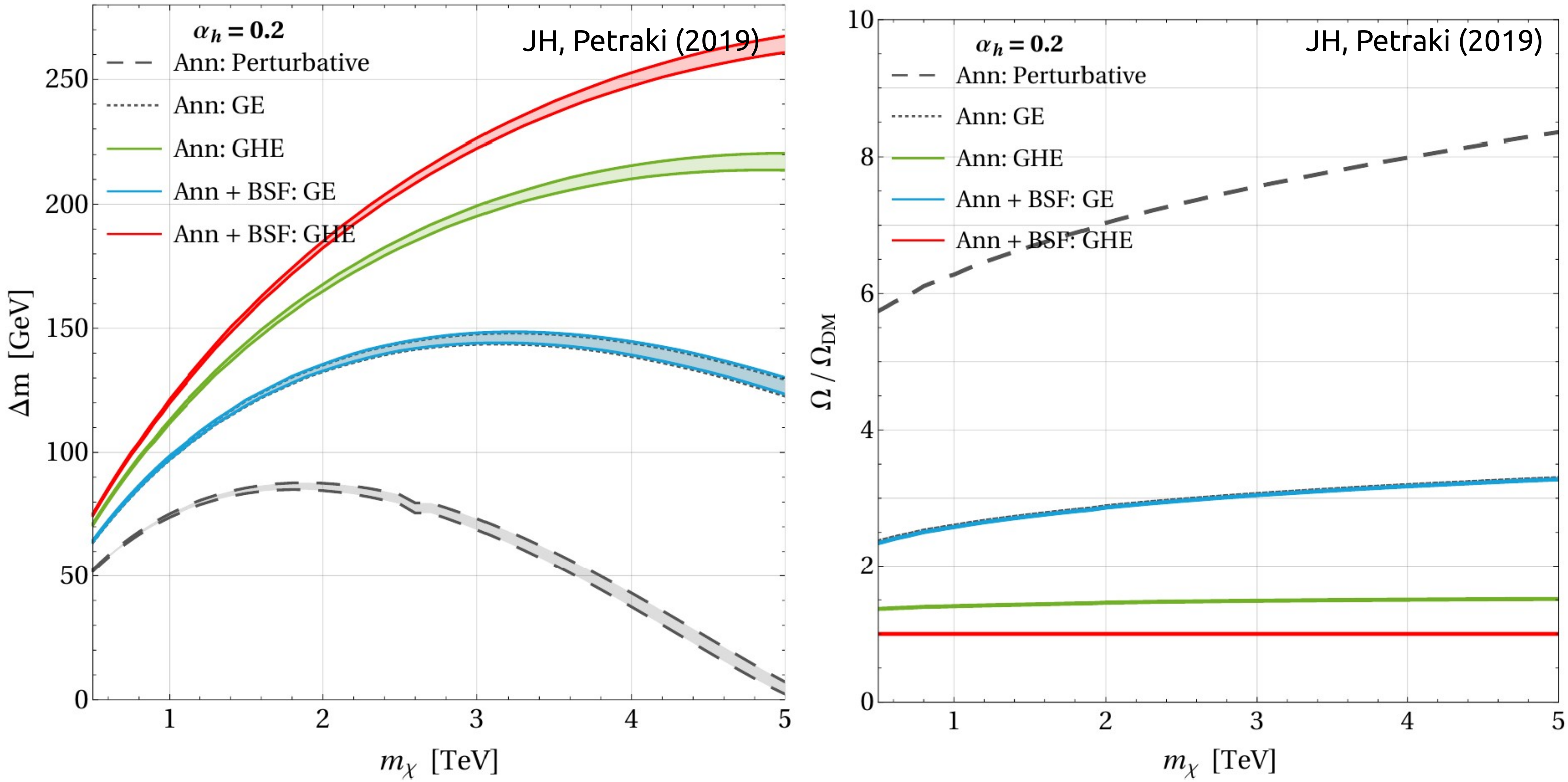


Impact on the relic density (with Higgs exchange)

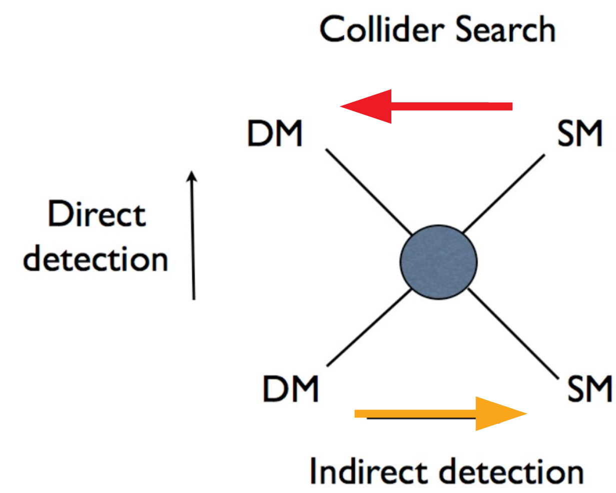


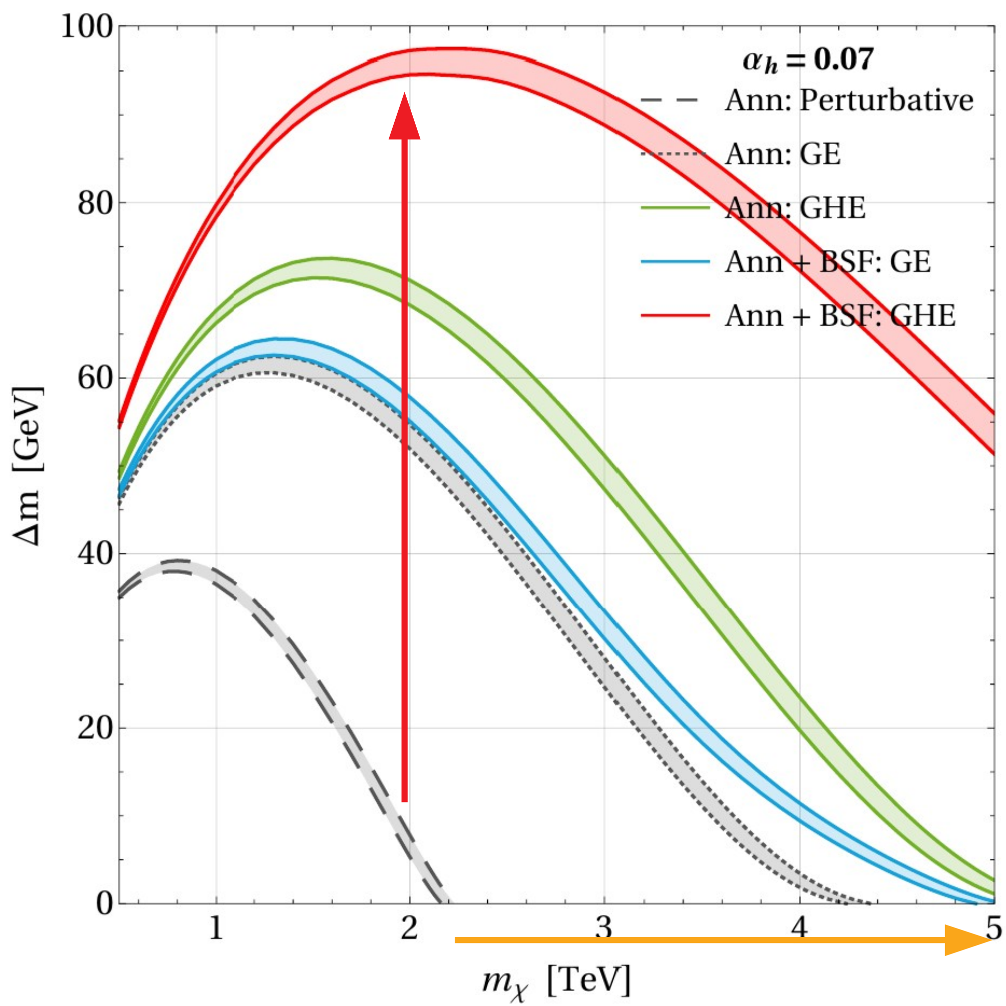


Impact on the relic density (with Higgs exchange)



Why relevant?





- More precise theoretical predictions
- Increased predicted mass splitting → **improved detection prospects with respect to multi-/mono-jet searches**
- DM can be heavier than anticipated → **interesting multi-TeV regime to be probed with indirect detection**

Conclusions

Software tools do not include most of these effects!

- *Sommerfeld enhancement* leads to huge corrective factors
- *Higgs boson* can similarly lead to sizable *Sommerfeld enhancement* in colored coannihilation scenarios
- *formation of bound states and their subsequent decay in unbroken non-abelian theories* can alter the relic abundance prediction more than previously estimated
- *Higgs boson* can similarly impact *bound state formation* and thus the relic density prediction
- important *implications on experimental studies* (collider, indirect detection)

Thank you for your attention!

Simplified model

- **Boltzmann equation:**

$$\tilde{Y} \equiv Y_\chi + Y_X + Y_{X^\dagger} = Y_\chi + 2Y_X$$

$$\frac{d\tilde{Y}}{dx} = -\sqrt{\frac{\pi}{45}}\frac{M_{\rm Pl} m_X\, g_{*,\rm eff}^{1/2}}{x^2}\langle \sigma_{\rm eff}\, v_{\rm rel}\rangle \left(\tilde{Y}^2 - \tilde{Y}_{\rm eq}^2\right)$$

$$Y_\chi^{\rm eq} = \frac{90}{(2\pi)^{7/2}}\frac{g_\chi}{g_{*,S}}\, x^{3/2}\, e^{-x}$$

$$Y_X^{\rm eq} = Y_{X^\dagger}^{\rm eq} = \frac{90}{(2\pi)^{7/2}}\frac{g_\chi}{g_{*,S}}\left[(1+\Delta)x\right]^{3/2} e^{-(1+\Delta)x}$$

- **Assumption:**

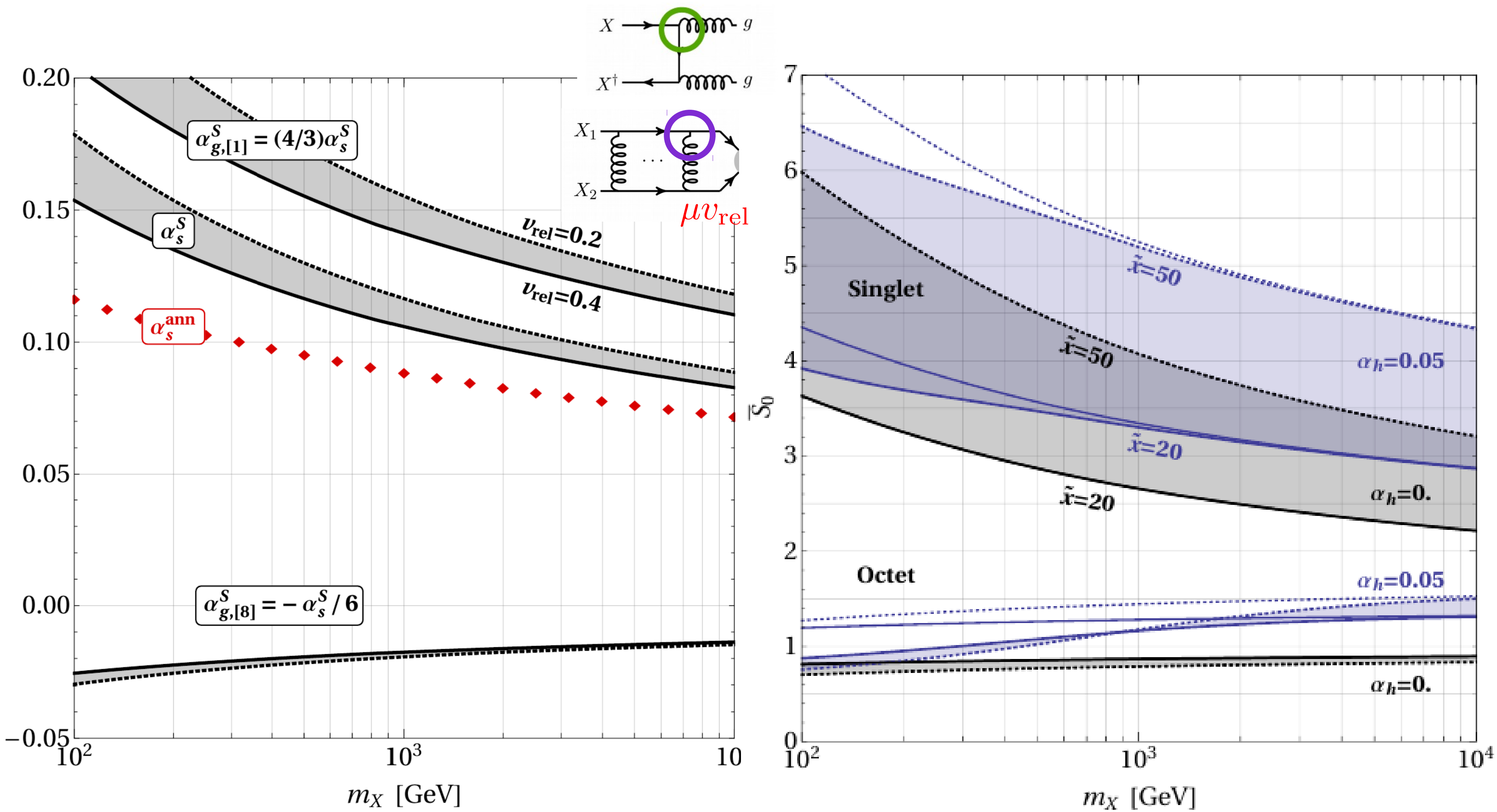
co-annihilating particle dominates DM abundance

$$\Gamma(X + {\rm SM} \leftrightarrow \chi + {\rm SM}) \gg H$$

$$\sigma(X + {\rm SM} \leftrightarrow \chi + {\rm SM}) \gg \frac{17\, x_{\rm dec}}{m_\chi M_{\rm Pl}} \sim 6\times 10^{-11}\ {\rm pb}\left(\frac{\rm TeV}{m_\chi}\right)$$

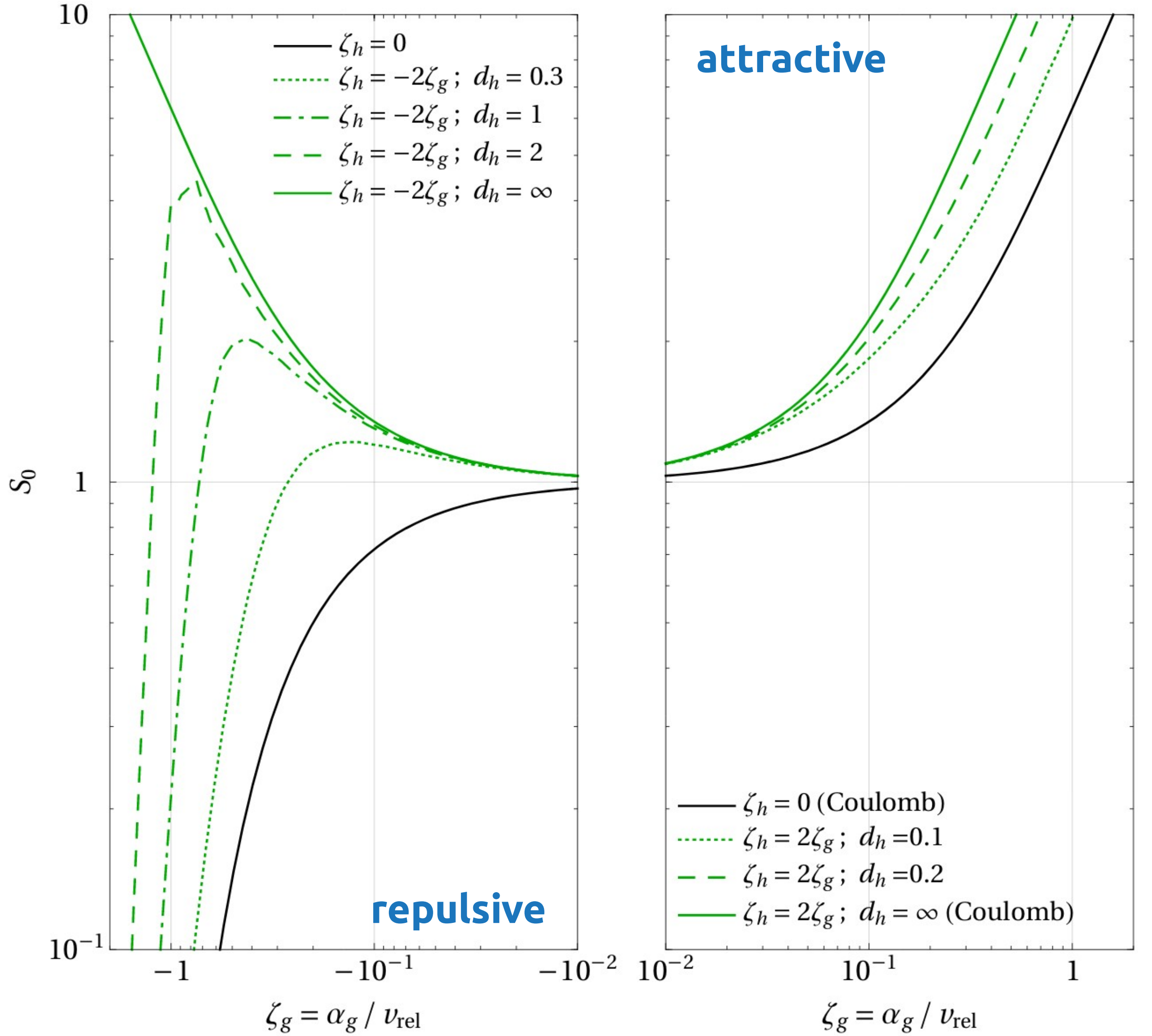
$$x_{\rm dec} = m_\chi / T_{\rm dec}$$

Running of the strong coupling – scattering states



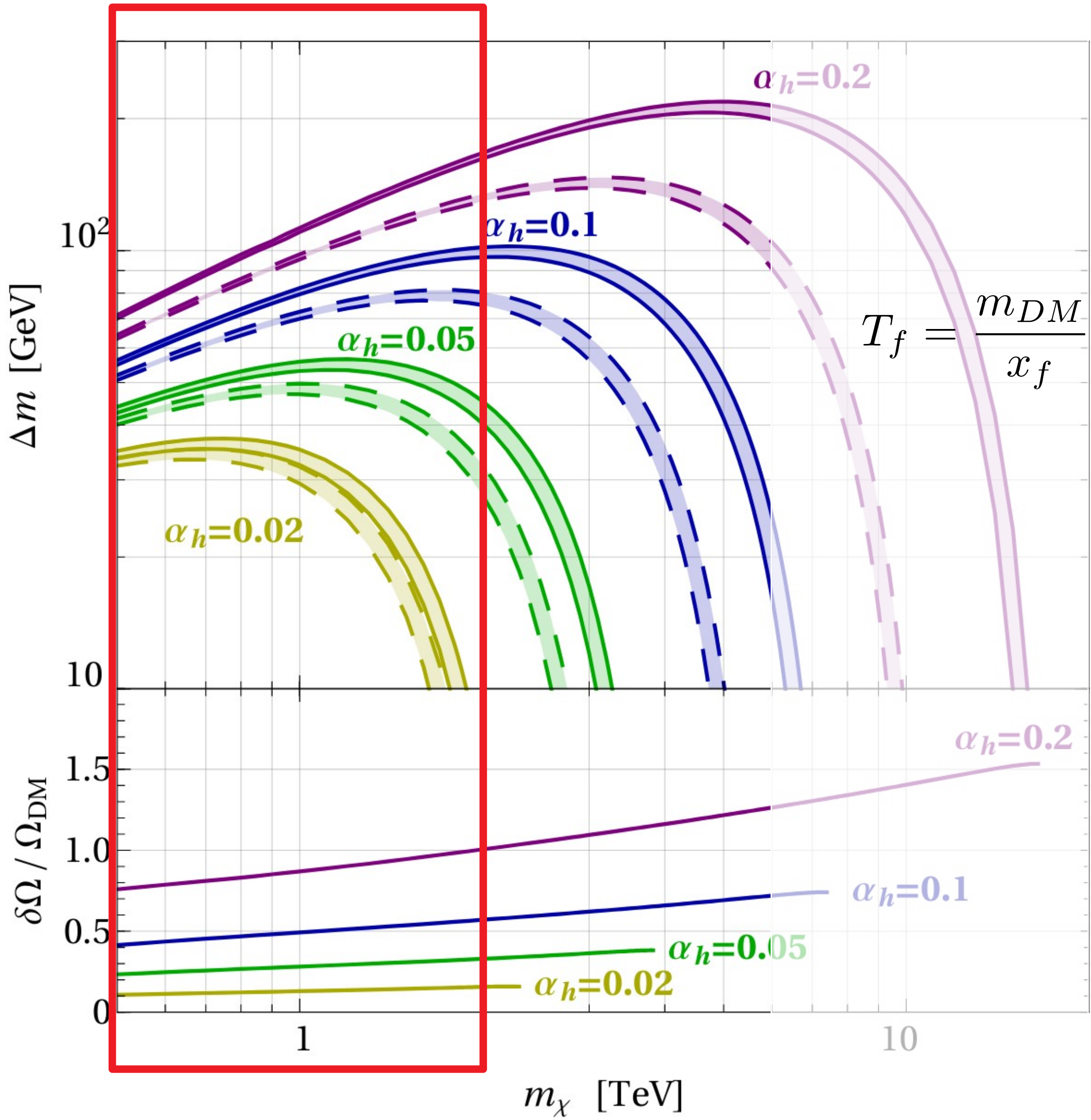
→ **for large enough m_X, α_h can significantly enhance the attraction or ameliorate the repulsion**

Impact of Higgs enhancement on the relic density



← small velocities **small velocities →**

Impact of Higgs enhancement on the relic density



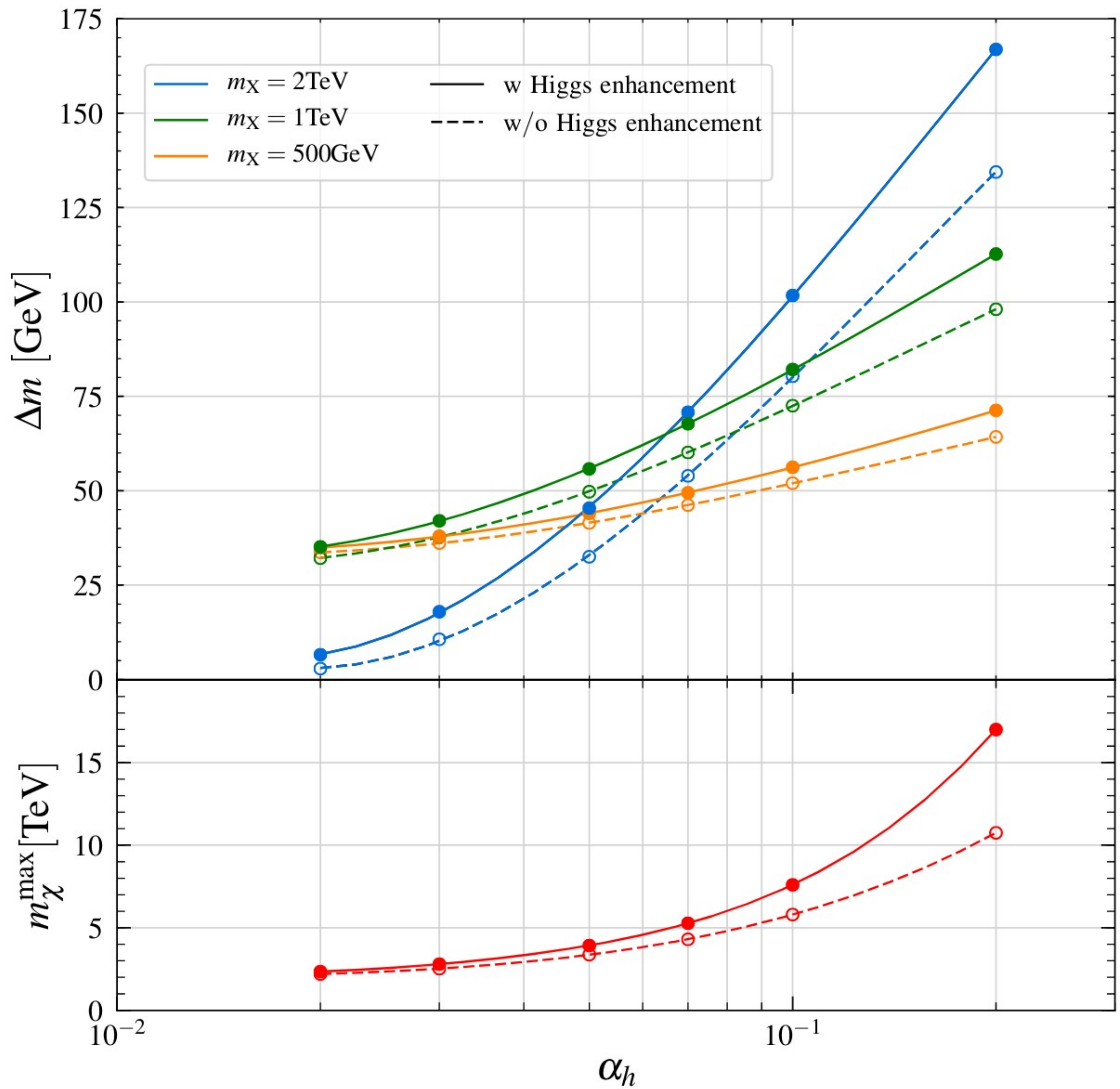
$$x_f = \frac{m_{DM}}{T_f} \approx 30$$

$$T_f = \frac{m_{DM}}{x_f} \lesssim T_{EWSB} \approx 170\text{GeV}$$

$$\rightarrow m_{DM} \lesssim 5\text{TeV}$$

JH, Petraki, (2018)

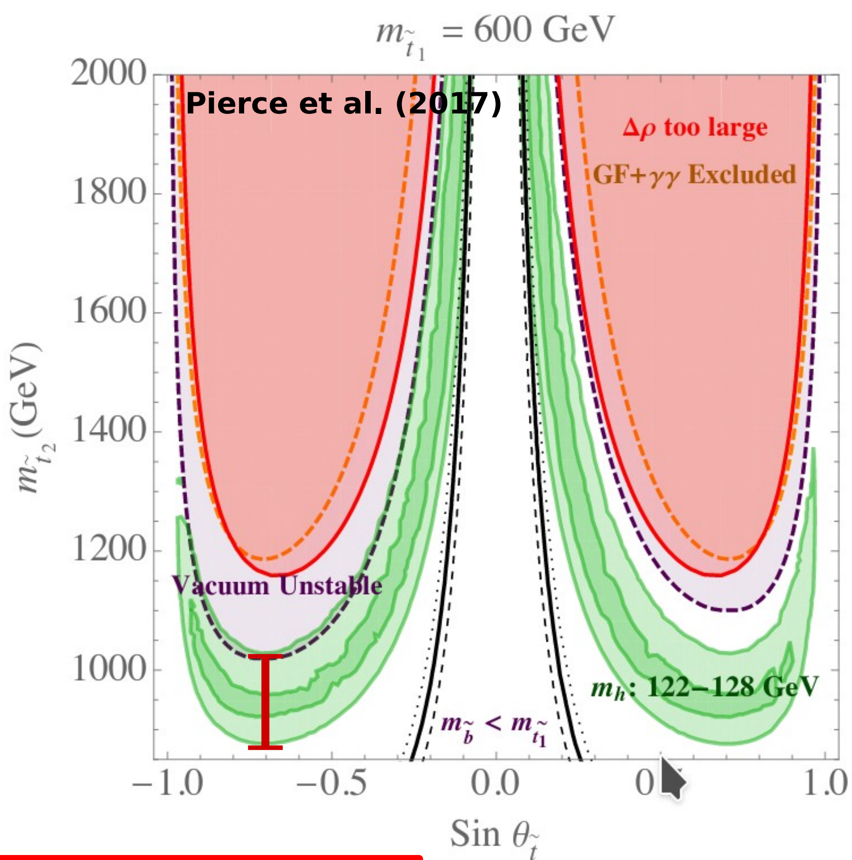
Impact of Higgs enhancement on the relic density

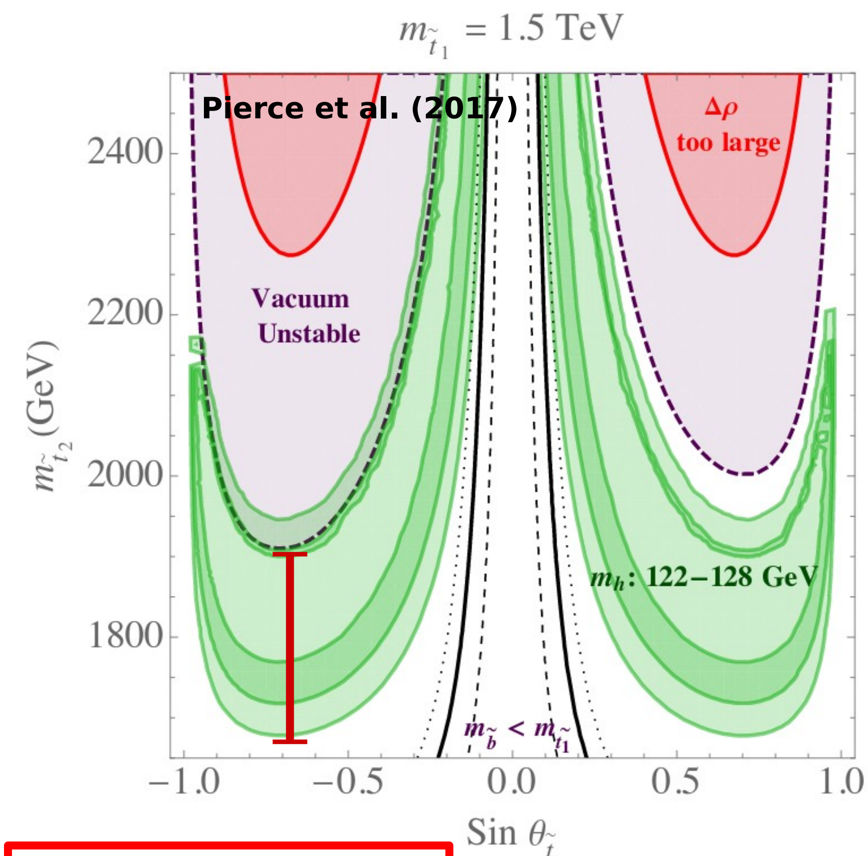


JH, Petraki, (2018)

Vacuum stability?

Example: realization within the MSSM:





$$\boxed{\alpha_h \approx (0.02 - 0.07)}$$

$$\boxed{\alpha_h \approx (0.01 - 0.05)}$$

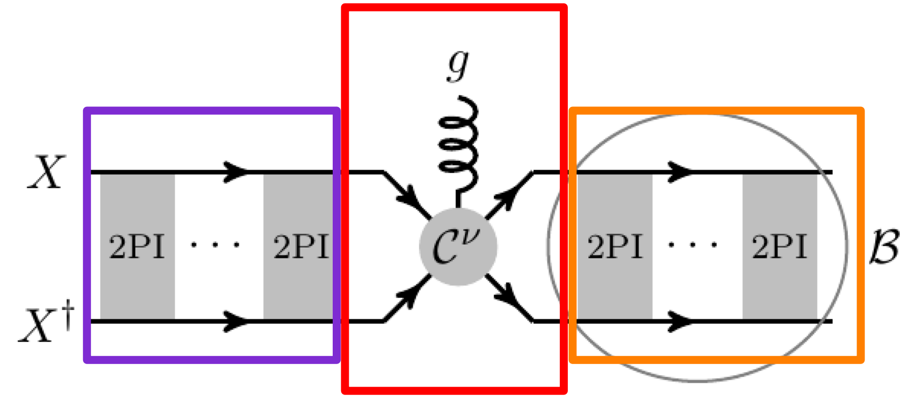
We checked explicitly one MSSM scenario as example with *vevacious*

$$\alpha_h \approx 0.15$$

$$m_{\tilde{\chi}_1^0} = 982.5 \text{ GeV}$$
$$m_{\tilde{t}_1} = 1066.1 \text{ GeV}$$

Bound state formation

$$\left[-\frac{\nabla^2}{2\mu} + V_{\rm scatt}(\mathbf{r})\right] \phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\, \phi_{\mathbf{k}}(\mathbf{r})$$



scattering state

radiative transition

bound state

Kinetic energy

$$\mathcal{E}_{\mathbf{k}} \equiv \frac{\mathbf{k}^2}{2\mu} = \frac{\mu v_{\rm rel}^2}{2} > 0$$

Scattering potential

$$\mathrm{V}_{\rm scatt}(r) = -\frac{\alpha_g^S}{r} - \frac{\alpha_h}{r}\, e^{-m_h r}$$

with $\alpha^S_{g,[\mathbf{1}]} = \frac{4\alpha_s^S}{3}$ and $\alpha^S_{g,[\mathbf{8}]} = -\frac{\alpha_s^S}{6}$

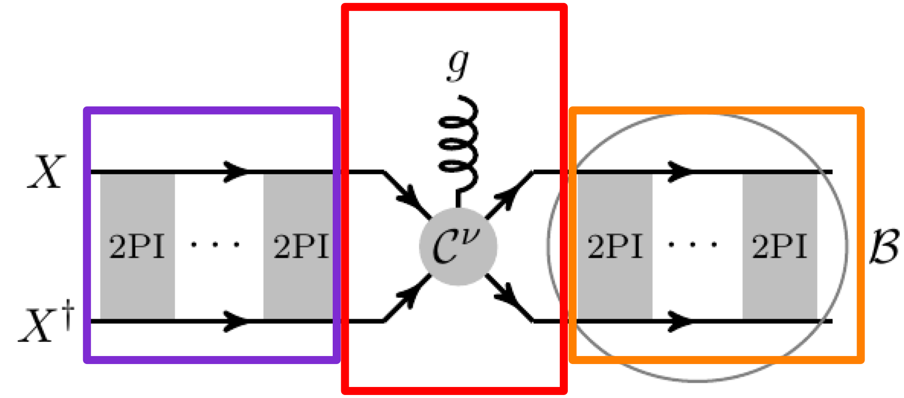
at scale $Q = \frac{m_X v_{\rm rel}}{2}$

scattering state

Bound state formation

$$\left[-\frac{\nabla^2}{2\mu}+V_{\rm scatt}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r})=\mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r})$$

$$\left[-\frac{\nabla^2}{2\mu}+V_{\rm bound}(\mathbf{r})\right]\psi_{n\ell m}(\mathbf{r})=\mathcal{E}_{n\ell}\,\psi_{n\ell m}(\mathbf{r})$$



scattering state

radiative transition

bound state

Binding energy

$$\mathcal{E}_{n\ell}\equiv-\gamma_{n\ell}^2\times\frac{\kappa^2}{2\mu}=-\frac{1}{2}\mu\,(\alpha_g^B+\alpha_h)^2\,\gamma_{n\ell}^2<0$$

$$V_{\rm bound}(r)=-\frac{\alpha_g^B}{r}-\frac{\alpha_h}{r}\,e^{-m_h r}$$

with Bohr momentum:

$$\kappa\equiv\mu\alpha$$

with $\alpha^B_{g,[\mathbf{1}]}=\frac{4\alpha_s^B}{3}$ and $\alpha^B_{g,[\mathbf{8}]}=-\frac{\alpha_s^B}{6}$

Coulomb limit: $\gamma^C=\frac{1}{n}$

at scale

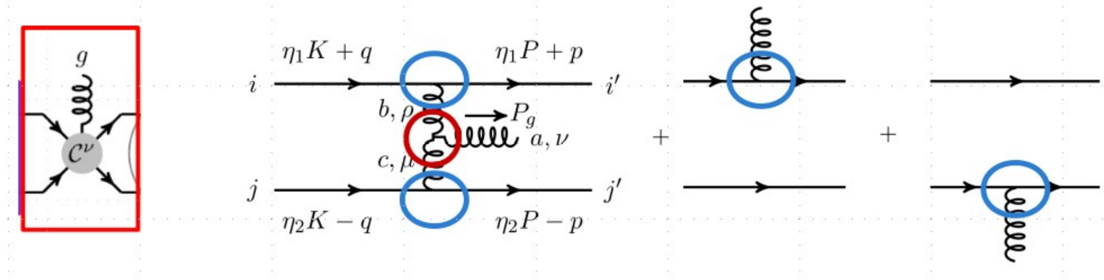
$$Q=\mu\alpha\gamma_{nl}$$

$$=\frac{m_X}{2}\left(\alpha_h+\alpha^B_{g,\{[\mathbf{1}],[\mathbf{8}]\}}\right)\times\gamma_{n\ell}\left(\frac{\alpha_{g,\{[\mathbf{1}],[\mathbf{8}]\}}}{\alpha_h},\ d_h\right)$$

bound state

Bound state formation

transition amplitude



$$\boxed{[\mathcal{M}^{\nu}_{\mathbf{k}\to\{n\ell m\}}]^a_{ii',jj'} = \frac{1}{\sqrt{2\mu}} \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3}\; \tilde{\psi}^*_{n\ell m}(\mathbf{p})\; \tilde{\phi}_{\mathbf{k}}(\mathbf{q})\; [\mathcal{M}^{\nu}_{\text{trans}}(\mathbf{q},\mathbf{p})]^a_{ii',jj'}}$$

$$[\boldsymbol{\mathcal{M}}_{\text{trans}}]^a_{ii',jj'} \simeq -g_S^{\text{BSF}}\,4M \Big\{ -if^{abc}(T_1^b)_{i'i}(T_2^c)_{j'j} \times 8\pi\mu\alpha_S^{NA} \frac{\mathbf{q}-\mathbf{p}}{(\mathbf{q}-\mathbf{p})^4}$$
$$+\,\eta_2\,(T_1^a)_{i'i}\,\delta_{j'j} \times \mathbf{p}\;(2\pi)^3\,\delta^3(\mathbf{q}-\mathbf{p}-\eta_2\mathbf{P}_g) - \eta_1\,\delta_{i'i}\,(T_2^a)_{j'j} \times \mathbf{p}\;(2\pi)^3\,\delta^3(\mathbf{q}-\mathbf{p}+\eta_1\mathbf{P}_g)\Big\}$$

with: $\mathbf{R}_1 = \mathbf{R}$ and $\mathbf{R}_2 = \bar{\mathbf{R}}$ $\qquad T_1^a = T^a \qquad T_2^a = -T^{a\,*}$

$$[\boldsymbol{\mathcal{M}}_{\mathbf{k}\to\{n\ell m\}}]^a_{ii',jj'} = -(2^5\pi\alpha_s^{\text{BSF}} M^2/\mu)^{1/2}\left[\left(\eta_2\,T^a_{i'i}\,\delta_{j'j} + \eta_1\delta_{i'i}\,T^a_{jj'}\right)\,\boldsymbol{\mathcal{J}}_{\mathbf{k},\{n\ell m\}} + if^{abc}T^b_{i'i}T^c_{jj'}\,\boldsymbol{\mathcal{Y}}_{\mathbf{k},\{n\ell m\}}\right]$$

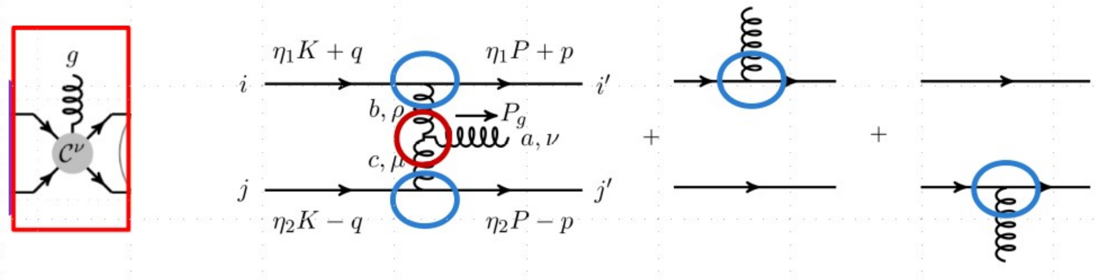
with the corresponding overlap integrals:

$$\boldsymbol{\mathcal{J}}_{\mathbf{k},\{n\ell m\}}(\mathbf{b}) \equiv \int \frac{d^3p}{(2\pi)^3}\;\mathbf{p}\,\tilde{\psi}^*_{n\ell m}(\mathbf{p})\,\tilde{\phi}_{\mathbf{k}}(\mathbf{p}+\mathbf{b})$$

$$\boldsymbol{\mathcal{Y}}_{\mathbf{k},\{n\ell m\}} \equiv 8\pi\mu\alpha_s^{NA} \int \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\;\frac{\mathbf{q}-\mathbf{p}}{(\mathbf{q}-\mathbf{p})^4}\;\tilde{\psi}^*_{n\ell m}(\mathbf{p})\,\tilde{\phi}_{\mathbf{k}}(\mathbf{q})$$

Bound state formation

transition amplitude



$$\boxed{\frac{1}{d_{\mathbf{R}}^2} \left| \mathcal{M}_{\mathbf{k}\to\{n\ell m\}}^{[\mathbf{adj}]\to[\mathbf{1}]} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{C_2(\mathbf{R})}{d_{\mathbf{R}}^2} \left| \mathcal{J}_{\mathbf{k},\{n\ell m\}}^{[\mathbf{adj},\mathbf{1}]} + \frac{C_2(\mathbf{G})}{2} \mathcal{Y}_{\mathbf{k},\{n\ell m\}}^{[\mathbf{adj},\mathbf{1}]} \right|^2}$$

with: $\dfrac{\mathcal{Y}_{\mathbf{k},\{100\}}}{\mathcal{J}_{\mathbf{k},\{100\}}(\mathbf{0})} = \dfrac{\alpha_s^B}{\alpha_h + \alpha_g^B}$

$$\frac{1}{d_{\mathbf{R}}^2} \left| \mathcal{M}_{\mathbf{k}\to 100}^{[\mathbf{adj}]\to[\mathbf{1}]} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{C_2(\mathbf{R})}{d_{\mathbf{R}}^2} \left[1 + \frac{C_2(\mathbf{G})}{2} \left(\frac{\alpha_s^B}{\alpha_h + \alpha_g^B} \right) \right]^2 \left| \mathcal{J}_{\mathbf{k},100}^{[\mathbf{adj},\mathbf{1}]} \right|^2$$

with: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

$$\frac{1}{9} \left| \mathcal{M}_{\mathbf{k}\to 100}^{[\mathbf{8}]\to[\mathbf{1}]} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{4}{27} \left[1 + \frac{3}{2} \left(\frac{\alpha_s^B}{\alpha_h + \alpha_g^B} \right) \right]^2 \left| \mathcal{J}_{\mathbf{k},100}^{[\mathbf{8},\mathbf{1}]} \right|^2$$

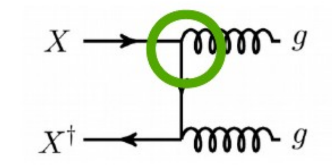
$\alpha_h \to 0$

$$\frac{1}{9} \left| \mathcal{M}_{\mathbf{k}\to 100}^{[\mathbf{8}]\to[\mathbf{1}]} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{4}{27} \left[1 + \frac{9}{8} \right]^2 \left| \mathcal{J}_{\mathbf{k},100}^{[\mathbf{8},\mathbf{1}]} \right|^2$$

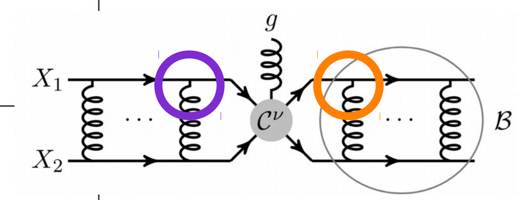
will have significant effect!

Running of the strong coupling

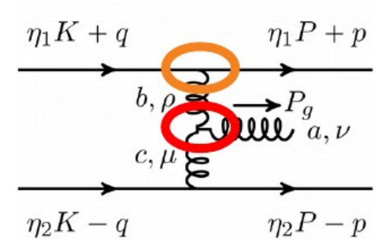
Vertices	α_s	α_g	Average momentum transfer Q
Annihilation: gluon emission	α_s^{ann}		m_X
Scattering-state wavefunction (ladder diagrams)	α_s^S	Colour-singlet $\alpha_{g,[\mathbf{1}]}^S = \frac{4\alpha_s^S}{3}$	$\frac{m_X\, v_{\text{rel}}}{2}$
		Colour-octet $\alpha_{g,[\mathbf{8}]}^S = -\frac{\alpha_s^S}{6}$	
Colour-singlet bound-state wavefunction (ladder diagrams)	$\alpha_{s,[\mathbf{1}]}^B$	$\alpha_{g,[\mathbf{1}]}^B = \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}$	$\kappa_{[\mathbf{1}]}\ \gamma_{n\ell}\left(\lambda_{[\mathbf{1}]}, d_h\right) = \frac{m_X}{2}\left(\alpha_h + \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}\right) \times \gamma_{n\ell}\left(\frac{4\alpha_{s,[\mathbf{1}]}^B}{3\alpha_h},\ d_h\right)$
Colour-octet bound state wavefunction (ladder diagrams)	$\alpha_{s,[\mathbf{8}]}^B$	$\alpha_{g,[\mathbf{8}]}^B = -\frac{\alpha_{s,[\mathbf{8}]}^B}{6}$	$\kappa_{[\mathbf{8}]}\ \gamma_{n\ell}\left(\lambda_{[\mathbf{8}]}, d_h\right) = \frac{m_X}{2}\left(\alpha_h - \frac{\alpha_{s,[\mathbf{8}]}^B}{6}\right) \times \gamma_{n\ell}\left(-\frac{\alpha_{s,[\mathbf{8}]}^B}{6\alpha_h},\ d_h\right)$
Formation of colour-singlet bound states: gluon emission	$\alpha_{s,[\mathbf{1}]}^{\text{BSF}}$		$\frac{m_X}{4}\left[v_{\text{rel}}^2 + \left(\alpha_h + \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}\right)^2 \times \gamma_{n\ell}^2\left(\frac{4\alpha_{s,[\mathbf{1}]}^B}{3\alpha_h},\ d_h\right)\right]$
Formation of colour-octet bound states: gluon emission	$\alpha_{s,[\mathbf{8}]}^{\text{BSF}}$		$\frac{m_X}{4}\left[v_{\text{rel}}^2 + \left(\alpha_h - \frac{\alpha_{s,[\mathbf{8}]}^B}{6}\right)^2 \times \gamma_{n\ell}^2\left(-\frac{\alpha_{s,[\mathbf{8}]}^B}{6\alpha_h},\ d_h\right)\right]$



μv_{rel}

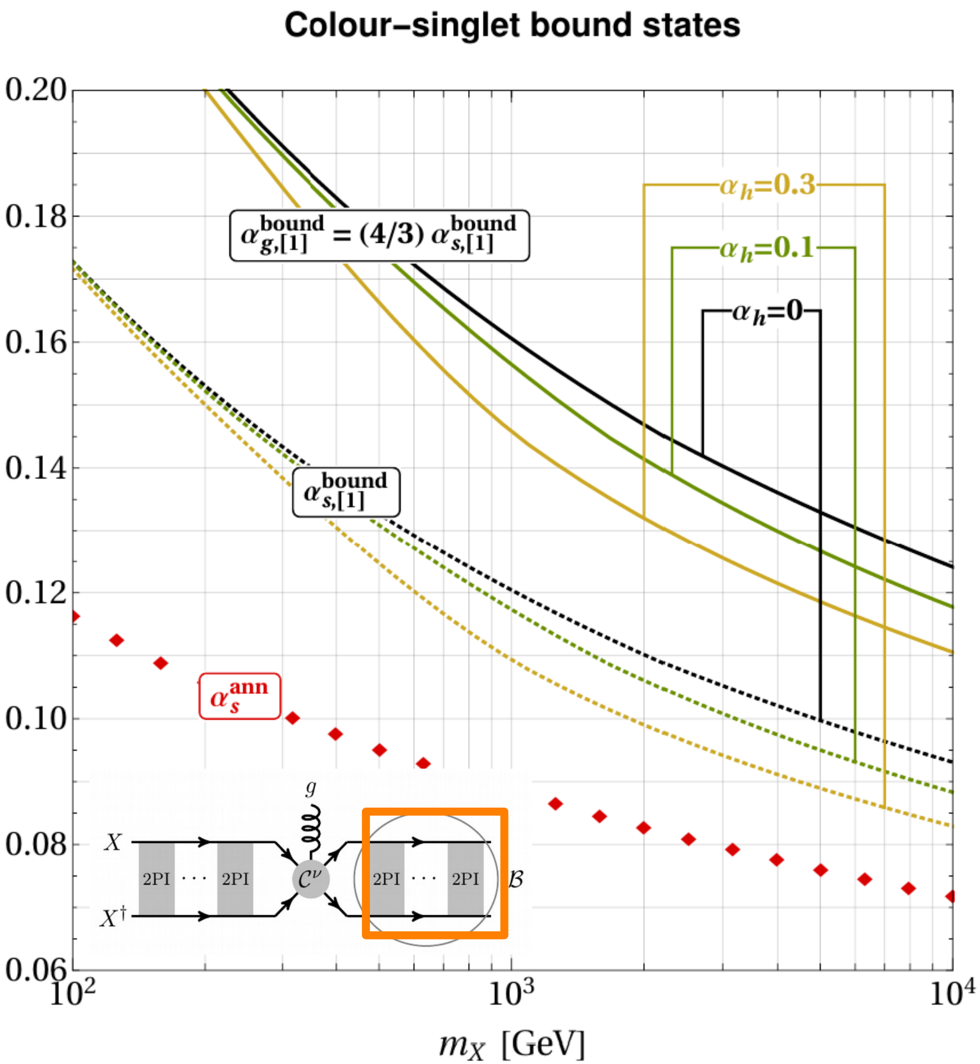


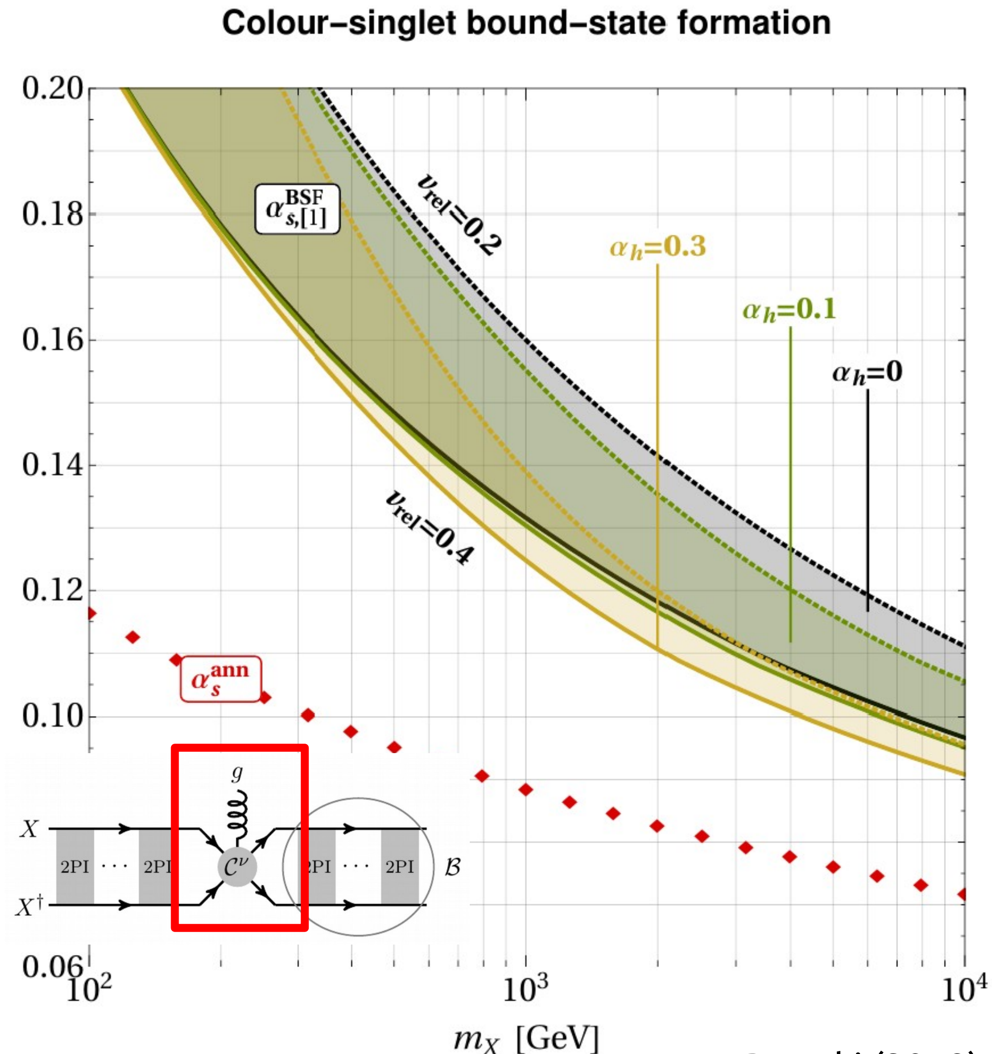
$\mu \alpha_g^B$



$| \mathbf{P_g} | = \mathcal{E}_\mathbf{k} - \mathcal{E}_{\mathbf{n}\ell}$

Running of the strong coupling – bound state





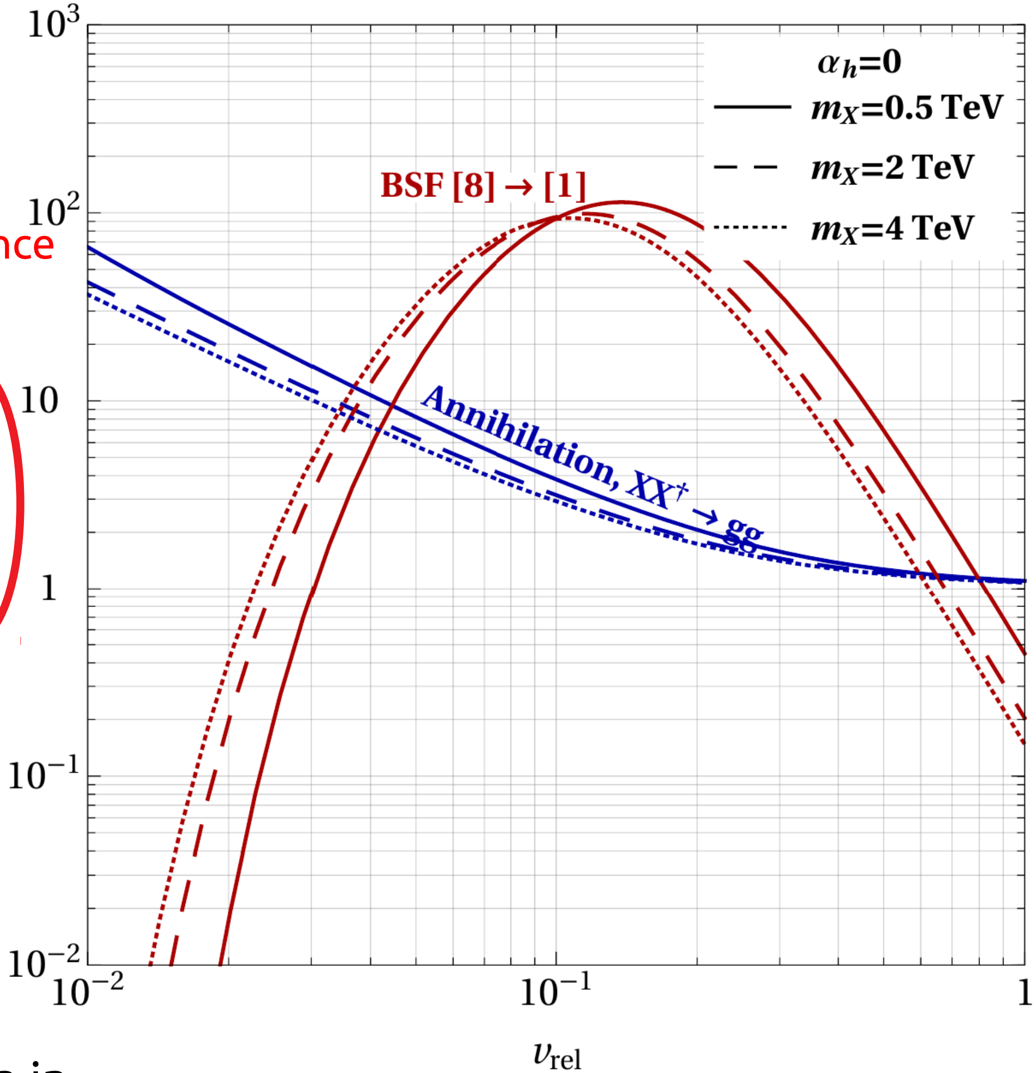
JH, Petraki (2019)
JH, Petraki (2018)

→ Higgs interaction decreases α_g considerably

Annihilation vs. BSF cross section

gluon exchange only

direct mass dependence cancels
→ only indirect scale dependence



$\alpha_s/v_{\rm rel} << 1$

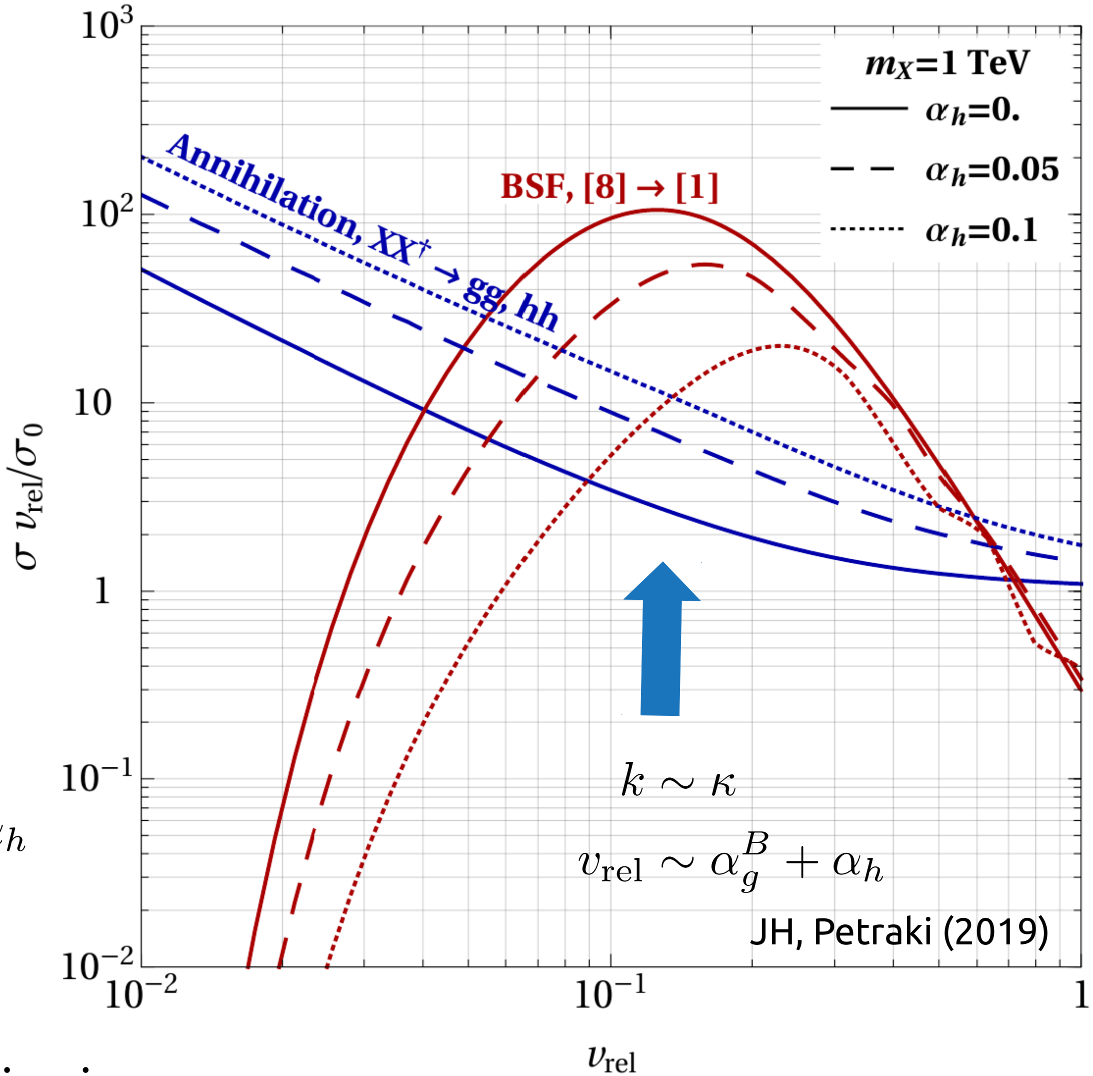
$\sigma_{\rm BSF}^{[\mathbf{8}]\to[\mathbf{1}]} \propto (\alpha_s/v_{\rm rel})^4$

$\alpha_s/v_{\rm rel} >> 1$

Coulomb repulsion in the scattering state

→ scale dependence has an strong impact on BSF and annihilation cross section!

Annihilation vs. BSF cross section



with Higgs exchange

$v_{\rm rel} \lesssim \alpha_g^S + \alpha_h$

$$\sigma_{\rm BSF} v_{\rm rel} \propto (\kappa/k)^4 \approx [(\alpha_g^B + \alpha_h)/v_{\rm rel}]^4$$

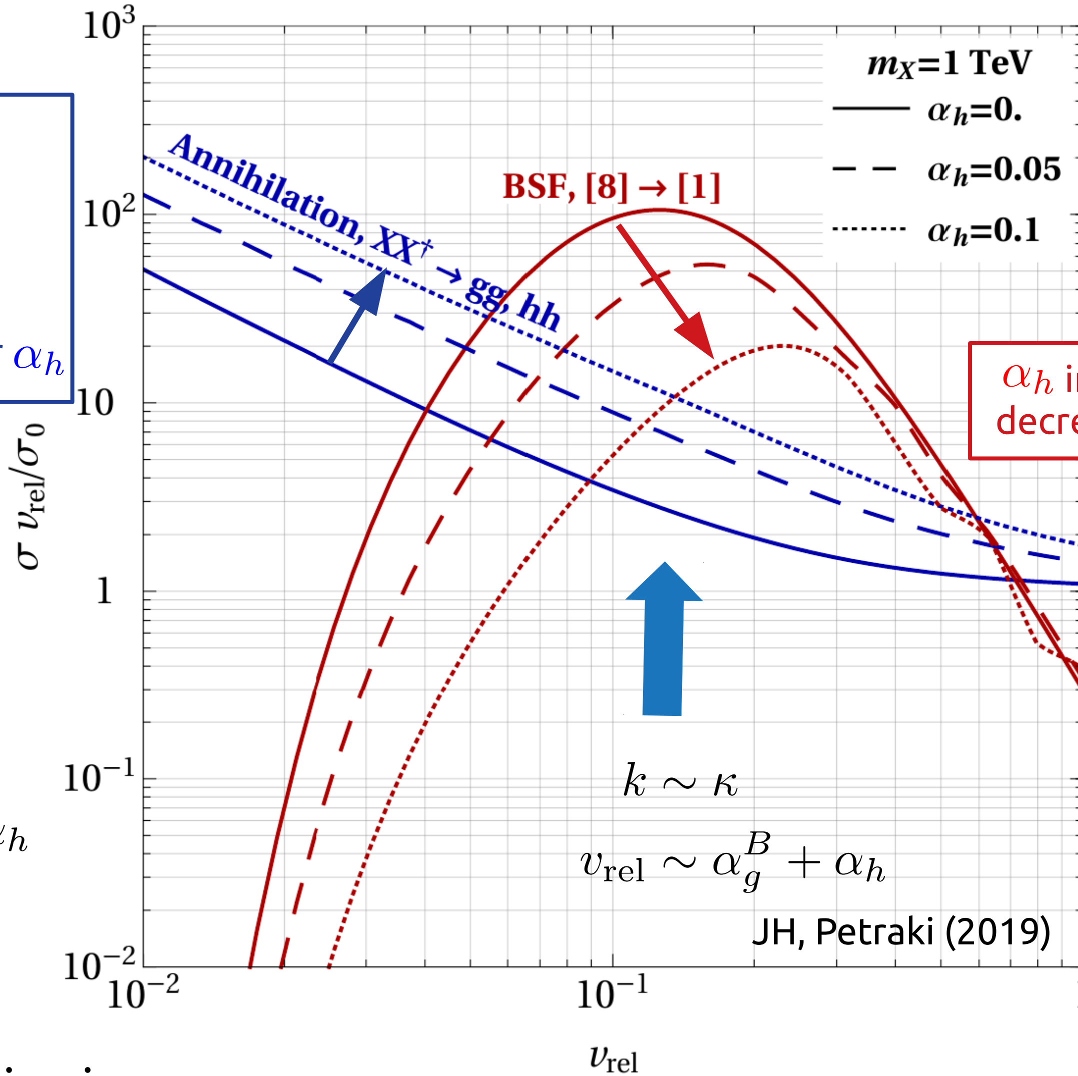
Coulomb repulsion in the scattering state

Annihilation vs. BSF cross section

with Higgs exchange

new annihilation channel

pert. annihilation + Sommerfeld effect increases with larger α_h



α_h increases scale and decreases α_g

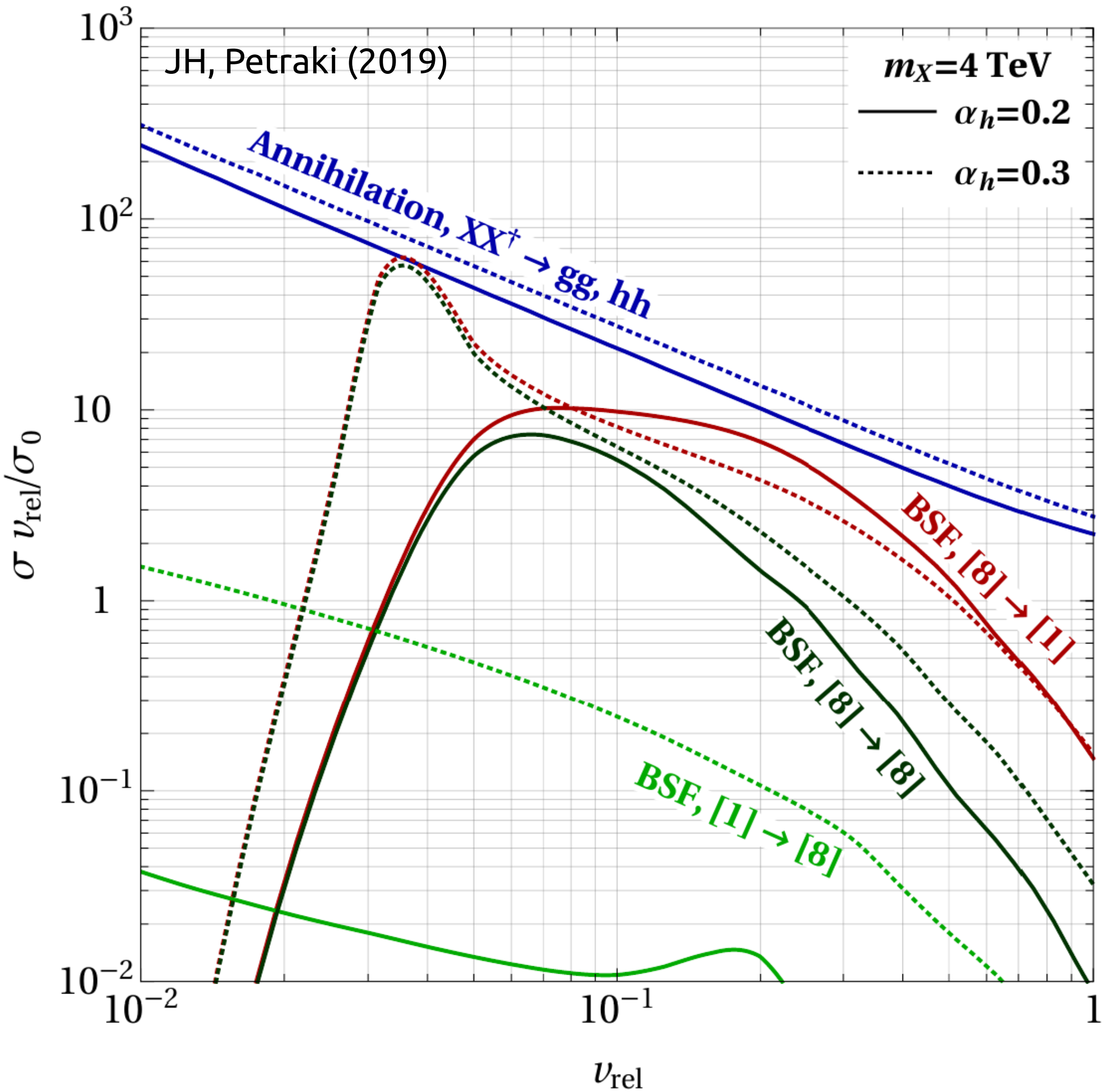
$v_{\rm rel} \lesssim \alpha_g^S + \alpha_h$

$$\sigma_{\rm BSF} v_{\rm rel} \propto (\kappa/k)^4 \approx [(\alpha_g^B + \alpha_h)/v_{\rm rel}]^4$$

Coulomb repulsion in the scattering state

→ relative strength of BSF seems to diminish, however, BSF peaks at later times!

Impact of the Higgs on the BSF cross section



with Higgs exchange

at a certain mass and coupling, the formation of octets are possible