Relaxation and evolution in fuzzy dark matter halos

Bar-Or, Tremaine & Fouvry (2019 and in preparation)

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Fuzzy dark matter

- extremely light bosons having galaxy-sized de Broglie wavelength

$$\frac{\lambda}{2\pi} = \frac{\hbar}{mv} = 1.9\,\mathrm{kpc}\frac{10^{-22}\,\mathrm{eV}}{m}\frac{10\,\mathrm{km\ s}^{-1}}{v}$$

- most interesting range to solve small-scale structure problems is m ~ 10^{-22} to 10^{-21} eV — larger masses are OK but look like CDM
- many particles in the same state so the dark matter can be described as a classical scalar field satisfying the Schrödinger-Poisson equations

$$-\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi(\mathbf{x})\psi = i\hbar\frac{\partial\psi}{\partial t}, \qquad \nabla^2\Phi = 4\pi G|\psi|^2.$$

"classical" ⇒ Planck's constant enters the equations only as $\hbar/m$

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Widrow & Kaiser (1993)
Sin (1994)
Goodman (2000)
Hu, Barkana & Gruzinov (2000)
Schive, Chiueh & Broadhurst (2014)
Hlozek, Grin, Marsh & Ferreira (2015)
Marsh (2016)
Hui + (2017)

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Fuzzy dark matter

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- most interesting range is $m \sim 10^{-22}$ to 10^{-21} eV
- can be described as a classical scalar field satisfying the Schrödinger-Poisson equations
- simplest models have negligible self-interaction on scales $\gtrsim 1$ pc
- the dynamics is identical to CDM on large scales (>> de Broglie wavelength), while the Heisenberg uncertainty principle suppresses small-scale structure
- therefore fuzzy dark matter might reduce or resolve alleged problems with CDM on small scales if the mass m is tuned to do so

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Fuzzy dark matter has MORE small-scale structure than cold dark matter

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The small-scale structure in a CDM halo

- is composed of bound sub-halos
- gradually disappears as the sub-halos are disrupted by tidal forces

The small-scale structure in an FDM halo

- arises from a set of traveling waves with random phases that is band-limited at $k \sim 2\pi/\lambda$
- can be thought of as arising from quasi-particles of mass $\sim\rho(\lambda/2\pi)^3$
- lasts forever

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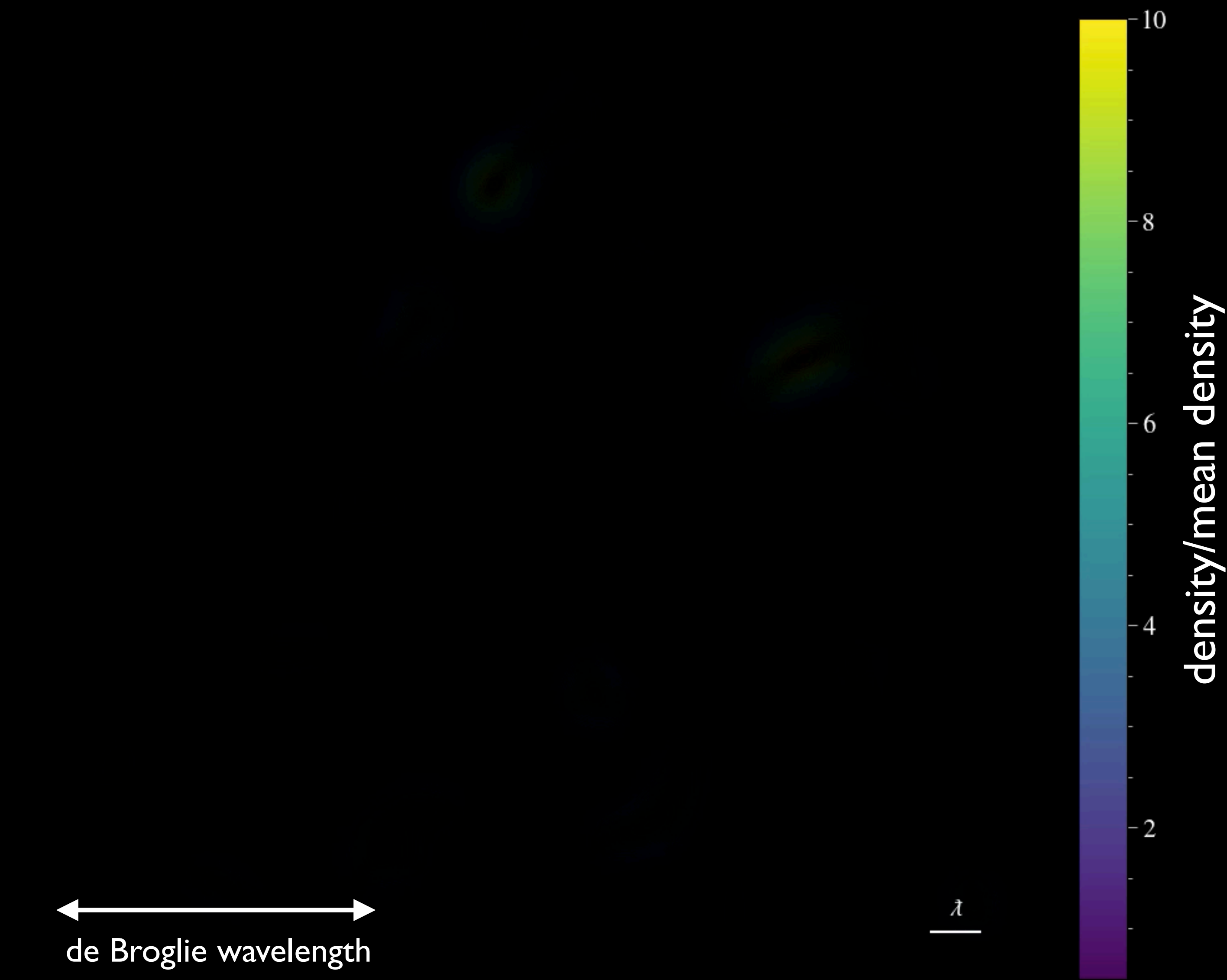
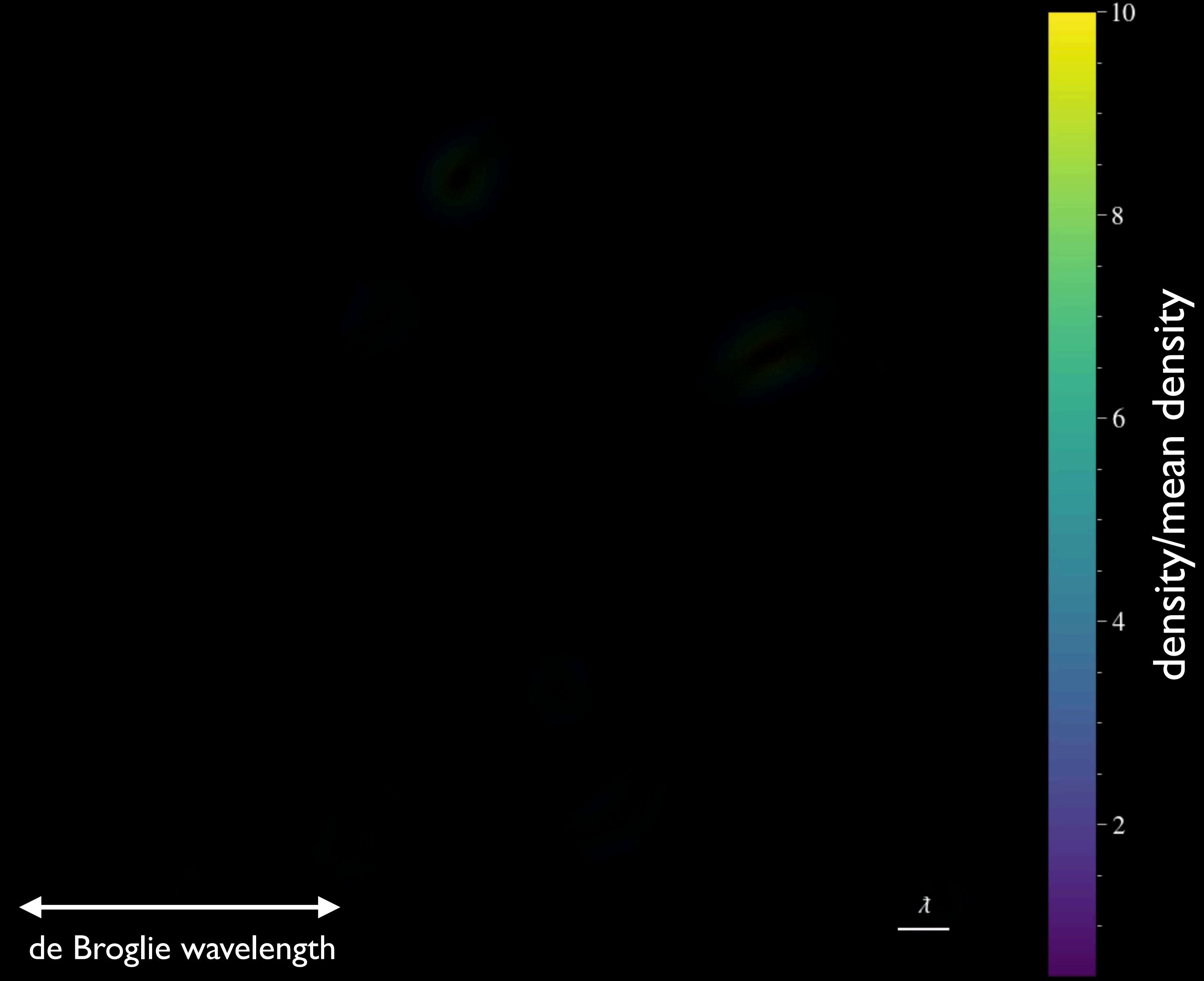


animation by
Ben Bar-Or



de Broglie wavelength

animation by
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Relaxation due to collisions between classical particles in a homogeneous system is described by the Boltzmann equation:

$$\partial_t f(p_1) = \int dp_2 dp_3 dp_4\, S(p_1, p_2, p_3, p_4)\left[f(p_3)f(p_4) - f(p_1)f(p_2)\right]$$

where p is momentum, f(p) is the distribution function in momentum space, and S describes the transition probability or cross-section.

The quantum-mechanical generalization of the Boltzmann equation is the Uehling-Uhlenbeck (1933) equation:

$$\partial_t f(p_1) = \int dp_2 dp_3 dp_4\, S(p_1, p_2, p_3, p_4)$$
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where ε = 0 for classical particles, +1 for bosons, -1 for fermions

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This can be derived rigorously if f(p) is defined as the Wigner (1932) distribution function

$$f(\mathbf{r}, \mathbf{v}, t) = \frac{1}{(2\pi)^3} \int d\mathbf{s} \, \psi\left(\mathbf{r} + \tfrac{1}{2}\hbar\mathbf{s}/m, t\right) \psi^*\left(\mathbf{r} - \tfrac{1}{2}\hbar\mathbf{s}/m, t\right) e^{-i\mathbf{v}\cdot\mathbf{s}}$$

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where ε = 0 for classical particles, +1 for bosons, -1 for fermions

For FDM, ε = 0 and $h^3 f(p) \gg 1$ so

$$\partial_t f(p_1) = h^3 \int dp_2 dp_3 dp_4 \, S(p_1, p_2, p_3, p_4)$$
$$\times \left[f(p_3)f(p_4)f(p_1) + f(p_3)f(p_4)f(p_2) - f(p_1)f(p_2)f(p_3) - f(p_1)f(p_2)f(p_4) \right]$$

Levkov + (2018), Bar-Or + (2019b)

For a halo having a Maxwellian distribution function with density ρ and one-dimensional velocity dispersion σ, the relaxation time is

$$t_{\rm relax} \simeq 0.34 \frac{\sigma^3}{G^2 m \rho \log \Lambda}$$

if the halo is composed of classical particles of mass m

$$t_{\rm relax} \simeq 0.34 \frac{\sigma_{\rm eff}^3}{G^2 m_{\rm eff} \rho \log \Lambda}$$

if the halo is composed of FDM

where $\sigma_{\rm eff} = \dfrac{\sigma}{\sqrt{2}}$ $\quad m_{\rm eff} = \rho (f\lambda)^3$ where $\lambda = \dfrac{h}{m\sigma}, \; f = \dfrac{1}{2\pi^{1/2}} = 0.28$

$$m_{\rm eff} = \simeq 1 \times 10^7 M_\odot \left(\frac{1 \ \text{kpc}}{r}\right)^2 \left(\frac{200 \text{ km s}^{-1}}{v_{\rm circ}}\right) \left(\frac{10^{-22} \text{ eV}}{m}\right)^3 .$$

Bar-Or + (2019b)

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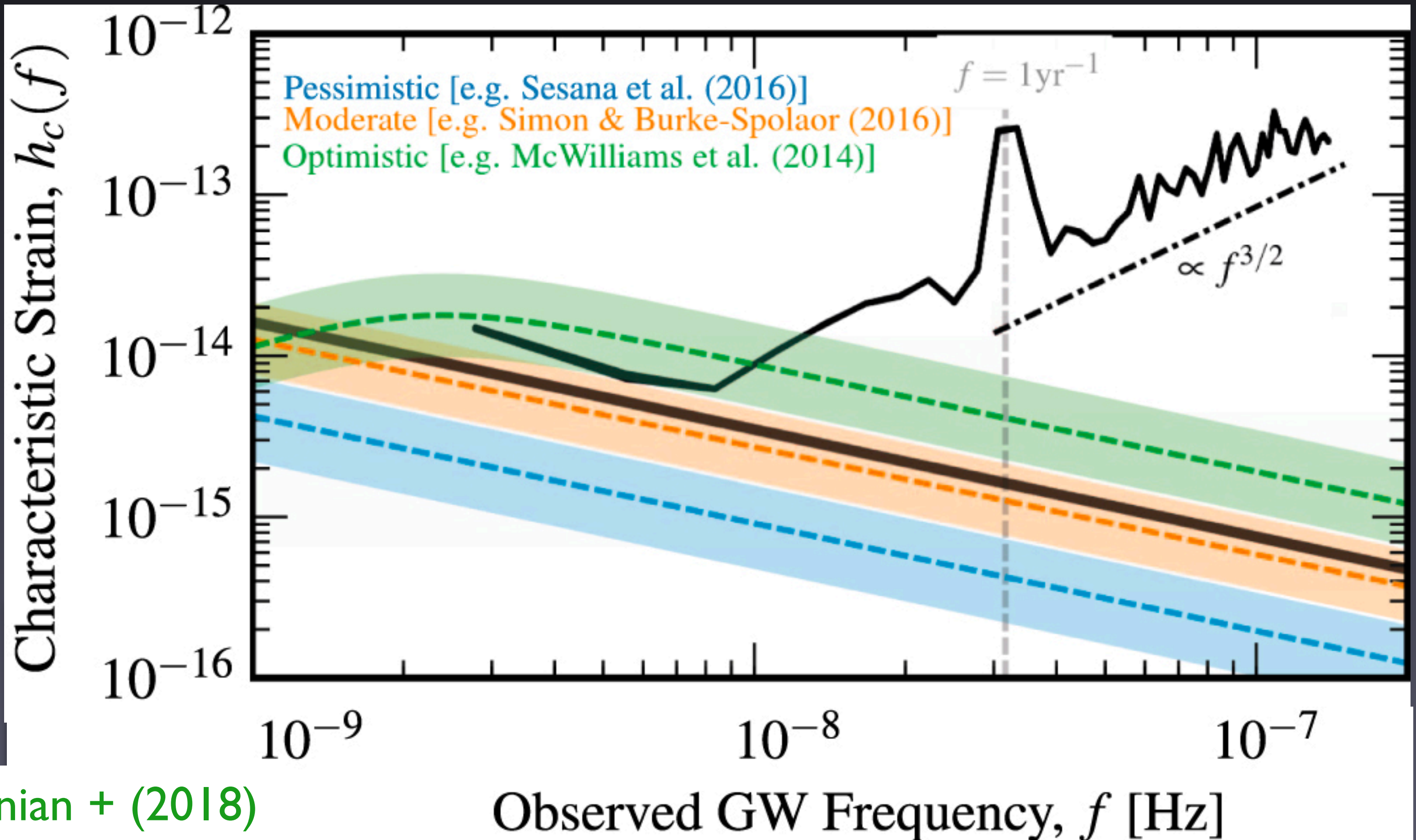
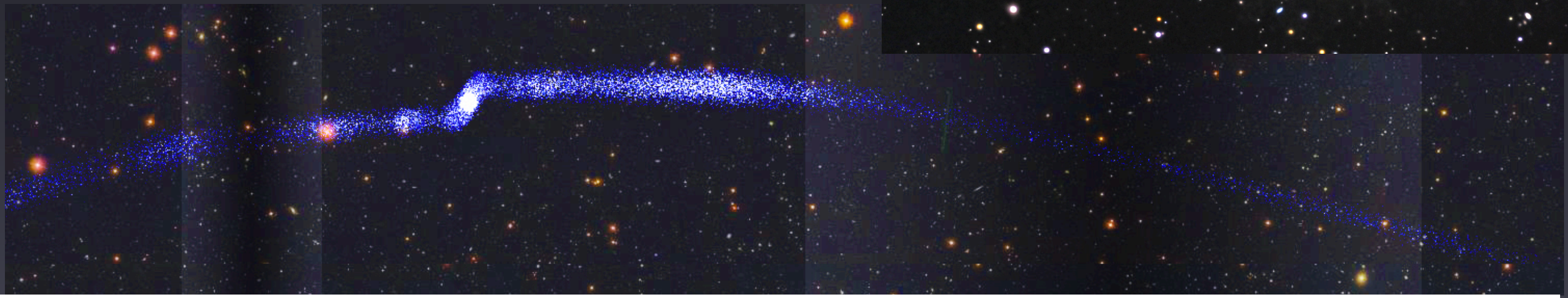
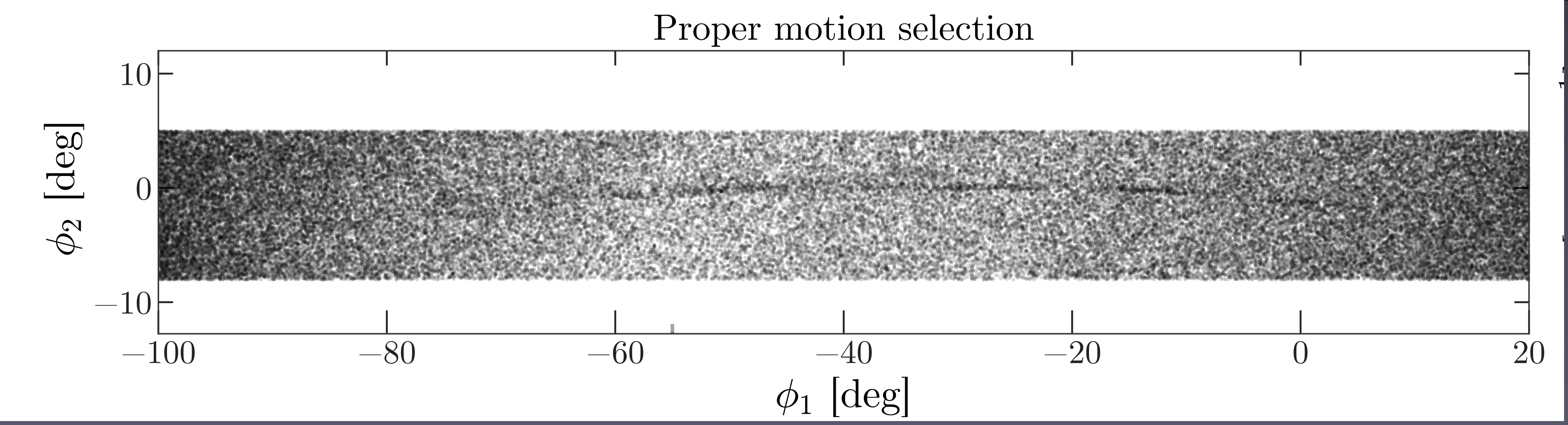
Interesting question #1: does FDM stall the inspiral of supermassive black holes when they reach equipartition with the quasiparticles?



Arzoumanian + (2018)

Interesting question #2: is FDM consistent with the long, smooth tidal streams observed in the halo?



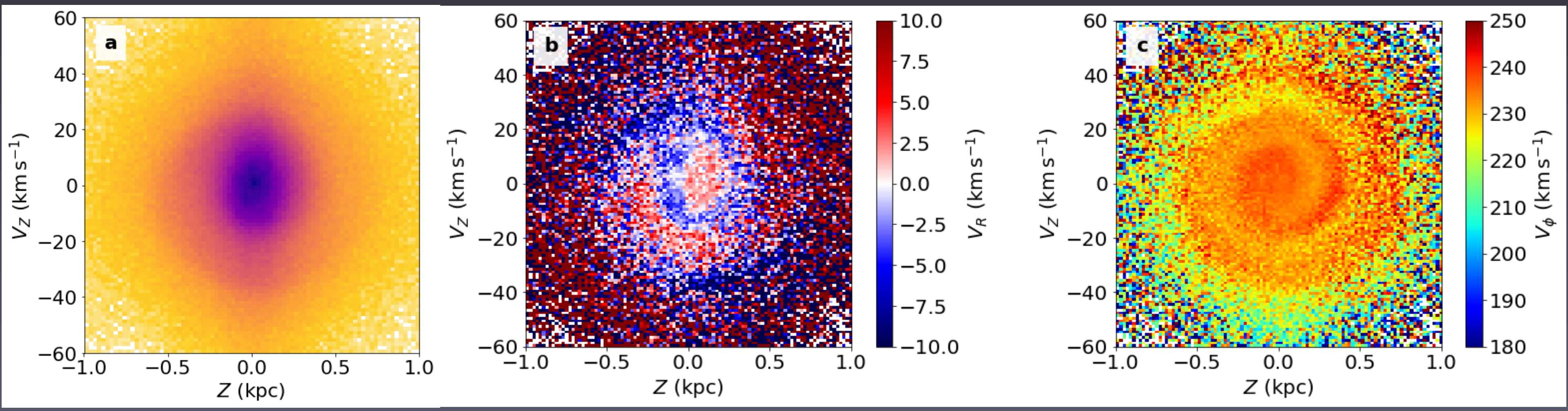


Interesting question #3: does relaxation from an FDM halo thicken galactic disks?

- answer depends strongly on whether the vertical energy input from FDM fluctuations is dumped locally or propagated away as bending waves in the disk
- if dumped locally, FDM heating may dominated the evolution of the disk thickness (Church + 2019)
- if propagated away the thickening is negligible in the solar neighborhood but may be significant at smaller radii (Hui + 2017)

Interesting question #4: is an FDM halo responsible for the non-equilibrium structure seen in the local Milky Way disk?

- Gaia provides plenty of evidence for recent perturbations to the vertical structure of the local disk, e.g., the Gaia snail (Antoja + 2018)
- these are unlikely to be due to CDM sub-halos as these are mostly destroyed at the distance of the Sun (Kelley + 2019)



Interesting question #5: how is relaxation related to the formation of central solitons?

- all excited states of Schrödinger-Poisson equations are unstable — lose energy by emitting mass rather than photons. Thus
  - isolated CDM halo survives forever
  - isolated FDM halo always eventually collapses to a soliton

The rate is probably governed by the relaxation time

$$t_{\rm relax} \sim 1 \times 10^{10}\,{\rm yr}\left(\frac{v}{100\ \ {\rm km\ s}^{-1}}\right)^2\left(\frac{r}{5\,{\rm kpc}}\right)^4\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^3$$

⇒ centers of FDM halos will condense into ground state, a.k.a. soliton; outer parts will behave like CDM

Fuzzy dark matter

- the soliton is the ground state of the Schrödinger-Poisson equations (Kaup 1968, Ruffini & Bonazzola 1969)

$$-\frac{\hbar^2}{2mr^2}\frac{d}{dr}r^2\frac{d\psi(r)}{dr} + m[\Phi(r) - E]\psi(r) = 0, \qquad \frac{d\Phi}{dr} = \frac{4\pi G}{r^2}\int_0^r x^2|\psi(x)|^2 dx.$$

- solutions form a one-parameter family of equilibrium bound systems. Central density and half-mass radius depend on mass M as

$$\rho_c = 0.0044\left(\frac{Gm^2}{\hbar^2}\right)^3 M^4 = 7.0\,M_\odot\,\mathrm{pc}^{-3}\left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^6\left(\frac{M}{10^9\,M_\odot}\right)^4$$

$$r_{1/2} = 3.93\frac{\hbar^2}{GMm^2} = 0.34\,\mathrm{kpc}\frac{10^9 M_\odot}{M}\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)^2.$$

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radius *decreases* as mass *increases*

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$$\rho(r) = \frac{\rho_c}{[1 + 0.19(r/r_{1/2})^2]^8}$$

empirical fit

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Relaxation in FDM

- centers of FDM halos will condense into solitons; outer parts will behave like CDM

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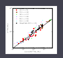
- at low masses (small velocities and radii) the soliton occupies most of the halo
- at high masses (large velocities and radii) the soliton is much smaller and denser than the halo
- from numerical simulations (Schive + 2014)

$$M_{\rm soliton} \sim M_{\rm min}^{2/3} M_{\rm halo}^{1/3}, \quad M_{\rm min} = 4 \times 10^7 M_\odot \left(\frac{10^{-22}\,{\rm eV}}{m}\right)^{3/2}$$



which implies peak rotation speed in soliton = peak rotation speed in halo (Bar + 2018)

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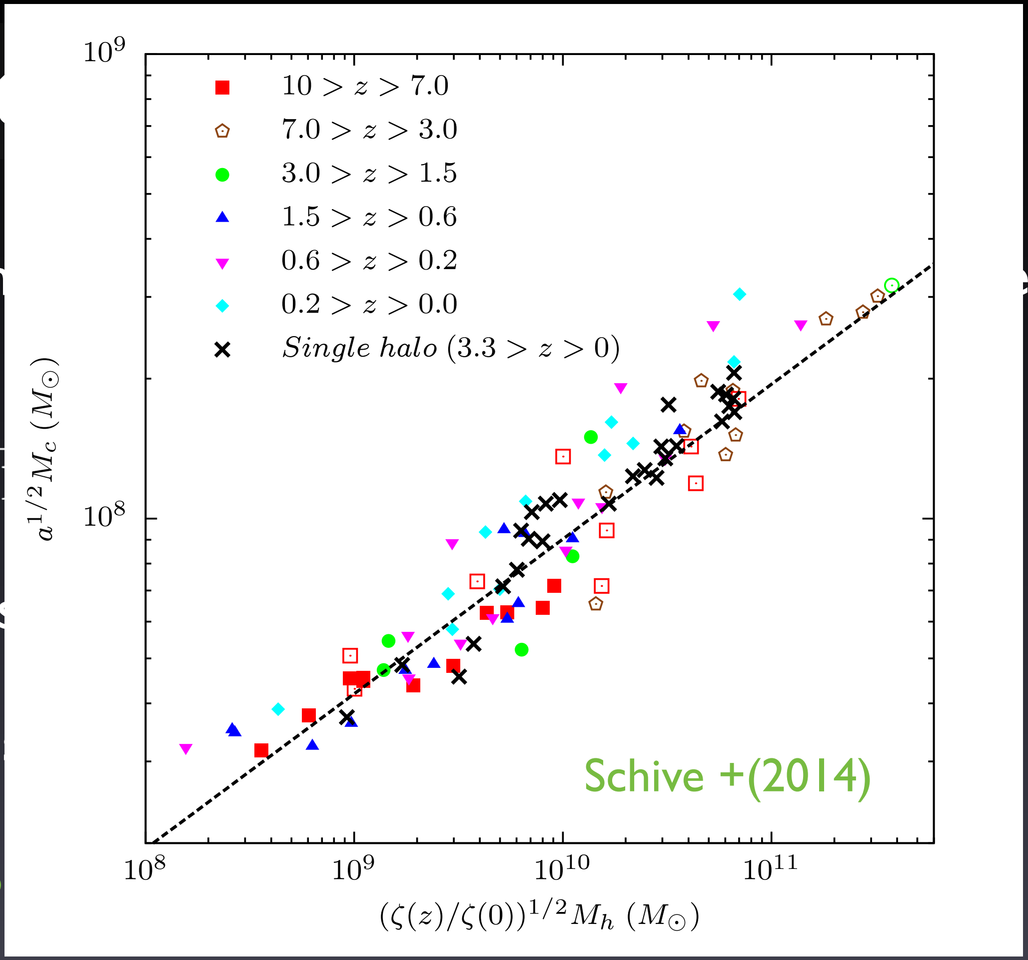

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Summary

- relaxation in an FDM halo can be described by the Uehling-Uhlenbeck equation
  - leads to a modified Fokker-Planck equation in which the collision term is cubic, rather than quadratic, in the distribution function
  - behavior is identical to the behavior of classical particles in a halo with effective dispersion $\sigma_{\mathrm{eff}} = \sigma/\sqrt{2}$ and mass $m_{\mathrm{eff}} = \rho\,(f\lambda)^3$ where $f=0.28$ and λ is the de Broglie wavelength at velocity σ
- many unanswered questions:
  - does FDM stall the inspiral of supermassive black holes?
  - is FDM consistent with the long, smooth tidal streams observed in the halo?
  - does relaxation from an FDM halo thicken galactic disks?
  - what is the relation of relaxation to the formation of central solitons?