



ALP cosmology and its experimental implications

July 16 2019@CERN

"Axions in the Lab and in the Cosmos"

Fumi Takahashi
(Tohoku)

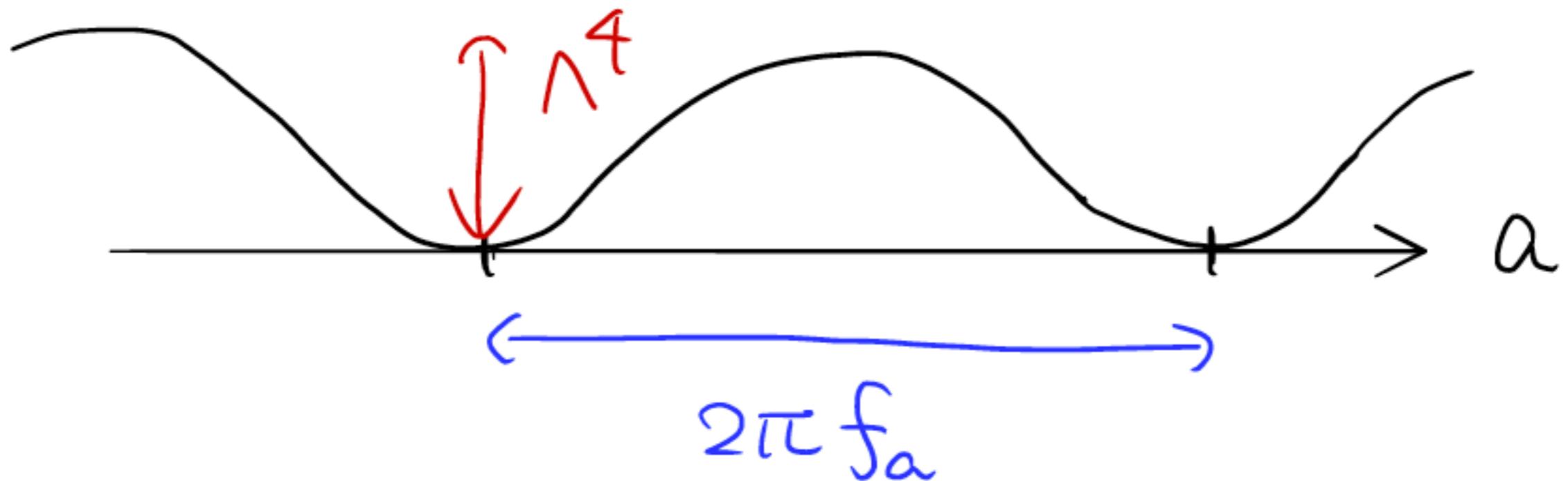
Axion-like particle (ALP)

Here ALP is a pNG boson that enjoys a discrete shift symmetry,

$$a \rightarrow a + 2\pi f_a$$

It is light, and its self-interaction is weak:

$$m_a \sim \frac{\Lambda^2}{f_a}, \quad \lambda \sim \frac{\Lambda^4}{f_a^4}$$



What is the cosmological role of ALP?

1. Dark matter

Misalignment mechanism $\theta_* = \mathcal{O}(1)$?
strings/walls?

Bunch-Davies distribution.

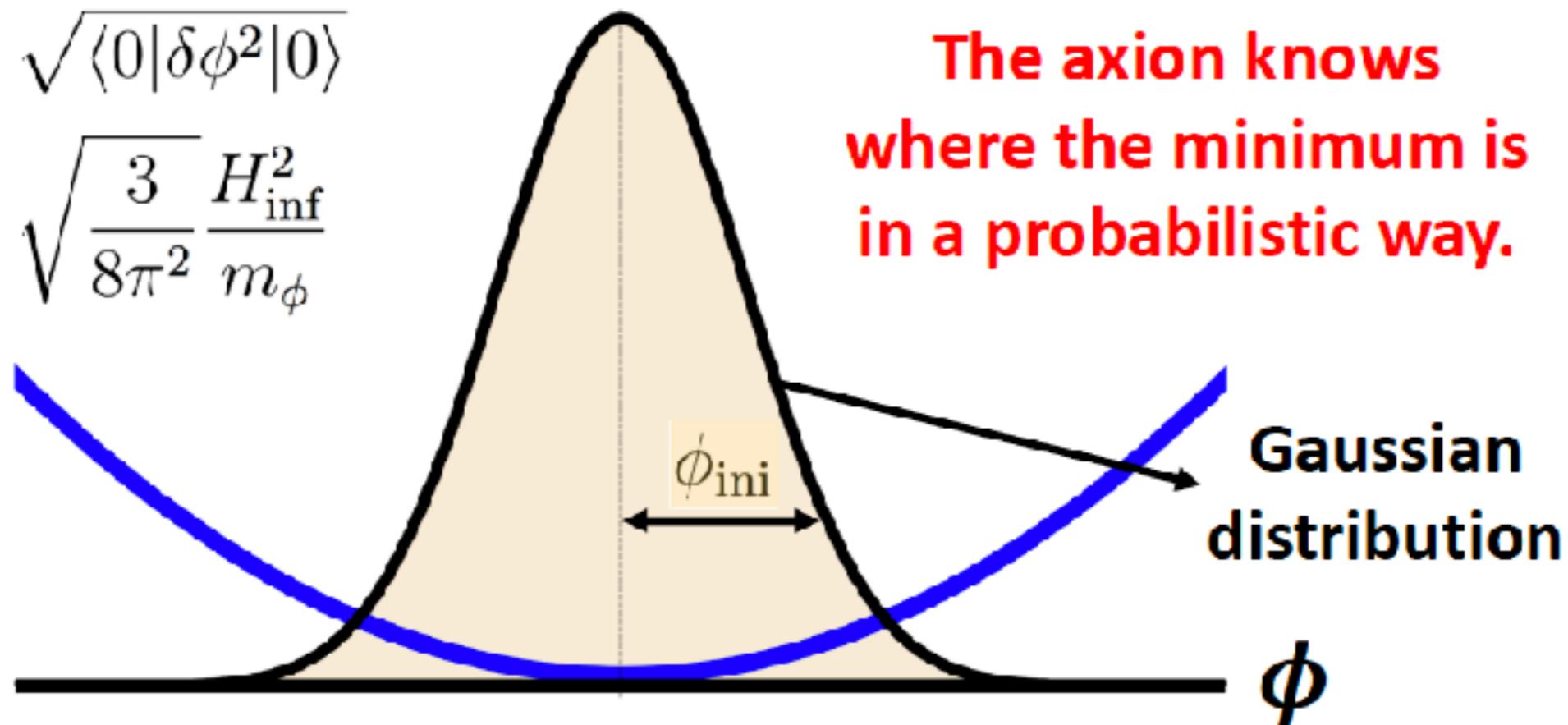
Peter W. Graham, Adam Scherlis, 1805.07362,
FT, Wen Yin, Alan H. Guth, 1805.08763
S-Y Ho, FT, Wen Yin 1901.01240

Bunch-Davies distribution

quantum fluctuations \longleftrightarrow classical motion

$$\begin{aligned}\phi_{\text{ini}} &\simeq \sqrt{\langle 0 | \delta\phi^2 | 0 \rangle} \\ &\simeq \sqrt{\frac{3}{8\pi^2} \frac{H_{\text{inf}}^2}{m_\phi}}\end{aligned}$$

The axion knows
where the minimum is
in a probabilistic way.



What is the cosmological role of ALP?

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Misalignment mechanism $\theta_* = \mathcal{O}(1)$?
strings/walls? Bunch-Davies distribution.

String and wall evolution depends on the UV completion:
e.g. clockwork axion model

See also Sikivie '86 Kim, Nilles, Peloso, hep-ph/0409138 Choi, Kim, Yun, 1404.6209, Higaki, FT, 1404.6923
Harigaya and Ibe, 1407.4893, Choi and Im, 1511.00132, Kaplan and Rattzzi, 1511.01827, Giudice and McCullough
[1610.07962](#)

$$V = \sum_{i=1}^N \left(-m_i^2 |\Phi_i|^2 + \lambda_i |\Phi_i|^4 \right) + \sum_{i=1}^{N-1} \epsilon \left(\Phi_i \Phi_{i+1}^3 + \text{h.c.} \right) \quad \Rightarrow \quad f_a \sim 3^N f$$

- Phase transition takes place at lower $T \sim f \ll f_a$
- Strings and walls form complicated network

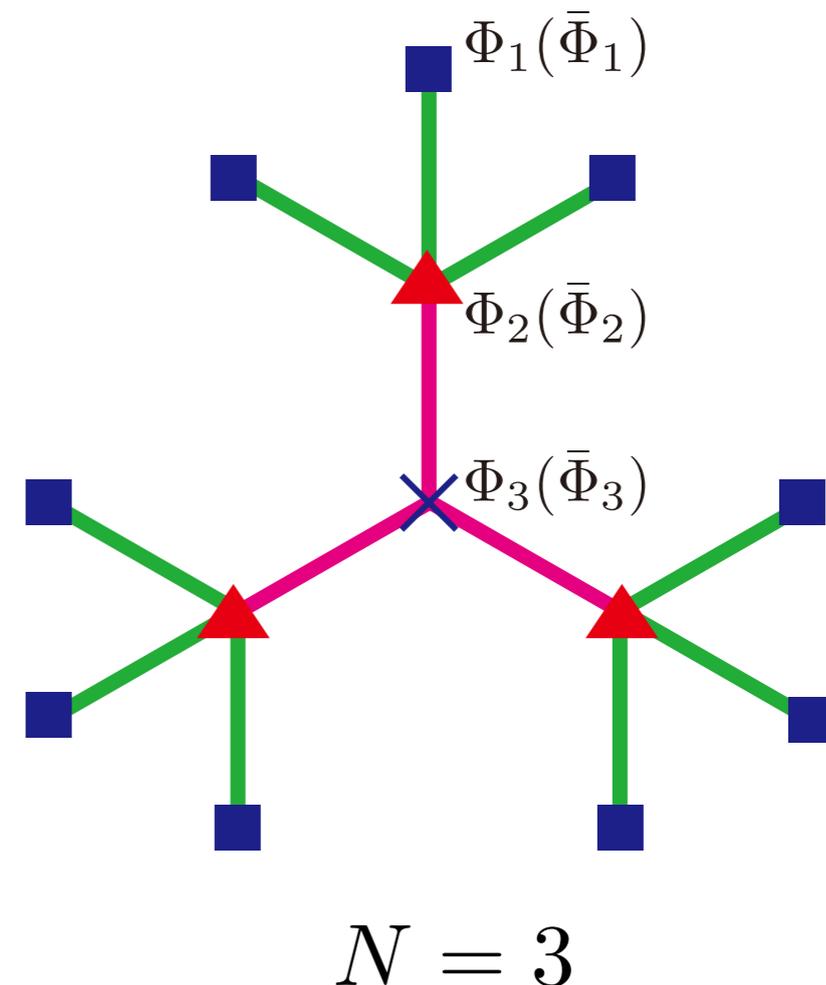
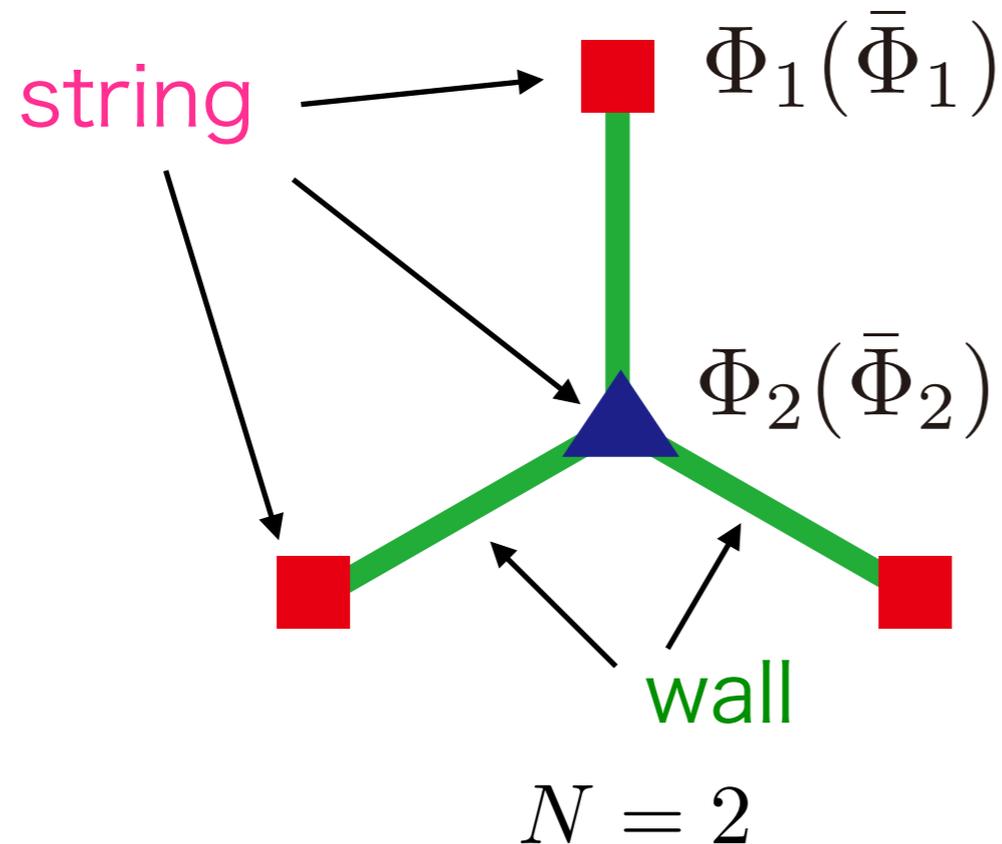
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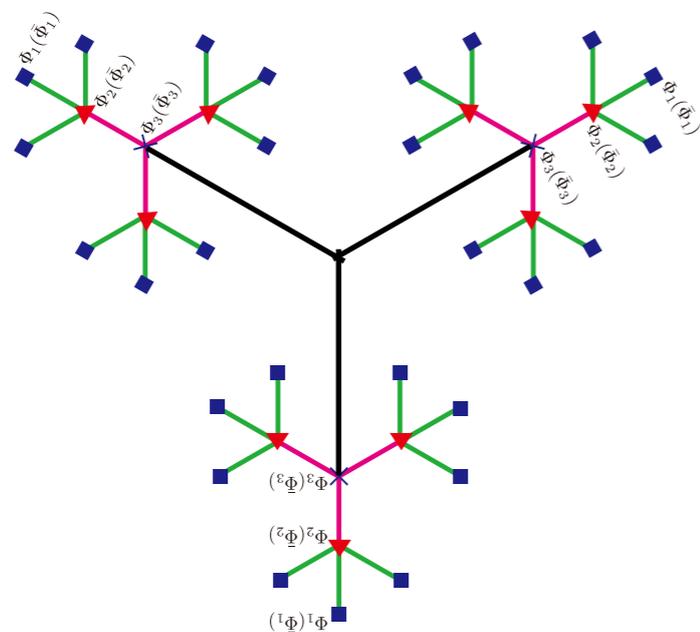
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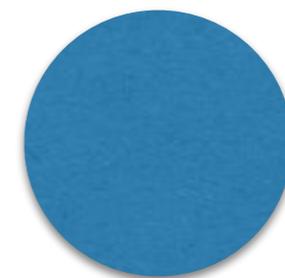
Misalignment mechanism $\theta_* = \mathcal{O}(1)$?
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String and wall evolution depends on the UV completion:
e.g. clockwork axion model



\simeq



$$\mu_{\text{eff}} \simeq \pi (3^{2(N-1)} f_1^2 + \dots + 3^2 f_{N-1}^2 + f_N^2) \ln \left(\frac{R}{\delta} \right) = \pi F_a^2 \ln \left(\frac{R}{\delta} \right)$$

However, it is unclear if such isolated string bundles are actually formed.

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Bunch-Davies distribution.

2. Dark energy

$m_a \lesssim 10^{-33}$ eV? Anthropic explanation?

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Thermal or non-thermal production
Anthropic explanation??

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CMB normalization, successful reheating,
any testable predictions?

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5. Portal to DM

6. Nothing special

ALP = DM

Axion abundance

based on the misalignment mechanism

QCD axion

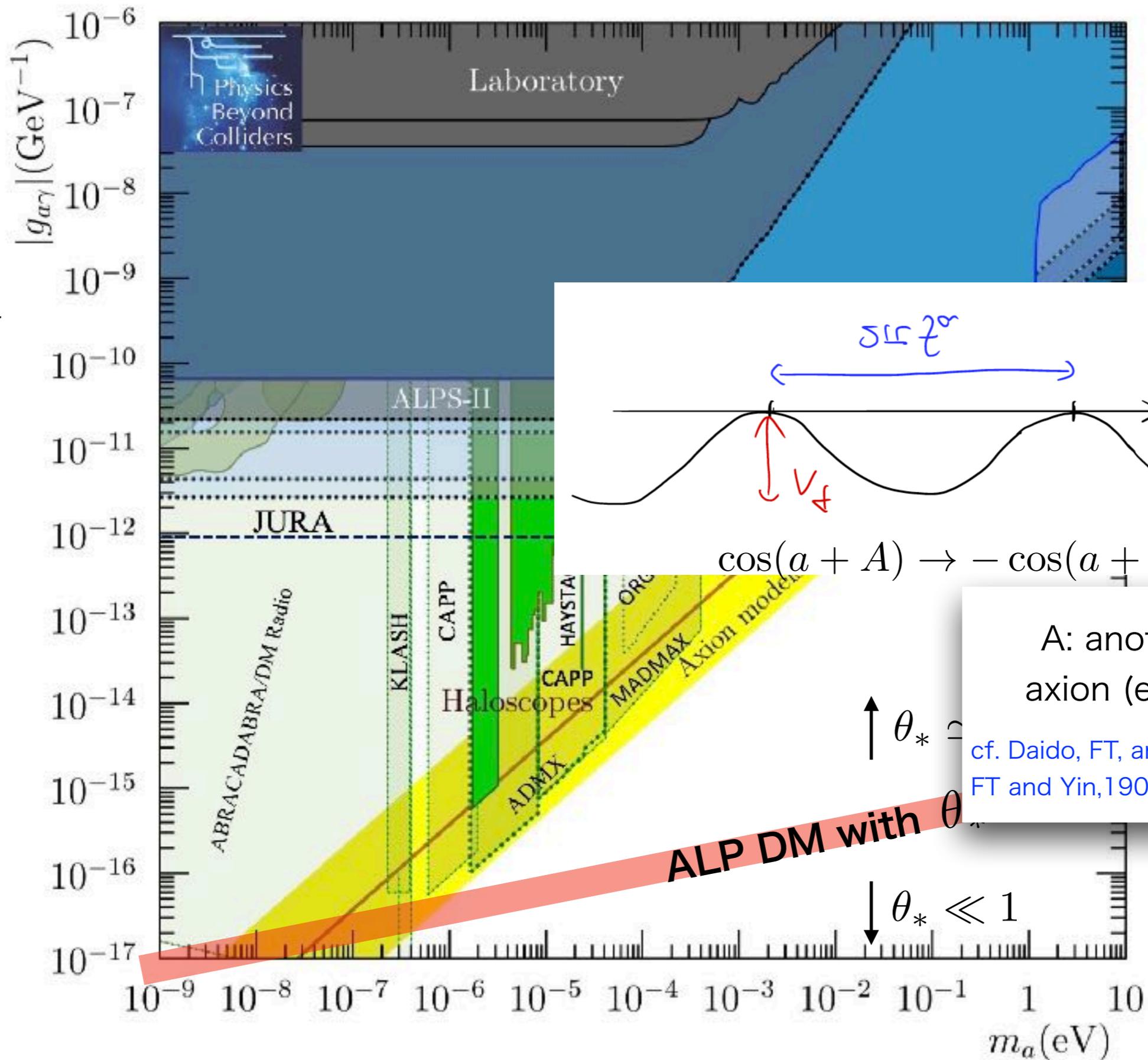
$$\Omega_a h^2 \simeq 0.14 \theta_*^2 \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{1.17}$$

“Classical axion window”: $10^8 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$

ALP

$$\Omega_\varphi h^2 \simeq 0.3 \theta_*^2 \left(\frac{m_\varphi}{1 \text{ eV}} \right)^{1/2} \left(\frac{f_\varphi}{10^{12} \text{ GeV}} \right)^2$$

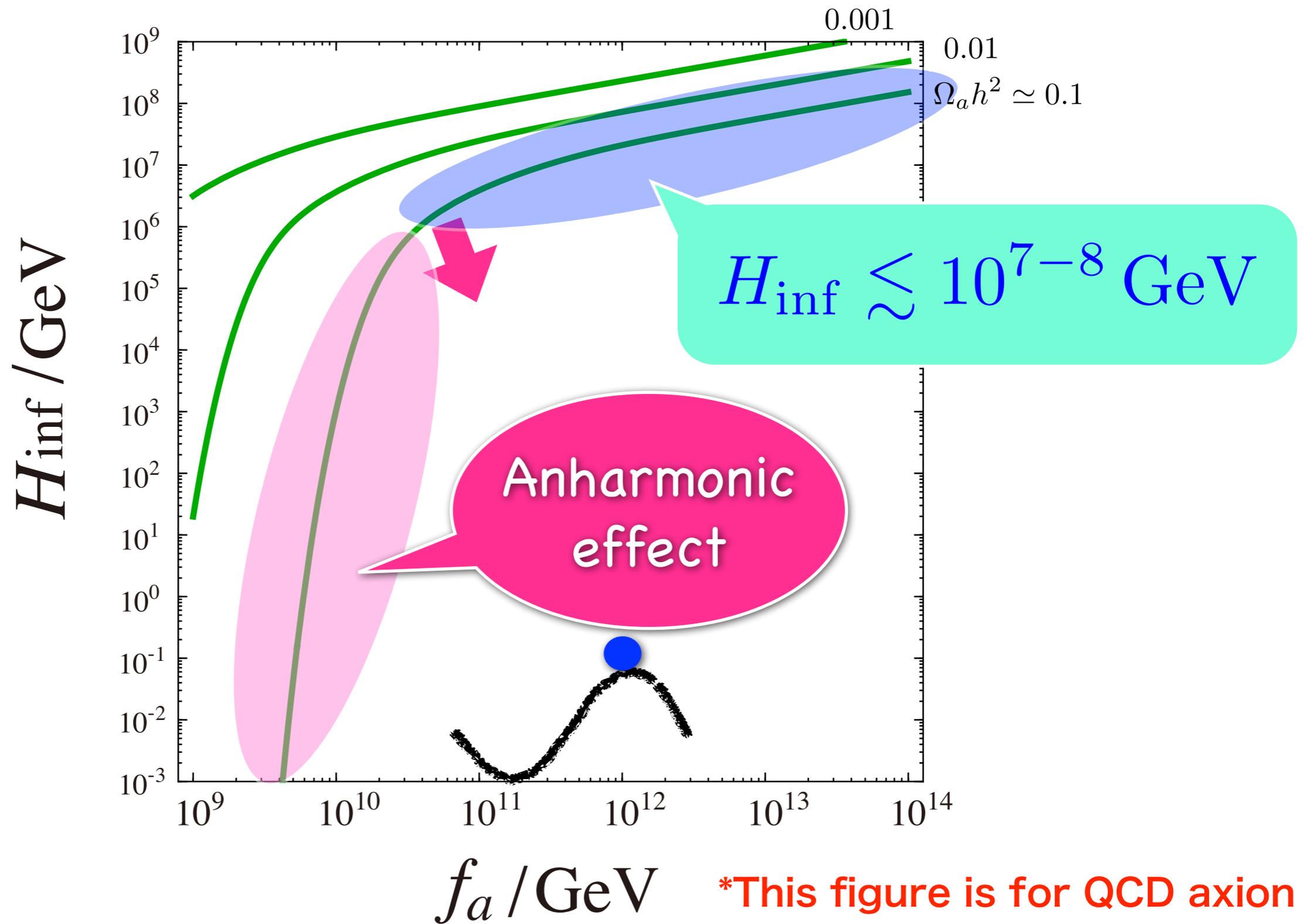
$$g_{a\gamma} = \frac{c_\gamma \alpha}{\pi f_a}$$



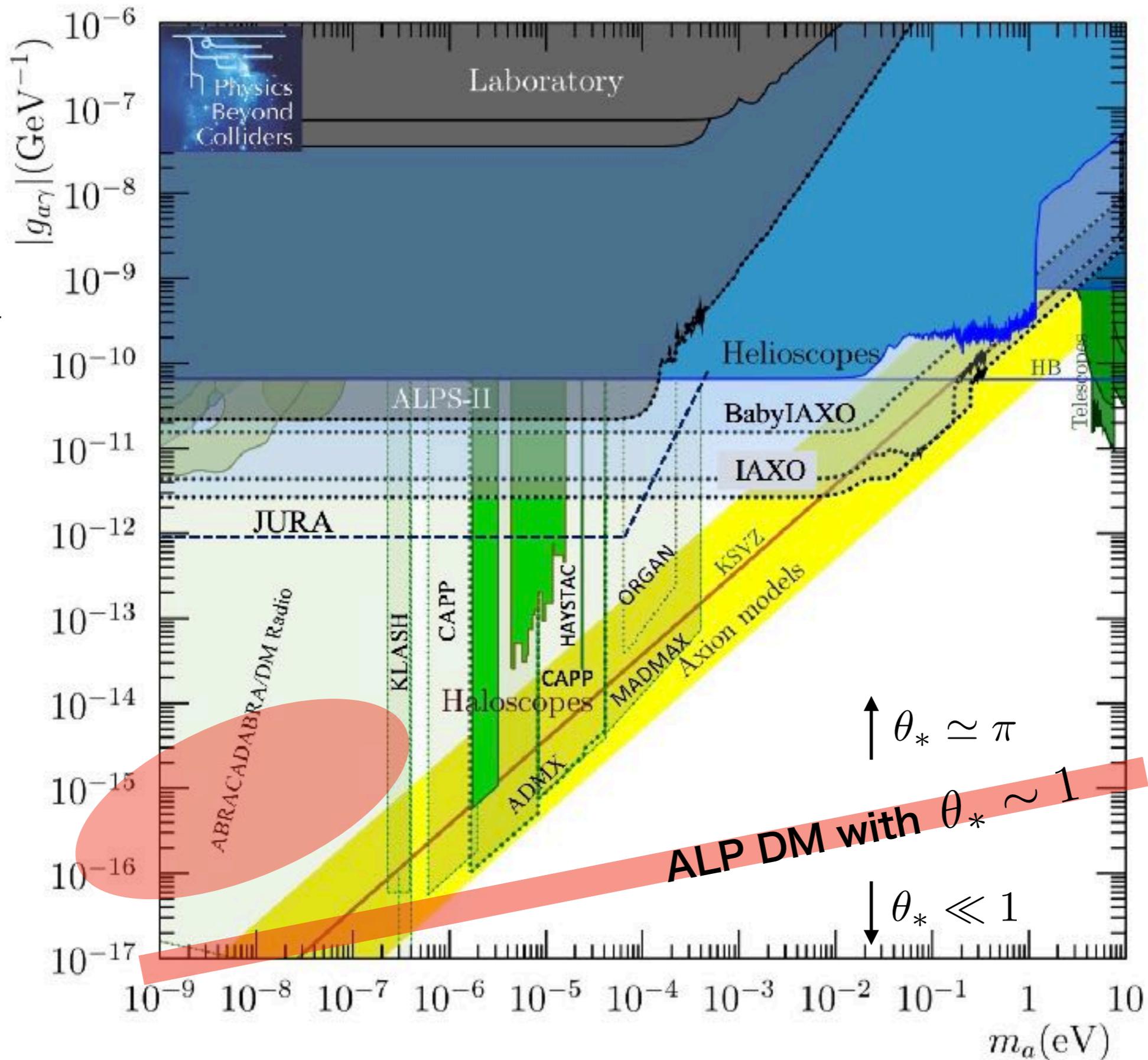
A: another heavy axion (e.g. inflaton)
 cf. Daido, FT, and Yin 1702.03284, FT and Yin, 1903.00462

Taken from "Physics Beyond Colliders at CERN: Beyond the Standard Model Working Group Report," arXiv:1901.09966 [hep-ex].

Isocurvature constraint on H_{inf}

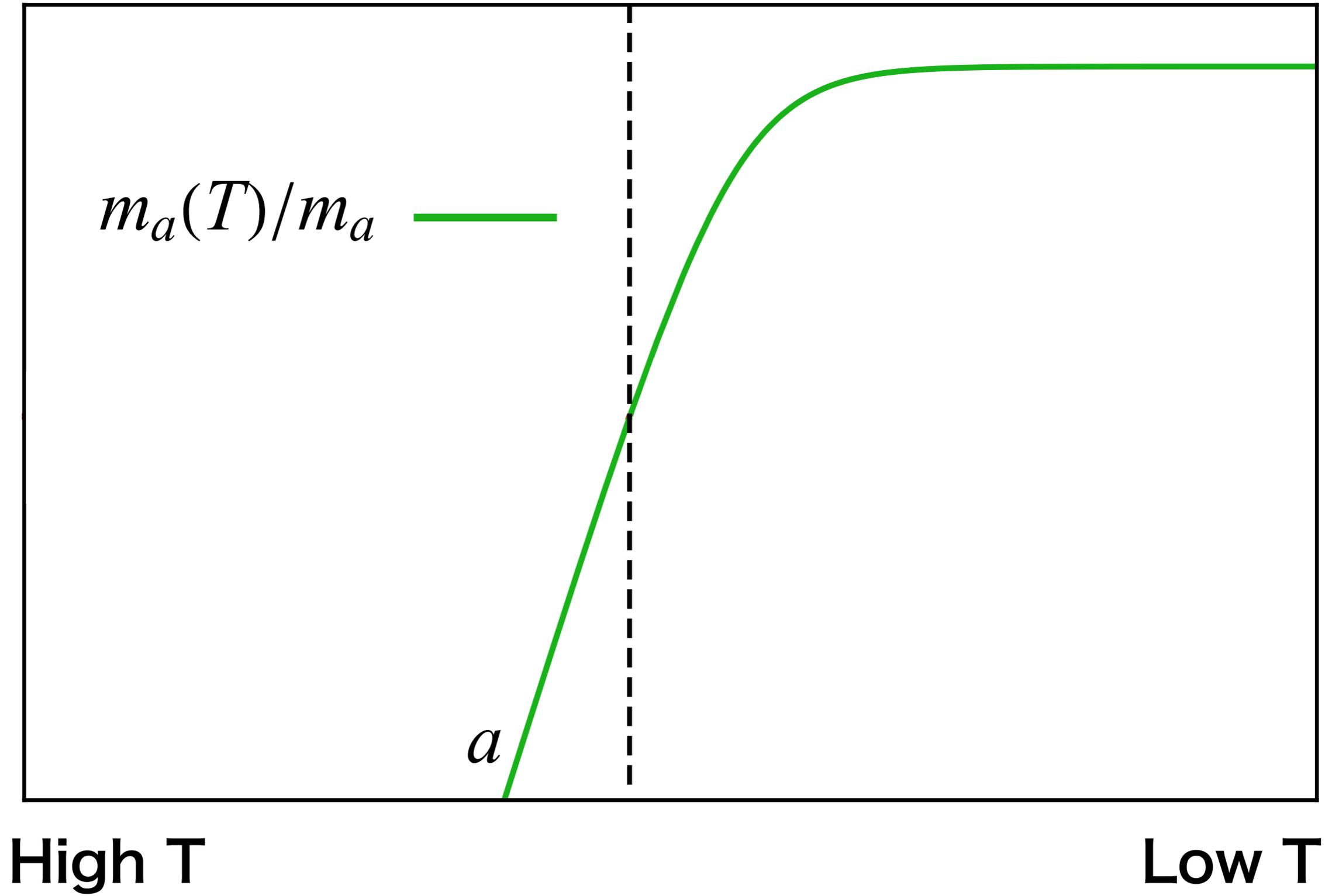


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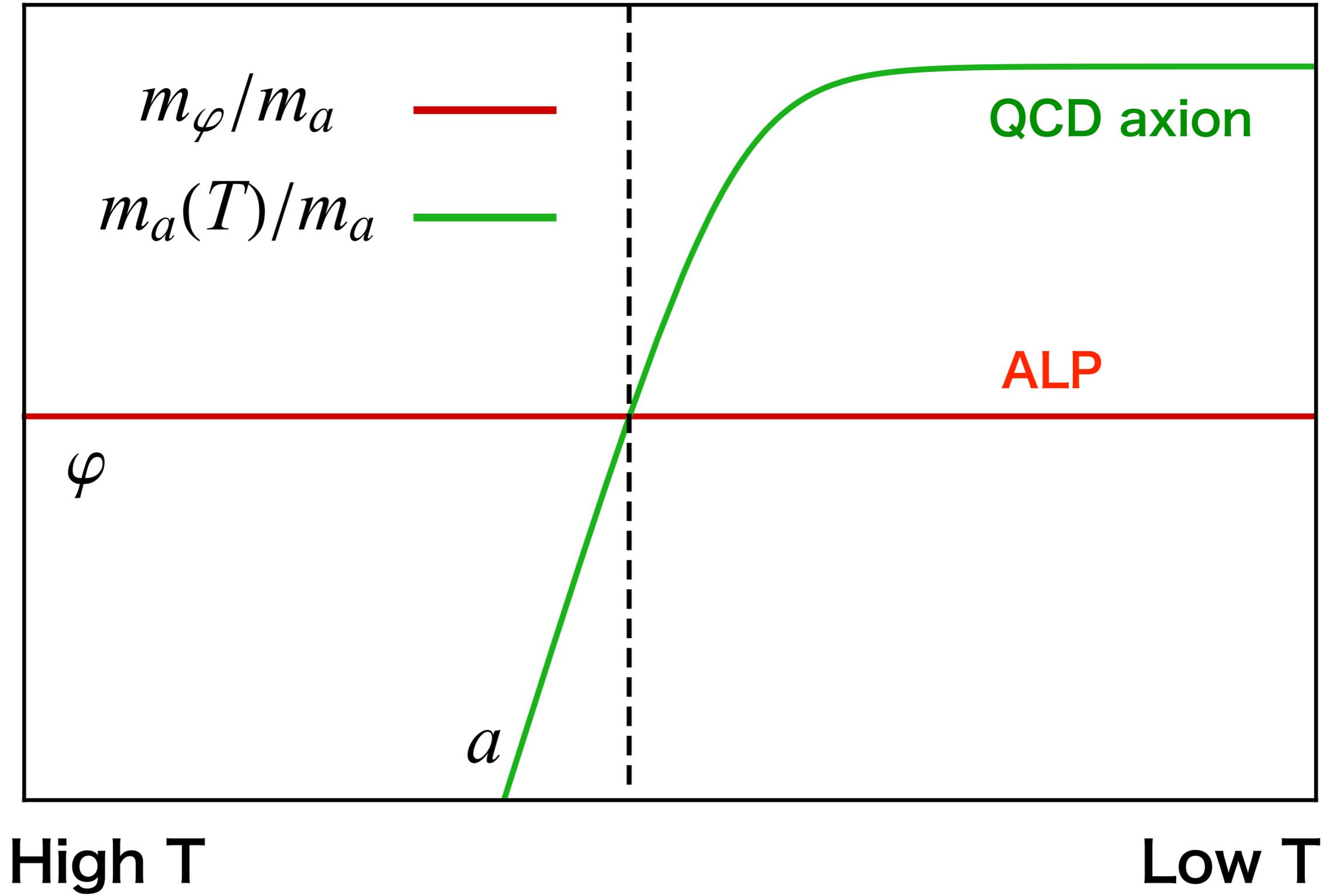


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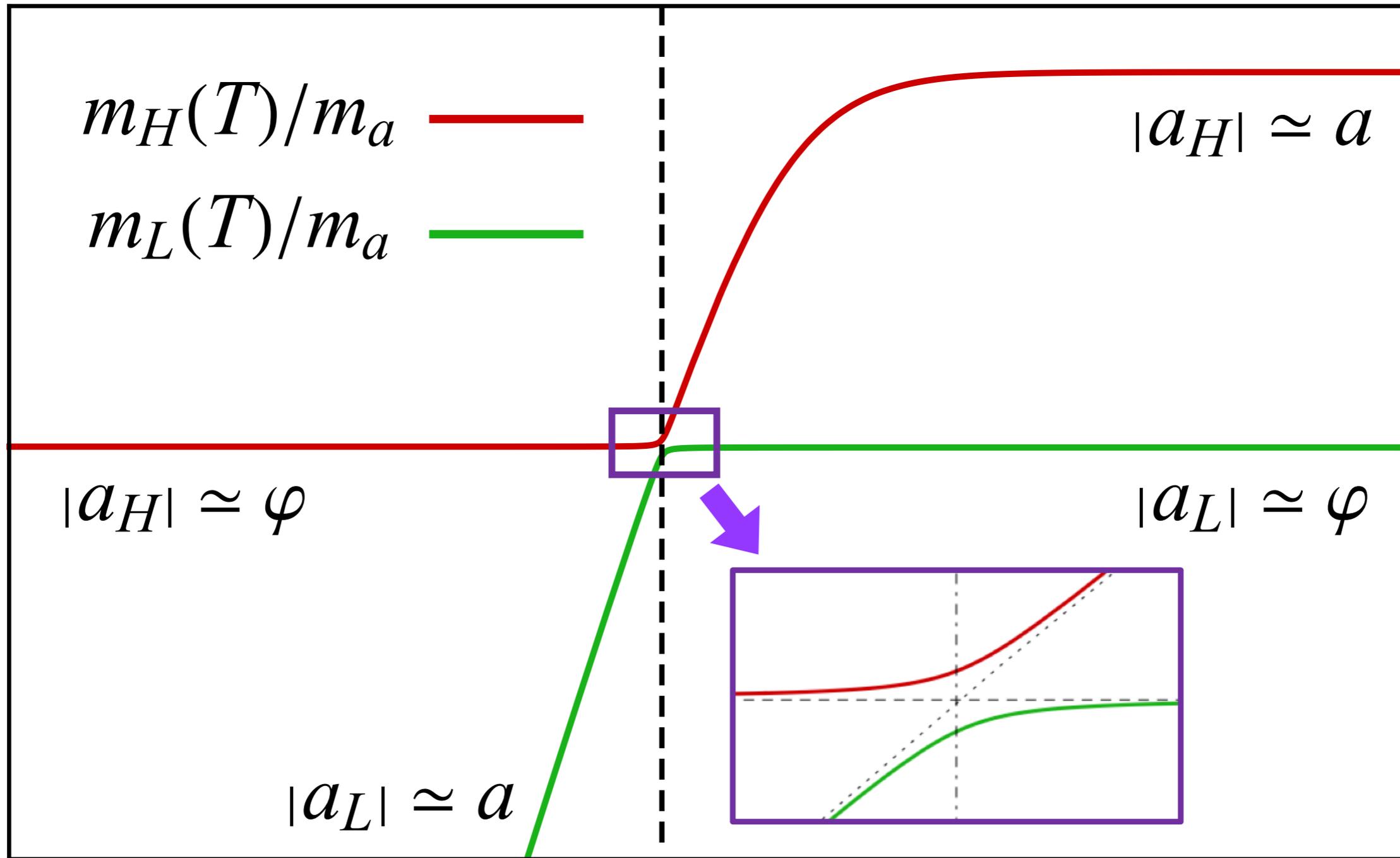
QCD axion



QCD axion + ALP with $m_\varphi < m_a(T=0)$



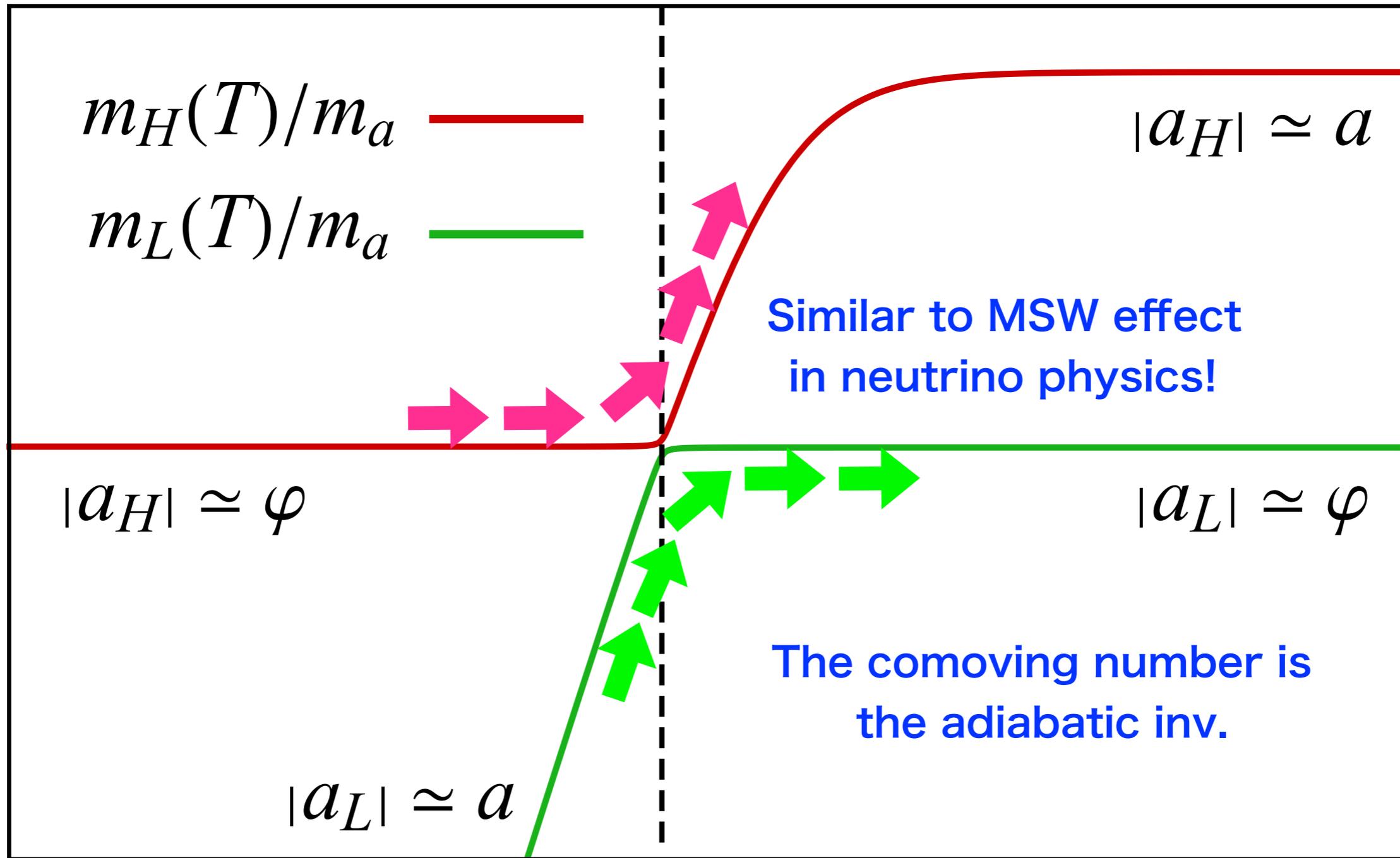
QCD axion + ALP with $m_\phi < m_a(T=0)$ + mass mixing



High T

Low T

QCD axion + ALP with $m_\phi < m_a(T=0)$ + mass mixing



High T

Kitajima and FT, [1411.2011](#)
 cf. Hill and Ross, '88

Low T

Set-up

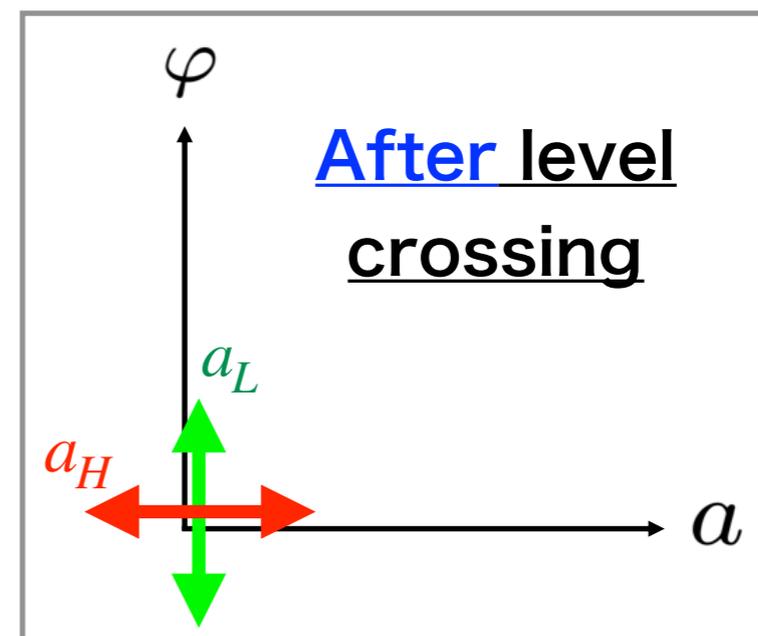
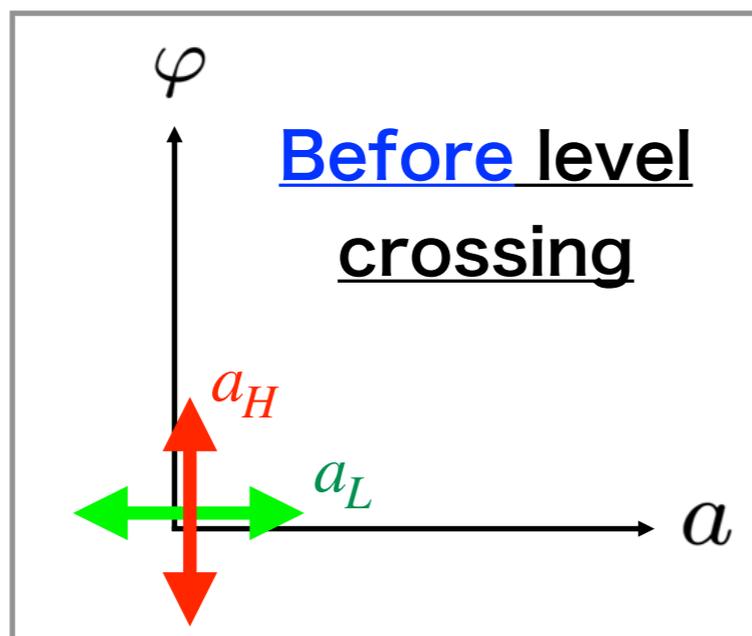
• Axion potential

Mass mixing

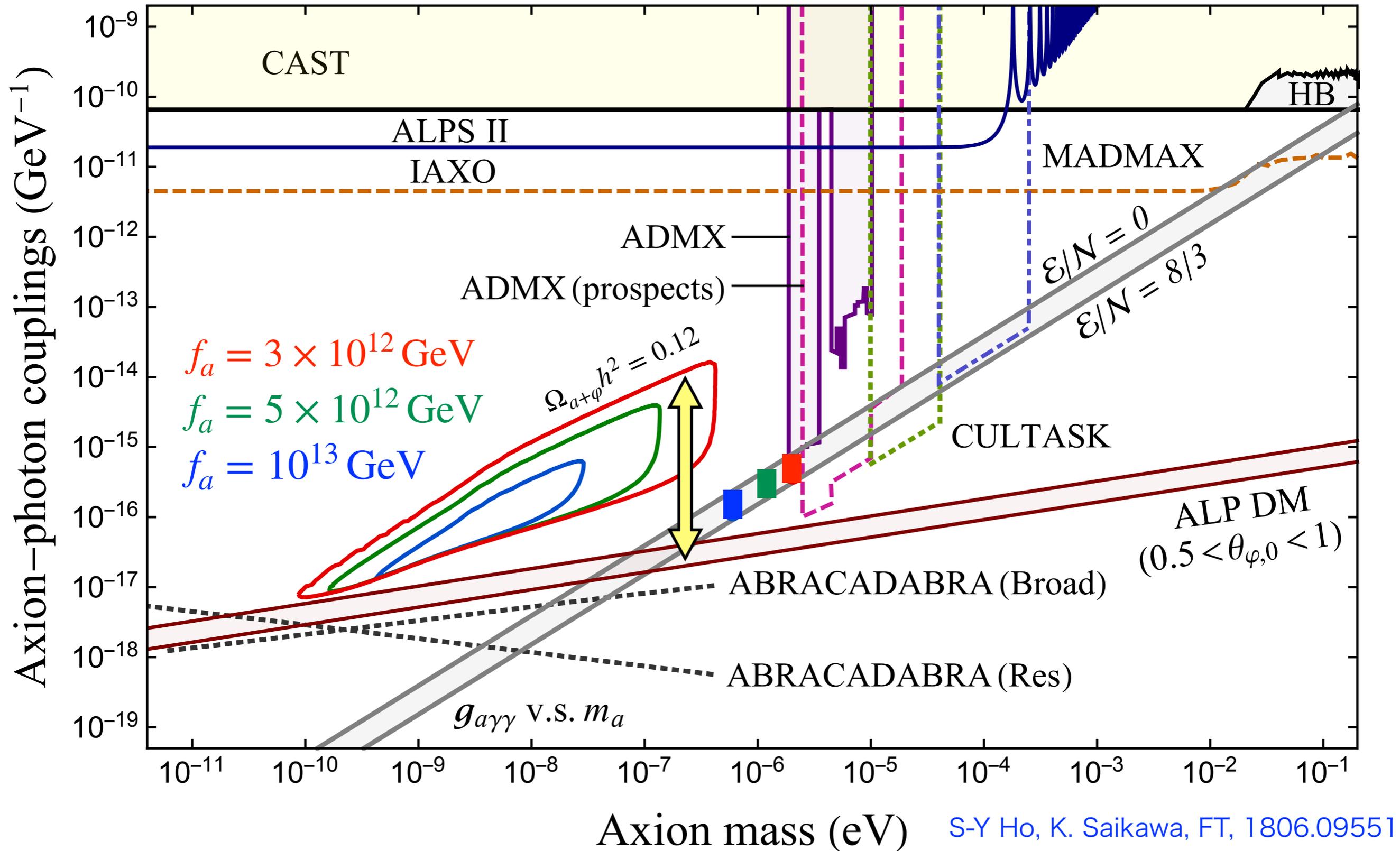
$$V_{\text{QCD}}(a) = m_a^2(T) f_a^2 \left[1 - \cos \left(\frac{a}{f_a} \right) \right] \quad V_{\text{mix}}(a, \varphi) = m_\varphi^2 f_\varphi^2 \left[1 - \cos \left(\frac{a}{f_a} + \frac{\varphi}{f_\varphi} \right) \right]$$

Mass matrix:
$$M^2 = \begin{pmatrix} 0 & 0 \\ 0 & m_a^2(T) \end{pmatrix} + m_\varphi^2 \begin{pmatrix} 1 & f_\varphi/f_a \\ f_\varphi/f_a & (f_\varphi/f_a)^2 \end{pmatrix}$$

$$\begin{pmatrix} a_H \\ a_L \end{pmatrix} = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} \varphi \\ a \end{pmatrix}$$



Axion coupling to photons



ALP coupling to photons can be enhanced by a few orders of magnitude.

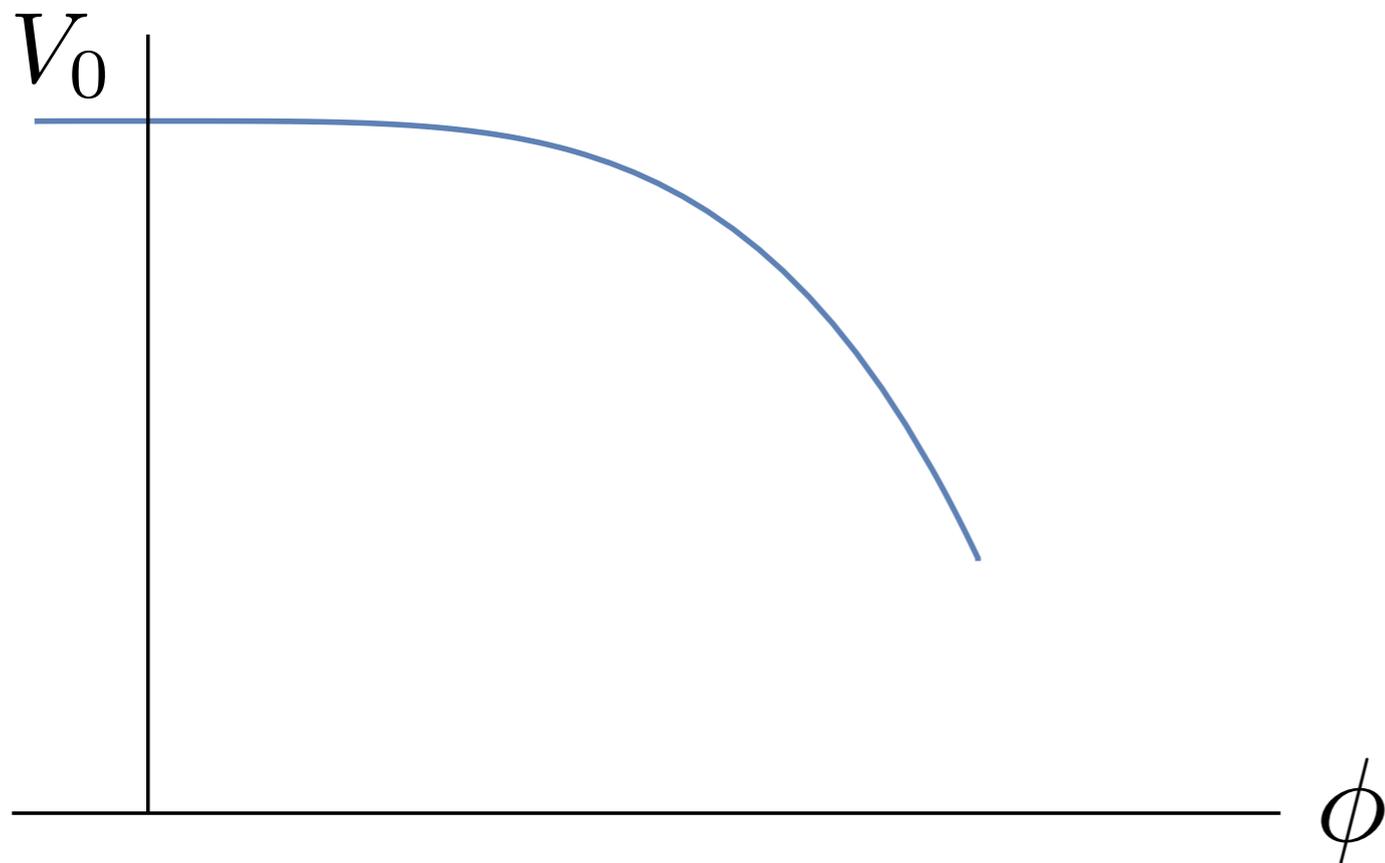
ALP = inflaton

Hilltop inflation

If the inflaton field excursion is (much) smaller than M_P , the inflaton potential must be very flat during inflation.

$$\frac{\delta\rho}{\rho} \sim \left| \frac{V^{3/2}}{V' M_P^3} \right| \sim 10^{-5}$$

e.g. Quartic hilltop inflation $V \simeq V_0 - \lambda\phi^4 + \dots$



CMB normalization:

$$\lambda \sim 10^{-13} \ll 1$$

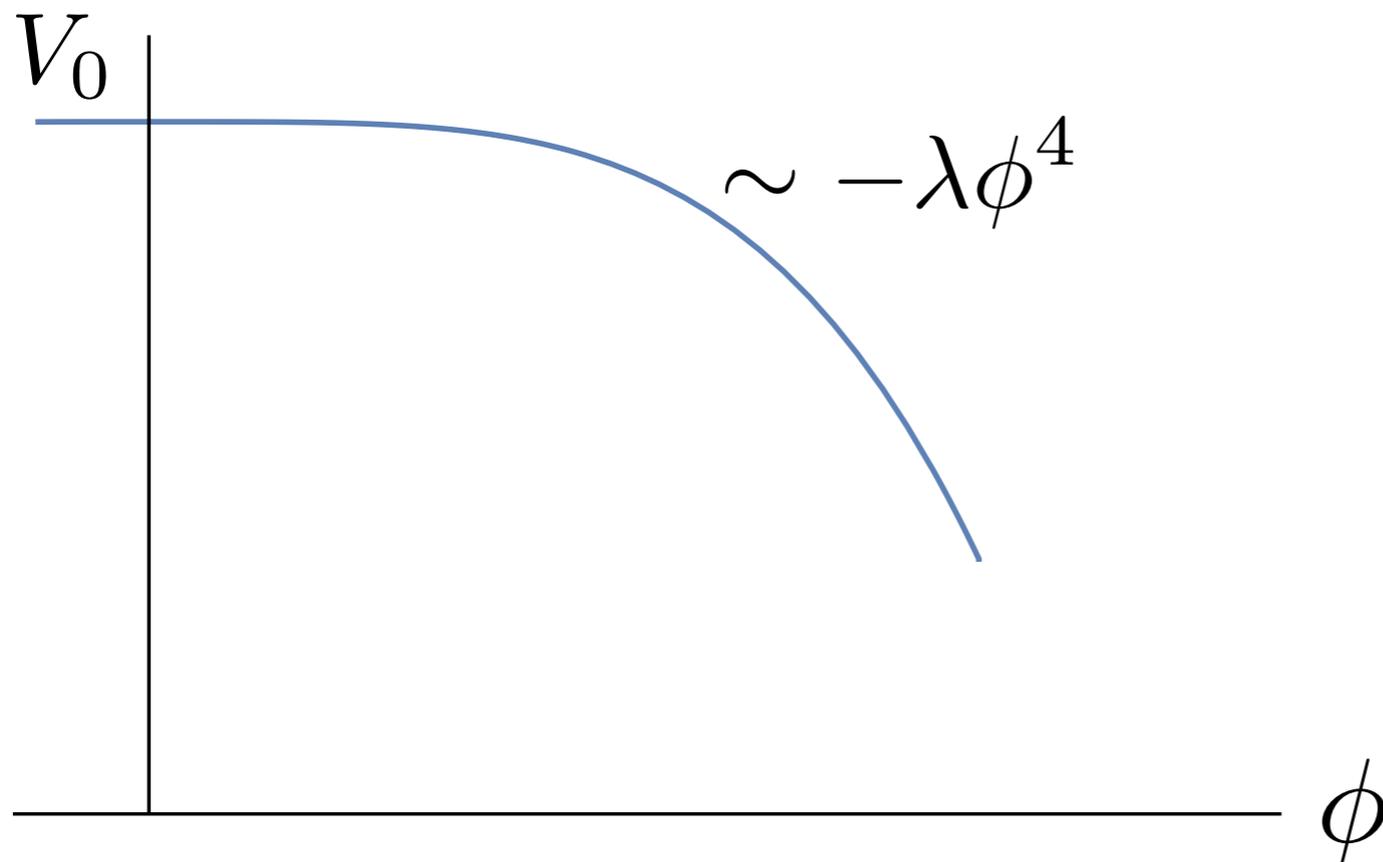
• Axion hilltop inflation

Czerny, FT 1401.5212,
Czerny, Higaki, FT 1403.0410, 1403.5883
Croon and Sanz, 1411.7809

Low-scale axion inflation can be realized with **at least two cosine terms**: “*Multi-natural inflation*”

$$V_{\text{inf}}(\phi) = \Lambda^4 \left(\cos \left(\frac{\phi}{f} + \theta \right) - \frac{\kappa}{n^2} \cos \left(\frac{n\phi}{f} \right) \right) + \text{const.}$$

$$= \boxed{V_0 - \lambda\phi^4} - \theta \frac{\Lambda^4}{f} \phi + (\kappa - 1) \frac{\Lambda^4}{2f^2} \phi^2 + \dots \quad \lambda \sim \frac{\Lambda^4}{f^4}$$



CMB normalization:

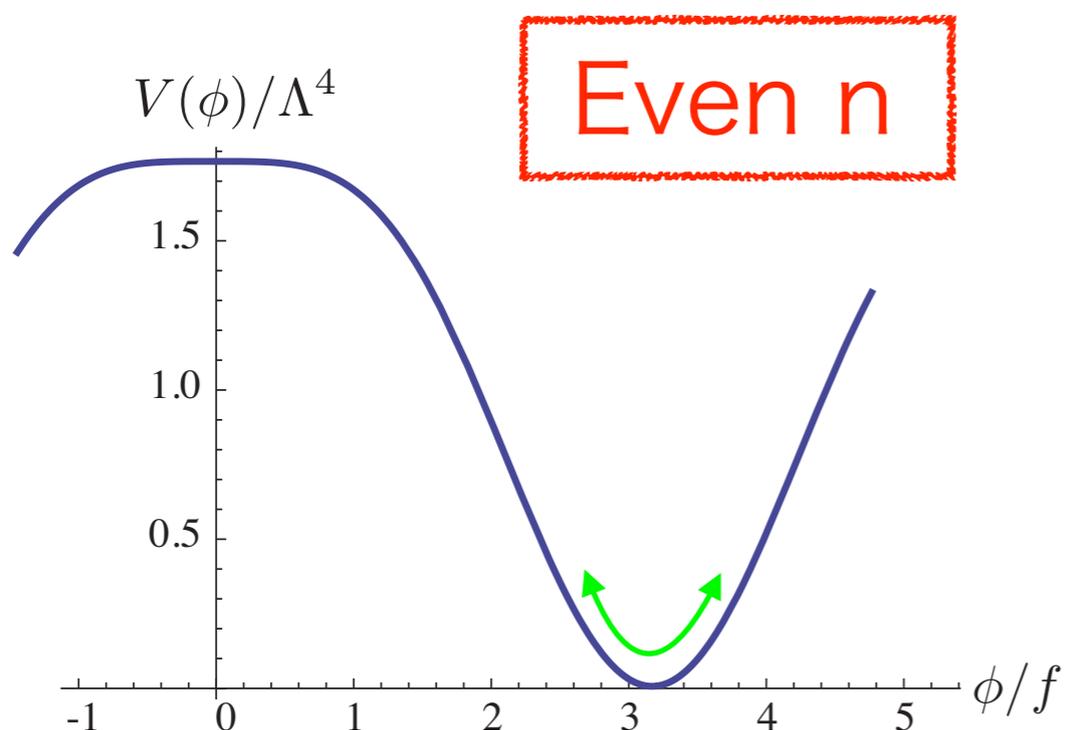
$$\lambda \sim \left(\frac{\Lambda}{f} \right)^4 \sim 10^{-13}$$

• Axion hilltop inflation

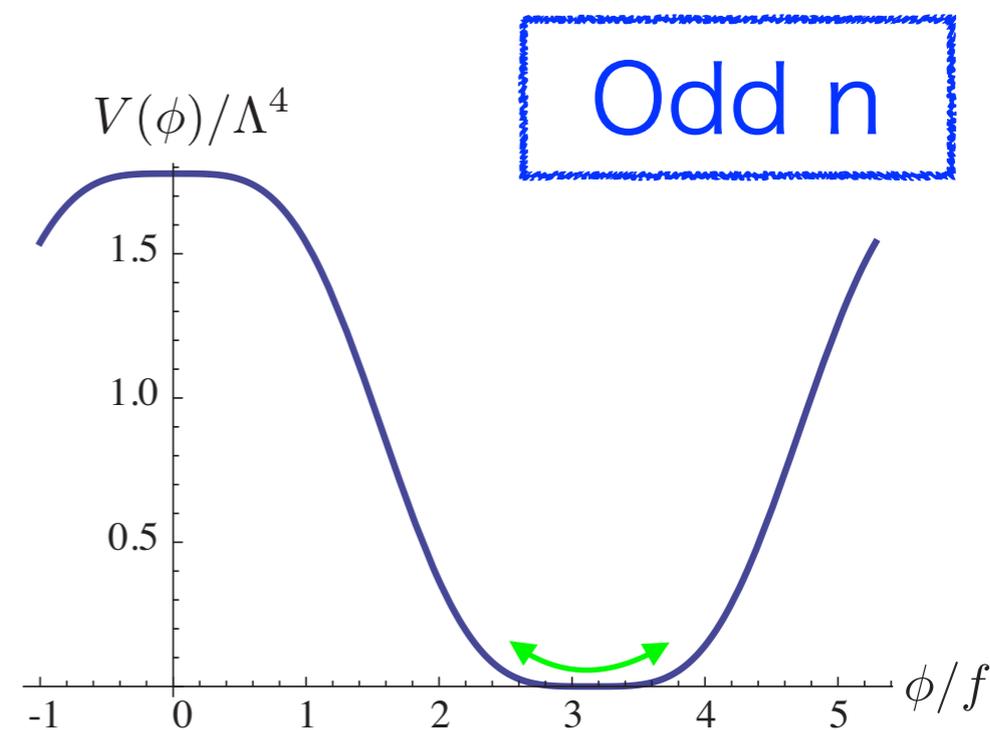
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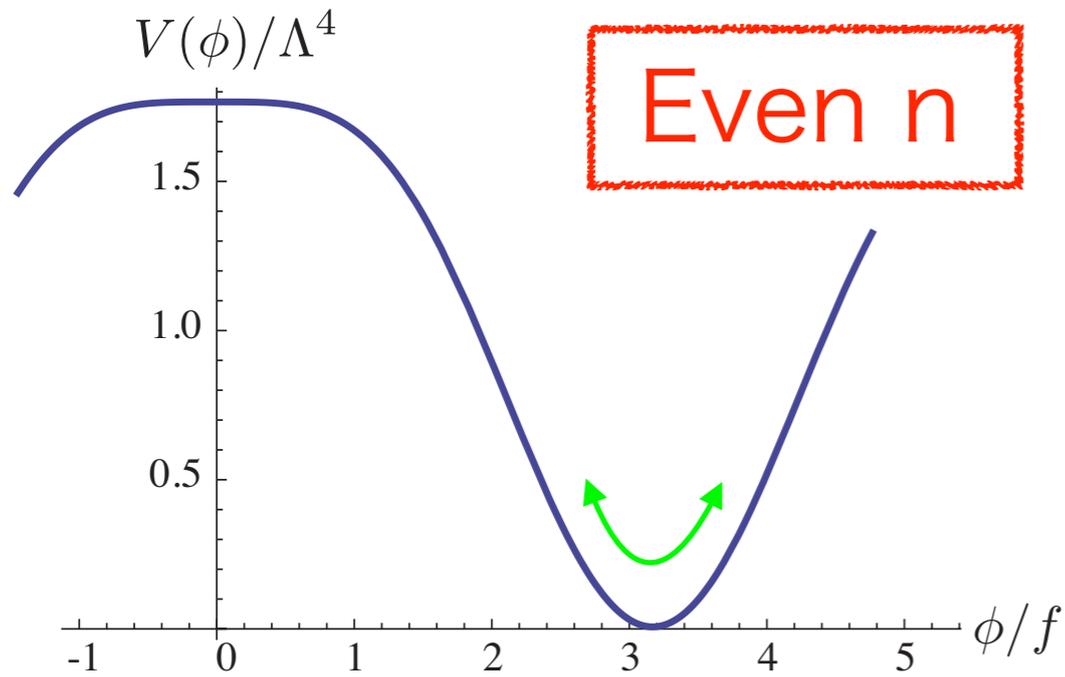
The inflaton mass at the minimum, m_ϕ , depends on n .



$$m_\phi \sim \Lambda^2 / f$$



$$m_\phi \ll \Lambda^2 / f$$



The potential is flat only around the potential maximum.

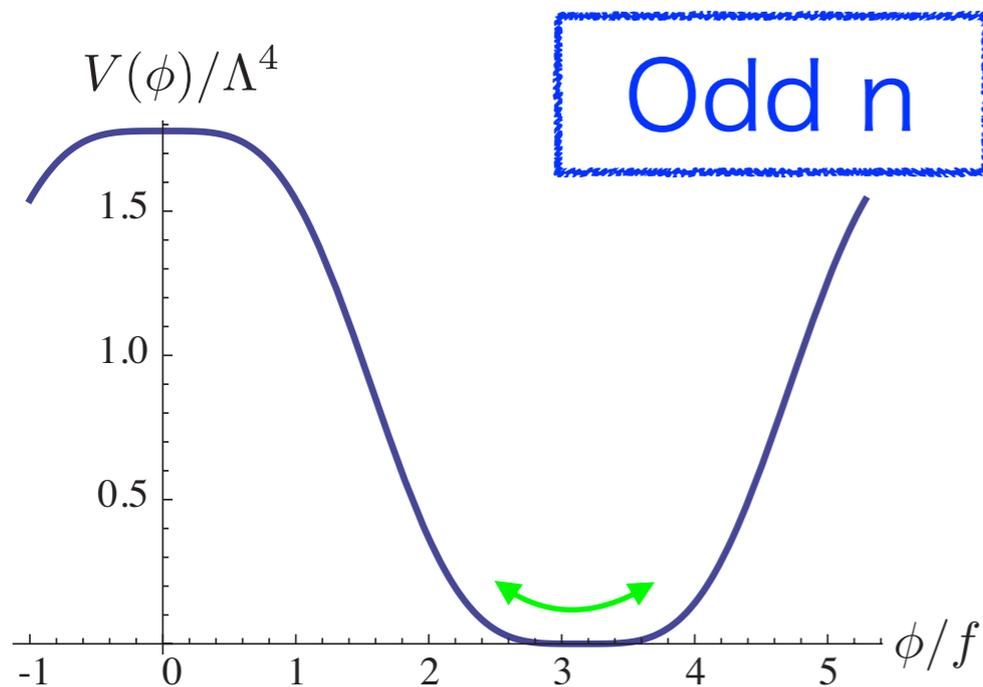
$$m_\phi \sim \frac{\Lambda^2}{f}$$

$$\lambda \sim \left(\frac{\Lambda}{f}\right)^4 \sim 10^{-13} \quad : \text{CMB norm}$$



$$f \sim 10^6 m_\phi$$

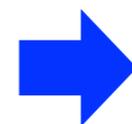
Czerny, Higaki, FT 1403.0410, FT and Yin, 1903.00462



The potential is flat both around the maximum and minimum.

$$1 - n_s = -2M_p^2 \frac{V''}{V} \simeq \frac{2}{3} \frac{m_\phi^2}{H_{\text{inf}}^2} \simeq 0.04$$

So $m_\phi \sim 0.1 H_{\text{inf}} \sim 0.1 \frac{\Lambda^2}{M_p}$ + CMB norm.

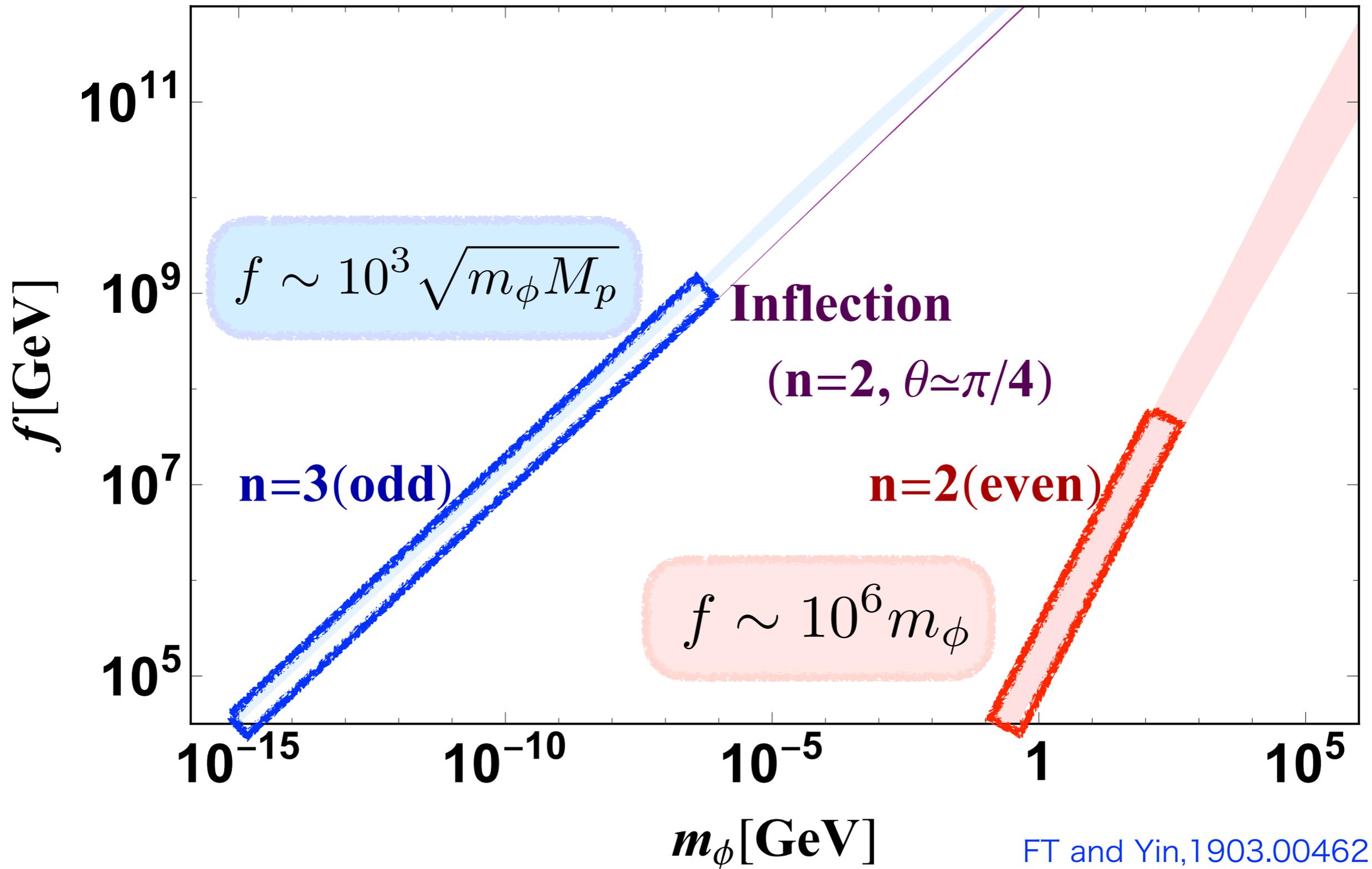


$$f \sim 10^3 \sqrt{m_\phi M_p}$$

Daido, FT, Yin, 1702.03284, 1710.11107

ALP mass and decay constant

cf. $f \sim 10^{12} \text{ GeV} (m_a / 6 \mu\text{eV})^{-1}$ for QCD axion



ALP mass and decay constant

In the case of the ALP coupled to photons

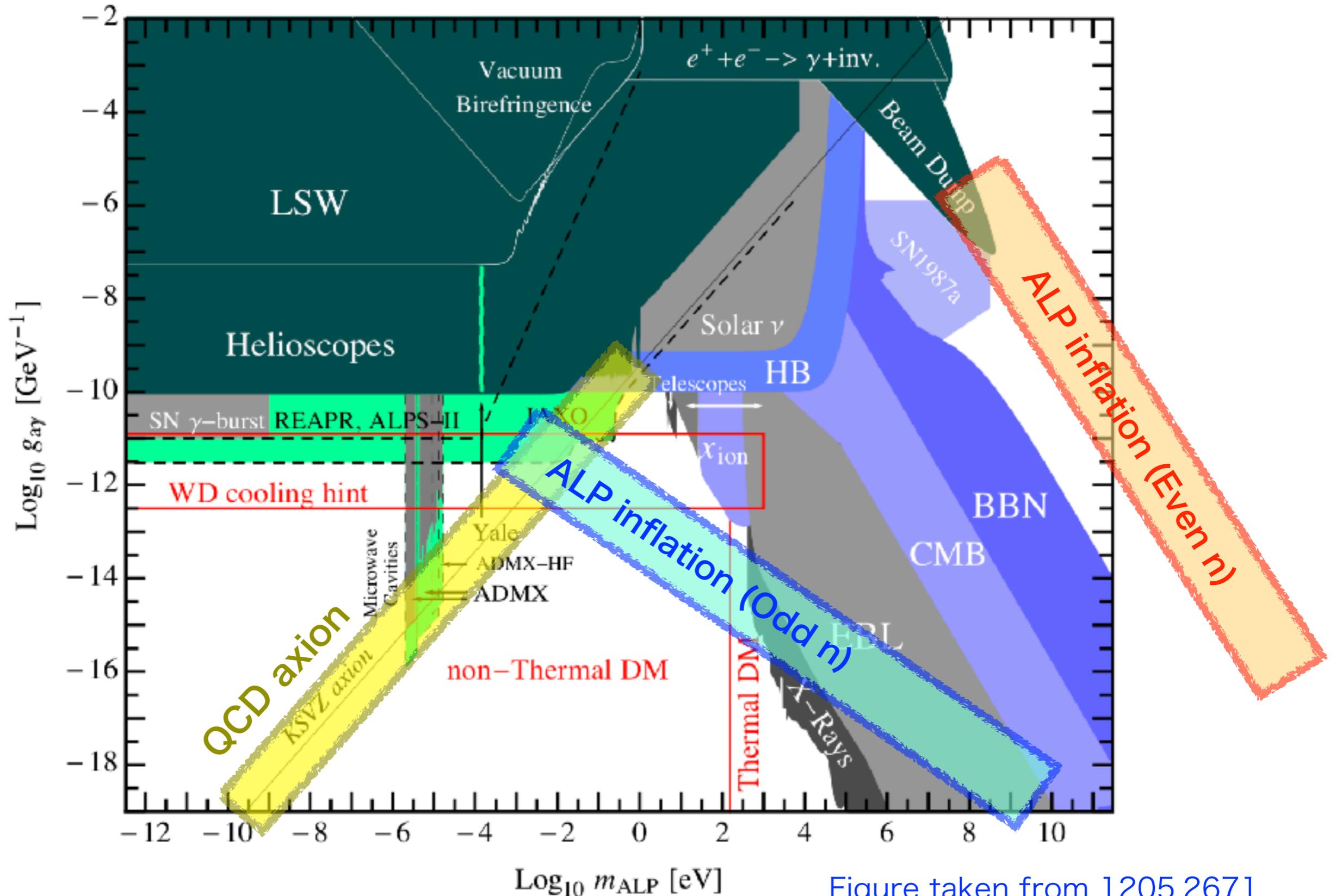


Figure taken from 1205.2671

We consider the following ALP inflaton couplings to the SM:

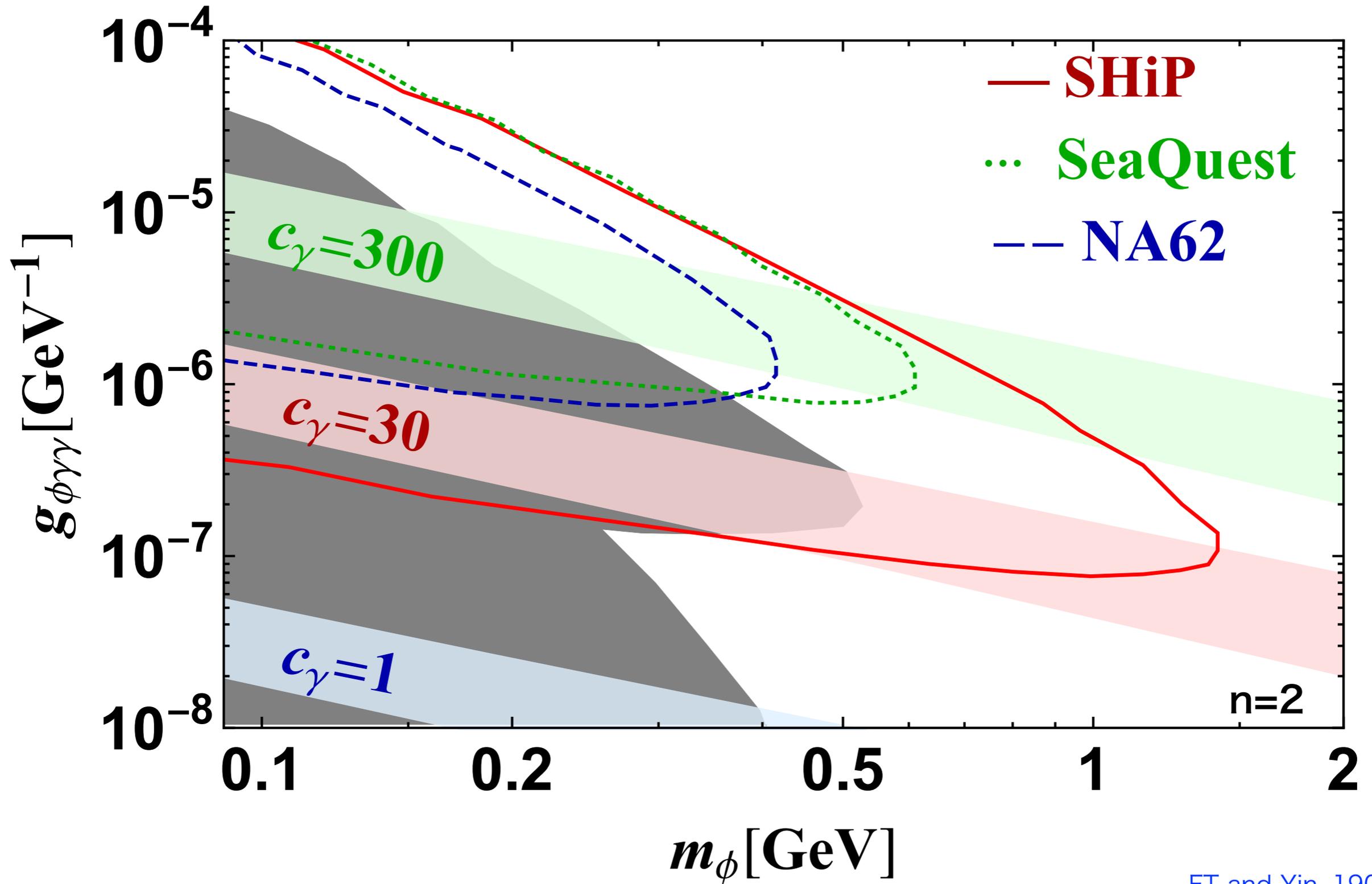
(1) Coupling to photons (or weak gauge bosons at high T)

$$\mathcal{L} = c_\gamma \frac{\alpha}{4\pi} \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} \equiv \frac{1}{4} g_{\phi\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu},$$

(2) Couplings to the SM fermions

$$\mathcal{L} = \sum_k i \frac{c_k m_k}{f} \phi \bar{\psi}_k \gamma_5 \psi_k,$$

Parameter regions predicted by ALP inflation (with even n)

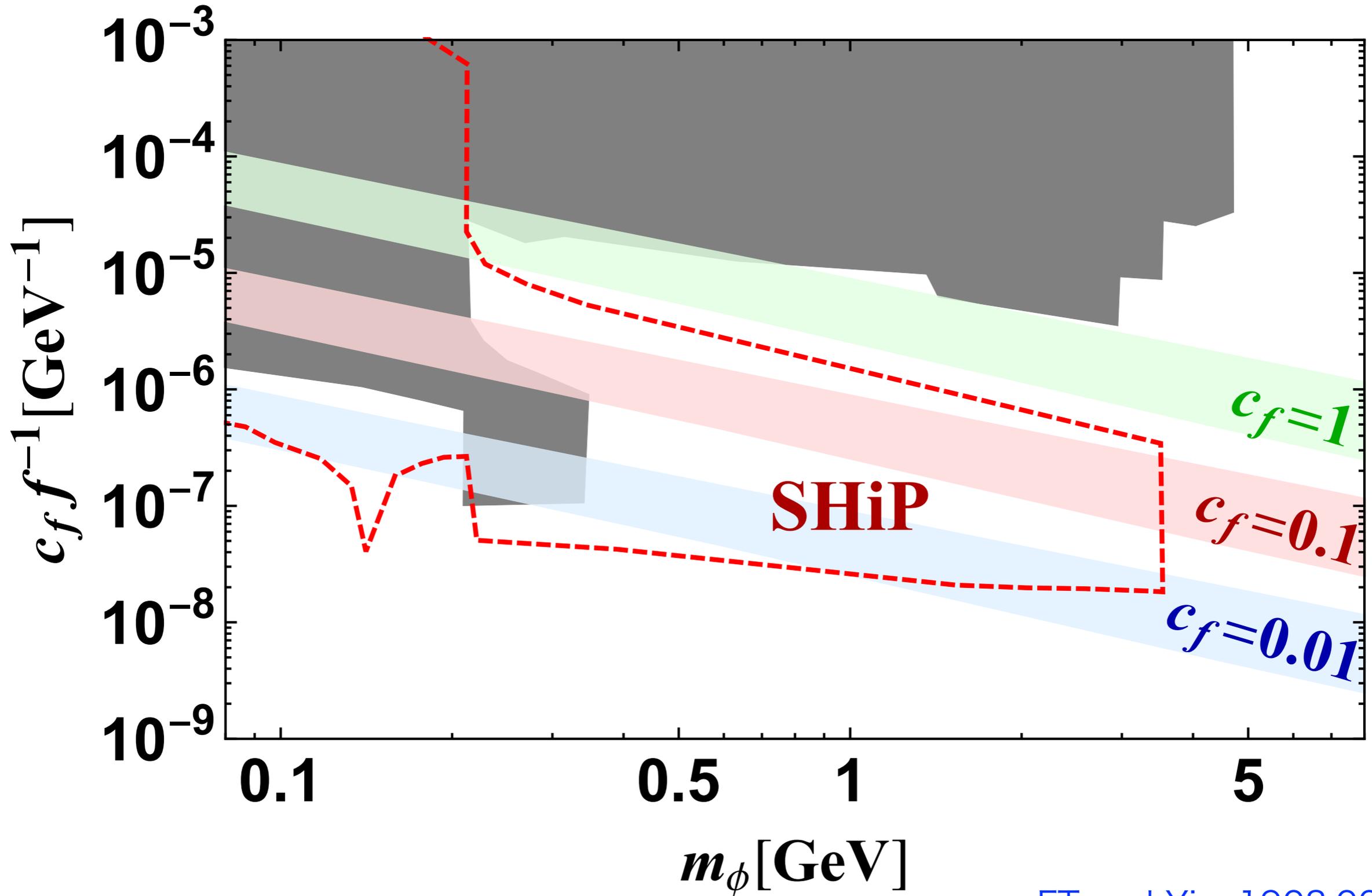


FT and Yin, 1903.00462,

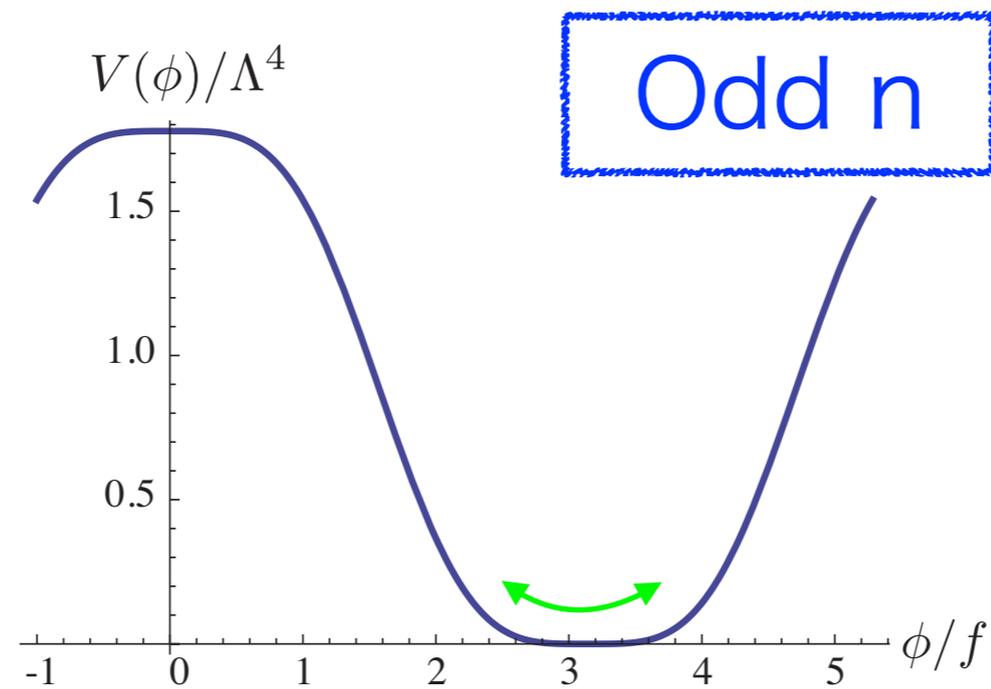
Limits adopted from Harland-Land et al 1902.04878.

For $c_\gamma \gtrsim 10$, the predicted region overlaps with the SHiP sensitivity.

In the case that the inflaton (ALP) has universal Yukawa-like interactions, i.e., $c_i = c_f$



ALP = inflaton = DM



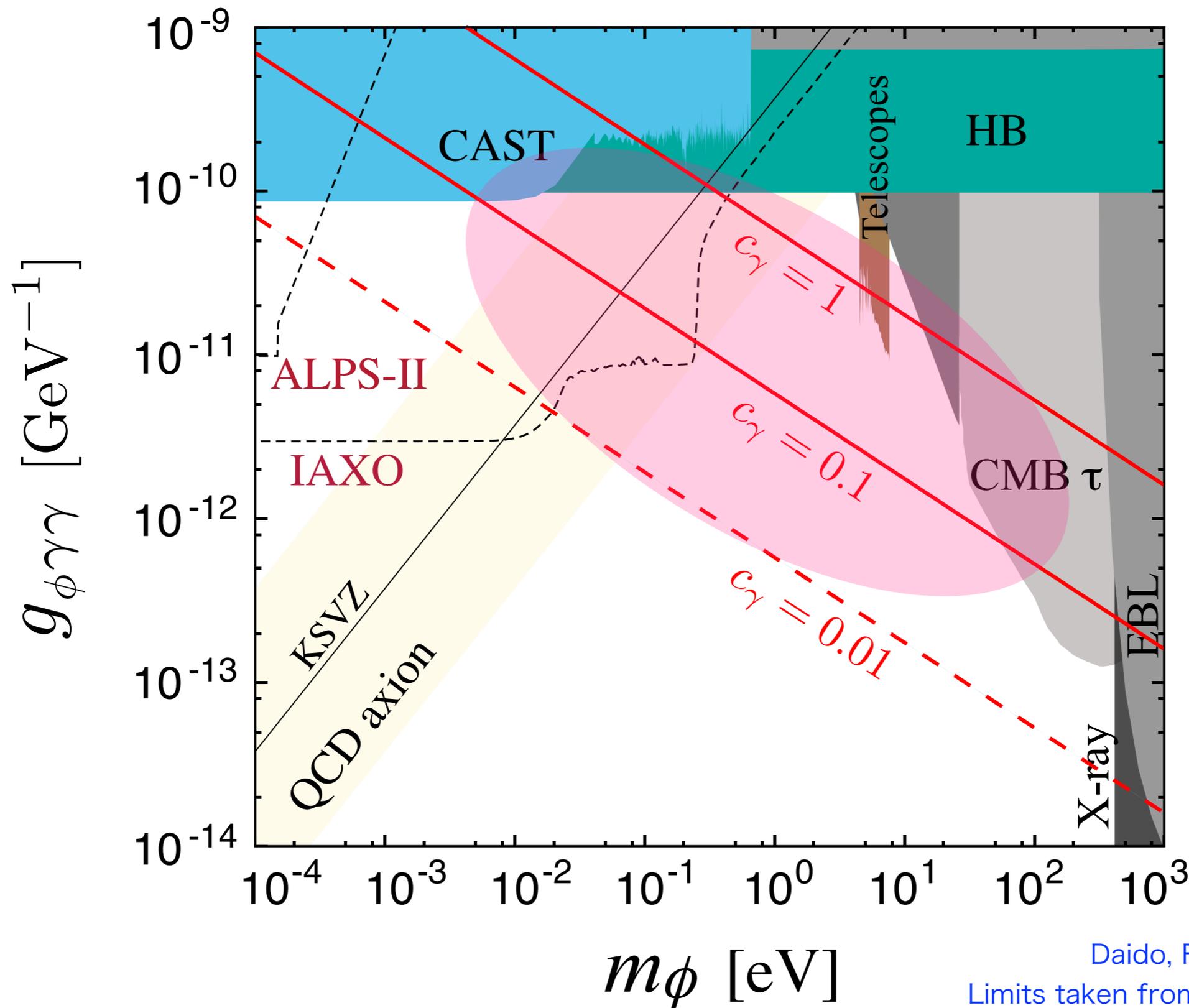
Inflaton mass and coupling to photons

$$\mathcal{L} = \frac{g_{\phi\gamma\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \quad g_{\phi\gamma\gamma} = \frac{c_\gamma \alpha}{\pi f}$$

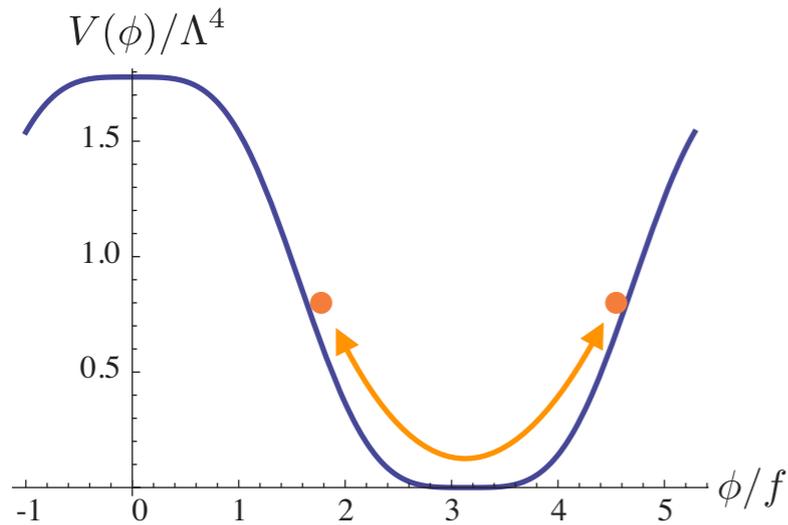
$$c_\gamma = \sum_i q_i Q_i^2$$

$$\psi_i \rightarrow e^{i\beta q_i \gamma_5 / 2} \psi_i$$

$$\phi \rightarrow \phi + \beta f$$



Reheating and ALP DM



Inflaton (ALP)
condensate

$$\mathcal{L} = \frac{g_{\phi\gamma\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\xi \equiv \left. \frac{\rho_\phi}{\rho_\phi + \rho_R} \right|_{\text{after reheating}}$$

Decay &
dissipation

Photons,
SM particles

Thermalized

ALP Dark Radiation
or HDM

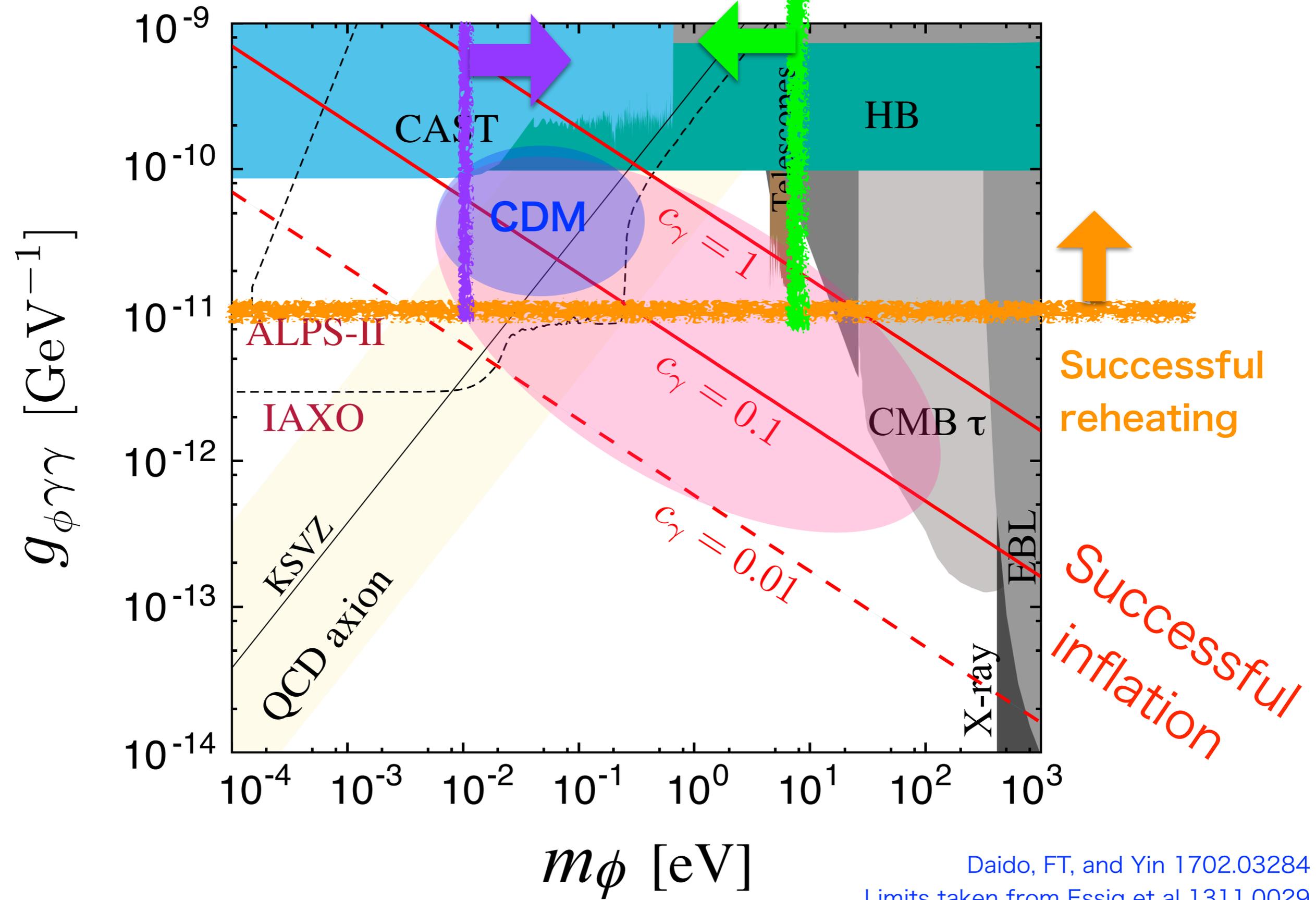
Remnant

ALP Dark Matter

As we shall see, $\xi = \mathcal{O}(0.01)$ is required to explain DM.

Small-scale structure
constraint on ALP CDM

HDM constraint on
thermalized ALP

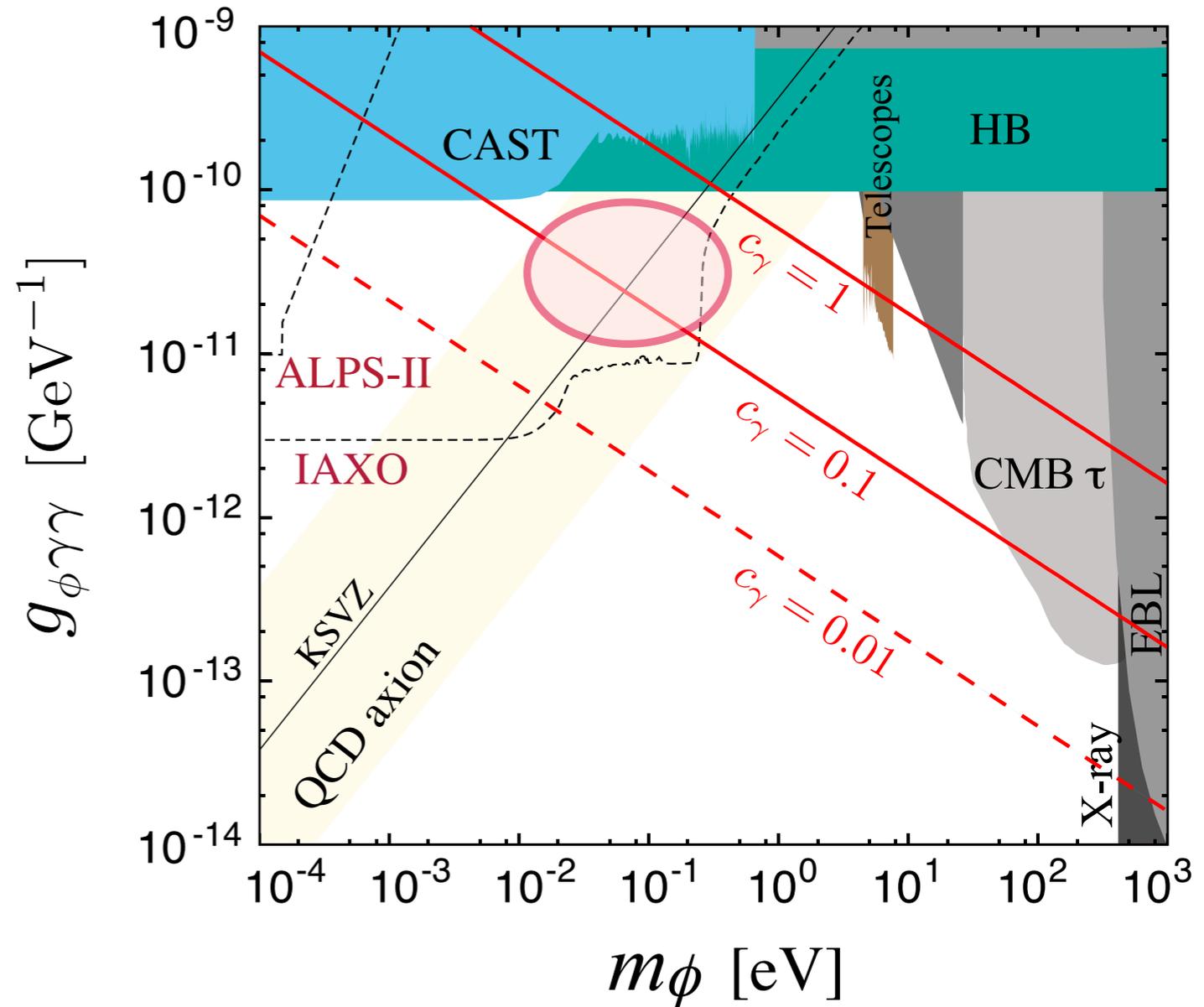


Inflaton = DM = ALP



$$m_\phi = \mathcal{O}(0.01 - 1) \text{ eV}$$
$$g_{\phi\gamma\gamma} = \mathcal{O}(10^{-11}) \text{ GeV}^{-1}$$

within the reach of future axion helioscopes and laser experiments.



“An ALP miracle”

*Plus, there is a preference for extra cooling of HB stars

$$g_{\phi\gamma\gamma} = (0.29 \pm 0.18) \times 10^{-10} \text{ GeV}^{-1}$$

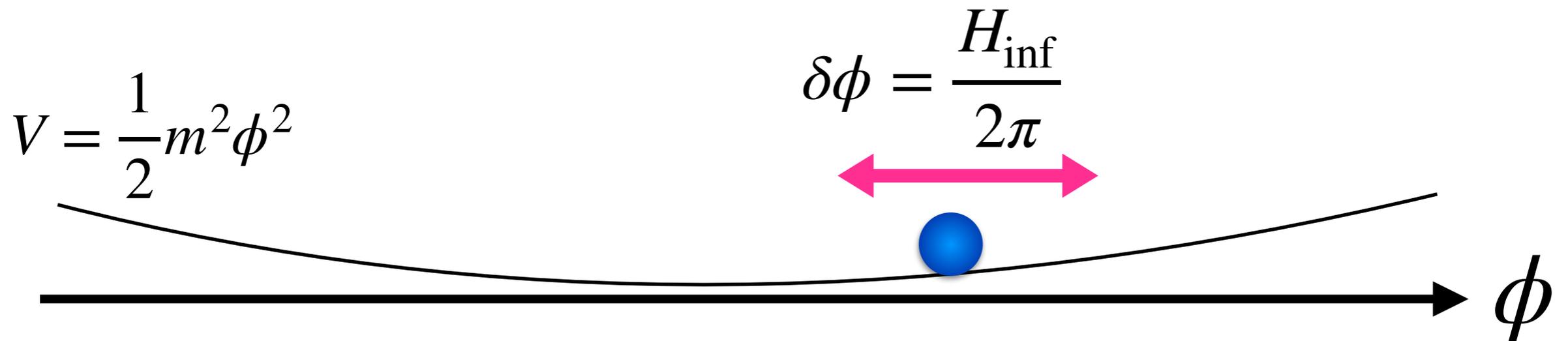
ALP = ?

Back-ups

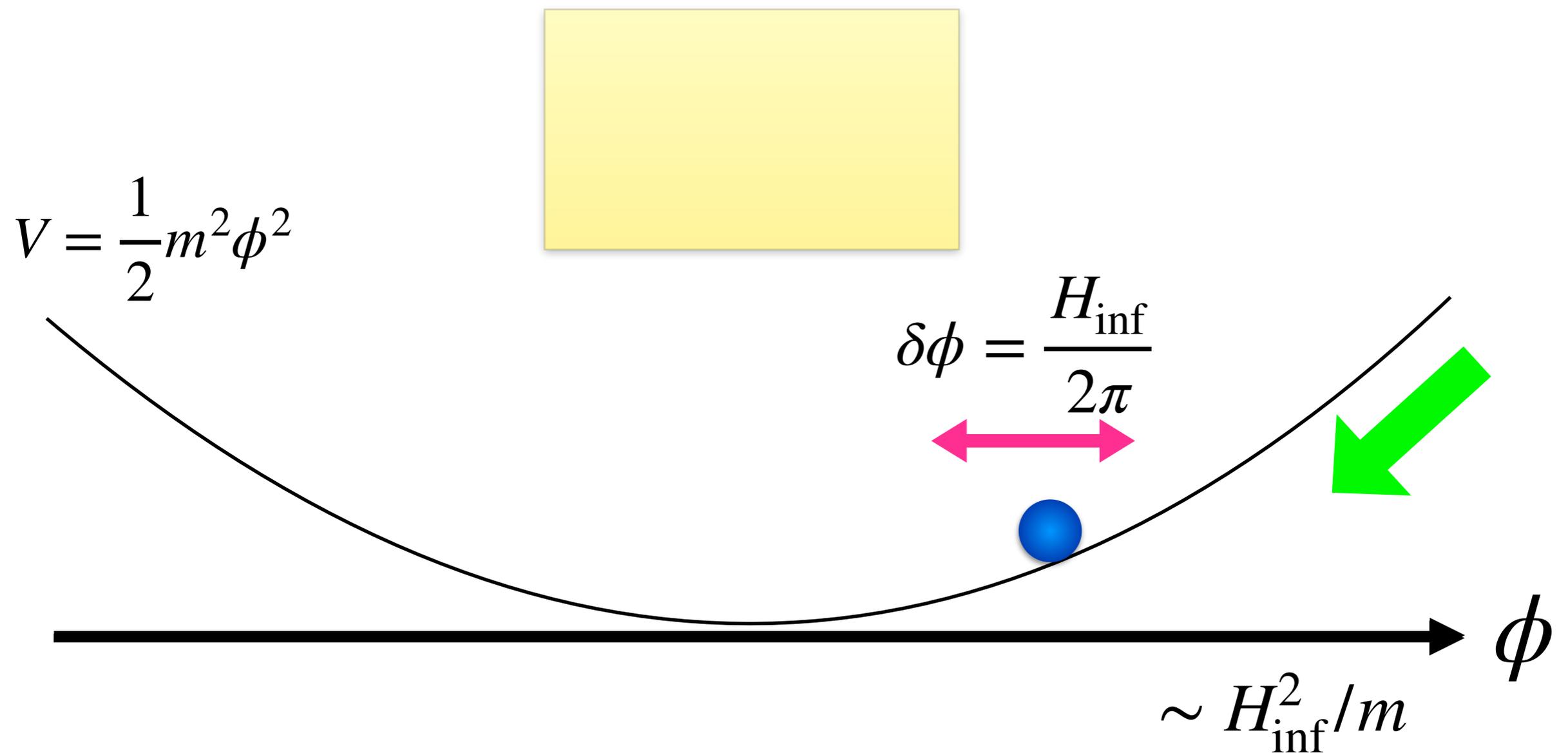
Consider a general light scalar during inflation.

$$m \ll H_{\text{inf}}$$

Then, the scalar acquires a quantum fluctuation. Even if it initially sits near the minimum, it soon goes away due to the fluctuations.



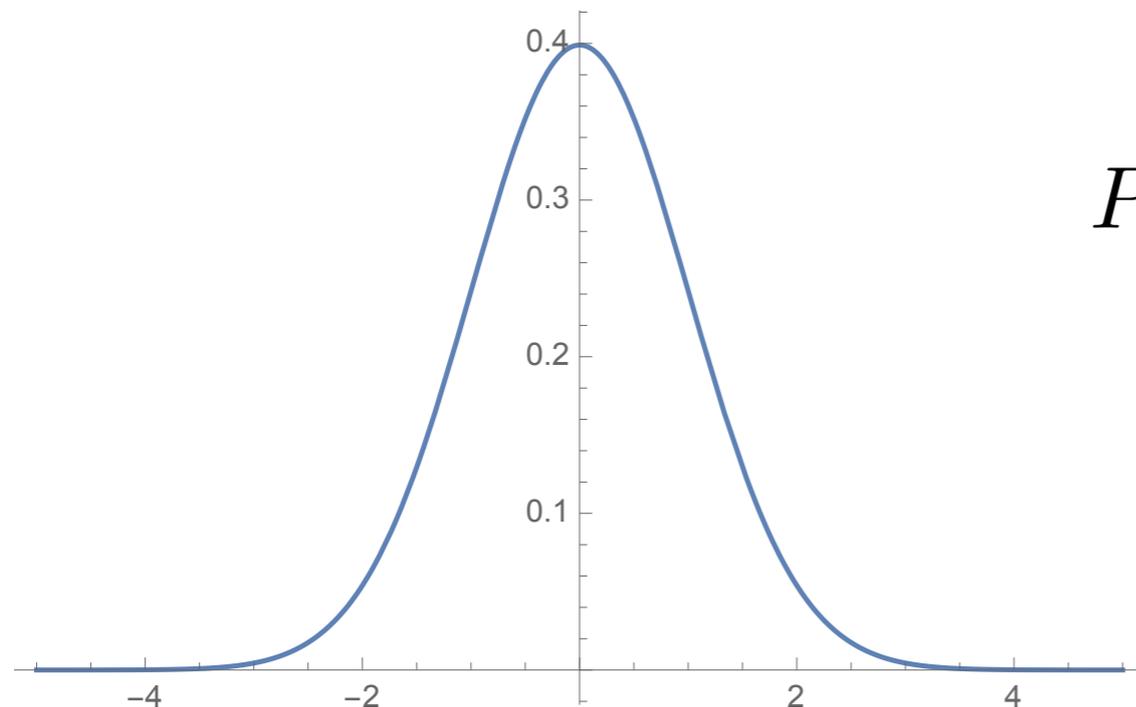
However, this does not continue forever. In the end, quantum dissipation is balanced by the classical motion, if the inflation lasts sufficiently long.



Bunch-Davis distribution

Bunch and Davies '78

$$\langle 0 | \delta\phi(\mathbf{x}, t)^2 | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} |\delta\phi_k|^2 \simeq \int \frac{k^2 dk}{2\pi^2} \frac{H^2}{2k^3} \left(\frac{k}{aH} \right)^{\frac{2m^2}{3H^2}} \simeq \frac{3H^4}{8\pi^2 m^2}$$



$$P[\phi] = \frac{1}{\sqrt{2\pi \langle \delta\phi^2 \rangle}} \exp\left(-\frac{\phi^2}{2 \langle \delta\phi^2 \rangle}\right)$$

The scalar knows where the minimum is.

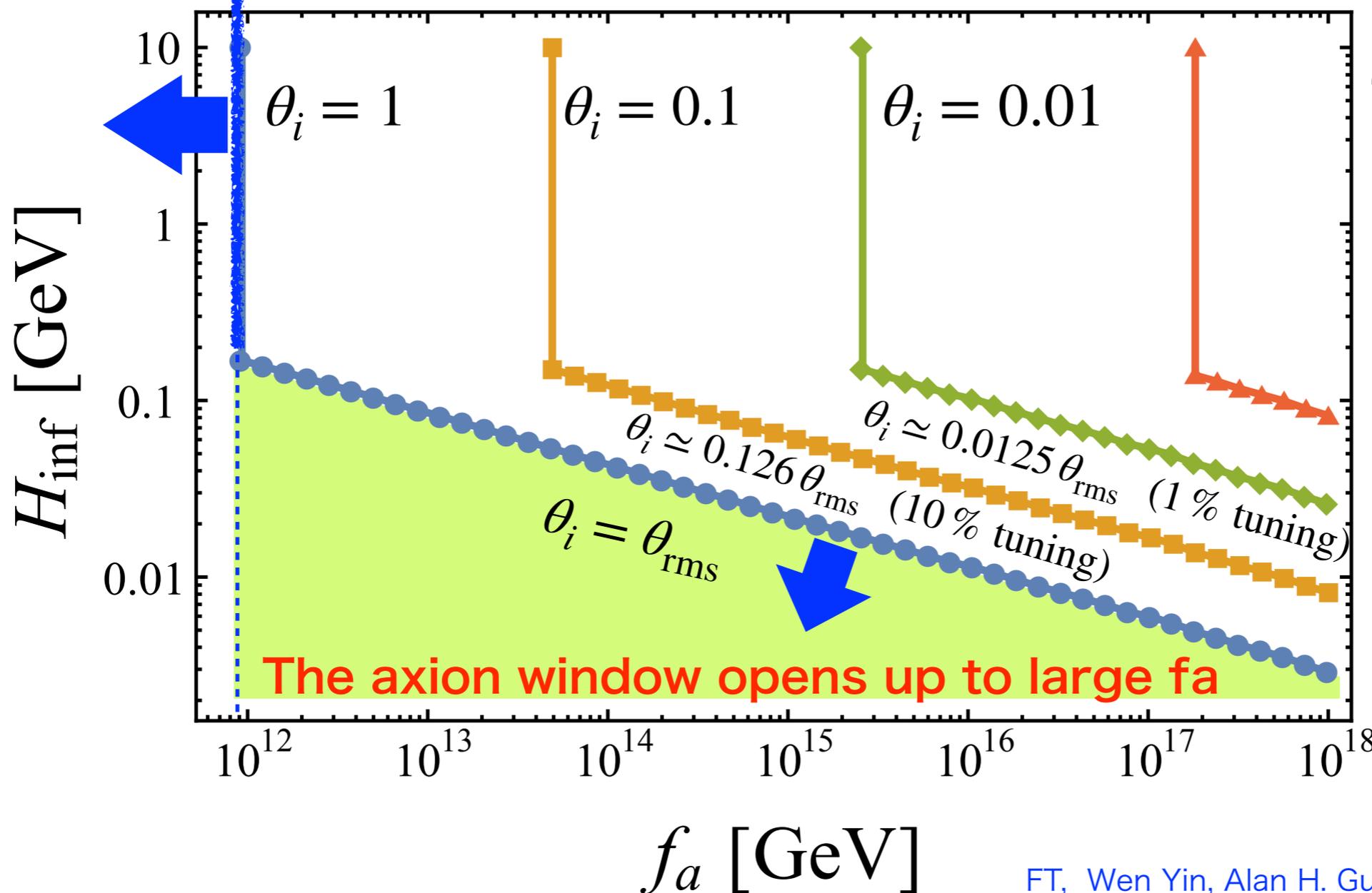
In the case of the QCD axion, the mass is T-dependent.

We set $m_a = m_a(T)$ with $T = \frac{H_{\text{inf}}}{2\pi}$: Gibbons-Hawking temperature

The upper bound of the QCD axion window can be relaxed in low-scale inflation with $H_{\text{inf}} \lesssim \Lambda_{\text{QCD}}$.

Conventional axion window, $f_a \lesssim 10^{12}$ GeV

Peter W. Graham, Adam Scherlis, 1805.07362, FT, Wen Yin, Alan H. Guth, 1805.08763



“Bunch-Davis distribution”

$$\theta_{\text{rms}} = \sqrt{\frac{3}{8\pi^2}} \frac{H_{\text{inf}}^2}{f_a m_a(T_{\text{inf}})}$$

Assumptions:

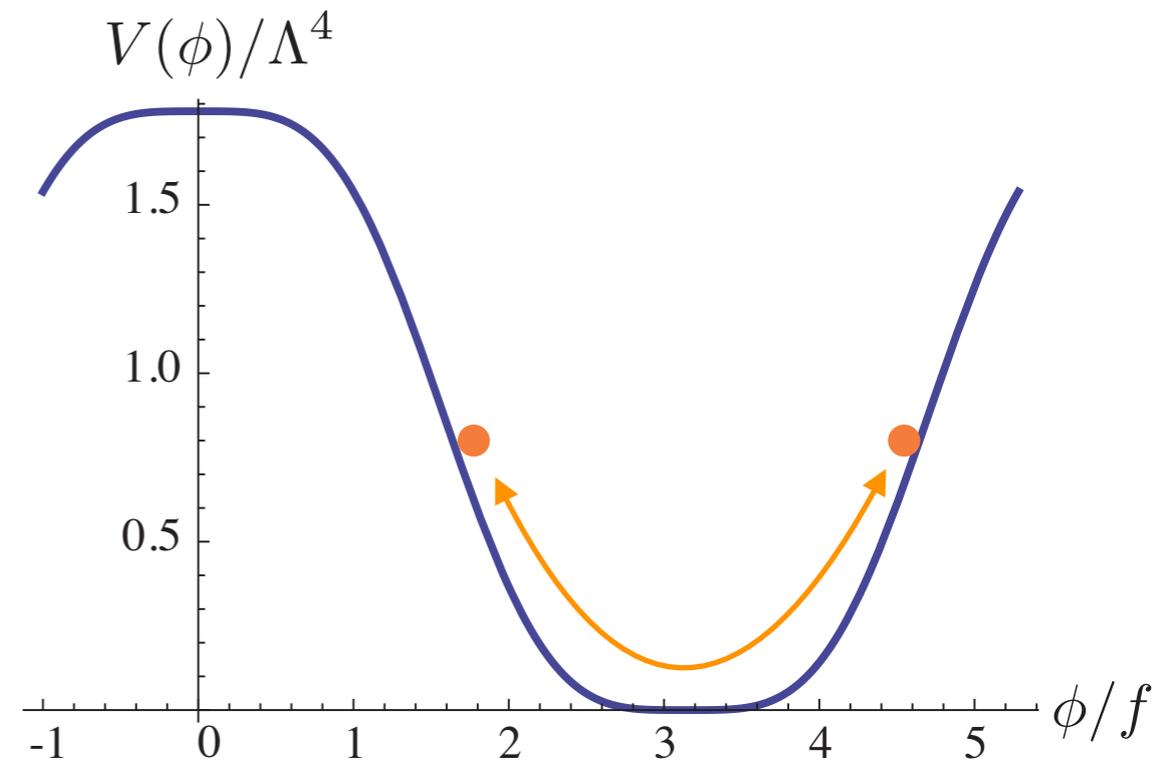
- Long inflation
 $N \gg H_{\text{inf}}^2/m^2$
- No extra contribution to θ .

• Decay and dissipation

✓ The decay rate into two photons :

$$\Gamma_{\text{dec}}(\phi \rightarrow \gamma\gamma) = \frac{c_\gamma^2 \alpha^2}{64\pi^3} \frac{m_{\text{eff}}^3}{f^2} \sqrt{1 - \left(\frac{2m_\gamma^{(th)}}{m_{\text{eff}}}\right)^2}$$

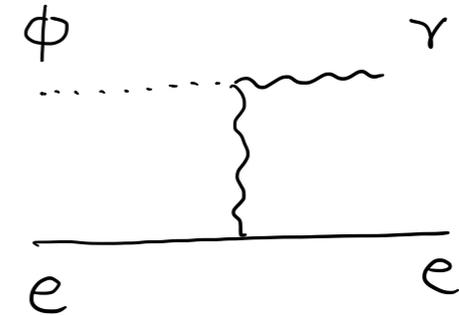
where $m_{\text{eff}}^2(t) = V''(\phi_{\text{amp}}) = 12\lambda\phi_{\text{amp}}^2$



✓ The dissipation rate is roughly given by

$$\Gamma_{\text{dis},\gamma} = C \frac{c_\gamma^2 \alpha^2 T^3}{8\pi^2 f^2} \frac{m_{\text{eff}}^2}{e^4 T^2}$$

Moroi, Mukaida, Nakayama and Takimoto, 1407.7465
cf. Salvio, Strumia, Xue, 1310.6982



Here C represents an uncertainty of the order-of-magnitude estimate as well as spatial inhomogeneities due to tachyonic preheating.

See Lozanov, Amin, 1710.06851

• Decay and dissipation

At $T > 100$ GeV, one should consider couplings to weak gauge bosons instead of photons:

$$\mathcal{L} = c_2 \frac{\alpha_2}{8\pi} \frac{\phi}{f} W_{\mu\nu} \tilde{W}^{\mu\nu} + c_Y \frac{\alpha_Y}{4\pi} \frac{\phi}{f} B_{\mu\nu} \tilde{B}^{\mu\nu},$$

with

$$c_2 = \sum_i q_i, \quad c_Y = \sum_j q_j Y_j^2, \quad c_\gamma = \frac{c_2}{2} + c_Y$$

We adopt the following dissipation rate at $T > 100$ GeV

$$\Gamma_{\text{dis,EW}} = C' \frac{c_2^2 \alpha_2^2 T^3}{32\pi^2 f^2} \frac{m_{\text{eff}}^2}{g_2^4 T^2} + C'' \frac{c_Y^2 \alpha_Y^2 T^3}{8\pi^2 f^2} \frac{m_{\text{eff}}^2}{g_Y^4 T^2},$$

• Decay and dissipation

Due to the decay and dissipation,

$$\xi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_R} \Big|_{\text{after reheating}} = \mathcal{O}(0.01 - 0.1)$$

for $g_{\phi\gamma\gamma} = \mathcal{O}(10^{-11}) \text{ GeV}^{-1}$ and $c_2, c_Y = \mathcal{O}(1)$, $C, C', C'' = \mathcal{O}(1 - 10)$

For successful reheating with $\xi \lesssim \mathcal{O}(0.1)$, one needs

$$g_{\phi\gamma\gamma} \gtrsim 10^{-11} \text{ GeV}^{-1}$$

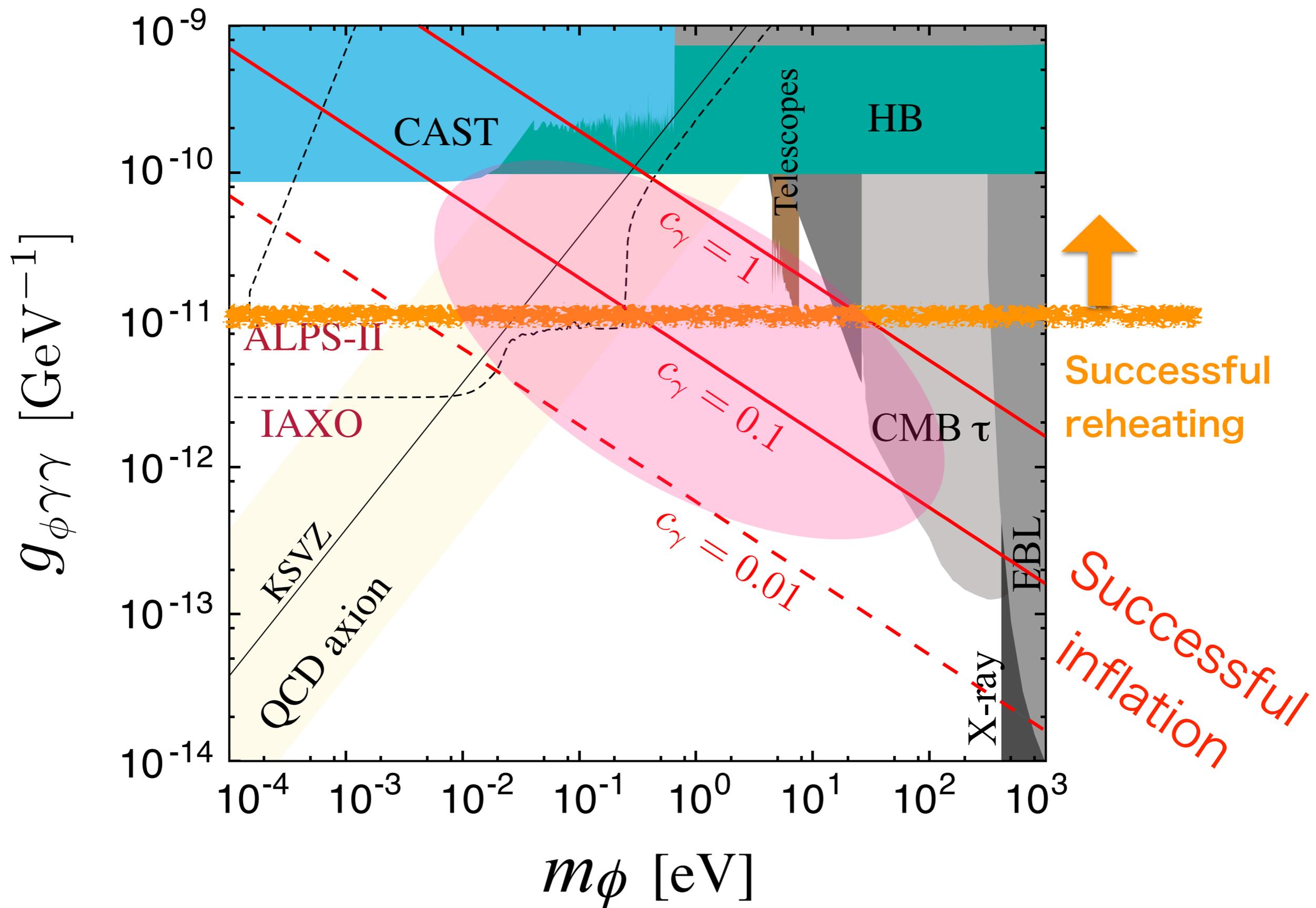
cf. BBN bound

$$\Delta N_{\text{eff}} < 1$$

→ $\xi \lesssim 0.26$

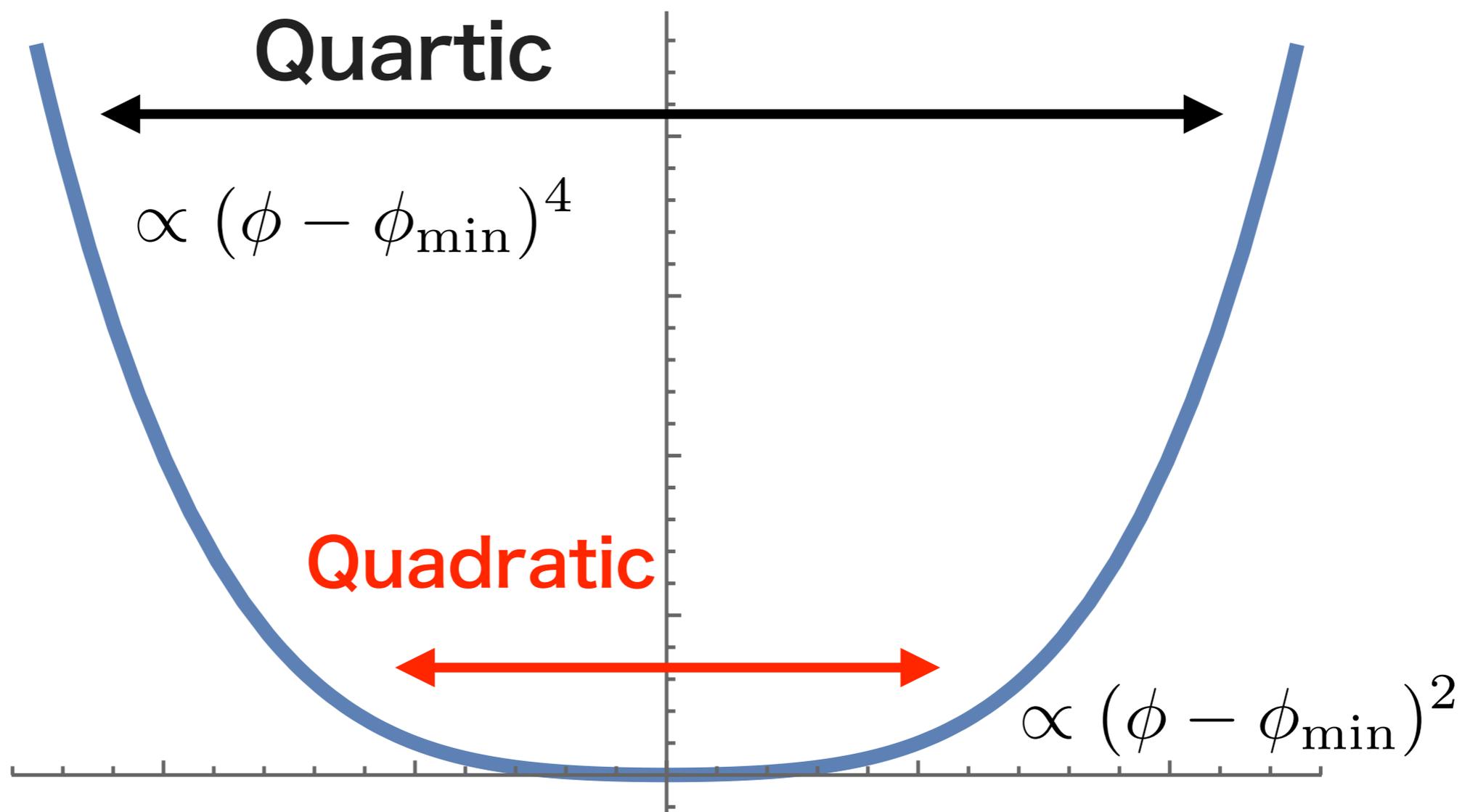
Note that the inflaton mass is then bounded above.

The typical reheating temperature $T_R \sim \mathcal{O}(10) \text{ TeV} \left(\frac{m_\phi}{1 \text{ eV}} \right)^{\frac{1}{2}}$



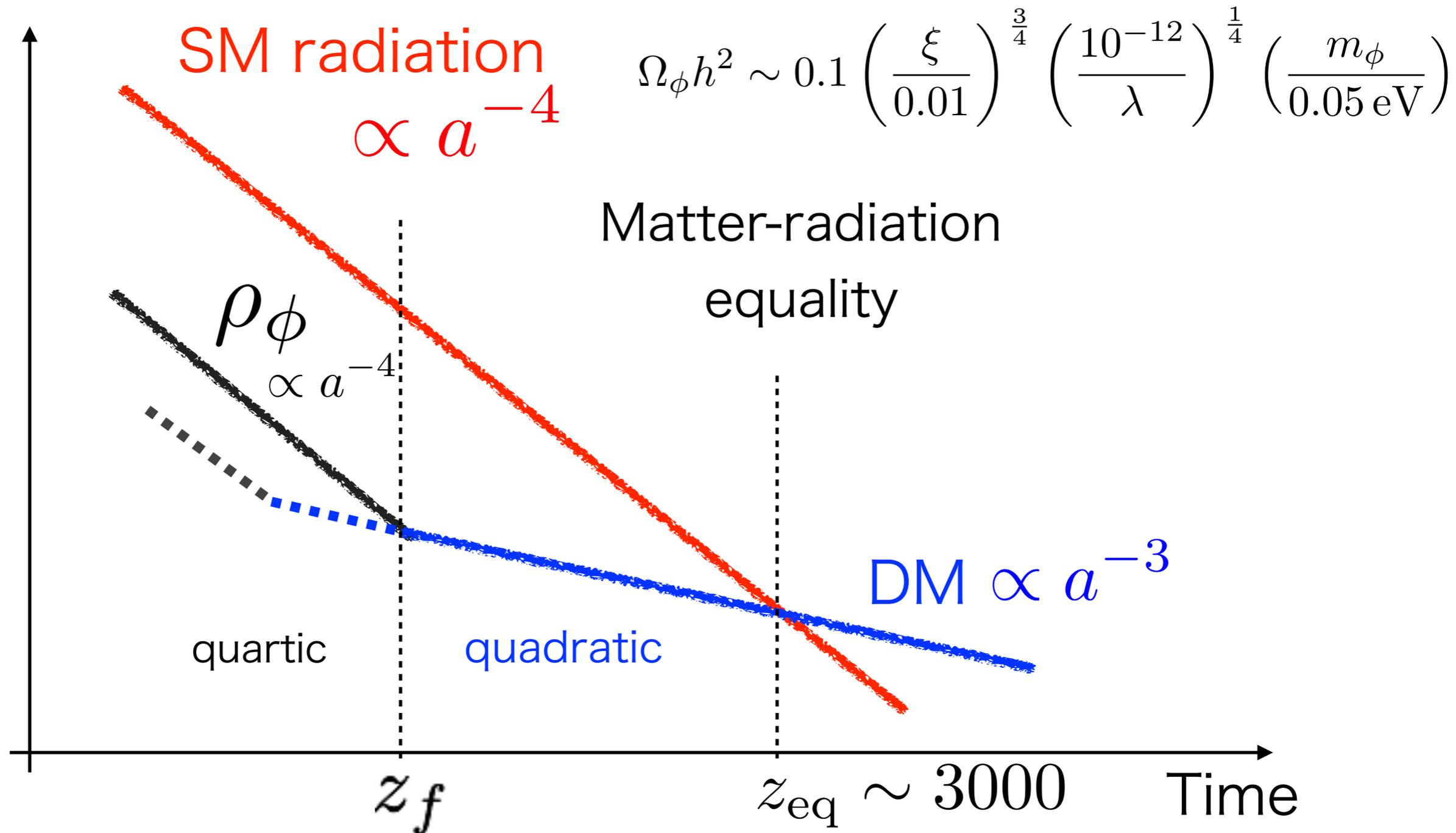
• ALP condensate as CDM

ρ_ϕ decreases like radiation when oscillating in a quartic potential and becomes matter-like in a quadratic potential.

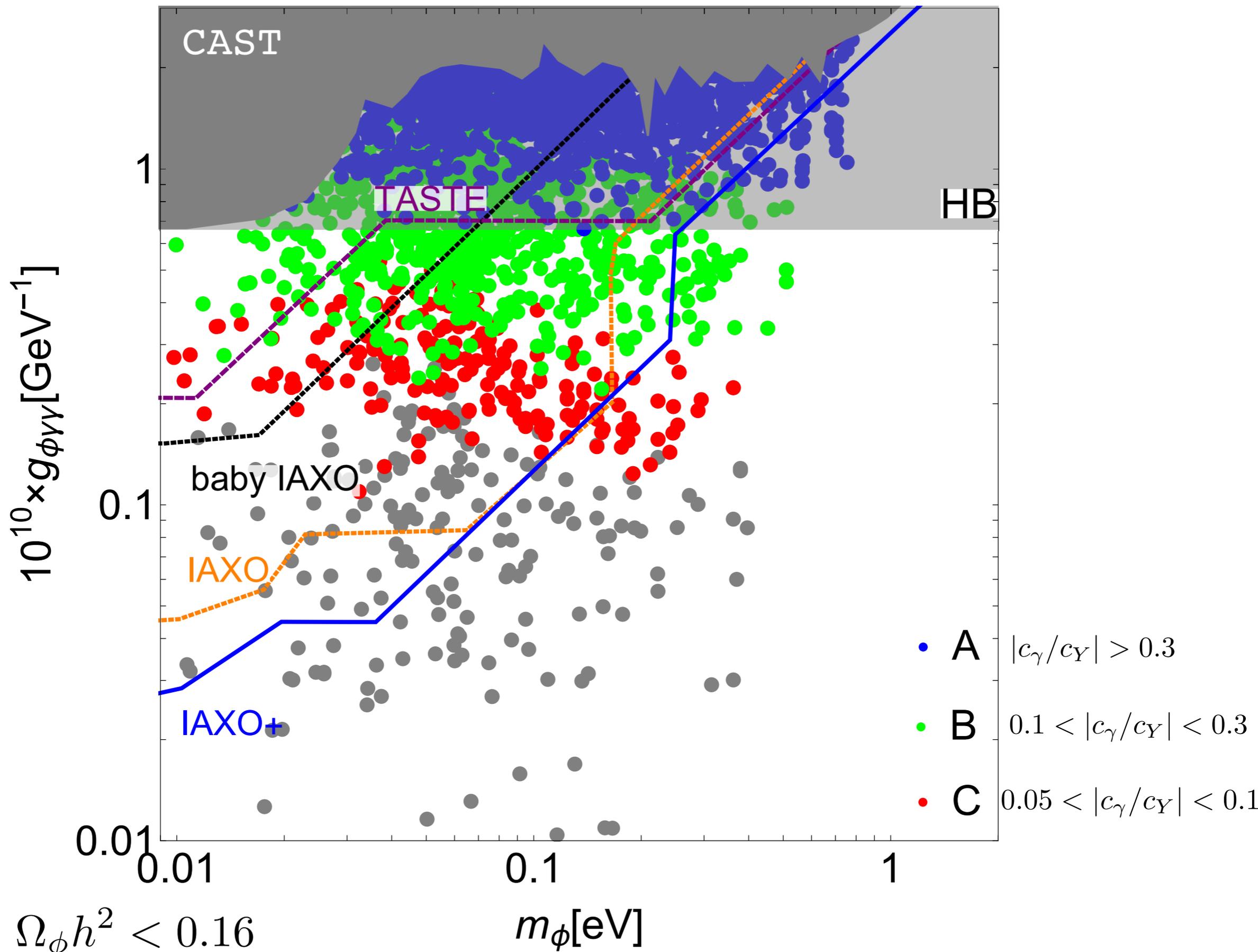


The transition takes place earlier (later) for the heavier (lighter) ALP mass.

• ALP condensate as CDM

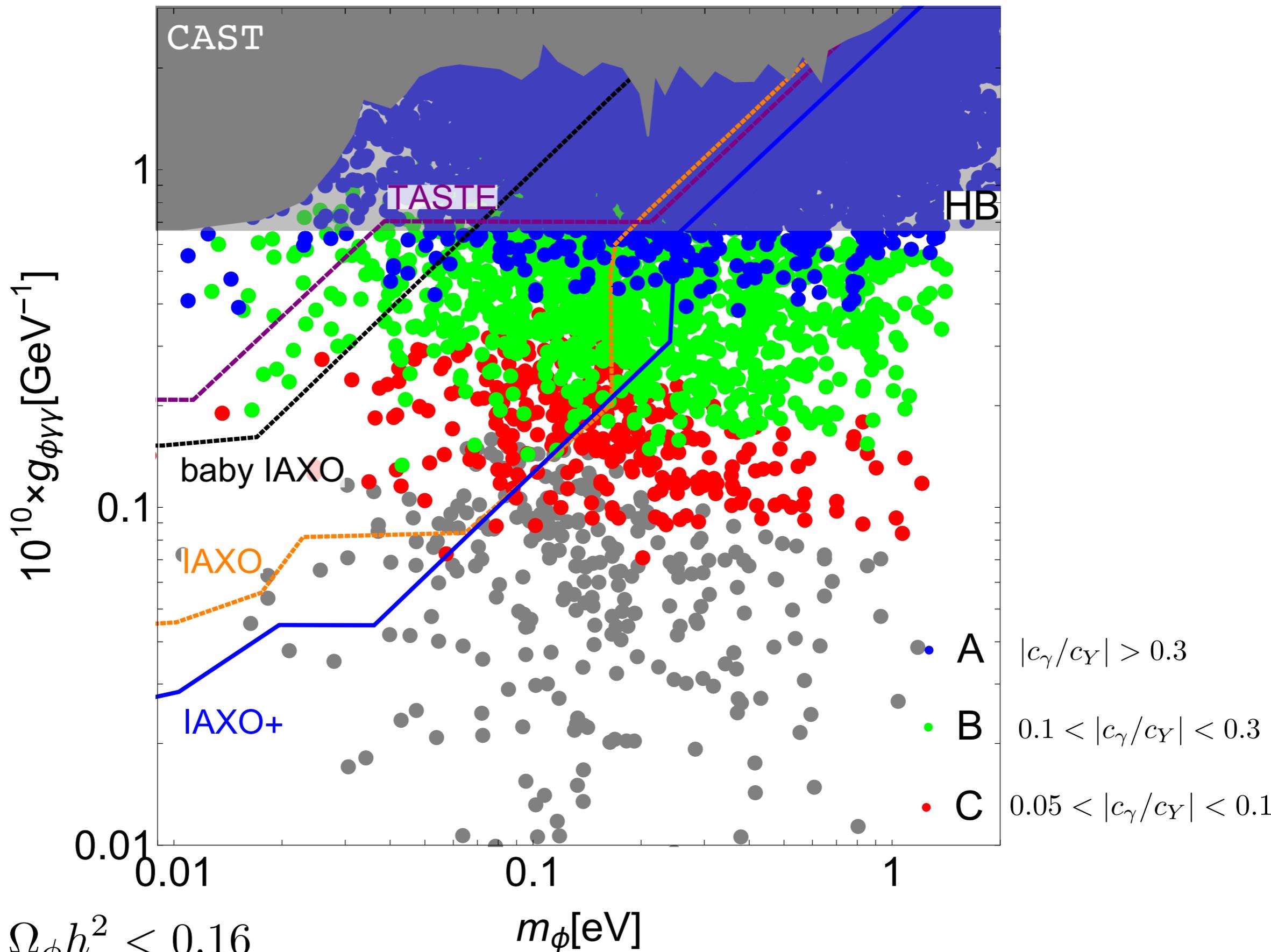


$z_f \gtrsim \mathcal{O}(10^5)$ by SDSS and Ly-alpha $\rightarrow m_\phi \gtrsim \mathcal{O}(0.01) \text{ eV}$



$$0.08 < \Omega_\phi h^2 < 0.16$$

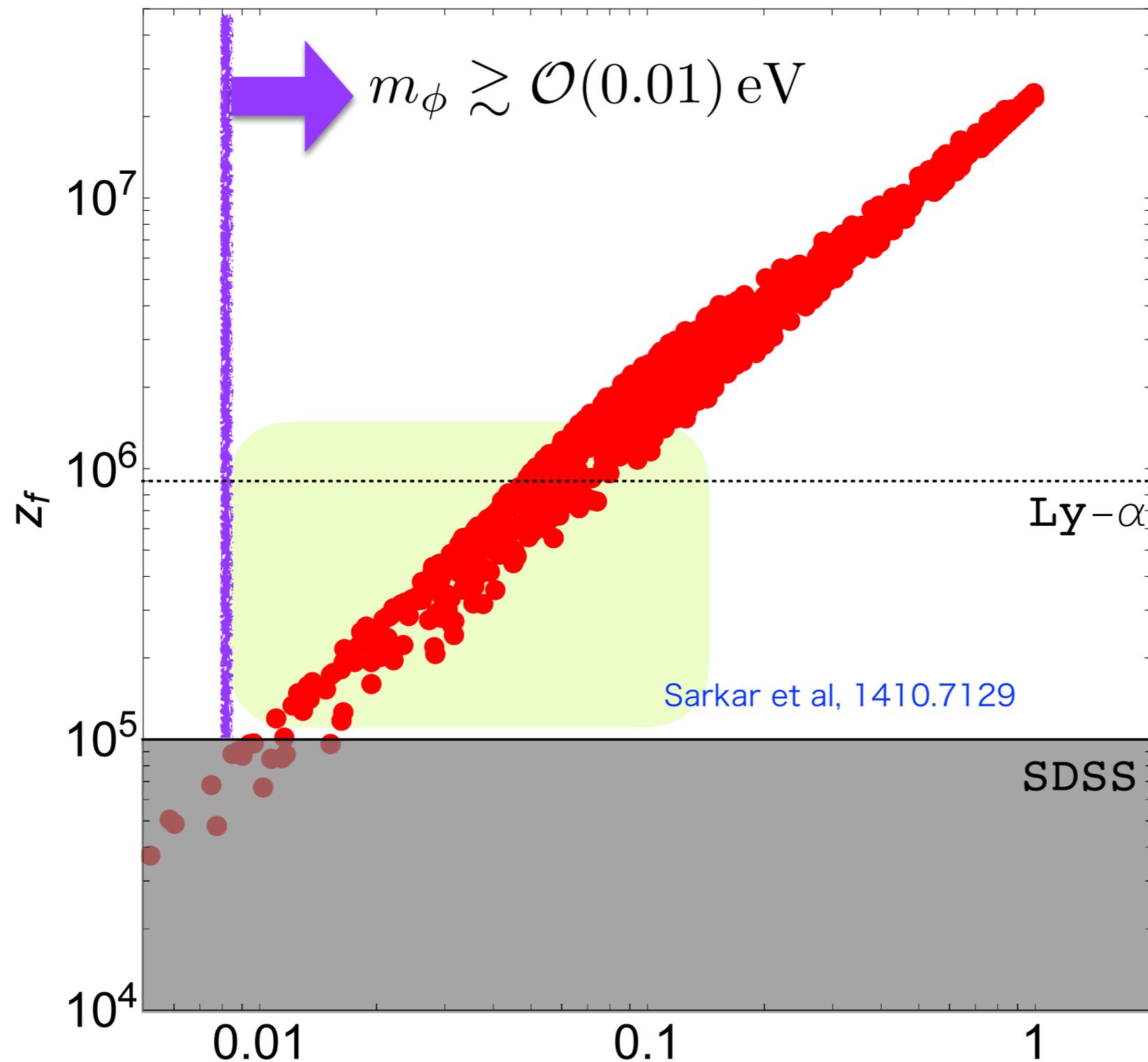
$$10^6 \text{ GeV} < f < 5 \times 10^7 \text{ GeV} \quad C = 10 \quad \text{Varied the other params by } O(1).$$



$$0.08 < \Omega_\phi h^2 < 0.16$$

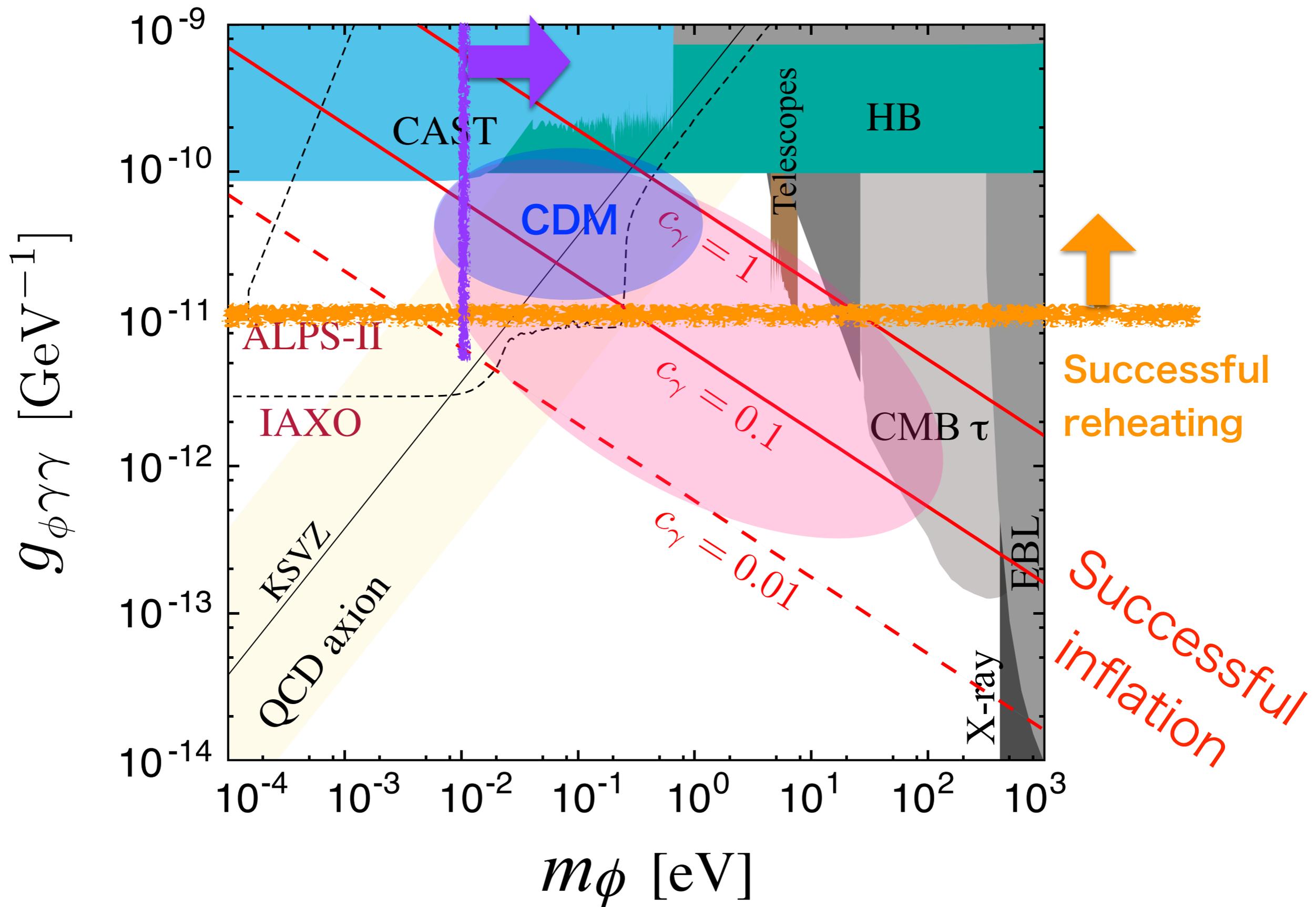
$$10^6 \text{ GeV} < f < 5 \times 10^7 \text{ GeV} \quad C = 30 \quad \text{Varied the other params by } O(1).$$

The red-shift at the transition vs. the ALP mass



$$1 + z_f = \left(\frac{1}{\Omega_\phi} \frac{\rho_f}{\rho_c} \right)^{\frac{1}{3}} \sim 7 \times 10^5 \left(\frac{0.12}{\Omega_\phi h^2} \right)^{\frac{1}{3}} \left(\frac{10^{-12}}{\lambda} \right)^{\frac{1}{3}} \left(\frac{m_\phi}{0.05 \text{ eV}} \right)^{\frac{4}{3}}$$

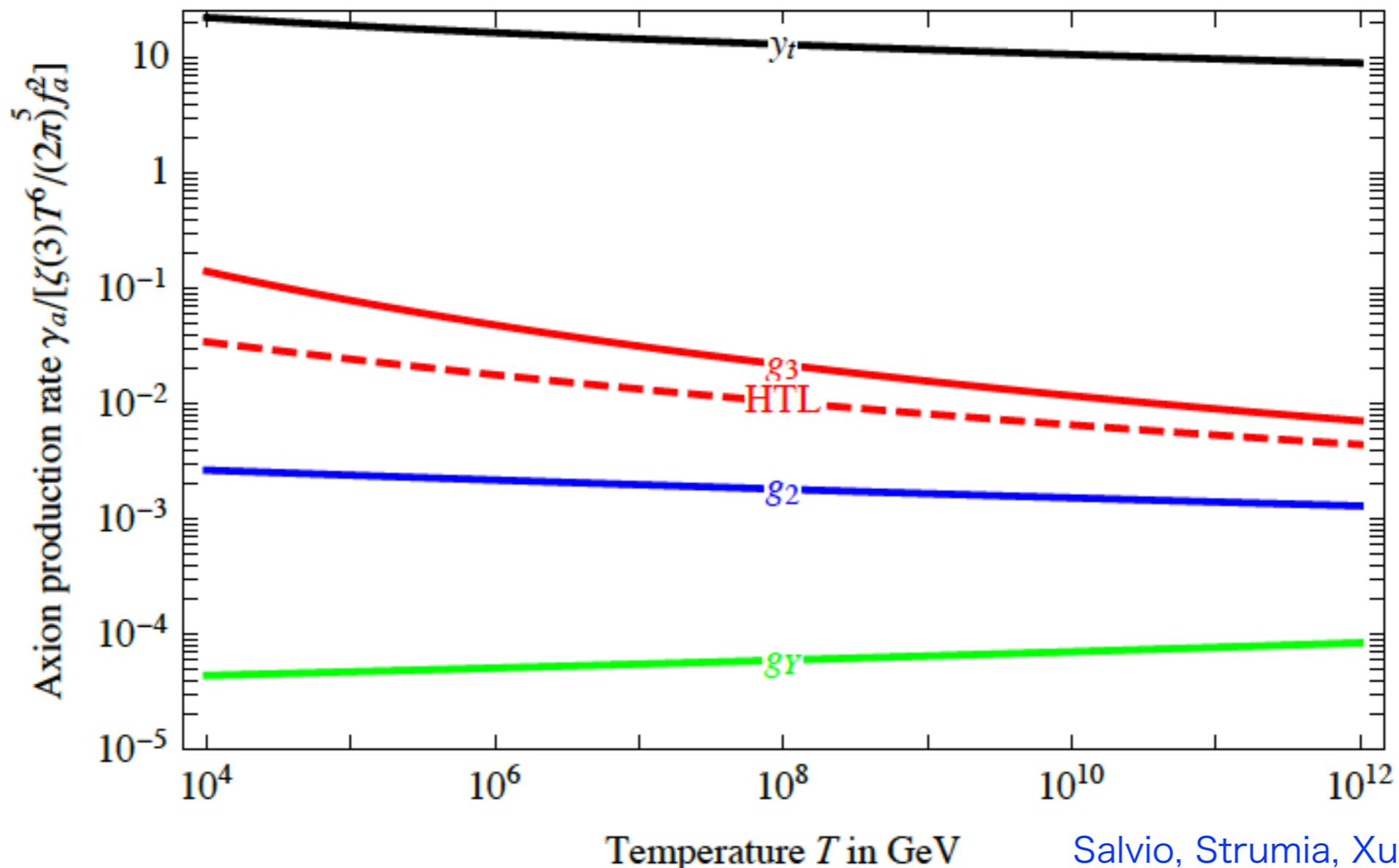
Small-scale structure constraints



• Thermalized ALPs as HDM

The ALP is thermalized if $r > 1$:

$$r = \frac{2.4}{Y_a^{\text{eq}}} \frac{\gamma_a}{Hs} \Big|_{T=T_{\text{RH}}} = 1.7 \frac{T_{\text{RH}}}{10^7 \text{ GeV}} \left(\frac{10^{11} \text{ GeV}}{f_a} \right)^2 \frac{\gamma_a}{T^6 \zeta(3)/(2\pi)^5 f_a^2} \Big|_{T=T_{\text{RH}}}$$



• Thermalized ALPs as HDM

In our case, the ALP is thermalized until the temperature drops down to the weak scale,

$$r \sim \left(\frac{T}{80 \text{ GeV}} \right) \left(\frac{8 \times 10^6 \text{ GeV}}{f/c_2} \right)^2$$

The thermalized ALPs contribute to DR/HDM with $\Delta N_{\text{eff}} \simeq 0.03$

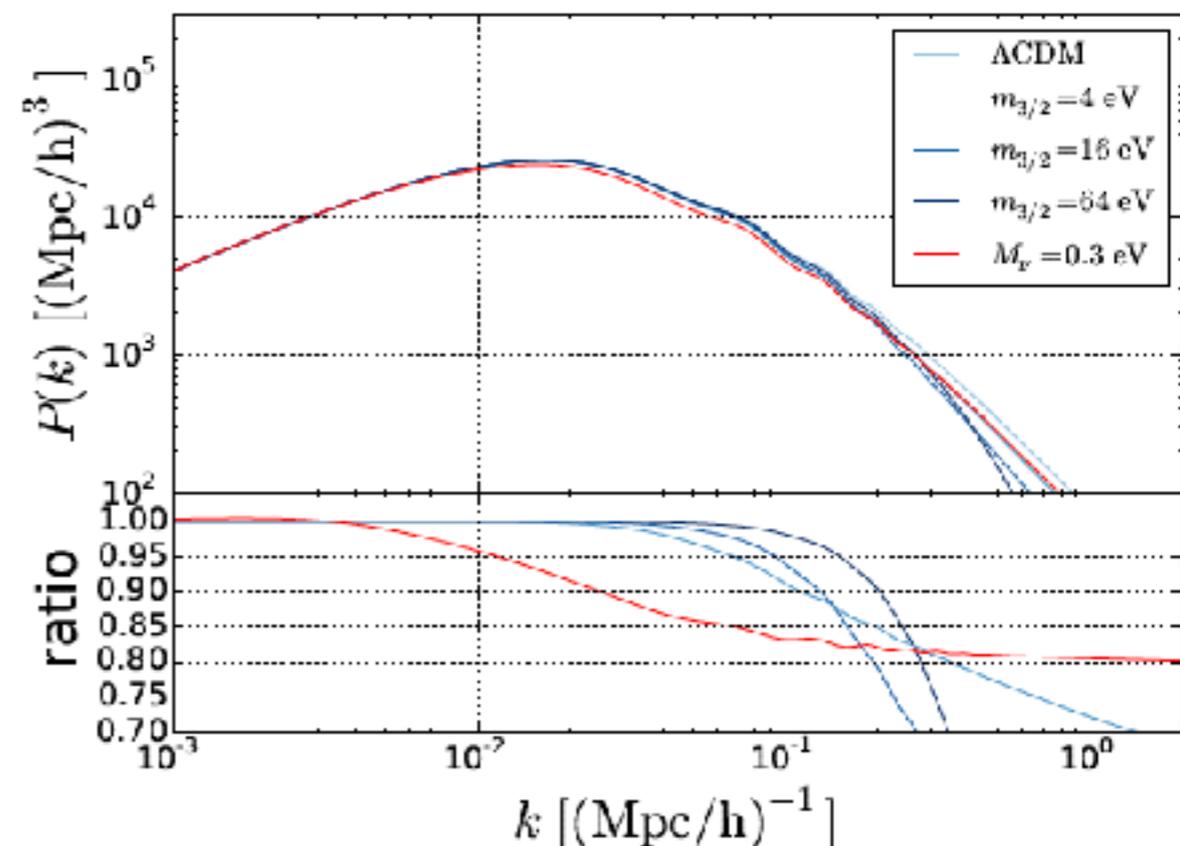
HDM abundance:

$$\Omega_{\phi}^{(th)} h^2 \simeq 0.007 \left(\frac{m_{\phi}}{10 \text{ eV}} \right) \left(\frac{g_{*s}}{106.75} \right)^{-1}$$

Upper bound on the mass:

$$m_{\phi} \lesssim 7 \text{ eV}$$

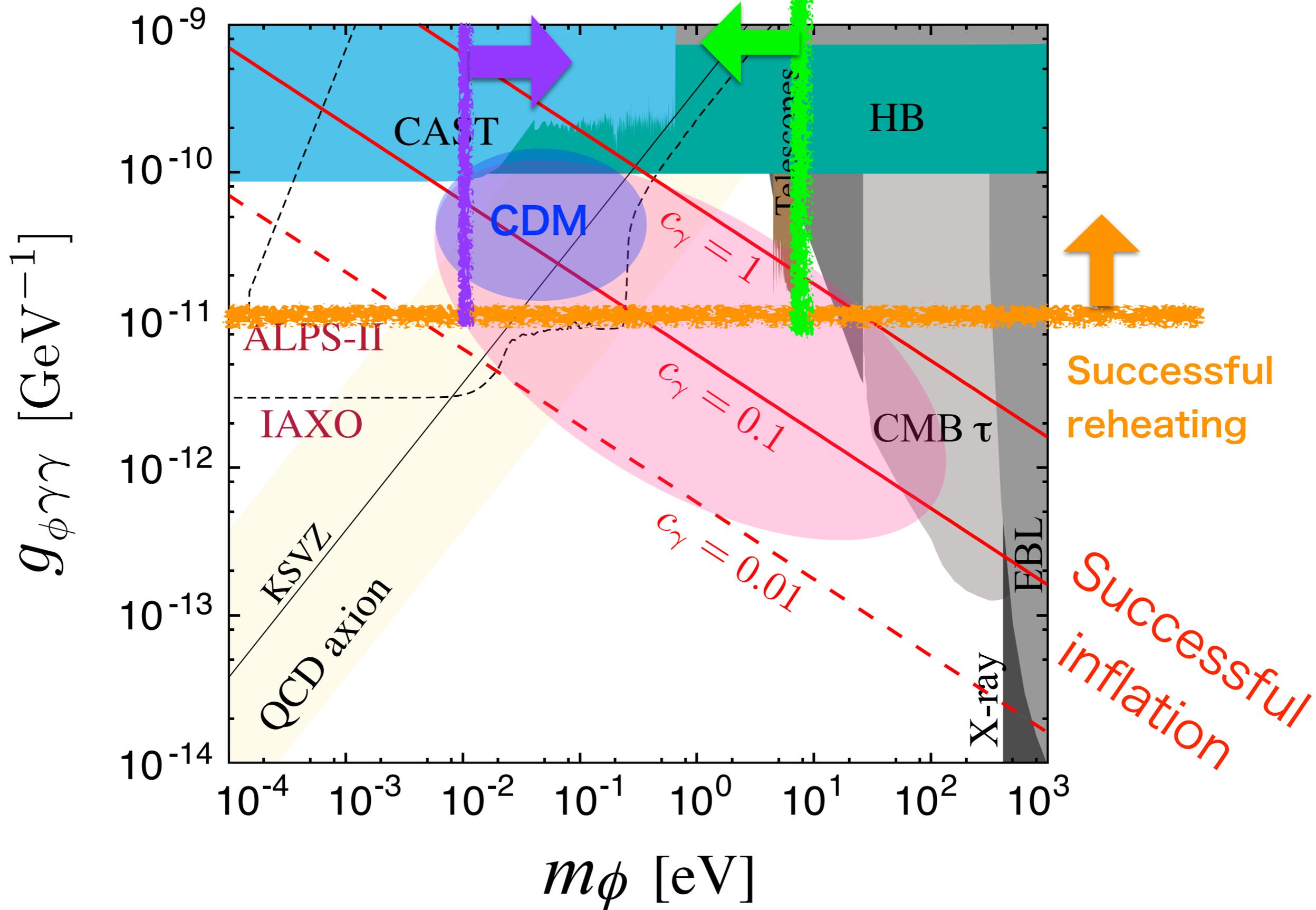
K. Osato, T. Sekiguchi, M. Shirasaki, et al, 1601.07386



*The thermalized ALP is close to DR for the parameters of our interest.

Small-scale structure
constraint on ALP CDM

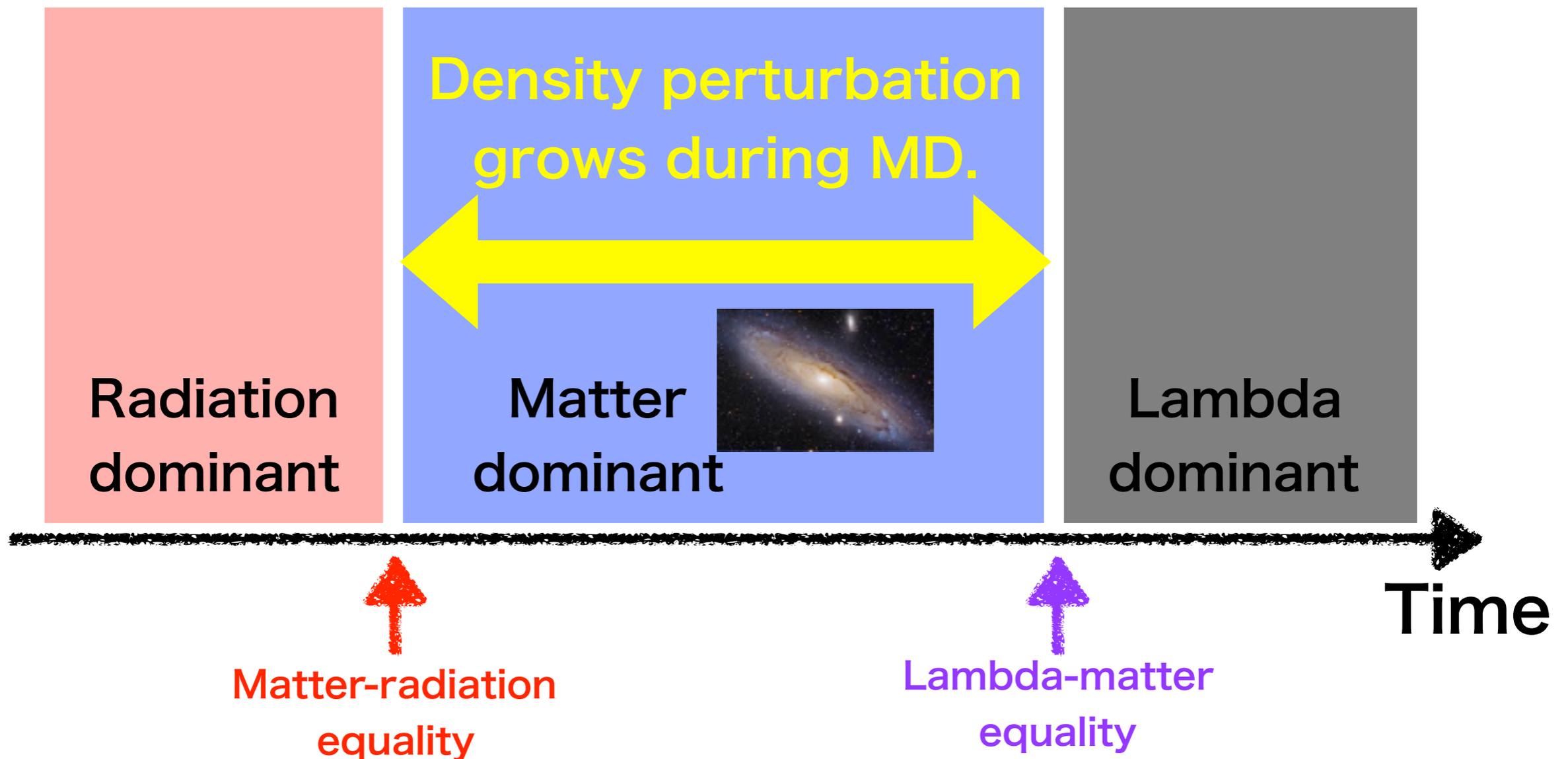
HDM constraint on
thermalized ALP



Anthropic argument for Λ

S. Weinberg '87, P. Davies and S. Unwin '81, J. D. Barrow '82,
A. Linde '87, J. D. Barrow and F. J. Tipler '88,

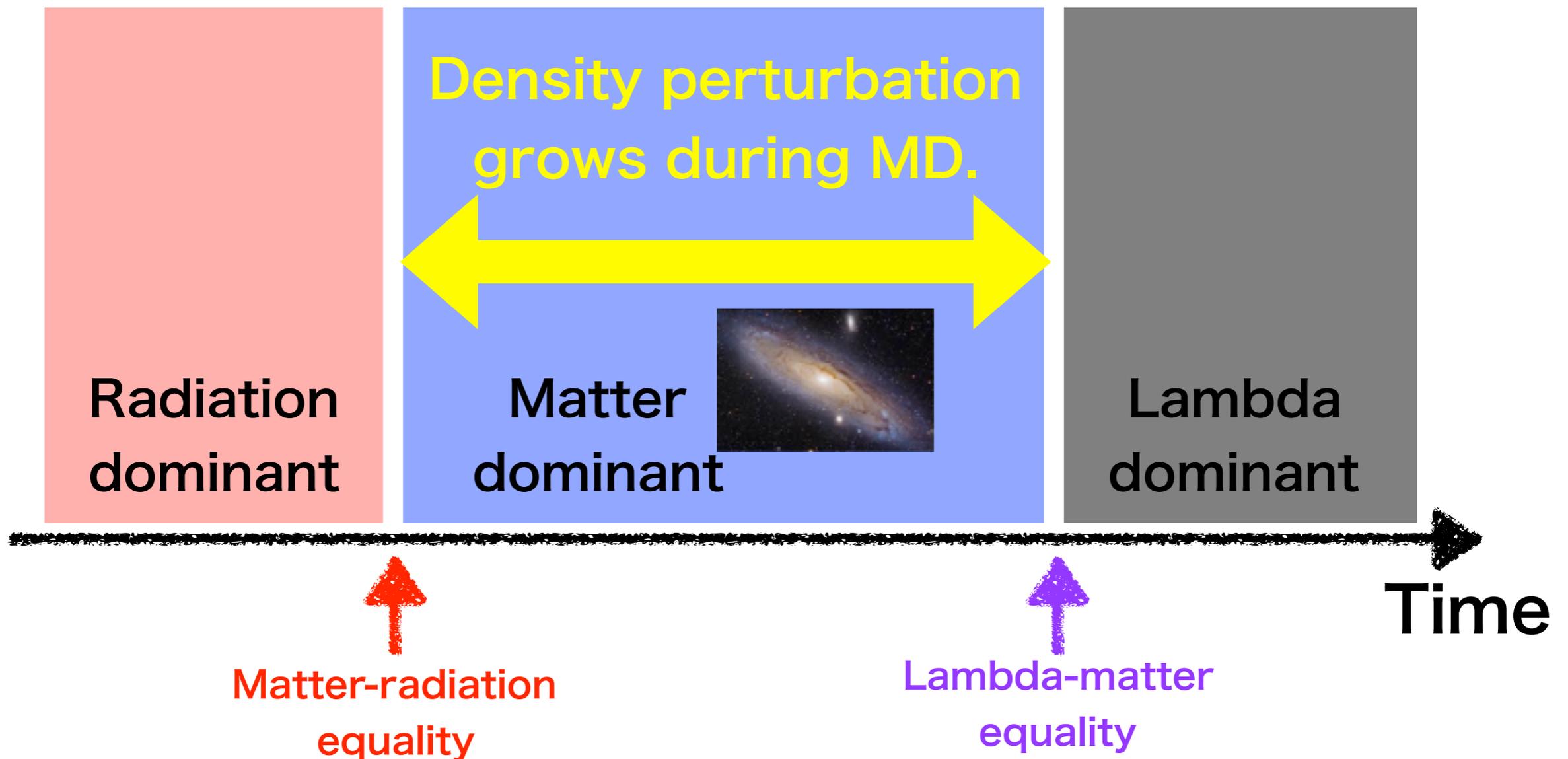
The anthropic argument seems successful to explain the observed value of Λ .



Anthropic argument for Λ

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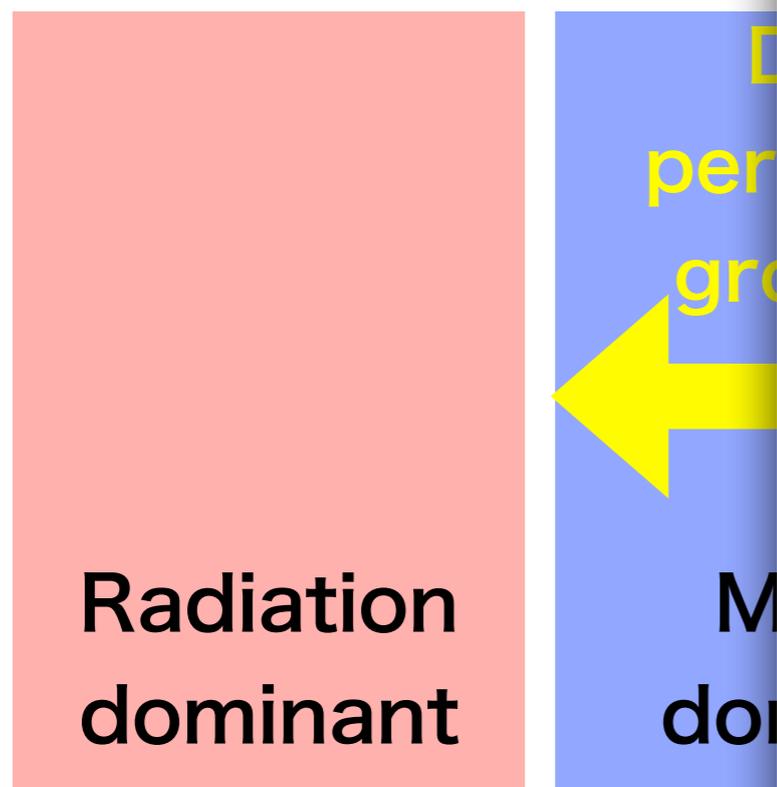
If Λ were larger, there would be less time for the density perturbation to grow.



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Matter-radiation
equality

TABLE 1
PROBABILITY THAT A RANDOM ASTRONOMER WOULD OBSERVE A
VACUUM ENERGY DENSITY AS SMALL AS THE VALUE ρ_V^* IN OUR
SUBUNIVERSE^a FOR VARIOUS VALUES OF ρ_V^*

λ_0	ρ_V^*/ρ_0	$R_G = 1 \text{ Mpc}$		$R_G = 2 \text{ Mpc}$	
		σ	$\mathcal{P}(\leq \rho_V^*)$	σ	$\mathcal{P}(\leq \rho_V^*)$
0.1.....	0.11	0.0067	0.0005	0.0042	0.0019
0.2.....	0.25	0.0063	0.0013	0.0040	0.0045
0.3.....	0.43	0.0059	0.0025	0.0038	0.0084
0.4.....	0.67	0.0054	0.0049	0.0036	0.015
0.5.....	1.00	0.0048	0.0097	0.0032	0.027
0.6.....	1.50	0.0041	0.021	0.0029	0.054
0.7.....	2.33	0.0033	0.054	0.0024	0.12
0.8.....	4.00	0.0023	0.19	0.0017	0.35
0.9.....	9.00	0.0011	0.90	0.0008	0.98

^a For $s = 1$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $n = 1$.
Martel, Shapiro, Weinberg '98

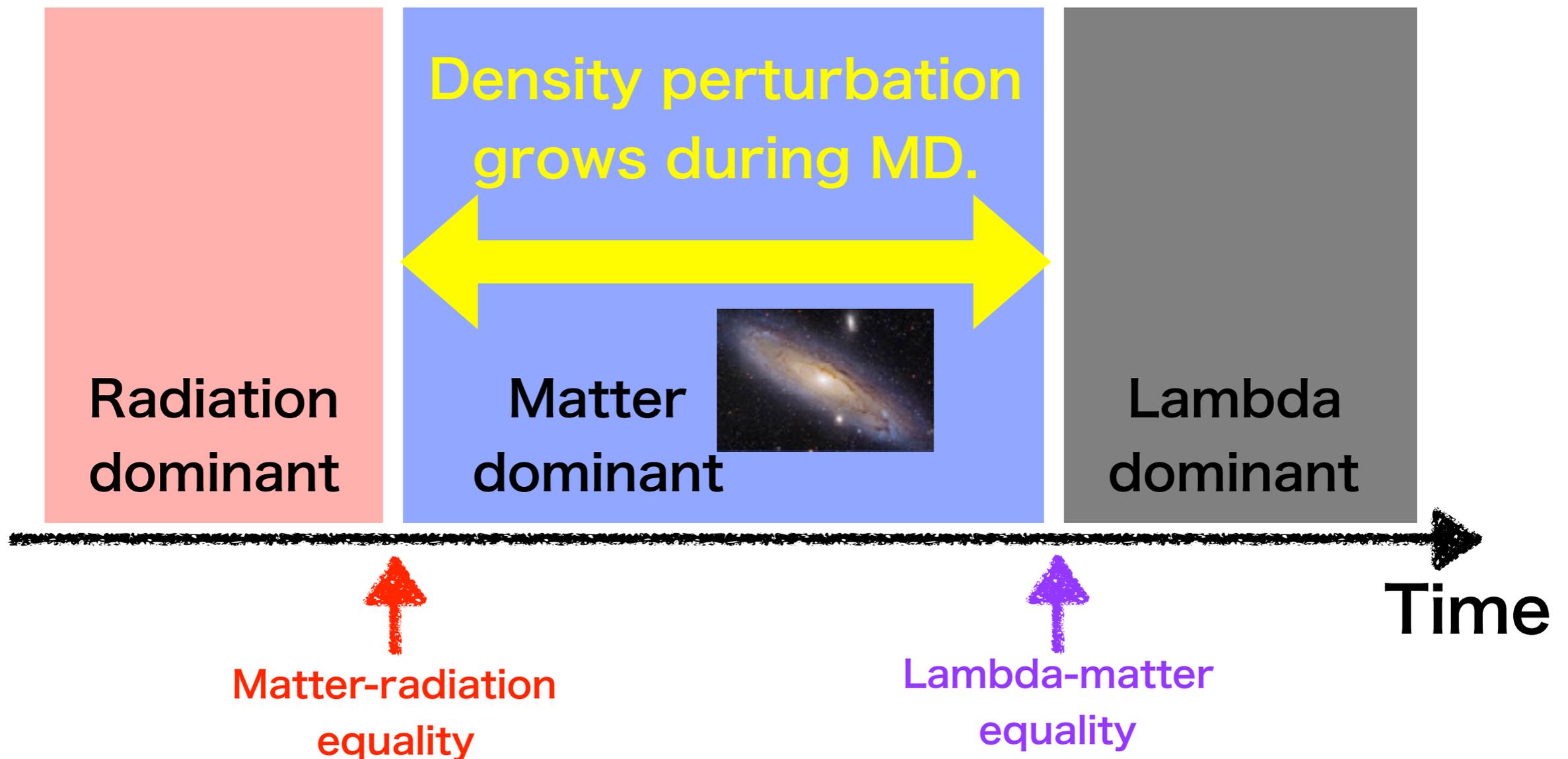
equality



Anthropic argument for N_{eff}

FT and M. Yamada, 1904.12864

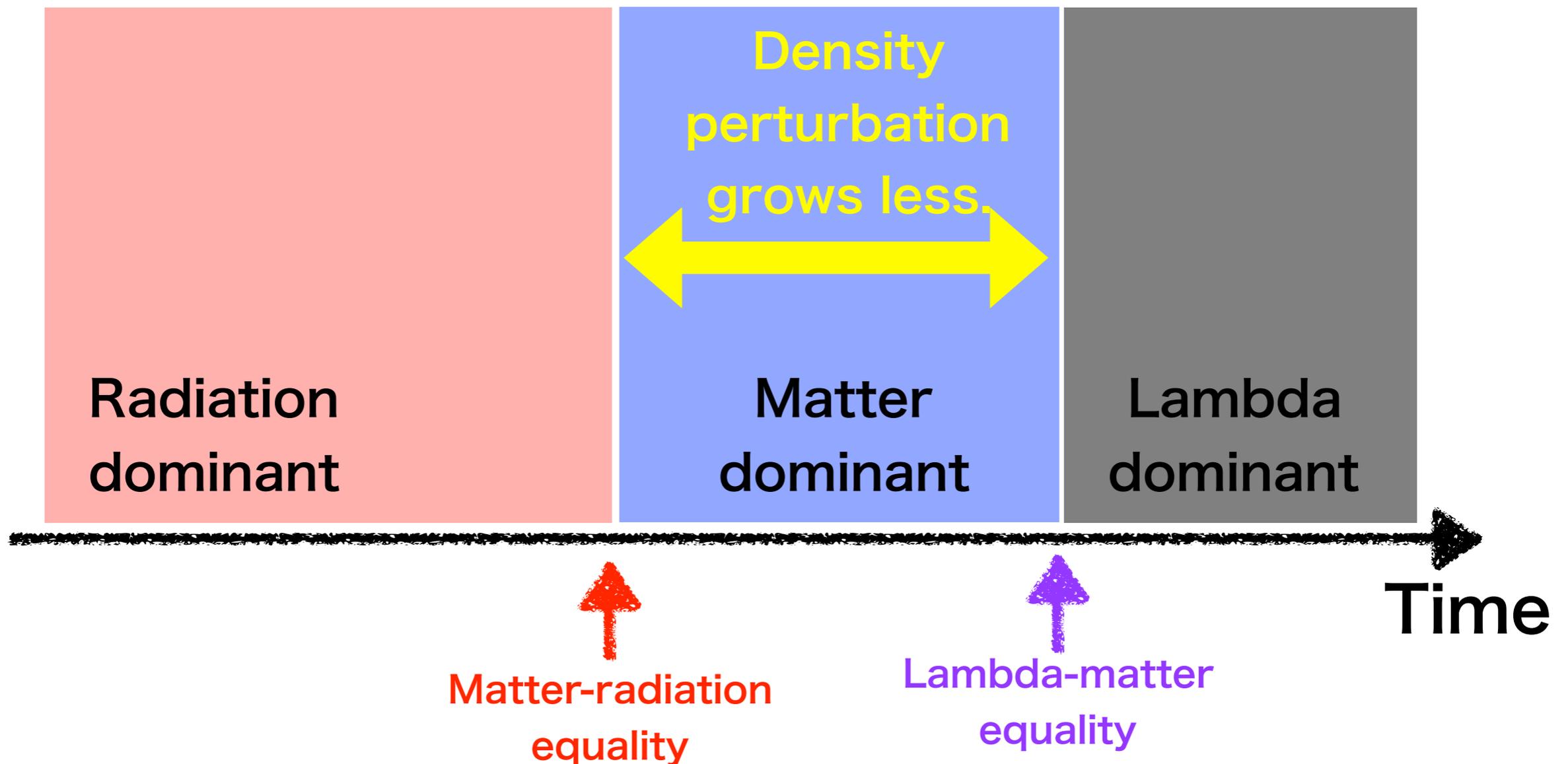
If N_{eff} were larger, there would be less time for the density perturbation to grow.



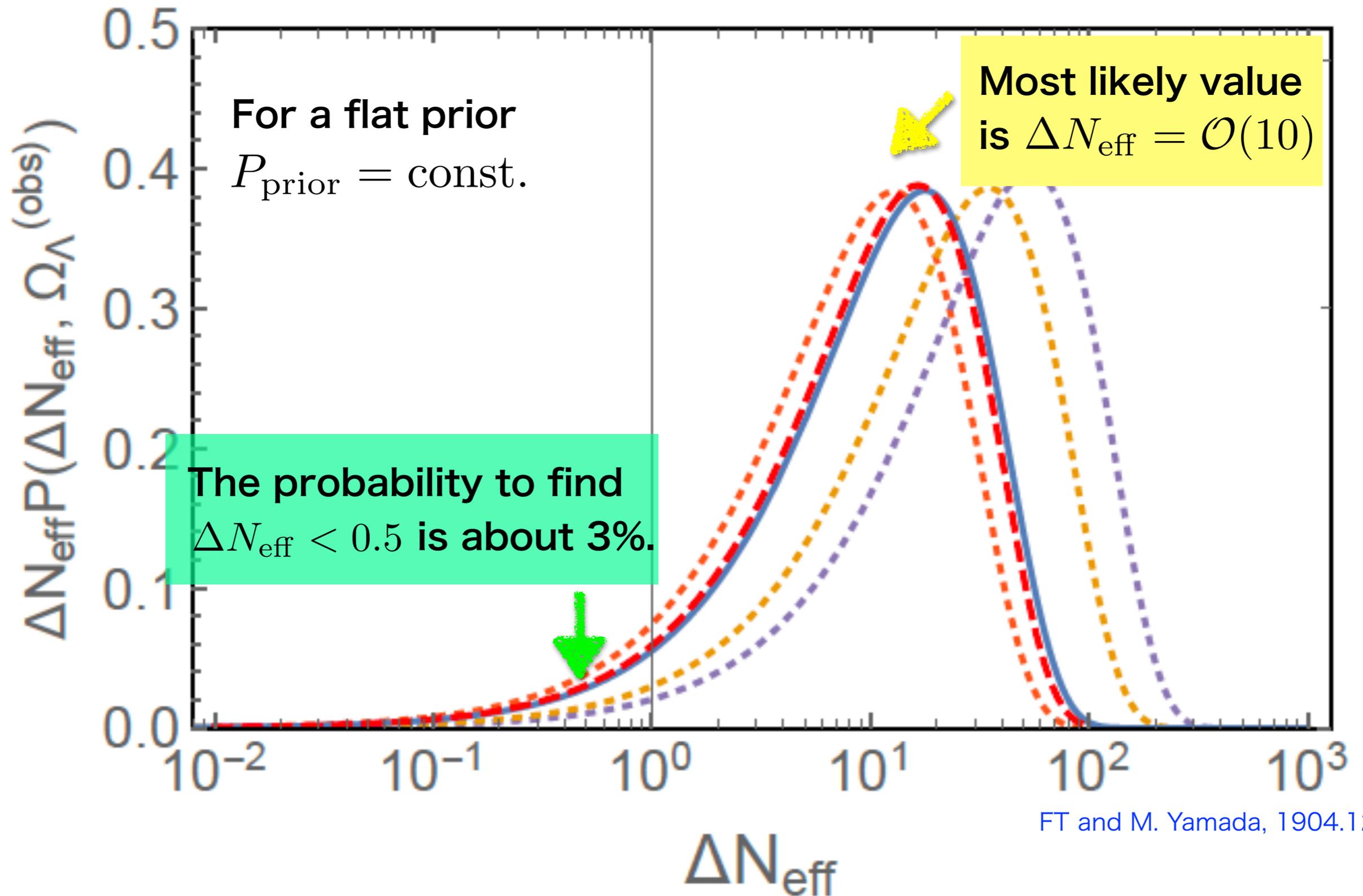
Anthropic argument for N_{eff}

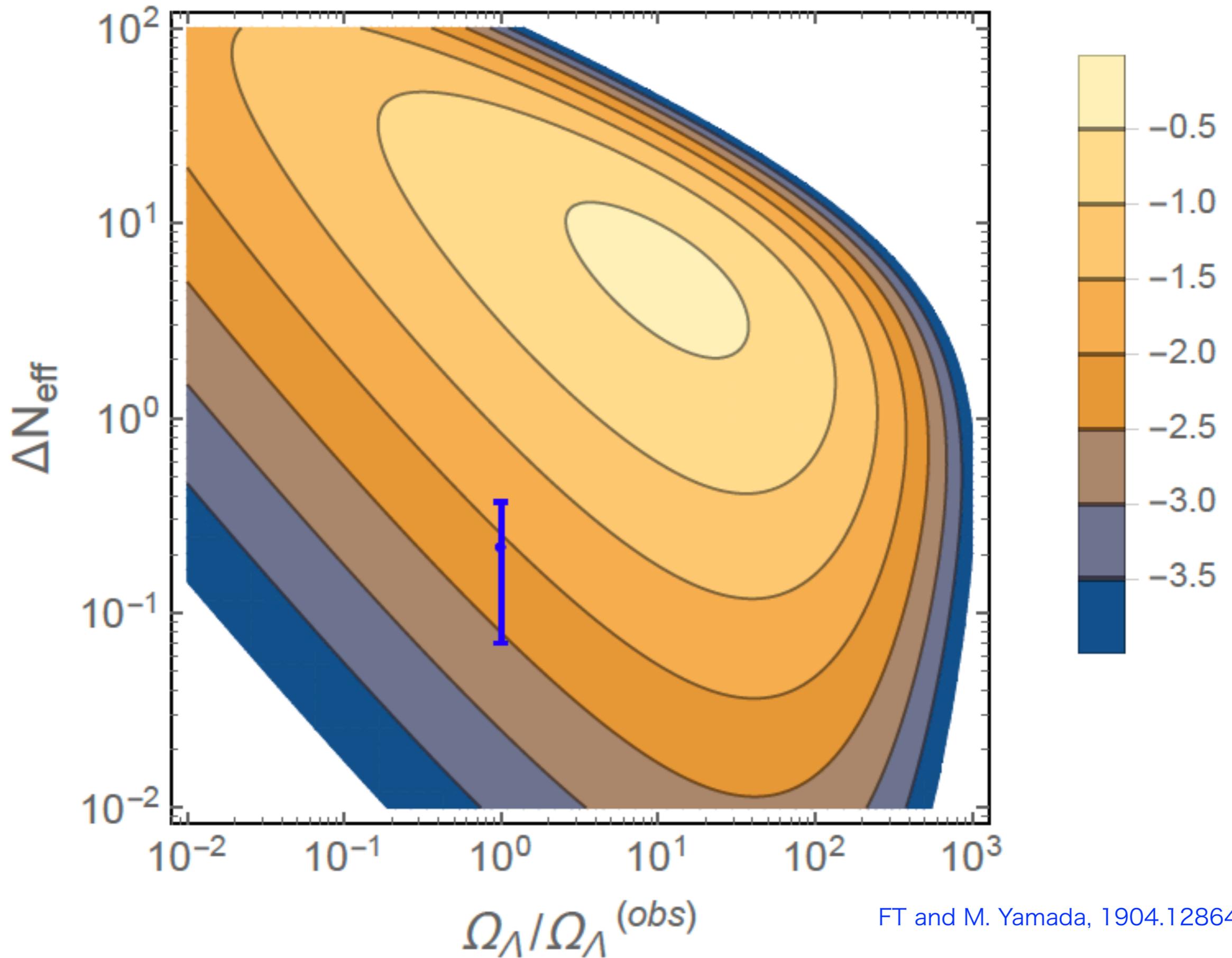
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Anthropic argument for N_{eff}





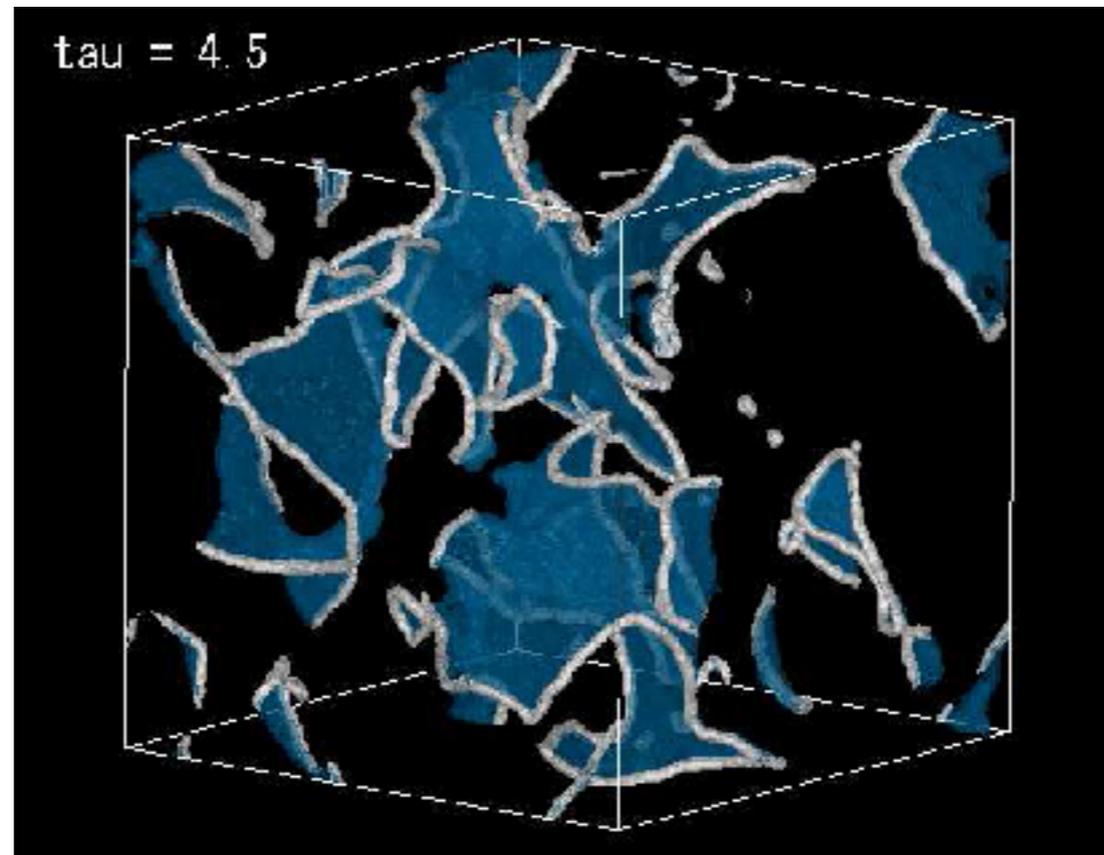
Solutions to isocurvature problem

1) Restoration of Peccei-Quinn symmetry during inflation.

Linde and Lyth '90 Lyth and Stewart '92

- Axions are produced from domain walls and axion DM is possible for $f_a = 10^{10}\text{GeV}$.

Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851,1207.3166



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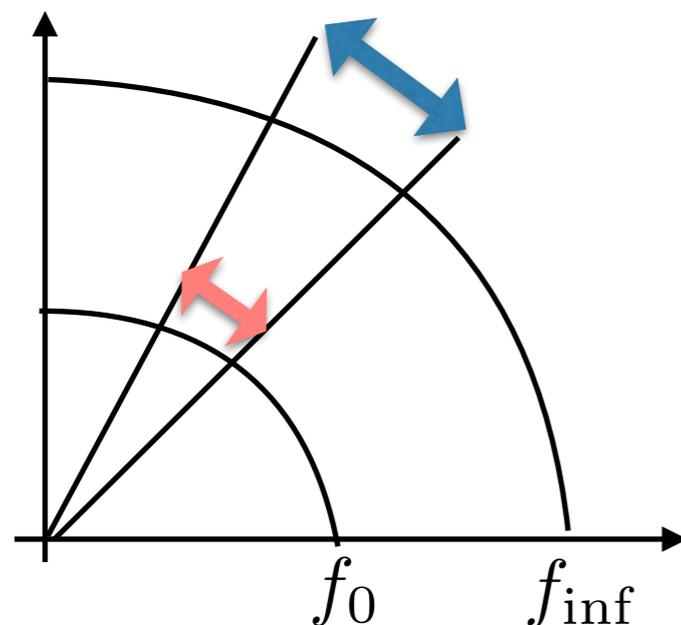
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Hiramatsu, Kawasaki, Saikawa and Sekiguchi, 1202.5851, 1207.3166

2) Dynamical axion decay constant

Linde and Lyth '90 Linde, '91



Axion: phase component
Saxion: radial component

$$S = \frac{f + \sigma}{\sqrt{2}} e^{ia/f}$$

$$\delta\theta = \text{const.} \quad \longrightarrow \quad \delta a = \delta a_{\text{inf}} \left(\frac{f_0}{f_{\text{inf}}} \right)$$

At small scales, however, axion fluctuations can be enhanced significantly!

Takeshi Kobayashi, FT, 1607.04294

Solutions to isocurvature problem

3) MSW-like resonance btw. axion and ALP.

Hill, Ross '88, Kitajima, FT 1411.2011

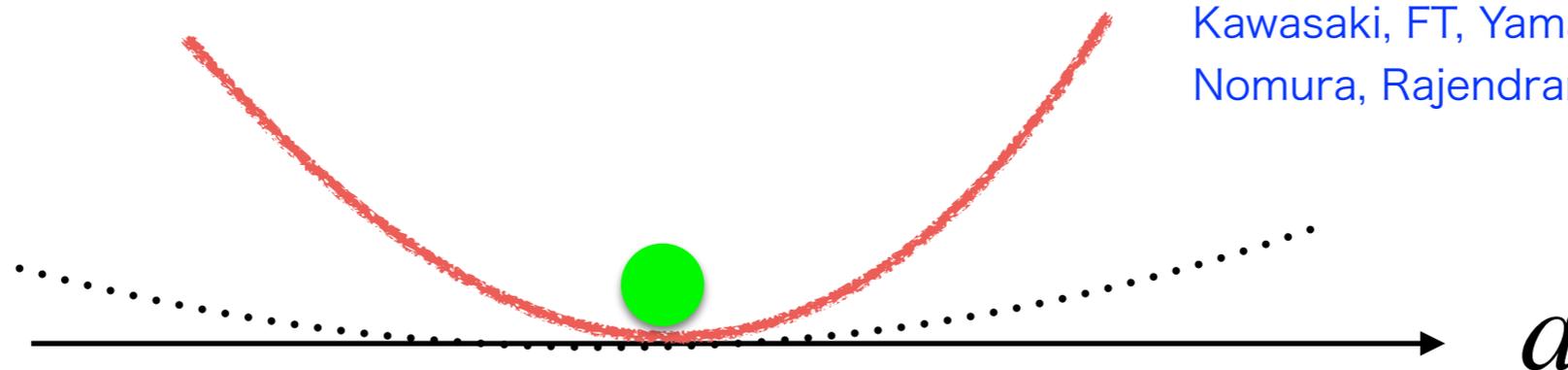
4) Heavy axions during inflation $m_a^2 \gtrsim H_{\text{inf}}^2$

- Stronger QCD during inflation

cf. Dvali, '95, Jeong, FT 1304.8131
Choi et al, 1505.00306

- Extra explicit PQ breaking
e.g. Witten effect

Dine, Anisimov hep-ph/0405256
Higaki, Jeong, FT, 1403.4186,
Barr and J.E.Kim, 1407.4311
FT and Yamada 1507.06387
Kawasaki, FT, Yamada 1511.05030
Nomura, Rajendran, Sanches, 1511.06347



The extra PQ breaking term must be sufficiently suppressed at present.