Quantum information in quantum gravity

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Quantum gravity and quantum information

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Quantum information

- a unifying theme in physics
- much discussed in quantum gravity
- *important to raise our standards of how we think about it in gravity*

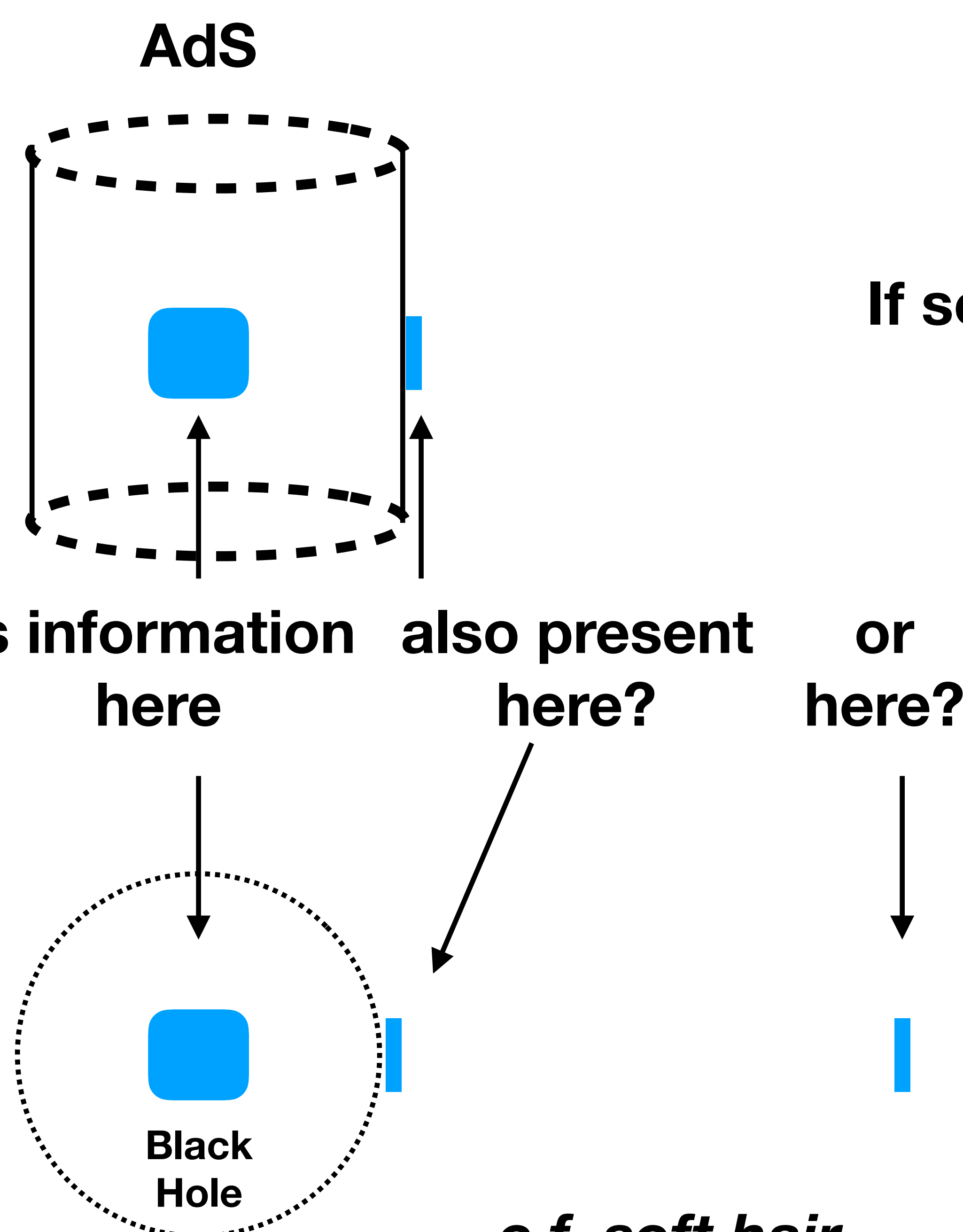
A key, initial, question: how is it *localized*?

Then, how does it evolve, etc.

Localization of information:

- part of foundational, axiomatic structure of local quantum field theory (LQFT)
  (*c.f.* algebraic QFT)
- prelude to discussing entanglement, complexity, entropy, …
- important role in puzzles of gravity

holography:



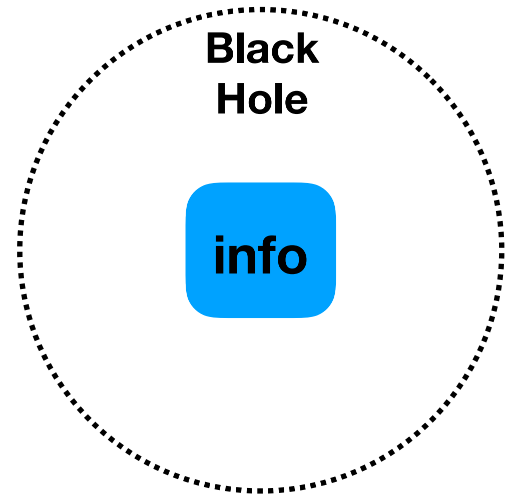
If so, how to describe/reconcile with bulk localization?

black holes:

How to describe/understand?

c.f. soft hair

Important for one of the biggest puzzles: black hole evolution



If:

1. Information can be localized inside a BH (e.g. BHs are “subsystems”)
2. BHs shrink and disappear (Hawking)
3. Physics is unitary

Then:

Information must *transfer out* of the BH

~~LQFT~~

How to describe, and what tells us about dynamics?

(Key Q for QG?)

But first: examine 1…

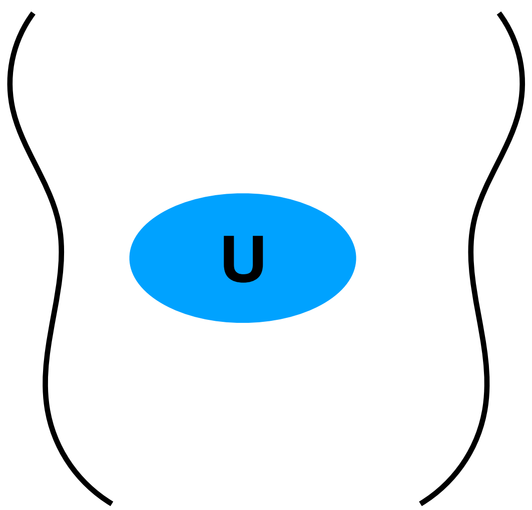
Describing localization: *subsystems*

Finite quantum systems: $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$ **Tensor product**

LQFT: $\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$ **(vN type III: "infinite enganglement")**

Instead, commuting subalgebras of observables, associated with open regions

M



$U \leftrightarrow \mathscr{A}_U$ **e.g.** $\phi_f = \int d^4x\, f(x) \phi(x)$

If U and U' are spacelike separated:

$$[A_U, A_{U'}] = 0$$

… locality

Subalgebras *define* "subsystems" **key role for observables**

Then, e.g., evolution describes transfer between subsystems

What about gravity? **Study in *perturbative approximation***

E.g. $$\mathscr{L} = \frac{2}{\kappa^2}R - \frac{1}{2}\left[(\nabla\phi)^2 + m^2\phi^2\right] \qquad \kappa = \sqrt{32\pi G}$$

$\phi(x)$ **: not gauge invariant, not physical observable**

$$\delta_\xi \phi(x) = -\kappa \xi^\mu \partial_\mu \phi(x)$$

Two approaches:

(discussion Friday)

1. Relational observables, e.g.

$$\sim \int d^4x \mathcal{O}(x) \prod_a \delta[Z^a(x) - \xi] \qquad \langle Z^a(x) \rangle \sim \lambda \delta^a_\mu x^\mu$$ **(*c.f.* inflation)**

2. Dressed observables

The latter comes closest to similar algebraic structure

Dressed observables:

Given $\phi(x)$, can we promote it to a gauge-invariant observable?

Compare QED, w/ charge q scalar:

$$\Phi(x) = \phi(x)e^{i\Lambda(x)}$$

gauge invariant

dressing

E.g. Faraday line

[Dirac, 1955]

$$\Lambda(x) = q\int_x^\infty A$$

Gravity:

Work perturbatively: $g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)$

$$\delta_\xi \phi(x) = -\kappa \xi^\mu \partial_\mu \phi(x) \qquad \delta h_{\mu\nu} = -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

[1503.08207; 1507.07921, 1607.01025 w/Donnelly]

$\mathcal{O}(\kappa)$: **Construct "dressing:"** $V^\mu[h, x]$ **so that** $\delta V^\mu(x) = \kappa \xi^\mu(x)$ ***(key property!)***

Then: $\Phi(x) = \phi(x^\mu + V^\mu(x))$ **is diff invariant!**

$$\Phi(x) = \phi(x^\mu + V^\mu(x)) \qquad \delta h_{\mu\nu} = -\,\partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

$V^\mu[h,x]$ **?** **There are *many* possible choices**

~allowed grav fields of ϕ particle

One useful choice: take Γ to be a curve from x to ∞

$$V^\Gamma_\mu(x) = \frac{\kappa}{2}\int_x^\infty dx'^\nu \left\{ h_{\mu\nu}(x') + \int_{x'}^\infty dx''^\lambda \left[\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')\right]\right\}$$

$\phi(x)$

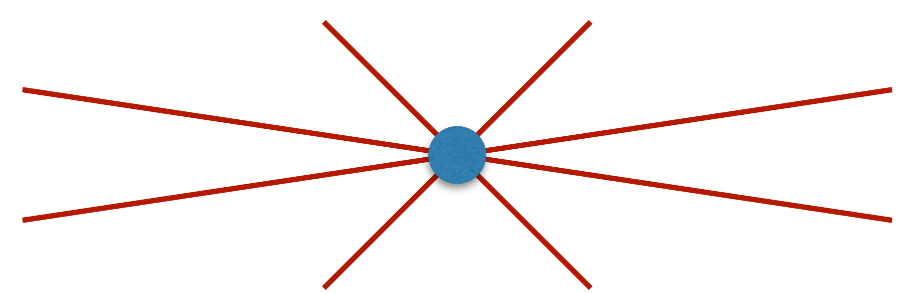
"gravitational line" **[1507.07921, w/Donnelly; 1805.06900]**

Create particle, + grav field

(*one* way to understand: shooting geodesics)

$\delta V^\mu(x) = \kappa \xi^\mu(x)$ **: diff-invariant to** $\mathcal{O}(\kappa)$

Another: $V^C_\mu(x)$ **"Coulomb" dressing**



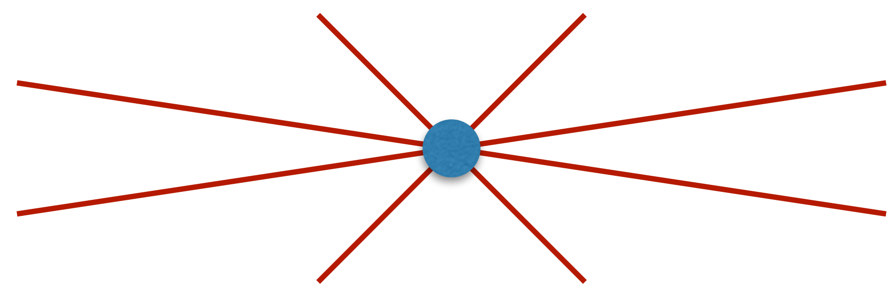
(e.g. spherical average of line)

***Both* satisfy** $\delta V^\mu(x) = \kappa \xi^\mu(x)$

Alternately: commute w/ constraints: $[C_\mu(x), \Phi(x')] = 0$

With $C_\mu(x) = G_{0\mu}(x) - 8\pi G\, T_{0\mu}(x)$

vs.



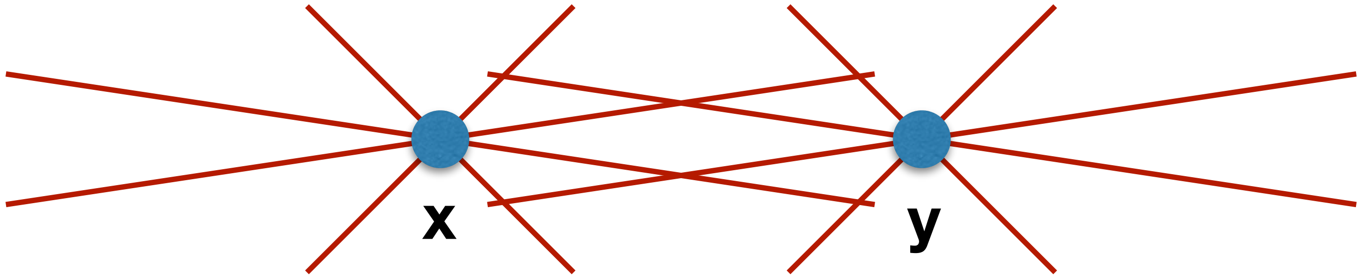
?
+ radiation

general difference between dressings

An apparently important consequence:

These no longer satisfy a local algebra

$$[\Phi(x), \Phi(y)] \neq 0 \qquad (x-y)^2 > 0$$



Intrinsic gravitational nonlocality: already see in perturbative gravity!

In NR limit, mass m: $$[\partial_t \Phi_C(x), \Phi_C(y)] \simeq \frac{Gm}{|x-y|} \partial_t \phi(x) \phi(y)$$

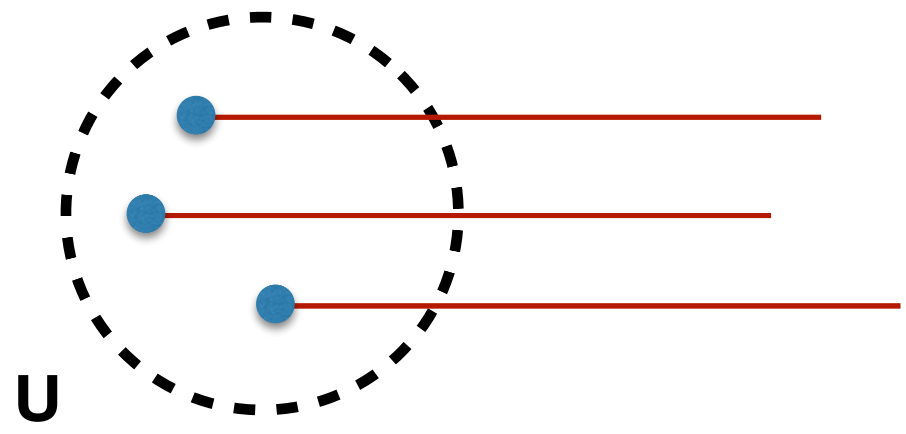
[1507.07921, w/Donnelly; "locality bound," hep-th/0103231 w/ Lippert]

This is likely fundamentally important.

But then, (how) can quantum information be localized?

Asymptotic observations and soft charges

Naively, dressing implies information not localized:



$\longleftarrow$ **Can detect charge or energy distribution asymptotically?**

Concrete example: soft charges

(c.f. Hawking, Perry, Strominger)

EM $$Q_\epsilon = \oint d\Omega\, \epsilon(\theta)\, [r^2 F_{0r}(\theta)]_{|\infty}$$

Gravity $$Q_\epsilon \sim \oint d\Omega \epsilon\ [rh + \cdots]_{|\infty}$$

Soft charges of dressed operators

[1903.06160]

EM: **Faraday line** $\Lambda(x) = q\int_x^\infty A$

a little singular, but regulate in cone: $\Lambda(x) = \int d^3x\, \bar{E}^i A_i$ **e.g.** $\bar{E}^r = \dfrac{f(\theta^A)}{r^2}$

$\Phi(x) = \phi(x)e^{i\Lambda(x)}$: $$[Q_\epsilon, \Phi] = i\int d\Omega\, \epsilon(\theta)\, [r^2\bar{E}^r(\theta)]_{|\infty}\, \Phi$$

$f(\theta)$

(note: configurations without antipodal matching!)

Grav:

$\Phi(x) = \phi(x^\mu + V^\mu(x))$: $$[Q_\epsilon, \Phi(x)] = [Q_\epsilon, V^\mu(x)]\ \partial_\mu\Phi(x)$$

These depend on profile of dressing.

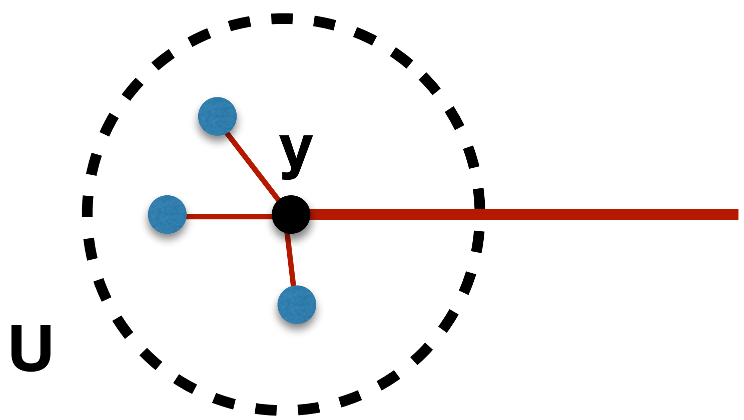
However, the dressing is highly non-unique

~ add arbitrary radiation (sourceless) field

The soft charges — and other asymptotic EM/grav field observables — depend on the details of the radiation field we add

Is there any *necessary* dependence on the charge/energy distribution?

Not much: e.g. can dress



Only detect $Q = \sum_i Q_i$

Gravity?

$\mathcal{O}(\kappa)$ construction for gravity:

First, dress general operator: **[1805.11095 w/ Donnelly]**

$$\phi(x) \to \Phi(x) = \phi(x + V^\mu(x))$$

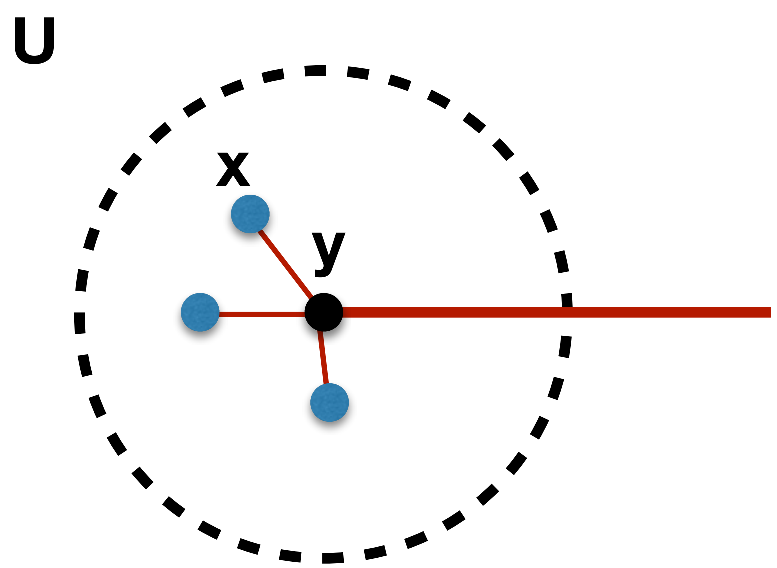
$$\longrightarrow \qquad A \to \hat{A} = e^{i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)}\; A\; e^{-i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)} + \mathcal{O}(\kappa^2)$$

and state:

$$\longrightarrow \qquad |\Psi\rangle \to |\,\widehat{\Psi}\,\rangle = e^{i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)}|\Psi\rangle + \mathcal{O}(\kappa^2)$$

Construct dressing $V^\mu(x)$ **:**

[1805.11095 w/Donnelly]



Pick $y \in U$

Let $V_S^\mu(y)$ **= any chosen "standard" dressing for y**

satisfies $\delta V_S^\mu(y) = \kappa \xi^\mu(y)$

Define

$$V_\mu^L(x,y) = -\frac{\kappa}{2}\int_y^x dx'^\nu \left\{ h_{\mu\nu}(x') - \int_y^{x'} dx''^\lambda \left[\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')\right]\right\}$$

Then:

$$V^\mu(x) = V_L^\mu(x,y) + V_S^\mu(y) + \frac{1}{2}(x-y)_\nu[\partial^\nu V_S^\mu(y) - \partial^\mu V_S^\nu(y)]$$ **satisfies** $\delta V^\mu(x) = \kappa \xi^\mu(x)$

Creates "standard" grav. field outside U: $\tilde{h}^\mu_{\lambda\sigma}(x)$

$$\hat{A} = e^{i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)}\ A\ e^{-i\int d^3x\, V^\mu(x)\, T_{0\mu}(x)} + \mathcal{O}(\kappa^2)$$

$$V^\mu(x) = V^\mu_L(x,y) + V^\mu_S(y) + \frac{1}{2}(x-y)_\nu[\partial^\nu V^\mu_S(y) - \partial^\mu V^\nu_S(y)]$$

Consider outside observations, e.g. of soft charges:

For local A,

$$[Q_\epsilon, \hat{A}] = i\int d^3x\, [Q_\epsilon, V^\mu(x)]\, [T_{0\mu}(x), A]$$

Let $[Q_\epsilon, V^\mu_S(y)] = q^\mu_{\epsilon,S}(y)$

"soft charges" of standard dressing

Then:

$$[Q_\epsilon, \hat{A}] = -iq^\mu_{\epsilon,S}(y)\, [P_\mu, A] - \frac{i}{2}\partial^\mu q^\nu_{\epsilon,S}(y)\, [M_{\mu\nu}, A]$$

Only depends on A through P_μ and $M_{\mu\nu}$

I.e. total Poincare charges

(compare EM)

Likewise

$$\langle \hat{\Psi}' | Q_1 \cdots Q_N | \hat{\Psi} \rangle = i \langle \Psi' | \left(q^{\mu_1}_{1,S} P_{\mu_1} + \frac{1}{2} \partial^{\mu_1} q^{\nu_1}_{1,S} M_{\mu_1 \nu_1} \right) \cdots \left(q^{\mu_N}_{N,S} P_{\mu_N} + \frac{1}{2} \partial^{\mu_N} q^{\nu_N}_{N,S} M_{\mu_N \nu_N} \right) | \Psi \rangle + \cdots$$

This depends on the moments of the total Poincare charges, and on the soft charges of the standard dressing

Similarly for other asymptotic measurements.

Also works nonperturbatively, in classical theory!

[Carlotto-Schoen; Corvino-Schoen]

So, to summarize so far:

[1903.06160]

1) Measurements at i^0 (and soft charges) don't detect charge or energy/momentum distribution in U.

2) Such measurements/charges *do* detect aspects of the radiation field superposed on the distribution

3) We can always choose initial states such that the distribution and the radiation field are correlated in a certain way, but we can also choose states where they are correlated in a different way.

No necessary linkage (except total charges: Q, or P, M)

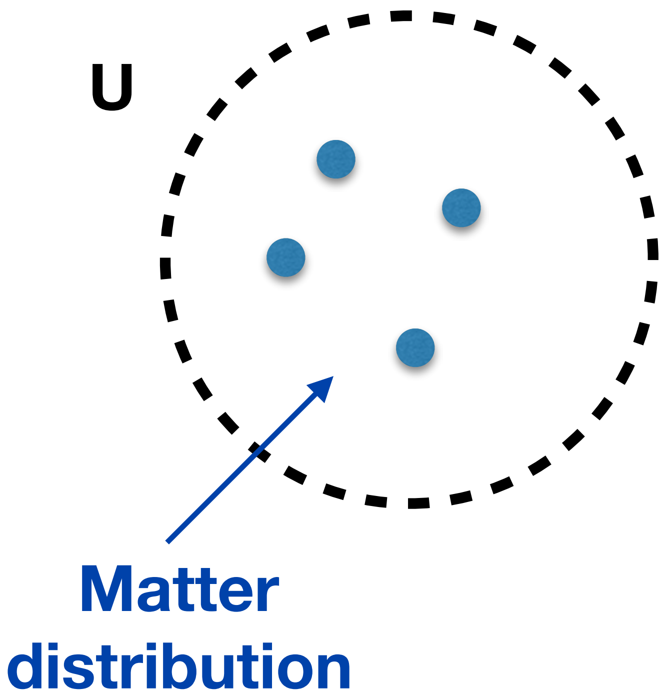
Soft charges are decoupled from information in charge or energy distribution.

So, suggests: **Information can be localized in EM/gravity.** (perturbatively)

"We can make localized EM/gravitational 'qubits'"

(E.g. if states with same CM wavefunction, different internal state)

And, suggests:



- Don't have asymptotic access to information in U (whether or not the matter is in a BH), at least in perturbative analysis

- Soft hair not relevant for BH information

[1706.03104 w/ Donnelly;
1903.06160;
see also Bousso and Porrati,
Compere, Long, Riegler, 1903.01812]

More discussion tomorrow!

If not, how is BH evaporation made unitary/consistent w/ QM?

quite possibly through *other* effects…

but first …

What is this telling us about quantum gravity?

Let's suppose that our aim is a *quantum-mechanical* theory describing gravity

So, how do properties of gravity fit into postulates of quantum mechanics?

What are the postulates of QM?

Hilbert space $\mathscr{H}$

Observables $\mathscr{A}$

Unitarity

"Universal quantum mechanics"
[0711.0757]

Don't necessarily start with spacetime.

"Quantum-first" approach to gravity

[0711.0757; 1803.04973; also Carroll & collabs]

Further guides:

What is it?

Discussion

1. **Additional mathematical structure on** $\mathscr{H}$
2. **Correspondence: match LQFT + GR in "weak gravity" limit**

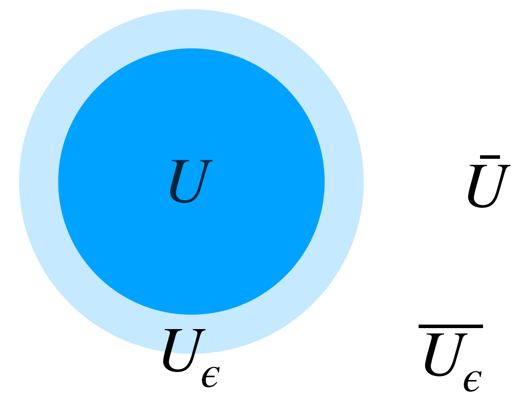
Typical quantum theories: begin w/ subsystem structure (localization)

"Einstein separability"

$\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$ **, or local subalgebras in LQFT**

In gravity, no obvious local subalgebras!

Alternate approach in LQFT:



$\mathscr{H} \neq \mathscr{H}_U \otimes \mathscr{H}_{\bar{U}}$

But, split vacuum: $|U_\epsilon\rangle$ **[Haag, and refs. therein]**

For $A \in \mathscr{A}_U$ **,** $A' \in \mathscr{A}_{\overline{U_\epsilon}}$ **:** $\langle U_\epsilon | A A' | U_\epsilon \rangle = \langle 0 | A | 0 \rangle \langle 0 | A' | 0 \rangle$

"disentangles" degrees of freedom

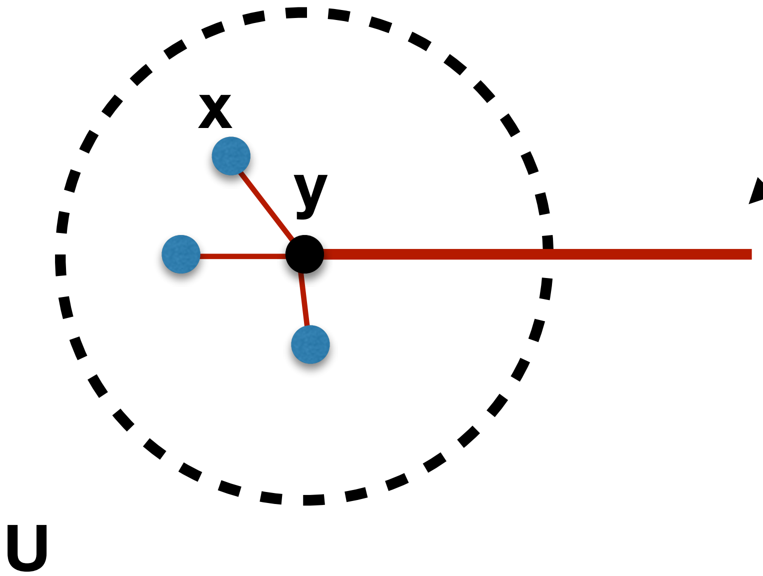
$A_I | U_\epsilon \rangle$ **,** $A_J | U_\epsilon \rangle$ **indistinguishable via measurements in** $\overline{U_\epsilon}$

$A_I, A_J \in \mathscr{A}_U$

~ "localized qubit"

Corresponding mathematical structure: $\mathscr{H}_U \otimes \mathscr{H}_{\overline{U_\epsilon}} \hookrightarrow \mathscr{H}$

We have done something like this in EM, gravity: dress operators, states as above



"expresses" Q

or gravity, P,M

$$\bigoplus_{Q} \mathscr{H}_{U,Q} \otimes \mathscr{H}_{\overline{U}_\epsilon,Q} \hookrightarrow \mathscr{H}$$

"Electromagnetic splitting"

$$\sim \bigoplus_{P_\mu,s,s_z} \mathscr{H}_{U,P_\mu,s,s_z} \otimes \mathscr{H}_{\overline{U}_\epsilon,P_\mu,s,s_z} \hookrightarrow \mathscr{H}$$

"Gravitational splitting"

[1706.03104, 1805.11095, 1903.06160]

Mathematical structure: network of such inclusions (replacing net of subalgebras)?

important strong-field subtleties [1803.04973]

Brief comment: connections to, and puzzles for, holography

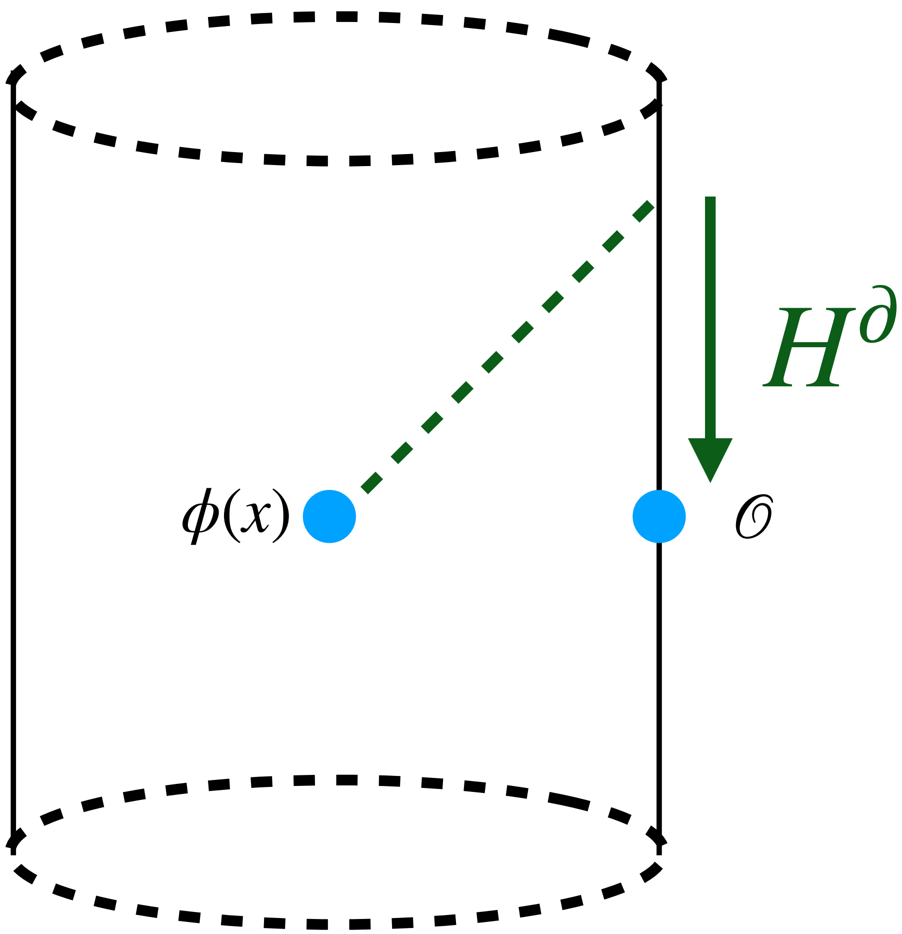
Discussion: Thursday

How does holography work?

Best argument on market: Marolf [0808.2842, 1308.1977] (+ Jacobson, …):

By virtue of gravitational constraints.

AdS:

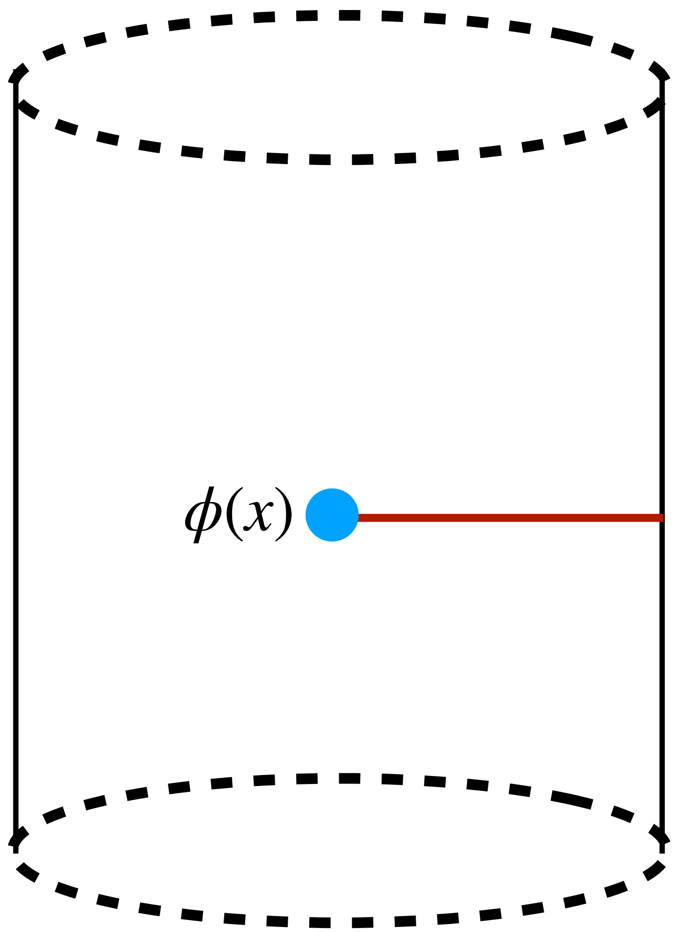


When solve constraints,
H is a surface term H^∂

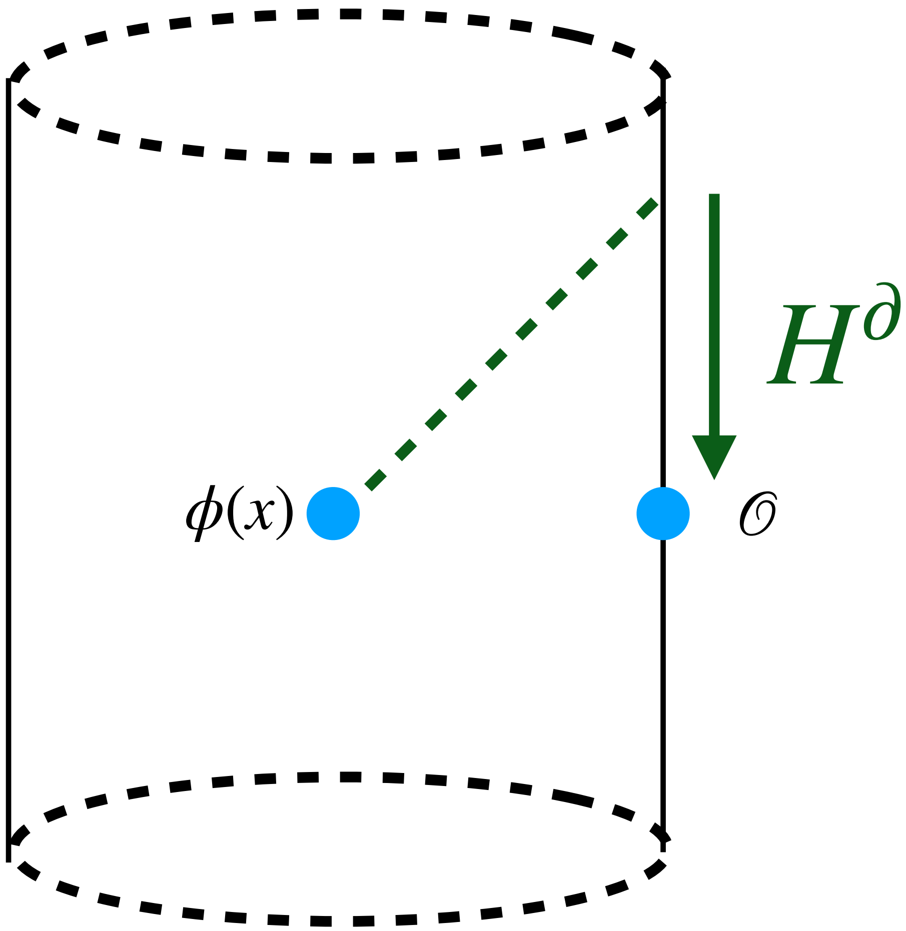
Solving constraints = finding gravitational dressing.

Explored in 1802.01602, with Kinsella

e.g.



$$\Phi(x) = \phi(x^\mu + V^\mu(x))$$



Holographic map: translate back, or

$$\lim_{a\to\infty} e^{ia^\mu P^\partial_\mu}\, \Phi(x)\, e^{-ia^\mu P^\partial_\mu}$$

Either way: apparently need to solve grav. constraints to *all orders* in κ

$\longleftrightarrow$ **Finding unitary bulk evolution***

Argues: need to determine unitary bulk evolution *in order to construct holographic map*

(Don't get unitary bulk evolution for "free")

So, what about unitary evolution?

(*similarly ent wedge reconstruction? HRT. Discuss?)

Some reasonable postulates for quantum gravity:

I) Principles of QM - already discussed

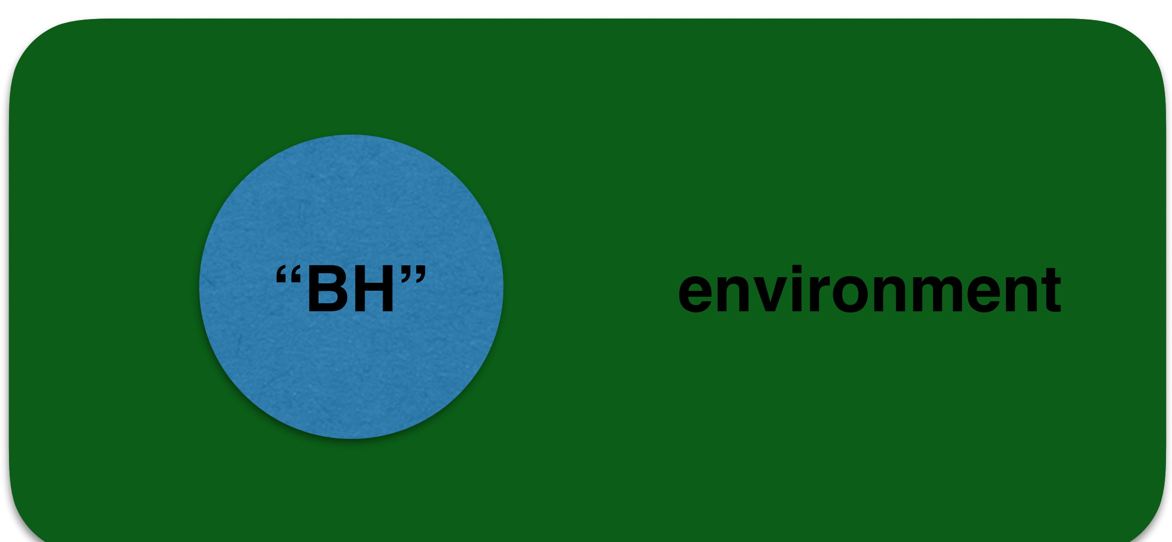
II) Subsystem structure - subtleties, but $\sim \mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$

III) Correspondence - also noted

(seek ‘minimal’ departure from LQFT+GR)

Consider BHs; challenge of unitarity

II) $\Rightarrow$



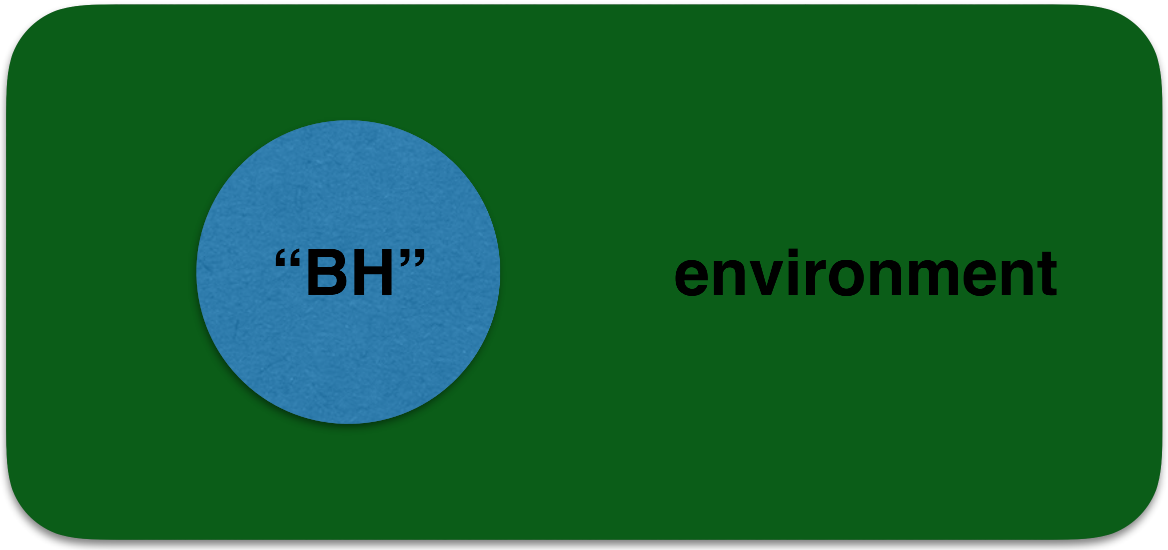
(“Schrodinger pic.”)

$$|K, M; \psi_e, T\rangle$$

BH states

~LQFT (by III)

$$N = e^{S_{bh}}$$



$$|K, M; \psi_e, T\rangle$$

$$H_I$$

I (QM) $\Rightarrow$ **must evolve unitarily**

Infinitesimal evolution: $$H = H_{bh} + H_{env} + H_I$$

(note can write LQFT evolution this way — but has wrong H)

H_{env} **: ~ LQFT**

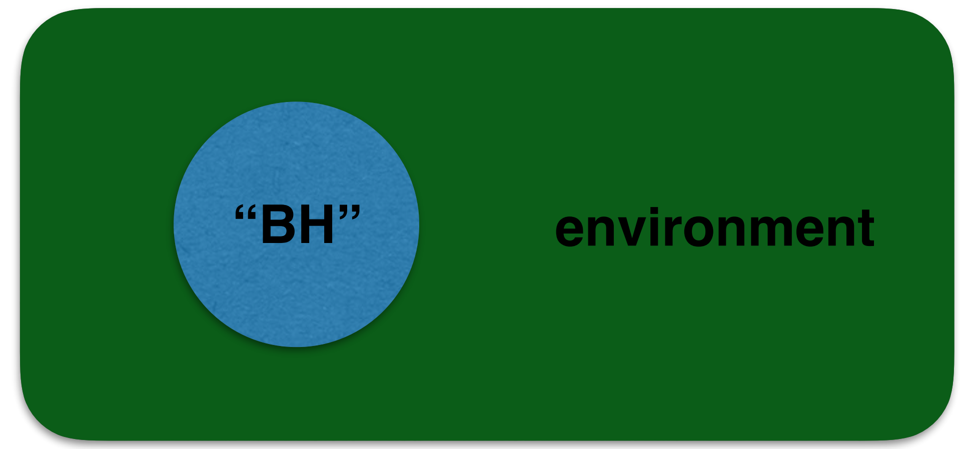
H_{bh} **: unknown, remain agnostic**

H_I **: must transfer information, by I)**

H_I : effective description; “parameterize our ignorance”

$$H = H_{bh} + H_{env} + H_I$$

$$|K, M; \psi_e, T\rangle$$



Simplest information transfer:

$$H_I = \sum_{Ab} \int dV \; \lambda^A \; O^b(x) \; G_{Ab}(x)$$

U(N) generators
(basic matrices
between BH states)

Act on “environment”

e.g. $\langle K|\lambda^A|J\rangle = \begin{pmatrix} 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ & & \cdots & & \end{pmatrix}$

One further postulate:

IV) Universality: new effects beyond LQFT couple *universally* to matter and gauge fields

(motivations: gravity; mining; ~BH thermo.)

$$H_I = \sum_{Ab} \int dV \; \lambda^A \; G_{Ab}(x) \; O^b(x)$$

$$\downarrow$$

$$H_I = \int dV \underbrace{\sum_A \; \lambda^A G_A^{\mu\nu}(x)}_{H_{\mu\nu}(x)} \; T_{\mu\nu}(x)$$

"BH state-dependent metric perturbation"

Further constraints:

$$H_I = \int dV \, H^{\mu\nu}(x) \, T_{\mu\nu}(x)$$

III, correspondence: $H^{\mu\nu}(x)$

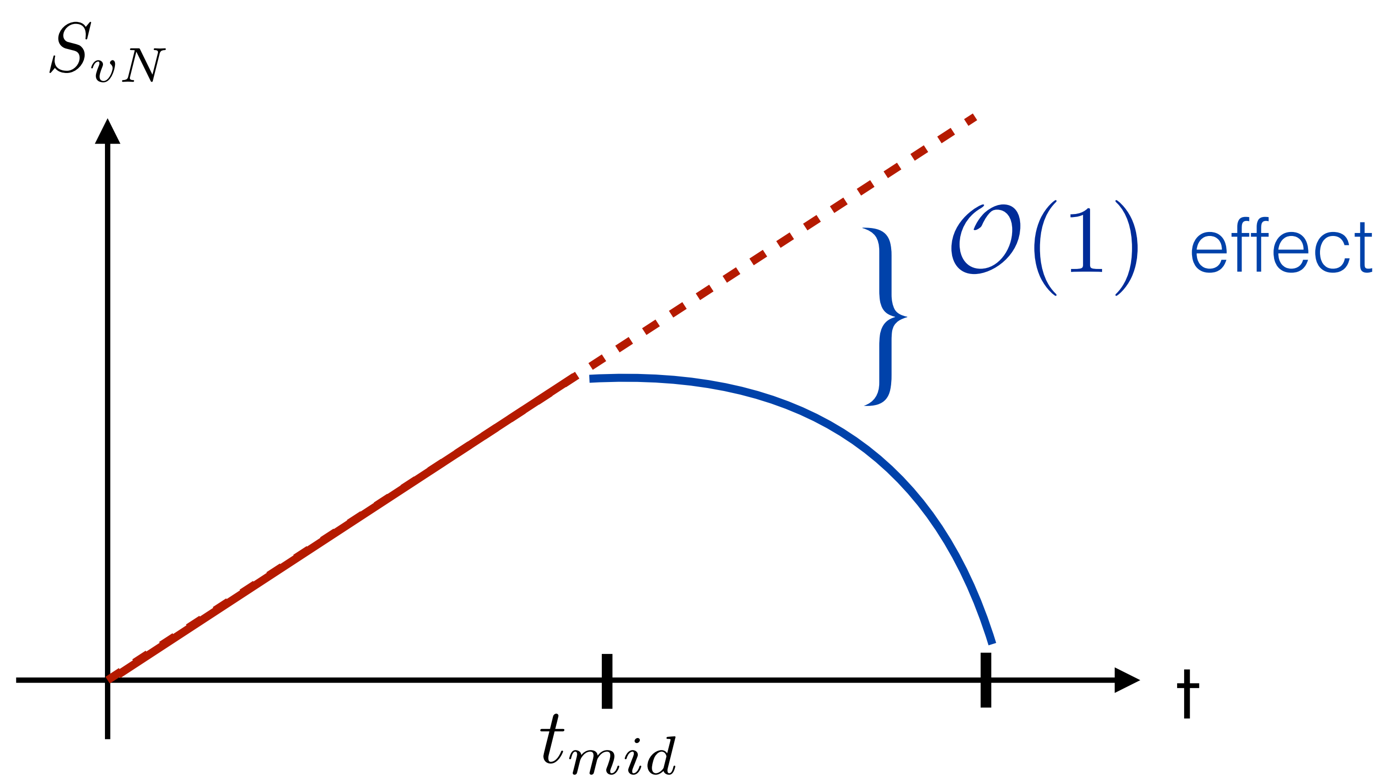
1) localized near BH

2) long wavelength, e.g. ~R

+ low energy, e.g. ~1/R

(e.g. avoid “firewall”)

I, QM (unitarity):



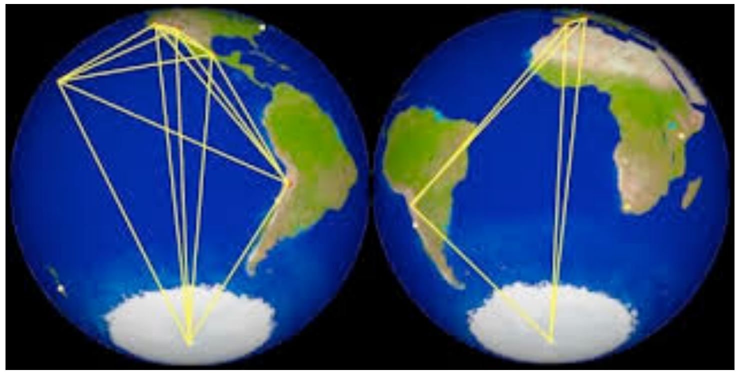
$$\frac{dI}{dt} \sim 1 \text{ qubit/R}$$

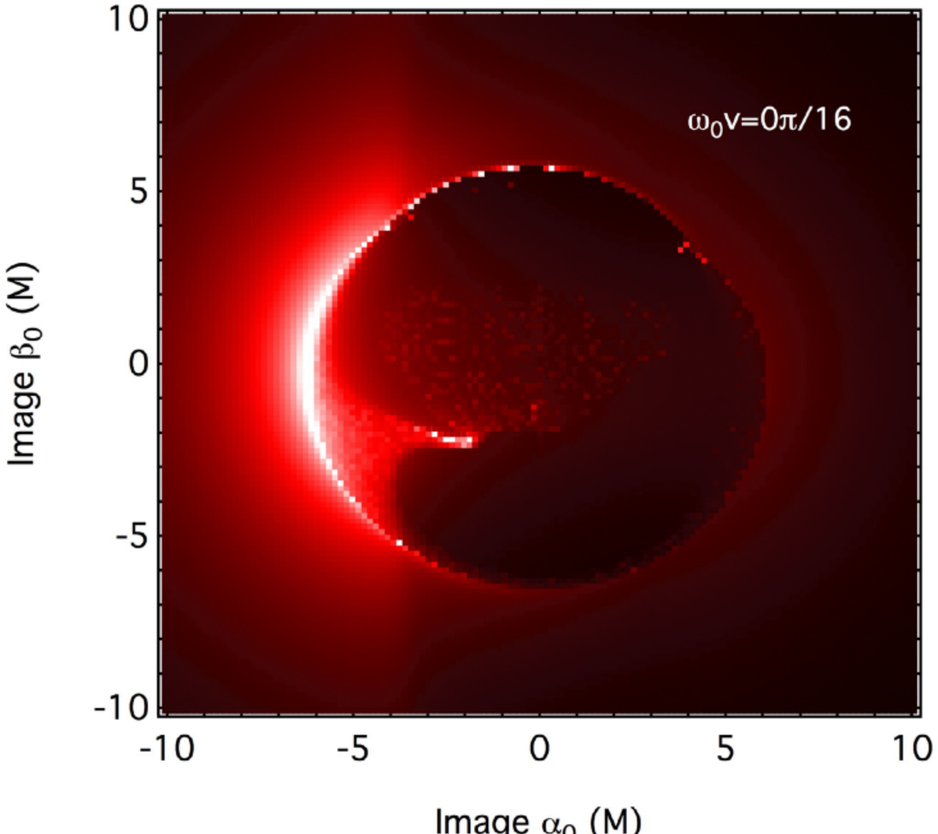
Sufficient condition: $\langle \psi, T | H^{\mu\nu}(x) | \psi, T \rangle \sim 1$

(distance, time scales ~ R)

I.e. O(1) metric perturbations!

This could also produce observable effects, e.g. to Event Horizon Telescope! (Sgr A*, M87)





[SG/Psaltis]
1606.07814

Necessary condition?

(Example of an interesting general problem in qinfo theory:

$$H = H_A + H_B + H_I \;)$$

arXiv:1701.08765

Rota; Discussion

Turns out: $\langle H_{\mu\nu} \rangle \sim e^{-S_{bh}/2}$ ***apparently suffices***

typical matrix element

Argument ~ Fermi's rule:

$$\frac{dI}{dT} \sim \frac{dP}{dT} = 2\pi\rho(E_f)|H_I|^2$$

$$H_I = \int dV \sum_A H^{\mu\nu}(x)\, T_{\mu\nu}(x)$$

$$\rho_{bh}(E) \sim e^{S_{bh}}$$

While effects can be "weak," two lessons:

1) **BHs are intrinsically quantum objects — *at horizon scales***

2) **Similar argument indicates O(1) modification to scattering amps of $\lambda \sim R$ modes: even weak scenario has GW signatures?!**

But an important question:
What is this telling us about the underlying dynamics of quantum gravity?

Summary/conclusions

When we better understand information for gravity, we will better understand quantum gravity; need to raise our standards

Localization/subsystems: key structural question; apparently different from LQFT

- **Observables nonlocal**
- **Perturbative localization of information: localized *states***
- **Insensitivity of soft charges, other asymptotic observables**
- **Plausibly part of foundational "Quantum-first" description**

Holography: nonperturbative "delocalization?"

- **but, appears to rely on unitary nonperturbative bulk evolution**

Unitary evolution:

- **in BHs, apparently possible via "exp small corrections"   observable??**
- **departure from locality (but not causality!) of LQFT**
- **fundamental description??**