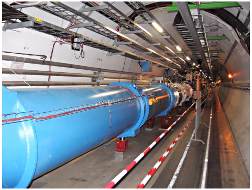


Gerard 't Hooft

Quantum Black Hole from Standard Physics

Presented at the CERN Workshop
on Quantum Gravity and Quantum Information

Institute for Theoretical Physics,
Science Faculty, Utrecht University,
POBox 80.089, 3508 TB, Utrecht

March 18 - 22, 2019

To describe a quantum system, we need:

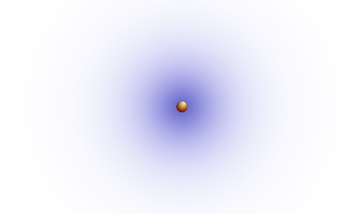
- A *Hilbert space*, spanned by an orthonormal basis of elementary states,
- An indication as to what these basis elements mean physically: what are the *observables*?
- A unitary *evolution operator* that tells us how the system evolves.
- Unitarity of this operator demands that the states evolve entirely within this Hilbert space, so we must be sure we have the entire Hilbert space needed to describe the system (*"completeness"*)

This can be done for a black hole. The Hilbert space is derived from the system of quantised fields (including grav. fields) in the black hole background. But completeness requires that we modify the *boundary conditions.*

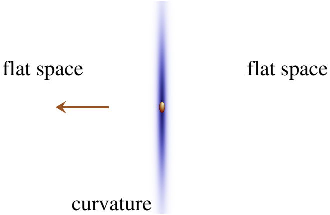
It all begins with the gravitational back reaction.

The *gravitational effect* of a fast, massless particle is easy to understand:

Schwarzschild metric of a particle with tiny rest mass $m \ll M_{\mathrm{Planck}}$:

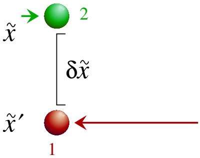


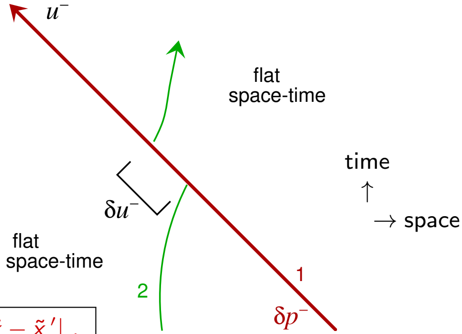
And now apply a strong *Lorentz boost*, so that $E/c^2 \gg M_{\mathrm{Planck}}$:



The gravitational backreaction:

Calculate the *Shapiro time delay* caused by the grav. field of a fast moving particle: simply Lorentz boost the field of a particle at rest:



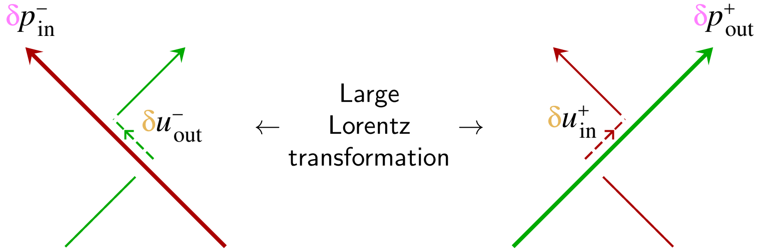


$$\boxed{\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}')\, \log|\tilde{x}-\tilde{x}'|\ .}$$

P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. **2** (1971) 303,
W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

The firewall transformation

The positions of *all* out-particles are *identified* with the momenta of *all* in-particles, and *vice versa*, such that the commutation rules are preserved: $[u^{\pm}, p^{\mp}] = i$.



Thus, we count the out-particles <u>not independently</u> from the in-particles; instead, our states in Hilbert space are defined *either* by counting all in-particles, *or* all out-particles.

These states are related by the BH scattering matrix.

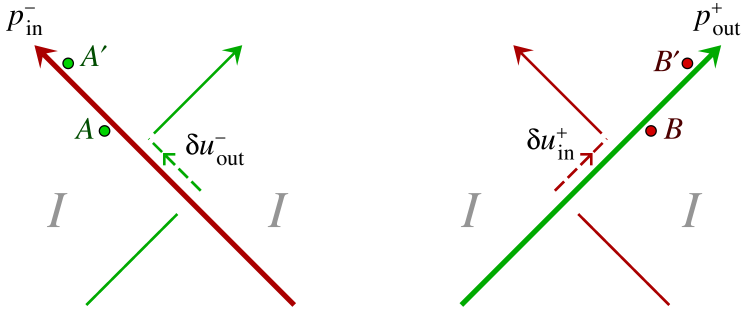
The best way to count the particles is: to count a particle
as in-going if $|p^-_{\rm in}| < M_{\rm Planck}$ and $|u^+_{\rm in}| > L_{\rm Planck}$, and
as out-going if $|p^+_{\rm out}| < M_{\rm Planck}$ and $|u^-_{\rm out}| > L_{\rm Planck}$.

Then these particles always keep their momenta small, so that they do not cause space-time curvature, while their positions are always greater than $L_{\rm Planck}$ so that "ordinary physics" should apply to them.

However, replacing momentum by position has a subtle effect on *space-time topology.*

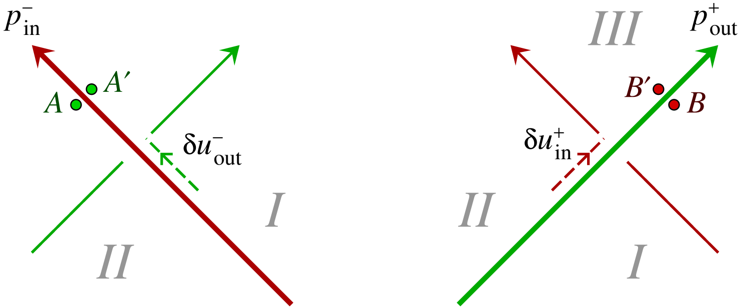
Which space-time points must be regarded as neighbors?

Standard GR formalism says that points are connected such that there is a deltra distribution of curvature:



Which space-time points must be regarded as neighbors?

But the firewall transformation treats
space-time as locally flat:

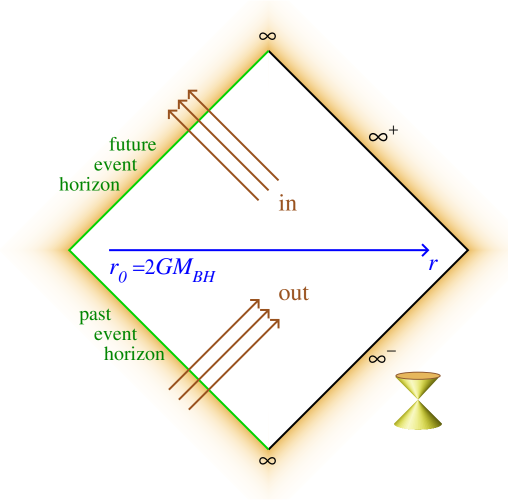


Thus, the firewall transformation

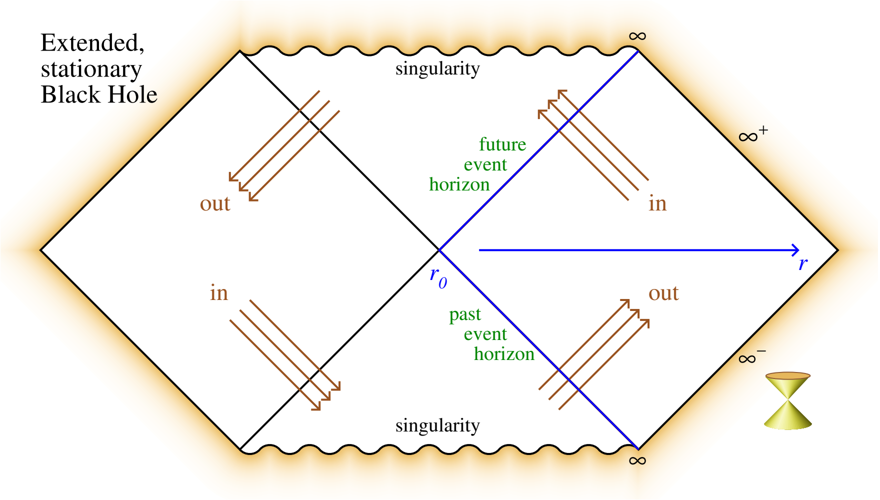
1) removes the firewalls by transforming hard in-particles into soft out-particles, and
2) this way it restores local smoothness of space-time.

To describe the evolution of all quantum states in the Schwarzschild geometry, we can now limit ourselves to the *maximal analytic extension* for the *eternal black hole*.

Black Hole



Extended, stationary Black Hole



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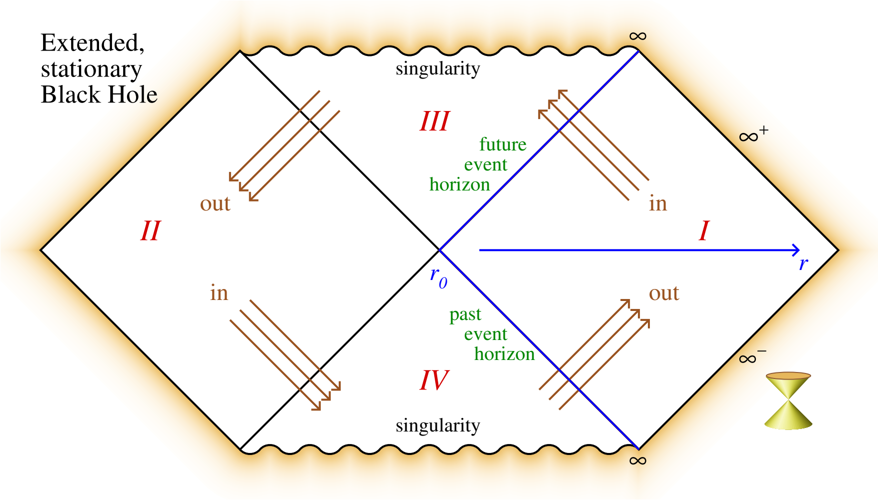
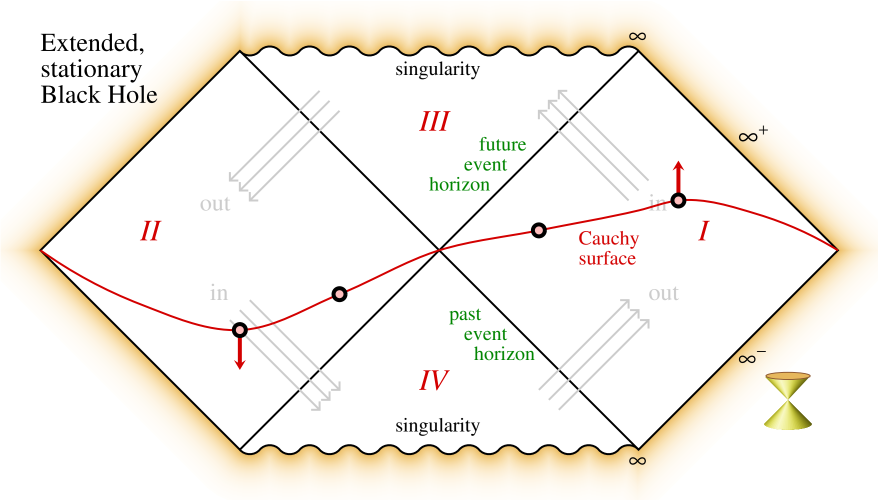
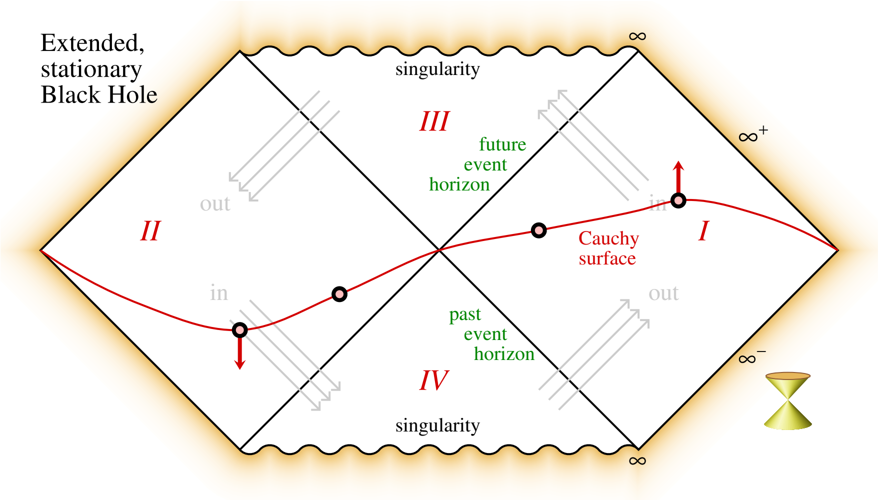


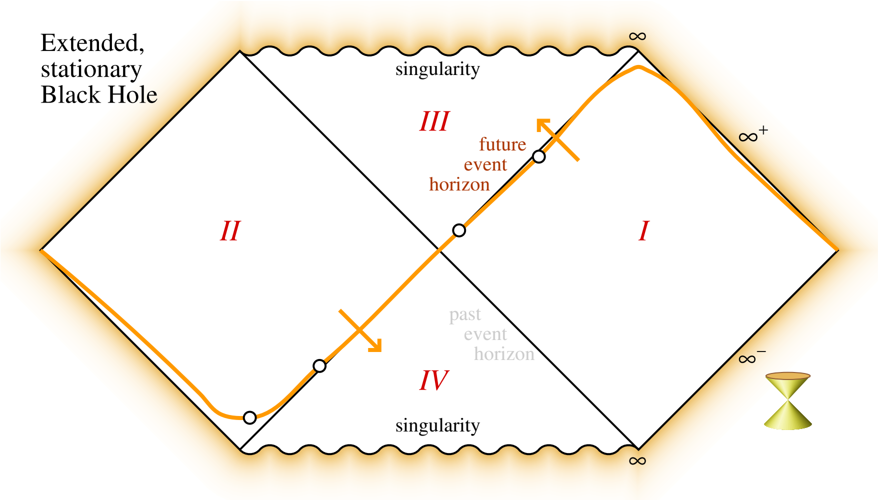


Extended,
stationary
Black Hole
∞
singularity
III
future
event
horizon
out
II
in
∞+
in
I
Cauchy
surface
out
past
event
horizon
∞−
IV
singularity
∞



$$\frac{x}{y} = e^{2\tau}$$

Extended,
stationary
Black Hole



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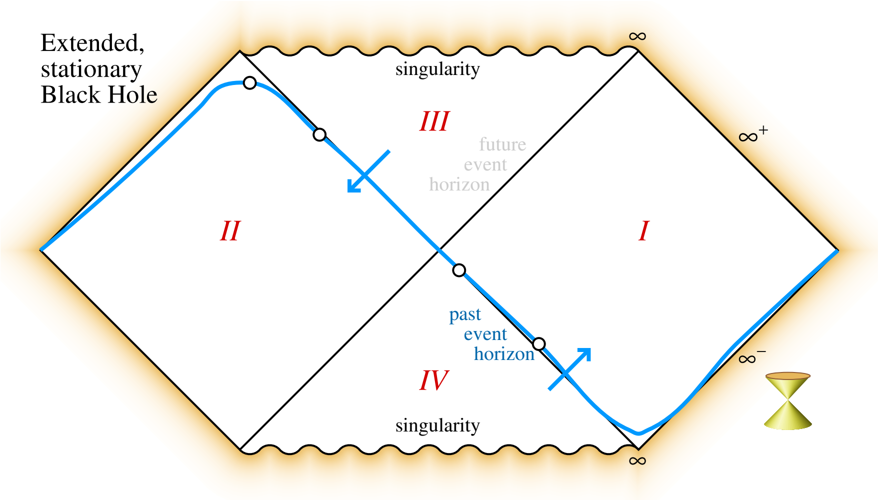
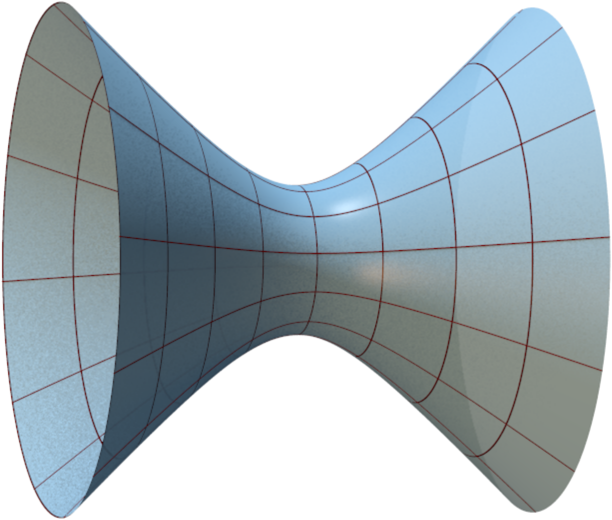


$$\frac{x}{y} = e^{2\tau}$$

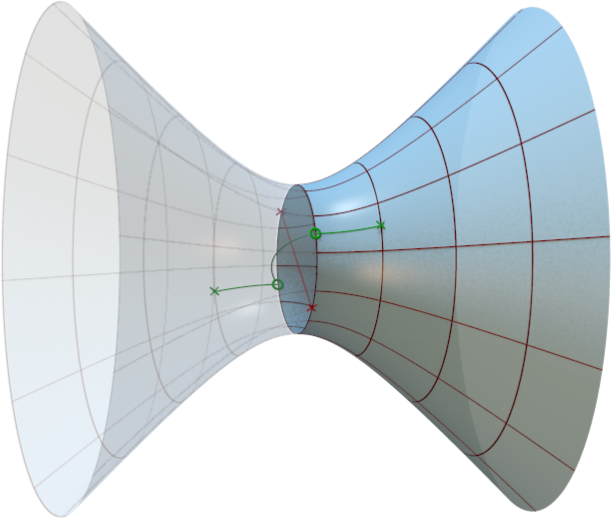
The antipodal identification: only points ON the horizon

II



I

Regions *I* and *II* describe *different* points in the *same* universe.



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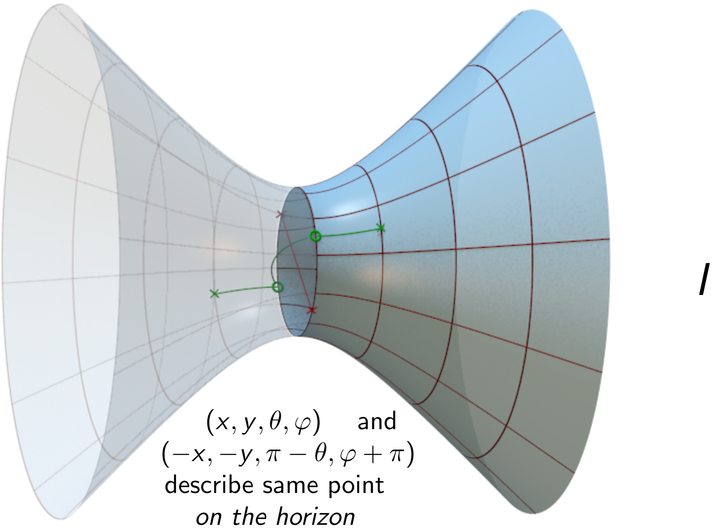
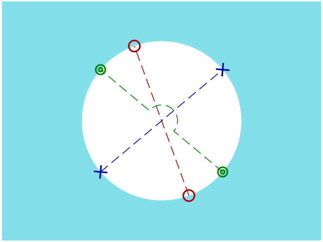


Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time: *an instanton*

N.Gaddam, O.Papadoulaki, P.Betzios (Utrecht PhD students)

Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

This scheme only requires CPT symmetry, no other combinations of C, P and/or T.

Entanglement of Hawking particles

The hartle-Hawking state,

$$|\,HH\,\rangle \;= C\sum_{E,n} e^{-\frac{1}{2}\beta E}\,|E,n\rangle_I\,|E,n\rangle_{II}$$

where $II \;=\;$ antipode of $\;I$,

is now a *pure* quantum state, where regions I and II are entangled.
It is *not* a thermal state.

Only if we do not look at states II, the states in I seem to form a perfect thermal mixture.

Only those General coordinate transformations are allowed that are one-to-one, so that no doubling takes place for the asymptotic regions.

Philosophy: what is needed is the evolution law over small time stretches.

If we have that, we can integrate the equations to get the large-time behaviour. Not the other way around!

During the small time interval, the black hole may be considered as eternal. This is why we consider the Penrose diagram of the stationary black hole.

The *imploding matter* and the *final explosive evaporation* do not play a role here. They are left out. Later we worry about how they can occur at other epochs of time.

During this interval we consider only excitations due to soft particles (these are the particles that do not (yet) affect the space-time metric as their grav. fields are weak).

Soft particles are soft for the local, freely falling observer.

The complete Hilbert space of the black hole will be spanned by

1) The Hartle Hawking (HH) state (which is just a single state describing a steady stream of in-going and out-going particles)
   For the local free-fall observer, it is just the vacuum state.

2) Other states, obtained by creating soft particles in HH as seen by the local free-fall observer.
   These operators undergo Bogolyubov transformations to create and annihilate particles as seen by the distant observer.

3) As time proceeds, soft particles turn into hard particles. But these leave their gravitational footprints in the bath of soft particles, as these are being displaced across the Cauchy surface.

By replacing the momentum operators $p_{\rm in}^-$ by the position operators $u_{\rm out}^-$ of the out-going particles, we exchange soft and hard particles. Thus, no hard particles will be left.

When $p^{\rm in}$ becomes VERY hard, the associated $u_{\rm out}^-$ coordinate of the out-particle will become so large that it leaves the system. Thus, the hard particles can be accommodated for by ordinary, soft particles states.

Note that, this way, out-particles can carry away energy out of the BH, so that, on long time scales, the mass is not constant.

The total energy is now exactly conserved (time translation invariance). The BH mass changes when a particle moves so far out that it is no longer part of the BH.

Expand in Spherical harmonics:

$$u^{\pm}(\Omega) = \sum_{\ell,m} u_{\ell m} Y_{\ell m}(\Omega) \ , \qquad p^{\pm}(\Omega) = \sum_{\ell,m} p^{\pm}_{\ell m} Y_{\ell m}(\Omega) \ ;$$

$$[u^{\pm}(\Omega),\, p^{\mp}(\Omega')] = i\delta^2(\Omega,\,\Omega') \ , \qquad [u^{\pm}_{\ell m},\, p^{\mp}_{\ell' m'}] = i\delta_{\ell\ell'}\delta_{mm'} \ ;$$

$$u^-_{\rm out} = \frac{8\pi G}{\ell^2+\ell+1} p^-_{\rm in} \ , \qquad u^+_{\rm in} = -\frac{8\pi G}{\ell^2+\ell+1} p^+_{\rm out} \ ,$$

$p^{\pm}_{\ell m}$ = total momentum in of ${}^{\rm out}_{\rm in}$-particles in (ℓ, m)-wave ,

$u^{\pm}_{\ell m}$ = (ℓ, m)-component of c.m. position of ${}^{\rm in}_{\rm out}$-particles .

Because we have linear equations, all different ℓ, m waves decouple, and for one (ℓ, m)-mode we have just the variables $u^{\pm}$ and $p^{\pm}$. They represent only one independent coordinate u^+, with $p^- = -i\partial/\partial u^+$.

The evolution law applies to every (ℓ, m) mode separately (in our approximation, there is no cross-talk). In each (ℓ, m) mode, the energy κ (seen by the distant observer) is separately conserved — but only when regions *I* and *II* are taken together: the states are *entangled* over *I* and *II*. So there is one wave function $\psi_\sigma(|u^+|)$ where $\sigma = I$ or II. The out-states are obtained from the in-states by Fourier transformation, which is unitary by construction:

The evolution equation, at given energy κ, is:

$$\psi^{\mathrm{out}} = \begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix} e^{-i\kappa \log\left(8\pi G/(\ell^2+\ell+1)\right)} \psi^{\mathrm{in}} , \qquad \text{where}$$

$$F_\pm(\kappa) = \int_0^\infty \frac{\mathrm{d}y}{y} y^{\frac{1}{2}-i\kappa}\, e^{\mp iy} = \Gamma(\tfrac{1}{2} - i\kappa)\, e^{\mp \frac{i\pi}{4} \mp \frac{\pi}{2}\kappa} .$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+F_-^* = -F_-F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

The integration kernal vanishes for large values of the argument, so this interaction is approximately local in time. The (ℓ, m) waves do not spend much time in the black hole. the Hartle-Hawking particles do stay there forever.

The operators $u(\theta,\varphi)$ and $p(\theta,\varphi)$ change signs if we go from pode to antipode. therefore, only odd values of ℓ contribute. This is counter-intuitive, but remember that u and p are **not second-quantized**.

Note that the local freely falling observer will find it obvious that u and p switch sign if we interchange regions *I* and *II*.

The identification of our position- or momentum-states as elements of the Fock space of the Standard Model is highly non-trivial

(and needs to be studied further.)

The $\ell = 0$ "dust shell" is not a legal state here. One must consider all its myriads of dust particles separately.
The *energy* is defined in the regions *I* and *II* separately, and *that* can be in odd and even ℓ states.

Our procedure it totally invariant under time translations.
Total energy is exactly conserved,

Not conserved: black hole mass =
total energy minus energies of all distant particles.

The particles in their (ℓ, m) modes do not spend much time in the black hole, but the Hartle-Hawking background is there eternally.

During formation and during final evaporation, more particles are outside, so the black hole starts out and ends up very light.

With our, calculable and unitary, evolution operator the system became totally internally consistent.

But ther relation between QFT Fock states and our spherical waves of momentum distribution requires further study.

Conjecture: high ℓ values distinguish different SM particles.

The Fourier transform in $x,\ p$ space is non-local:

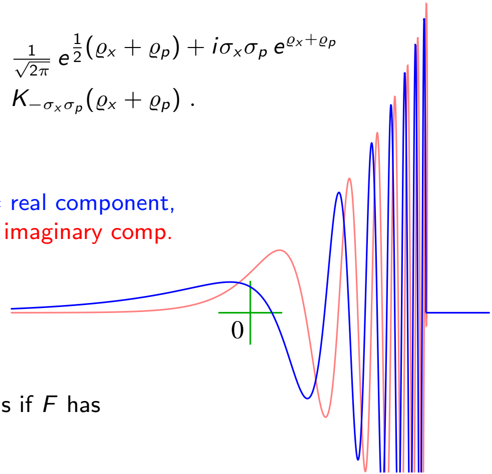
$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi}}\, e^{ipx}$$

But if we write $x = \sigma_x\, e^{\varrho_x}$ and $p = \sigma_p\, e^{\varrho_p}$, where σ_x and σ_p are signs $\pm$, then the relation becomes:

$$\begin{aligned}\langle \varrho_x, \sigma_x | \varrho_p, \sigma_p\rangle &= \frac{1}{\sqrt{2\pi}}\, e^{\frac{1}{2}(\varrho_x + \varrho_p) + i\sigma_x\sigma_p\, e^{\varrho_x+\varrho_p}} \\ &= K_{-\sigma_x\sigma_p}(\varrho_x + \varrho_p)\ .\end{aligned}$$

Blue = real component,
Red = imaginary comp.

$K_+(x)$:



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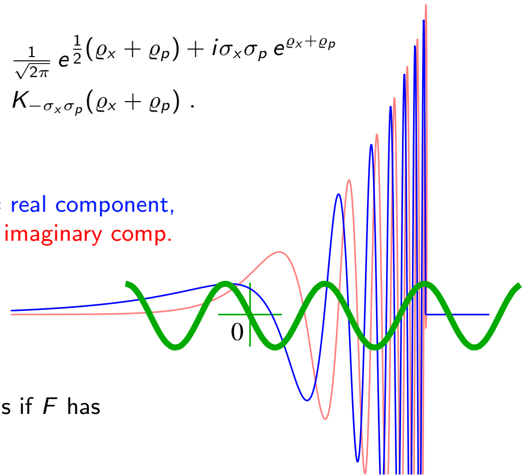

In practice it will appear as if F has a finite support.

THANK YOU

So, a fast particle moving in will shift all particles on their way out; some of them will move from region *I* to *II* or back.

This is why we cannot ignore the particles of region II .

Furthermore, region *II* is an *exact copy* of region *I*. It has asymptotic regions. Therefore, it represents an entire universe.

What universe is that ?

Only one answer makes sense:

It is the other side of the same black hole.

This is a topological twist without singularities,

because nowhere in regions I or II, the radius is less than $2GM_{\mathrm{BH}}$.

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