Netta Engelhardt

Princeton University

Spring Equinox 2019

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Recall: Why Study Black Hole Entropy

Black holes must have entropy, else you can decrease the entropy of the universe by throwing matter into a black hole.

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Grav. thermo is a hint in the IR lagrangian of gravity about its UV completion.

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- 1 Setup of the Problem
- 2 Attempts at a Solution
- 3 Resolution, Part I: Replacing the Event Horizon

- 4 Resolution, Part II: Entropy of BHs
- 5 Applications, Extensions, Implications...

6 Conclusion

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The Proposal: Event Horizon and Entropy

Bekenstein-Hawking:

$$S_{BH} = rac{A(ext{EH})}{4l_P^2}$$

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where A(EH) is the area of the black hole event horizon.

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Bekenstein-Hawking:

$$S_{BH} = rac{A(ext{EH})}{4l_P^2}$$

where A(EH) is the area of the black hole event horizon.

Confirmation for (certain) stationary BHs:

- Strominger-Vafa microstate counting
- Ryu-Takayanagi

AdS-Schwarzschild



$$S_{BH} = \frac{\text{Area}[X]}{4G\hbar} = S_{vN}[\rho_L] = S_{vN}[\rho_R]$$

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Stationary vs. Time Evolving

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Entropy of evolving black holes

What does the entropy of generic black holes measure, and how do we compute it?

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What does the entropy of generic black holes measure, and how do we compute it?

This problem is 45 years old. Until very recently, we didn't have the tools to solve it.

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Coarse-Grained Entropies

Even if the state of a system is pure, if our knowledge of the state is incomplete, we can define various notions of a *coarse-grained* entropy.

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Even if the state of a system is pure, if our knowledge of the state is incomplete, we can define various notions of a *coarse-grained* entropy.

For example, if we know some data \mathcal{D} about the state (e.g. expectation values of certain operators), we can maximize the von Neumann entropy over all states with this data. _{Janyes 1957}

$$S^{
m coarse} \equiv \max_{\mathcal{D}} S_{vN}$$

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Meaning that we maximize S_{vN} while keeping \mathcal{D} fixed.

Expectations: Coarse-Grained BH Entropy

Expectation

Want: S_{BH} to be the coarse-grained entropy of our ignorance of the BH interior, subject to knowledge of the exterior. Something like:

 $S_{BH} \stackrel{?}{=} \max_{\text{exterior}} S_{vN}$

Most Recent Claim to Fame

We can define a cross-section $\sigma(t)$ of the event horizon via a time-slice t of the boundary:



The region between $\sigma(t)$ and \mathscr{I} is the causal wedge of t, $C_W[t]$.

slight abuse of terminology here

Most Recent Claim to Fame

Kelly, Wall:

$$rac{\mathrm{Area}[\sigma(t)]}{4\ell_P^2} = \max_{\mathrm{C}_{\mathrm{W}}[t]} \mathcal{S}_{vN} \equiv \mathcal{S}^{(1)}[t].$$

Since C_W can be reconstructed from 1-pt functions work starting with HKLL, so this is really a maximization of S_{vN} under the constraint of the 1-pt functions being fixed.

Attempts at a Solution

Kelly-Wall, Debunked

Counterexample: NE, Wall



Contradiction!

Positivity and (assumed) uniqueness of CFT vacuum/rigidity theorem for vacuum in GR implies only pure AdS is consistent with grey region: $S^{(1)} = 0$. But area of *B* is nonzero.

... back at Square 1?

 It seems that we are back at zero for understanding the issue of black hole entropy (for non-stationary BHs)

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- It seems that we are back at zero for understanding the issue of black hole entropy (for non-stationary BHs)
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- How about rethinking what we mean by the BH exterior?
- Turns out that defining BH exterior via event horizons is problematic for other reasons as well.

Table of Contents

- 1 Setup of the Problem
- 2 Attempts at a Solution
- 3 Resolution, Part I: Replacing the Event Horizon

- 4 Resolution, Part II: Entropy of BHs
- 5 Applications, Extensions, Implications...

6 Conclusion

Recall: defn of an Event Horizon

Black hole event horizon

A black hole is a region of the spacetime that cannot send light rays to any future-infinite observer. The boundary of this region is the event horizon.

Recall: defn of an Event Horizon

Black hole event horizon

A black hole is a region of the spacetime that cannot send light rays to any future-infinite observer. The boundary of this region is the event horizon.

... that means that the event horizon cannot be observed by any observer at finite time; it also doesn't exist unless there's an asymptotic future.

List of drawbacks to the event horizon

1 Entropy is not defined if there is no asymptotic future

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List of drawbacks to the event horizon

1 Entropy is not defined if there is no asymptotic future

2 Uncomfortable acausality: even if there is an asymptotic future (and we have no way of determining whether there is one), the entropy of a black hole now should not depend on matter falling into it in 10 million years. "Black hole teleology problem".

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- It gets worse: event horizons don't usually exist in cosmologies even when there is an asymptotic future (unless the spacetime is asymptotically de Sitter)

Resolution, Part I: Replacing the Event Horizon

New Horizons: A Wishlist



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1 Capture a notion of light being trapped in a black hole

Resolution, Part I: Replacing the Event Horizon

New Horizons: A Wishlist



Capture a notion of light being trapped in a black hole
 Depend only on data which is local in time



Resolution, Part I: Replacing the Event Horizon

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- 6 Satisfy a broadly applicable area law

Local Notions of "Light Can't Escape"

2 ways to fire light from a surface to the future, given by 2 vectors:



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Local Notions of "Light Can't Escape"

Generate two lightlike surfaces:



Outgoing light goes out, ingoing light goes in - but only in (approximately) flat space!

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Trapped Surfaces

 Near a black hole singularity (or close to a big crunch), spacetime volume contracts: both ingoing and outgoing light rays contract. These are called trapped surfaces.

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Trapped Surfaces

- Near a black hole singularity (or close to a big crunch), spacetime volume contracts: both ingoing and outgoing light rays contract. These are called trapped surfaces.
- Right after a big bang singularity, spacetime volume expands rapidly: both ingoing and outgoing light rays expand. These are called anti-trapped surfaces.

Local Black Hole Boundaries

If black hole = trapped surfaces, and outside of the black = normal surfaces, then the boundary of a black hole is the region between trapped surfaces and normal surfaces. These have one contracting direction of light rays and one stationary direction: neither expanding nor contracting; called marginally trapped.

(Aside: works in time reverse for expanding cosmology: marginally anti-trapped surfaces.)

Future/Past Holographic Screens

A future holographic screen is a surface that can be foliated by marginally trapped surfaces s(r) (where r indexes the foliation). Every point on a holographic screen lies on precisely one of these marginally trapped surfaces. Bousso, NE



Past holographic screen: foliating by marginally anti-trapped surfaces.

Visual Construction of Holographic Screens from Bousso '99

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Visual Construction of Holographic Screens from Bousso '99





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Example of past holographic screens: cosmology



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6 Satisfy a broadly applicable area law

An Area Law for Holographic Screens

Area Theorem Bousso, NE

The area of cross-sections of a holographic screen increases monotonically with flow along the holographic screen.

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An Area Law for Holographic Screens

Area Theorem Bousso, NE

The area of cross-sections of a holographic screen increases monotonically with flow along the holographic screen.



Assumes Null Energy Condition and a genericity condition.

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- 5 Area \propto coarse-grained entropy of exterior (valid in those cases where event horizon entropy is accounted for.)

6 Satisfy a broadly applicable area law

New Horizons: A Wishlist

- 1 Capture a notion of light being trapped in a black hole
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So only thing left to do is solve a 45-year-old problem.

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Area as a Coarse-Grained Entropy

Want: entropy that is associated to our ignorance of the region inside the black hole subject to a complete knowledge of the spacetime outside of the black hole.





Area as a Coarse-Grained Entropy

Want: entropy that is associated to our ignorance of the region inside a slice *s* of the holographic screen subject to a complete knowledge of the spacetime outside of *s*.



Let's be Precise

We define the region outside of s and between s and \mathscr{I} as the outer wedge: $O_W[s]$.



We call the entropy associated with our ignorance of the interiof of s the outer entropy NE, Wall:

$$S^{\text{outer}}[s] \equiv \max_{O_W[s]} S_{vN}$$

Let's be Precise

We're going to specialize to a subset of holographic screens (roughly speaking, the outermost spacelike component of a holographic screen). In particular, a leaf s of a holographic screen is called a *minimar* surface [NE, Wall] if

• Every surface $\sigma \subset \Sigma_{out}$ circumscribing *s* has larger area:

$\operatorname{Area}[\sigma] > \operatorname{Area}[s]$

 s is strictly stable: θ_{ℓ,k} < 0 (requires genericity condition or inequality can be saturated); this is equivalent to requiring that s lie on a spacelike component of a future holographic screen Ashtekar, Galloway

Area = Coarse-Grained Entropy NE, Wall

$$S^{ ext{outer}}[s] \equiv \max_{O_W[s]} S_{vN} = rac{ ext{Area}[s]}{4\ell_P^2}$$

where s is a minimar surface (slice of outermost spacelike component of holographic screen).

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where s is a minimar surface (slice of outermost spacelike component of holographic screen).

The area of the (quasi-local) black hole horizon is the maximum entropy that is compatible with its exterior.

Proof Sketch

 The area of the HRT surface X in any aAdS spacetime computes S_{vN}

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Proof Sketch

- **1** The area of the HRT surface X in any aAdS spacetime computes S_{vN}
- 2 We prove that in any classical spacetime that we can put behind a minimar surface s, the area of the HRT surface is bounded from above by the area of s: Area[X] ≤ Area[s]; this is done by a combination of focusing and applying Wall's maximin formalism.

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4 Then we construct a spacetime in which the areas are equal.

Coarse-Grained Spacetime

What geometry is the maximizing "coarse-grained" state dual to?



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Details

We specify initial data on N_{-k} (e.g. $\theta_k = 0 = T_{kk}$, \mathcal{R} is constant...). Get:

$$abla_k \theta_\ell \big|_{\lambda = \text{const}} < 0$$



Here we've introduced coordinates u and v, with $\ell^a = (d/du)^a$ and $k^a = (d/dv)^a$.

Location of X

Location of X is given by:

$$0 = \theta_{\ell}[s] + L[v]$$

where v is a function of the transverse coordinates and L is an elliptic operator.

The Operator L

The stability operator is *invertible* whenever s is (strictly) stable Andersson-Mars-Simon, so the equation has a solution, and the solution has $\nu < 0$ Krein-Rutman theorem.

Aside: a vector generalization of L is very useful for understanding properties of surfaces (particularly extremal

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surfaces) in general; upcoming work w/ S. Fischetti.

Altogether

There exists a spacetime with outer wedge $O_W[s]$ whose CFT dual ρ_1 satisfies:

$$S_{\nu N}[
ho_1] = rac{\operatorname{Area}[X]}{4G\hbar} = rac{\operatorname{Area}[S]}{4}$$

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Resolution, Part II: Entropy of BHs

Altogether

There exists a spacetime with outer wedge $O_W[s]$ whose CFT dual ρ_1 satisfies:

$$S_{\nu N}[
ho_1] = rac{\operatorname{Area}[X]}{4G\hbar} = rac{\operatorname{Area}[s]}{4}$$

Since we've already shown that for any state ρ with a CFT dual containing $O_W[s]$, $S_{vN}[\rho] \leq \text{Area}[s]/4$, this means that

$$\frac{\text{Area}[s]}{4} = \max_{O_W[s]} S_{vN} = S^{\text{outer}}[s]$$

Resolution, Part II: Entropy of BHs

New Horizons: A Wishlist

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The Entropy of Pure State Black Holes

Applications, Extensions, Implications...

Explanation of the Area Law as a Second Law



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The Entropy of Pure State Black Holes

Applications, Extensions, Implications...

Explanation of the Area Law as a Second Law



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Penrose Inequality in AdS

Briefly discussed on Monday, the Penrose Inequality is a conjecture that the area of apparent horizons is bounded by the ADM mass.

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Penrose Inequality in AdS

Briefly discussed on Monday, the Penrose Inequality is a conjecture that the area of apparent horizons is bounded by the ADM mass. It is considered to be a test of cosmic censorship. In 4D asympt. flat space:

$$M \ge \left(rac{\operatorname{Area}[\sigma]}{16\pi}
ight)^{1/2}$$

Equivalently:

$$\operatorname{Area}[\sigma] < \operatorname{Area}[\operatorname{Static BH w}/ \text{ mass } M]$$

Penrose Inequality in AdS

The construction of the coarse-grained spacetime can be used to prove an AdS Penrose Inequality NE, Horowitz:

General AdS Penrose Inequality

Let σ be an apparent horizon in D dims, and let M be the spacetime mass. Then:

Area[σ] \leq Area[Static AdS BH w/ mass M].

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Stock-Taking

1 Black holes must have entropy, and this entropy must be coarse-grained.

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Stock-Taking

- Black holes must have entropy, and this entropy must be coarse-grained.
- Holographic screens have the same nice properties that make event horizons attractive: area monotonicity theorem, agreement with old microstate counting arguments in stationary case

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- Period Physics Phys
- Solved a very old problem: area of spacelike holographic screens in black holes is the entropy of our ignorance of their interior.

Stock-Taking

- Black holes must have entropy, and this entropy must be coarse-grained.
- Pholographic screens have the same nice properties that make event horizons attractive: area monotonicity theorem, agreement with old microstate counting arguments in stationary case
- Solved a very old problem: area of spacelike holographic screens in black holes is the entropy of our ignorance of their interior.
- 4 This has a number of nice applications, from a thermodynamic explanation of the (spacelike) holographic screen area law and a proof of the AdS Penrose inequality.

To Do List

- Understand if the infalling matter can always be removed via simple local sources;
- Generalize to include quantum corrections upcoming: Bousso,

Chandrasekaran, Shahbazi Moghaddam

- Understand boundary-anchored version see also discussion in Marolf-Grado-White ;
- If boundary-anchored version can be understood, would give an interpretation of the area of non-minimal extremal surfaces
- Properties of the coarse-grained entropy on the boundary;
- What about the area law for timelike component of holographic screens?

Boundary Description

Can we compute S^{outer} from the CFT?

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Boundary Description

Can we compute S^{outer} from the CFT?

To do so, we need to find a way of fixing the outer wedge from boundary data.

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Boundary Description

Can we compute S^{outer} from the CFT?

To do so, we need to find a way of fixing the outer wedge from boundary data.

This boils down to reconstructing the data on N from the CFT:



Boundary Description

If the black hole is in equilibrium after N, then this is the same as reconstructing the data in the causal wedge.



 $S^{outer}[\mu] = \mathcal{S}^{(1)}[t]$

Also works if black hole is approximately in equilibrium - i.e. after it settles down.

Boundary Description Out of Equilibrium

Black hole not in equilibrium \Rightarrow there is infalling matter.

Idea

Suppose that by turning on certain local sources (that propagate causally in the bulk) σ , we can remove this matter. Can reconstruct causal wedge in new state from the one point functions $\langle O \rangle_{\sigma}$. Then we can turn σ off. This gives us the data on N.



Boundary Description Out of Equilibrium

But we don't necessarily know which sources correspond to removing the infalling matter.

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Boundary Description Out of Equilibrium

But we don't necessarily know which sources correspond to removing the infalling matter.

Theorem: We can never recover *more* than $O_W[s]$ no matter which sources we turn on. Hawking

$$egin{aligned} S^{outer}[s] &= \max_{O_W[s]} S_{vN} \ &\leq \max_{\langle O(t'>t)
angle_\sigma} S_{vN} \end{aligned}$$

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If the infalling matter can be removed by some local bulk-causal source, then it is an equality.