

Extracting the Universe from the Wave Function

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CERN Workshop on
Quantum Gravity and
Quantum Information,
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Quantum gravity is hard, but we're not completely lost.

Guiding principles:

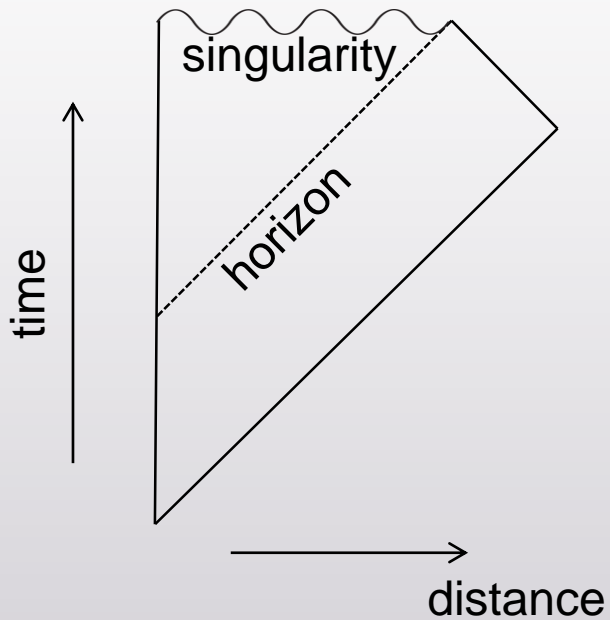
- What a quantum theory is (H, H , A)
- Classical gravity, $R_{\mu\nu} - (1/2)Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$
- Quantum field theory
- Black hole entropy & evaporation

Save time by not worrying about:

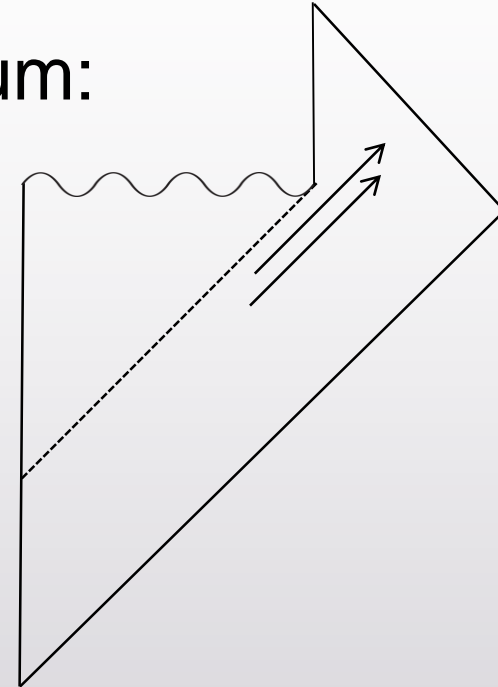
- String theory, loop quantum gravity, AdS/CFT...

Hawking's Legacy: black hole radiation, evaporation, entropy, and unitarity

Classical black hole:



Quantum:

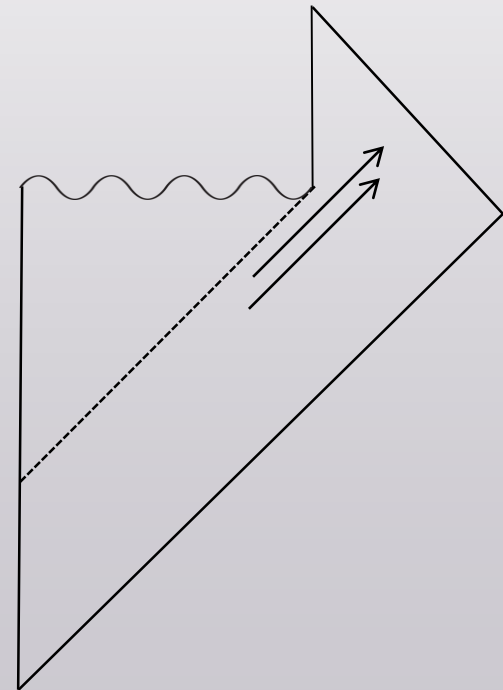


Black hole temperature: $T = \frac{1}{8\pi GM}$ $M = \text{mass}$

Black hole entropy: $S = \frac{A}{4G}$ $A = \text{horizon area}$

Issues/questions from black hole evaporation

- Generalized 2nd Law of Thermo: $\delta \left(\frac{A}{4G} + S_{\text{matter}} \right) \geq 0$
- What are the degrees of freedom underlying S_{bh} ?
- Is evaporation unitary?
Or is information lost?
- Is there a firewall at the horizon, or a fuzzball inside?



Two routes to progress

1. **Semiclassical.** Consider quantum fields on a background spacetime, perhaps with quantum fluctuations thereof.

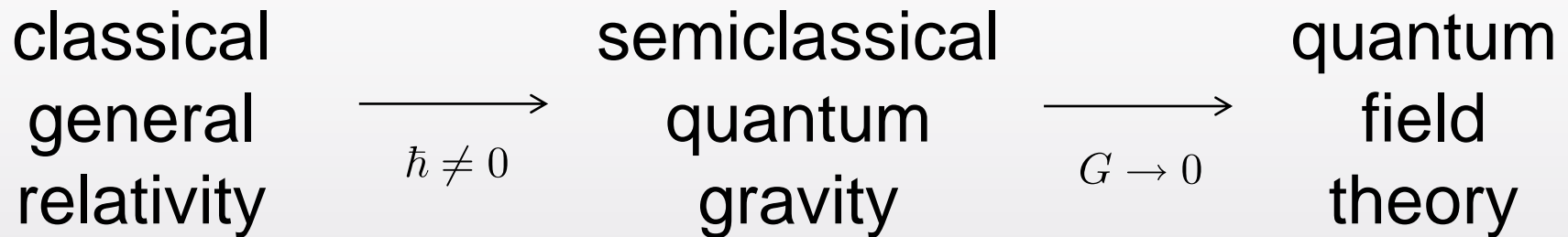
Focus: understanding BH info, putting it to use.

2. **Fully quantum.** Wave function of the universe. Space (and time?) are emergent.

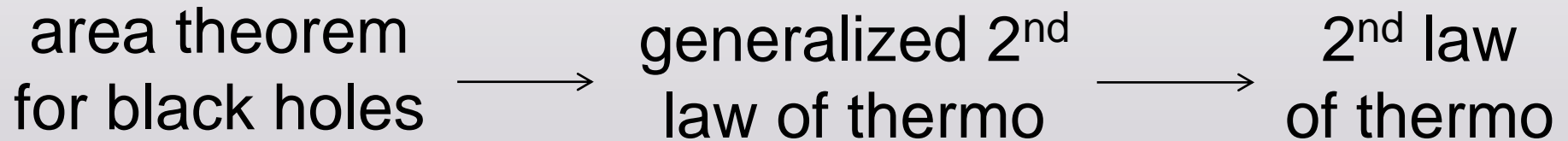
Focus: understanding locality and structure of Hilbert space.

QFT theorems from quantum gravity (Bousso)

Paper-generating algorithm:



e.g.

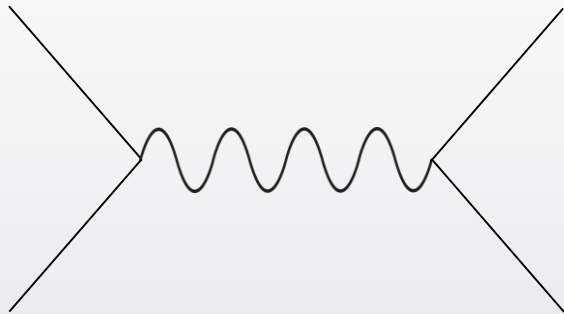


$$\delta A \geq 0 \qquad \delta \left(\frac{A}{4G} + S_{\text{matter}} \right) \geq 0 \qquad \delta S_{\text{matter}} \geq 0$$

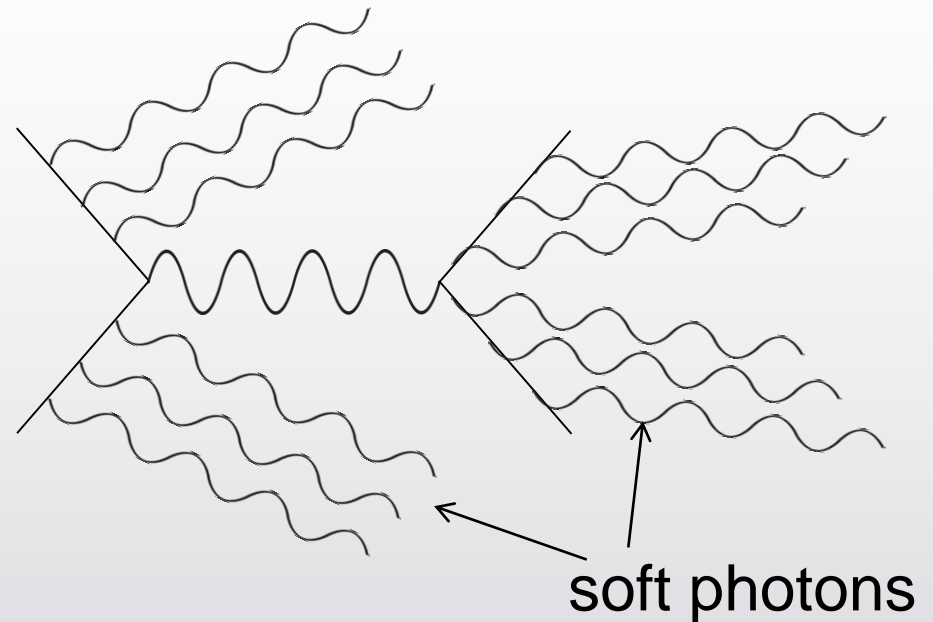
Also novel results, e.g. “Quantum Null Energy Condition.”

Soft modes/hair (Semenoff, Puhm, Comprere)

Scattering in QED:



is really



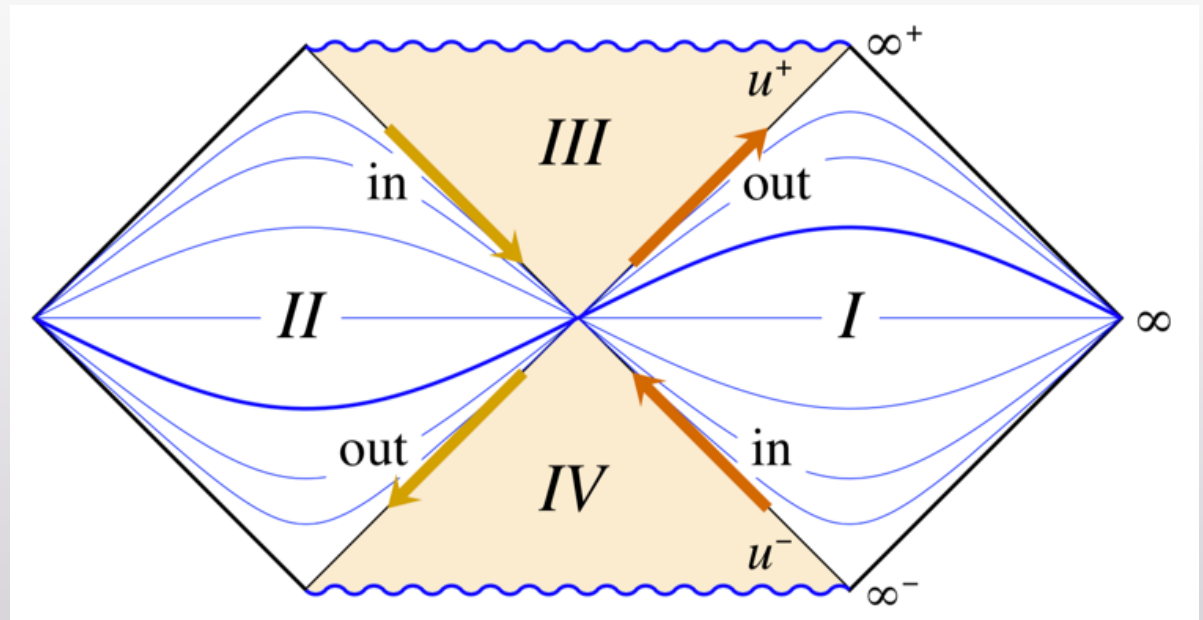
- Tracing out soft photons causes decoherence for hard modes.
- Can soft gravitational modes help carry information out of black holes?

Black Hole microstates from QFT ('t Hooft)

Consider low-energy modes in an eternal BH background.

Antipodal
identification:
 $I, III \leftrightarrow II, IV$

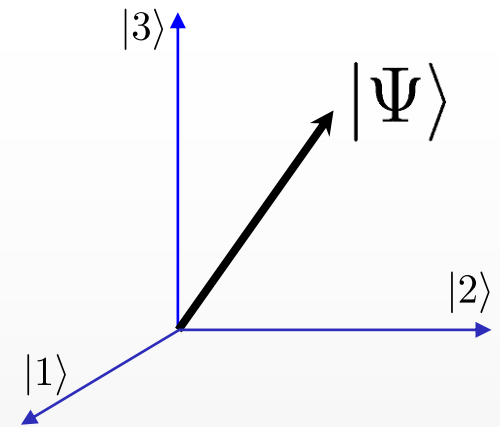
Hilbert space
spanned by soft
in or out modes



Can calculate a unitary S-matrix connecting incoming and outgoing particles.

Quantum-first gravity:

$|\Psi\rangle$, not $\Psi(x)$.



- The universe is just a vector evolving smoothly in a Hilbert space. Not a function “of” anything.
- **Fundamental concepts:** states, tensor products, factors, density operators, entropy, entanglement, mutual information, Hamiltonian...
- **Derived (emergent) concepts:** particles, fields, forces, space, metric, locality, (time?)...

The challenge: finding our universe in the wave function.

Three angles on quantum-first gravity

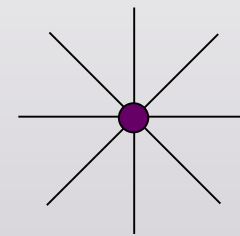
- AdS/CFT. Bulk geometry arises from boundary entanglement. (van Raamsdonk, Swingle, others)
- Learn what we can from gauge QFT's. Hilbert-space structure, algebra of observables, etc. (Giddings)
- Start with locally-finite-dim. Hilbert spaces. Use entanglement to define geometry. Space, locality, field theory are emergent, ultimately from the Hamiltonian. (Carroll, Bao, Cao, Chatwin-Davies, Singh, &c.)

Non-Locality and structure of Hilbert Space in Quantum Gravity (Giddings)

We tend to think we can factorize Hilbert space into a tensor product of regions, $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$.

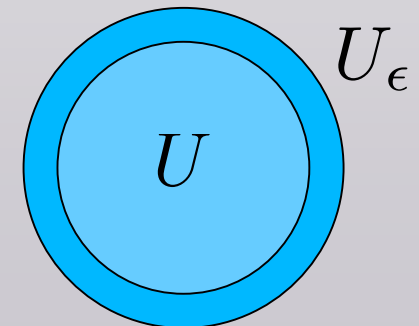
But that's not really true, even in QFT. Define subsystems using commuting subalgebras of operators.

In gravity, can't even confine gauge-invariant operators to a region.



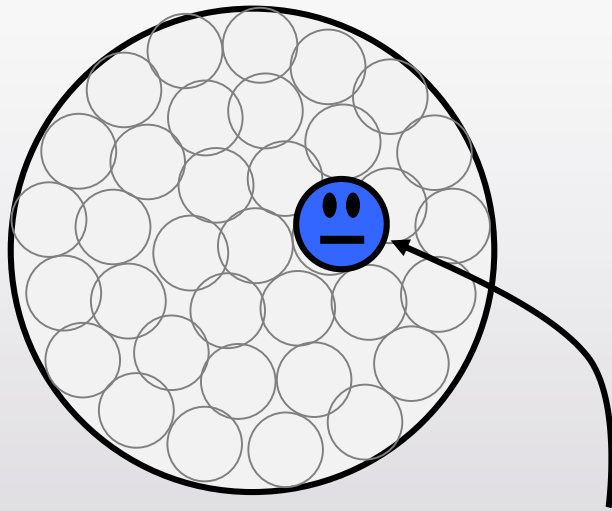
Instead, “split structure”:

$$\bigoplus_{p_\mu, s} \mathcal{H}_{U, p_\mu, s, s_z} \otimes \mathcal{H}_{\bar{U}_\epsilon} \hookrightarrow \mathcal{H}$$

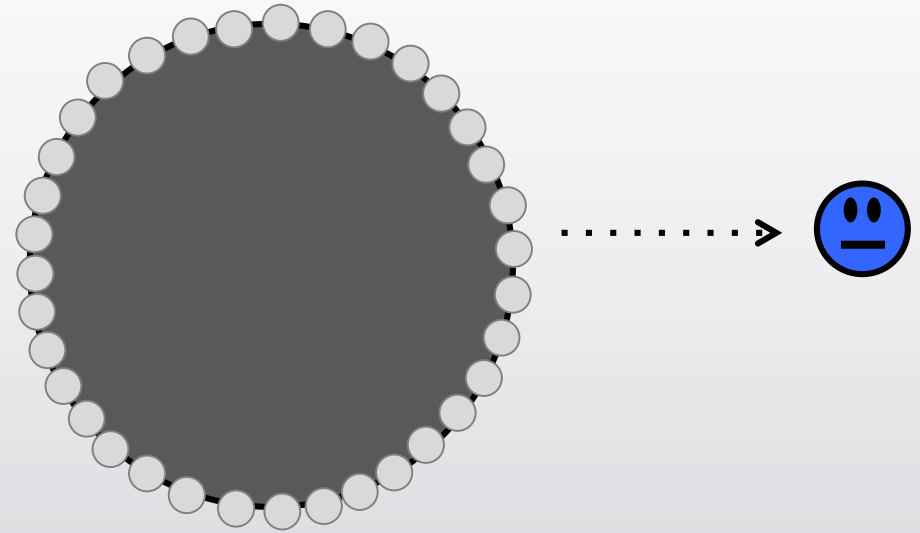


Hints of non-locality: holography, complementarity, unitarity

Two views of a black hole and its degrees of freedom:



infalling observer: local
quantum field theory



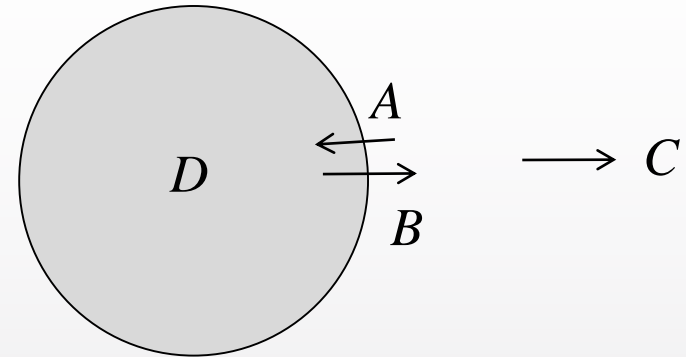
external observer: degrees of freedom
spread holographically across horizon

Radiation from horizon starts very high-energy, so it's
measuring the BH in a different way.

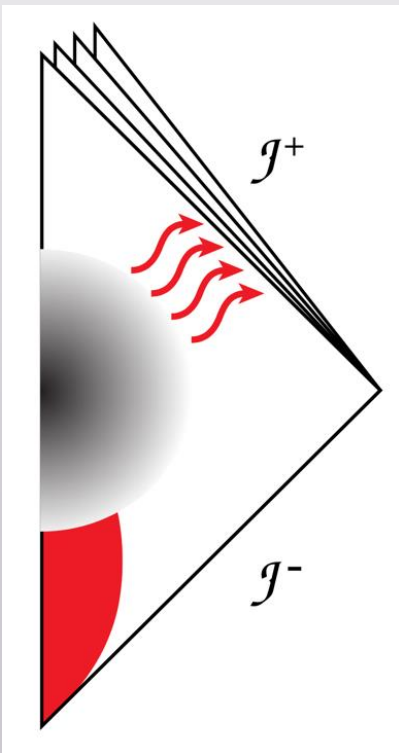
Firewalls?

Standard lore:

- $A+B$ must be maximally entangled for no firewall;
- $B+C$ must be maximally entangled for unitarity.
- B can't be maximally entangled with two things at once



[Almheiri, Marolf,
Polchinski & Sully 2012]



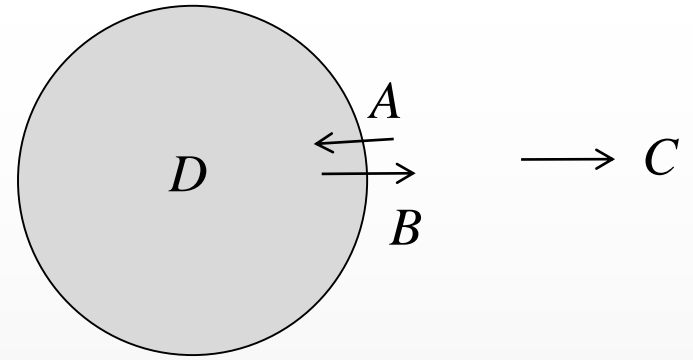
But:

“Unitarity” is a statement about the global wave function of the universe.

“No firewall” is a statement about branches describing semiclassical spacetimes.

[Hertog & Hartle 2017]

No contradiction between B+C being entangled in the global wave function, and A+B being entangled on every branch.



Bell states: $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$

Toy model for global wave function:

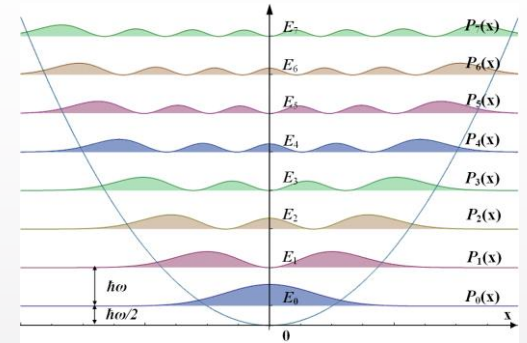
$$|\Psi\rangle = |\phi^+\rangle_{AD} \otimes |\phi^+\rangle_{BC}$$

If C+D is the “environment,” branching structure can have A+B entangled on each branch. No logical contradiction.

$$|\Psi\rangle = |\phi^+\rangle_{AB} \otimes |\phi^+\rangle_{CD} + |\phi^-\rangle_{AB} \otimes |\phi^-\rangle_{CD} \\ + |\psi^+\rangle_{AB} \otimes |\psi^+\rangle_{CD} - |\psi^-\rangle_{AB} \otimes |\psi^-\rangle_{CD}$$

Locally-finite Hilbert space

QFT has $\dim(\mathcal{H}) = \infty$ because energy can be infinitely large.



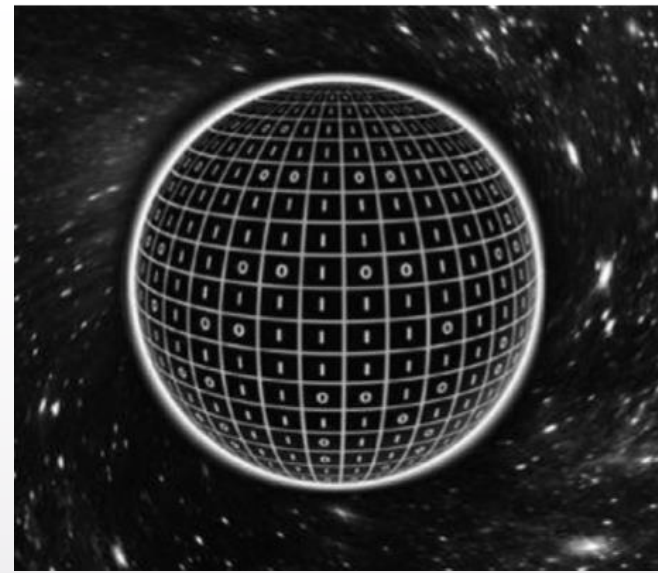
But with gravity there's a cutoff: try to fit too much energy in a region (more than $E = L/2G$), and you just make a black hole. Try to fit in more, and the black hole grows larger than L . [Bekenstein]

With gravity, only a finite # of things can happen in a region. **Therefore we expect that $\dim(\mathcal{H}) = \text{finite}$.**

Indeed we know what $\dim(\mathcal{H})$ is.

Black holes have entropy proportional to the area of the event horizon:

$$S_{BH} = \frac{A}{4G} \sim \left(\frac{L}{L_P} \right)^2$$



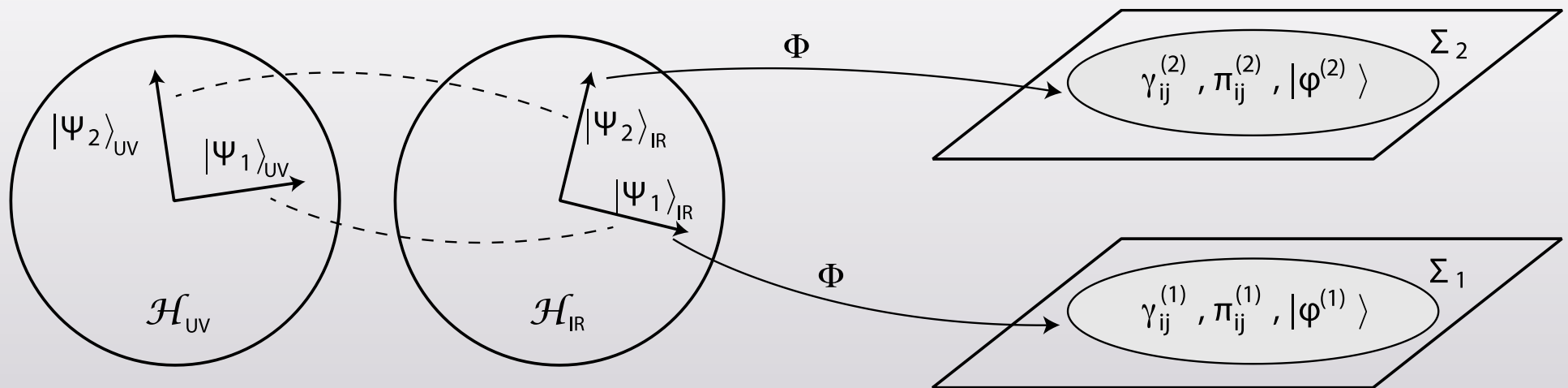
This relates to dimensionality of Hilbert space of the region:

$$\dim(\mathcal{H}) \sim e^S \sim e^{(L/L_P)^2}.$$

Suggests that regions of space are described by finite-dim factors of Hilbert space.

Obvious challenges for Lorentz invariance, gauge theory, etc.

“Locally” is tricky if space itself is emergent. Divide full Hilbert space into system (infrared, accessible degrees of freedom) and environment (ultraviolet or far-away ones). The system will have **pointer states**, robust under environmental monitoring.

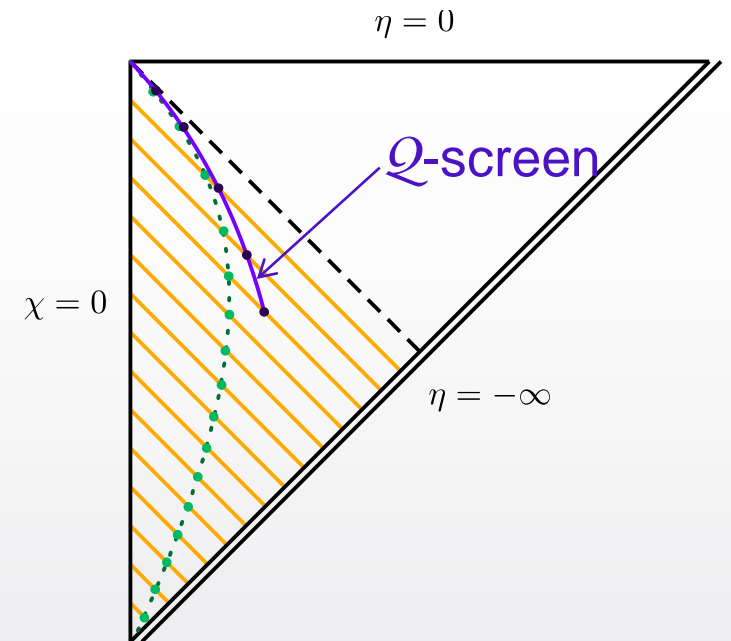


These pointer states describe emergent macroscopic regions of space. “Locally finite” means that finite-dimensional factors of \mathcal{H}_{IR} describe compact regions of the emergent space in some particular branch.

Same goes for our universe, which by the Cosmic No-Hair Theorem is asymptoting to **de Sitter space**.

And Cosmic No-Hair can be derived from the Generalized Second Law of Thermodynamics:

$$S_{\text{gen}} = S_{\text{thermo}} + \frac{A}{4G}$$



[Bousso & Engelhardt 2015;
Carroll & Chatwin-Davies 2017]

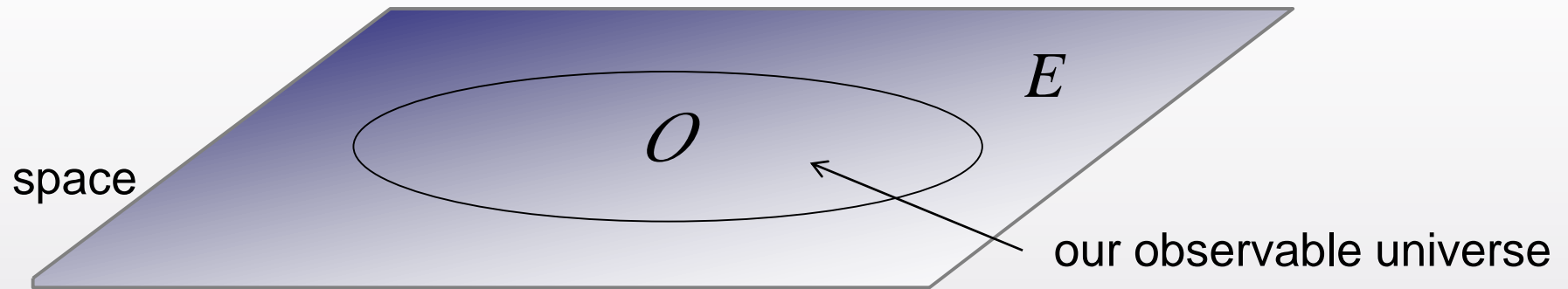
Our de Sitter patch will be a maximum-entropy state with

$$L_{\mathcal{O}} \sim H_0^{-1} \sim 10^{28} \text{ cm}, \quad S_{\mathcal{O}} \sim \left(\frac{L_{\mathcal{O}}}{L_P} \right)^2 \sim 10^{120},$$

$$\therefore \dim(\mathcal{H}_{\mathcal{O}}) \sim \exp(10^{120}).$$

[Dyson, Kleban & Susskind; Banks & Fischler]

So we have the following picture:



$$\mathcal{H} = \mathcal{H}_O \otimes \mathcal{H}_{\text{ext}}$$

↗ ↖

dim = $\exp(10^{120})$ dim = ?

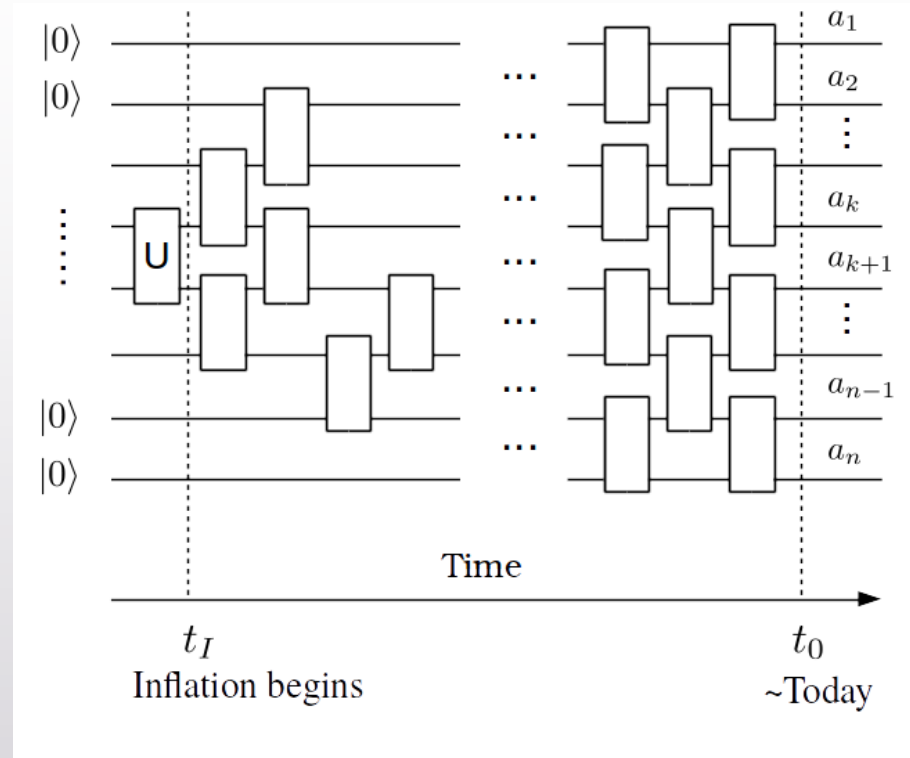
Hilbert space is locally finite-dimensional.

Don't need to specify an algebra of observables – it's simply "the set of all Hermitian operators."

Cosmic expansion and degrees of freedom

Consider a comoving region of flat FRW cosmology with matter/energy.

Model evolution of the region by a quantum circuit.



“Expansion of space”



Entanglement of initially unentangled ancilla degrees of freedom.

Mad-Dog Everettianism:



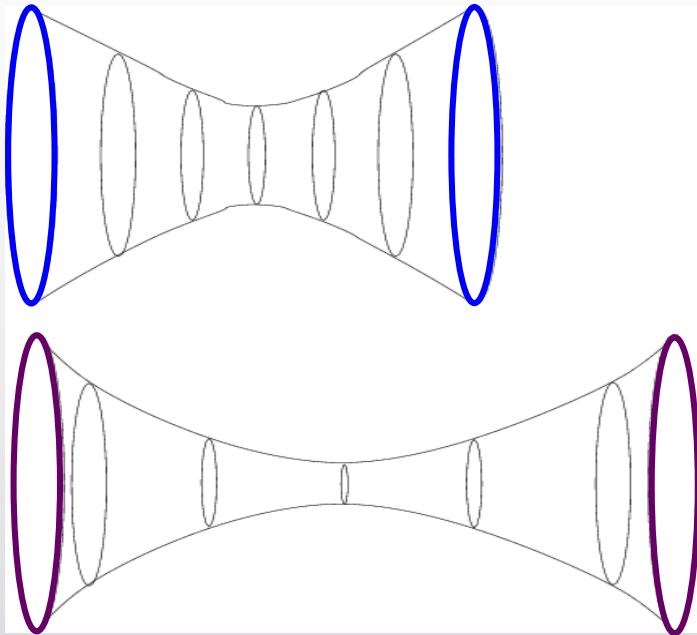
- Hilbert space \mathcal{H}_O , Hamiltonian H , and state $|\psi\rangle$.
- Derive everything else, including space itself.

The meager data we are given:

- an integer $d = \dim(\mathcal{H}_O)$
- d real numbers, the energy eigenvalues of H
$$\{E_0, E_1, E_2, \dots\}, \quad \hat{H}|n\rangle = E_n|n\rangle,$$
- d complex numbers, components of some state $|\psi\rangle$.

$$\{\psi_0, \psi_1, \psi_2, \dots\}, \quad |\psi\rangle = \sum_n \psi_n |n\rangle.$$

Spacetime geometry from quantum entanglement



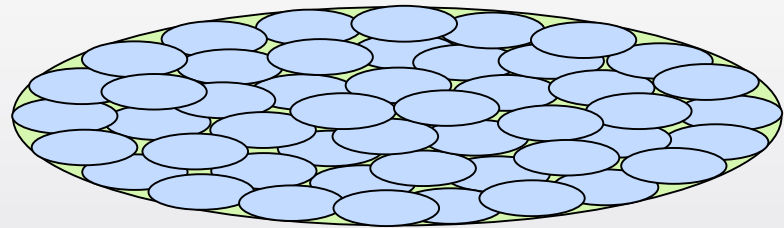
AdS/CFT approach.
Wormholes in anti-de Sitter:
boundaries move apart as
entanglement between them
is reduced.

When looking for the key to quantum gravity,
AdS/CFT is the world's brightest lamppost.

Finding bulk space in Hilbert space

Consider a Hilbert space factored into finite-dim pieces.
Can we factor to represent local regions of space?

$$\mathcal{H} = \bigotimes_a \mathcal{H}_a$$



We can write the Hamiltonian as a sum of operators connecting all possible combinations of factors.

$$\hat{H} = \sum_a h_a \hat{O}_a^{(\text{self})} + \sum_{ab} h_{ab} \hat{O}_{ab}^{(2\text{-pt})} + \sum_{abc} h_{abc} \hat{O}_{abc}^{(3\text{-pt})} + \dots$$

A “ k -local Hamiltonian” is one where any factor only interacts with k other factors, its “nearest neighbors.”

Cotler, Penington, and Renard, 2017:

- Most Hamiltonians have no local factorization.
- When it exists, locality is (mostly) unique.

E.g. for an n -qubit Hilbert space we would have:

$$\sum_{j=0}^k 3^j \binom{n}{j} \ll 2^n$$

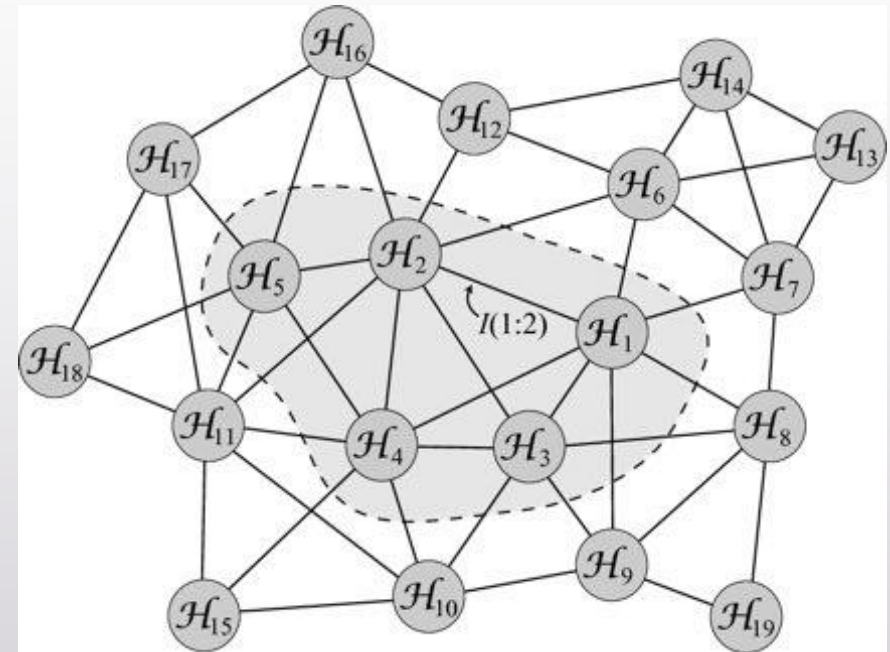
parameters in a k -local Hamiltonian energy eigenvalues
(dim H)

So if the Hamiltonian can be written in a local form at all (which we assume it can), locality (basically) fixes what the factorization will be.

This notion of locality gives a graph structure and a topology to a factorized Hilbert space. Geometry?

Given a pure state vector $|\Psi\rangle$, the state on one factor (or a collection thereof) is given by its **reduced density matrix**:

$$\rho_p = \text{Tr}_{q \neq p} |\Psi\rangle\langle\Psi|.$$



How entangled one region is with the rest of the world is characterized by its **von Neumann entropy**:

$$S_p = -\text{Tr} \rho_p \log \rho_p$$

Quantifying entanglement in QFT

How entangled two regions are with each other is characterized by their **mutual information**:

$$I(p : q) = S_p + S_q - S_{pq}$$

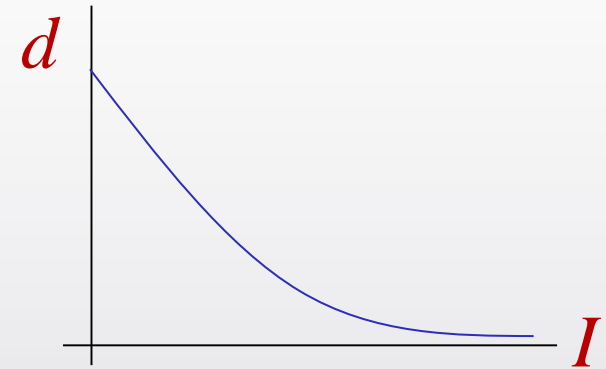
And mutual information provides an operator-independent way of characterizing two-point correlation functions:

$$\begin{aligned} I(p : q) &\equiv S_p + S_q - S_{pq} \\ &\geq \frac{(\langle \mathcal{O}_p \mathcal{O}_q \rangle - \langle \mathcal{O}_p \rangle \langle \mathcal{O}_q \rangle)^2}{2 \|\mathcal{O}_p\|^2 \|\mathcal{O}_q\|^2} \end{aligned}$$

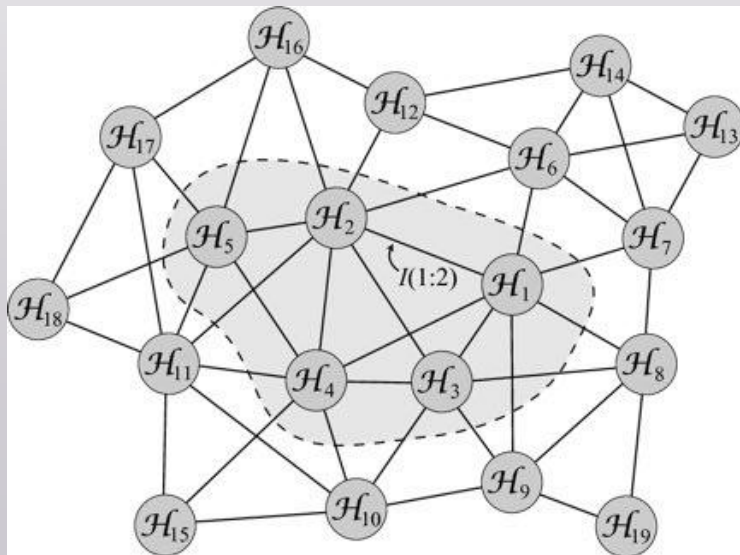
So nearby = higher mutual information.

Boldly conjecture that we can define an **effective metric** on the information graph associated with our factorized-Hilbert-space states:

$$d(p, q) \propto -\log I(p : q)$$



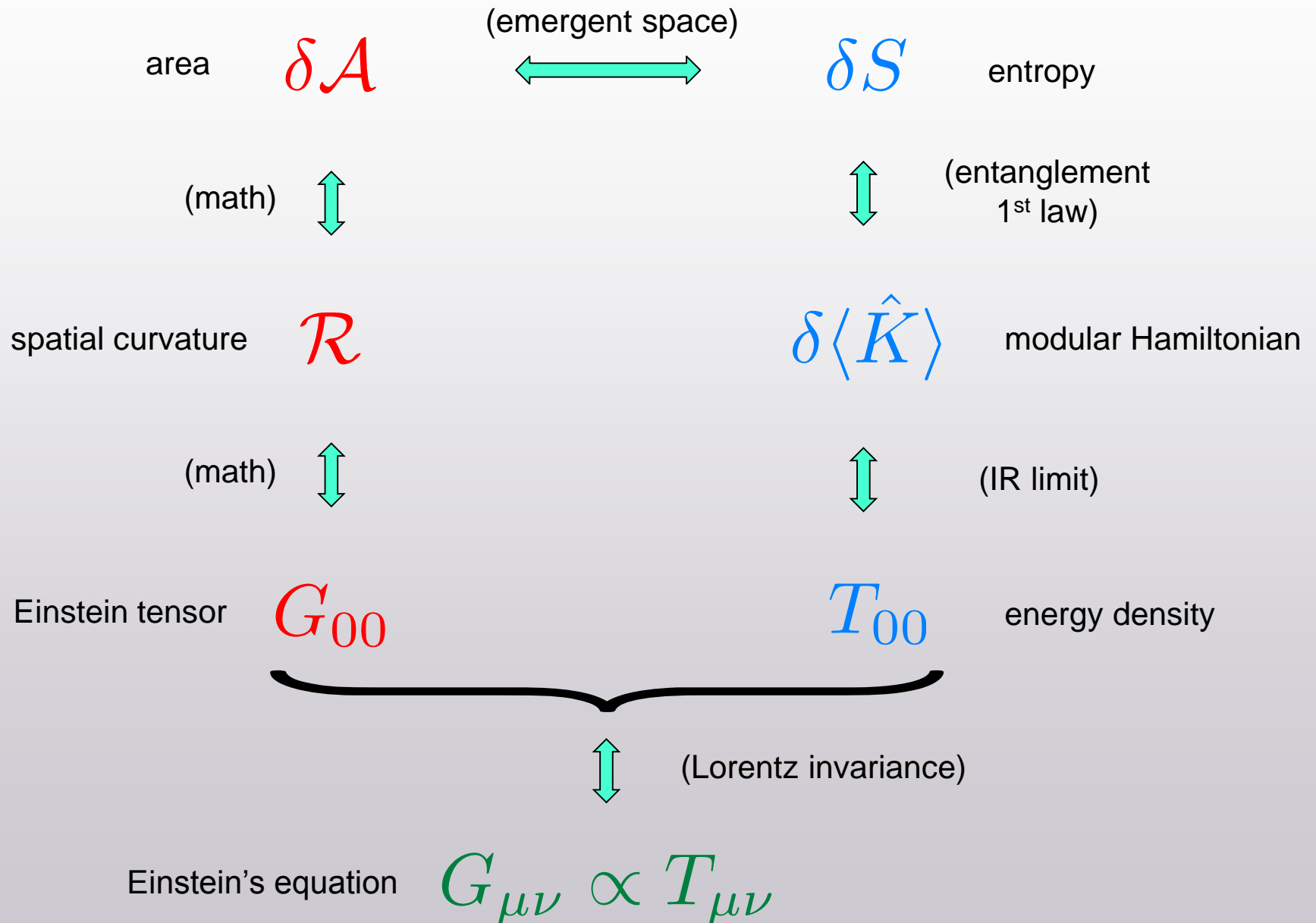
Less entanglement = further away.



Our graph of Hilbert-space factors now has a **geometry**.

Use math to find the best-fit smooth manifold.

Einstein's Equations from Entanglement (after Jacobson)



Emergent gravity in the bulk

We have space; don't have a Hamiltonian, dynamics, time.

But every density matrix ρ defines a “modular energy”:

$$\hat{\rho} \equiv e^{-\hat{K}} \quad \rightarrow \quad \hat{K} = -\log \hat{\rho}$$

Perturbing the quantum state changes both the entropy and the modular energy. They obey the **Entanglement First Law**:

$$\hat{\rho} \rightarrow \hat{\rho} + \delta\hat{\rho} \quad \Longrightarrow \quad \delta S[\hat{\rho}] = \delta \langle \hat{K} \rangle_{\hat{\rho}}$$

Change in the entropy of a region is proportional to the change in modular energy.

Aside: conjecture from this workshop

Quantum Null Energy Condition:

$$\langle T_{kk} \rangle \geq \frac{S''_{\text{out}}}{2\pi A}$$

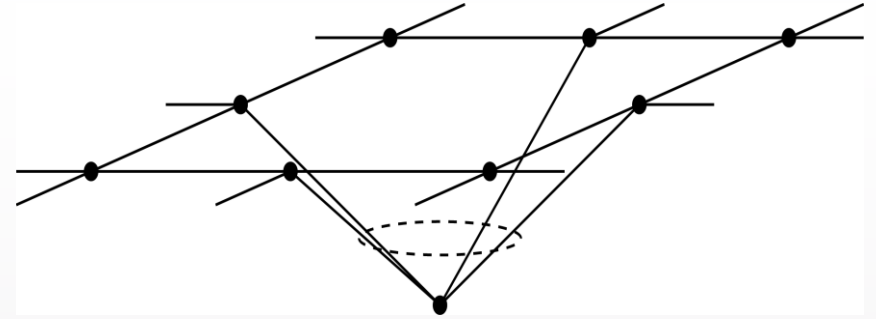
[Bousso, Fisher, Koeller, Leichenauer, and Wall
2013]

Is there a Hilbert-space version?

Perhaps related to the Entanglement 1st Law:

$$\langle T_{kk} \rangle = \langle T_{00} \rangle + \langle T_{11} \rangle \geq \langle T_{00} \rangle = \delta \langle \hat{K} \rangle_{\hat{\rho}} = \delta S$$

Decreasing entropy decreases entanglement, pushing points away, creating spatial curvature.



Therefore, **spatial curvature** at p is related to modular energy:

$$\mathcal{R}_p = -\zeta \langle \hat{K}_p \rangle$$

In long-distance (infrared) limit, our theory should behave like a CFT. Then modular energy is related to ordinary energy:

$$\langle \hat{K}_p \rangle = \langle T_{00}^{(CFT)} \rangle$$

Covariant version would be Einstein's equation:

$$G_{\mu\nu} = \zeta \langle T_{\mu\nu}^{(CFT)} \rangle$$

Prospects

- Semiclassical gravity is clearly interesting and useful.
- Hints of non-locality in BH evaporation.
- We ultimately want to full quantum theory.
- Maybe get there from Hilbert space directly, rather than by quantizing a classical model.
- Spacetime can plausibly emerge from entanglement, complete with Einstein's equation.
- Hilbert space structure etc. still far from understood.