RENORMALONS & TOP-MASS MEASUREMENTS

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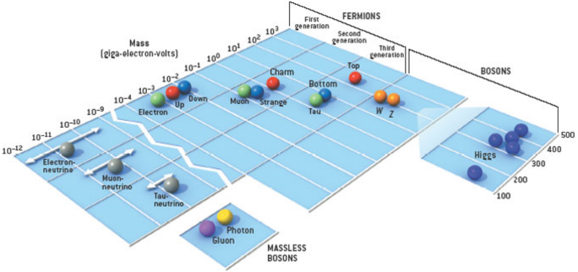
*In collaboration with P. Nason and C. Oleari [arxiv:1810.10931]

The top pole-mass

- Top quark = **heaviest particle** in the SM: a precise determination of $\mathbf{m_t}$ is desirable.
  - CMS: $m_t = 172.44 \pm 0.13\ (\text{stat}) \pm 0.47\ (\text{syst})$ GeV
  - ATLAS: $m_t = 172.51 \pm 0.27\ (\text{stat}) \pm 0.42\ (\text{syst})$ GeV



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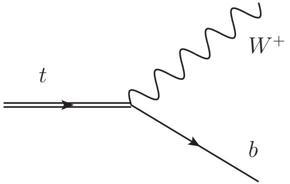
- The top is a **resonance**: $t \to W b$



Complex pole scheme:
$\mathbf{p^2 = m_t^2 - i\Gamma_t m_t}$

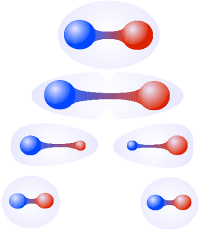
1. Inclusion of finite decay width effects;
2. Gauge invariant;
3. Straightforward to apply.

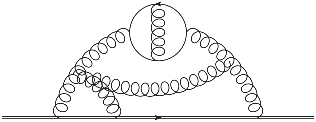
$\Rightarrow$ $W\,b\,j$ @ NLO QCD [arXiv:1305.7088]
$\Rightarrow$ $b\bar{b}\ell^-\bar{\nu}_\ell l^+\nu_l$ @ NLO QCD [arXiv:1012.4230], NLO QCD (+PS) [arXiv:1607.04538], NLO QED [arXiv:1607.05571]
$\Rightarrow$ $b\bar{b}j\ell^-\bar{\nu}_\ell l^+\nu_l$ @ NLO QCD [arXiv:1710.07515]

Pole-mass renormalons

- The pole mass is not very well-defined for a **coloured** object.

pole mass = location of the pole in the Feynman propagator, that corresponds to an **asymptotic state**. But there is **confinement!**



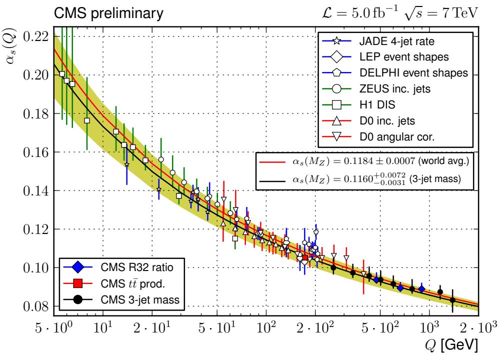


Radiative corrections do not displace the location of m_t: the pole mass counterterm absorbs both UV and **IR** contributions of the self energy Σ

IR Renormalons

- QCD is affected by **infrared slavery**:

$$\alpha_s(k) = \frac{\alpha_s(Q)}{1 + 2b_0\alpha_s(Q)\log\left(\frac{k}{Q}\right)} = \frac{1}{2b_0 \log\left(\frac{k}{\Lambda_{QCD}}\right)}; \quad b_0 = \frac{11C_A}{12\pi} - \frac{n_l T_R}{3\pi} > 0$$



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- All orders contribution coming from low-energy region

$$\underbrace{\int_0^Q dk\, k^{p-1} \alpha_s(Q)}_{\text{NLO}} \Longrightarrow \underbrace{\int_0^Q dk\, k^{p-1} \alpha_s(k)}_{\text{all orders}} = \boxed{Q^p \times \alpha_s(Q) \sum_{n=0}^{\infty} \left(\frac{2\, b_0}{p}\, \alpha_s(Q)\right)^n n!}$$

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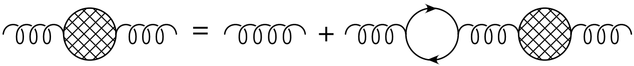
- **Asymptotic series**

  $\Rightarrow$ Minimum for $n_{\min} \approx \frac{p}{2b_0\alpha_s(Q)}$

  $\Rightarrow$ Size $Q^p \times \alpha_s(Q)\sqrt{2\pi n_{\min}}e^{-n_{\min}} \approx \boxed{\Lambda^p_{\mathrm{QCD}}}$

  We are interested in $p = 1$, i.e. in **linear renormalons**

- All-orders computation can be carried out exactly in the large number of flavour n_f limit



$$\frac{-ig^{\mu\nu}}{k^2+i\eta} \to \frac{-ig^{\mu\nu}}{k^2+i\eta} \times \frac{1}{1+\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct}}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} = \alpha_s(\mu)\left(-\frac{n_f \mathrm{T_R}}{3\pi}\right)\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - \frac{5}{3}\right]$$

Large n_f limit

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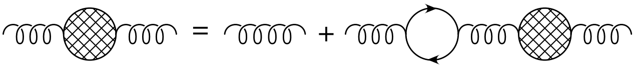

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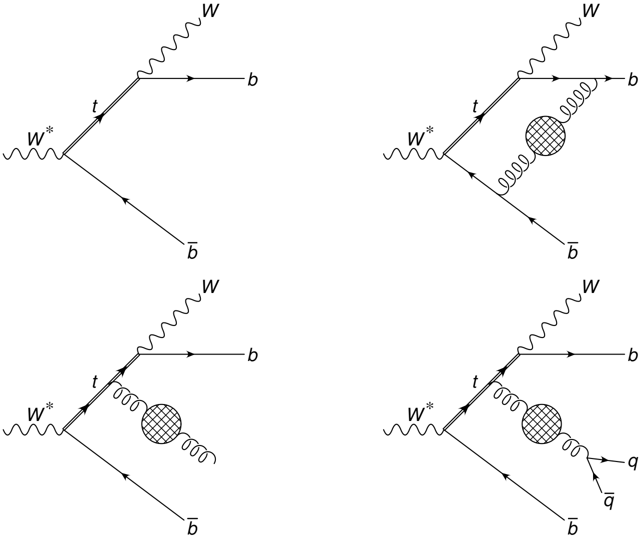
- **naive non-abelianization** at the end of the computation

$$n_f \to \quad n_l - \frac{11\mathrm{C_A}}{4\mathrm{T_R}} \qquad \frac{5}{3} \to C = \frac{(67-3\pi^2)\mathrm{C_A} - 20/3 n_l \mathrm{T_R}}{3(11\mathrm{C_A} - 4n_l\mathrm{T_R})}$$

$$\Pi(k^2+i\eta,\mu^2)-\Pi_{\rm ct} \to \quad \alpha_s(\mu)\underbrace{\left(\frac{11\mathrm{C_A}}{12\pi} - \frac{n_l\mathrm{T_R}}{3\pi}\right)}_{b_0}\left[\log\left(\frac{|k^2|}{\mu^2}\right) - i\pi\theta(k^2) - C\right]$$

Single-top production

$W^* \to t\bar{b} \to Wb\bar{b}$ at all orders using the (complex) pole scheme



Integrated cross section

Integrated cross section (with cuts $\Theta(\Phi)$):

$$\sigma = \int \mathrm{d}\Phi \, \frac{\mathrm{d}\sigma(\Phi)}{\mathrm{d}\Phi} \, \Theta(\Phi)$$

$$= \sigma_{\mathrm{LO}} \quad - \frac{1}{\pi b_0} \int_0^\infty \mathrm{d}\lambda \, \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{T(\lambda)}{\alpha_s(\mu)} \right] \arctan \left[\pi \, b_0 \, \alpha_s \left(\lambda e^{-C/2} \right) \right]$$

- $T(0) = \quad \sigma_{\mathrm{NLO}}$
- $T(\lambda) = \boxed{\sigma_{\mathrm{NLO}}(\lambda)} + \dfrac{3\lambda^2}{2\mathrm{T_R}\alpha_s} \displaystyle\int \mathrm{d}\Phi_{g^*} \, \mathrm{d}\Phi_{\mathrm{dec}} \, \frac{\mathrm{d}\sigma^{(2)}_{q\bar{q}}(\lambda, \Phi)}{\mathrm{d}\Phi} \left[\Theta(\Phi) - \underbrace{\Theta(\Phi_{g^*})}_{q\bar{q}\to g^*} \right]$

  with $\lambda =$ gluon mass
- $T(\lambda) \xrightarrow{\lambda\to\infty} \dfrac{1}{\lambda^2}$
- $\alpha_s(\lambda \, e^{-C/2}) = \dfrac{\alpha_s(\lambda)}{1 - b_0 C \alpha_s(\lambda)} \approx \underbrace{\alpha_s(\lambda) \left[1 + b_0 C \alpha_s(\lambda) \right] = \alpha_s^{\mathrm{CMW}}(\lambda)}_{b_0 \, C = \frac{1}{2\pi} \left[\left(\frac{67}{18} - \frac{\pi^2}{6} \right) \mathrm{C_A} - \frac{10}{9} n_l \mathrm{T_R} \right]}$

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So, if

$$\left. \frac{\mathrm{d}T(\lambda)}{\mathrm{d}\lambda} \right|_{\lambda=0} = A \neq 0$$

the low-λ contribution takes the form

$$\langle O \rangle \sim -A \sum_{n=0}^{\infty} \int_0^m \mathrm{d}\lambda \left[-2b_0 \, \alpha_s(m) \log \left(\frac{\lambda^2}{m^2} \right) \right]^n = -Am \sum_{n=0}^{\infty} \left(2 \, b_0 \, \alpha_s(m) \right)^n n!$$

Linear k term $\leftrightarrow$ Linear renormalons

The size of the linear renormalon is independent from C.

IR-safe observables

Average value of an observable O (e.g. reconstructed-top mass, W-boson energy, ...)

$$\langle O \rangle = \frac{1}{\sigma} \int \mathrm{d}\Phi \, \frac{\mathrm{d}\sigma(\Phi)}{\mathrm{d}\Phi} O(\Phi)$$

$$= \langle O \rangle_{\rm LO} - \frac{1}{\pi b_0} \int_0^\infty \mathrm{d}\lambda \, \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{\widetilde{T}(\lambda)}{\alpha_s(\mu)} \right] \arctan \left[\pi \, b_0 \, \alpha_s \left(\lambda e^{-C/2} \right) \right]$$

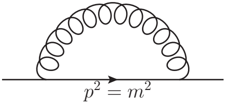
- $\widetilde{T}(0) = \langle O \rangle_{\rm NLO}$
- $\widetilde{T}(\lambda) = \boxed{\langle O(\lambda) \rangle_{\rm NLO}} + \frac{3\lambda^2}{2 n_f \mathrm{T_R} \alpha_s} \int \mathrm{d}\Phi_{g^*} \mathrm{d}\Phi_{\rm dec} \frac{\mathrm{d}\sigma^{(2)}_{q\bar{q}}(\lambda, \Phi)}{\mathrm{d}\Phi} \Big[\overline{O}(\Phi) - \underbrace{\overline{O}(\Phi_{g^*})}_{q\bar{q} \to g^*} \Big]$

  with $\lambda =$ gluon mass, $\quad \overline{O}(\Phi) = [O(\Phi) - O_{\rm LO}] \, \Theta(\Phi) / \sigma_{\rm LO}$
- $\widetilde{T}(\lambda) \xrightarrow{\lambda \to \infty} \frac{1}{\lambda^2}$

pole-$\overline{\text{MS}}$ mass relation

$\overline{m}(\mu)$ $\Rightarrow$UV-divergent contribution of self-energy corrections

m_{pole} $\Rightarrow$UV-divergent + $\underbrace{\text{IR (finite)}}_{\alpha_s^{n+1} n!}$ contributions

At $\mathcal{O}(\alpha_s)$: $\quad m_{\text{pole}} - \overline{m}(\mu)$ =Fin $\Big[i \times$  $\Big]$

At all-orders:

$$\frac{m_{\text{pole}} - \overline{m}(\mu)}{m} = -\frac{1}{\pi b_0} \int_0^{\infty} \mathrm{d}\lambda \frac{\mathrm{d}}{\mathrm{d}\lambda} \left[\frac{r_{\text{fin}}(\lambda)}{\alpha_s(\mu)} \right] \arctan \left[\pi b_0 \alpha_s (\lambda e^{-C/2}) \right] + \dots$$

$$\boxed{r_{\text{fin}}(\lambda) \xrightarrow{\lambda \ll 1} \underbrace{-\alpha_s(\mu) \frac{\mathrm{C_F}}{2} \lambda}_{\textbf{linear renormalon}}}, \qquad r_{\text{fin}}(\lambda) \xrightarrow{\lambda \to \infty} \mathcal{O}\left(\frac{m^2}{\lambda^2} \right)$$

Changing the mass scheme

Neglecting terms of the order $\mathcal{O}\left(\alpha_s^2(\alpha_s b_0)^n\right)$

$$O = O_{\mathrm{LO}}(m_{\mathrm{pole}}) + \alpha_s \left[\sum_{i=0}^{\infty} O_{i+1}(m_{\mathrm{pole}}) \times (\alpha_s b_0)^i\right]$$

$$= O_{\mathrm{LO}}(\overline{m}) + 2\mathrm{Re}\left[\frac{\partial O_{\mathrm{LO}}(\overline{m})}{\partial \overline{m}} \underbrace{(m_{\mathrm{pole}} - \overline{m})}_{\mathcal{O}(\alpha_s)}\right] + \alpha_s \left[\sum_{i=0}^{\infty} O_{i+1}(\overline{m}) \times (\alpha_s b_0)^i\right]$$

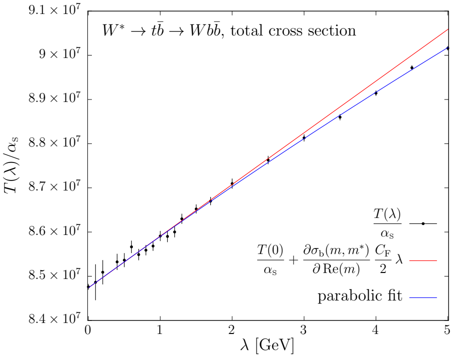
$$= O_{\mathrm{LO}}(\overline{m}) + 2\mathrm{Re}\left[\frac{\partial O_{\mathrm{LO}}(m_{\mathrm{pole}})}{\partial m_{\mathrm{pole}}} (m_{\mathrm{pole}} - \overline{m})\right] + \alpha_s \left[\sum_{i=0}^{\infty} O_{i+1}(m_{\mathrm{pole}}) \times (\alpha_s b_0)^i\right]$$

$$\approx O_{\mathrm{LO}}(\overline{m}) - \frac{1}{\pi b_0}\int_0 \mathrm{d}\lambda \frac{\mathrm{d}}{\mathrm{d}\lambda}\left\{-\frac{C_{\mathrm{F}}\lambda}{2} \times 2\mathrm{Re}\left[\frac{\partial O_{\mathrm{LO}}(m_{\mathrm{pole}})}{\partial m_{\mathrm{pole}}}\right] + \frac{T(\lambda)}{\alpha_s}\right\}$$

$$\times \arctan\left[\pi b_0 \alpha_s (\lambda e^{-C/2})\right] + \dots$$

Total cross section

$\sigma^{\rm tot}(\overline{m}(\mu))$ is renormalon free: $\underbrace{\frac{T(\lambda)}{\alpha_s}}_{\rm pole} \to \underbrace{\frac{T(\lambda)}{\alpha_s} - \frac{\partial \sigma_b}{\partial {\rm Re}(m)} \frac{{\rm C_F}}{2}\lambda}_{\overline{\rm MS}}$

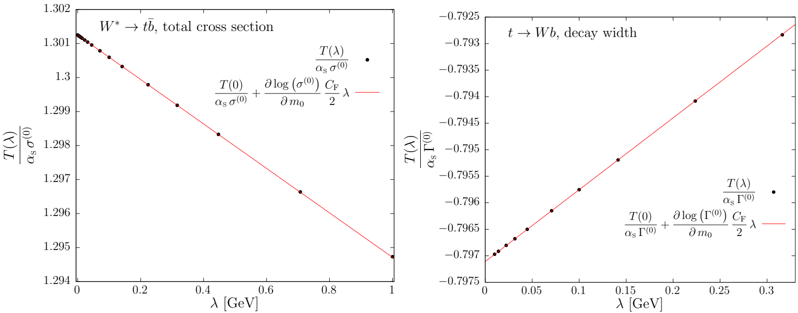


$\Rightarrow$ If a complex mass is used, the top can never be on-shell and the only term that can develop a linear λ sensitivity is the mass counterterm.

Total cross section in NWA

For $\Gamma_t \to 0$ the cross section factorizes

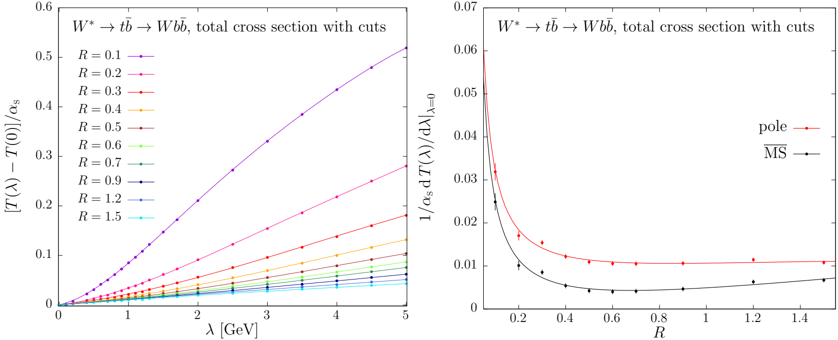
$$\sigma(W^* \to W\, b\, \bar{b}) = \sigma(W^* \to t\bar{b}) \times \frac{\Gamma(t \to W\, b)}{\Gamma_t}$$



Since both terms are free from linear renormalons, also $\sigma(W^* \to W\, b\, \bar{b})$ is free from linear renormalons.

Total cross section with cuts

Cuts: a b jet and a separate $\bar{b}$ jet with $k_\perp > 25$ GeV (anti-$k_\perp$ jets).

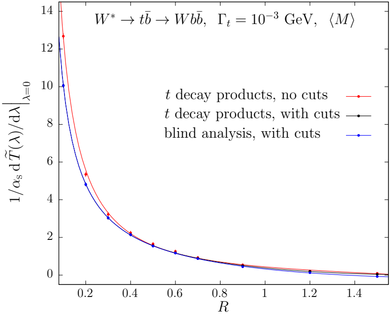


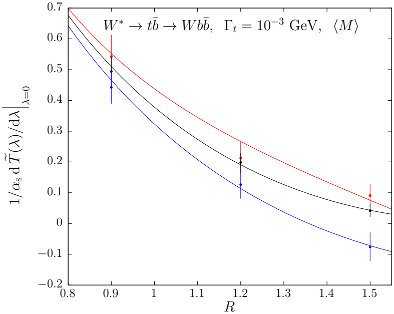
$$\text{Small } R\text{:} \quad \left.\frac{dT(\lambda)}{d\lambda}\right|_{\lambda=0} \propto \frac{1}{R} \Rightarrow \textbf{jet renormalon};$$

Large R: small slope for $\overline{\text{MS}}$.

Reconstructed-top mass in NWA

$$O = M = \sqrt{(p_W + p_{b_j})^2}$$

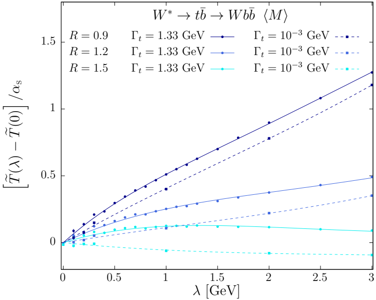


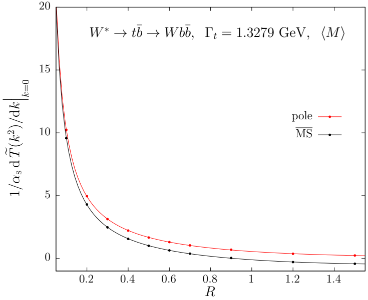


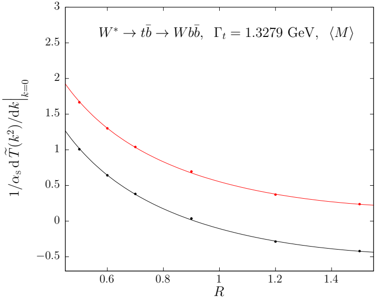
- For $\Gamma_t \to 0$, we can define the "top-decay products"
- For large R, $\langle M \rangle \approx m_{\text{pole}}$ and $T'(0) = 0$: no linear renormalon
- If we move to $\overline{\text{MS}}$ we add $-\frac{C_F}{2} \frac{\partial \langle M \rangle}{\text{Re}(m)} \approx -0.67$: physical linear renormalon

Reconstructed-top mass

For the blind analysis, restoring $\Gamma_t = 1.3279$ GeV only slightly changes this picture







Reconstructed-top mass: some numbers

$$M = \sum_{i=0}^{\infty} c_i \alpha_s^i$$

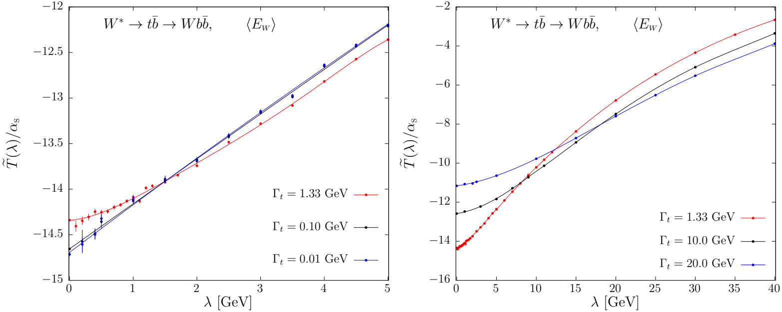
	$c_i \alpha_s^i$ [MeV]		
i	$\mathrm{Re}(m_{\mathrm{pole}} - \overline{m}(\mu))$	$\langle M \rangle_{\mathrm{pole}},\ R = 1.5$	$\langle M \rangle_{\overline{\mathrm{MS}}},\ R = 1.5$
5	+89	−10(1)	+79(1)
6	+60	−11(1)	+49(1)
7	+47	−11(1)	+35(1)
8	+44	−12(1)	+31(1)
9	+46	−15(1)	+31(1)
10	+55	−19(1)	+36(1)

More accurate estimates of $m_{\mathrm{pole}} - \overline{m}(\mu)$ (e.g. inclusion of b and c mass effects) can be found in

- [Beneke, Marquad, Nason, Steinhauser, arXiv:1605.03609]: Δm =110 MeV
- [Hoang, Lepenik, Preisser, arXiv:1802.04334]: $\Delta m = 250$ MeV

Energy of the W boson, pole scheme (lab frame)

$\mathbf{E_W}$ = simplified **leptonic observable**. In absence of cuts, is this observable free from renormalons?



When the pole scheme is used we always have renormalons

- Vanishing Γ_t (left): slope ≈ 0.5 near 0;
- Large Γ_t (right): slope ≈ 0.06 near 0;

Energy of the W boson, $\overline{\text{MS}}$ scheme (lab frame)

$\mathbf{E_W}$ = simplified **leptonic observable**. In absence of cuts, is this observable free from physical renormalons?

Γ_t	slope (pole)	$\frac{\partial \langle E_W \rangle_b}{\partial \, \text{Re}(m)}$	$-\frac{C_F}{2} \frac{\partial \langle E_W \rangle_b}{\partial \, \text{Re}(m)}$	slope ($\overline{\text{MS}}$)
NWA	0.53 (2)	0.10 (3)	−0.066 (4)	0.46 (2)
10 GeV	0.058 (8)	0.0936 (4)	−0.0624 (3)	0.004 (8)
20 GeV	0.061 (2)	0.0901 (2)	−0.0601 (1)	0.001 (2)

Yes, if a **finite width** is used!

Warning!

Despite the fact the energy of the W boson is not affected by linear renormalons, an accurate determination of the top mass is limited by the reduced sensitivity on the top-mass value:

$$2\mathrm{Re}\left[\frac{\partial\langle E_W\rangle_{\mathrm{LO}}}{\partial m}\right] = 0.1$$

$$2\mathrm{Re}\left[\frac{\partial\langle M\rangle_{\mathrm{LO}}}{\partial m}\right] = 1$$

for $E = 300$ GeV, $m_W = 80.4$ GeV, $m_t = 172.5$ GeV ($\beta = 0.5$)

Conclusions

- We devised a simple method that enables us to investigate the presence of linear infrared renormalons in **any infrared safe observable**.
- The **inclusive cross section** and $\mathbf{E_W}$ are free from physical renormalons if $\Gamma_t > 0$ (for σ also in NWA).
- Once jets requirements are introduced, the **jet renormalon** leads to an unavoidable ambiguity.
- For large R, $\langle \mathbf{M} \rangle \approx \mathbf{m}_{\text{pole}}$. This observable has a **physical renormalon**.