Developments in scattering amplitudes for three-jet production at NNLO

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work in progress with Dmitry Chicherin, Thomas Gehrmann
Johannes Henn, Pascal Wasser, Yang Zhang

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Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

European Research Council

High-precision experiments require high-precision predictions

- Ever improving experimental precision at the LHC
- Ever increasing accuracy in the theoretical predictions
  - fixed-order scattering amplitudes
  - resummation of large logarithms
  - parton showering and hadronization

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- Les Houches 2017 **precision wish list**: many processes require unknown two-loop $2 \to 3$ scattering amplitudes

process	known	desired
$pp \to 2\,\text{jets}$	$\text{N}^2\text{LO}_{\text{QCD}}$ $\text{NLO}_{\text{QCD}}+\text{NLO}_{\text{EW}}$	
$pp \to 3\,\text{jets}$	NLO_{QCD}	$\text{N}^2\text{LO}_{\text{QCD}}$

Precision wish list: jet final states [Les Houches 2017]

Outline

Topic. Analytic calculation of two-loop five-point non-planar master integrals for massless scattering

Goal. Virtual corrections to three-jet production at NNLO

- Function space: pentagon functions
- Calculation of the last missing topology
- Outlook

State of the art

Available two-loop five-gluon amplitudes in pure Yang Mills

- Numerical results: **planar** amplitudes for all helicity configurations [Badger, Brønnum-Hansen, Hartanto, Peraro '17] [Abreu, Cordero,Ita, Page, Zeng '17]
- Analytic results: **planar** master integrals and **planar** $+++++$ amplitude [Gehrmann, Henn, Lo Presti '15], **planar** $++++-$ amplitudes [Badger, Brønnum-Hansen, Hartanto, Peraro '18], **planar** amplitudes for all helicity configurations [Abreu, Dormans, Cordero, Ita, Page '18]

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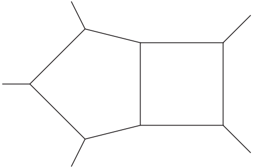
What about **non-planar** amplitudes?

- Master integrals of the **non-planar** hexa-box topology [Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser '18]
- Integrands: $+++++$ amplitude [Badger,Frellesvig, Zhang '13] [Badger, Mogull, Ochirov, O'Connell '15]

Five-point master integral topologies



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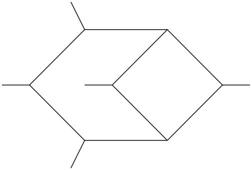

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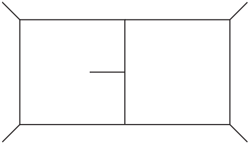

in progress...

* [Gehrmann, Henn, Lo Presti '15]

** [Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser '18]

*** [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia '19**?**]

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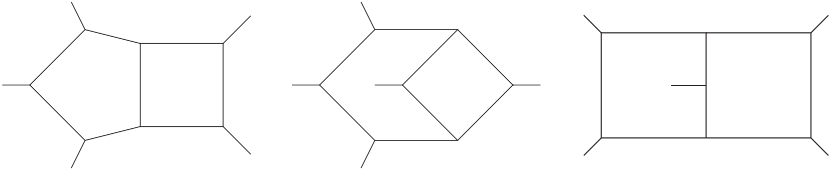

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However...

the **function space** is entirely known!

Pentagon functions

Iterated integrals

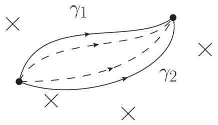
- Chen iterated integrals along path γ

$$\mathcal{I}[\gamma] = \int_\gamma \omega_n \circ \ldots \circ \omega_2 \circ \omega_1 = \int_0^1 dt_1 f_1(t_1) \int_0^{t_1} dt_2 f_2(t_2) \ldots \int_0^{t_{n-1}} dt_n f_n(t_n)$$

ω_i differential 1-form $\qquad \gamma^*(\omega_i) = f_i(t_i) dt_i$

- "d-log" forms $\omega_i = d \log W_i$
- **Alphabet** $=$ set of all independent **letters** $\mathbb{A} = \{W_i\}$
- **Transcendental weight** $=$ number of integrations
- Homotopy invariance

$\gamma_1 \sim \gamma_2$



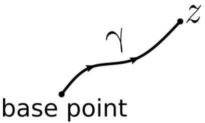
$\Longrightarrow \quad \mathcal{I}[\gamma_1] = \mathcal{I}[\gamma_2]$ if $\omega_i \wedge \omega_{i+1} = 0$

Iterated integrals: examples

- Dilogarithm

$$\mathrm{Li}_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2} = \int_\gamma d\log(1-z) \circ d\log(z)$$

$$\gamma : [0,1] \to \mathbb{R},$$
$$\gamma(0) = \text{base-point} = 0$$
$$\gamma(1) = z$$



Simplest choice: a straight line $\gamma(t) = zt$

$$\mathrm{Li}_2(z) = -\int_0^1 \frac{dt}{t} \int_0^t dt' \frac{zdt'}{1-zt'}$$

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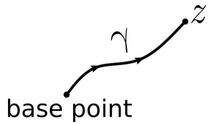

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- Multiple polylogarithms

$$\mathrm{Li}_{1,1}(x,y) = \sum_{0<k_1<k_2} \frac{x^{k_1} y^{k_2}}{k_1 k_2} =$$

$$= \int_\gamma \left[d\log\left(\frac{x(1-y)}{1-x}\right) \circ d\log(1-xy) + d\log(1-x) \circ d\log(1-y) \right]$$

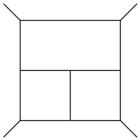
The alphabet: examples

- On-shell massless 4-pt Feynman integrals

  Variables: $x = s/t$

$$\mathbb{A} = \{x, 1 + x\}$$

  Functions: Harmonic Polylogarithms

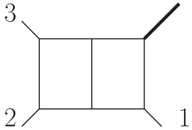


- Massless 4-pt Feynman integrals with one off-shell and three on-shell legs

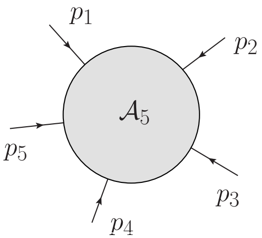
  Variables: $x = s_{12}/s_{123}$ and $y = s_{23}/s_{123}$

$$\mathbb{A} = \{x, 1 - x, y, 1 - y, 1 - x - y, x + y\}$$

  Functions: 2dHPLs



Kinematics of five-particle massless scattering



massless particles $p_i^2 = 0$

Mandelstam invariants

$$s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j$$

- Five independent Mandelstam invariants

$$s_{12},\ s_{23},\ s_{34},\ s_{45},\ s_{51}$$

- One pseudo-scalar invariant

$$\epsilon(p_1, p_2, p_3, p_4) := \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$$

New feature of 5-particle scattering!

Pentagon alphabet

The planar case 26 letters: 21 parity even and 5 parity odd

- $W_1 = 2p_1 \cdot p_2$ and 4 cyclic
- $W_6 = 2p_4 \cdot (p_3 + p_5)$ and 4 cyclic
- $W_{11} = 2p_3 \cdot (p_4 + p_5)$ and 4 cyclic
- $W_{16} = -2p_1 \cdot p_3$ and 4 cyclic
- $W_{26} = \frac{\mathrm{tr}\left((1-\gamma_5)\not{p}_1\not{p}_2\not{p}_4\not{p}_5\right)}{\mathrm{tr}\left((1+\gamma_5)\not{p}_1\not{p}_2\not{p}_4\not{p}_5\right)}$ and 4 cyclic
- $W_{31} = \epsilon(p_1, p_2, p_3, p_4)$

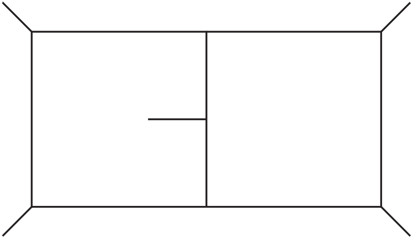
[Gehrmann, Henn, Lo Presti '15]

Pentagon alphabet

The non-planar case 31 letters: 26 parity even and 5 parity odd

- $W_1 = 2p_1 \cdot p_2$ and 4 cyclic
- $W_6 = 2p_4 \cdot (p_3 + p_5)$ and 4 cyclic
- $W_{11} = 2p_3 \cdot (p_4 + p_5)$ and 4 cyclic
- $W_{16} = -2p_1 \cdot p_3$ and 4 cyclic
- $W_{21} = 2p_3 \cdot (p_1 + p_4)$ and 4 cyclic
- $W_{26} = \frac{\mathrm{tr}\left((1-\gamma_5)\not{p}_1\not{p}_2\not{p}_4\not{p}_5\right)}{\mathrm{tr}\left((1+\gamma_5)\not{p}_1\not{p}_2\not{p}_4\not{p}_5\right)}$ and 4 cyclic
- $W_{31} = \epsilon(p_1, p_2, p_3, p_4)$

Conjectured [Chicherin, Henn, Mitev '17], now confirmed! ✓



How do we calculate it?

Double pentagon topology

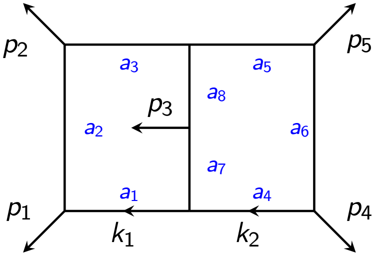
$$\mathcal{I}_{a_1,\ldots,a_{11}} = \int \frac{d^D k_1 d^D k_2}{(i\pi^{D/2})^2} \frac{P_9^{-a_9} P_{10}^{-a_{10}} P_{11}^{-a_{11}}}{P_1^{a_1} P_2^{a_2} \ldots P_8^{a_8}}$$

Propagators

$$P_1 = k_1^2$$
$$P_2 = (k_1 - p_1)^2$$
$$P_3 = (k_1 - p_1 - p_2)^2$$
$$P_4 = k_2^2$$
$$P_5 = (k_2 - p_1 - p_2 - p_3)^2$$
$$P_6 = (k_2 - p_1 - p_2 - p_3 - p_4)^2$$
$$P_7 = (k_1 - k_2)^2$$
$$P_8 = (k_1 - k_2 + p_3)^2$$

ISPs

$$P_9 = (k_1 - p_1 - p_2 - p_3 - p_4)^2$$
$$P_{10} = (k_2 - p_1)^2$$
$$P_{11} = (k_2 - p_1 - p_2)^2$$



IBP reduction

- Not all the integrals of the family are independent

$$\int \frac{d^D k_1 d^D k_2}{(i\pi^{D/2})^2} \left(\frac{\partial}{\partial k_i} \xi^\mu \right) \frac{P_9^{-a_9} P_{10}^{-a_{10}} P_{11}^{-a_{11}}}{P_1^{a_1} P_2^{a_2} \dots P_8^{a_8}} = 0$$

$\Rightarrow$ **Integration by Parts Identities** [Chetyrkin, Tkachov '81]

- Any integral of the family can be "reduced" via IBPs to **108 master integrals** $g_i(s, \epsilon)$

$$\mathcal{I}_{a_1,\dots,a_{11}} = \sum_{i=1}^{108} c_i(a_1, \dots, a_{11}; s, \epsilon) g_i(s, \epsilon)$$

- Public codes to solve IBPs via Laporta algorithm: `FIRE5`, `Kira`, `LiteRed`, `Reduze2`
- What is the most natural choice of basis?

Differential equations

$$\frac{\partial}{\partial s} g_i(s,\epsilon) = \text{linear combination of } \mathcal{I}_{a_1,\ldots,a_{11}} \overset{\text{IBP}}{=} \sum_j c_j(s,\epsilon) g_j(s,\epsilon)$$

[Chetyrkin, Tkachov '81]

$$d\vec{g}(s,\epsilon) = dA(s,\epsilon)\vec{g}(s,\epsilon)$$

"Messy" solution

$$g(s,\epsilon) = \sum_{p=0}^{\infty} \frac{1}{\epsilon^{4-p}} \sum_k r_k(s) \sum_{w=0}^{p} h_{k,p}^{(w)}(s)$$

$r_k(s)$ algebraic functions of s_{ij}

$h_{k,p}^{(w)}(s)$ transcendental weight-w iterated integral

- Change basis: $\vec{g} \to \vec{f} = M\vec{g}$

$$d\vec{f}(s,\epsilon) = \underbrace{\left(M dA M^{-1} + M^{-1} dM\right)}_{d\widetilde{A}(s,\epsilon)} \vec{f}(s,\epsilon)$$

Differential equations in the canonical form

$$\boxed{d\vec{f}(s,\epsilon) = \epsilon \; d\widetilde{A}(s)\vec{f}(s,\epsilon)}$$

[Henn '13]

$$d\widetilde{A}(s) = \sum_{i=1}^{31} c_i \, d\log W_i(s)$$

c_i constant matrices
$W_i(s)$ letters of the alphabet

- Apparent singularity structure
- The dependence on ϵ is factorized
- Elegant solution $\Rightarrow$ **uniform transcendental weight**

$$\vec{f}(s,\epsilon) = \mathbb{P}\exp\left(\epsilon \int_{\gamma(s_0\to s)} d\widetilde{A}\right)\vec{f}(s_0,\epsilon) \longrightarrow \frac{1}{\epsilon^4}\sum_{w=0}^{\infty}\epsilon^w h^{(w)}(s)$$

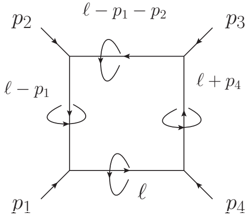
$h^{(w)}(s)$ transcendental weight-w iterated integrals

How do we find a uniform transcendental weight basis?

Leading singularities

The box integral

$$\int \frac{d^4\ell}{P_1 P_2 P_3 P_4} = \int_{-\infty}^{\infty} d\ell^0 \int_{-\infty}^{\infty} d\ell^1 \int_{-\infty}^{\infty} d\ell^2 \int_{-\infty}^{\infty} d\ell^3 \frac{1}{P_1 P_2 P_3 P_4}$$



$$\frac{1}{(2\pi i)^4} \oint_{P_1=0} d\ell^0 \oint_{P_2=0} d\ell^1 \oint_{P_3=0} d\ell^2 \oint_{P_4=0} d\ell^3 \frac{1}{P_1 P_2 P_3 P_4} = \frac{1}{st}$$

$$\int_{\mathbb{R}^{1,3}} (\text{4D integrand}) \to \int_{\mathbb{T}^4} (\text{4D integrand}) = \text{leading singularity}$$

[Arkani-Hamed, Cachazo, Cheung, Kaplan '09]

Leading singularities

IntegraND with unit leading singularities $\Rightarrow$ **dlog form**

$$\frac{d^4\ell\ st}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2(\ell+p_4)^2} =$$
$$d\log\frac{\ell^2}{(\ell-\ell^*)^2}\wedge d\log\frac{(\ell-p_1)^2}{(\ell-\ell^*)^2}\wedge d\log\frac{(\ell-p_1-p_2)^2}{(\ell-\ell^*)^2}\wedge d\log\frac{(\ell+p_4)^2}{(\ell-\ell^*)^2}$$

where ℓ^* is the solution of the maximal cut

$$(\ell^*)^2 = (\ell^*-p_1)^2 = (\ell^*-p_1-p_2)^2 = (\ell^*+p_4)^2 = 0$$

Integrals with d-log integraND evaluate to $\mathbb{Q}$-linear combinations of iterated integrals of uniform weight

[Arkani-Hamed, Bourjaily, Cachazo, Trnka '10]

$\Rightarrow$ Algorithmic construction of dlog basis integrals

✓ We know how to find an integrals basis $\vec{f}$ s.t.

$$d\vec{f}(s,\epsilon) = \epsilon \; d\widetilde{A}(s)\vec{f}(s,\epsilon)$$

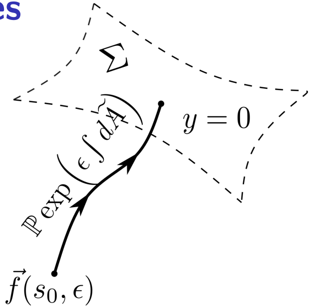
✓ We know how to integrate the differential equation

$$\vec{f}(s,\epsilon) = \mathbb{P}\exp\left(\epsilon \int_{\gamma(s_0 \to s)} d\widetilde{A}\right) \vec{f}(s_0,\epsilon)$$

How to fix the boundary values $\vec{f}(s_0,\epsilon)$?

- Numerically $\to$ public codes: `pySecDec`, `FIESTA`
- Analytically $\to$ require regularity in spurious singularities

Boundary values



- $\vec{f}(s,\epsilon)$ are **finite** at $\epsilon < 0$
- Spurious singularities where $W_i = 0$

$$d\vec{f}(s,\epsilon) = \epsilon \underbrace{\left(\sum_{i=1}^{31} c_i d\log W_i\right)}_{d\widetilde{A}} \vec{f}(s,\epsilon)$$

- Consider e.g. $y = W_1 = 2p_1 \cdot p_2 \to 0$

$$d\widetilde{A} = \epsilon a_1 d\log y + \mathcal{O}(y) \implies \vec{f}(s,\epsilon)|_{y\sim 0} = \exp\left(\epsilon a_1 \log y\right) \vec{J}(\epsilon) + \mathcal{O}(y)$$

- $\vec{f}(s,\epsilon)$ finite at $y \to 0 \implies$ constraints on $\vec{J}(\epsilon)$

- Transport $\vec{f}(s_0,\epsilon)$ to the singular hypersurface Σ where $y = W_1 = 0$ and match

$$\mathbb{P}\exp\left(\epsilon \int_{\gamma(s_0\to\Sigma)} d\widetilde{A}\right) \vec{f}(s_0,\epsilon) = \exp\left(\epsilon a_1 \log y\right) \vec{J}(\epsilon) + \mathcal{O}(y)$$

Summary

Analytic calculation of master integrals for the virtual corrections to 3-jet production at NNLO

- Function space $\Rightarrow$ **pentagon functions**
  - iterated integrals
  - 31-letter alphabet
- Work in progress on the last missing topology
  - IBPs & differential equations
  - How to find a "nice" basis

    Integrands with **unit leading singularities**
    $\Rightarrow$ integrals with uniform transcendental weight
  - How to fix the **boundary values**

    Singularity structure of the DEs $+$ physical behaviour

Outlook

Short term

- ▷ complete calculation of double-pentagon
- ▷ from iterated integrals to Li_4, $\mathrm{Li}_{2,2}$ (fast numeric evaluation)

Medium term

full analytic QCD 5-gluon amplitudes for 3-jet production

Long term

amplitudes for $H + 2j$ and $V + 2j$

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STAY TUNED!