Higher Order Radiative Corrections in QCD: Why and How

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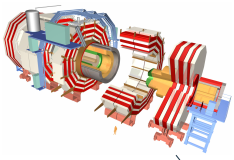
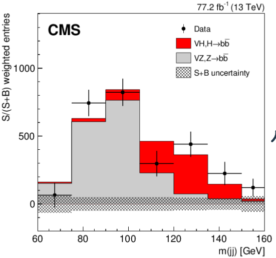
Outline

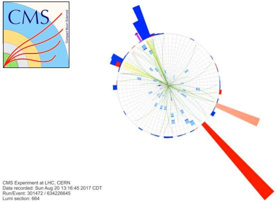
1. Higher orders: why?
2. Higher orders: how?
3. CoLoRFulNNLO
4. CoLoRFulNNLO at work
5. Outlook and Conclusions

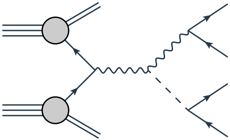
Higher orders: why?

- LHC has now produced > 3 years of $13\ \text{TeV}$ data
- Expected integrated luminosity $> 150\ \text{fb}^{-1}$ by the end of 2018
- Exceeded design peak luminosity by factor of 2
- Excellent machine availability $> 50\%$ of time in stable operation









Precision matters

Excellent machine and detector performance in tough environment

- data taking efficiency $\sim$ 94%, at or above 90% used for physics
- average pile-up $\sim$ 38 in 2017 and 2018

Experimental precision reached

- SM benchmark processes (e.g., W, Z production) measured to 1% exp. precision (important tests of and constraints on theory)
- jets also doing great: total experimental systematic uncertainty in the cross section $\sim$ 6%, at low rapidities ($|y| < 2$)

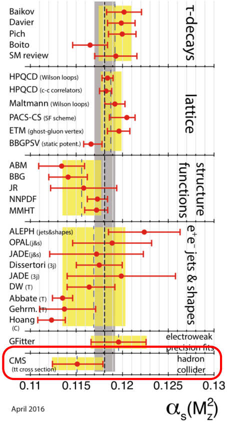
There is lots more data to come

- HL-LHC approved with integrated luminosity goal of 3000 fb^{-1}
- So far, only a fraction of foreseen data registered and analyzed

Must take up the challenge of high precision also on the theory side

Precision from jets: α_s at the LHC

[S. Bethke, Nucl. Part. Phys. Proc. 282-284 (2017) 149]



α_S from jet measurements at the LHC

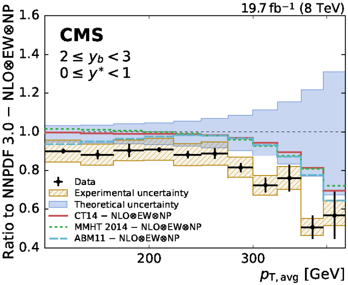
- Several determinations based on jet measurements at 7 and 8 TeV
- Typically NLO theory is used, except for the $t\bar{t}$ total cross section, which based on NNLO
- Uncertainties already dominated by theory, e.g., α_S from transverse energy-energy correlation at 8 TeV

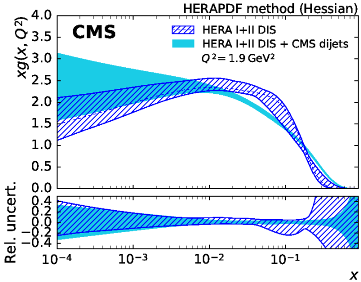
$$\alpha_S(M_Z) = 0.1162 \pm 0.0011\ (\text{exp.})\ {}^{+0.0076}_{-0.0061}\ (\text{scale}) \pm 0.0018\ (\text{PDF}) \pm 0.0003\ (\text{NP})$$

[ATLAS Coll., Eur. Phys. J. 77 (2017) 872, Phys. Lett. B750 (2015) 427-447]

Precision from jets: constraining PDFs

- Triple-differential dijet jet cross section at $\sqrt{s} = 13$ TeV
- Experimental uncertainties small enough to constrain PDFs
- Largest impact on the high-x region



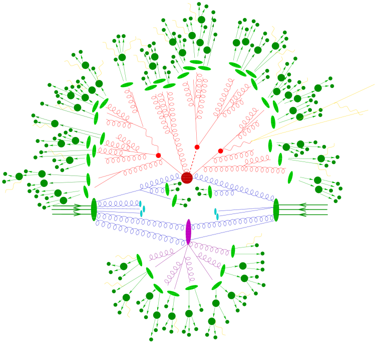


[CMS Coll., Eur. Phys. J. C 77 (2017) 746]

[CMS-SMP-16-011, CERN-EP-2017]

To fully exploit the physics potential of the LHC requires **precision**. QCD must be understood/modeled as best as feasible

- parton model – beams of partons
- radiation off incoming partons
- primary hard scattering ($\mu \simeq Q \gg \Lambda_{\mathrm{QCD}}$)
- radiation off outgoing partons ($Q > \mu > \Lambda_{\mathrm{QCD}}$)
- hadronization and heavy hadron decay ($\mu \simeq \Lambda_{\mathrm{QCD}}$)
- multiple parton interactions, underlying event



[S. Höche, arXiv:1411.4085]

QCD at LHC

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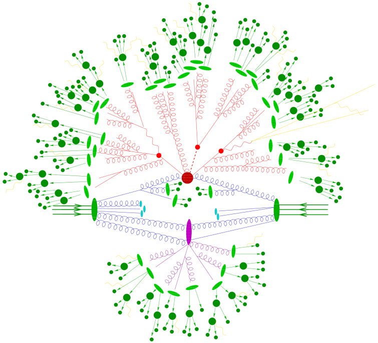


[S. Höche, arXiv:1411.4085]

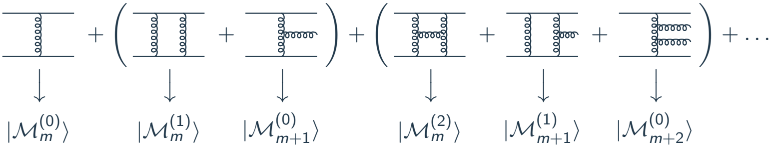
One particular aspect of precision: calculation of **exact higher order corrections** to physical observables in perturbation theory

The hard process in perturbation theory

The scale of the hard scattering is $\mu \gg \Lambda_{\mathrm{QCD}}$, so by **asymptotic freedom**, we can use perturbation theory – i.e., expansion in powers of $\alpha_{\mathrm{S}}(\mu)$ – to compute it

$$\sigma_m = \alpha_{\mathrm{S}}^p(\mu)\Big[\sigma_m^{\mathrm{LO}} + \alpha_{\mathrm{S}}(\mu)\sigma_m^{\mathrm{NLO}} + \alpha_{\mathrm{S}}^2(\mu)\sigma_m^{\mathrm{NNLO}} + \dots\Big]$$

Representative Feynman-diagrams



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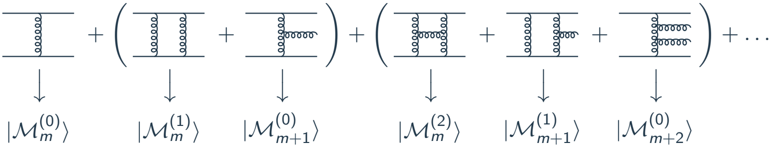


How many terms to compute?

The precision frontier

LO: leading order QCD predictions give only **order of magnitude** estimates for rates and rough estimates for shapes of distributions

- large dependence on unphysical scale choices (renormalization, factorization)
- jets $\neq$ partons: jet structure appears only beyond LO
- conceptually and computationally 'easy', has been completely automated for years: `Helac`, `MadEvent`, `Sherpa`,...

NLO: at least next-to-leading order corrections are **required** to obtain more realistic estimates of cross sections and better pictures of relevant distributions

- general methods for organizing the computation well-known
- powerful new methods to compute one-loop matrix elements based on unitarity, recursion relations
- also implemented into automated tools (`aMC@NLO`, `GoSam`, `Helac-NLO`, `MadLoop`,...)

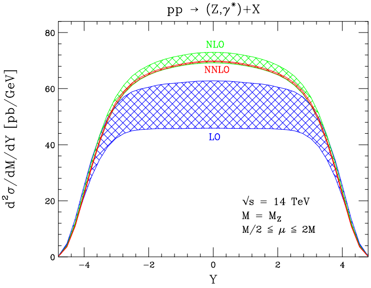
Computing **NLO** corrections for general processes is essentially solved

NNLO: the precision frontier

- computing new two-loop matrix elements still difficult (but great progress!)
- truly general methods for organizing the computation efficiently still missing (personal opinion)

Why higher order corrections?

- Convergence is slow ($\alpha_s \sim 0.1$), corrections are 'large'
- Dependence on unphysical scales considerably reduced at higher orders
- Reliable estimate of theoretical uncertainties
- Benchmark processes measured with high experimental accuracy
- The lack of striking signals of new physics at LHC suggests that BSM effects will be accessible only through precision studies



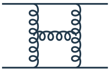
[Anastasiou, Dixon, Melnikov, Petriello, Phys. Rev. D 69 (2004) 094008]

Higher orders: how?

NNLO ingredients – matrix elements

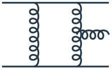
A generic m-jet cross section at NNLO involves

- 2-loop (VV)

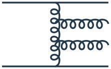
  - m-parton kinematics, two loops (double virtual)
  - $2 \to 2$ available (including VV production)
  - More legs still difficult, but huge progress, $2 \to 3$ on the way

- 1-loop (RV)

  - $m + 1$ parton kinematics, one loop (real-virtual)
  - NLO complexity, 'doable'

- tree (RR)

  - $m + 2$-parton kinematics, tree level (double real)
  - LO complexity, 'easy'

NNLO ingredients – matrix elements

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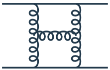

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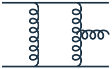

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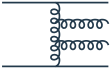


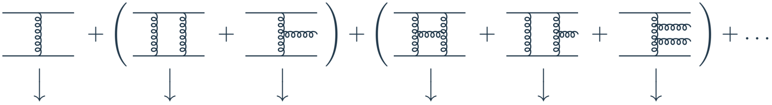
  - $m+2$-parton kinematics, tree level (double real)
  - LO complexity, 'easy'

Assuming we know the relevant matrix elements, can we use those matrix elements to compute cross sections?

The cross section involves the **squared** matrix element

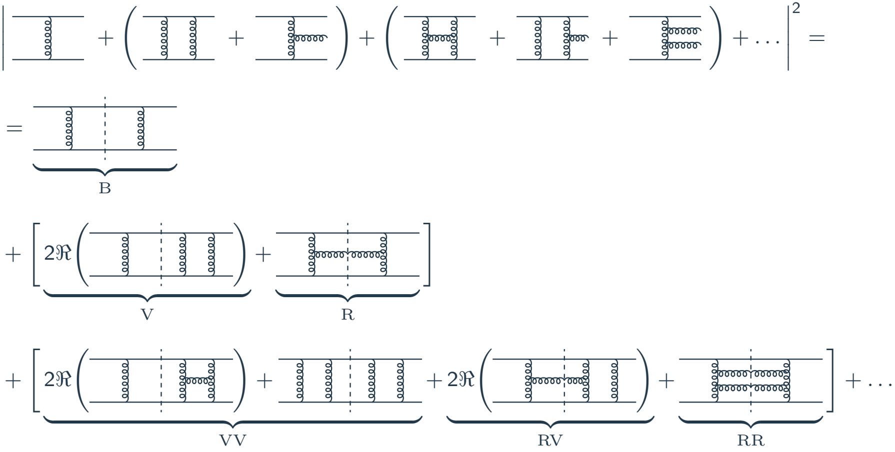
$$\text{cross section} = \int |\text{matrix element}|^2 \times \text{phase space}$$

Recall the structure of the matrix element



$$|\mathcal{M}_m^{(0)}\rangle \quad |\mathcal{M}_m^{(1)}\rangle \quad |\mathcal{M}_{m+1}^{(0)}\rangle \quad |\mathcal{M}_m^{(2)}\rangle \quad |\mathcal{M}_{m+1}^{(1)}\rangle \quad |\mathcal{M}_{m+2}^{(0)}\rangle$$

Squaring the matrix element, we find



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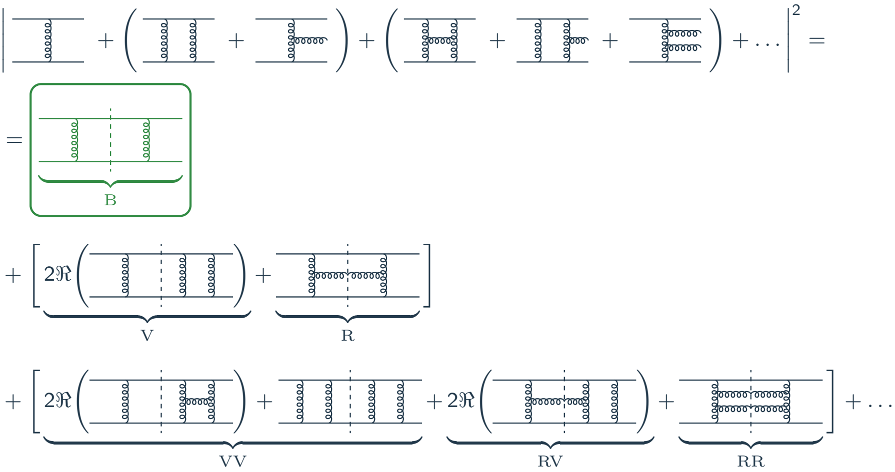

The three lines on the r.h.s. correspond to the **LO**, NLO and NNLO contributions

- LO = B (Born)

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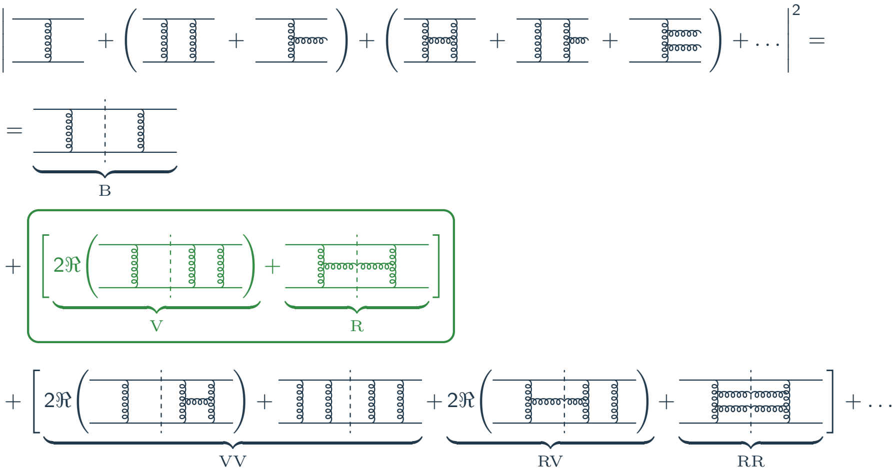

The three lines on the r.h.s. correspond to the LO, **NLO** and NNLO contributions

- $\mathrm{LO} = \mathrm{B}$ (Born)
- $\mathrm{NLO} = \mathrm{R} + \mathrm{V}$ (Real + Virtual)

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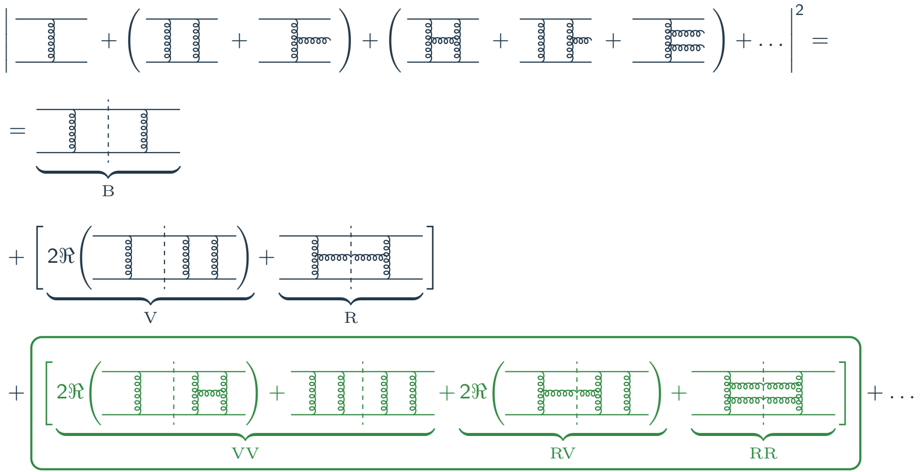

The three lines on the r.h.s. correspond to the LO, NLO and **NNLO** contributions

- $\mathrm{LO} = \mathrm{B}$ (Born)
- $\mathrm{NLO} = \mathrm{R} + \mathrm{V}$ (Real + Virtual)
- $\mathrm{NNLO} = \mathrm{RR} + \mathrm{RV} + \mathrm{VV}$ (Double Real + Real-Virtual + Double Virtual)

The NNLO correction

The NNLO correction to a generic m-jet observable is the sum of three terms

$$\sigma^{\mathrm{NNLO}} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m} \,.$$

with **different final-state multiplicities**:

1. Double real
$$\mathrm{d}\sigma^{\mathrm{RR}}_{m+2} = \mathrm{d}\phi_{m+2}\, |\mathcal{M}^{(0)}_{m+2}|^2$$
2. Real-virtual
$$\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} = \mathrm{d}\phi_{m+1}\, 2\Re\langle\mathcal{M}^{(0)}_{m+1}|\mathcal{M}^{(1)}_{m+1}\rangle$$
3. Double virtual
$$\mathrm{d}\sigma^{\mathrm{VV}}_{m} = \mathrm{d}\phi_{m} \left[2\Re\langle\mathcal{M}^{(0)}_{m}|\mathcal{M}^{(2)}_{m}\rangle + |\mathcal{M}^{(1)}_{m}|^2\right]$$

Above, J_n is the **jet function**, it defines the precise physical observable we are computing. Its form can be (arbitrarily) complicated, containing Θ functions, δ functions, etc.

Aside: unresolved partons

'**Unresolved parton(s)**' or '**parton(s) become(s) unresolved**': refers to a kinematical configuration that is **degenerate**, i.e., either some momenta are collinear or some are soft (or a combination).

Physically, if p partons are unresolved in an $n+p$-parton event, then the $n+p$ momenta are indistinguishable from the momenta of an n-parton event.

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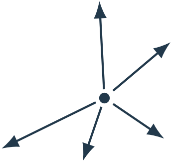
Examples:

- 5 resolved partons, 0 unresolved partons
- all momenta are 'well separated' and 'hard'



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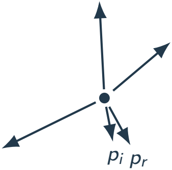
Examples:

- 4 resolved partons, 1 unresolved parton
- one momentum pair is collinear, $p_i || p_r$



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Examples:

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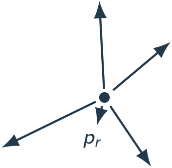
- one momentum is soft, $p_r \to 0$



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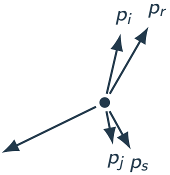
Examples:

- 3 resolved partons, 2 unresolved partons
- two momentum pairs are collinear, $p_i||p_r$ and $p_j||p_s$



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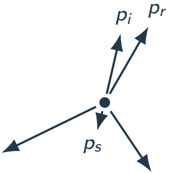
- one momentum pair is collinear, a third momentum is soft, $p_i || p_r$ and $p_s \to 0$



The problem - IR singularities

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Double real

- Tree level squared MEs with $m+2$-parton kinematics
- MEs diverge as one or two partons unresolved
- **phase space integral divergent** (up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration in dim. reg.)
- no loops, so no explicit ϵ poles in dim. reg.

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Real-virtual

- One-loop squared MEs with $m+1$-parton kinematics
- MEs diverge as one parton unresolved
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- one loop, **explicit ϵ poles** up to $\mathrm{O}(\epsilon^{-2})$ from loop integration in dim. reg.

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Double virtual

- Two-loop squared MEs with m-parton kinematics
- jet function screens divergences in MEs as partons become unresolved
- phase space integral is finite
- two loops, **explicit ϵ poles** up to $\mathrm{O}(\epsilon^{-4})$ from loop integration in dim. reg.

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and naively (i.e., in $d = 4$ dims.) **all three are divergent**!

Kinoshita-Lee-Nauenberg theorem

Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive ('IR and collinear safe') observables. I.e., the full correction is finite for appropriately defined quantities.

However

The various contributions (RR, RV and VV) usually need to be computed numerically. (Recall J_n can be 'arbitrarily' complicated.) Hence the cancellation of infrared singularities must be made explicit $\Rightarrow$ need a method to deal with divergent phase space integrals.

Handling singularities: phase space slicing - a caricature

Want to evaluate (at $\epsilon \to 0$)

$$\sigma = \int_0^1 \mathrm{d}\sigma^{\mathrm{R}}(x) + \sigma^{\mathrm{V}} \qquad \text{where} \qquad \begin{aligned} \mathrm{d}\sigma^{\mathrm{R}}(x) &= \mathrm{d}x\, x^{-1-\epsilon} R(x)\,, \quad R(0) = R_0 < \infty \\ \sigma^{\mathrm{V}} &= R_0/\epsilon + V\,, \quad V < \infty \end{aligned}$$

- split integration into singular and non-singular regions

$$\int_0^1 \mathrm{d}\sigma^{\mathrm{R}}(x) = \int_\delta^1 \mathrm{d}\sigma^{\mathrm{R}}(x) + \int_0^\delta \mathrm{d}\sigma^{\mathrm{R}}(x)$$

- approximate the sum of the singular real + virtual

$$\begin{aligned} \sigma(\delta) &\simeq \int_\delta^1 \left[\mathrm{d}\sigma^{\mathrm{R}}(x)\right]_{\epsilon=0} + \left[\int_0^\delta \frac{R_0}{x^{1+\epsilon}} + \sigma^{\mathrm{V}}\right]_{\epsilon=0} \\ &= \int_\delta^1 \mathrm{d}x\, \frac{R(x)}{x} + \left[-\frac{\delta^{-\epsilon}}{\epsilon} R_0 + \frac{R_0}{\epsilon} + V\right]_{\epsilon=0} = \int_\delta^1 \mathrm{d}x\, \frac{R(x)}{x} + R_0 \ln\delta + V \end{aligned}$$

- The last integral is finite, computable with standard numerical methods and

$$\sigma = \lim_{\delta \to 0} \sigma(\delta)$$

Handling singularities: phase space slicing

Phase space slicing: split phase space according to singular configurations

$$\int_0^1 |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V = \underbrace{\int_\delta^1 |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R}_{\substack{\text{regularized} \\ \text{by cutoff}}} + \underbrace{\int_0^\delta |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V}_{\substack{\text{can be obtained from} \\ \text{resummation framework}}}$$

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- Not used at NLO
- Generates large numerical cancellations on cutoff (must check independence)
- Can use existing NLO calculations as basis (X+jet)
- Local subtractions for NLO-like singularities
- Simpler to implement (resummation)

Handling singularities: phase space slicing

Two approaches based on different resummation frameworks

- q_{T} subtraction [Catani, Cieri, de Florian, Ferrera, Grazzini]
- N-jettiness subtraction [Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh]

q_{T} or jettiness used to disentangle “pure” NNLO regions

So far only for “simpler” configurations: one/zero colored particle in the final state

In principle works at N^kLO for X production if we can compute ($X+n$ jet) production at N^{k-n}LO and can carry out the resummation to N^kLL

Handling singularities: subtraction method - a caricature

Want to evaluate (at $\epsilon \to 0$)

$$\sigma = \int_0^1 \mathrm{d}\sigma^{\mathrm{R}}(x) + \sigma^{\mathrm{V}} \qquad \text{where} \qquad \begin{aligned} &\mathrm{d}\sigma^{\mathrm{R}}(x) = \mathrm{d}x\, x^{-1-\epsilon} R(x)\,, \quad R(0) = R_0 < \infty \\ &\sigma^{\mathrm{V}} = R_0/\epsilon + V\,, \quad V < \infty \end{aligned}$$

- define the counterterm

$$\mathrm{d}\sigma^{\mathrm{R,A}}(x) = \mathrm{d}x\, x^{-1-\epsilon} R_0$$

- use it to reshuffle singularities between real and virtual contributions

$$\begin{aligned} \sigma &= \int_0^1 \left[\mathrm{d}\sigma^{\mathrm{R}}(x) - \mathrm{d}\sigma^{\mathrm{R,A}}(x)\right]_{\epsilon=0} + \left[\sigma^{\mathrm{V}} + \int_0^1 \mathrm{d}\sigma^{\mathrm{R,A}}(x)\right]_{\epsilon=0} \\ &= \int_0^1 \mathrm{d}x \left[\frac{R(x) - R_0}{x^{1+\epsilon}}\right]_{\epsilon=0} + \left[\frac{R_0}{\epsilon} + V - \frac{R_0}{\epsilon}\right]_{\epsilon=0} \\ &= \int_0^1 \mathrm{d}x\, \frac{R(x) - R_0}{x} + V \end{aligned}$$

- The last integral is finite, computable with standard numerical methods

Handling singularities: subtraction method

Subtraction method: use local counterterm to rearrange singularities

$$\int_0^1 |\mathcal{M}_R|^2 \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V = \underbrace{\int_0^1 (|\mathcal{M}_R|^2 - \mathcal{D}) \,\mathrm{d}\phi_R}_{\text{integrable}} + \underbrace{\int_0^1 \mathcal{D} \,\mathrm{d}\phi_R + \int |\mathcal{M}_V|^2 \,\mathrm{d}\phi_V}_{\text{poles cancel analytically}}$$

Handling singularities: subtraction method

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- Method of choice at NLO
- Subtractions can be completely local (good convergence)
- At NNLO lots of singular configurations with overlaps
- Integration of subtraction term quite complicated (can be numerical)

Handling singularities: subtraction method

Definition of the subtraction term is not unique, several approaches

- Sector decomposition [Anastasiou, Melnikov, Petriello; Binoth, Heinrich]
- Antenna subtraction [Gehrmann, Gehrmann-de Ridder, Glover]
- Sector-improved residue subtraction (STRIPPER) [Czakon; Boughezal, Melnikov, Petriello]
- Projection-to-born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi]
- CoLoRFulNNLO subtraction [Del Duca, GS, Trócsányi]

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- CoLoRFulNNLO subtraction [Del Duca, GS, Trócsányi]

Personal opinion: general solution not yet available

CoLoRFulNNLO

Basic idea of subtraction

Idea

Reshuffle singular pieces between RR, RV and VV by subtracting and adding back suitably defined **approximate cross sections**, such that this rearrangement renders each piece finite individually in $d = 4$ dims.

CoLoRFulNNLO - the structure

The NNLO correction to some m-jet observable J is the sum of three pieces

$$\sigma^{\mathrm{NNLO}}[J] = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

The three contributions are separately IR **divergent** in $d = 4$

- RR: double and single unresolved real emission
- RV: single unresolved real emission $\oplus$ ϵ-poles from $m + 1$ parton one-loop
- VV: ϵ poles from m parton two-loop

The approximate cross sections must account for all types of unresolved real emission without double counting

For the RR contribution subtractions are needed to regularize one- and two-parton emissions

$$\sigma_{m+2}^{\mathrm{NNLO}} = \int_{m+2} \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} J_m \right] \right\}_{d=4}$$

- A_1 and A_2 have overlapping singularities $\Rightarrow$ A_{12} is needed to cancel

For the RV contribution there are only one-parton emissions but from one-loop-tree interference

$$\sigma_{m+1}^{\mathrm{NNLO}} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] J_m \right\}_{d=4}$$

- Notice the integrated A_1 from RR which is still singular $\Rightarrow$ subtraction is needed (last term)

The m-parton contribution contains the double virtual and integrated subtractions

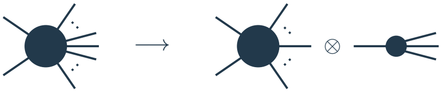
$$\sigma_{m}^{\mathrm{NNLO}} = \int_{m} \left\{ \mathrm{d}\sigma_{m}^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_{12}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR},\mathrm{A}_1} \right)^{\mathrm{A}_1} \right] \right\}_{d=4} J_m$$

Building approximate cross sections

The subtraction terms must match the singularity structure of real emission pointwise (in d dimensions) $\Rightarrow$ phase space integrals over real radiation rendered convergent

- Singularity structure of real emission is **universal**: factorization formulae

$$|\mathcal{M}_{m+q}(\{p_m, p_q\})|^2 \xrightarrow{\{p_q\}\text{ unresolved}} (8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon})^q\, \mathit{Sing}_q(\{p_q\}) \otimes |\mathcal{M}_m(\{p_m\})|^2$$



- Explicit form of factorization formulae **known** for all unresolved limits at NNLO

[Campbell, Glover 1997; Catani, Grazzini 1998; Del Duca, Frizzo, Maltoni 1999; Kosower 2002]

[Bern, Dixon, Dunbar, Kosower 1994; Bern, Del Duca, Kilgore, Schmidt 1998-9;Kosower, Uwer 1999; Catani, Grazzini 2000; Kosower 2003]

Define subtraction terms based on factorization formulae $\Rightarrow$ the result is trivially general and explicit

The following three problems must be addressed

1. Matching of limits to avoid multiple subtraction in overlapping singular regions of PS. Easy at NLO: collinear limit + soft limit - collinear limit of soft limit.

$$\mathbf{A}_1|\mathcal{M}_{m+1}^{(0)}|^2 = \sum_i \left[\sum_{i\neq r}\frac{1}{2}\mathbf{C}_{ir} + \mathbf{S}_r - \sum_{i\neq r}\mathbf{C}_{ir}\mathbf{S}_r\right]|\mathcal{M}_{m+1}^{(0)}|^2$$

2. Extension of IR factorization formulae over full PS using momentum mappings that respect factorization and delicate structure of cancellations in all limits.

$$\{p\}_{m+1} \xrightarrow{r} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+1}(\{p\}_{m+1}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{1,m}]$$
$$\{p\}_{m+2} \xrightarrow{r,s} \{\tilde{p}\}_m : \quad \mathrm{d}\phi_{m+2}(\{p\}_{m+2}; Q) = \mathrm{d}\phi_m(\{\tilde{p}\}_m; Q)[\mathrm{d}p_{2,m}]$$

3. Integration of the counterterms over the phase space of the unresolved parton(s).

Subtraction terms from limit formulae

Specific issues at NNLO

1. Matching is cumbersome if done in a brute force way. However, an efficient solution that works at any order in PT is known.

2. Extension is delicate. E.g., **counterterms** for single unresolved real emission (unintegrated and integrated) **must have universal IR limits**. This is **not guaranteed** by QCD factorization.

3. Choosing the counterterms such that integration over the unresolved phase space is (relatively) straightforward generally conflicts with the delicate cancellation of IR singularities.

General features of CoLoRFulNNLO

CoLoRFulNNLO: Completely **Lo**cal subt**R**actions for **Ful**ly differential **NNLO**

Explicit formulae for general processes with colorless initial state

- Automation is possible
- Inclusion of hadronic initial states on the way

Fully differential in phase space, completely local subtractions

- Can compute any IR and collinear-safe observable in $d = 4$ dims.
- Azimuthal and color correlations correctly taken into account in unresolved emissions

Poles of integrated subtraction terms computed analytically

- Can check pole cancellation in (double) virtual contribution explicitly

Subtractions built using universal IR limit formulae and exact PS factorization

- Altarelli-Parisi splitting functions, soft currents
- Phase space factorizations based on momentum mappings that can be generalized to any number of unresolved partons

MCCSM is a **M**onte **C**arlo for the **C**oLoRFulNNLO **S**ubtraction **M**ethod

- Completely **general** and fully **automatic**
- Highly **flexible** and **tunable**
- Phase space is recursively constructed, MINT is used for Monte Carlo integration
- Histogram output in YODA format through an interface to YODA
- Written in standard fortran90 (by Á. Kardos)
- User must provide only the squared MEs (including color- and spin-correlated)

CoLoRFulNNLO at work

CoLoRFulNNLO at work

The CoLoRFulNNLO subtraction scheme has been used to compute NNLO QCD corrections to

- Higgs boson decay into bottom quarks

  [Del Duca, Duhr, GS, Tramontano, Trócsányi 2015]

- event shapes and groomed event shapes in $e^+e^- \to 3$ jets

  [Del Duca, Duhr, Kardos, GS, Trócsányi 2016;
  Del Duca, Duhr, Kardos, GS, Szőr, Trócsányi, Tulipánt 2016
  Kardos, GS, Trócsányi 2018]

- energy-energy correlation in $e^+e^- \to 3$ jets

  [Del Duca, Duhr, Kardos, GS, Trócsányi 2016;
  Tulipánt, Kardos, GS 2017]

Will discuss two applications

- New measurement of $\alpha_\mathrm{s}(M_Z)$ from energy-energy correlations in e^+e^- collisions

  [Kardos, Kluth, GS, Tulipánt, Verbytskyi 2018]

- Full NNLO QCD corrections to VH production with $H \to b\bar{b}$ decay at the LHC

  [Ferrera, GS, Tramontano 2017]

Energy-energy correlation

Energy weighted distribution of angles χ between particles

$$\frac{1}{\sigma_{\mathrm{t}}}\frac{\mathrm{d}\Sigma(\chi)}{\mathrm{d}\cos\chi} \equiv \frac{1}{\sigma_{\mathrm{t}}}\int\sum_{i,j}\frac{E_i E_j}{Q^2}\mathrm{d}\sigma_{e^+e^-\to\, ij+X}\,\delta(\cos\chi - \cos\theta_{ij})$$

Was measured extensively at LEP and predecessors

Accurate theory predictions available

- NNLO fixed order from CoLoRFulNNLO
- NNLL resummation in back-to-back region [de Florian, Grazzini 2005]

Potential for yapa (yet another precision $\alpha_{\mathrm{s}}(M_Z)$)

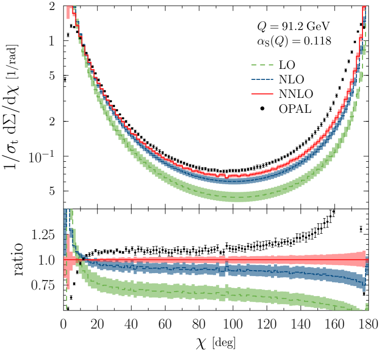
EEC data

- EEC one of the oldest event shapes [Basham, Brown, Ellis, Love 1978]
- However, no measurements after LEP1...
- Transverse EEC in multijet events used successfully at LHC to determine α_s at NLO [ATLAS coll., Eur. Phys. J. C77 (2017) 872, Phys. Lett. B750 (2015) 427-447]

Experiment	$\sqrt{s}$, GeV, data	$\sqrt{s}$, GeV, MC	Events
SLD	91.2(91.2)	91.2	60000
OPAL	91.2(91.2)	91.2	336247
OPAL	91.2(91.2)	91.2	128032
L3	91.2(91.2)	91.2	169700
DELPHI	91.2(91.2)	91.2	120600
TOPAZ	59.0 – 60.0(59.5)	59.5	540
TOPAZ	52.0 – 55.0(53.3)	53.3	745
TASSO	38.4 – 46.8(43.5)	43.5	6434
TASSO	32.0 – 35.2(34.0)	34.0	52118
PLUTO	34.6(34.6)	34.0	6964
JADE	29.0 – 36.0(34.0)	34.0	12719
CELLO	34.0(34.0)	34.0	2600
MARKII	29.0(29.0)	29.0	5024
MARKII	29.0(29.0)	29.0	13829
MAC	29.0(29.0)	29.0	65000
TASSO	21.0 – 23.0(22.0)	22.0	1913
JADE	22.0(22.0)	22.0	1399
CELLO	22.0(22.0)	22.0	2000
TASSO	12.4 – 14.4(14.0)	14.0	2704
JADE	14.0(14.0)	14.0	2112

EEC predictions at NNLO

- NLO correction is large as judged by scale variation ⇒ must go to NNLO
- Higher order predictions improve agreement with data
- Fixed order prediction diverges in the forward and back-to-back regions ⇒ resummation is required
- Sizeable deviations from data even at NNLO ⇒ must take into account hadronization corrections



[Tulipánt, Kardos, GS, Eur. Phys. J. C 77 (2017) no.11, 749]

Hadronization corrections

Point-by-point multiplicative correction factors were derived using modern MC tools

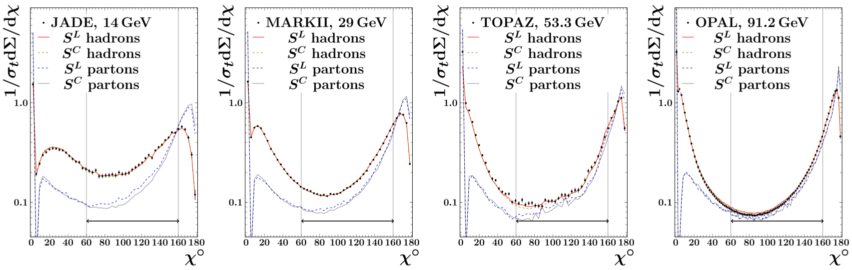
- `Sherpa2.2.4` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 jets at NLO using `AMEGIC`, `COMIX` and `GoSam`, Lund (S^L) or cluster (S^C) hadronization
- `Herwig7.1.1` for $e^+e^- \to 2, 3, 4, 5$ jets, 2 jets at NLO using `MadGraph5` and `GoSam`, cluster (H^M) hadronization only

Hadronization corrections are ratios of hadron to parton level distributions in the MCs

Simulated samples were reweighted to data at hadron level on an event-by-event basis to assure a better description of data ('poor man's tuning')

Simultaneously allows for the estimation of the missing statistical correlations of data points

MC predictions at parton and hadron level after reweighting

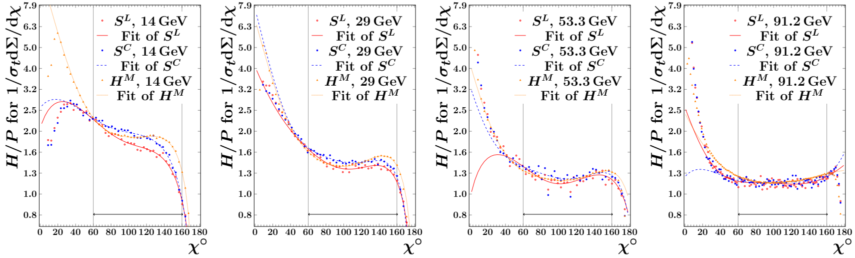


[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

- Hadronization corrections decrease as $\sim 1/Q$, O(10)% at 91.2 GeV

Hadronization corrections

Hadron/parton ratios after reweighting at hadron level

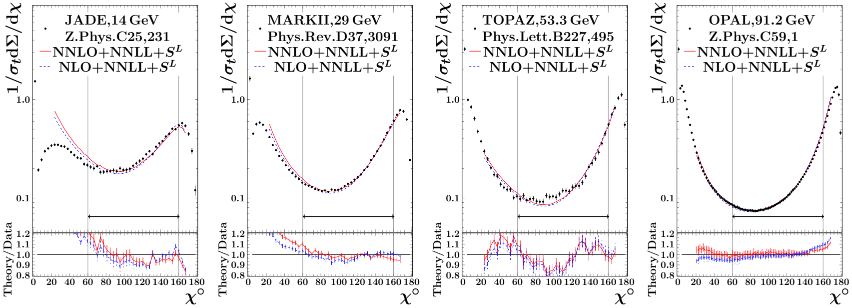


[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

- Hadronization corrections are parametrized using smooth functions to tame statistical fluctuations (the parametrization is valid only in the fit range)

Fits to data

Fits to data of NNLO+NNLL and NLO+NNLL predictions in the S^L setup



[Kardos, Kluth, GS, Tulipánt, Verbytskyi
Eur. Phys. J. C 78 (2018) no.6, 498]

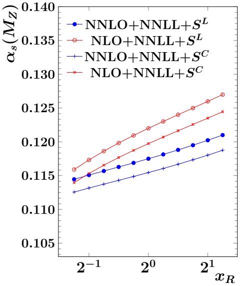
- Fit range $[60^\circ, 160^\circ]$, chosen to avoid regions where the theoretical prediction or hadronization corrections become unreliable
- The result is insensitive to a $\pm 5^\circ$ change in fit range

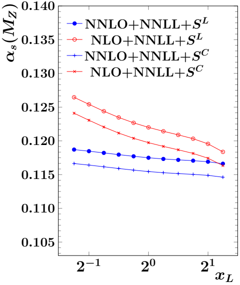
Fit uncertainties

Estimated the uncertainty by

- Varying the renormalization scale $x_R = \mu_R/Q \in [1/2, 2]$: $(ren.)$
- Varying the resummation scale $x_L \in [1/2, 2]$: $(res.)$
- Varying the hadronization model S^L vs. S^C: $(hadr.)$
- Considering the fit uncertainty from the $\chi^2 + 1$ criterion as implemented in `MINUIT2`: $(exp.)$

Notice reduced slope at NNLO+NNLL





Result

Main result from global fit at NNLO+NNLL with S^L setup

$$\alpha_s(M_Z) = 0.11750 \pm 0.00018(exp.) \pm 0.00102(hadr.) \pm 0.00257(ren.) \pm 0.00078(res.)$$
$$\alpha_s(M_Z) = 0.11750 \pm 0.00287(comb.)$$

Note using NLO+NNLL only (i.e., no NNLO), we find

$$\alpha_s(M_Z) = 0.12200 \pm 0.00023(exp.) \pm 0.00113(hadr.) \pm 0.00433(ren.) \pm 0.00293(res.)$$
$$\alpha_s(M_Z) = 0.12200 \pm 0.00535(comb.)$$

Inclusion of **NNLO corrections crucial** in reducing uncertainty: factor of 1/2!

The result is **consistent** with the world average ($\alpha_s(M_Z) = 0.1175 \pm 0.0029$ vs. 0.1181 ± 0.0011) and **competitive** with other precision event shapes ($1 - T$, C, etc.)

VH production with $H \to b\bar{b}$ decay at the LHC

Motivations

- Associated VH production is most sensitive production mode to search for $H \to b\bar{b}$
  - leptons, missing E_{T} to trigger
  - high p_{T} V to suppress backgrounds
- Unique opportunity to study both the Higgs boson coupling to vector bosons and down-type quarks
- $H \to b\bar{b}$ has the largest branching ratio (58%) for $m_H = 125$ GeV
- Drives the uncertainty of the total Higgs boson width

Theory: narrow width approximation very accurate ($\Gamma_H \ll m_H$), so need fully differential calculations for production and decay

- VH production with leptonic V decays known in NNLO QCD (using q_{T} subtraction)

  [Ferrera, Grazzini, Tramontano 2011]

- $H \to b\bar{b}$ known in NNLO QCD (using sector decomposition and CoLoRFulNNLO)

  [Anastasiou, Herzog, Lazopoulos 2012;
  Del Duca, Duhr, GS, Tramontano Z. Trócsányi 2015]

$VH(b\bar{b})$ in full NNLO QCD

Consider $pp \to VH + X \to l_1 l_2 b\bar{b} + X$ in the narrow width approximation

$$d\sigma_{pp\to VH\to Vb\bar{b}} = d\sigma_{pp\to VH} \times \frac{d\Gamma_{H\to b\bar{b}}}{\Gamma_H} = \left[\sum_{k=0}^{\infty} d\sigma^{(k)}_{pp\to VH}\right] \times \left[\frac{\sum_{k=0}^{\infty} d\Gamma^{(k)}_{H\to b\bar{b}}}{\sum_{k=0}^{\infty} \Gamma^{(k)}_{H\to b\bar{b}}}\right] \times \mathrm{Br}(H\to b\bar{b})$$

For full NNLO, expand up to second order

$$\begin{aligned} d\sigma^{\mathrm{NNLO}}_{pp\to VH\to Vb\bar{b}} = \Bigg[& d\sigma^{(0)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}} + d\Gamma^{(2)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}} + \Gamma^{(2)}_{H\to b\bar{b}}} \\ & + d\sigma^{(1)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}}} \\ & + d\sigma^{(2)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}}}\Bigg] \times \mathrm{Br}(H\to b\bar{b}) \end{aligned}$$

$VH(b\bar{b})$ in full NNLO QCD

Previous partial NNLO calculations did not consider NNLO corrections in decay

[Ferrera, Grazzini, Tramontano 2014-5
Campbell, Ellis, Williams 2016]

$$d\sigma^{\mathrm{NNLO(prod)+NLO(dec)}}_{pp\to VH\to Vb\bar{b}} = \left[d\sigma^{(0)}_{pp\to VH} \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}} + d\Gamma^{(1)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}} + \Gamma^{(1)}_{H\to b\bar{b}}} + \left(d\sigma^{(1)}_{pp\to VH} + d\sigma^{(2)}_{pp\to VH} \right) \times \frac{d\Gamma^{(0)}_{H\to b\bar{b}}}{\Gamma^{(0)}_{H\to b\bar{b}}} \right] \times \mathrm{Br}(H\to b\bar{b})$$

New: include NNLO contributions in decay and the combination of NLO contributions for production and decay

Results: cross sections

Kinematical selection cuts

$pp \to W^+H + X \to l\nu_l b\bar{b} + X$

- $p_{\mathrm{T}}^{l} > 15$ GeV, $|\eta_l| < 2.5$
- $E_{\mathrm{T}}^{miss} > 30$ GeV
- $p_{\mathrm{T}}^{W} > 150$ GeV
- at least two b-jets with $p_{\mathrm{T}}^{b} > 25$ GeV and $|\eta_b| < 2.5$

$pp \to ZH + X \to \nu\nu b\bar{b} + X$

- $E_{\mathrm{T}}^{miss} > 150$ GeV
- at least two b-jets with $p_{\mathrm{T}}^{b} > 25$ GeV and $|\eta_b| < 2.5$

Results: cross sections

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$pp \to ZH + X \to \nu\nu b\bar{b} + X$

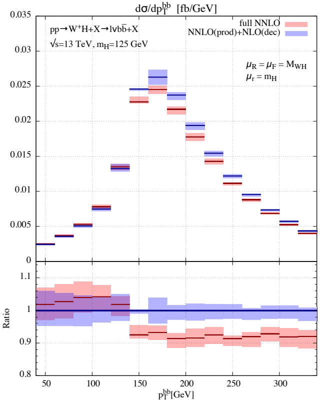
- $E_T^{miss} > 150$ GeV
- at least two b-jets with $p_T^b > 25$ GeV and $|\eta_b| < 2.5$

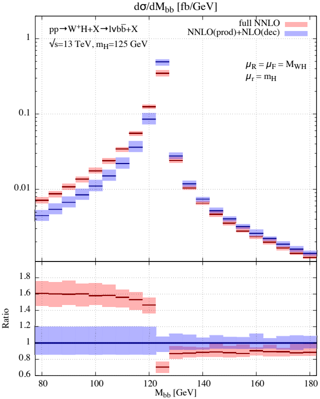
Cross section predictions at the LHC with $\sqrt{s} = 13$ TeV

σ (fb)	NNLO(prod)+NLO(dec)	full NNLO
$pp \to W^+H + X \to l\nu_l b\bar{b} + X$	$3.94^{+1\%}_{-1.5\%}$	$3.70^{+1.5\%}_{-1.5\%}$
$pp \to ZH + X \to \nu\nu b\bar{b} + X$	$8.65^{+4.5\%}_{-3.5\%}$	$8.24^{+4.5\%}_{-3.5\%}$

- Cross sections reduced by $\sim$ 5–6% at full NNLO wrt. NNLO(prod)+NLO(dec)
- Uncertainties correspond to scale variation

Transverse momentum and invariant mass of leading b-jet pair: $W^+H(b\bar{b})$



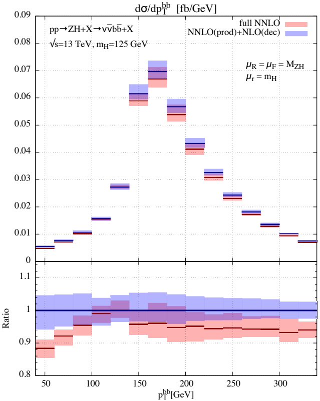


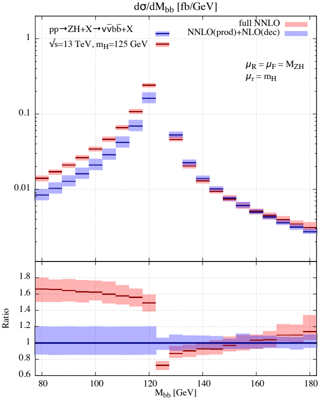
[Ferrera, GS, Tramontano Phys. Lett. B 780 (2018) 346-351]

- Contributions included in full NNLO produce important effects on the shapes: -8% – $+5\%$ corrections in $p_T^{b\bar{b}}$, -30% – $+60\%$ corrections in $M_{b\bar{b}}$!

Results: distributions

Transverse momentum and invariant mass of leading b-jet pair: $ZH(b\bar{b})$





[Ferrera, GS, Tramontano Phys. Lett. B 780 (2018) 346-351]

- Contributions included in full NNLO produce important effects on the shapes: $-10\% - -5\%$ corrections in $p_T^{b\bar{b}}$, $-30\% - +70\%$ corrections in $M_{b\bar{b}}$!

Outlook and Conclusions

The current situation

Incredible performance of LHC experiments demands a corresponding improvement of theoretical predictions

Techniques and tools for performing NNLO QCD calculations quickly maturing

- Two-loop amplitudes for $1 \to 3$ and $2 \to 2$ processes known
- Several approaches for dealing with IR singularities and organizing the cross section calculation proposed, more in the works
- Some public codes: `FEWZ`, `DYNNLO`, `HNNLO`, `EERAD3`, `MATRIX`, ...
- Benchmark processes computed: $e^+e^- \to 2, 3$ jets, $ep \to 2$ jets, $pp \to V/H$, $pp \to V/H +$ jet, $pp \to VV$, $pp \to 2$ jets, $pp \to t\bar{t}$

Analyses are now making use of NNLO results

- Extraction of α_s, constraining PDFs, searches,...
- First comparison of LHC jet data with NNLO

Going beyond

More processes: the NNLO wish list

[Les Houches 2017: Physics at TeV Colliders Standard Model Working Group Report]

- $pp \to H + 2$ jets
- $pp \to 3$ jets
- $pp \to V + 2$ jets
- $pp \to \gamma\gamma +$ jet
- $pp \to t\bar{t} +$ jet

These are very challenging, reaching new bottlenecks

- Two-loop (massive) amplitudes: great progress, but still (very) difficult
- Numerics for double real radiation will be challenging for any method

More loops: N^{3}LO,...

- First results for N^{3}LO for simplest kinematics: $pp \to H$ and $ep \to$ jet

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2015; Currie, Gehrmann, Glover, Huss, Niehues, Vogt 2018]

- Case-by-case computations or based on 'projection to Born' method that exploits the special kinematics of the process

Conclusions

Amazing progress in fixed order calculations in the past decade

- Automation of NLO
- NNLO for 3 jets at lepton colliders
- NNLO for $2 \to 2$ processes at hadron colliders
- Even N^{3}LO for simplest kinematics

NNLO results are being used for analyses

But reaching new bottlenecks, in particular NNLO still very challenging beyond $2 \to 2$

Will need significant developments: new understanding, new ideas, new tools

The future is challenging but exciting!