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Statistical Issues on the Neutrino Mass Hierarchy with $\Delta\chi^2$

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Abstract

The Neutrino Mass Hierarchy Determination (ν MHD) is one of the main goals of the major current and future neutrino experiments. The statistical analysis usually proceeds from the standard method, single dimensional estimator ($1D-\Delta\chi^2$). This method shows some draw-backs and concerns, together with a debatable strategy. The issues of the standard method on ν MHD for the neutrino reactor experiments are explained.

Theory: $\bar{\nu}_e$ Survival Probability Using Reactor Spectrum

$$p^{\lambda}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}(1 - cos\frac{\Delta m^2_{atm}L}{2E}) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_{12}(1 - cos\frac{\delta m^2_{sol}L}{2E})$$

$$+ \frac{1}{2}\sin^2 2\theta_{13}\left[cos^2\left(\theta_{12} + \frac{\pi}{2}\lambda\right)\right](cos\frac{L}{2E}(\Delta m^2_{atm} - \delta m^2_{sol}) - cos\frac{L\Delta m^2_{atm}}{2E})$$

$$\lambda = 0 \longrightarrow p^{\lambda}(\bar{\nu}_e \to \bar{\nu}_e) = p_{IH}(\bar{\nu}_e \to \bar{\nu}_e)$$

$$\lambda = 1 \longrightarrow p^{\lambda}(\bar{\nu}_e \to \bar{\nu}_e) = p_{NH}(\bar{\nu}_e \to \bar{\nu}_e)$$

Issue I: $|\overline{\Delta\chi^2}|$ oscillations with $|\Delta m^2|_{inj}$

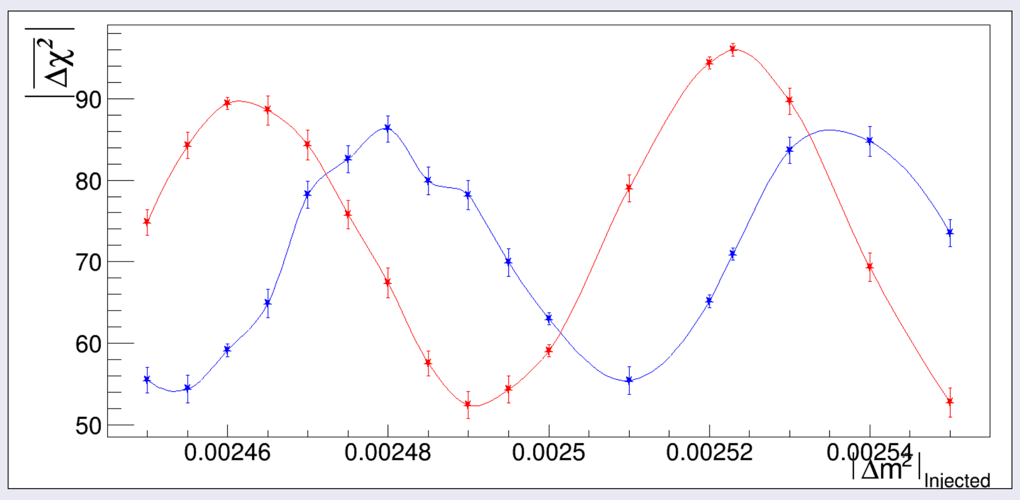


Figure 1: 200(NH) + 200 (IH) JUNO-toy simulations for $36GW$ and 6 years running assuming an **infinite energy resolution**. Blue line is for NH sample and red line for IH sample.

This leads to a question, how this figure come up. The fitting procedures and the minimization of χ^2 are done by the ROOT minimization libraries (the TMinuit algorithm). In the minimization procedure all the oscillation parameters were fixed to the best-fitting values of [1]. A total of 108357 signal events are processed for each toy-simulations. The official version of JUNO Software "J17v1r1" is used.

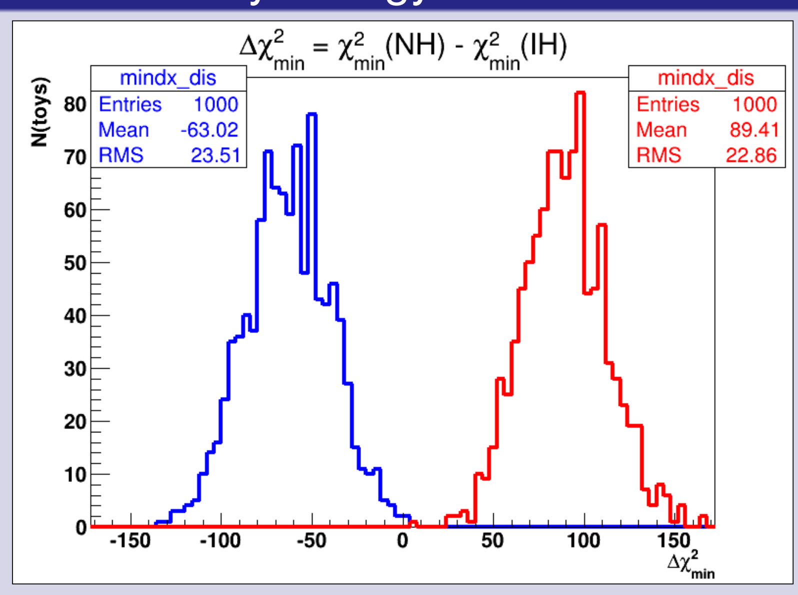
To conclude this point:

The oscillation of $|\overline{\Delta\chi^2}|$ with $|\Delta m^2|_{inj}$ means that ν MHD significance using $\Delta\chi^2$ strongly depends on the values of the parameter $|\Delta m^2|_{inj}$. Consequently, the experimental sensitivity is controlled by the assumed value of the neutrino atmospheric mass difference.

Issue II: The Limited Power of $\Delta\chi^2$

So, first of all we have to determine at which value the simulations will run. NH sample is generated at $\Delta m^2 = 2.500 \times 10^{-3} ev^2$ and IH sample is generated at $\Delta m^2 = -2.460 \times 10^{-3} ev^2$. Our JUNO-toy simulations are preformed in a single event basis. We use $\frac{3\%}{\sqrt{E}}$ relative energy resolution via a Gaussian distribution centered at the expected mean with the standard deviation of energy uncertainty.

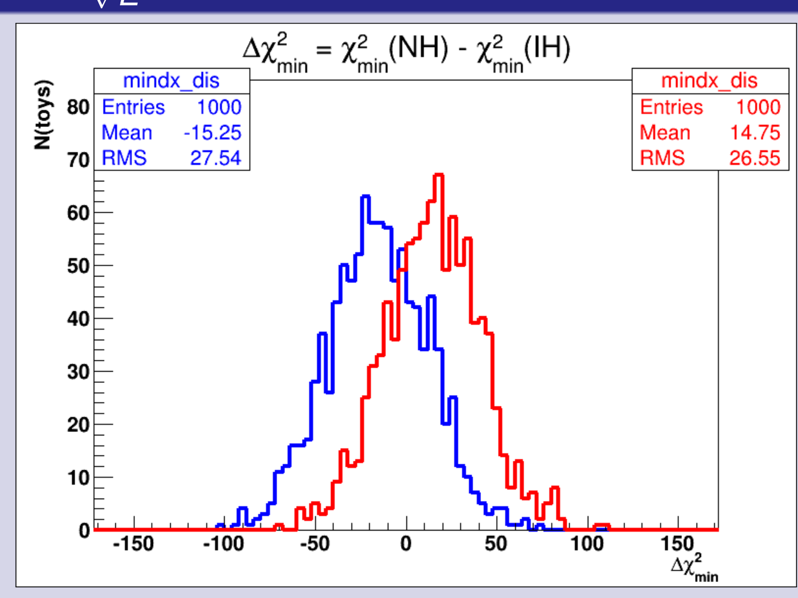
Infinity Energy Resolution



$$\sigma_{\Delta\chi^2} \equiv 2\sqrt{\Delta\chi^2}$$

Infinity energy resolution		
μ_{NH}	-63.02	
σ_{NH}	23.51	
μ_{IH}	89.39	
σ_{IH}	22.83	
n"σ"(NH)	6.485(z-test)	7.94(approximation)
n"σ"(IH)	6.676(z-test)	9.45(approximation)

$\frac{3\%}{\sqrt{E}}$ Relative Energy Resolution



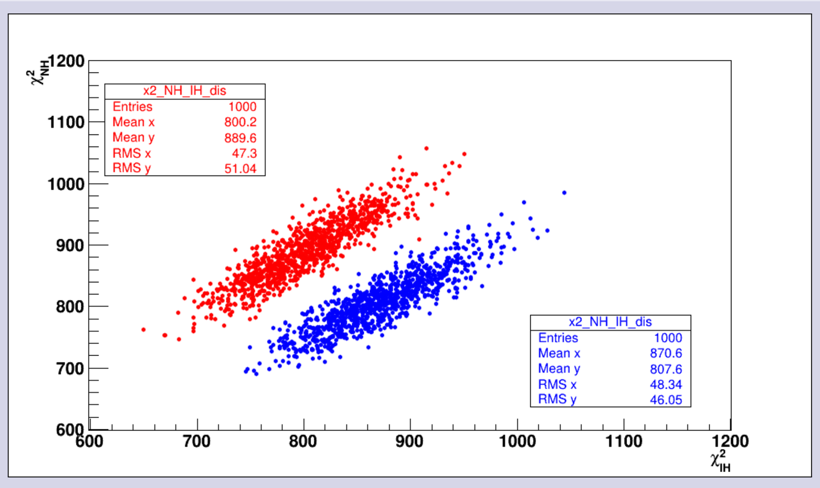
$$\sigma_{\Delta\chi^2} \neq 2\sqrt{\Delta\chi^2}$$

relative energy resolution $\frac{3\%}{\sqrt{E}}$		
μ_{NH}	-15.21	
σ_{NH}	27.52	
μ_{IH}	14.69	
σ_{IH}	26.55	
n"σ"(NH)	1.086(z-test)	3.9(approximation[1])
n"σ"(IH)	1.120(z-test)	3.8(approximation)[1]

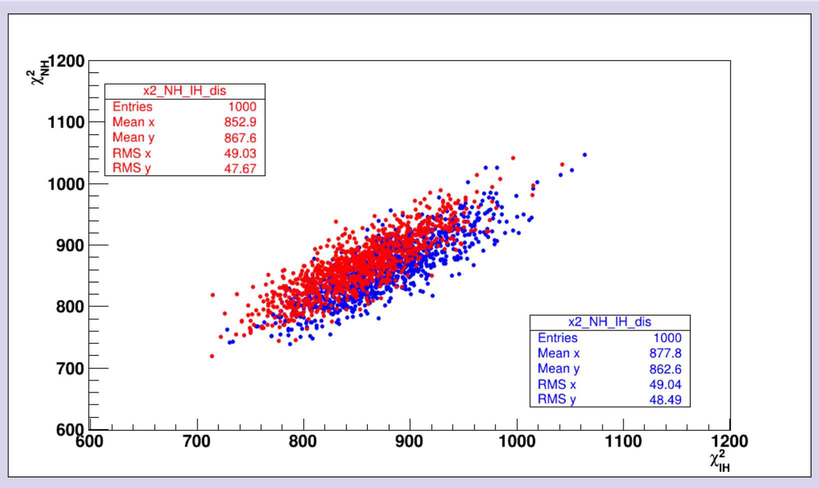
To conclude this point:

The $\Delta\chi^2$ estimator provides us with different results due to different simulation procedures. When the simulation is performed on a single event basis and not on a semi-analytical basis [1], it does not take into account the correlation between the bins due to systematic uncertainties, the significance drastically drops. The systematic uncertainties due to the $\frac{3\%}{\sqrt{E}}$ relatively energy resolution causes unbalanced immigration effect between bins that consequently create side-bin correlations leading to significant reduction in the experiment sensitivity . That invalids the use of the standard approximation.

Issue III: Non-bright Results using χ^2 as a Bi-Dimensional



Infinity energy resolution



3% relatively energy resolution

The two islands of χ^2 for 1000(NH) + 1000 (IH) simulations, generated at $\Delta m^2 = 2.500 \times 10^{-3}$ for NH hypothesis (blue island) and $\Delta m^2 = -2.460 \times 10^{-3}$ for IH hypothesis (red island) for the usual conditions.

To conclude this point:

When $\chi^2_{min(NH)}$ and $\chi^2_{min(IH)}$ are drawn in two dimensional map, their strong positive correlation manifests χ^2 as a bi-dimensional estimator. This strong positive correlation leading to an overlap between the χ^2 distributions of the two hypotheses results in reduction of the experimental sensitivity.

	NH	IH
μ_{NH}	853.5	828
σ_{NH}	44	44
μ_{IH}	862	867
σ_{IH}	43	42
r	0.85	0.85
p-Value(NH)	0.331	
nσ(NH)	0.437 σ	
p-Value(IH)	0.310	
nσ(IH)	0.496 σ	

The results at $\frac{3\%}{\sqrt{E}}$

Summary and Conclusions

That is why evaluating the used statistical methods and updating them is a necessary step in building a robust statistical analysis for the new physics problems. To summarize, the evaluation and draw-backs of the standard method run through three main issues. Firstly, the experimental sensitivity strongly depend on the value of the neutrino atmospheric mass difference as shown in the oscillation of $|\overline{\Delta\chi^2}|$ with $|\Delta m^2|_{inj}$. Secondly, when the energy systematic error is taking into account, the statistical assumptions are not valid any more and the limited power of the $\Delta\chi^2$ manifests itself. Thirdly, the strong positive correlation between the two components $\chi^2_{min(NH)}$ and $\chi^2_{min(IH)}$ leads to an overlap instead of maximize the separation between them.

References

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What next?

An alternative method can largely recover the experimental sensitivity by also maximizing the separation due to the strong anti-correlation effec [2].