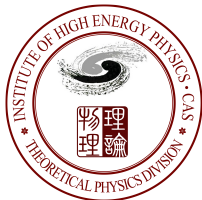


Positivity Constraints on Vector Boson Scattering

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LHC EWWG discussion

Based on 1808.00010 with Shuang-Yong Zhou

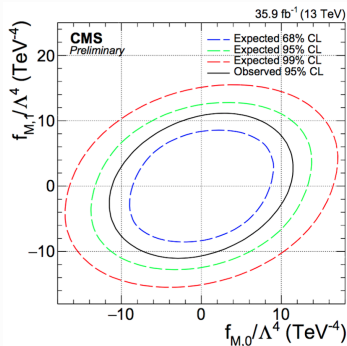
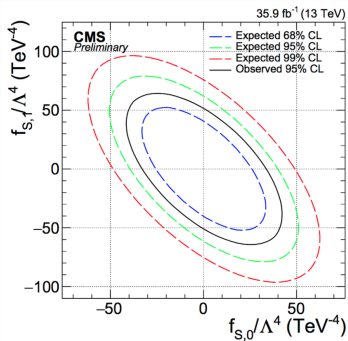
TH framework

- We study the anomalous Quartic Gauge-boson Couplings (aQGC) parametrized by 18 dim-8 operators. *[O. J. P. Eboli et al. '06]*

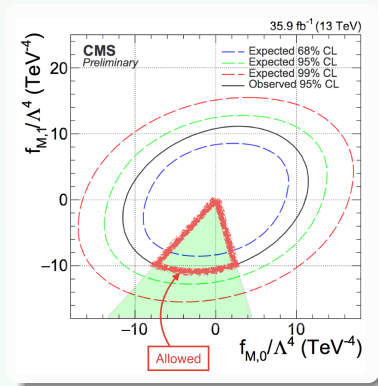
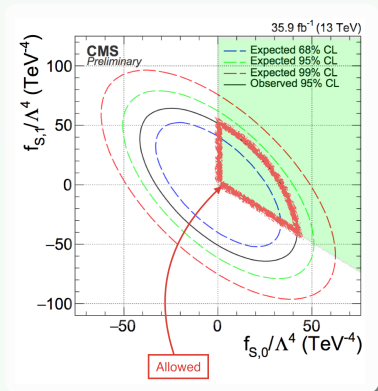
$$\begin{aligned}
 O_{S,0} &= [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi] \\
 O_{S,1} &= [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi] \\
 O_{S,2} &= [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi] \\
 O_{M,0} &= \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right] \\
 O_{M,1} &= \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\nu\beta} \right] \times \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right] \\
 O_{M,2} &= \left[\hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \right] \times \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right] \\
 O_{M,3} &= \left[\hat{B}_{\mu\nu} \hat{B}^{\nu\beta} \right] \times \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right] \\
 O_{M,4} &= \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\mu \Phi \right] \times \hat{B}^{\beta\nu} \\
 O_{M,5} &= \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\nu \Phi \right] \times \hat{B}^{\beta\mu} (+h.c.) \\
 O_{M,7} &= \left[(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\mu} D^\nu \Phi \right]
 \end{aligned}$$

$$\begin{aligned}
 O_{T,0} &= \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \text{Tr} \left[\hat{W}_{\alpha\beta} \hat{W}^{\alpha\beta} \right] \\
 O_{T,1} &= \text{Tr} \left[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta} \right] \times \text{Tr} \left[\hat{W}_{\mu\beta} \hat{W}^{\alpha\nu} \right] \\
 O_{T,2} &= \text{Tr} \left[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta} \right] \times \text{Tr} \left[\hat{W}_{\beta\nu} \hat{W}^{\nu\alpha} \right] \\
 O_{T,5} &= \text{Tr} \left[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \right] \times \hat{B}_{\alpha\beta} \hat{B}^{\alpha\beta} \\
 O_{T,6} &= \text{Tr} \left[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta} \right] \times \hat{B}_{\mu\beta} \hat{B}^{\alpha\nu} \\
 O_{T,7} &= \text{Tr} \left[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta} \right] \times \hat{B}_{\beta\nu} \hat{B}^{\nu\alpha} \\
 O_{T,8} &= \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \times \hat{B}_{\alpha\beta} \hat{B}^{\alpha\beta} \\
 O_{T,9} &= \hat{B}_{\alpha\mu} \hat{B}^{\mu\beta} \times \hat{B}_{\beta\nu} \hat{B}^{\nu\alpha},
 \end{aligned}$$

- TGC and QGC are fully correlated at dim-6. To parametrize independent QGC couplings not constrained by TGC measurements, we need the above operators.



CMS-PAS-SMP-18-001



What are positivity constraints

- In this work, we derive a set of “positivity constraints” on the 18 operator coefficients, **by assuming that the SMEFT has a UV completion.**
- What are “positivity constraints”:

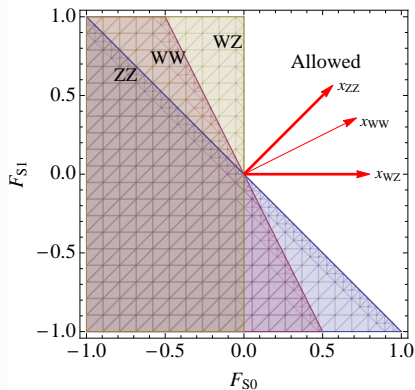
- ▶ A linear combination of coefs. $(F_{S,0}, F_{S,1}, F_{S,2}, \dots)$ must be positive.
- ▶ Or equivalently, consider a vector $\vec{c} = (F_{S,0}, F_{S,1}, F_{S,2}, \dots)$. Positivity says that \vec{c} has to be positive upon projection on a certain direction \vec{x}_i , i.e.

$$\vec{c} \cdot \vec{x}_i \geq 0$$

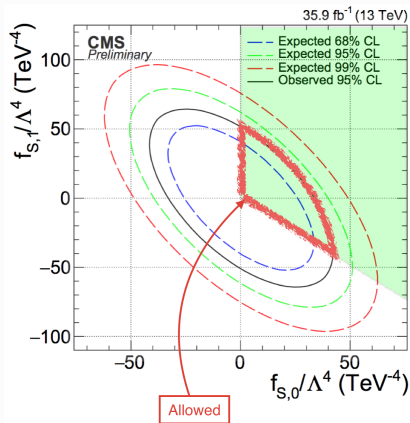
- ▶ \vec{x}_i come from the requirements that the VBS amplitudes (WW, ZZ, \dots) satisfy the fundamental principles of QFT (analyticity, unitarity, etc.), i.e. we have $\vec{x}_{WW}, \vec{x}_{ZZ}, \vec{x}_{WZ}, \dots$
 - But the resulting constraint apply regardless of the amplitude from which it is derived.

Implications on EXP results

$\tilde{\chi}_{WW,ZZ,WZ}$ and positivity bounds



Combined with measurements



Outline

- 1 Derivation
- 2 Implication
- 3 Conclusion

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Positivity approach

- First established in [\[A. Adams et al. JHEP '06\]](#): dispersion relation + optical theorem, forward 2-to-2 scattering.
- Non-forward generalization: [\[C. de Rham et al. Phys.Rev.D '17\]](#), [\[C. de Rham et al. JHEP '18\]](#)
- Application in collider pheno:
 - ▶ ZZ and $Z\gamma$: [\[B. Bellazzini and F. Riva '18\]](#)
 - ▶ Implications in Higgs physics under ceratin assumptions:
[\[I. Low et al. '09\]](#) [\[A. Falkowski et al. '12\]](#)
- In general the approach has strong implication on SMEFT dim-8 operators, which are important for the interpretation of VBS, so we should understand the constraints.

Analytic dispersion relation

- As an simplified version: consider the forward scattering ($t = 0$) of two identical particles with mass m , with possible heavy new physics.
(see [\[C. Cheung and G. N. Remmen '16\]](#) for a quick overview)
- If the UV completion exists, the amplitude $M(s, t = 0)$
 - ▶ is analytic and
 - ▶ satisfies Froissart unitarity bound $M(s, 0) \leq \mathcal{O}(s \ln^2 s)$.

Analytic dispersion relation

- Consider the contour integral:

$$f = \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{M(s, 0)}{(s - \mu^2)^3}$$

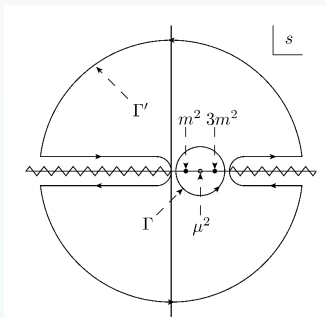
- Deform Γ to Γ' and notice that boundary contribution vanishes due to Froissart bound:

$$f = \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{M(s, 0)}{(s - \mu^2)^3} = \frac{1}{2\pi i} \left(\int_{-\infty}^0 + \int_{4m^2}^{\infty} \right) ds \frac{\text{Disc}M(s, 0)}{(s - \mu^2)^3}$$

i.e. sum of residues at low energy =

discontinuity along +x axis + discontinuity along -x axis

- Note that BSM (above Λ) enters the discontinuity, as poles (tree level) or branch cuts (heavy loops).



Derivation of positivity

- discontinuity along $\pm x$ axis must positive, because of optical theorem (disc. = $x\text{sec} > 0$) (plus crossing symmetry for $-x$)
- \Rightarrow sum of residues at low energy is positive.

We started with the amplitude in the full theory, but have reached a conclusion that only involves low energy, which can be computed in SMEFT:

$$\text{sum of residues at low energy} = \frac{d^2 M(s, 0)}{ds^2} = \sum_i c_i^{(8)} x_i + \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} > 0$$

- Conclusion: the above positivity condition must be satisfied, if
 - ▶ SMEFT has a UV completion, that satisfies unitarity, Lorentz symmetry, is analytic.
 - ▶ At low energy, the SMEFT is valid and tree level calculation is a good approximation, which anyway need to be assumed in a real measurement.

Dim-6 contributions can be removed

$$\sum_i c_i^{(8)} x_i \geq - \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j}$$

- In general, we expect dim-6 to be better constrained by other processes.
- But in any case, dim-6 doesn't matter, because by explicit calculation the **RHS is positive**.
- E.g. from WZ scattering:

$$\text{R.H.S} \propto a_3^2 b_3^2 \left[e^2 C_{DW} - s_W^2 c_W^2 C_{\varphi D} - 4s_W^3 c_W C_{\varphi WB} \right]^2 + 36(a_1 b_1 + a_2 b_2)^2 e^2 s_W^2 c_W^2 C_W^2$$

- and from WW:

$$\text{R.H.S} \propto a_3^2 b_3^2 s_W^2 \left(e^2 C_{DB} + c_W^2 C_{\varphi D} \right)^2 + e^2 c_W^2 [6(a_1 b_1 + a_2 b_2) s_W C_W + a_3 b_3 e C_{DW}]^2$$

$$\sum_i c_i^{(8)} x_i \geq - \sum_{i,j} c_i^{(6)} c_j^{(6)} y_{i,j} \geq 0$$

or simply:

$$\vec{c} \cdot \vec{x}_i \geq 0$$

Explicitly:

- Polarization matters. We will use

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

to denote the polarization of the two vector boson being scattered.

- As an example, $ZZ \rightarrow ZZ$ gives the following constraint:

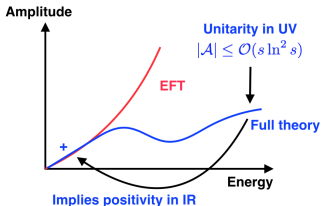
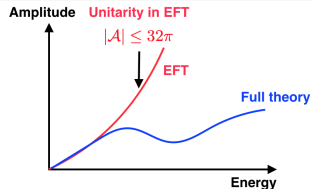
$$\begin{aligned} & 8a_3^2 b_3^2 t_W^4 (F_{S,0} + F_{S,1} + F_{S,2}) + \left[a_3^2 (b_1^2 + b_2^2) \right. \\ & \left. + (a_1^2 + a_2^2) b_3^2 \right] t_W^2 \left(-t_W^4 F_{M,3} + t_W^2 F_{M,5} - 2F_{M,1} + F_{M,7} \right) \\ & + \left[(a_1 b_1 + a_2 b_2)^2 + (a_1^2 + a_2^2) (b_1^2 + b_2^2) \right] \left(2t_W^8 F_{T,9} \right. \\ & \left. + 4t_W^4 F_{T,7} + 8F_{T,2} \right) + 8(a_1 b_1 + a_2 b_2)^2 \left[t_W^4 \left(t_W^4 F_{T,8} \right. \right. \\ & \left. \left. + 2F_{T,5} + 2F_{T,6} \right) + 4F_{T,0} + 4F_{T,1} \right] \geq 0 \end{aligned}$$

- Depending on \vec{a}, \vec{b} , there is a infinite number of constraints from $ZZ \dots$
- Other constraints from $W^\pm Z, W^\pm W^\pm, W^\pm W^\mp, W^\pm \gamma, Z\gamma, \gamma\gamma$.
- **These are the key results of this work.**

“Unitarity”

- It is well-known that unitarity violation can be a problem in SMEFT.
 - In VBS, unitarization techniques are needed.
 - However, here unitarity problem concerns **only the prediction of the SMEFT**, and only signals the breakdown of EFT.
- Our bounds are derived from a different information, i.e. the Froissart unitarity bound. **This unitarity refers to the behaviour of the UV theory at large energy.**
 - This is then connected to the IR (EFT) of the theory by the dispersion relation

i.e. Unitarity in UV (full theory) \Rightarrow Positivity in IR (EFT)



Outline

1 Derivation

2 Implication

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Example: simplified model

Consider the simplified model in [\[Brass, Fleper, Kilian, Reuter, Sekulla '18\]](#)

In the present paper, we do not refer to a specific scenario. We construct a simplified model with transverse couplings of a generic heavy resonance σ . The effective Lagrangian takes the following form,

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma(m_\sigma^2 - \partial^2)\sigma + \sigma(J_{\sigma\parallel} + J_{\sigma\perp}) \quad (19a)$$

$$J_{\sigma\parallel} = F_{\sigma H} \text{tr} \left[(\mathbf{D}_\mu \mathbf{H})^\dagger (\mathbf{D}^\mu \mathbf{H}) \right] \quad (19b)$$

$$J_{\sigma\perp} = g^2 F_{W\sigma} \sigma \text{tr} [\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] + g'^2 F_{B\sigma} \sigma \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}] \quad (19c)$$

with three independent coupling parameters.

In the low-energy limit, the scalar resonance can be integrated out, and we obtain the SMEFT Lagrangian with the following nonzero coefficients of the dimension-8 operators at leading order:

$$F_{S_0} = F_{\sigma H}^2 / 2m_\sigma^2 \quad (20a)$$

$$F_{M_0} = -F_{\sigma H} F_{\sigma W} / m_\sigma^2 \quad (20b)$$

$$F_{M_2} = -F_{\sigma H} F_{\sigma B} / m_\sigma^2 \quad (20c)$$

$$F_{T_0} = F_{\sigma W}^2 / 2m_\sigma^2 \quad (20d)$$

$$F_{T_5} = F_{\sigma W} F_{\sigma B} / m_\sigma^2 \quad (20e)$$

$$F_{T_8} = F_{\sigma B}^2 / 2m_\sigma^2. \quad (20f)$$

Example: simplified model

If we plug in the dim-8 coefficients into our positivity constraints, we see:

$$ZZ : (a_1 b_1 + a_2 b_2)^2 \left(s_W^4 F_{\sigma B} + 2c_W^4 F_{\sigma W} \right)^2 + a_3^2 b_3^2 s_W^4 c_W^4 e^{-4} F_{\sigma H}^2 > 0$$

$$W^\pm Z : a_3^2 b_3^2 F_{\sigma H}^2 > 0$$

$$W^\pm W^\pm : (a_1 b_1 + a_2 b_2)^2 F_{\sigma W}^2 + \left[(a_1 b_1 + a_2 b_2) F_{\sigma W} + a_3 b_3 s_W^2 e^{-2} F_{\sigma H} \right]^2 > 0$$

$$W^\pm W^\mp : (a_1 b_1 + a_2 b_2)^2 F_{\sigma W}^2 + \left[(a_1 b_1 + a_2 b_2) F_{\sigma W} - a_3 b_3 s_W^2 e^{-2} F_{\sigma H} \right]^2 > 0$$

$$ZA : (a_1 b_1 + a_2 b_2)^2 \left[s_W^2 F_{\sigma B} - 2c_W^2 F_{\sigma W} \right]^2 > 0$$

WA : none

$$AA : (a_1 b_1 + a_2 b_2)^2 (F_{\sigma B} + 2F_{\sigma W})^2 > 0$$

*up to factors of 2 that can be absorbed in the definitions of $F_{\sigma X}$

All inequalities are satisfied, as they are all sum of squares.

- In a **top-down approach**, positivity is automatically true, in different models, different ways — by asking for positivity, we are not restricting the UV models.
- In a **bottom-up approach**, we can derive the same constraints, **but without using model details**, and therefore we **restrict the parameter space without losing model-independence**.

1D limits

- Consider **one operator at a time**:

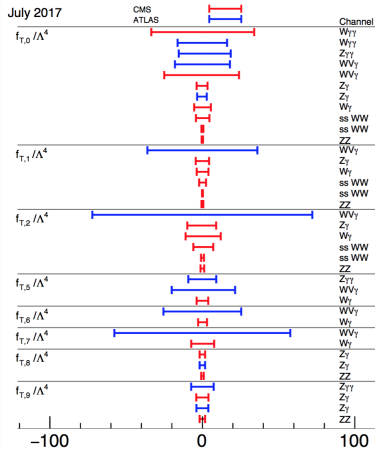
$f_{S,0}$	$f_{S,1}$	$f_{S,2}$	$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
+	+	+	✗	-	○	-	○	✗
$f_{M,7}$	$f_{T,0}$	$f_{T,1}$	$f_{T,2}$	$f_{T,5}$	$f_{T,6}$	$f_{T,7}$	$f_{T,8}$	$f_{T,9}$
+	+	+	+	✗	+	✗	+	+

+: positive -: negative
 ○: free ✗: forbidden

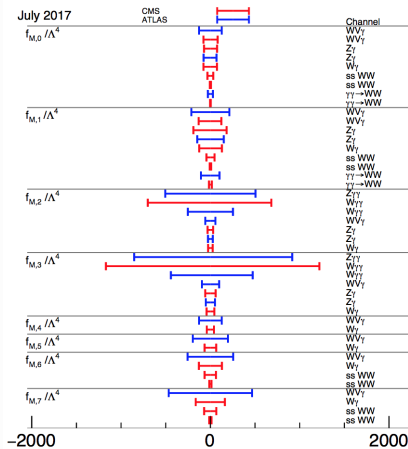
- Note there are coefficients that are **not individually allowed**.
 - E.g. F_{T5} . In the simplified model $F_{T5} \propto F_{\sigma W} F_{\sigma B}$, cannot take nonzero value independent of $F_{T0} \propto F_{\sigma W}^2$ and $F_{T8} \propto F_{\sigma B}^2$.

1D limits: EXP

Individual limits on transversal coefficients



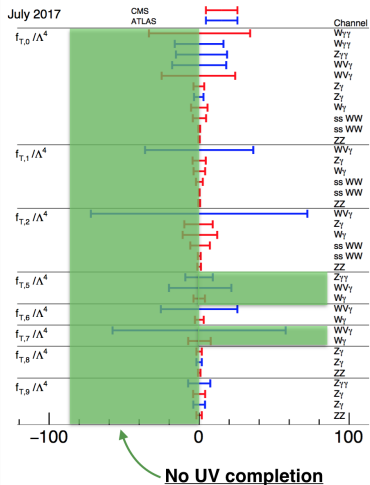
Individual limits on mixed coefficients



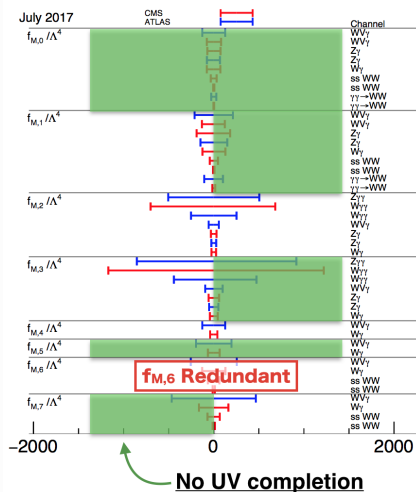
https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSMPaTGC#aQGC_Results

1D limits: EXP+positivity

Transversal coefficients, positivity



Mixed coefficients, positivity



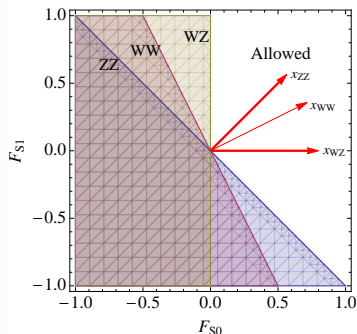
2D limits: Longitudinal case

As a first example, consider $O_{S,0}$ and $O_{S,1}$.

- Coef. vector: $\vec{c} \equiv (F_{S,0}, F_{S,1})$. Positivity: $\vec{c} \cdot \vec{x}_i \geq 0$.
- There are 3 useful constraints, from WW , ZZ , WZ scattering

$$\vec{x}_{WZ} = a_3^2 b_3^2 (1, 0), \quad \vec{x}_{WW} = a_3^2 b_3^2 (2, 1), \quad \vec{x}_{ZZ} = a_3^2 b_3^2 (1, 1)$$

\vec{x} and corresponding exclusion



- Note that \vec{x}_{WW} is **between** \vec{x}_{ZZ} and \vec{x}_{WZ} , so positivity on \vec{x}_{WW} does not give new independent information.
- **In general: if \vec{x} is a positive linear combination of a set of \vec{x}_i , i.e.**

$$\vec{x} = \alpha_i \vec{x}_i, \quad \alpha_i \geq 0 \quad \forall i$$

Then positivity along \vec{x} does not lead to additional exclusion.

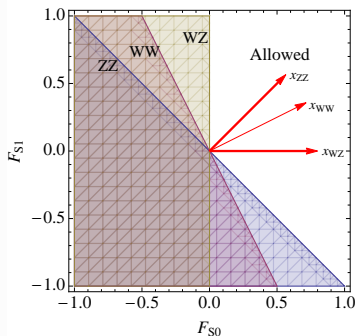
2D limits: longitudinal case

As a first example, consider $O_{S,0}$ and $O_{S,1}$.

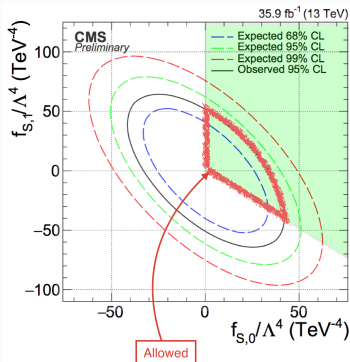
- Coef. vector: $\vec{c} \equiv (F_{S,0}, F_{S,1})$. Positivity: $\vec{c} \cdot \vec{x}_i \geq 0$.
- There are 3 useful constraints, from WW , ZZ , WZ

$$\vec{x}_{WZ} = a_3^2 b_3^2 (1, 0), \quad \vec{x}_{WW} = a_3^2 b_3^2 (2, 1), \quad \vec{x}_{ZZ} = a_3^2 b_3^2 (1, 1)$$

\vec{x} and corresponding exclusion



Combined with EXP



2D limits: longitudinal and transversal case

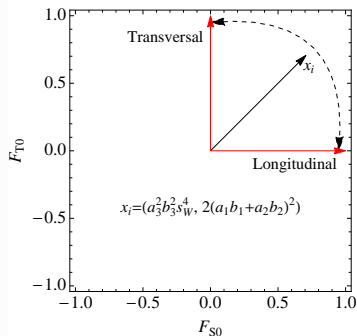
As a second example, consider $O_{S,0}$ and $O_{T,0}$.

- The constraint from WW is

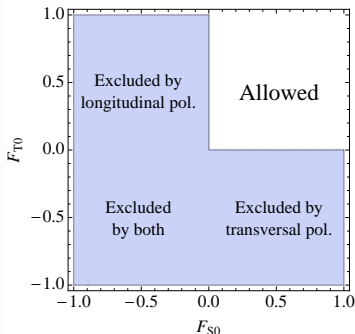
$$\vec{x}_{WW} = (a_3^2 b_3^2 s_W^4, 2(a_1 b_1 + a_2 b_2)^2)$$

- Depending on the chosen \vec{a}, \vec{b} , \vec{x}_{WW} can take **any direction in the first quadrant**.

\vec{x} can vary in the 1st quadrant



Corresponding exclusion



2D limits: mixed case

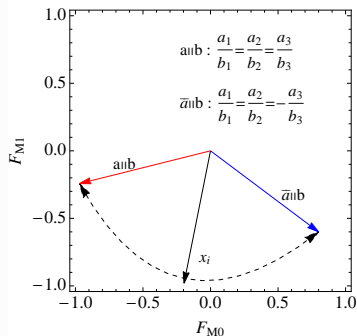
As a third example, consider $O_{M,0}$ and $O_{M,1}$.

- The most useful constraint is from WW

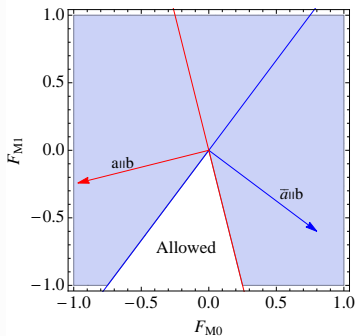
$$\vec{x}_{WW} = - \left(4(a_1 b_1 + a_2 b_2) a_3 b_3, (a_1^2 + a_2^2) b_3^2 - (a_1 b_1 + a_2 b_2) a_3 b_3 + (b_1^2 + b_2^2) a_3^2 \right)$$

- Varies between $(-4, -1)$ and $(4, -3)$.

\vec{x} sits between red and blue arrows



Corresponding exclusion



2D limits: mixed case

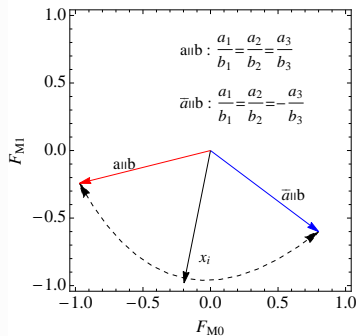
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- The most useful constraint is from WW

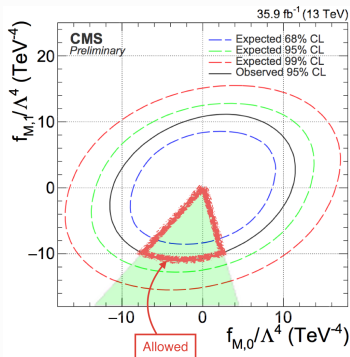
$$\vec{x}_{WW} = - \left(4(a_1 b_1 + a_2 b_2) a_3 b_3, (a_1^2 + a_2^2) b_3^2 - (a_1 b_1 + a_2 b_2) a_3 b_3 + (b_1^2 + b_2^2) a_3^2 \right)$$

- Varies between $(-4, -1)$ and $(4, -3)$.

\vec{x} sits between red and blue arrows



Corresponding exclusion



2D limits: summary

In 2D case, constraints are given by minimally **two key vectors**.

- $F_{S0} - F_{S1}$:

$$\vec{x}_{WZ} = (1, 0)$$

$$\vec{x}_{ZZ} = (1, 1)$$

- $F_{S0} - F_{T0}$:

$$\vec{x}_{WW}(\text{trans.}) = (0, 1)$$

$$\vec{x}_{WW}(\text{long.}) = (1, 0)$$

- $F_{M0} - F_{M1}$:

$$\vec{x}_{WW}(\vec{a} \parallel \vec{b}) = (-4, -1)$$

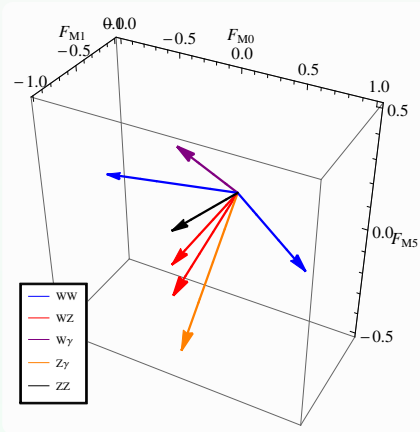
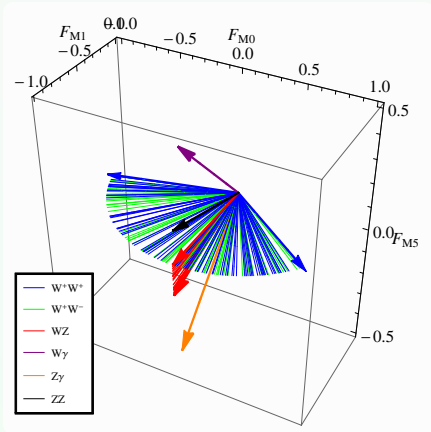
$$\vec{x}_{WW}(\vec{a} \perp \vec{b}) = (4, -3)$$

What happens in higher dimension parameter space?

3D limits

As a last example, consider $O_{M,0}$, $O_{M,1}$ and $O_{M,5}$.

- Scan the polarization space by randomly generating \vec{a} and \vec{b} .
- \vec{x} within the pyramid formed by other \vec{x}_i does not give new info!



3D limits

How to find the “key vectors” that characterize the bounds in general:

- Scan all possible polarizations \vec{a} , \vec{b} .
- Project to a 2D plane ($f_{M0} - f_{M5}$).
- Take the endpoints of \vec{x} .
- **Find the convex hull** of the set of points.
- The vertices corresponds to the key vectors:

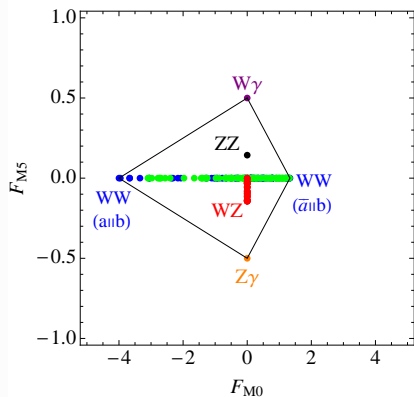
$$\vec{x}_{W\gamma} = (0, -2, 1),$$

$$\vec{x}_{Z\gamma} = (0, -2, -1),$$

$$\vec{x}_{WW}(\vec{a} \parallel \vec{b}) = (-4, -1, 0),$$

$$\vec{x}_{WW}(\vec{a} \perp \vec{b}) = (4, -3, 0).$$

\vec{x}_i projected on 2D plane



3D limits

- Allowed region is given by

$$-2F_{M1} + F_{M5} \geq 0,$$

$$-2F_{M1} - F_{M5} \geq 0,$$

$$-4F_{M0} - F_{M1} \geq 0,$$

$$4F_{M0} - 3F_{M1} \geq 0.$$

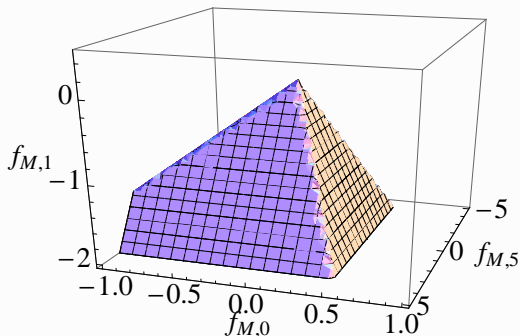
- Note that

$$F_{M5} \in [-2|F_{M1}|, 2|F_{M1}|],$$

$$F_{M0} \in [-\frac{3}{4}|F_{M1}|, \frac{1}{4}|F_{M1}|]$$

- In principle same approach applies for higher-D case: *the problem is equivalent to finding a **D-1 dimensional convex hull**.*

3D allowed region given by a pyramid



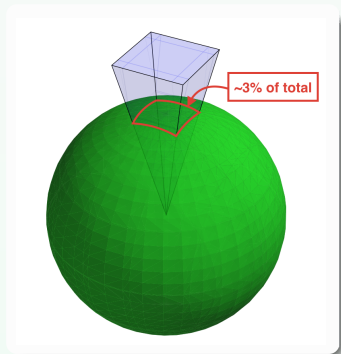
Recall:

$f_{M,0}$	$f_{M,1}$	$f_{M,2}$	$f_{M,3}$	$f_{M,4}$	$f_{M,5}$
X	-	O	-	O	X

Volume in full parameter space

When all 18 parameters are turned on, how much of the parameter space is excluded by positivity?

- By brutal force, randomly through points on a 18D sphere, uniformly distributed, and count how many of the them fall within constraints for all polarizations.
- We find that **only** $\sim 3\%$ **parameter space** is left.



How does positivity affect future experimental search?

We don't know, but some thoughts...

- It might help MC generation? Because less space need to be scanned? Does it help to move to higher dim? (i.e. towards global EFT fit)
 - ▶ 2D/3D benchmarks?
 - ▶ Or simply a MATHEMATICA script...

```
ForAll[{a1, a2, a3, b1, b2, b3}, PositivityConditions]
```

- Does it provide enough info to support a “guided search”? Instead of blindly searching for BSM in 18D space, can we make use of the fact that we know **BSM only exists in the 3% parameter space?**
- Presentation, e.g. 1D limits should be presented in a more reasonable form.

Outline

- 1 Derivation
- 2 Implication
- 3 Conclusion**

Conclusion

- Dim-8 aQGC operator coefficients satisfy a set of **positivity constraints**, if they are generated by a UV completion.
- They have strong implication, e.g. 18D parameter space reduced to 3%, independent of experimental precision.
- The shape of the allowed parameter space shows interesting structure.
- They should be taken into account for future aQGC studies.

Thank you!