Nuclear physics of dark matter direct detection

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PHYSTAT Dark Matter 2019

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MH, Klos, Menéndez, Schwenk PLB 746 (2015) 410, PRL 119 (2017) 181803, PRD 94 (2016) 063505, 99 (2019) 055031

Fieguth, MH, Klos, Menéndez, Schwenk, Weinheimer PRD 97 (2018) 103532

XENON collaboration + MH, Klos, Menéndez, Schwenk PRL 122 (2019) 071301

Direct detection of dark matter

- Rate for WIMP–nucleus scattering: **factorization**

$$\frac{dR}{dE_r} = \underbrace{\frac{\sigma_{\chi N}^{\text{SI}}}{m_\chi \mu_N^2}}_{\text{particle + hadronic physics}} \times \underbrace{\left|\mathcal{F}_+^M(q^2)\right|^2}_{\text{nuclear physics}} \times \underbrace{\rho_0 \int_{v_{\text{min}}}^{v_{\text{esc}}} \frac{f(\mathbf{v},t)}{v} d^3v}_{\text{astrophysics}}$$

- This talk:
  - **Nuclear responses**: $\mathcal{F}_+^M(q^2)$, ...
  - **Hadronic matrix elements**: relating $\sigma_{\chi N}^{\text{SI}}$ to BSM operators, ...
  - **Uncertainties**: known and unknown unknowns

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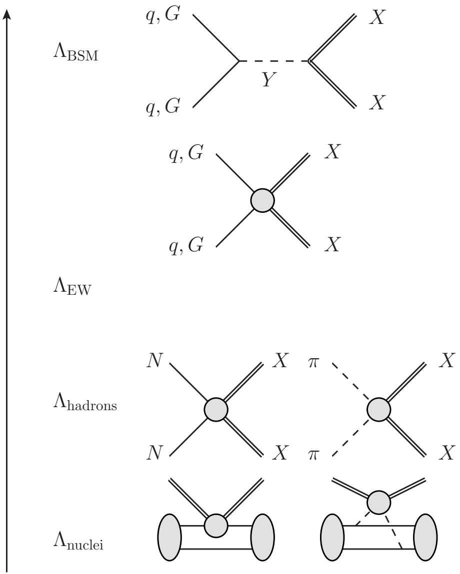
  - **Hadronic matrix elements**: relating $\sigma^{SI}_{\chi N}$ to BSM operators, ...
  - **Uncertainties**: known and unknown unknowns
- Ultimate aim: controlled approximations at all stages
  - Effective field theories (EFTs) to connect scales
  - EFTs + many-body methods for nuclear structure
  - EFTs/lattice QCD/dispersion relations for hadronic matrix elements

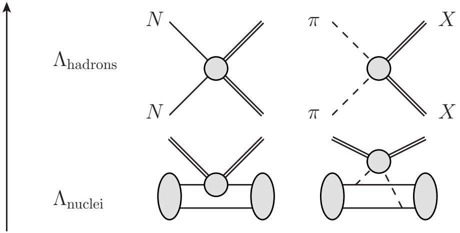
$\hookrightarrow$ Trace all of this back to QCD as far as possible **"*ab initio*"**

Direct detection of dark matter: scales



1. **BSM scale** Λ_{BSM}: $\mathcal{L}_{\text{BSM}}$
2. **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda^i_{\text{BSM}}} \mathcal{O}_{i,k}$
   SMEFT
3. Integrate out **EW physics**
4. **Hadronic scale**: nucleons and pions
   $\hookrightarrow$ effective interaction Hamiltonian H_I
   Chiral EFT
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$
   $\hookrightarrow$ nuclear wave function Chiral EFT, NREFT

Direct detection of dark matter: scales



4. **Hadronic scale**: nucleons and pions
   $\hookrightarrow$ effective interaction Hamiltonian H_I
   Chiral EFT

5. **Nuclear scale**: $\langle\mathcal{N}|H_I|\mathcal{N}\rangle$
   $\hookrightarrow$ nuclear wave function Chiral EFT, NREFT

- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\rm max}| = 2\mu_{\mathcal{N}\chi}|\mathbf{v}_{\rm rel}| \sim 200\,\text{MeV} \qquad |\mathbf{v}_{\rm rel}| \sim 10^{-3} \qquad \mu_{\mathcal{N}\chi} \sim 100\,\text{GeV}$$

- **Chiral EFT**: pions, nucleons, and WIMPs as degrees of freedom
  Prézeau et al. 2003, Cirigliano et al. 2012, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015, Bishara et al. 2017 ...

- **NREFT**: all degrees of freedom integrated out but nucleons and WIMPs
  Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013 ...

Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
  - Based on **chiral symmetry** of QCD
  - Expansion in M_π/Λ_χ, $\Lambda_\chi \sim 600\,\text{MeV}$
  - **Low-energy constants**
  - Hierarchy of multi-nucleon forces
  - Consistency of NN and $3N$

$\hookrightarrow$ modern theory of nuclear forces

- Long-range part related to **pion–nucleon scattering**

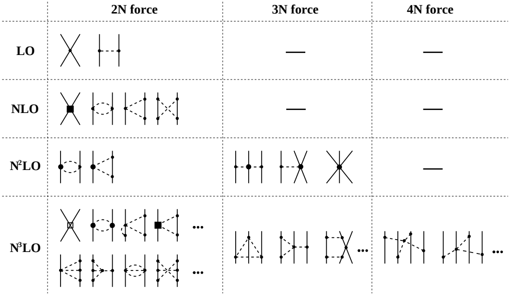
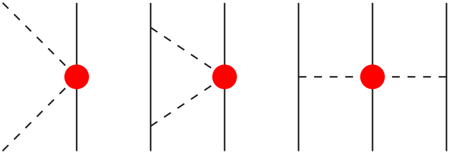


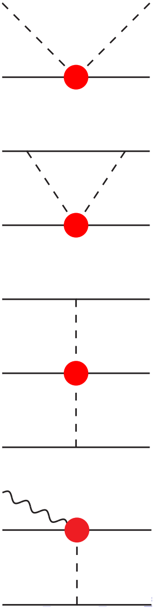
Figure taken from 1011.1343



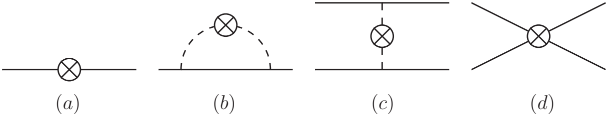
Chiral EFT: currents

- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**

  $\hookrightarrow$ β decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ, up to one-loop level
  - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. 2016, 2019
  - Without unitary transformations: Park et al. 2003, Pastore et al. 2008, Baroni et al. 2015
- For **dark matter** further currents: s, p, tensor, spin-2, θ^μ_μ

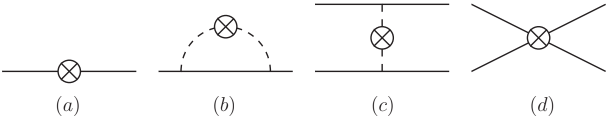


Chiral EFT for dark matter: diagrams



- One-body (1*b*) and two-body (2*b*) diagrams:
  - **Leading 1b responses** (*a*): standard spin-independent (SI) and spin-dependent (SD) interactions
  - **Subleading 1b responses** (*a*): non-relativistic expansion of nucleon form factors
  - **Radius corrections** (*b*): q^2 expansion of nucleon form factors
  - **Two-body currents** (*c*), (*d*): pion exchange, contact operators

Chiral EFT for dark matter: diagrams



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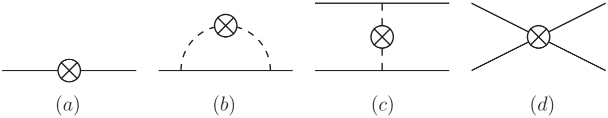
- Nuclear responses:
  - **SI**: coherent sum over all *A* nucleons ("*M*- response")
  - **SD**: proportional to $\langle \boldsymbol{S} \rangle$ (combination of Σ', Σ'')
  - **Spin-orbit**: coherent sum over partially filled shells (Φ'')
  - **Two-body currents**: coherent over all *A*, but suppressed in chiral counting

Chiral EFT vs. NREFT



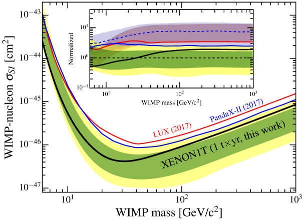
- NREFT
  - Integrate out the pion, expand in q and v
  - **Large number of operators** $\mathcal{O}_i$ already at $1b$ level, $> 2 \times 10$ at $\mathcal{O}(q^2, vq, v^2)$
  - ***2b* corrections** would require many more operators in addition

$\hookrightarrow$ misses most important effects (in the way it is typically applied)

- Chiral EFT
  - **Hierarchy** predicted from **chiral symmetry**
  - Including $2b$ corrections much more efficient

$\hookrightarrow$ fewer parameters

Case 1: spin-independent scattering



XENON1T 2018

- **SI scattering**: only keep isoscalar, fully coherent response

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma^{\text{SI}}_{\chi N}}{4\mu_N^2 v^2}\left|\mathcal{F}_+^M(q^2)\right|^2 \qquad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

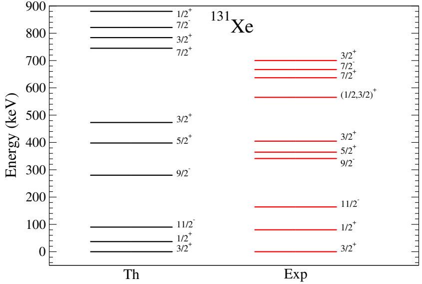
- Traditionally, "Helm form factor" Helm 1956, Lewin, Smith 1996

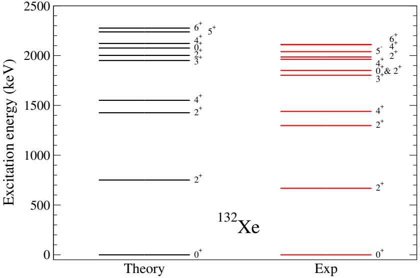
$$\mathcal{F}_+^M(q^2)\big|_{\text{Helm}} = A\frac{3j_1(qr_n)}{qr_n}e^{-\frac{1}{2}q^2s^2}$$

$\hookrightarrow$ purely phenomenological

- Modern many-body methods: significant step forward, but not fully "ab initio" yet

Spectra and shell-model calculation





- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- Currently phenomenological shell-model interaction

  $\hookrightarrow$ chiral-EFT-based interactions in the future
- Progress in ab-initio methods: In-Medium Similarity Renormalization Group . . .
- Already possible for “light” nuclei

Further cross checks: charge radii and neutron skin

	^{19}F	^{28}Si	^{40}Ar	^{74}Ge	^{132}Xe
$\sqrt{\langle r^2\rangle_{\text{ch}}}$ [fm] (th)	2.83	3.19	3.43	4.08	4.77
(exp)	2.898(2)	3.122(2)	3.427(3)	4.0742(12)	4.7808(49)
$\sqrt{\langle r_n^2\rangle} - \sqrt{\langle r_p^2\rangle}$ [fm]	0.02	0	0.11	0.17	0.28
shell-model interaction	USDB	USDB	SDPF.SM	RG	GCN

- Excellent agreement for charge radii

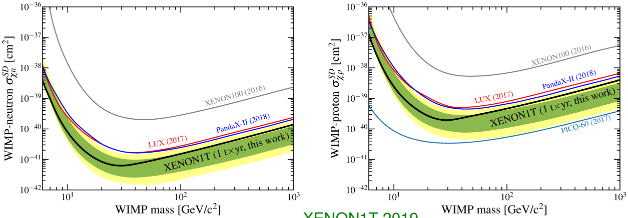
$$\langle r^2_{\text{ch}}\rangle = \langle r^2_p\rangle + \langle r^2_{E,p}\rangle + \frac{N}{Z}\langle r^2_{E,n}\rangle + \langle r^2_{\text{rel}}\rangle + \langle r^2_{\text{spin-orbit}}\rangle$$

- Point-neutron radii more uncertain, can be tested by measuring weak radii
- Related to structure factors by

$$\langle r^2_p\rangle = -\frac{3}{Z}\frac{\mathrm{d}}{\mathrm{d}q^2}\big(\mathcal{F}^M_+(q^2) + \mathcal{F}^M_-(q^2)\big)\big|_{q^2=0} \qquad \langle r^2_n\rangle = -\frac{3}{N}\frac{\mathrm{d}}{\mathrm{d}q^2}\big(\mathcal{F}^M_+(q^2) - \mathcal{F}^M_-(q^2)\big)\big|_{q^2=0}$$

- Further cross checks: electric quadrupole and magnetic dipole moments, transition matrix elements

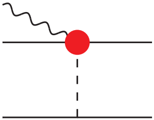
Case 2: spin-dependent scattering



XENON1T 2019

- **SD scattering**: typically proton- or neutron-only

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma^{SD}_{\chi N}}{3\mu_N^2 v^2}\frac{\pi}{2J+1}S_N(q^2)$$

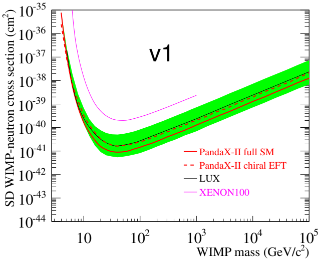


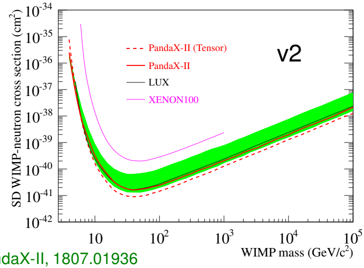
- Xe sensitive to proton spin due to **two-body currents**

  Klos, Menéndez, Gazit, Schwenk 2013

- Power of EFT: parameters (low-energy constants) fixed from related processes

A note on spin-dependent scattering





PandaX-II, 1807.01936

- Convention for SD scattering goes back at least to Engel, Pittel, Vogel 1992

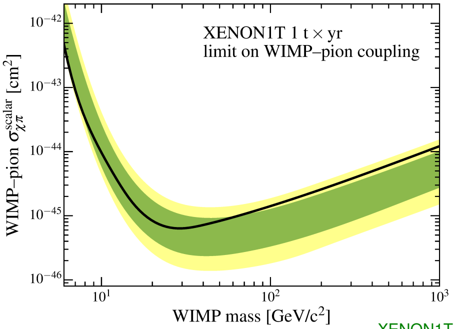
  $\hookrightarrow$ **axial-vector–axial-vector** current $\bar{\chi}\gamma^\mu\gamma_5\chi\, \bar{q}\gamma_\mu\gamma_5 q$ (motivated by SUSY)
- In QCD

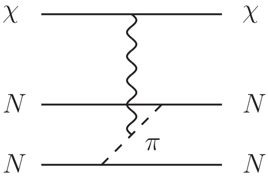
$$\langle N(p')|\bar{q}\gamma_\mu\gamma_5\tau^3 q|N(p)\rangle = \langle N(p')|\gamma^\mu\gamma_5 G_A(q^2)\tau^3 + \gamma_5\frac{q^\mu}{2m_N}G_P(q^2)\tau^3|N(p)\rangle$$

$$G_A(0) = g_A \qquad G_A(q^2) - \frac{q^2}{4m_N^2}G_P(q^2) = \mathcal{O}(M_\pi^2)$$

- **Induced pseudoscalar** $G_P(q^2)$ neglected in v1, "improving" the LUX limits

  $\hookrightarrow$ need to use consistent conventions for meaningful comparison!





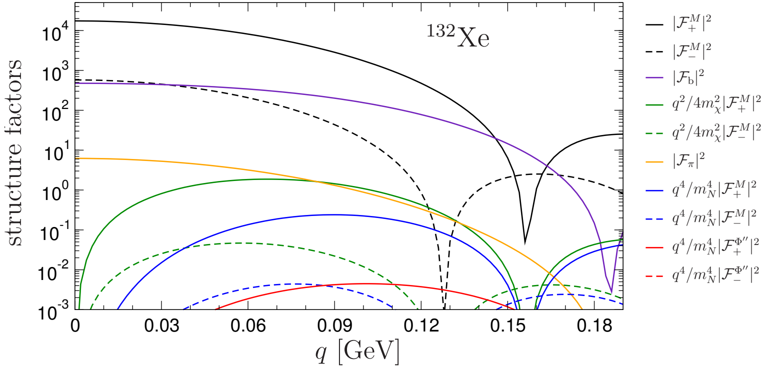
XENON1T + MH, Klos, Menéndez, Schwenk 2019

- One way to illustrate $2b$ effects: **WIMP–pion scattering**

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi\pi}^{\text{scalar}}}{\mu_\pi^2 v^2} \left|\mathcal{F}_\pi(q^2)\right|^2 \qquad \mu_\pi = \frac{m_\chi M_\pi}{m_\chi + M_\pi}$$

- Expression in terms of cross section depends on underlying operator, here for a scalar $\bar\chi\chi\bar q q$

Full set of coherent contributions



$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\bigg|\sum_{I=\pm}\Big(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\Big)\mathcal{F}_I^M(q^2) + c_\pi\mathcal{F}_\pi(q^2) + c_b\mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm}c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\bigg|^2$$

$$+\frac{1}{4\pi v^2}\sum_{i=5,8,11}\bigg|\sum_{I=\pm}\xi_i(q,v_T^\perp)c_I^{M,i}\mathcal{F}_I^M(q^2)\bigg|^2$$

$$+\frac{1}{v^2(2J+1)}\Big(|a_+|^2S_{00}(q^2) + \text{Re}\,(a_+a_-^*)S_{01}(q^2) + |a_-|^2S_{11}(q^2)\Big)$$

Role of the coefficients

- Coefficients c, a: combinations of **Wilson coefficients** (BSM operators) and **hadronic matrix elements**
- In above examples expressed in terms of **single-particle cross sections**

$$\sigma_{\chi N}^{\mathrm{SI}} = \frac{\mu_N^2}{\pi}\big|c_+^M\big|^2 \qquad \sigma_{\chi N}^{\mathrm{SD}} = \frac{3\mu_N^2}{\pi}\big|a_+\big|^2 \qquad \sigma_{\chi\pi}^{\mathrm{scalar}} = \frac{\mu_\pi^2}{4\pi}\big|c_\pi\big|^2$$

$\hookrightarrow$ can analyze experiment in terms of these coefficients

- But: to constrain a particular BSM operator/model: need to disentangle

  $\hookrightarrow$ **matching relations**

- Example: Dirac spin-$1/2$ WIMP, scalar operator $\mathcal{L}^{\mathrm{BSM}} = C_q^{SS}\bar\chi\chi m_q\bar q q$

$$c_\pm^M = \frac{1}{2}\big(f_p \pm f_n\big) + \cdots \qquad f_N = m_N \sum_{q=u,d,s} C_q^{SS} f_q^N + \cdots \qquad \langle N|m_q\bar q q|N\rangle = m_N f_q^N$$

$\hookrightarrow$ scalar couplings of the nucleon f_q^N

ChiralEFT4DM: a PYTHON package for dark matter

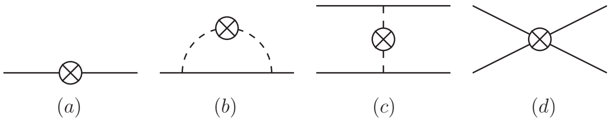
- ChiralEFT4DM: all results available as PYTHON package at

  `https://theorie.ikp.physik.tu-darmstadt.de/strongint/ChiralEFT4DM.html`
- Includes:
  - (Quasi-) Coherent structure factors for F, Si, Ar, Ge, Xe
  - Nucleon matrix elements and matching relations for spin-$1/2$ and spin-0 WIMP
  - 1b and 2b responses up to third chiral order
  - S, P, V, A, T, θ^{μ}_{μ}, and spin-2 effective operators that can lead to a coherent response
  - Convolution with Standard Halo Model

Uncertainties

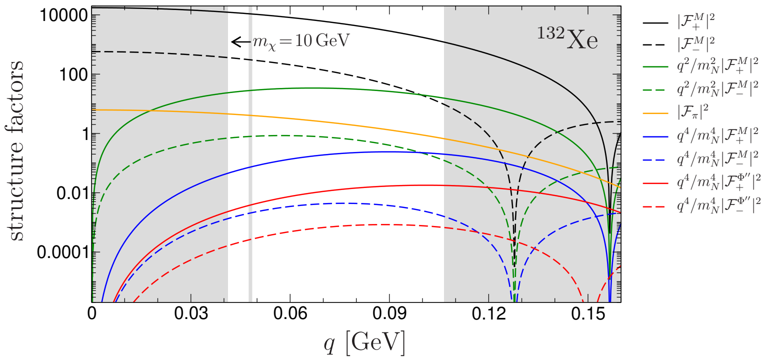
- **EFT truncations**:
  - Order-by-order convergence
  - Bayesian interpretation Furnstahl, Phillips, Wesolowski 2015 ...
- **Low-energy constants**:
  - Single-nucleon couplings mostly well determined from simpler processes
  - Two-nucleon parameters more uncertain
- **Many-body methods**:
  - Not fully systematic yet, but progress towards ab-initio calculations
  - Compare to observables: spectra, radii, EM transitions
- **Hadronic matrix elements**:
  - Vector and axial-vector couplings well known
  - Tension between lattice QCD and phenomenology for scalar couplings

Conclusions



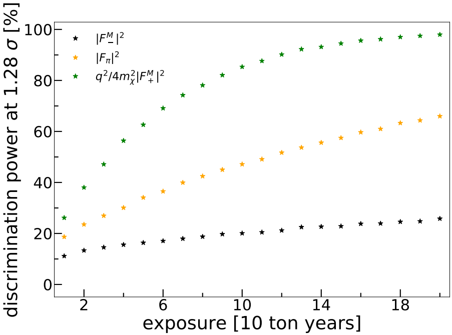
- **Chiral EFT** for WIMP–nucleus scattering
  - Connects nuclear and hadronic scales
  - Systematic approach to nuclear responses thanks to EFT
- Uncertainties
  - **EFT truncation errors**: currents, few-nucleon interaction
  - **Low-energy constants**: input from few-nucleon observables
  - **Many-body method**: towards ab-initio for “heavy” nuclei
  - **Hadronic matrix elements**: for BSM matching

Discriminating different response functions



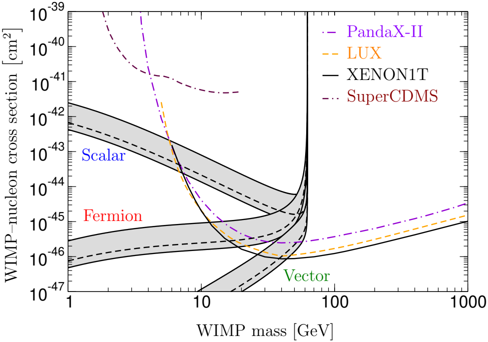
- White region accessible to XENON-type experiment
- Can one tell these curves apart in a realistic experimental setting?
- Consider XENON1T-like, XENONnT-like, DARWIN-like settings

Discriminating different response functions



- DARWIN-like setting, $m_\chi = 100\,\text{GeV}$
- q-dependent responses more easily distinguishable
- If interaction not much weaker than current limits, DARWIN could discriminate most responses from standard SI structure factor

- **Higgs Portal**: WIMP interacts with SM via the Higgs
  - **Scalar**: $H^\dagger H S^2$
  - **Vector**: $H^\dagger H V_\mu V^\mu$
  - **Fermion**: $H^\dagger H \bar{f} f$
- If $m_h > 2m_\chi$, should happen at the LHC
  $\hookrightarrow$ limits on **invisible Higgs decays**



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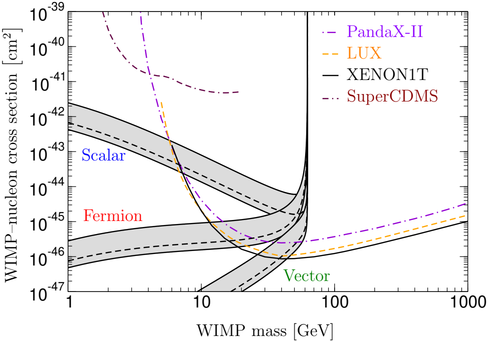
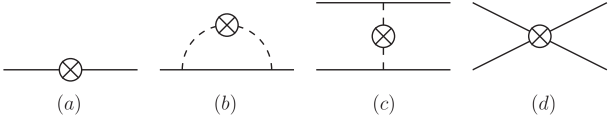


- Translation requires input for **Higgs–nucleon coupling**

$$f_N = \sum_{q=u,d,s,c,b,t} f_q^N = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q^N + \mathcal{O}(\alpha_s) \qquad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle$$

- Issues: input for $f_N = 0.260 \ldots 0.629$ outdated, 2b currents missing

Higgs–nucleon coupling



- **One-body contribution**

$$f_N^{1b} = 0.307(9)_{ud}(15)_s(5)_{\text{pert}} = 0.307(18)$$

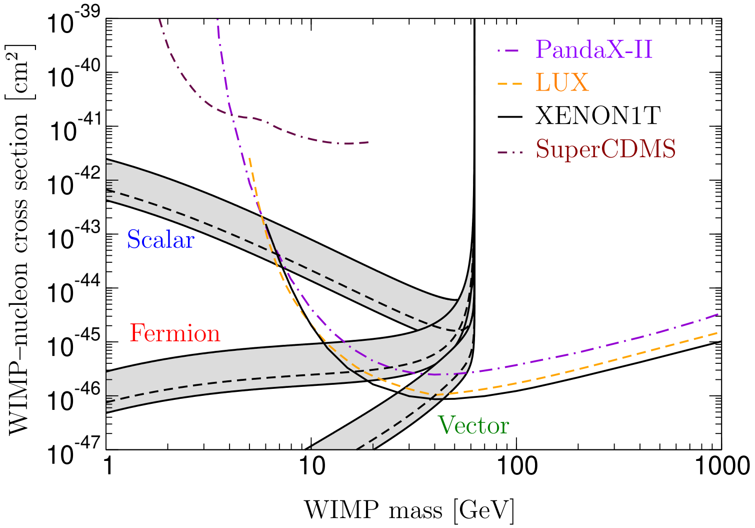
- Limits on WIMP–nucleon cross section subsume **2b effects**

  $\hookrightarrow$ have to be included for meaningful comparison

- **Two-body contribution**
  - Need s and θ^μ_μ currents
  - Treatment of θ^μ_μ tricky: several ill-defined terms combine to $\langle\Psi|T + V_{NN}|\Psi\rangle = E_b$
  - A cancellation makes the final result anomalously small

$$f_N^{2b} = \big[-3.2(0.2)_A(2.1)_{\text{ChEFT}} + 5.0(0.4)_A\big] \times 10^{-3} = 1.8(2.1) \times 10^{-3}$$

Improved limits for Higgs Portal dark matter



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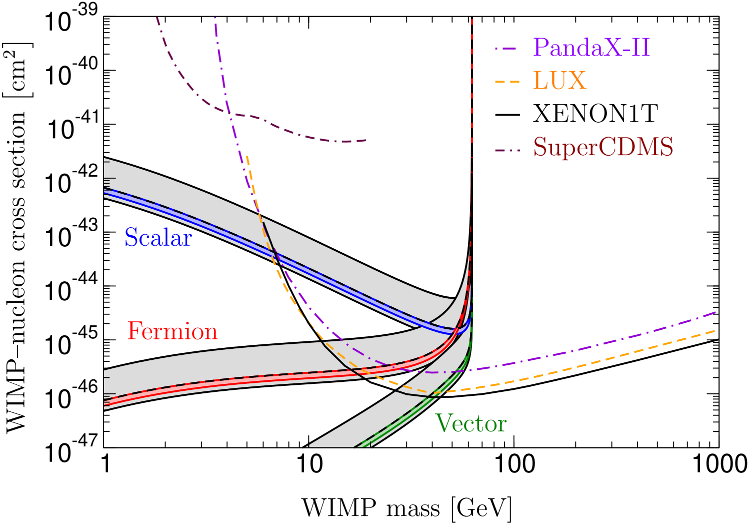

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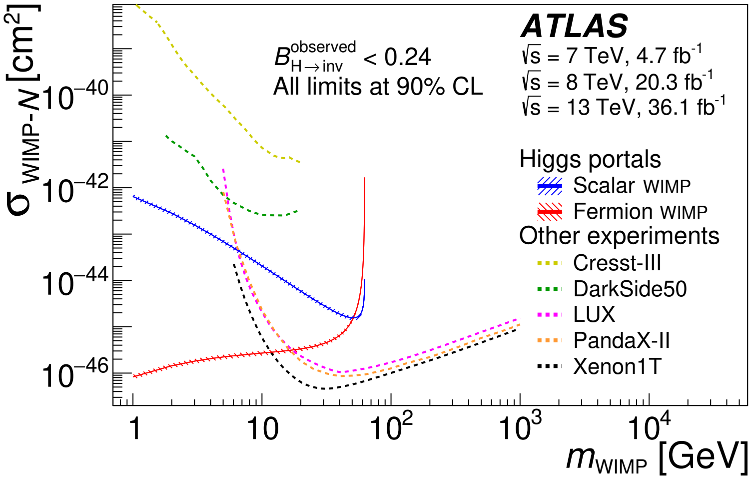

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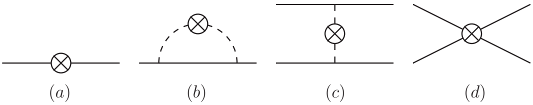
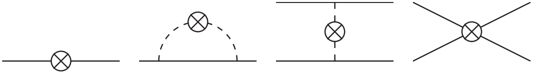
Cross section and nuclear structure factors



$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\bigg|\sum_{I=\pm}\Big(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\Big)\mathcal{F}_I^M(q^2) + c_\pi\mathcal{F}_\pi(q^2) + c_b\mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm}c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\bigg|^2$$
$$+ \frac{1}{4\pi v^2}\sum_{i=5,8,11}\bigg|\sum_{I=\pm}\xi_i(q, v_T^\perp)c_I^{M,i}\mathcal{F}_I^M(q^2)\bigg|^2$$
$$+ \frac{1}{v^2(2J+1)}\Big(|a_+|^2 S_{00}(q^2) + \text{Re}\,(a_+a_-^*)S_{01}(q^2) + |a_-|^2 S_{11}(q^2)\Big)$$

- Decomposition into **nuclear structure factors** $\mathcal{F}$, S_{ij} and coefficients c, a
- Three classes of contributions:
  - **(Sub-) Leading 1b responses** (a): $c_I^M\mathcal{F}_I^M(q^2)$, $c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)$, $|a_\pm|^2 S_{ij}(q^2)$
  - **Radius corrections** (b): $\dot{c}_I^M\mathcal{F}_I^M(q^2)$
  - **Two-body currents** (c), (d): $c_\pi\mathcal{F}_\pi(q^2)$, $c_b\mathcal{F}_b(q^2)$
- (a)+(b) essentially **nucleon form factors**, but (c)+(d) genuinely new effects

Cross section and nuclear structure factors



(a) (b) (c) (d)

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\left|\sum_{I=\pm}\left(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\right)\mathcal{F}_I^M(q^2) + c_\pi\mathcal{F}_\pi(q^2) + c_b\mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm}c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\right|^2$$
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- Nuclear structure interpretation:
  - $\mathcal{F}_I^M(q^2)$: $\mathbb{1} \Rightarrow$ charge distribution (coherent), $\mathcal{F}_\pm^M(0) = Z \pm N$
  - $\mathcal{F}_I^{\Phi''}(q^2)$: $i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \Rightarrow$ spin-orbit interaction (quasi-coherent)
  - $S_{ij}(q^2)$: $\mathbf{S}_\chi \cdot \mathbf{S}_N$, $\mathbf{S}_\chi \cdot \mathbf{q}\,\mathbf{S}_N \cdot \mathbf{q} \Rightarrow$ spin average (not coherent),
    $S_{00}(0) \pm S_{01}(0) + S_{11}(0) = \frac{(2J+1)(J+1)}{4\pi J}|\langle\mathbf{S}_{p/n}\rangle|^2$
- Coefficients: convolution of **Wilson coefficients** and **nucleon matrix elements**

$$c_\pm^M = \frac{\zeta}{2}(f_p \pm f_n) + \cdots \qquad f_N = \frac{m_N}{\Lambda^3}\sum_{q=u,d,s} C_q^{SS} f_q^N + \cdots \qquad \langle N|m_q\bar{q}q|N\rangle = m_N f_q^N$$

Chiral Perturbation Theory

- Effective theory of QCD based on **chiral symmetry**

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

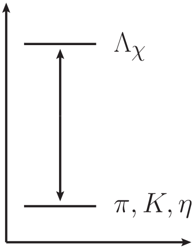
- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

  $\hookrightarrow$ **scale separation**

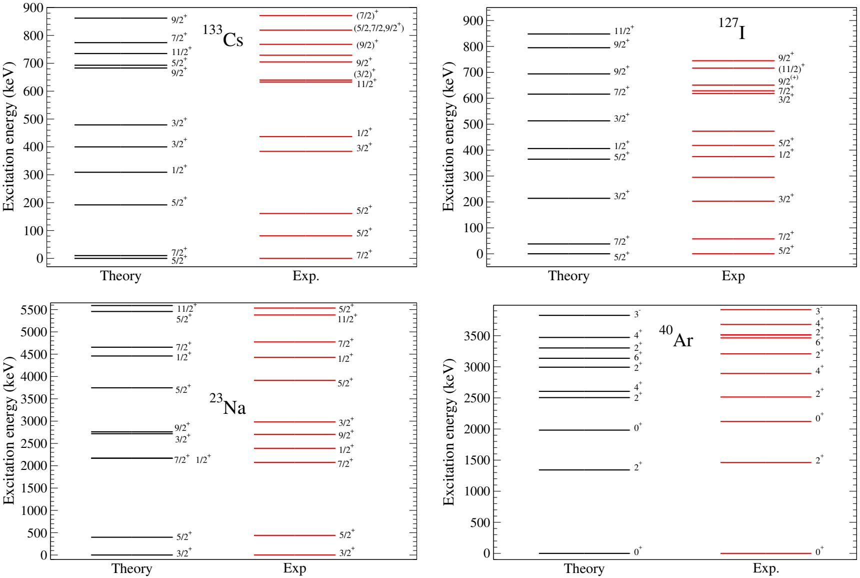
- Breakdown scale: $\Lambda_\chi = M_\rho \ldots 4\pi F_\pi \sim 1\,\text{GeV}$
- Two variants
  - ***SU*(2)**: *u*- and *d*-quark **dynamical**, m_s fixed at **physical value**

    $\hookrightarrow$ expansion in M_π/Λ_χ, usually nice convergence
  - ***SU*(3)**: *u*-, *d*-, and *s*-quark dynamical

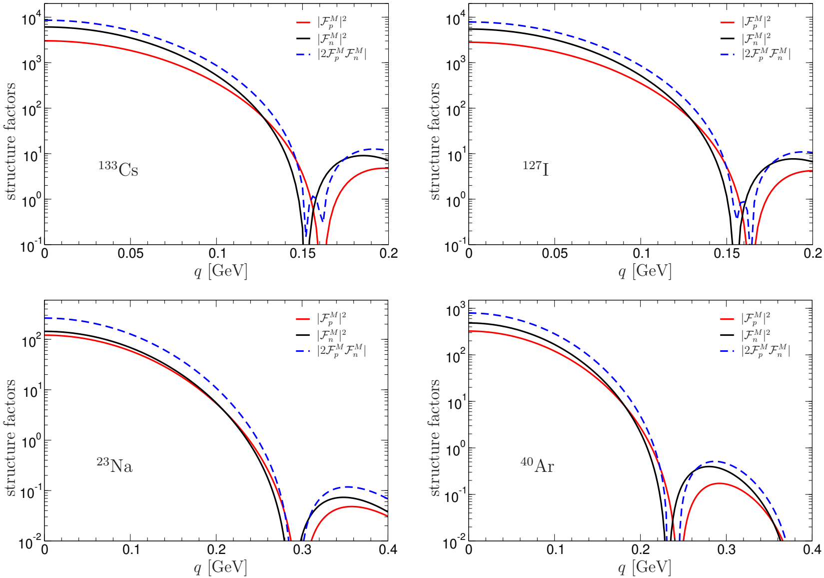
    $\hookrightarrow$ expansion in M_K/Λ_χ, sometimes tricky



Spectra


Excitation energy (keV)
133Cs
Theory
Exp.
9/2+ 7/2+ 11/2+ 5/2+ 9/2+ 3/2+ 3/2+ 1/2+ 5/2+ 7/2+ 5/2+
(7/2)+ (5/2,7/2,9/2+) (9/2)+ 9/2+ (3/2)+ 11/2+ 1/2+ 3/2+ 5/2+ 5/2+ 7/2+
127I
Theory
Exp
11/2+ 9/2+ 9/2+ 7/2+ 3/2+ 1/2+ 5/2+ 3/2+ 7/2+ 5/2+
9/2+ (11/2)+ 9/2(+) 7/2+ 3/2+ 5/2+ 1/2+ 3/2+ 7/2+ 5/2+
23Na
Theory
Exp
11/2+ 5/2+ 7/2+ 1/2+ 5/2+ 9/2+ 3/2+ 7/2+ 1/2+ 5/2+ 3/2+
5/2+ 11/2+ 7/2+ 1/2+ 5/2+ 3/2+ 9/2+ 1/2+ 7/2+ 5/2+ 3/2+
40Ar
Theory
Exp.
3- 4+ 2+ 6+ 2+ 4+ 2+ 0+ 2+ 0+
3- 4+ 2+ 6+ 2+ 4+ 2+ 0+ 2+ 0+

Structure factors



Chiral counting

- **Effective WIMP Lagrangian** for spin-$1/2$ SM singlet χ Goodman et al. 2010

$$\mathcal{L}_\chi = \frac{1}{\Lambda^3}\sum_q \Big[C_q^{SS}\bar\chi\chi\, m_q\bar q q + C_q^{PS}\bar\chi i\gamma_5\chi\, m_q\bar q q + C_q^{SP}\bar\chi\chi\, m_q\bar q i\gamma_5 q + C_q^{PP}\bar\chi i\gamma_5\chi\, m_q\bar q i\gamma_5 q\Big]$$
$$+ \frac{1}{\Lambda^2}\sum_q \Big[C_q^{VV}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu q + C_q^{AV}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu q + C_q^{VA}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu\gamma_5 q + C_q^{AA}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu\gamma_5 q\Big]$$
$$+ \frac{1}{\Lambda^3}\Big[C_g^S\bar\chi\chi\,\alpha_s G^a_{\mu\nu}G_a^{\mu\nu}\Big]$$

- **Chiral power counting**

$$\partial = \mathcal{O}(p), \qquad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \qquad a_\mu, v_\mu = \mathcal{O}(p), \qquad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

$\hookrightarrow$ construction of effective Lagrangian for nucleon and pion fields

$\hookrightarrow$ organize in terms of **chiral order ν**, $\mathcal{M} = \mathcal{O}(p^\nu)$

Chiral counting: summary

WIMP	Nucleon	V: t	V: $\mathbf{x}$	A: t	A: $\mathbf{x}$
	1b	0	1 + 2	2	0 + 2
V	2b	4	2 + 2	2	4 + 2
	2b NLO	—	—	5	3 + 2
	1b	0 + 2	1	2 + 2	0
A	2b	4 + 2	2	2 + 2	4
	2b NLO	—	—	5 + 2	3

WIMP	Nucleon	S	P
	1b	2	1
S	2b	3	5
	2b NLO	—	4
	1b	2 + 2	1 + 2
P	2b	3 + 2	5 + 2
	2b NLO	—	4 + 2

- $+2$ from NR expansion of WIMP spinors, terms can be dropped if $m_\chi \gg m_N$
- Red: all terms up to $\nu = 3$
- 1b: one-body (single-nucleon), 2b: two-body, 2b NLO: two-body at (nominally) next-to-leading order

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\bigg[i\gamma_\mu\big(\partial^\mu - i v^\mu\big) - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5\bigg(2a^\mu - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi}\bigg) + \cdots\bigg]\Psi$$

$\hookrightarrow$ **no scalar source**!

WIMP	Nucleon	S
	1b	2
S	2b	3

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\bigg[i\gamma_\mu\big(\partial^\mu - i v^\mu\big) - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5\bigg(2a^\mu - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi}\bigg) + \cdots\bigg]\Psi$$

$\hookrightarrow$ **no scalar source**!

- Scalar coupling

$$f_N = \frac{m_N}{\Lambda^3}\sum_{q=u,d,s} C_q^{SS} f_q^N + \cdots \qquad \langle N|m_q\bar{q}q|N\rangle = f_q^N m_N$$

$\hookrightarrow$ for $q = u, d$ related to **pion–nucleon σ-term** $\sigma_{\pi N}$

WIMP	Nucleon	S
	1b	2
S	2b	3

- Chiral expansion

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \qquad \dot{\sigma} = \frac{5g_A^2 M_\pi}{256\pi F_\pi^2} + \mathcal{O}(M_\pi^2)$$

$\hookrightarrow$ slow convergence due to strong $\pi\pi$ rescattering

$\hookrightarrow$ use phenomenology for the full scalar form factor!

Extracting $\sigma_{\pi N}$ from πN scattering: low-energy theorem

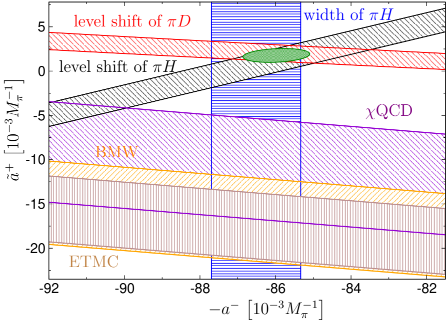
- No scalar probe, but still relation to experiment! How?

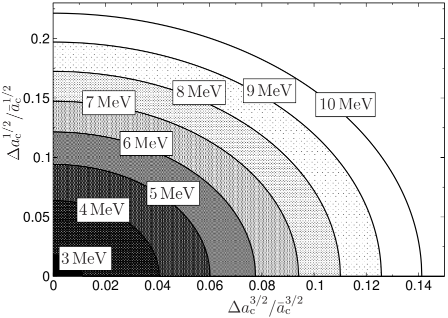
  $\hookrightarrow$ **low-energy theorem**
- Topic for another talk MH, Ruiz de Elvira, Kubis, Meißner PRL 115 (2015) 092301, PLB 760 (2016) 74, JPG 45 (2018) 024001
  - Goes back to Cheng, Dashen; Brown, Pardee, Peccei 1971
  - Relates $\sigma_{\pi N}$ to πN scattering amplitude, but at **unphysical kinematics**

    $\hookrightarrow$ analytic continuation to the Cheng–Dashen point
  - **No chiral logs** at one-loop order! Bernard, Kaiser, Meißner 1996
  - **Protected by *SU*(2)**

    $\hookrightarrow$ expected correction: $\sigma_{\pi N} M_\pi^2 / m_N^2 \sim 1$ MeV

πN σ-term: a lingering tension with lattice QCD



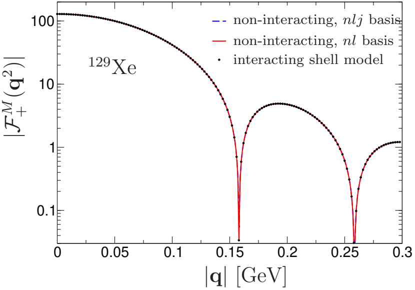


- Independent experimental constraints from **pionic atoms** and **low-energy cross sections** agree at the level of $\sigma_{\pi N} = 58(5)\,\text{MeV}$
- Tension can be illustrated in scattering-length plane
  $\hookrightarrow$ independent constraint from lattice calculation of $a_{0+}^{1/2}$ and $a_{0+}^{3/2}$
- This needs to be resolved: **rare opportunity** to benchmark lattice BSM matrix elements from experiment

Scalar two-body currents: oscillator model

$$\mathcal{F}_\pi(\mathbf{q}^2) = \frac{M_\pi}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \sum_{n_1 l_1 n_2 l_2} \sum_{\tau_1 \tau_2} \int \frac{d^3p_1 d^3p_2 d^3p_1' d^3p_2'}{(2\pi)^6} R_{n_1 l_1}(|\mathbf{p}_1'|) R_{n_2 l_2}(|\mathbf{p}_2'|) R_{n_1 l_1}(|\mathbf{p}_1|) R_{n_2 l_2}(|\mathbf{p}_2|)$$
$$\times \frac{(2l_1+1)(2l_2+1)}{16\pi^2} P_{l_1}(\hat{\mathbf{p}}_1' \cdot \hat{\mathbf{p}}_1) P_{l_2}(\hat{\mathbf{p}}_2' \cdot \hat{\mathbf{p}}_2)(2\pi)^3 \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2' - \mathbf{q})$$
$$\times (3 - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \frac{\mathbf{q}_1^{\text{ex}} \cdot \mathbf{q}_2^{\text{ex}}}{\left((\mathbf{q}_1^{\text{ex}})^2 + M_\pi^2\right)\left((\mathbf{q}_2^{\text{ex}})^2 + M_\pi^2\right)}$$

- Two-body current defines genuinely **new structure factor**
- Checked the oscillator model for 1b case
  $\hookrightarrow$ reproduces perfectly the $L = 0$ multipole



Scalar two-body currents: numerical estimates

- Early claims: could be as large as 60% Prézeau, Kurylov, Kamionkowski, Vogel 2003

$$\frac{\mathcal{A}_{\pi\pi}}{\mathcal{A}_{NN}} \simeq (0.21 \pm 0.08) r \frac{\mathcal{N}_{\pi\pi}}{A} \qquad r = \frac{S_u m_u + S_d m_d}{(S_u m_u + S_d m_d)\frac{\epsilon_s^0}{2} + \sum_{q=s,c,b,t} S_q m_q \epsilon_s^q}$$

- They find $1 < \frac{\mathcal{N}_{\pi\pi}}{A} < 2$ and consider r as large as 1.5. This brings you to 60%.

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- They find $1 < \frac{\mathcal{N}_{\pi\pi}}{A} < 2$ and consider r as large as 1.5. This brings you to 60%.
- Decomposition is actually scale dependent, they quote

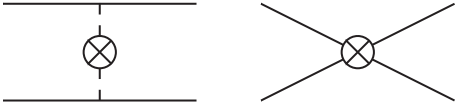
$$\epsilon_s^0 = \langle N|\bar{u}u + \bar{d}d|N\rangle \simeq 16 \pm 8$$

- Without such cancellations ($C_s^{SS} = C_g^{\prime S} = 0$) MH et al. 2016

$$2\frac{2f_\pi}{f_p + f_n}\frac{\mathcal{F}_\pi(0)}{A} = -9\%$$

- Claimed enhancement not at all related to power counting, but to cancellations in BSM parameter space
- To actually check power counting: **scalar current in light nuclei** + **lattice σ-terms** work in progress, NPLQCD 2013, 2018, Körber et al. 2017

Contact terms



- Scalar source suppressed for $(N^\dagger N)^2$

  $\hookrightarrow$ **long-range contribution dominant** (in Weinberg counting)

- Typical size **(5–10)%**

  $\hookrightarrow$ reflected by results for structure factors

  $\hookrightarrow$ more important in case of cancellations

- Contact terms do appear for other sources, e.g. θ^μ_μ

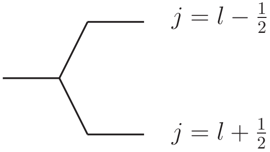
  $\hookrightarrow$ related to **nuclear binding energy** E_b

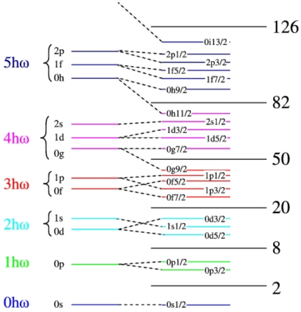
- Same structure factor in spin-2 two-body currents

Coherence effects

- Six distinct nuclear responses
  Fitzpatrick et al. 2012, Anand et al. 2013
  - $M \leftrightarrow \mathcal{O}_1 \leftrightarrow$ SI
  - $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow$ SD
  - $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
  - $\Delta, \tilde{\Phi}'$: not coherent
- **Quasi-coherence** of Φ''
  - Spin-orbit splitting
  - Coherence until mid-shell
  - About 20 coherent nucleons in Xe
  - Interference M–$\Phi'' \leftrightarrow \mathcal{O}_1$–$\mathcal{O}_3$
- Coherent 2b currents:
  - Scalar $\propto N + Z$
  - Vector $\propto N - Z$

$\hookrightarrow$ concentrate on scalar case





Spin-2 and coupling to the energy-momentum tensor

- Effective Lagrangian truncated at dim-7, but if WIMP heavy $m_\chi/\Lambda = \mathcal{O}(1)$

  $\hookrightarrow$ heavy-WIMP EFT Hill, Solon 2012, 2014

$$\mathcal{L} = \frac{1}{\Lambda^4}\bigg\{ \sum_q C_q^{(2)} \bar{\chi}\gamma_\mu i\partial_\nu \chi \frac{1}{2}\bar{q}\Big(\gamma^{\{\mu} iD_-^{\nu\}} - \frac{m_q}{2} g^{\mu\nu}\Big) q + C_g^{(2)} \bar{\chi}\gamma_\mu i\partial_\nu \chi \Big(\frac{g_{\mu\nu}}{4} G^a_{\lambda\sigma} G_a^{\lambda\sigma} - G_a^{\mu\lambda} G^\nu_{a\lambda}\Big)\bigg\}$$

  $\hookrightarrow$ leading order: **nucleon pdfs**

  $\hookrightarrow$ similar two-body current as in scalar case, pion pdfs, EMC effect

- Coupling of trace anomaly θ^μ_μ to $\pi\pi$

$$\theta^\mu_\mu = \sum_q m_q \bar{q}q + \frac{\beta_{\text{QCD}}}{2g_s} G^a_{\mu\nu} G_a^{\mu\nu} \quad \Leftrightarrow \quad \langle \pi(p')|\theta_{\mu\nu}|\pi(p)\rangle = p_\mu p'_\nu + p'_\mu p_\nu + g_{\mu\nu}(M_\pi^2 - p\cdot p')$$

  $\hookrightarrow$ probes gluon Wilson coefficient C_g^S