Including direct detection likelihoods in global fits

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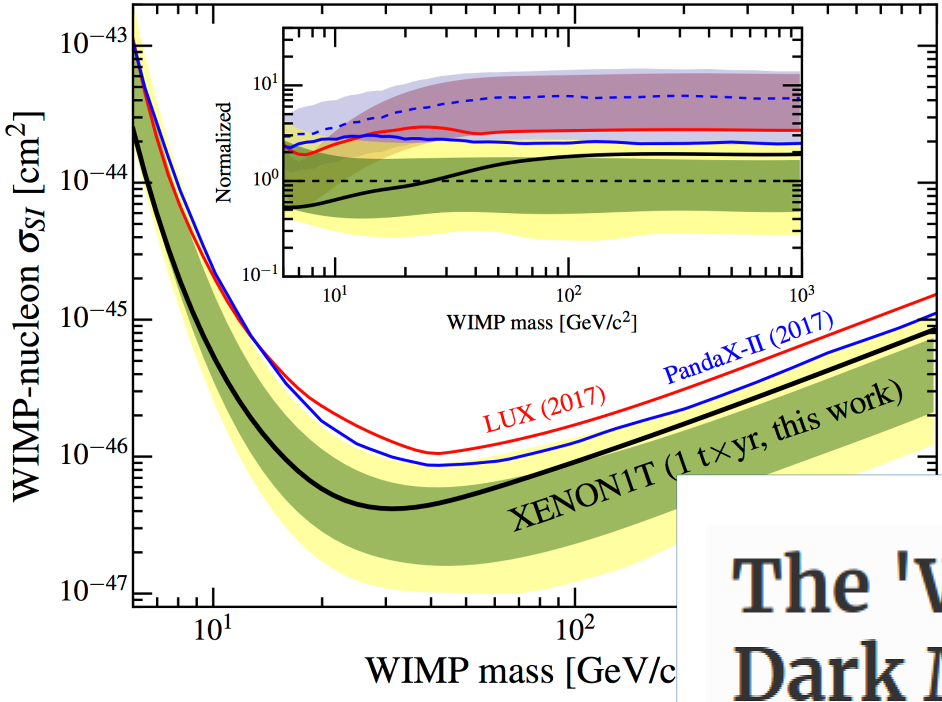
Where are the WIMPs?



So far we have not observed any conclusive evidence for WIMPs in direct, indirect or collider experiments

Frankfurter Allgemeine

Dunkle Materie

Wo sind die Wimps?

The 'WIMP Miracle' Hope For Dark Matter Is Dead

Forbes

SCIENTIFIC AMERICAN.

In the Dark about Dark Matter

Recent disappointments have physicists looking beyond WIMPs for dark matter particles

Is the WIMP idea in trouble?

To study the viability of WIMP models in a rigorous way, we need to

1) combine information from **many different data sets**
   - Cosmology
   - Astrophysics
   - Laboratory experiments
2) account for a **many different sources of uncertainties**
   - Astrophysical distributions
   - Experimental backgrounds
   - Detector calibration
   - Theoretical uncertainties
3) explore the parameter space of **many different WIMP models**

In short, we need a a very general and flexible global fitting framework!

GAMBIT

The Global And Modular BSM Inference Tool

- An **international community** with 40+ collaborators (10 experiments, 14 major theory codes)
- A **new software framework** for global fits developed over the past six years

- First **public code release** in May 2017, arXiv:1705.07908 (gambit.hepforge.org)
- So far **7 physics studies**:
  arXiv:1705.07917, arXiv:1705.07935
  arXiv:1705.07931, arXiv:1806.11281
  arXiv:1808.10465, arXiv:1809.02097,
  arXiv:1810.07192

  + many more in preparation

GAMBIT

- Apply **wide ranges of constraints** to a given model
  - Construction of composite likelihoods
  - Efficient scans of multi-dimensional parameter space
  - Consistent treatment of uncertainties and nuisance parameters

- Maximum of **flexibility and modularity** in terms of
  - Fast definition of new data sets and models
  - Plug and play of many popular theory tools* (dynamical adaptation to user's system)
  - Large database of models and observables (+ more to come)
  - Many statistical methods (frequentist & Bayesian)

- **Optimized** for parallel computing & fully open source

* GAMBIT supports backend codes in C/C++, Fortran, Python and Mathematica

GAMBIT modules

- A module provides GAMBIT with a range of capabilities (the ability to calculate a certain quantity)

- **DarkBit** (arXiv:1705.07920) – dark matter observables
- **ColliderBit** (arXiv:1705.07919) – collider observables (Higgs + SUSY searches from ATLAS, CMS, LEP)
- **FlavBit** (arXiv:1705.07933) – flavour physics ($g-2$, $b \to s\gamma$, B decays)
- **SpecBit** (arXiv:1705.07936) – RGE running, masses, mixings, …
- **DecayBit** (arXiv:1705.07936) – decay widths for all relevant particles
- **PrecisionBit** (arXiv:1705.07936) – SM likelihoods, electroweak precision tests
- **ScannerBit** (arXiv:1705.07959) – manages statistics, sampling and optimisation

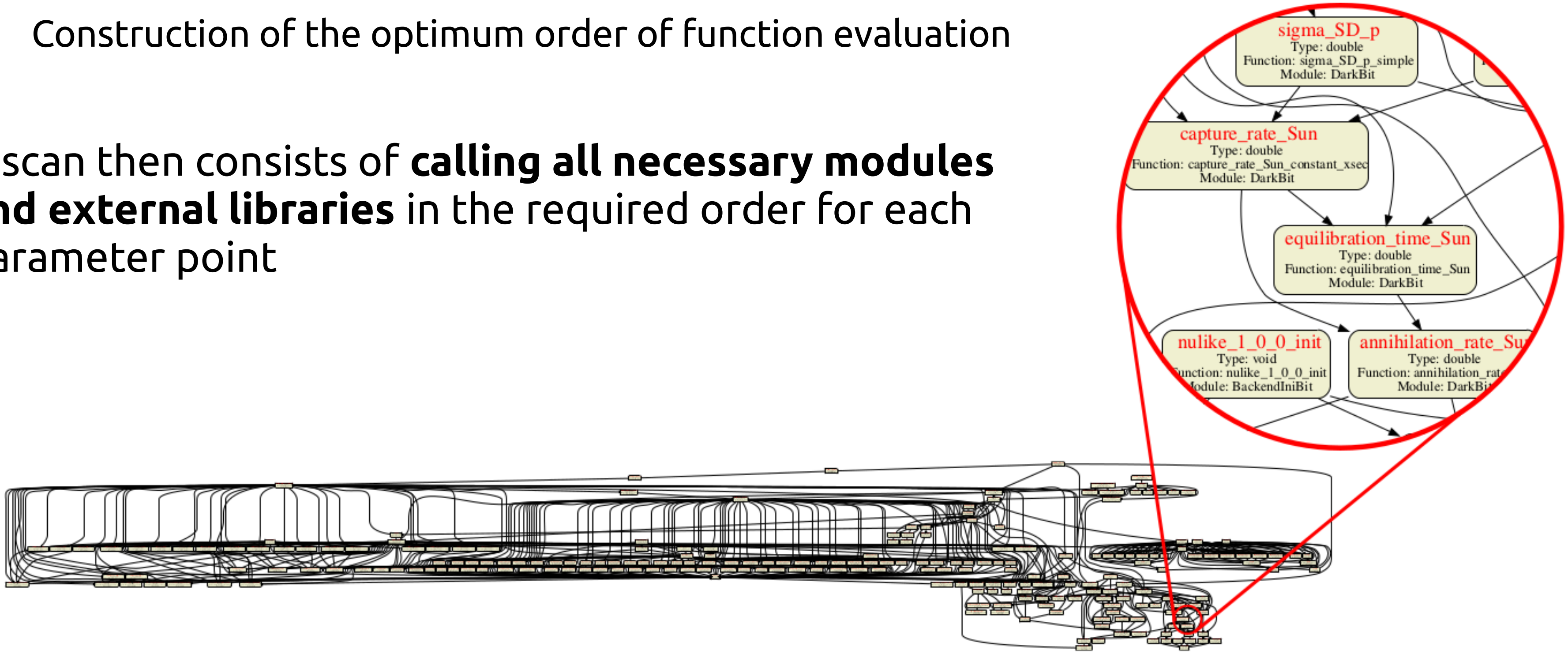
- Coming soon: **NeutrinoBit** & **CosmoBit**

How does GAMBIT work?

- User specifies the model, parameter space, observables and scanning technique

- GAMBIT then performs the **dependency resolution**
  - Identification of all functions necessary to calculate requested observables
  - Determination of the required inputs for each function
  - Construction of the optimum order of function evaluation

- A scan then consists of **calling all necessary modules and external libraries** in the required order for each parameter point



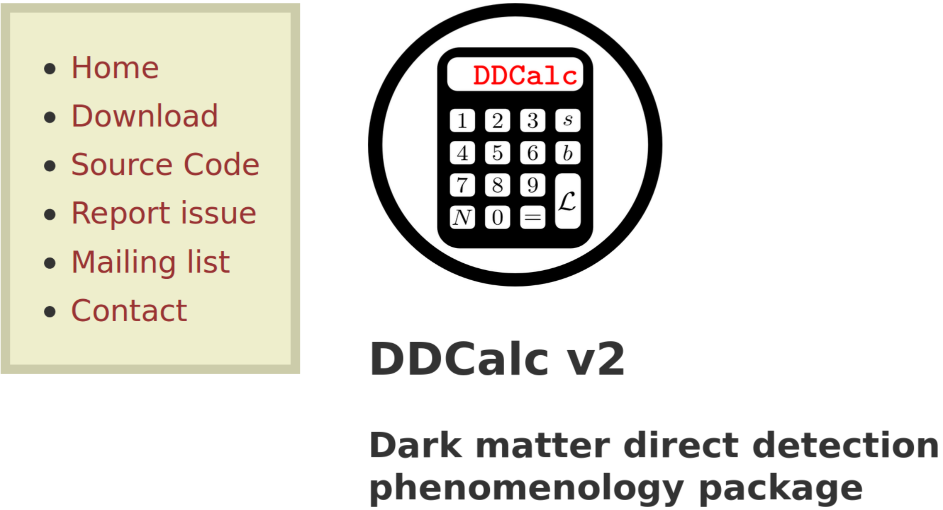
Direct detection likelihoods for global fits

- To incldue information from direct detection experiments, we need a code capable of calculating likelihoods of a given parameter points in the fraction of a second

- **Step 1:** Calculate **predicted physical recoil spectrum** for each isotope in target
  - Depends on assumed particle physics model and specific parameter point
  - Requires input from astrophysics (DM density & velocity distribution)
  - Important uncertainties from nuclear physics (nuclear matrix elements and form factors)

- **Step 2:** Calculate **predicted event rate** and compare with measurement
  - Convolute recoil spectrum with detector response
  - Integrate event rate for each signal region
  - Calculate likelihood based on signal prediction, background expectation and observation

DDCalc



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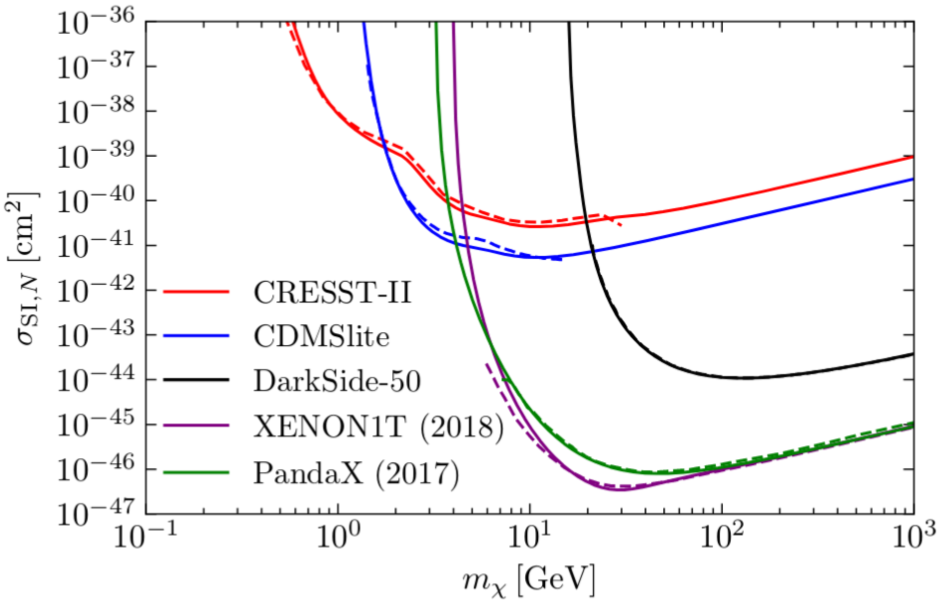
DDCalc v2

Dark matter direct detection phenomenology package

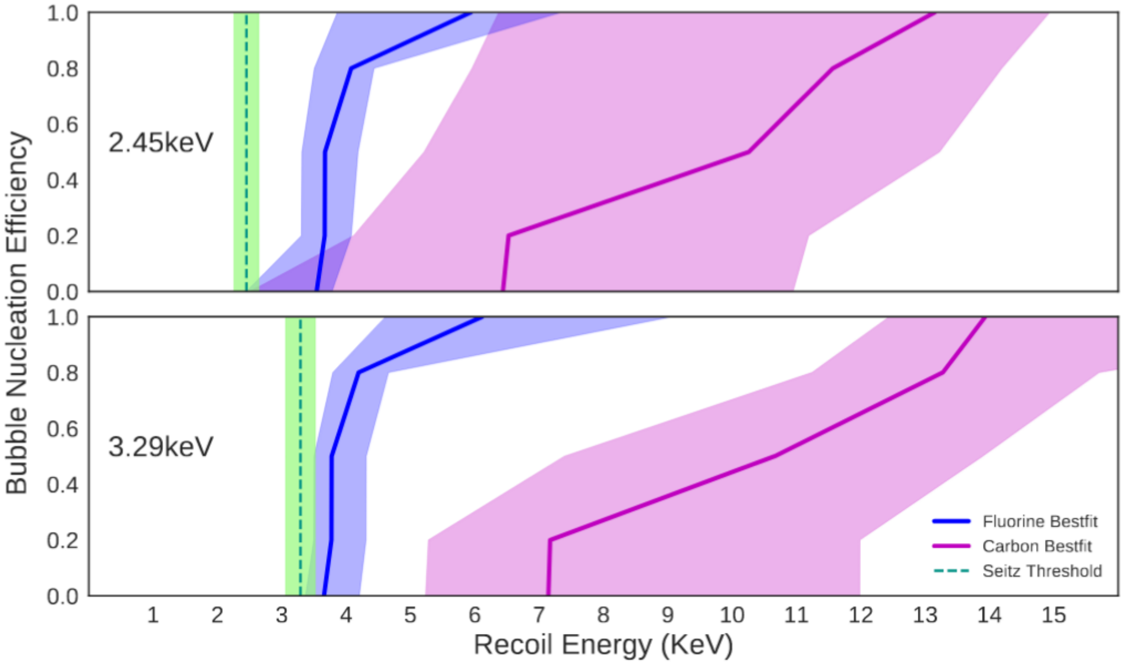


- Calculation of event rates and likelihoods for a **wide range of different particle physics models** (including the most general set of non-relativistic effective operators), different dark matter velocity distributions and nuclear form factors

- **User-friendly interface** (in Fortran, C and Python) and pre-compiled libraries for the use in external codes

Example 1: PICO

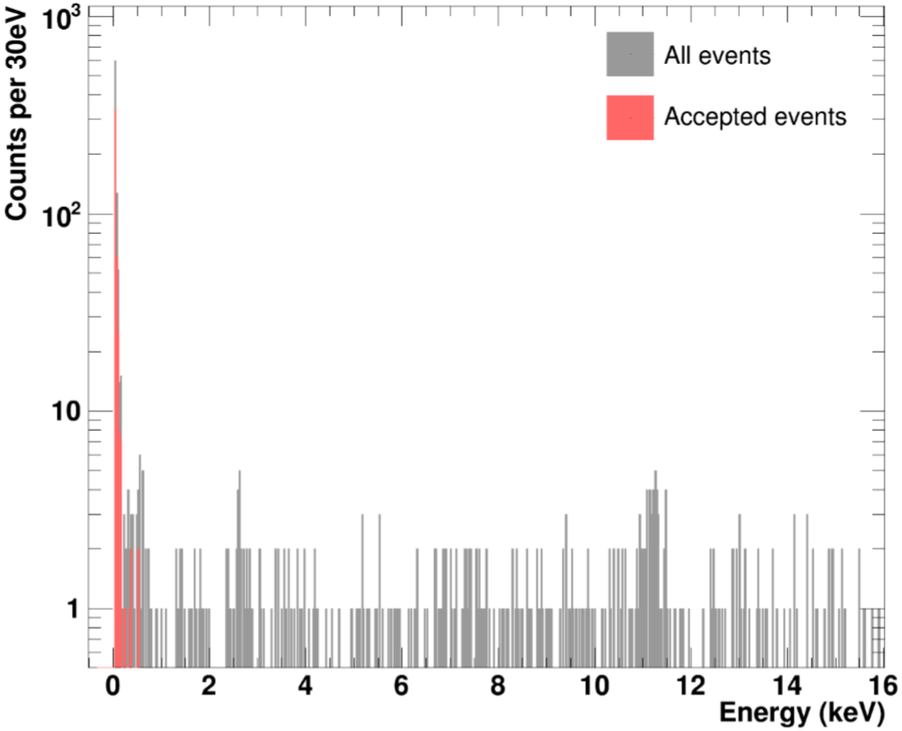
- No information on recoil energy recorded
- Only relevant input: **Acceptance function** (energy threshold)
- Multiply acceptance function with true recoil spectrum to obtain observable event rates



- Calculate total number of expected events and corresponding Poisson likelihood

$$\mathcal{L}_i(N_{\mathrm{p},i}|N_{\mathrm{o},i}) = \frac{(b_i + N_{\mathrm{p},i})^{N_{\mathrm{o},i}}\, e^{-(b_i+N_{\mathrm{p},i})}}{N_{\mathrm{o},i}!}$$

Example 2: CRESST-II



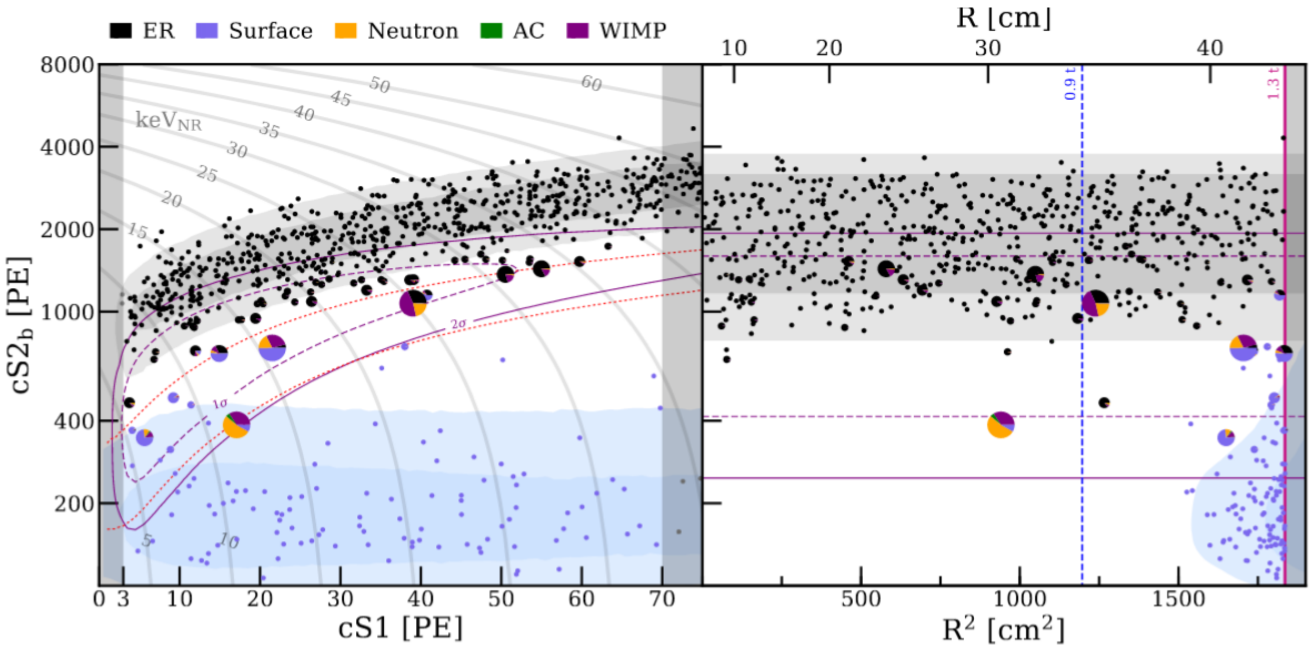
- Large number of observed events but very good energy resolution
- Define **large number of signal regions** (bins) and calculate Poisson likelihood in each bin
- Extra complication: **No background model**
  - Prediction < observation:

    No contribution to likelihood (log L = 0)
  - Prediction > observation:

    Calculate Poisson likelihood assuming no background
- Note: **Maximum gap method** does not give a likelihood → not useful for global fits

Example 3: XENON1T

- Background discrimination based on profile likelihood with three unbinned and one binned variable (and ~20 nuisance parameters)
- **Very challenging** to include all this information in global fits



- Current approach
  - Identify **small number of signal regions** with low background rates
  - Implement crude **Monte Carlo simulation** of the detector to estimate acceptance function
  - Estimate **background expectation** based on public information

Wishlist for the future

- To have a more realistic treatment, we plan to implement the **extended (i.e. unbinned) maximum likelihood method**
- Requires **differential acceptance functions and background likelihoods** (marginalised over nuisance parameters) for each observed event
- Impossible to extract this information from publicly available data
- We **need the help** of experimental collaborations!

- Another (less controversial?) option would be to continue with **binned likelihoods**, but substantially **increase the number of signal regions**
- Experimental collaborations could perform the **optimisation of these signal regions** themselves or together with us (using e.g. Fisher information)
- **Advantage:** Eliminates the “risk” that someone claims a DM signal based on public data (provided there is no significant excess in any signal region)

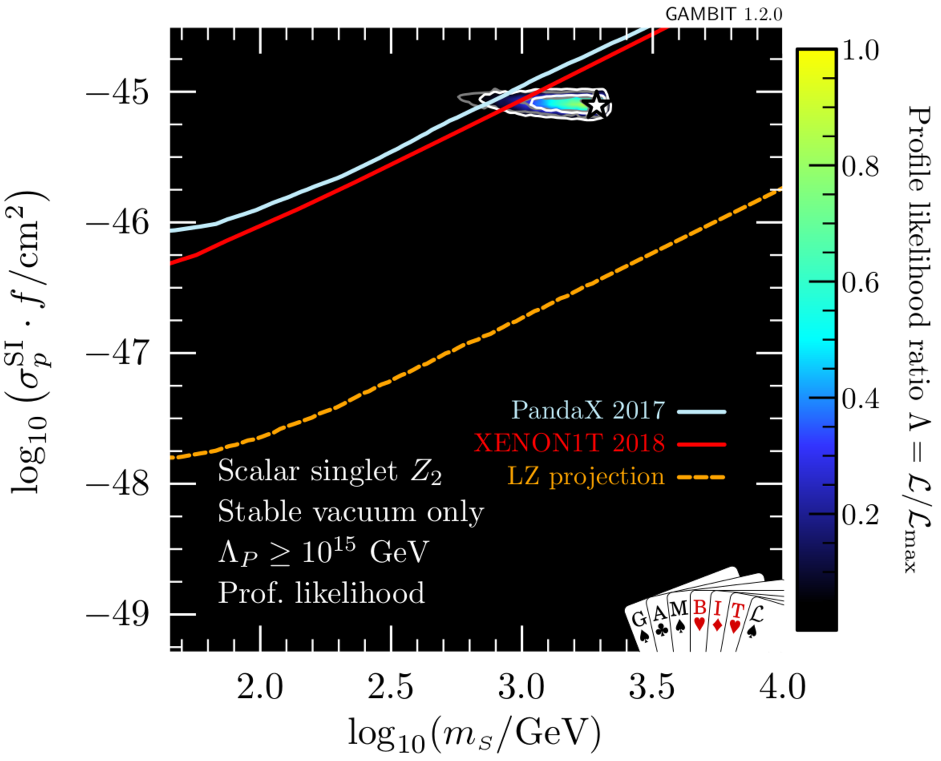
Application: Higgs portal dark matter models

- WIMPs that couple directly to the Standard Model *Z*-boson have long been ruled out experimentally
- What abound WIMPs that **couple to the Standard Model Higgs boson**?
- Three possibilities:
    - Scalar DM particles: $\frac{1}{2}\lambda_{hS} S^2 |H|^2$
    - Fermionic DM particles: $\frac{1}{2}\frac{\lambda_{h\chi}}{\Lambda_\chi}\left(\cos\theta\,\overline{\chi}\chi + \sin\theta\,\overline{\chi}i\gamma_5\chi\right)H^\dagger H$
    - Vector DM particles: $\frac{1}{2}\lambda_{hV} V_\mu V^\mu H^\dagger H$

Scalar Higgs portals

- Higgs portal coupling is dimensionless → model **fully renormalisable**

- Scalar Higgs portal models remain valid and perturbative up to the Planck scale (at least in some regions of parameter space)



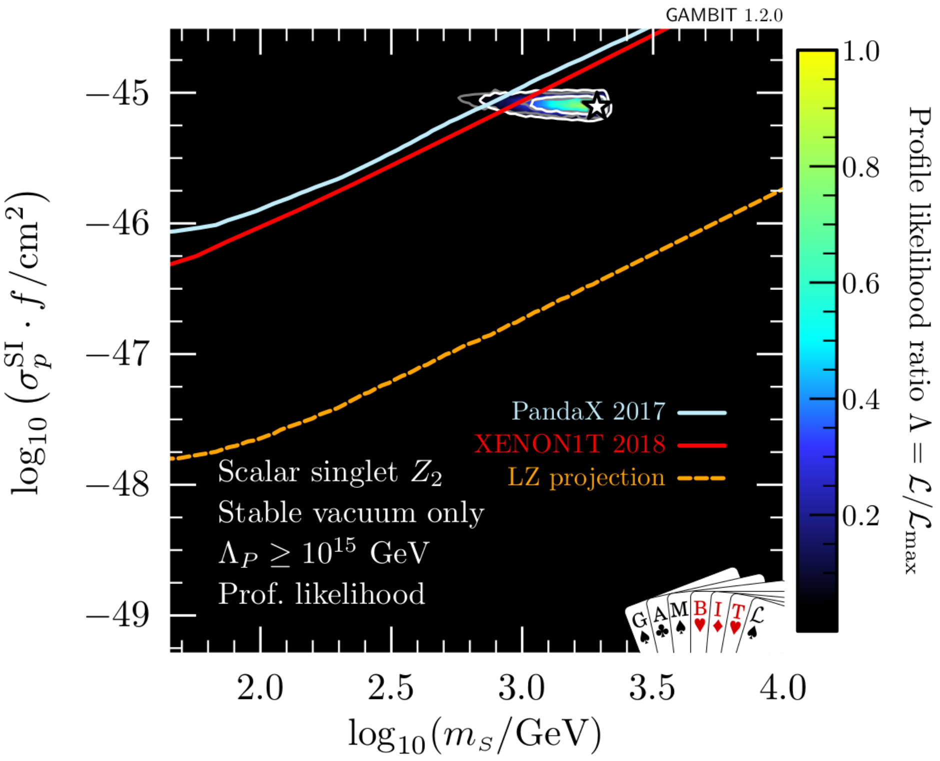
- The scalar DM particle prevents the Higgs self-coupling from running to negative values and thus **stabilises the electroweak vacuum**

- Small remaining parameter region where scalar singlets **can be all of DM**, evade all experimental constraints and stabilise the vacuum

Challenge: Goodness-of-fit

- Many likelihoods are **difficult to normalise**
- Cannot directly interpret **value of global likelihood** at best-fit point
- Very difficult to use Monte Carlo to study distribution of likelihood values (would require a global fit for each mock data set)
- Current strategy: **"Parameter goodness-of-fit"**
  - For each likelihood calculate ratio of its value at global best-fit point and its maximum
  - Result gives an estimate for the tension between data sets

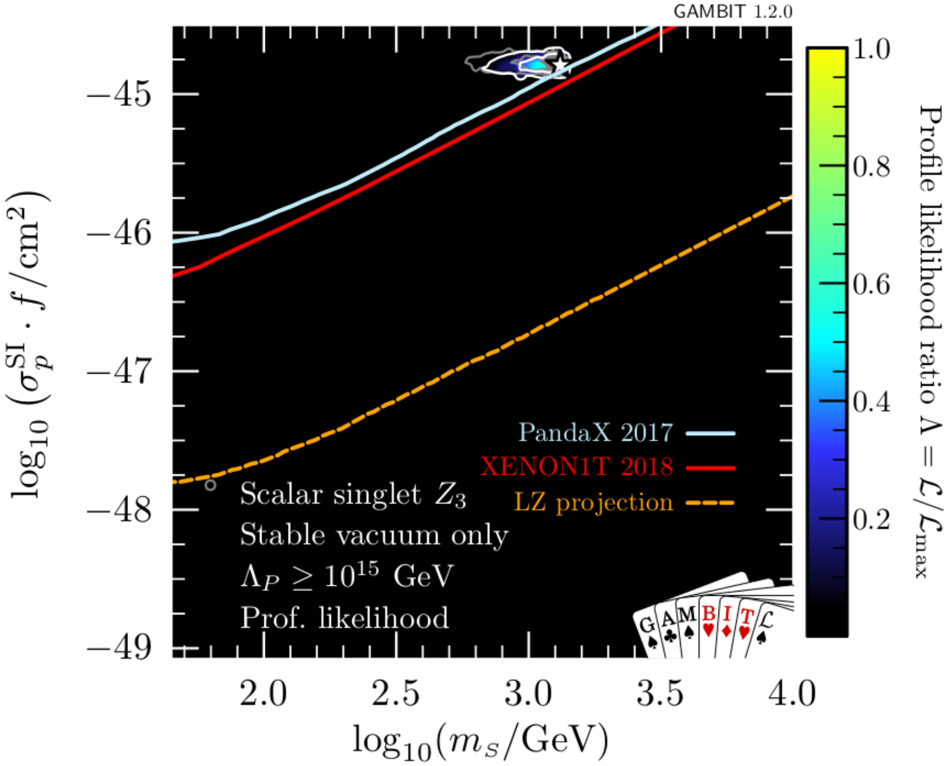
Scalar Higgs portals



Likelihood contribution	Ideal	$\Delta \ln \mathcal{L}$
Relic density	5.989	0.120
LUX Run II 2016	−1.467	0.207
PandaX 2016	−1.886	0.131
PandaX 2017	−1.550	0.280
XENON1T 2018	−3.440	0.179
γ rays (*Fermi*-LAT dwarfs)	−33.244	0.170
Higgs invisible width	0.000	0
Hadronic elements σ_s, σ_l	−6.625	0.019
Local DM density ρ_0	1.142	0.101
DM velocity v_0	−2.998	0.001
DM escape velocity v_{esc}	−4.474	0.005
α_s	5.894	0.002
Higgs mass	0.508	0.043
Top quark mass	−0.645	0.196
Vacuum stability	0.000	0
Total		1.455

Scalar Higgs portals

- Same analysis with a complex scalar instead of a real scalar



Likelihood contribution	Ideal	$\Delta \ln \mathcal{L}$
Relic density	5.989	0.142
LUX Run II 2016	−1.467	0.592
PandaX 2016	−1.886	0.380
PandaX 2017	−1.550	0.752
XENON1T 2018	−3.440	1.770
γ rays (*Fermi*-LAT dwarfs)	−33.244	0.207
Higgs invisible width	0.000	0
Hadronic elements σ_s, σ_l	−6.625	0.043
Local DM density ρ_0	1.142	0.499
DM velocity v_0	−2.998	0.013
DM escape velocity v_{esc}	−4.474	0
α_s	5.894	0.001
Higgs mass	0.508	0.004
Top quark mass	−0.645	0.041
Vacuum stability	0.000	0
Total		4.443

From likelihoods to p-values

- Far from clear that the test statistic obtained in this way **follows a χ^2 distribution**
- Even if asymptotics can be assumed, difficult to **estimate degrees of freedom**
- Naively: D.o.f. = number of experimental measurements – number of parameters
- In practice, only **very few experiments** have sensitivity to the best-fit point
- Conversely, some experimental constraints do not depend on all model parameters
- **Calculation of p-value difficult!**

Alternative: Bayesian model comparison

- We can compare different models by calculating **Bayesian evidences**:

$$\mathcal{Z}(\mathcal{M}) \equiv \int \mathcal{L}(D|\theta) P(\theta)\, d\theta$$

Prior distribution of θ

Likelihood of data D given parameter θ

- If the data *D* is in good agreement with the typical expectation for model *M*, the evidence will be large, otherwise it will be reduced

- We can then calculate the **odds ratio** between two different models M_1 and M_2:

$$\frac{P(\mathcal{M}_1|D)}{P(\mathcal{M}_2|D)} = \frac{\mathcal{Z}(\mathcal{M}_1)}{\mathcal{Z}(\mathcal{M}_2)} \frac{P(\mathcal{M}_1)}{P(\mathcal{M}_2)}$$

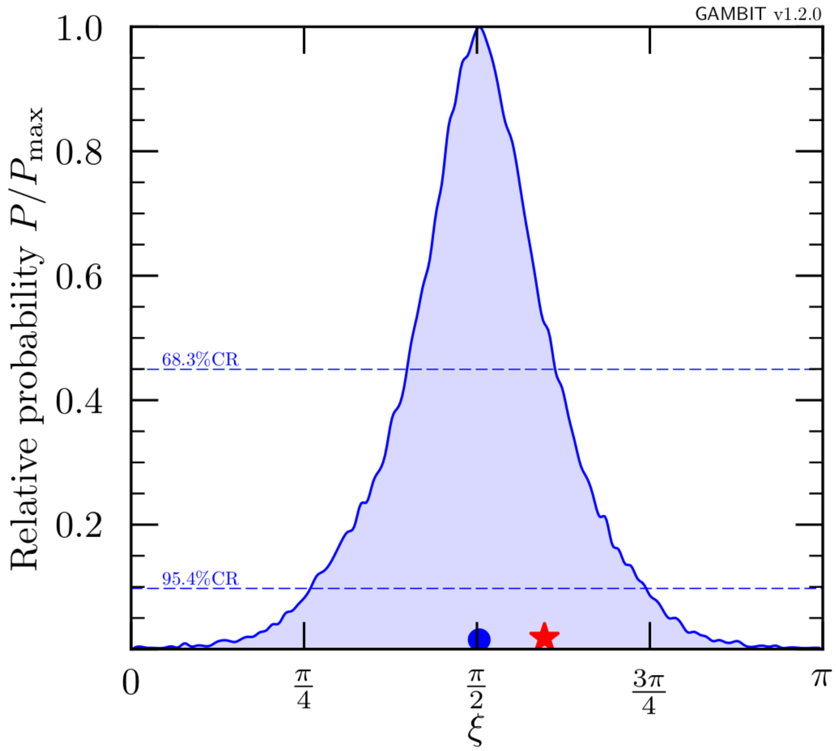
Prior beliefs

Example: Fermionic Higgs portal

$$\frac{1}{2}\frac{\lambda_{h\chi}}{\Lambda_\chi}\left[\cos\xi\,\overline{\chi}\chi+\sin\xi\,\overline{\chi}i\gamma_5\chi\right]\left(v_0 h+\frac{1}{2}h^2\right)$$

- Novel feature: The model contains a **phase ξ**
  - For $\xi = 0$ the model is CP-conserving, for $\xi \neq 0$ **CP is violated**
  - For $\xi \to \pi/2$ direct detection cross sections are strongly suppressed

$$\frac{d\sigma_{\rm SI}^X}{dq^2}=\frac{1}{v^2}\left(\frac{\lambda_{hX}}{\Lambda_X}\right)^2\frac{A^2F^2(E)f_N^2m_N^2}{4\pi m_h^4}\times\left(\cos^2\xi+\frac{q^2}{4m_X^2}\sin^2\xi\right),$$



Bayesian model comparison with GAMBIT

- Global fitting frameworks are ideally suited for calculating Bayesian evidences and performing model comparison

- If we take equal prior probability for the CP-conserving and CP-violating model, we find **strong evidence against** the CP-conserving case

- Since the CP-conserving model is nested inside the CP-violating model, this result is largely prior-independent

Model	Comparison model and priors			Odds
$\xi = 0$	m_χ: log	$\lambda_{h\chi}/\Lambda_\chi$: log	ξ: flat	70:1
$g_\mathrm{P}/\Lambda_\mathrm{P} = 0$	m_χ: log	$g_\mathrm{s}/\Lambda_\mathrm{s}$: log	$g_\mathrm{P}/\Lambda_\mathrm{P}$: log	140:1

- **Conclusion: Experimental constraints are pushing us away from the simplest (and most appealing?) WIMP models and towards models with more complexity**

Conclusions

- **Global fits** can provide answers to some of the **most pressing questions** of particle physics (“Have WIMPs been ruled out?”)

- To do so, we need to **construct fast global likelihood functions** that can be used to explore the parameter space of different models

- For many direct detection experimentss this can be done with DDCalc, but more work is needed to stay up to date with most **recent experimental developments**

- Example: Higgs portal model
  - Scalar Higgs portal still has **allowed parameter space** (where the electroweak vacuum can be stabilised), but the p-value of the best-fit point is **difficult to estimate**
  - A Bayesian analysis of the fermionic Higgs portal reveals the **preference for more complex DM models** with new ways of evading direct detection constraints

Backup

Higgs portal models: Experimental constraints

- Constraints
  - Relic density (underabundance OK)
  - LHC: Invisible Higgs decays
  - Direct detection: XENON1T, PandaX, …
  - Indirect detection: Fermi-LAT (dwarfs)
  - IceCube solar neutrinos
  - Perturbativity

- Uncertainties / nuisance parameters
  - Local DM density
  - DM velocity distribution
  - Nuclear physics parameters
  - Quark masses
  - Higgs mass
  - Gauge couplings

Comparing different Higgs portal models

- It is also possible to compare **Bayesian evidences for non-nested models**
- For example, we can calculate the odds ratios in favour of the scalar Higgs portal:

Model	Parameters and priors			Odds
S	m_S: log	λ_{hS}: log		1:1
V_μ	m_V: log	λ_{hV}: log		6:1
χ	m_χ: log	$\lambda_{h\chi}/\Lambda_\chi$: log	ξ: flat	1:1
ψ	m_ψ: log	$\lambda_{h\psi}/\Lambda_\psi$: log	ξ: flat	1:1

- We find no strong preference between fermionic and scalar Higgs portal
- The vector Higgs portal model is slightly disfavoured, but we expect this result to depend somewhat on the choice of priors