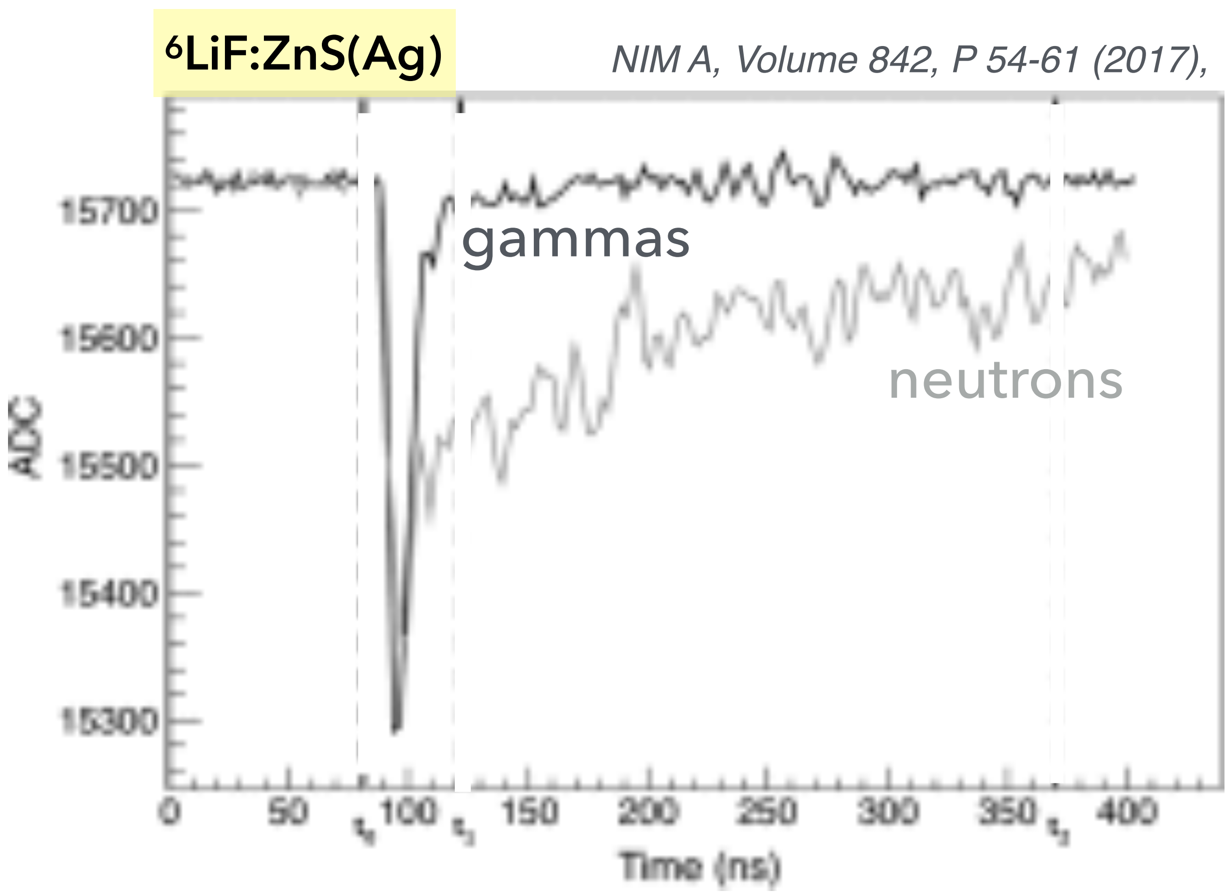
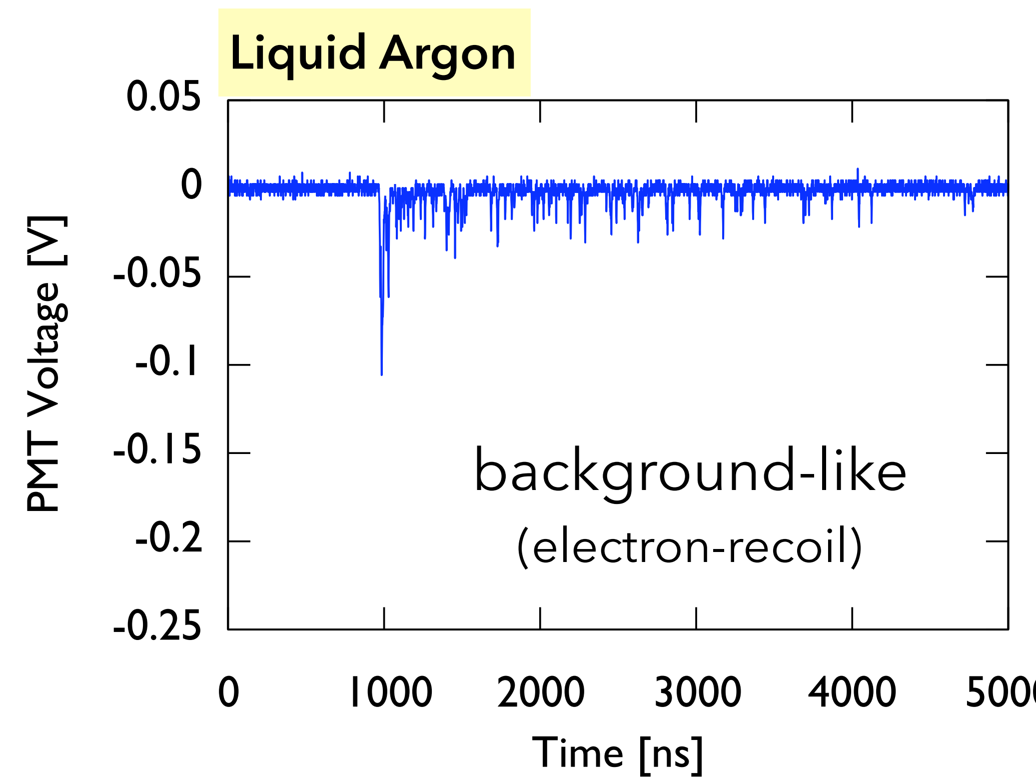
A statistical model for the leakage of backgrounds mitigated by pulseshape discrimination methods

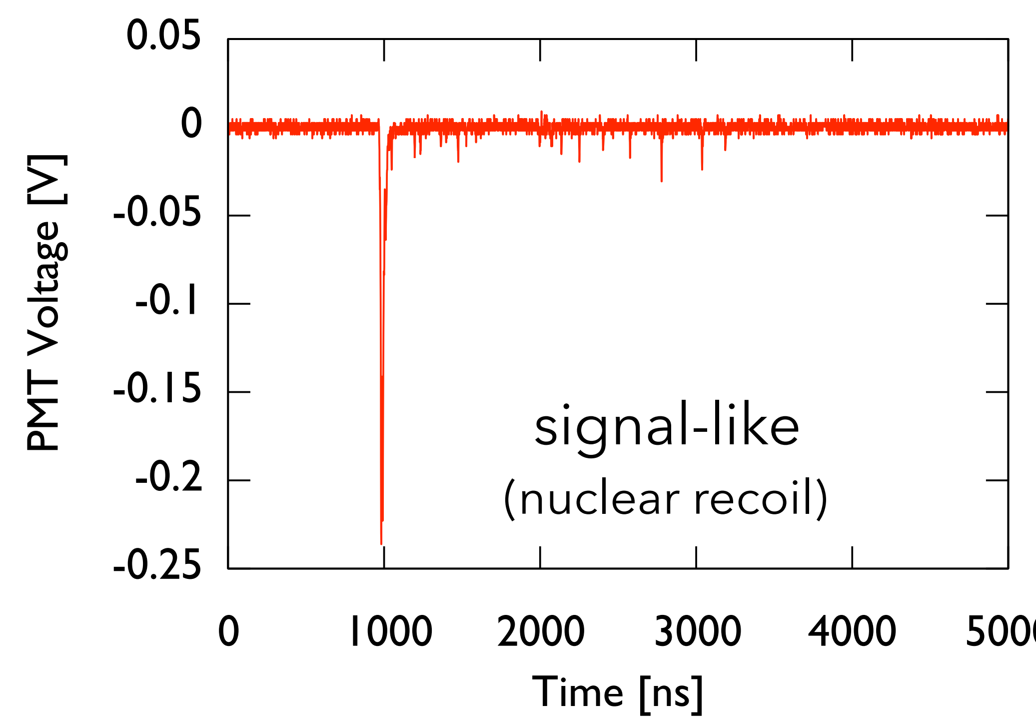
Tina Pollmann for the DEAP collaboration
PHYSTAT Dark Matter 2019
Stockholm Jul 31st to Aug 2nd 2019

Most scintillation detectors allow some discrimination between event classes based on the scintillation pulse shape.

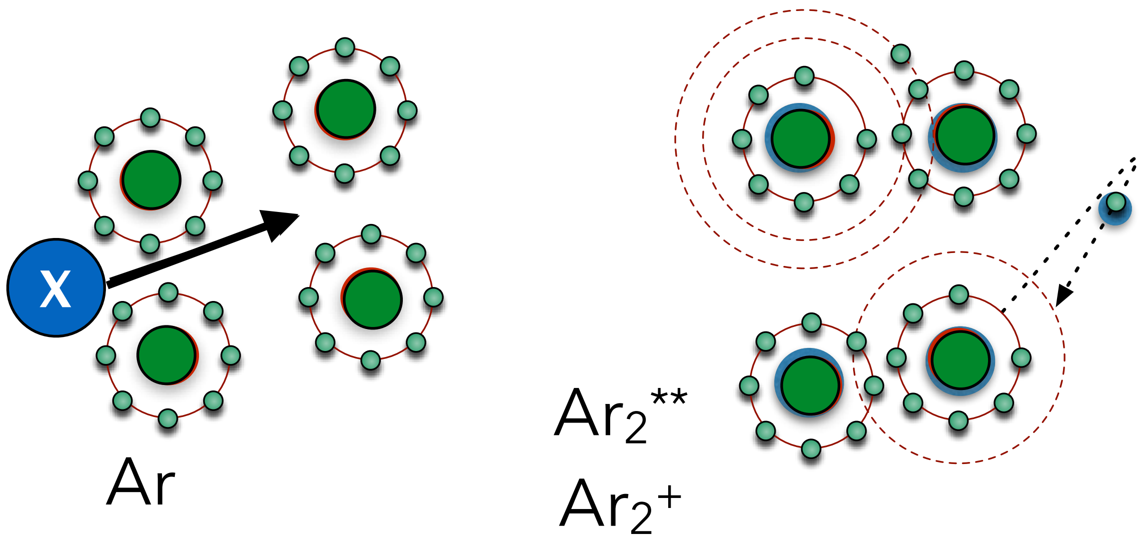


A fast and a slow decaying state are produced in abundances that differ by interaction type.





In liquid noble gas targets, the pulseshape is created by a singlet and a triplet excimer state with different lifetimes and relative abundance determined by the event class.



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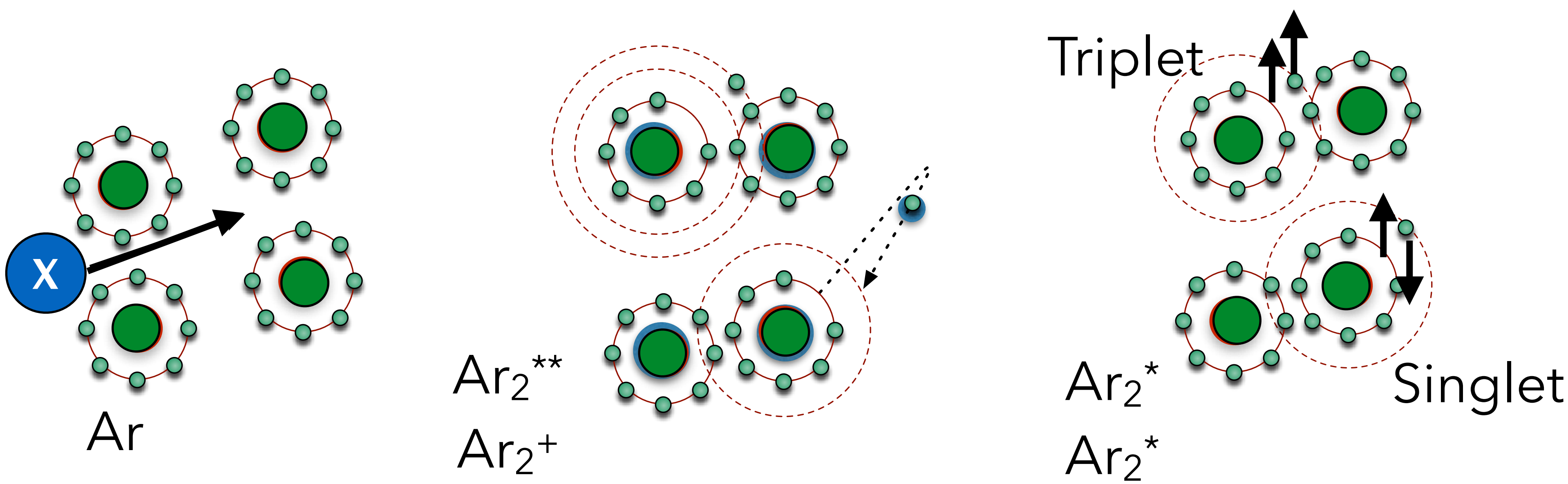

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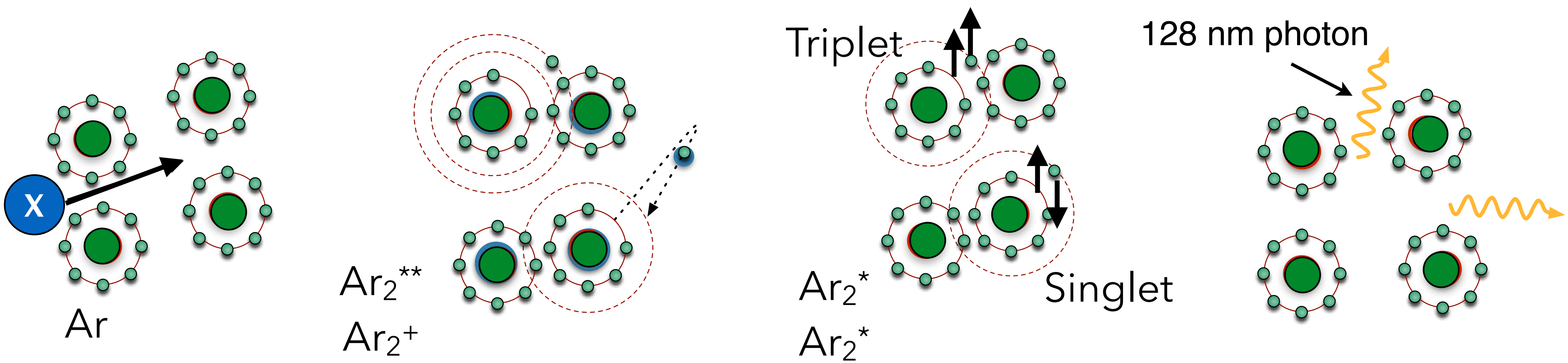

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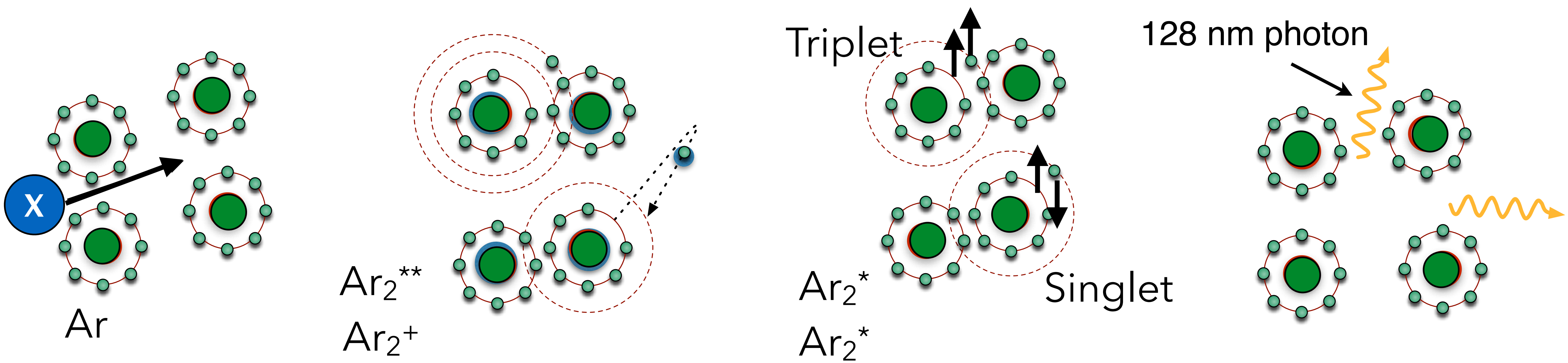

Discrimination relies on estimating the singlet/triplet ratio of each event. The larger the lifetime difference, the more reliable the estimate is(*).

Element/ Lifetime	τ singlet	τ triplet
Neon	<18 ns	14900 ns
Argon	6 ns	1400 ns
Xenon	4 ns	22 ns

(*)Neglecting some detector effects.

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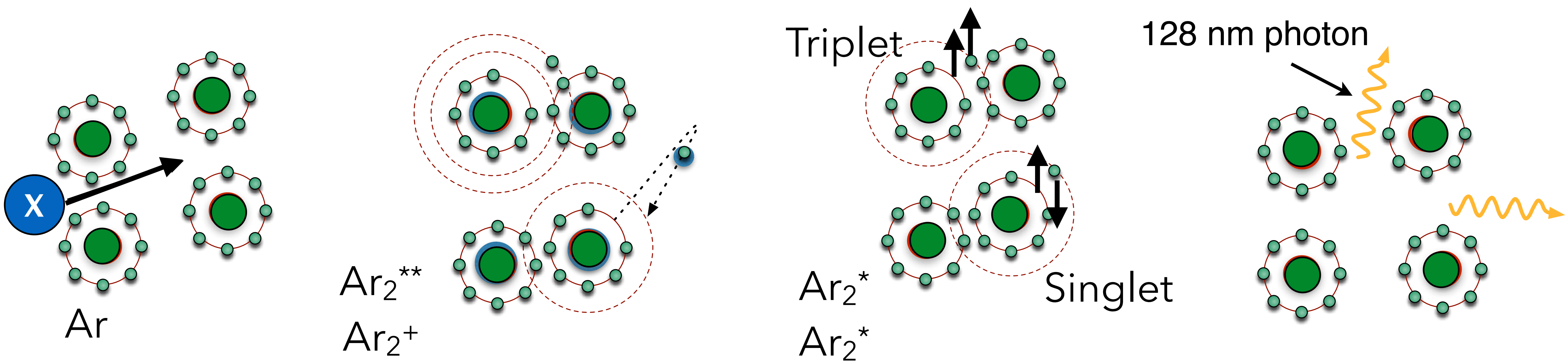

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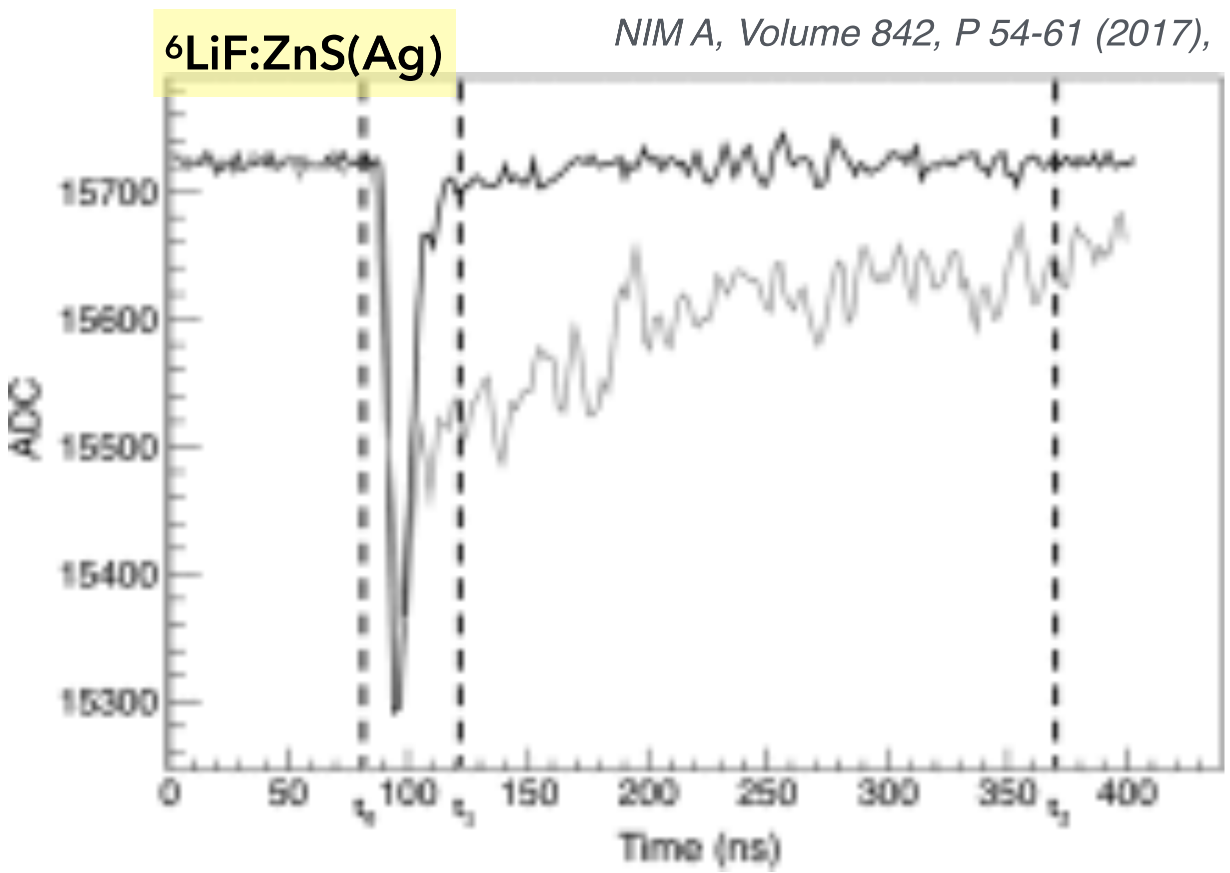
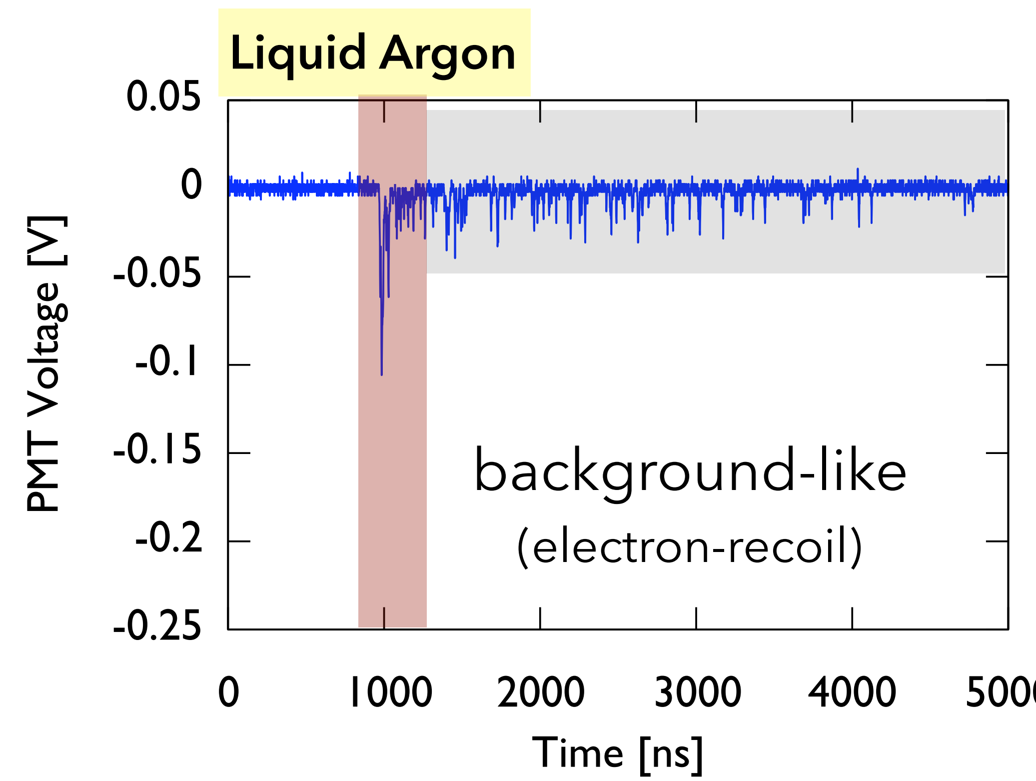
Element/ Lifetime	τ singlet	τ triplet	
Neon	<18 ns	14900 ns	PSD-only works
Argon	6 ns	1400 ns	PSD-only works
Xenon	4 ns	22 ns	need to extract ionization electrons to discriminate background

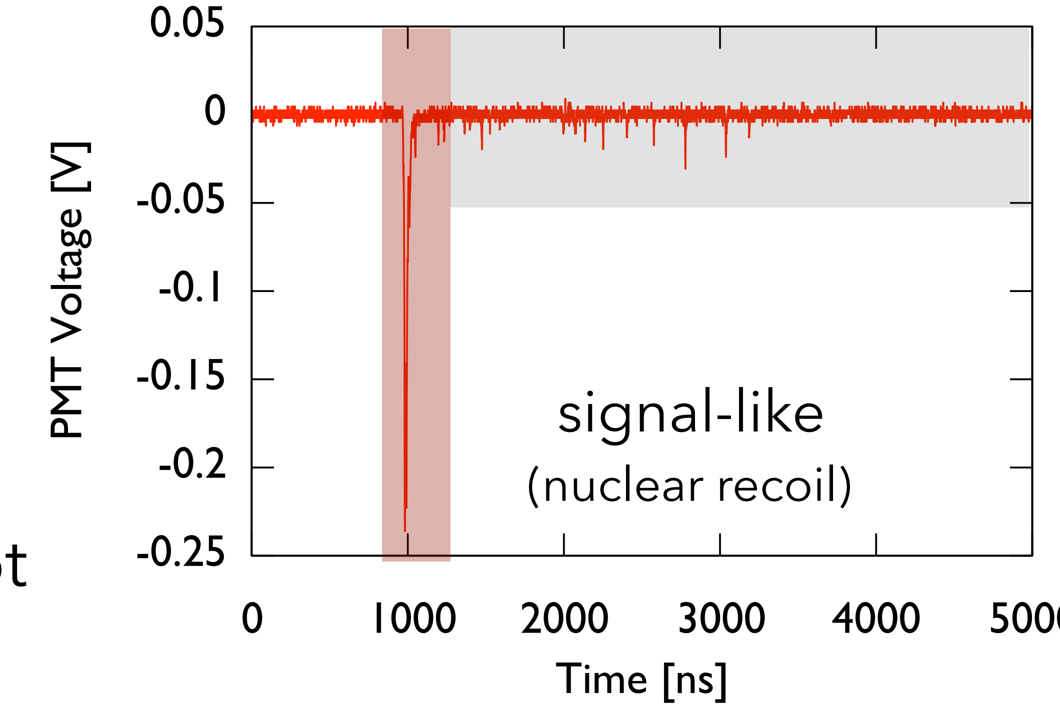
(*)Neglecting some detector effects.

A very common class of pulse shape discrimination parameters (PSP) are prompt- or late-fraction based.

NIM A, Volume 842, P 54-61 (2017),







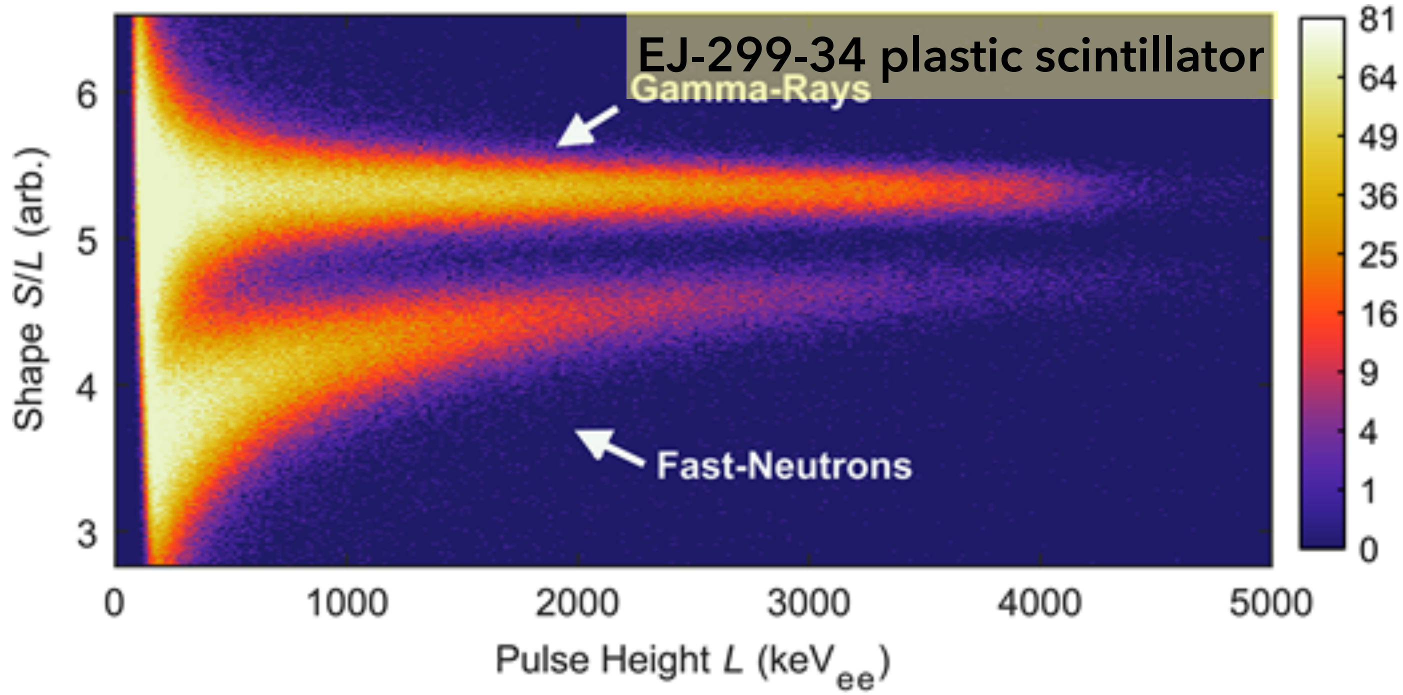
A fast and a slow decaying state are produced in abundances that differ by interaction type.

$$\mathrm{PSP} = Q_{tail}/Q_{full}.$$

$$\mathrm{PSP} = n_{\mathrm{prompt}}/(n_{\mathrm{prompt}} + n_{\mathrm{late}}) = \mathrm{fprompt}$$

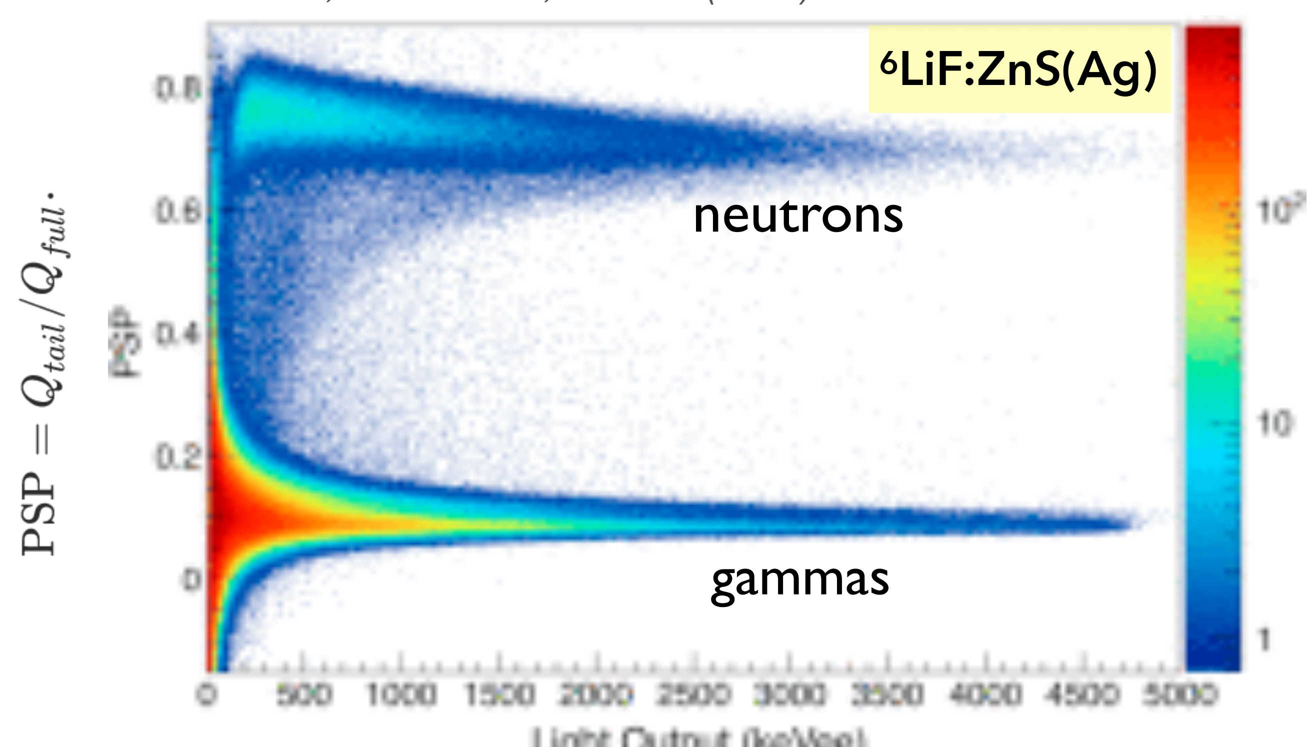
A very common class of pulse shape discrimination parameters (PSP) are prompt- or late-fraction based.

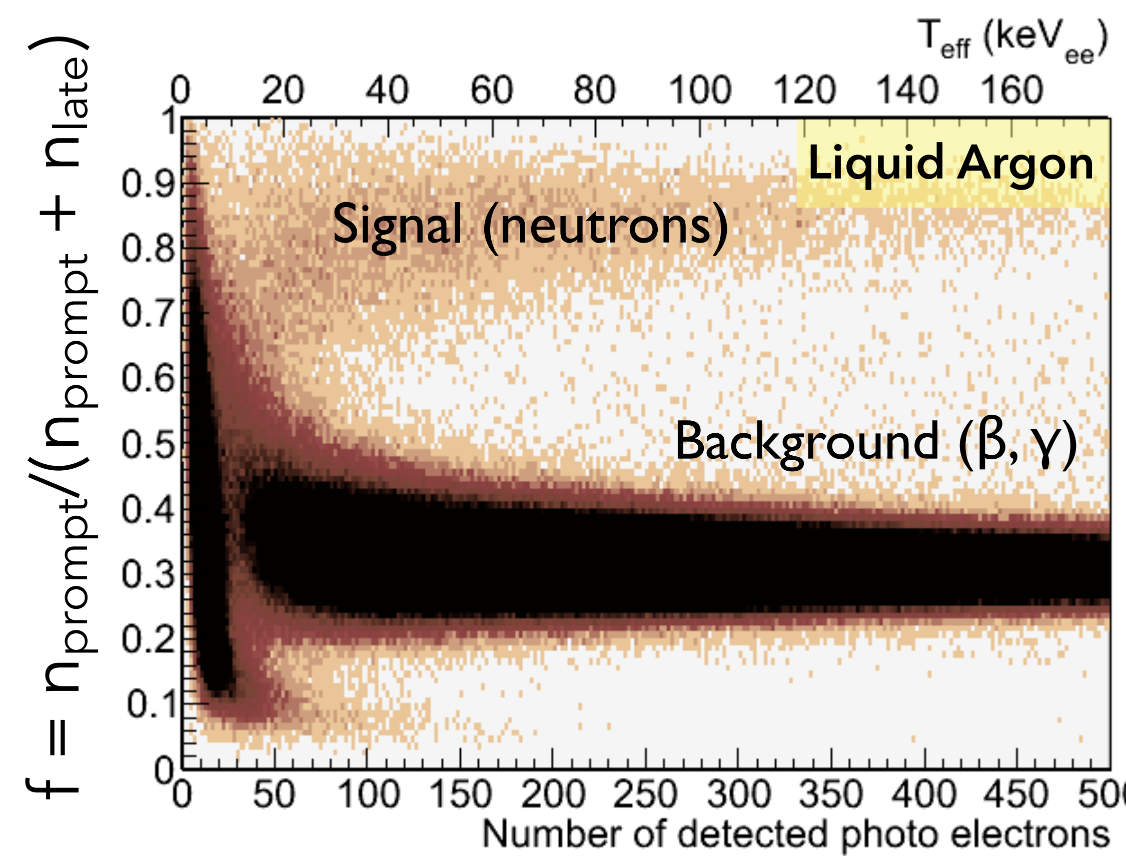
Radiation Protection Dosimetry, Volume 175, Issue 3, P 406–412 (2017)



PSD parameter (PSP) as a function of energy.

NIM A, Volume 842, P 54-61 (2017)

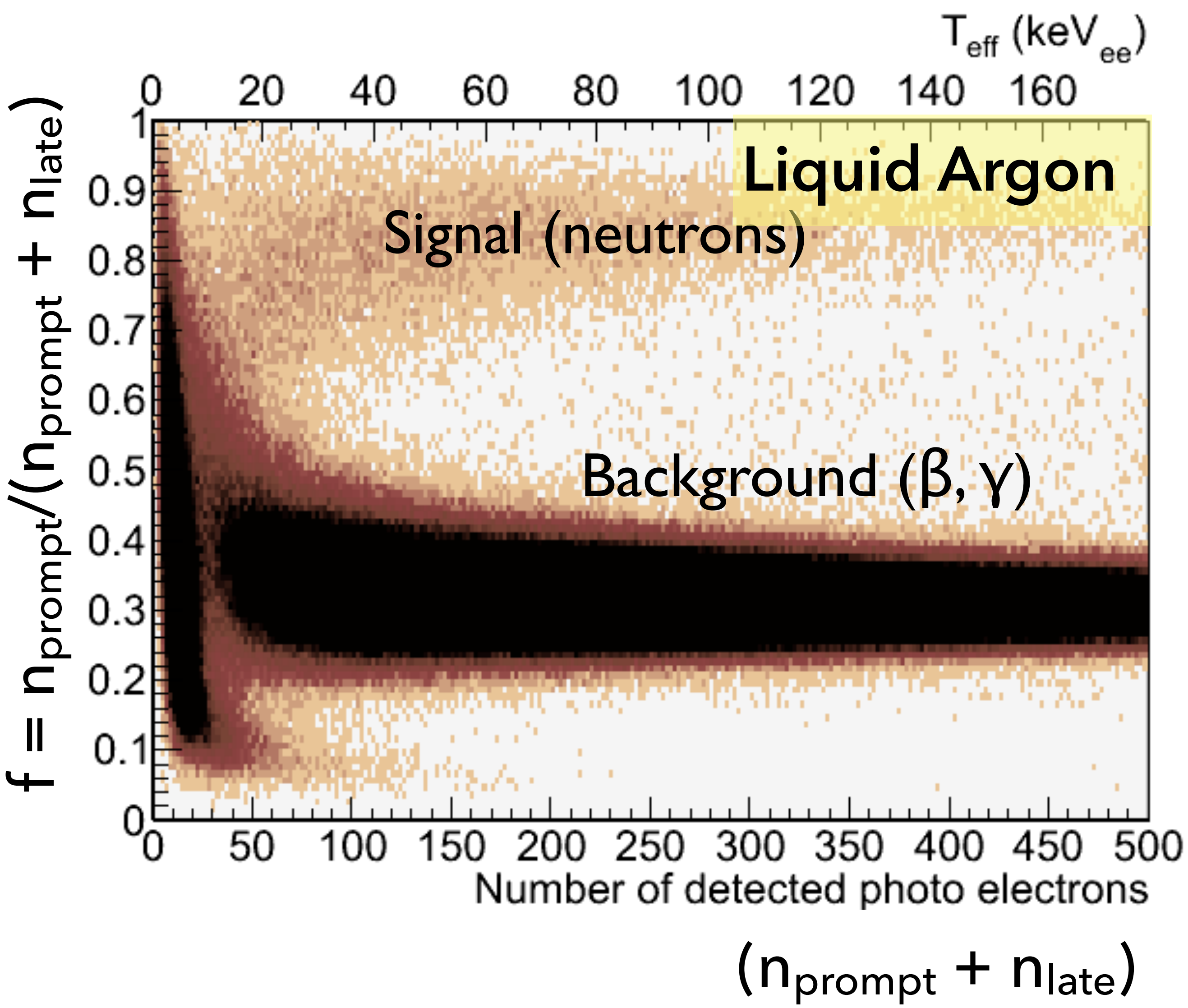


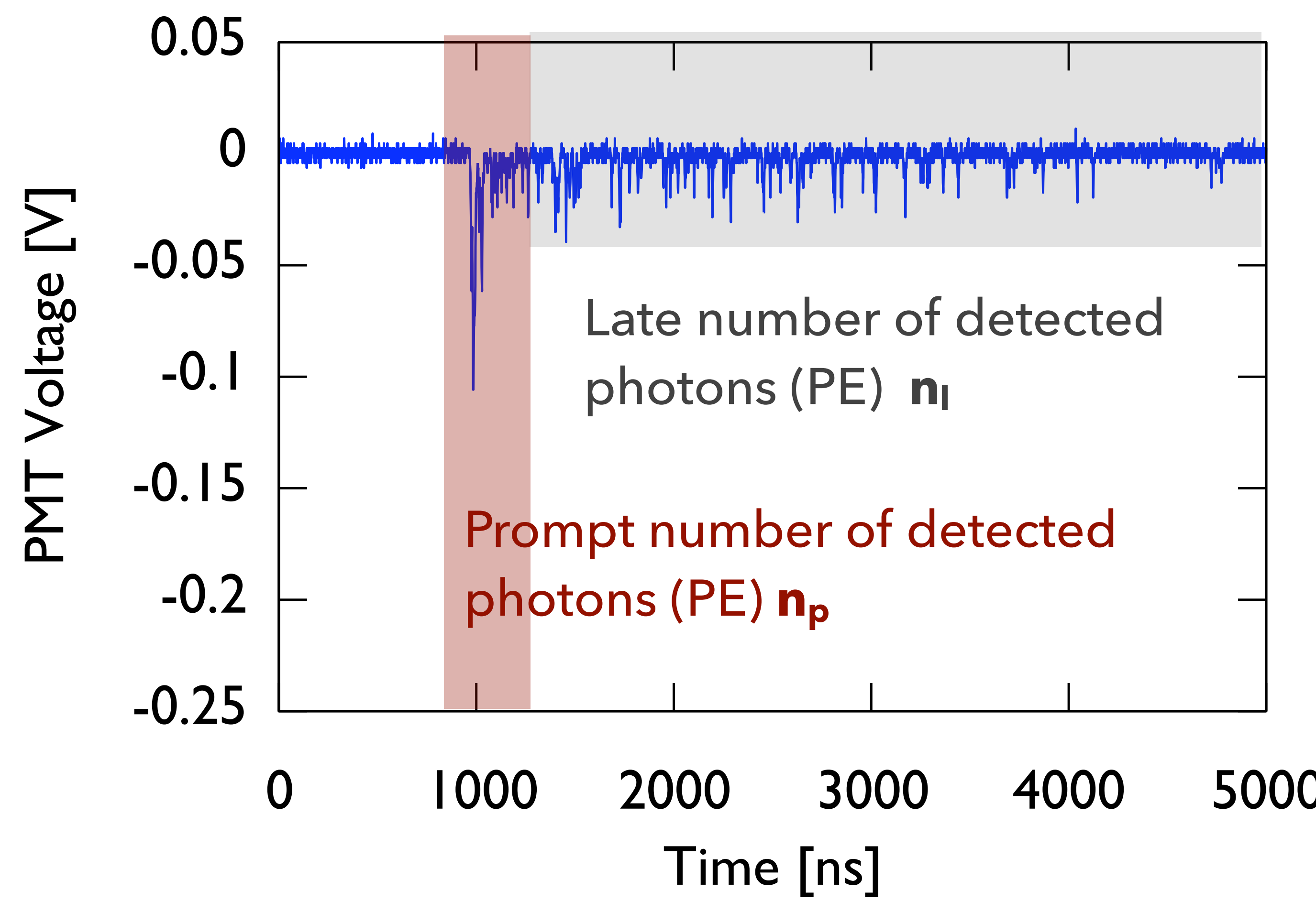


Astroparticle physics, V 85 (2016)

This talk will focus on the prompt/total PSP;

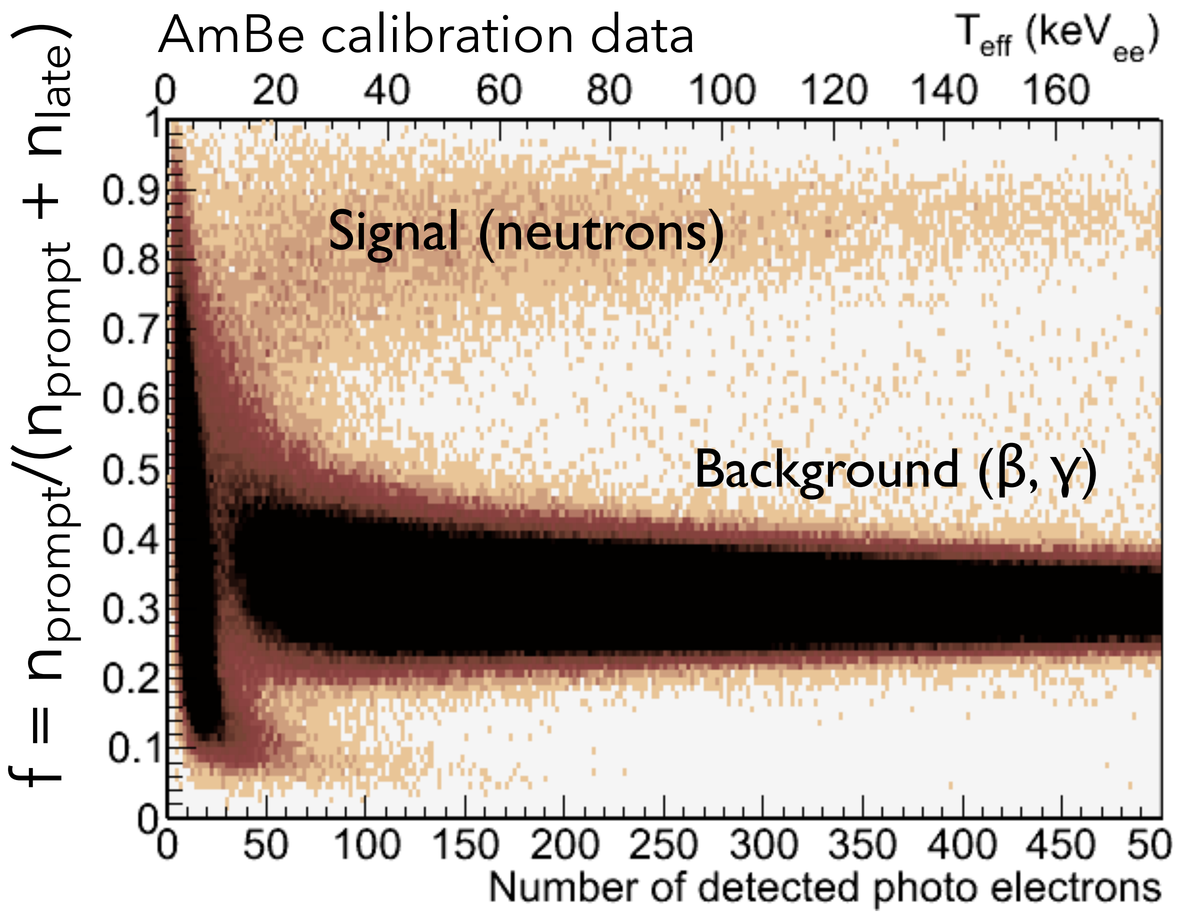
the math for other combinations is left as an excercise for the reader.





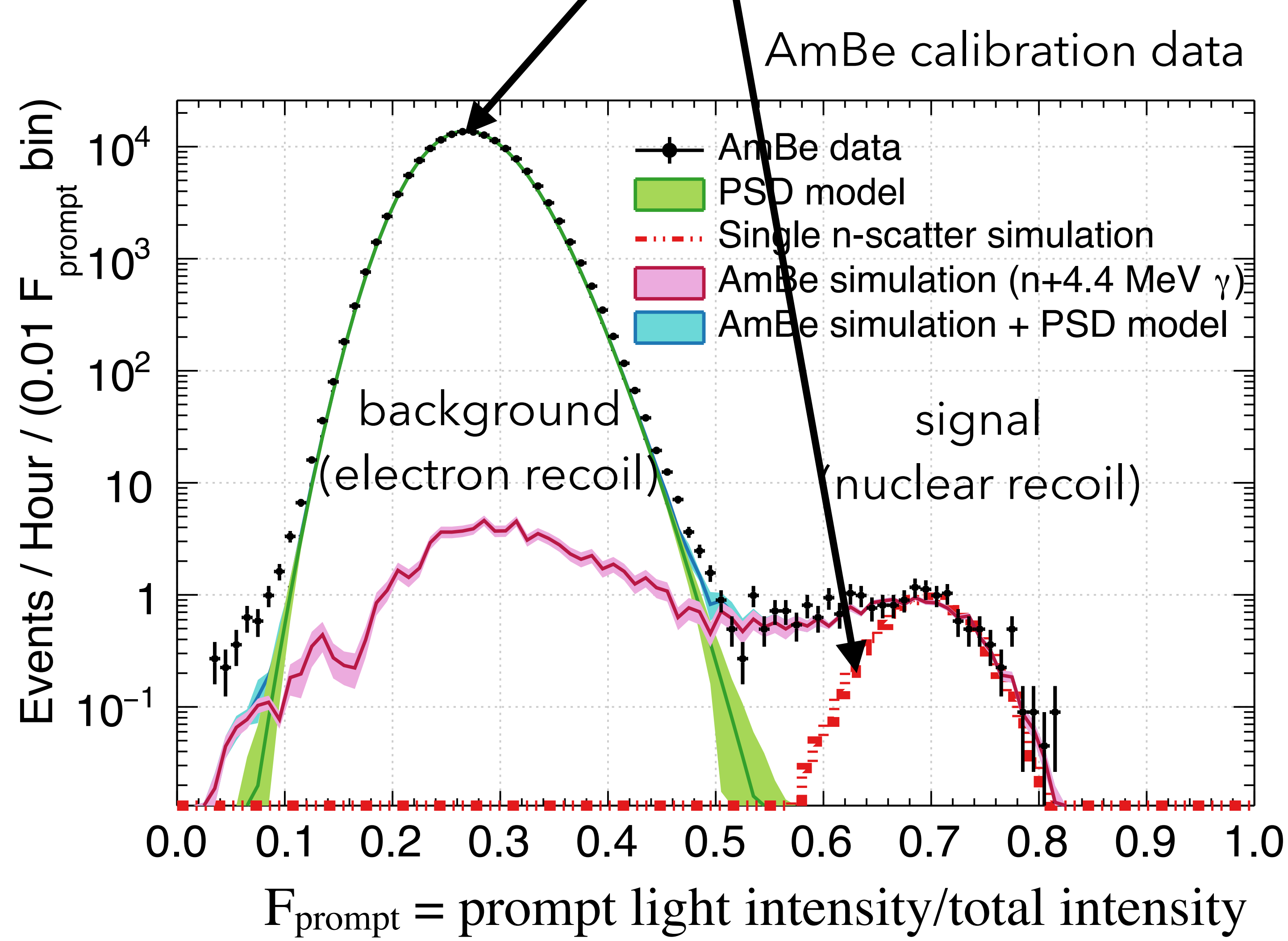
Astroparticle physics, V 85 (2016)

We want to derive the shape of the PSP distribution.



Astroparticle physics, V 85 (2016)

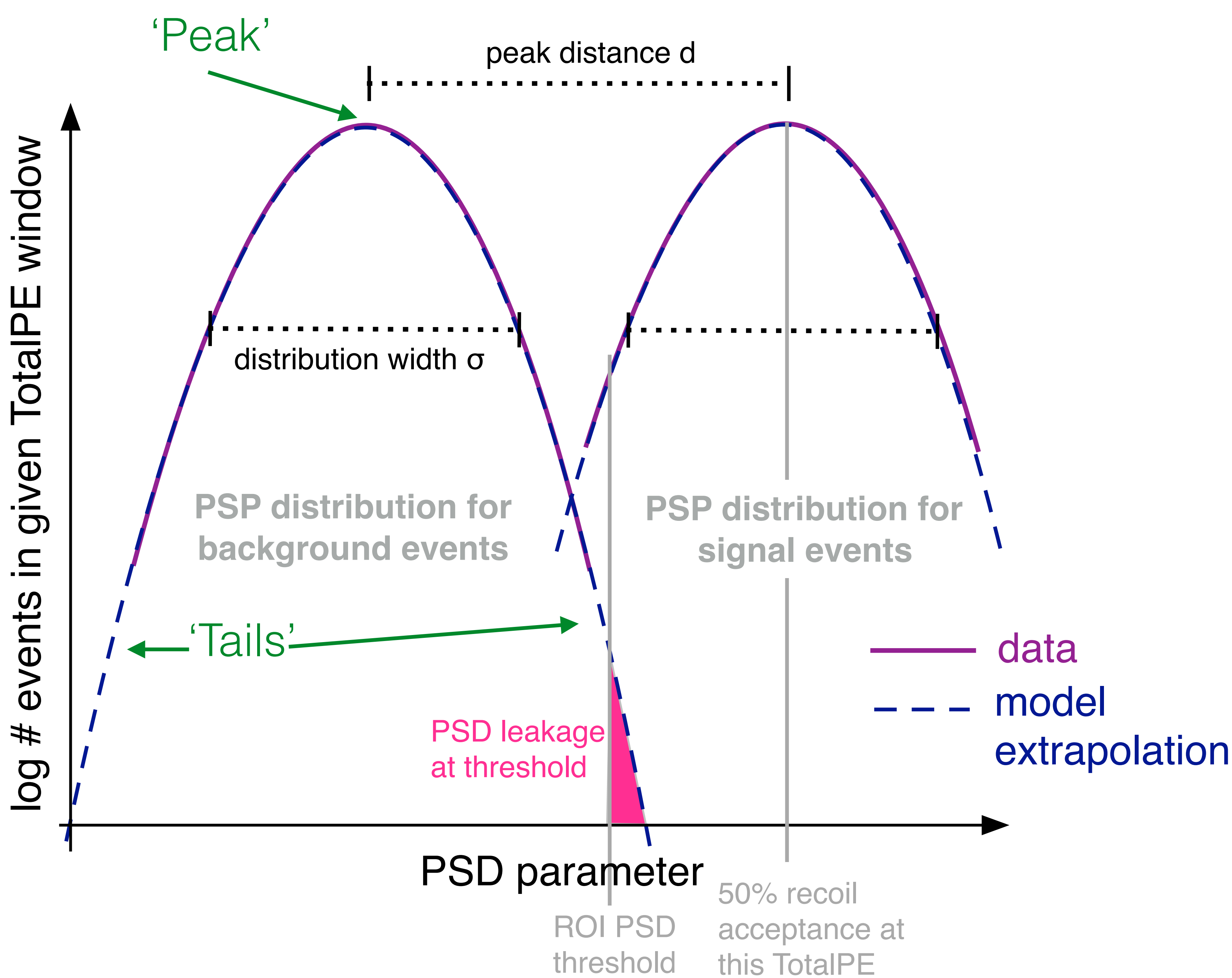
How is this shape described mathematically? How do detector properties such as energy resolution affect it?



arXiv:1902.04048

We care about the shape because it tells us how much background leaks into the signal region.

And how much of the signal is accepted by the cut.



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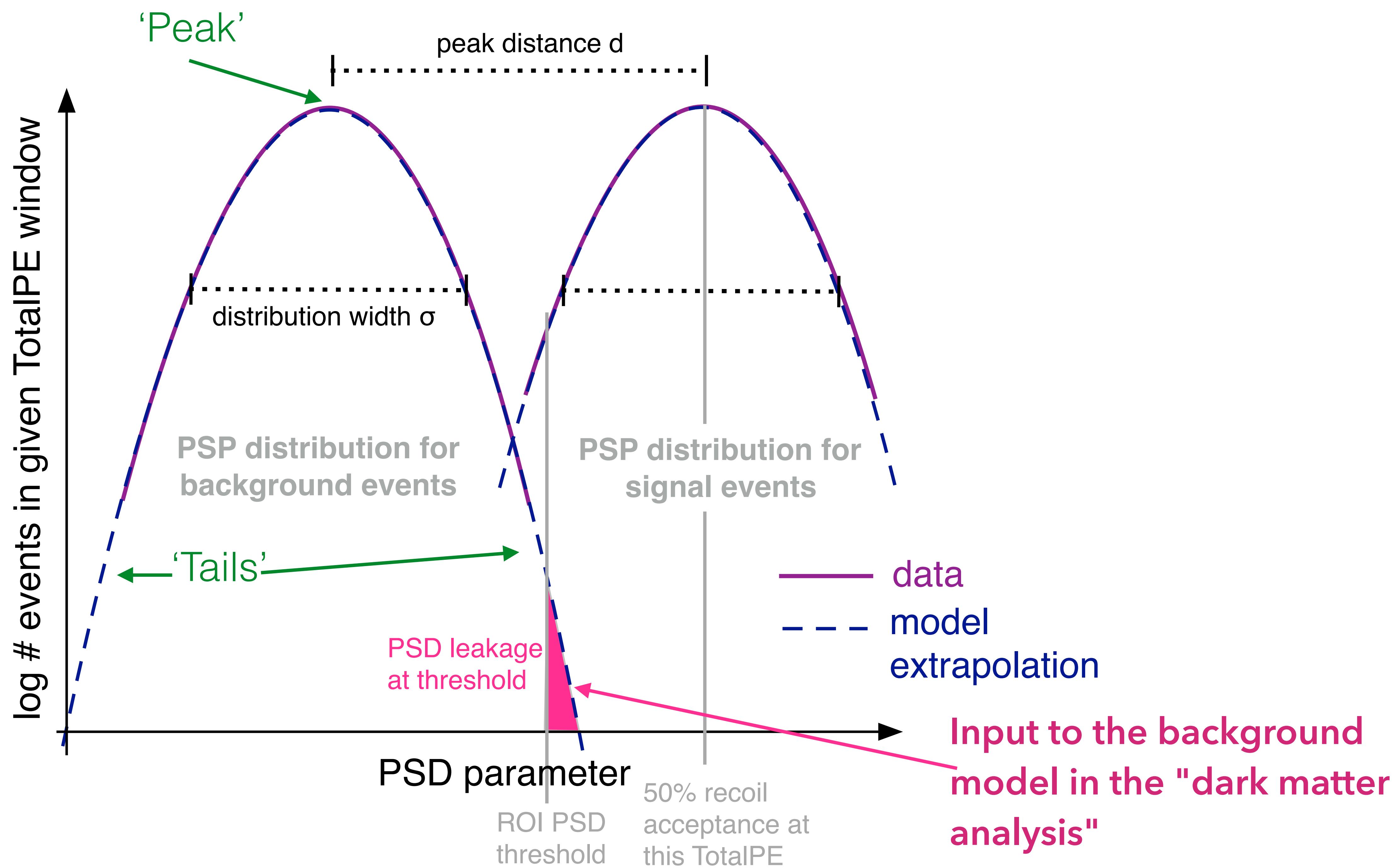
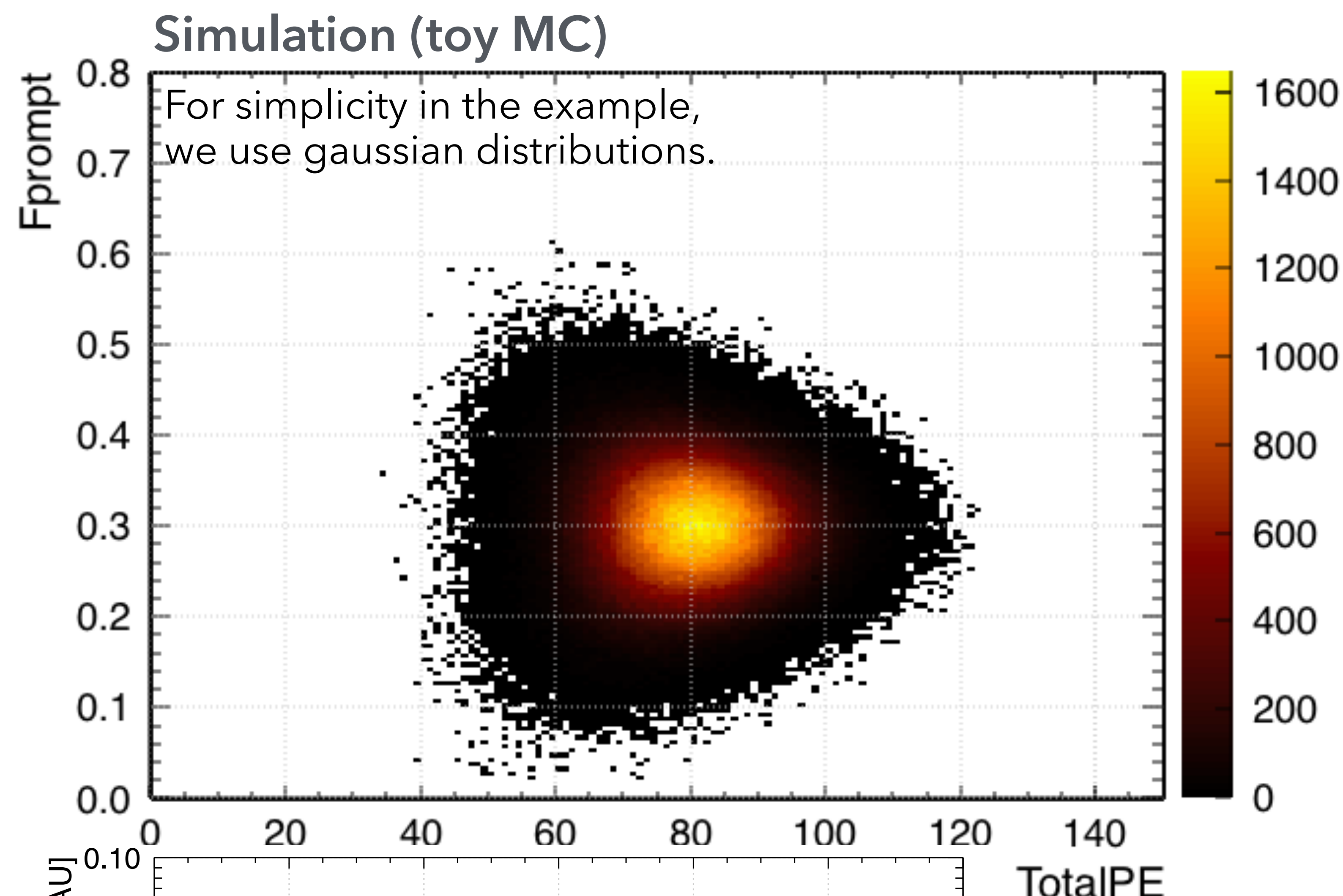


Start simple: a mono-energetic background source.

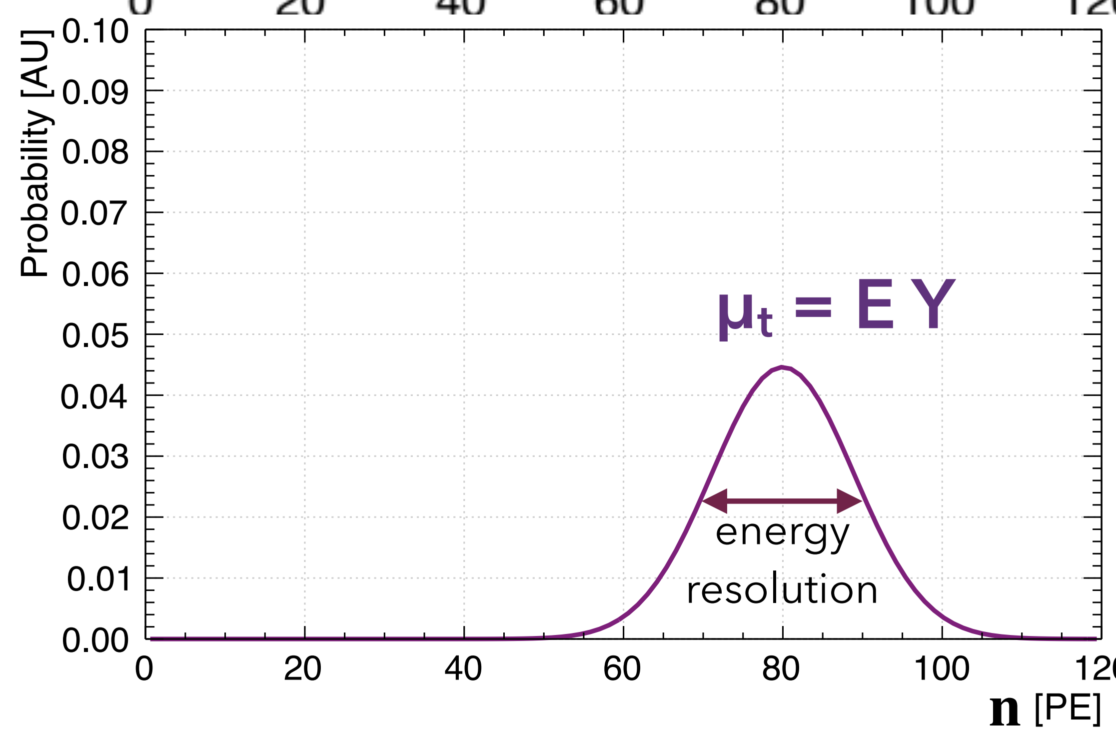
PSP:

$f = n_p/(n_p + n_l)$



Total number of detected photons (PE)

$n_t = n_p + n_l$



E: energy

Y: lightyield

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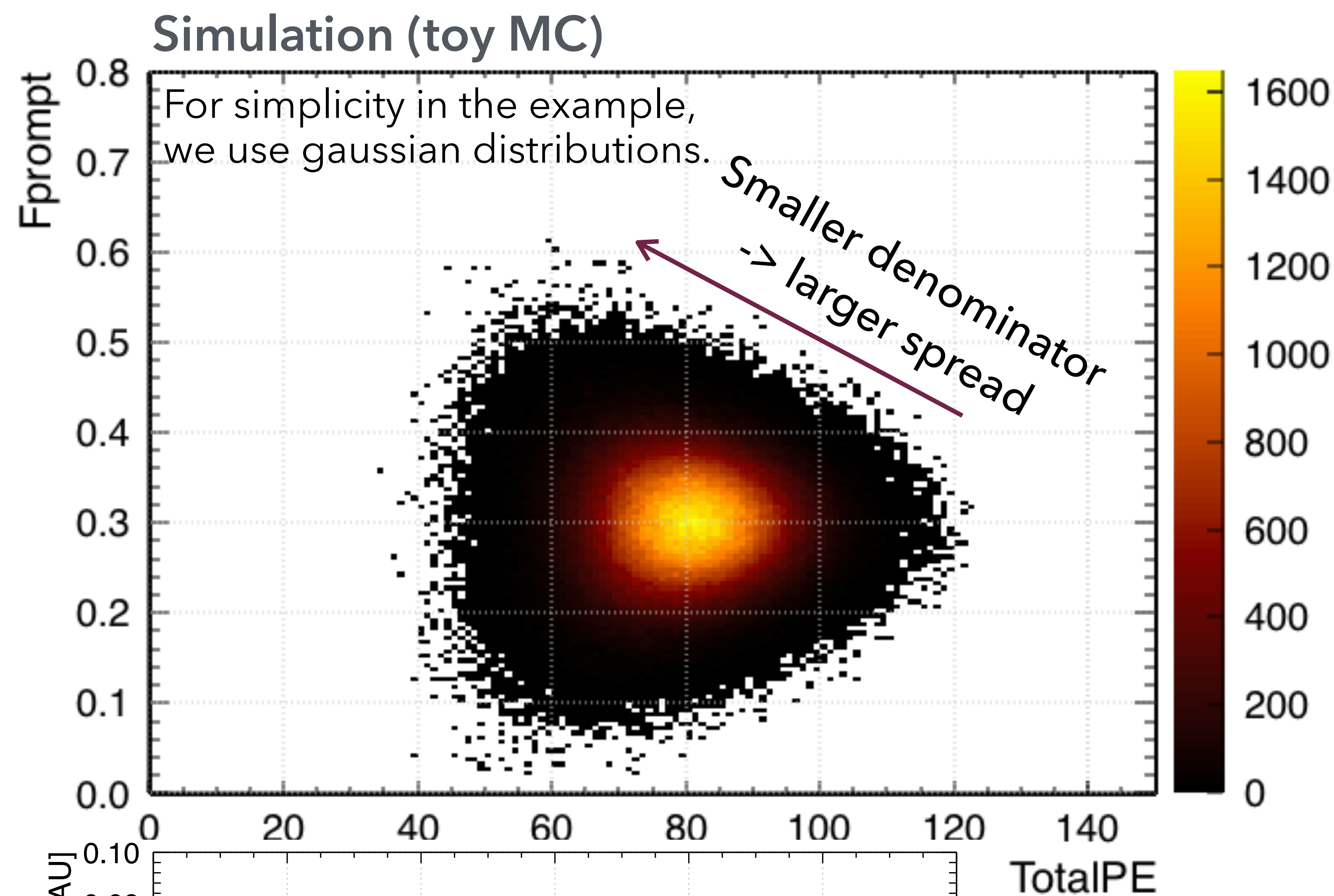

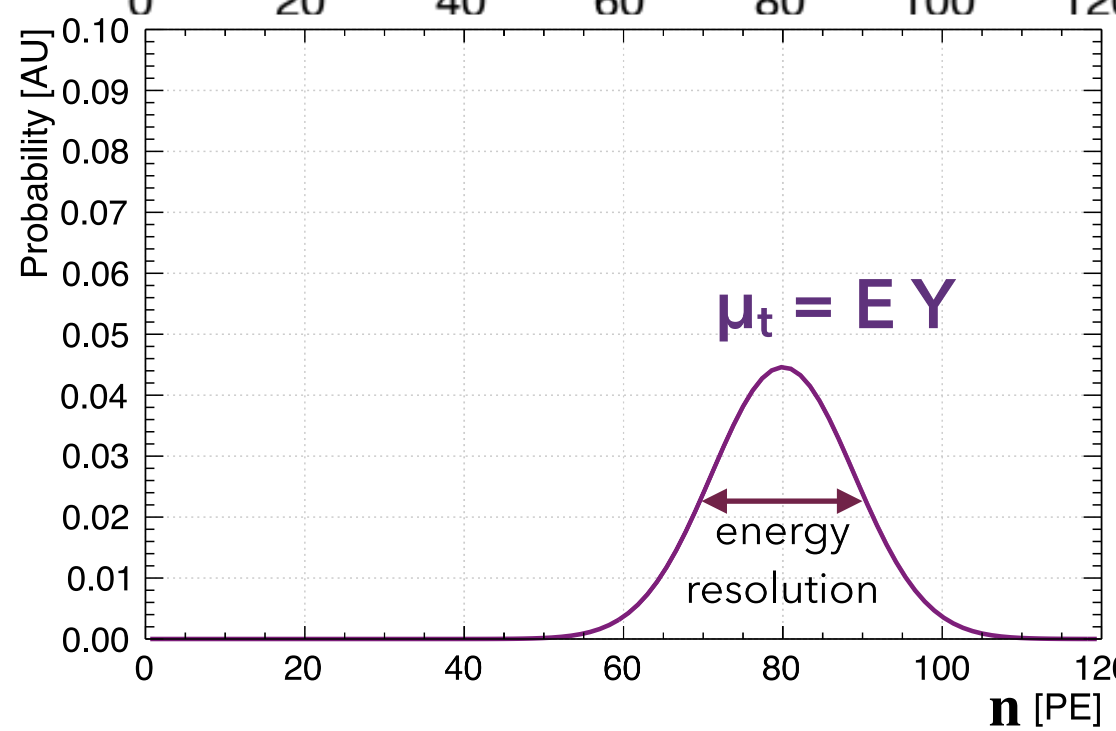

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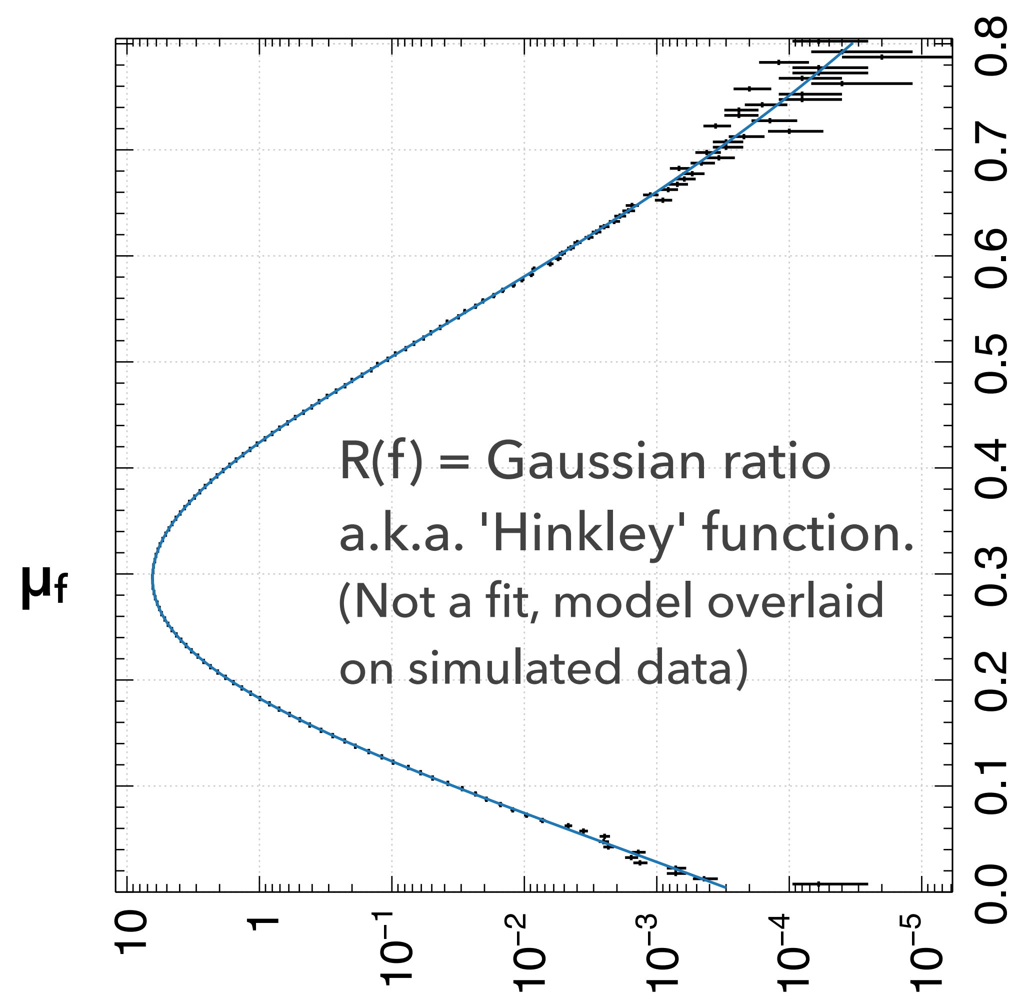
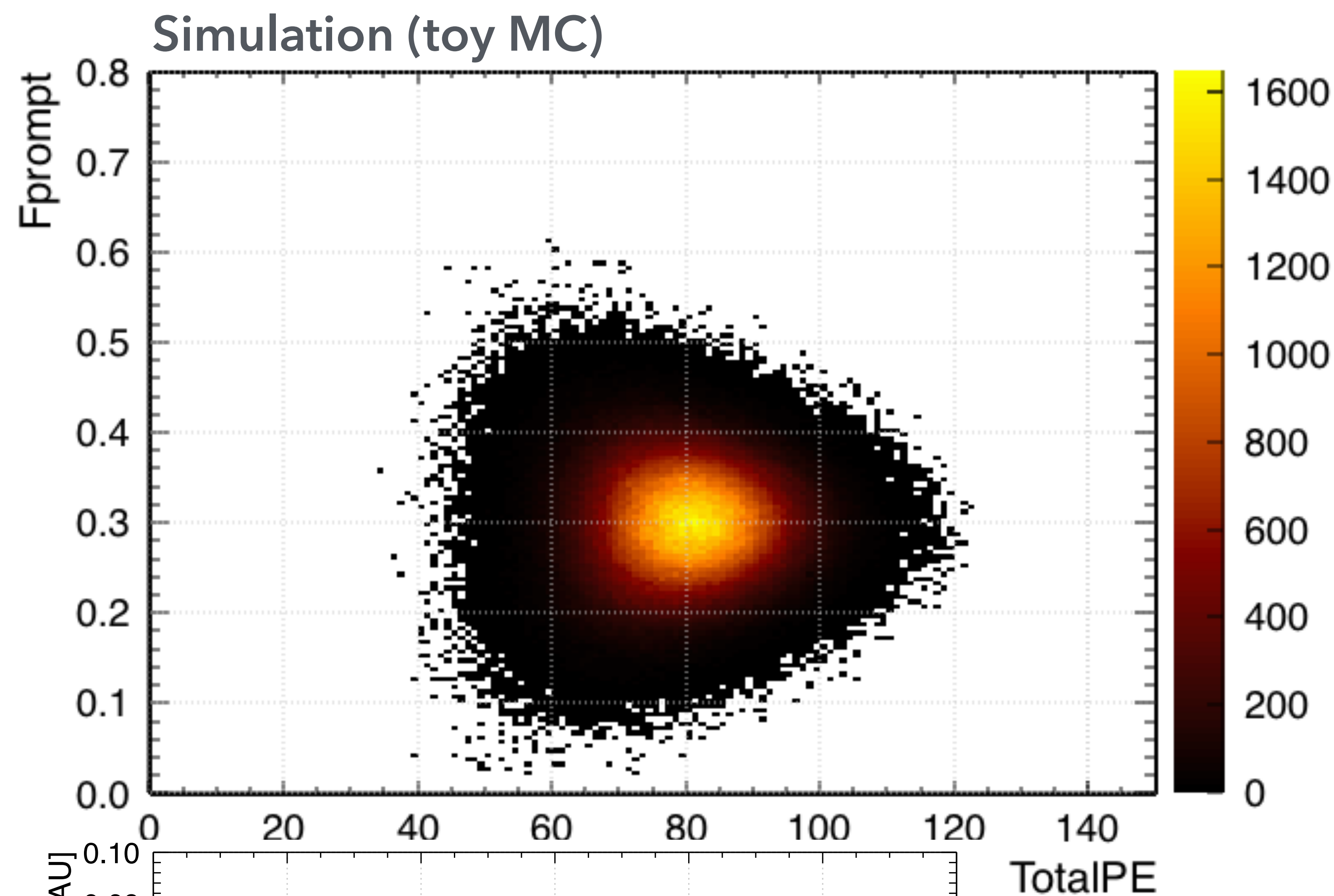
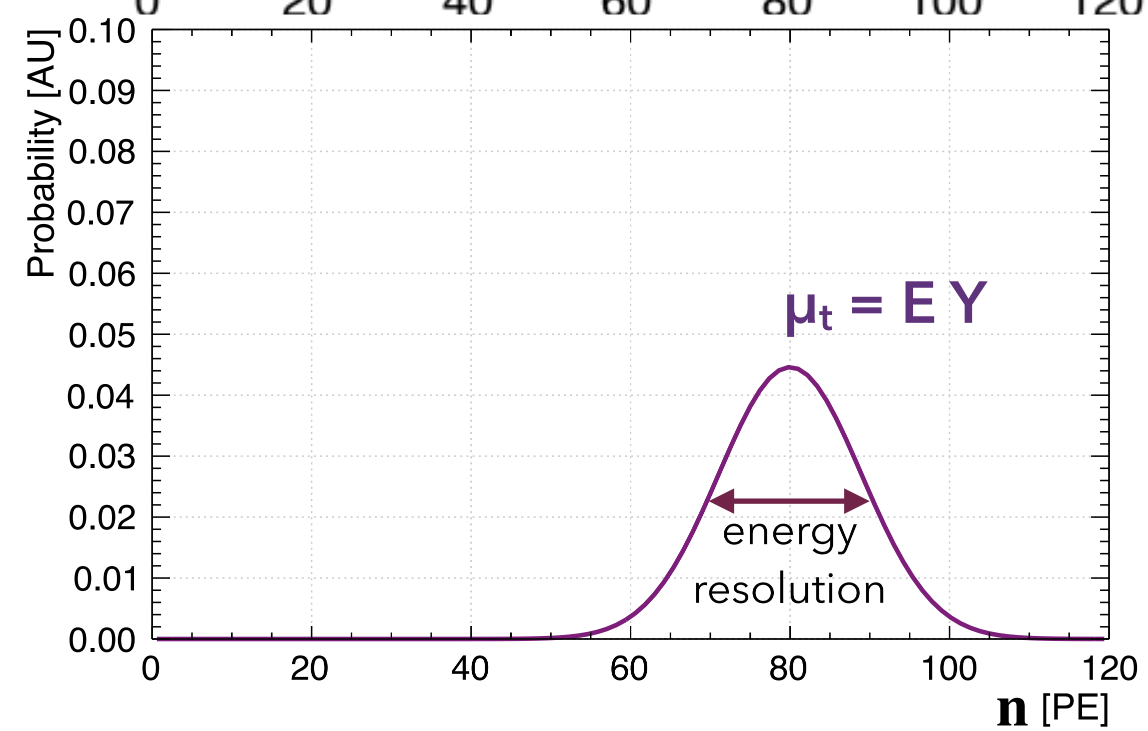
What is the shape of the PSP distribution, R(f)?







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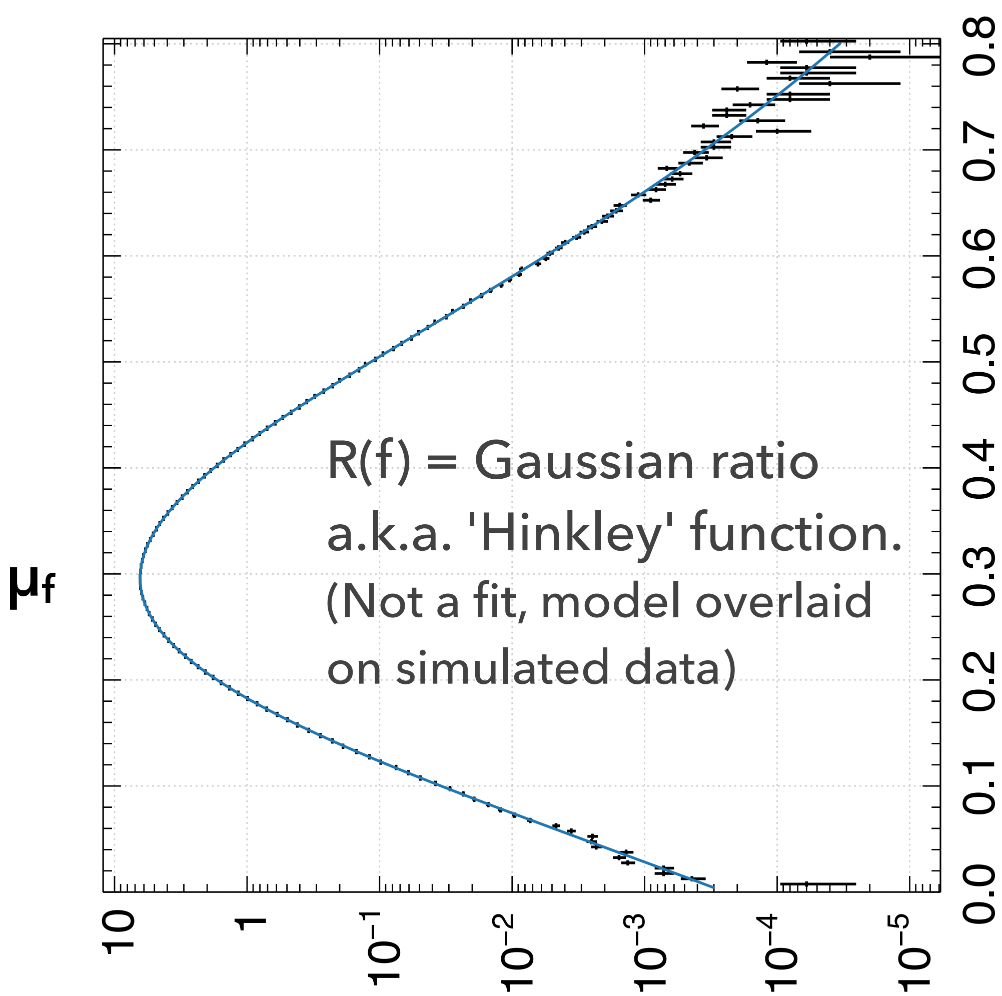
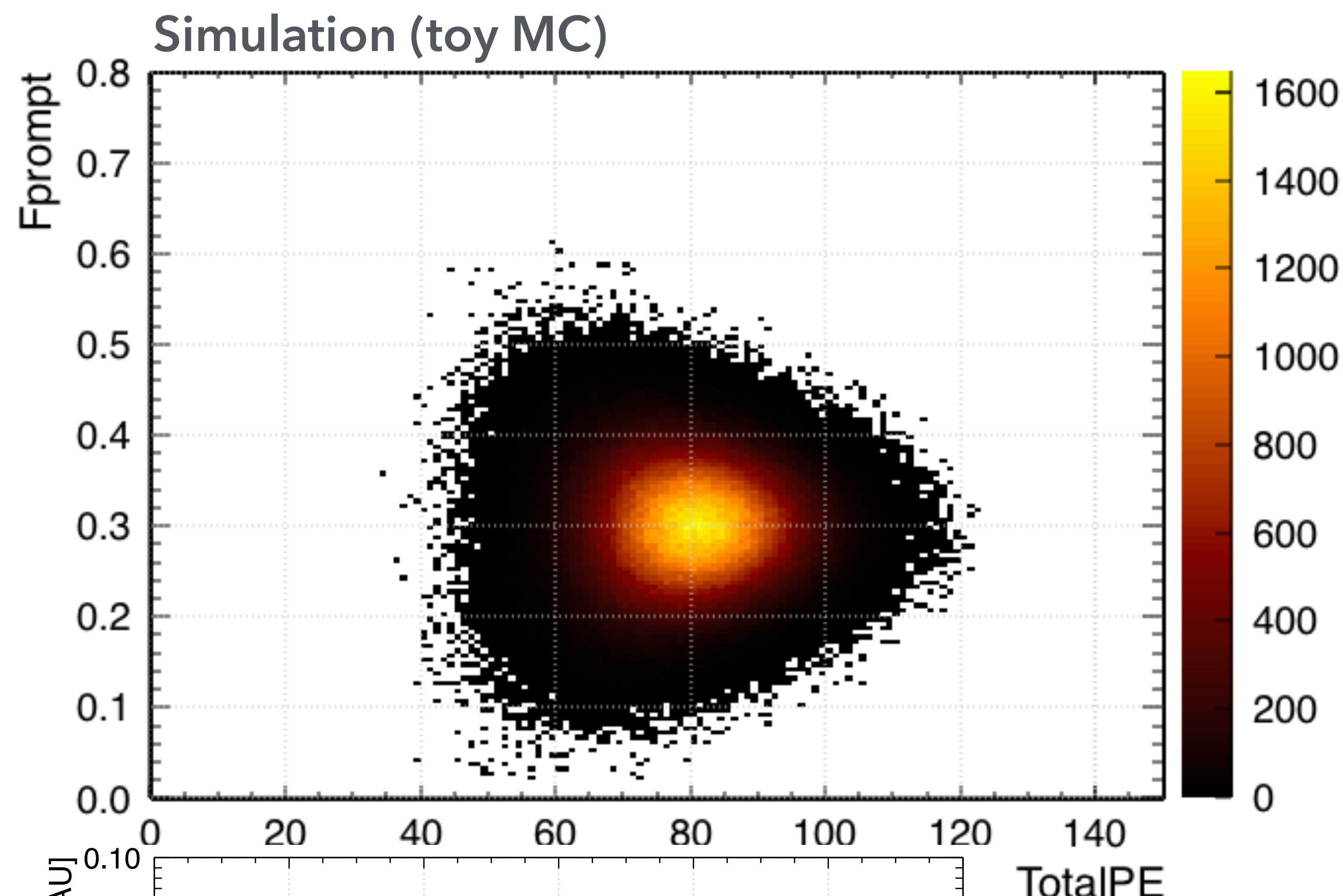
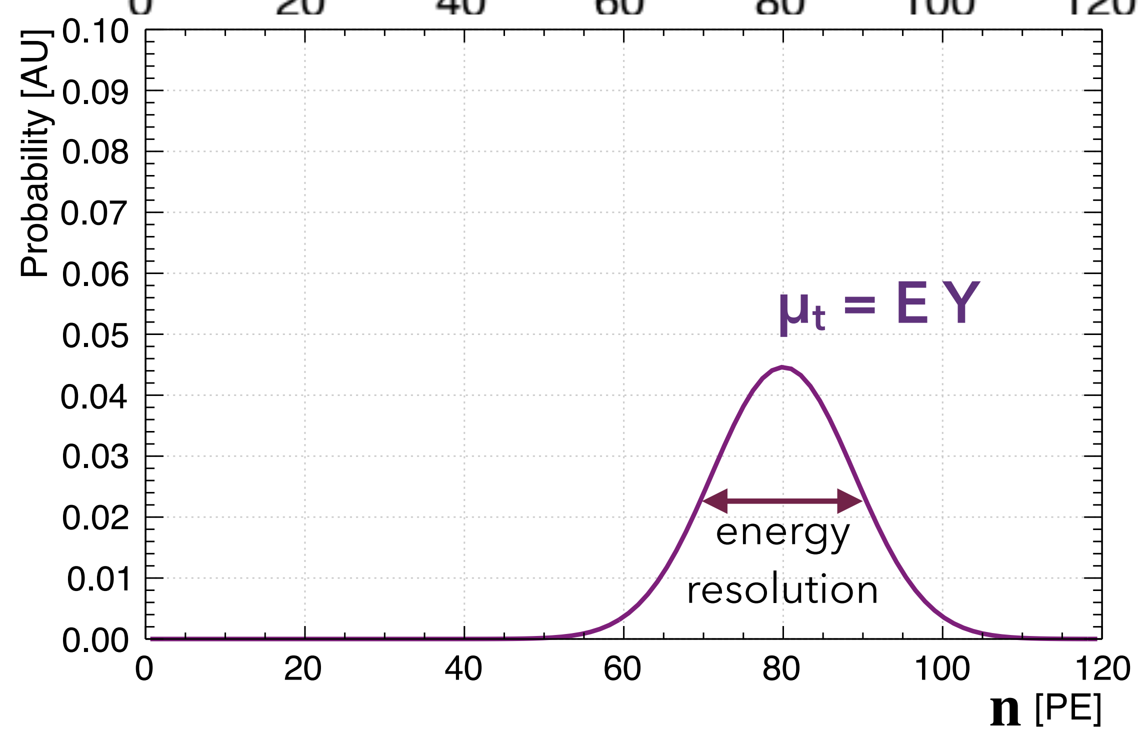


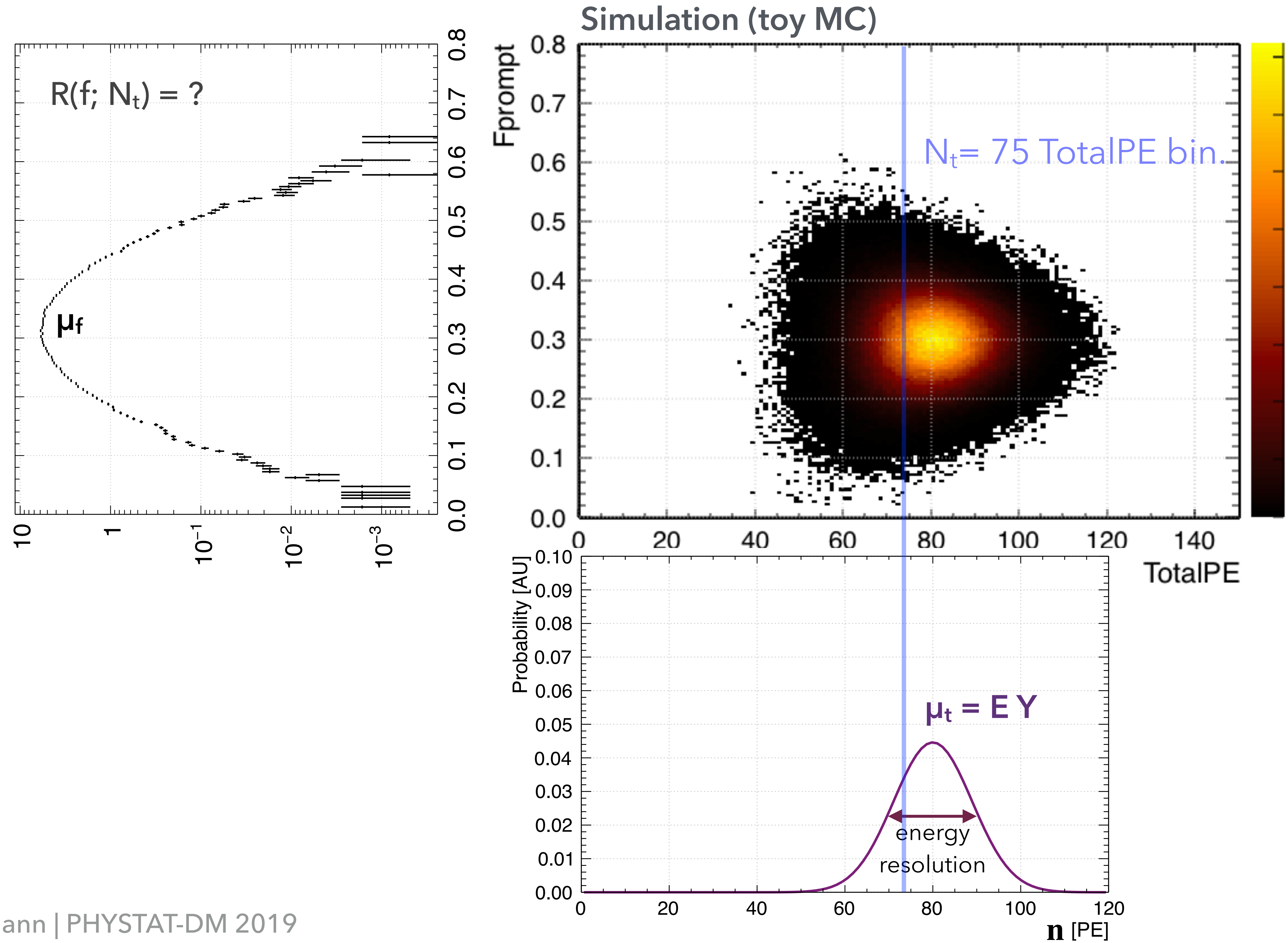




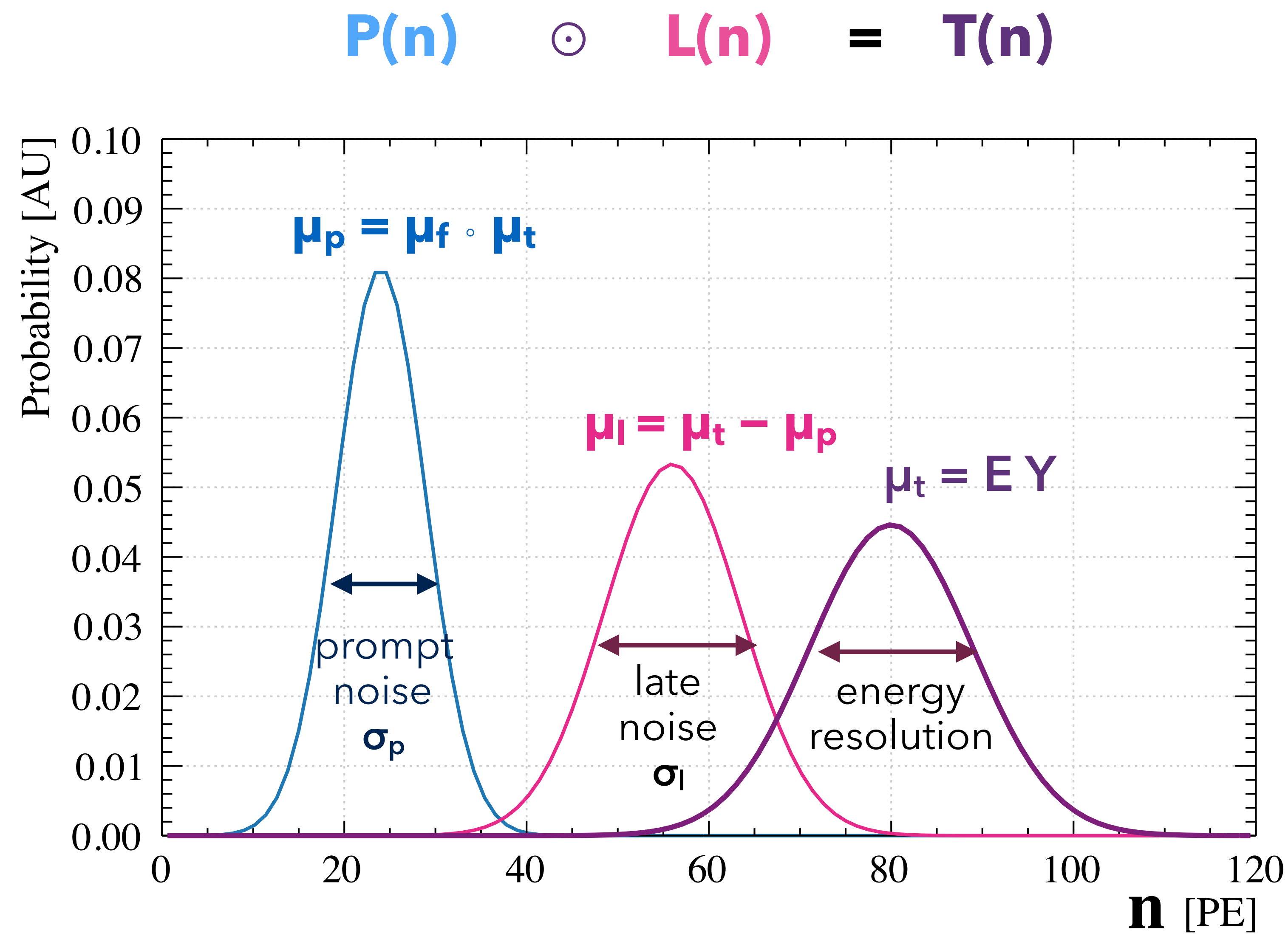
But:

1. The underlying distributions are not gaussian, especially at small numbers of PE.
2. We are typically not dealing with mono-energetic sources.
3. We select an energy ROI that removes some of the events.

R(f) differs in each TotalPE bin. What is the shape of the PSP distribution in a given TotalPE bin N_t, $R(f; N_t)$?



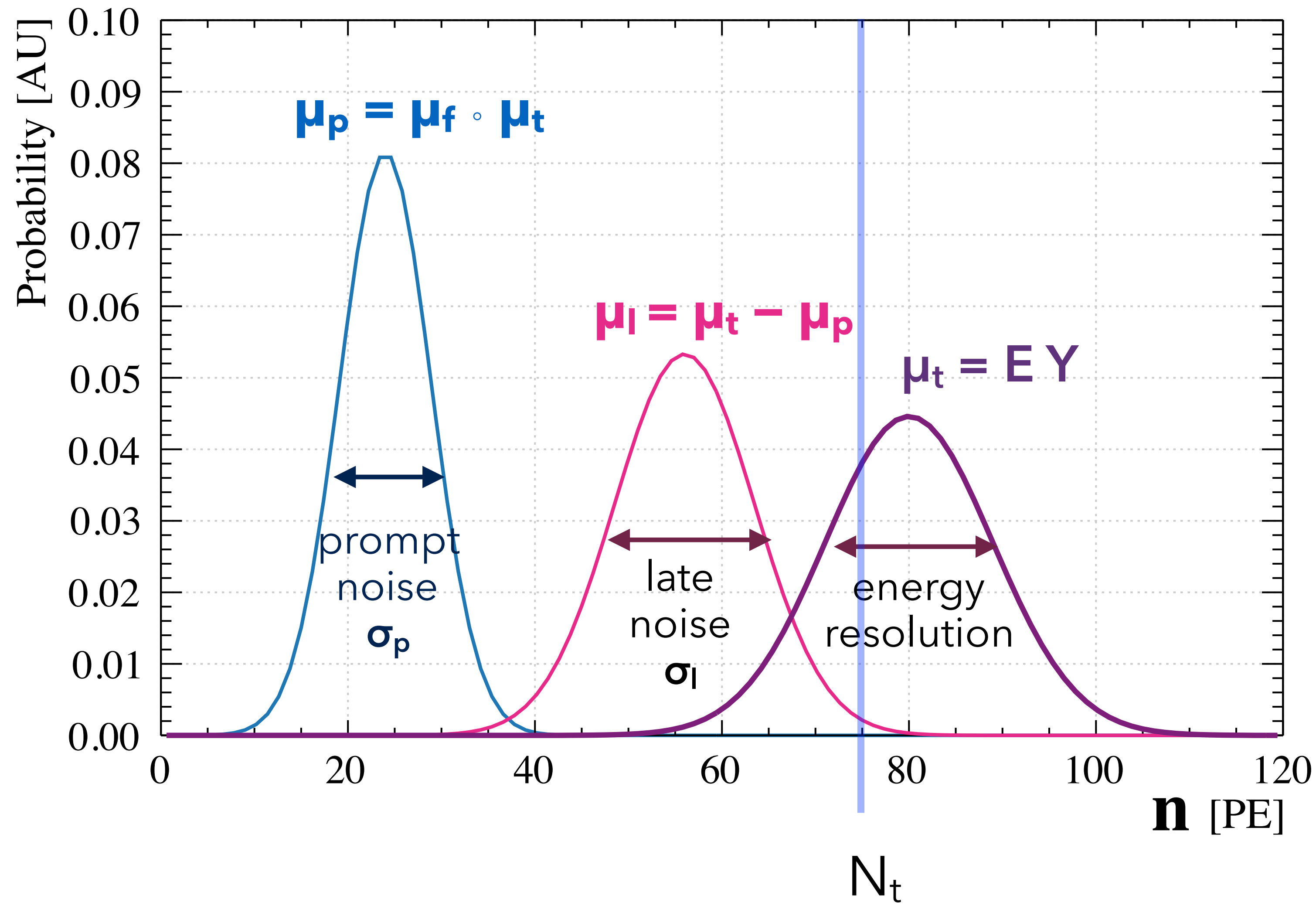
To derive $R(f; N_t)$, consider the distributions of prompt and late PE (for mono-energetic events).



To derive R(f; N_t), consider the distributions of prompt and late PE (for mono-energetic events).

$P(n) \odot L(n) = T(n)$

The events that contribute to N_t are not drawn from the "free" **$P(n_p)$** and **$L(n_l)$** distributions because we require **$n_p + n_l = N_t$**



To derive R(f; N_t), consider the distributions of prompt and late PE (for mono-energetic events).

For each N_t, an event is the union of the disjoint events
$[n_p]$ and $[n_l = N_t - n_p]$,
or alternatively,
$[n_p = N_t - n_l]$ and $[n_l]$

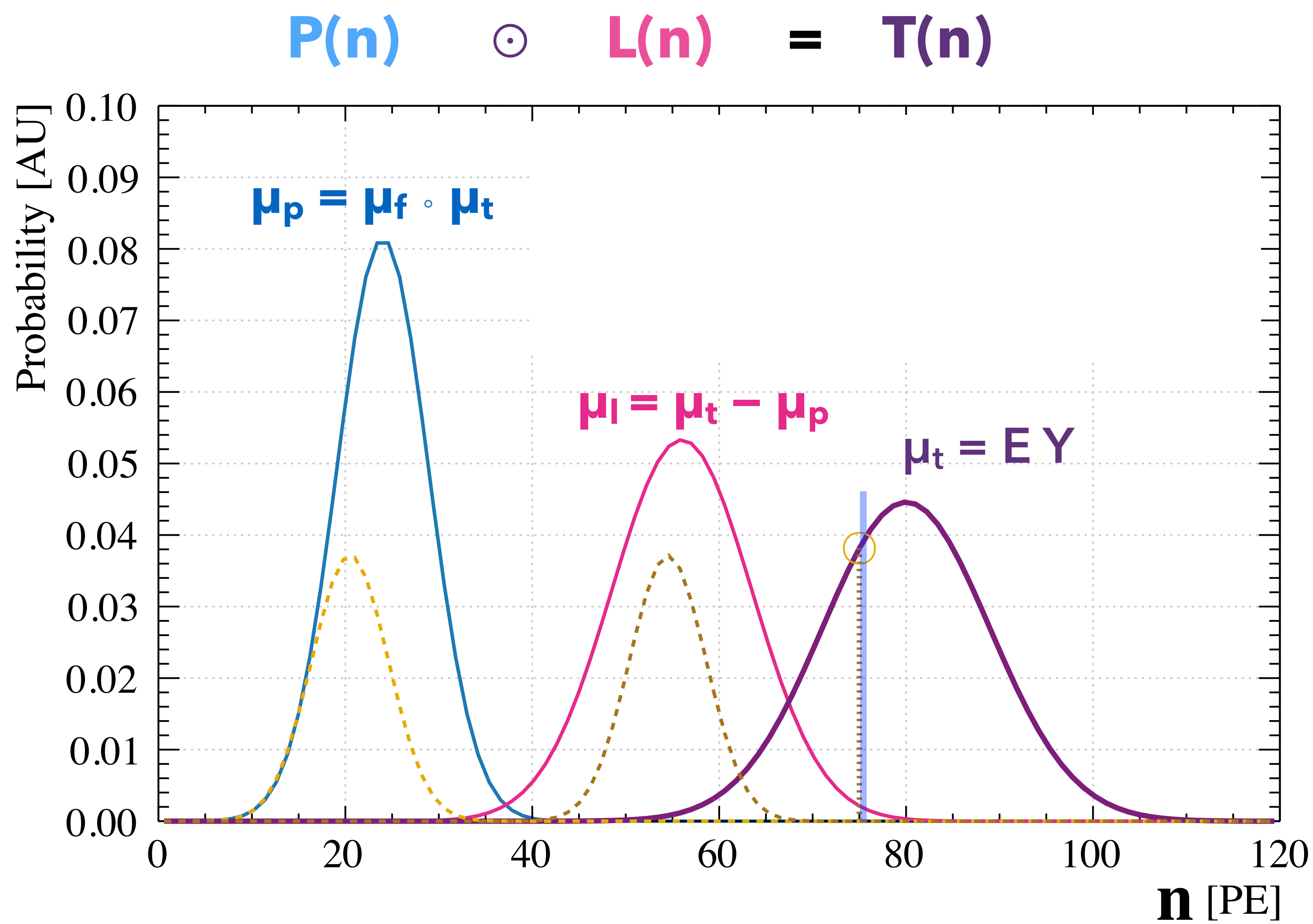
The probability distribution for such an event is:

$P'(n_p \mid n_l = N_t - n_p) = P(n) \cdot L(N_t - n)$

or

$L'(n_l \mid n_p = N_t - n_l) = P(N_t - n) \cdot L(n)$

These are the correlated distributions.



Astroparticle physics, V 85 (2016)

To derive $R(f; N_t)$, consider the distributions of prompt and late PE (for mono-energetic events).

Example:

$P(n) = \mathrm{Gaus}(n, \mu_p, \sigma_p)$

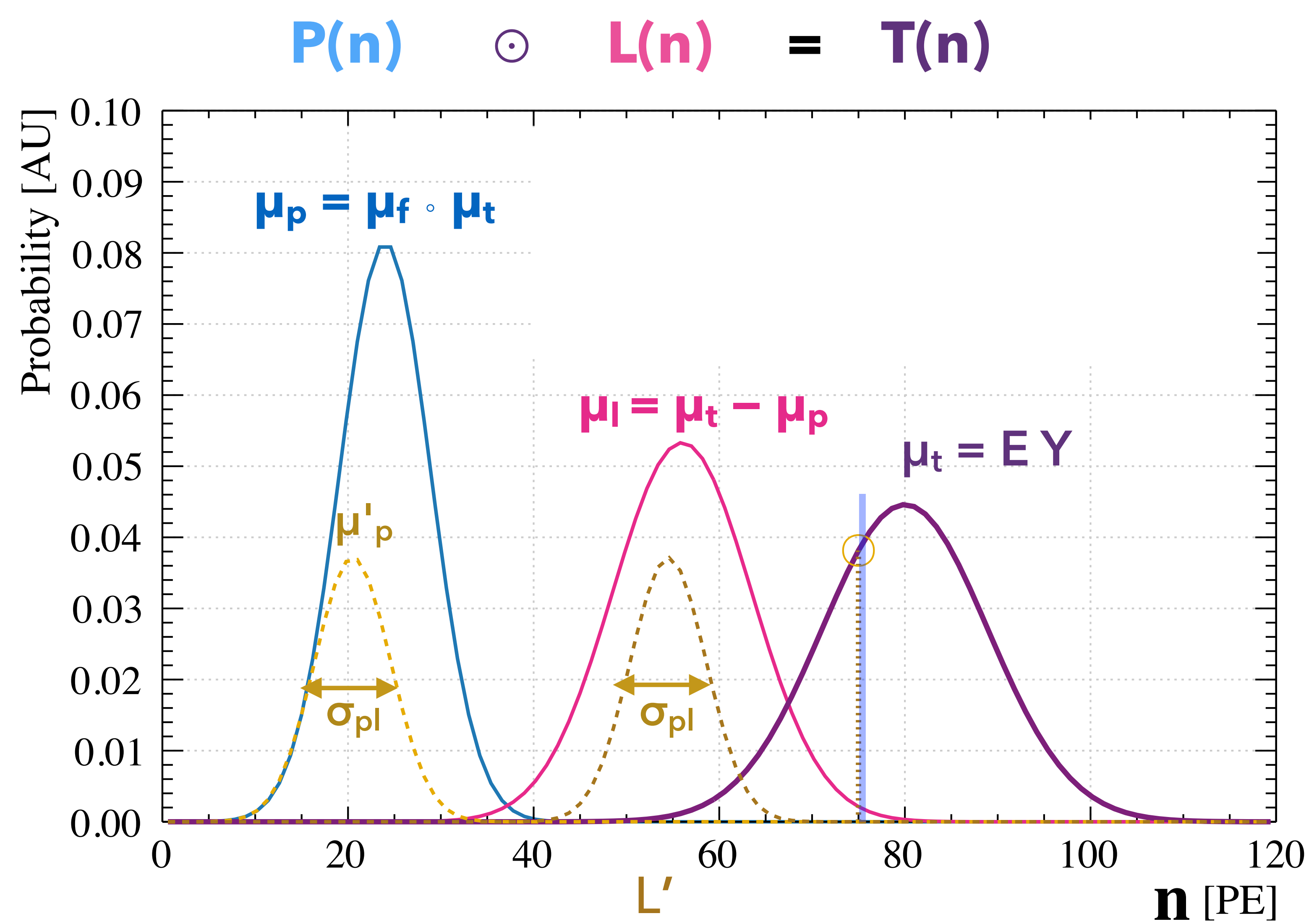
$L(n) = \mathrm{Gaus}(n, \mu_l, \sigma_l)$

$P'(n) = \mathrm{Gaus}(n, \mu_p, \sigma_p) \cdot \mathrm{Gaus}(N_t - n, \mu_l, \sigma_l)$

$P'(n) = \mathrm{Gaus}(n, \mu'_p, \sigma_{pl})$

$$\mu'_p = \frac{\mu_p \sigma_l^2 + (N_t - \mu_l)\sigma_p^2}{\sigma_p^2 + \sigma_l^2}$$

$$\sigma_{pl} = \sqrt{\frac{\sigma_p^2 \cdot \sigma_l^2}{\sigma_p^2 + \sigma_l^2}}$$



$P'(n; 75) = P(n) \cdot L(75 - n)$

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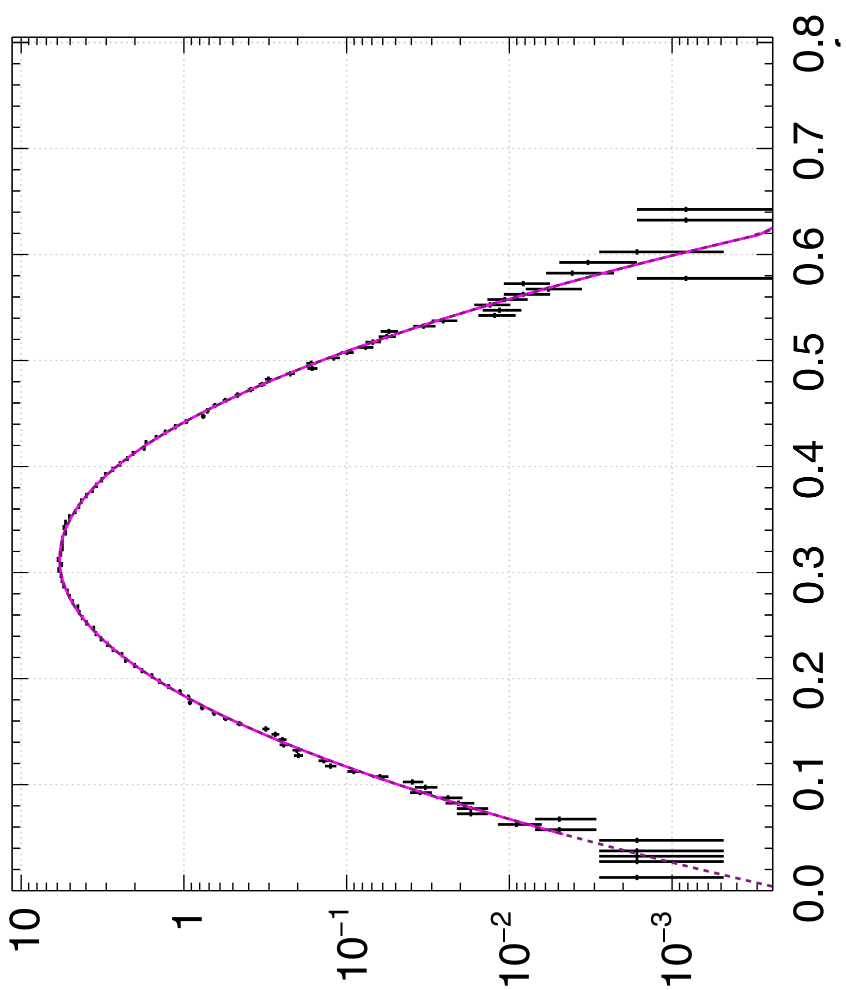
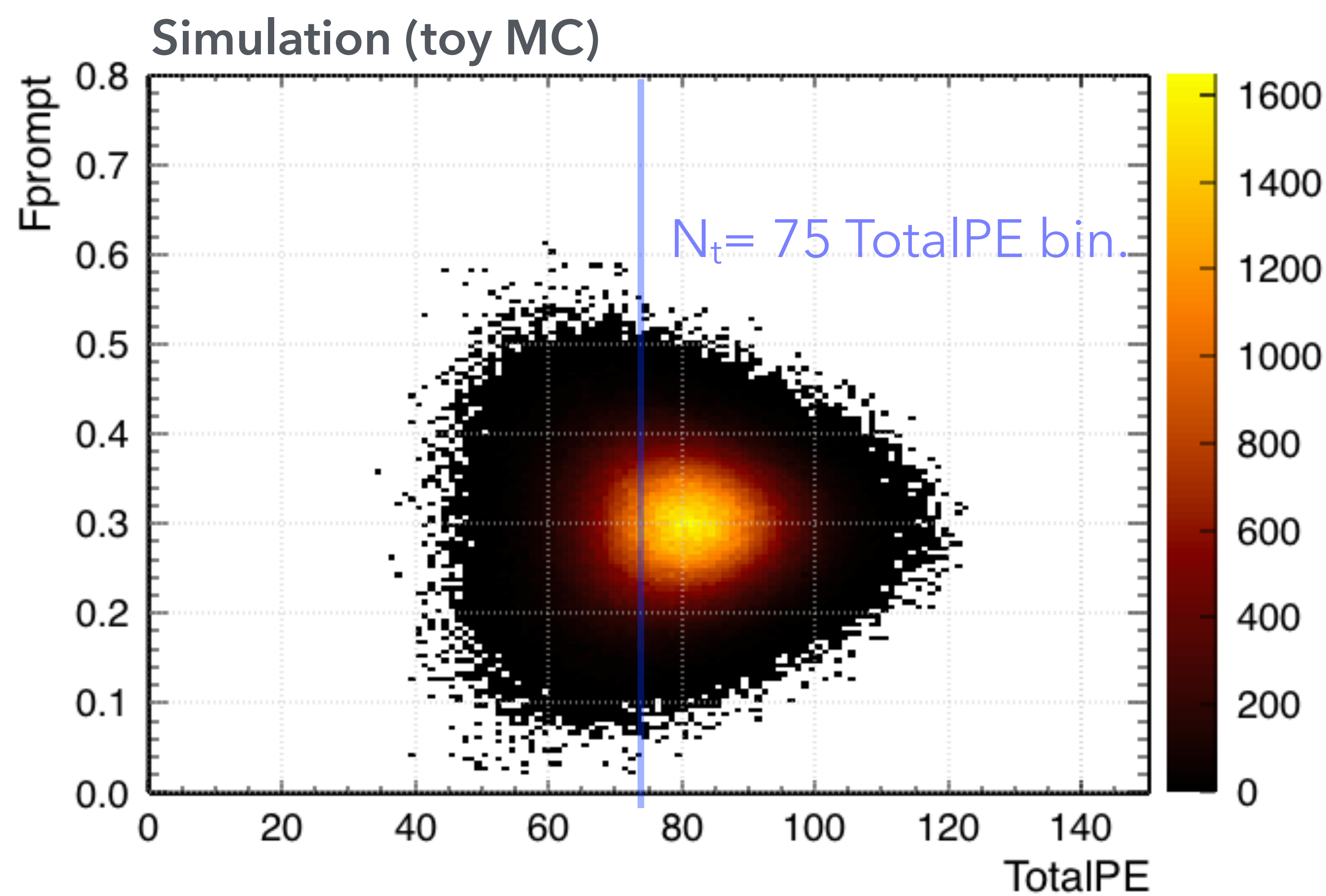
The Fprompt distribution R(f) in each TotalPE bin is $P'(n; N_t)$ after a variable transformation from $n \rightarrow f = n/N_t$.





$R(f; N_t) = N_t P'(f = n/N_t; N_t)$

Variable transformation

$\mathbf{P}(x) = \vartheta \cdot \mathbf{P}(x/\vartheta)$

The backgrounds in the detector are usually not mono-energetic, so build a sum over the contributions from all energies to a given TotalPE slice.

Fprompt distribution for events of N_t TotalPE

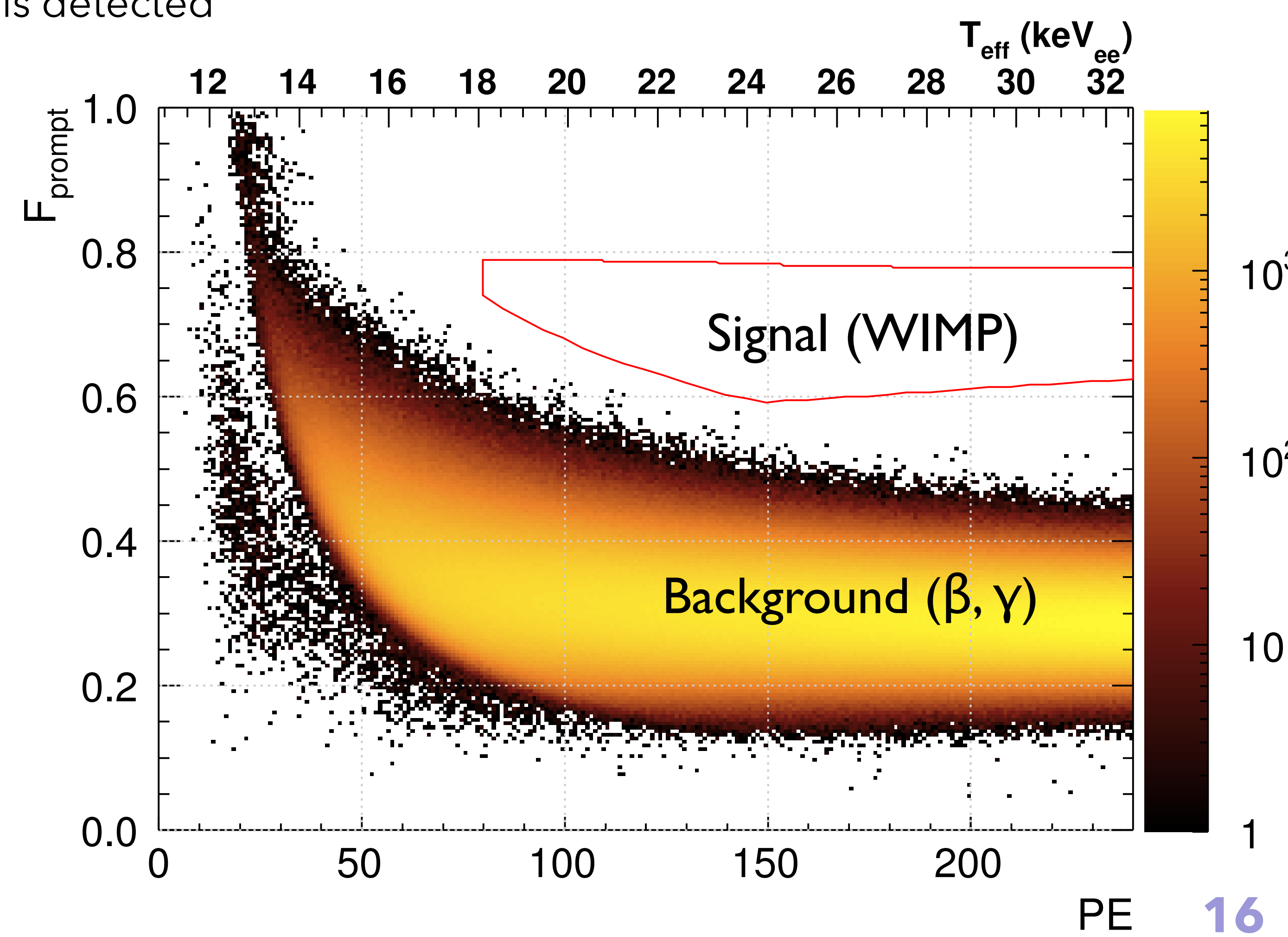
Number of events with this energy.

Correlated PromptPE distribution for events of true energy E.

$$R(f; N_t) = \Sigma_E T(E) \circ N_E(N_t) \circ N_t \circ P_E'(f; N_t)$$

Sum over all Energies.

Probability that an event of this energy is detected at N_t TotalPE



Phys. Rev. Lett. 121, 071801 (2018) *arXiv:1707.08042*

High-energy events fluctuating down are worse for PSD than low-energy events fluctuating up.

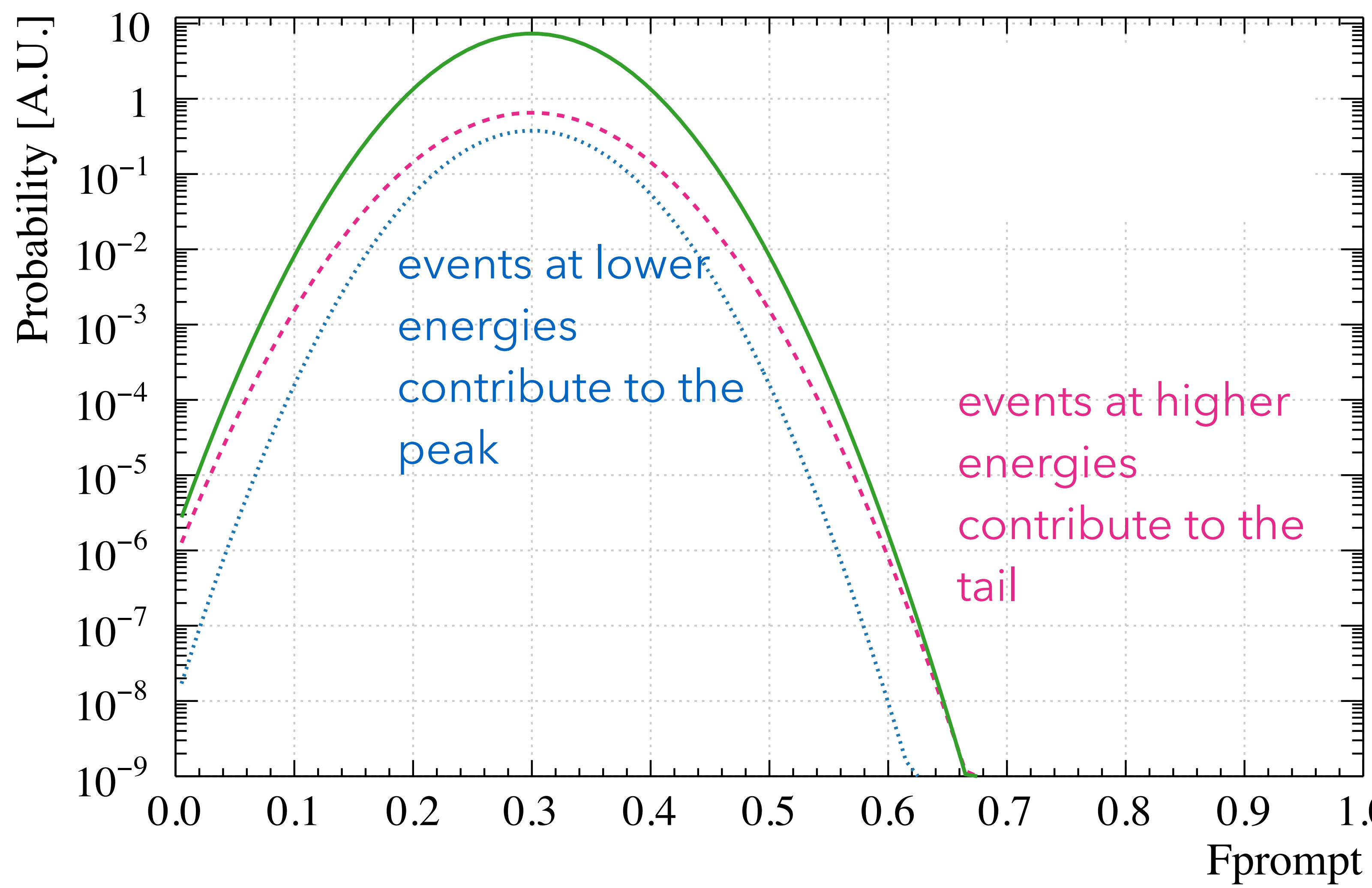
$$R(f; N_t) = \Sigma_E\, T(E) \circ N_E(N_t) \circ N_t \circ P_E'(f; N_t)$$

Simulation. Contributions to the Fprompt distribution at 80 TotalPE. Flat spectrum.

EY = 80 PE, measured energy = 80 PE

EY = 90 PE, measured energy = 80 PE

EY = 70 PE, measured energy = 80 PE



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$$R(f; N_t) = \Sigma_E\, T(E) \circ N_E(N_t) \circ N_t \circ P_E'(f; N_t)$$

The hard part:

$\mu_f = \mu_f(E, n)$	The relative fraction of prompt light is a function of both energy (due to underlying scintillation physics) and number of detected photons (due to instrumental effecs such as dark noise)
$\sigma = \sigma(YE)$	The detector resolution as a function of the number of detected photons for both the prompt and late comonent must be known.
$P(n) = ?$ $L(n) = ?$	The shape of the distribution of prompt and late PE, especially at small numbers of PE, is a convolution of different micro-physics and detector effects. There may also be correlations which have to be minded.

The backgrounds in the detector are usually not mono-energetic, so build a sum over the contributions from all energies to a given TotalPE slice.

$$R(f; N_t) = \Sigma_E \, T(E) \circ N_E(N_t) \circ N_t \circ P_E'(f; N_t)$$

Or use an 'effective model'

For approximately flat spectra and monotonic resolution function:

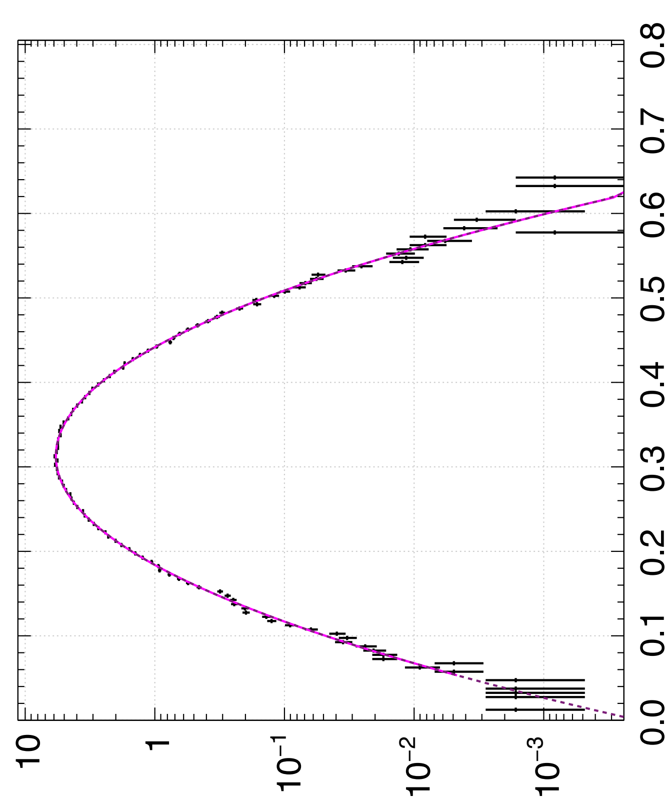
$R(f; N_t)$ = Fully correlated Hinkley function (with arbitrary 'width' parameters)
= Gamma distribution ⊙ Gaussian

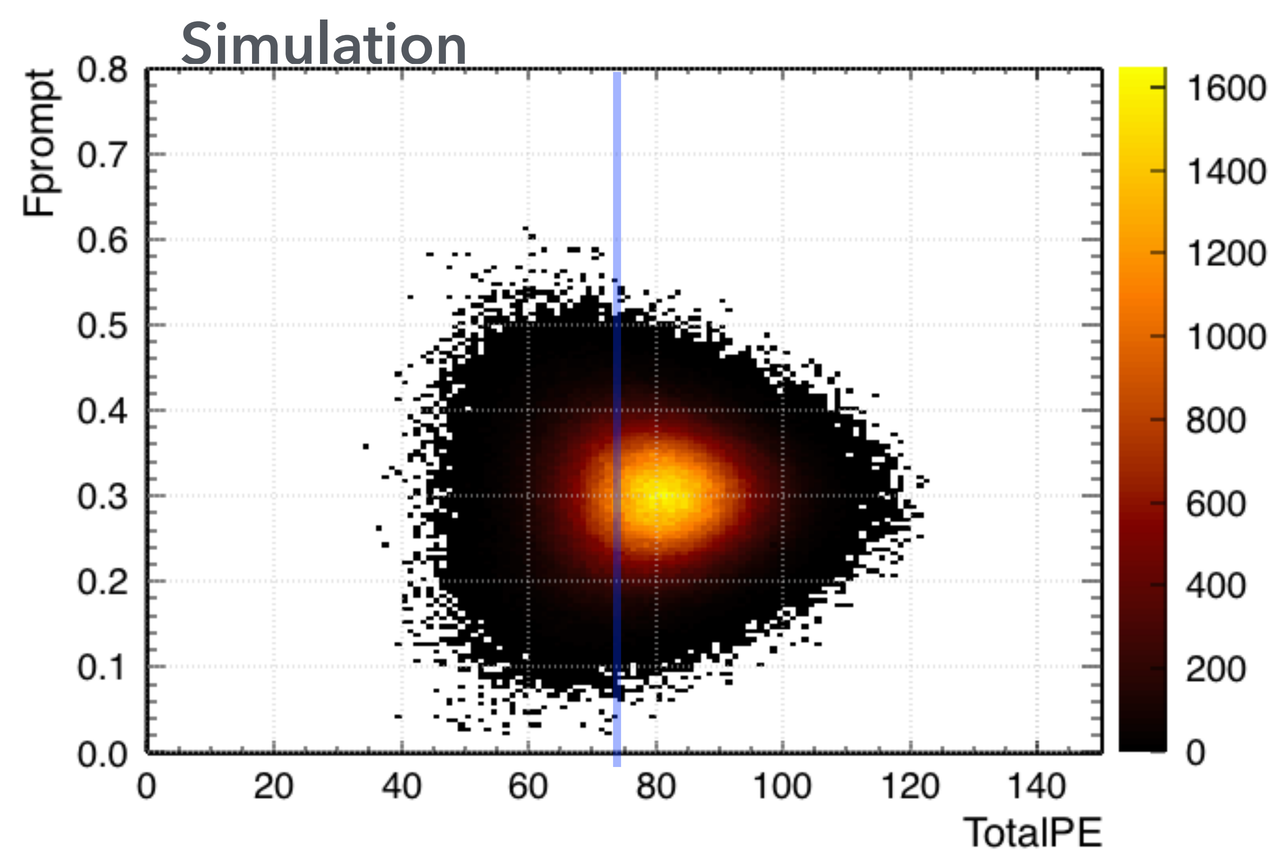
Conclusion

For "fast to total" (or "tail to total") PSP distributions:

- You probably don't want to use the (uncorrelated) Hinkley distribution.
- A statistical model with physical parameters can be created following the steps outlined here. It requires one to understand the detector well enough.
- Several analytic distributions can mimic the shape of the physical model, given effective parameters that no longer have a physical interpretation.

More details: *Astroparticle physics, V 85 (2016) arXiv:0904.2930v2*





Backup

$\mu_f = \mu_f(E, n)$

Instrumental effects affect the apparent singlet fraction.

The singlet fraction is also energy-dependent due to physics reasons. To create a proper PSD model and MC implementation, this must be understood.

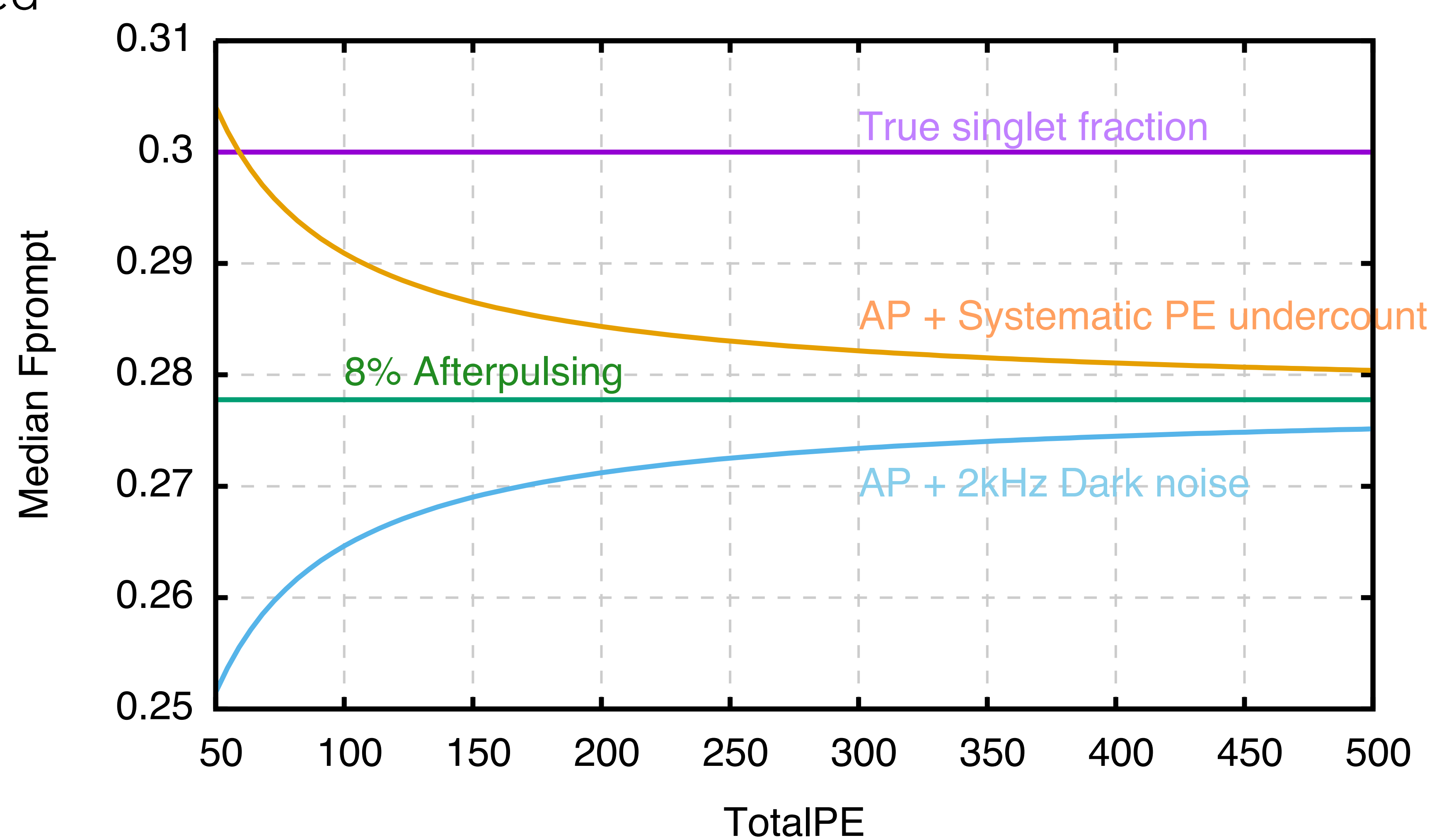
$$\hat{\mu}_p = a\mu_p + \delta_p$$
$$\hat{\mu}_l = a(\mu_l + r_{ap}\mu_t) + \delta_l$$

$$\hat{f}_p = \frac{\bar{f}_p \cdot (1 - \delta_t/\hat{\mu}_t)}{1 + r_{ap}} + \delta_p/\hat{\mu}_t$$

$\delta_p = \delta_t * 150\text{ns}/10000\text{ns}$

^ = observed

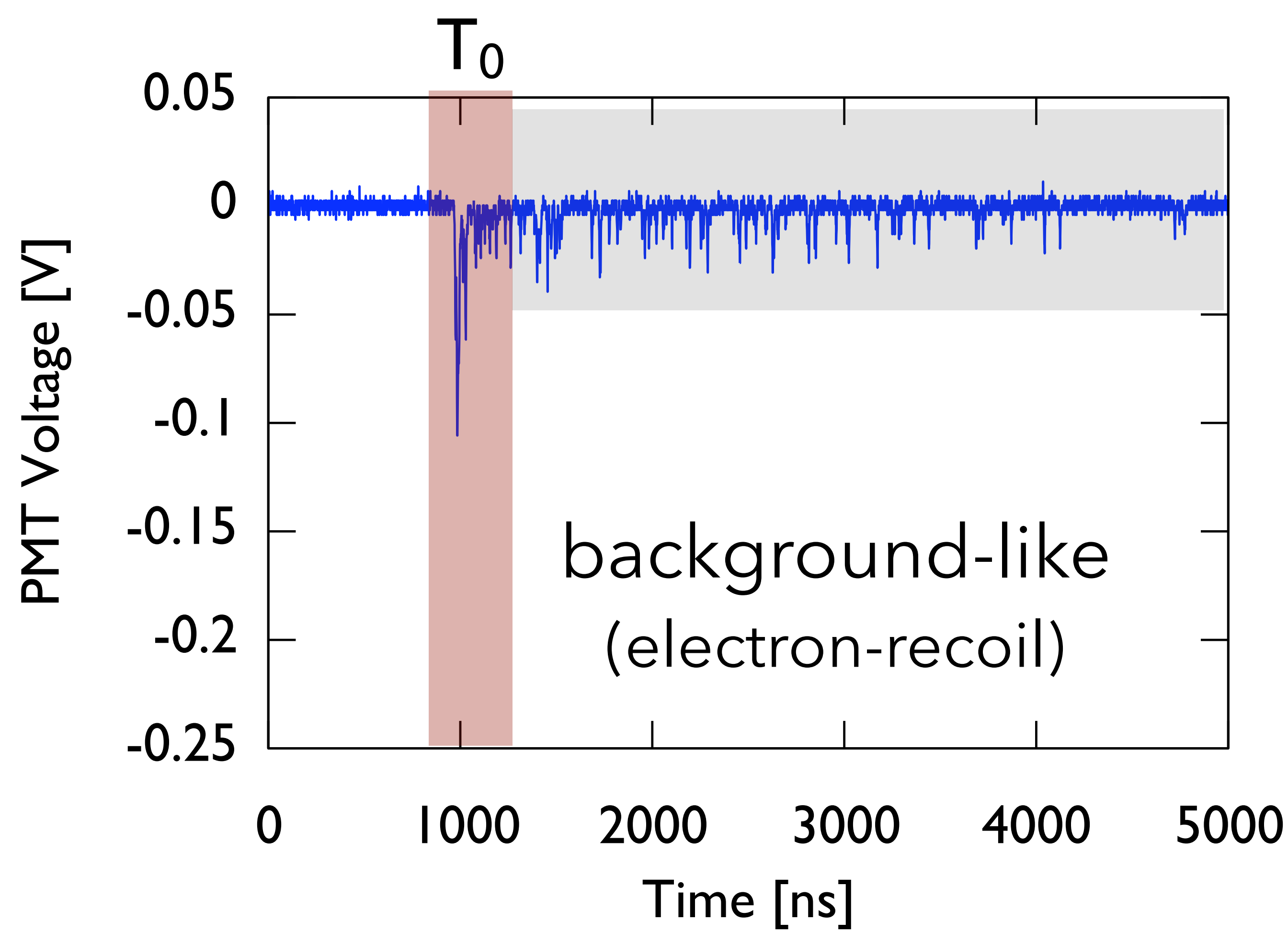
no ^ = true



$\sigma = \sigma(YE)$

In a fraction-based discriminator, in addition to the regular energy resolution terms, there is 'window noise'.

Uncertainty of T0 for each event leads to random shifts in the prompt (or late) window, moving events from the prompt to the late window, or vice versa, in a fully correlated manner.



P(n) = ?

Ideally, the shape of the free distributions is measured with a mono-energetic calibration source.

A Beta-Binomial distribution can sweep a lot of ignorance about microphysical processes under the rug.