Isospin breaking corrections to ϵ'/ϵ

Antonio Rodríguez Sánchez

KAON2019
11/09/2019

In collaboration with:
Vincenzo Cirigliano
Hector Gisbert
Antonio Pich

LUND UNIVERSITY

Basic expressions

Isospin decomposition

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}}\left(\mathcal{A}_{3/2} + \mathcal{A}_{5/2}\right) = A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}\,, \\ A(K^0 \to \pi^0\pi^0) &= \mathcal{A}_{1/2} - \sqrt{2}\left(\mathcal{A}_{3/2} + \mathcal{A}_{5/2}\right) = A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}\,, \\ A(K^+ \to \pi^+\pi^0) &= \frac{3}{2}\left(\mathcal{A}_{3/2} - \frac{2}{3}\mathcal{A}_{5/2}\right) = \frac{3}{2} A_2^+\, e^{i\chi_2^+}, \end{aligned}$$

ϵ' at first order in CP violation

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

Motivation

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right] = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega\, \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\left(1 - \frac{1}{\omega}\, \frac{\mathrm{Im}A_2}{\mathrm{Im}A_0}\right)$$

$\mathrm{Im}A_2 = 0$ in the isospin limit $(m_u = m_d = e^2 = 0)$. But,

- $\Delta I = 1/2$ rule! $\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = 1/22$
- Chiral enhancement of EW Penguins!

SM estimates include them even when they break isospin

Beyond the order of magnitude? $\rightarrow$ Chirally unenhanced contributions are a must

In this work we assess them using $\chi\mathrm{PT}$, updating Cirigliano '03

The short-distance effective Lagrangian

$K \to \pi\pi$ transitions $\to \Delta S = 1 \to$ EW origin!

$$M_W \xrightarrow{\quad ? \quad} M_{K,\pi}$$

As far as you can, use perturbative (OPE) methods

$$M_W \longrightarrow m_b \longrightarrow m_c \longrightarrow \mu \xrightarrow{?} M_{K,\pi}$$

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^* \sum_{i=1}^{10} C_i(\mu_{\text{SD}})\, Q_i(\mu_{\text{SD}})$$ Buchalla '95

We work at NLO, updating inputs for the running and the matching

Chiral Lagrangians

- We do not know how to analytically evaluate $\langle(\pi\pi)|Q_i(\mu_{SD})|K\rangle$
- However, we know how Q_i transform under $SU(3)_L \times SU(3)_R$
- Enough to build their low-energy realization $\rightarrow \chi\mathrm{PT}$

$$\mathcal{L}_{\rm strong} = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + \sum_{i=1}^{10} L_i\, O_i^{p^4} + F^{-2}\sum_{i=1}^{90} X_i\, O_i^{p^6} + \mathcal{O}(p^8)$$

$$\begin{aligned}\mathcal{L}^{\Delta S=1} = {} & G_8\, F^4\,\langle \lambda D^\mu U^\dagger D_\mu U\rangle + G_8\, F^2 \sum_{i=1}^{22} N_i\, O_i^8 \\ & + G_{27}\, F^4 \left(L_{\mu 23} L_{11}^\mu + \tfrac{2}{3} L_{\mu 21} L_{13}^\mu\right) + G_{27}\, F^2 \sum_{i=1}^{28} D_i\, O_i^{27} + \mathcal{O}(G_F p^6)\end{aligned}$$

$$\mathcal{L}_{\rm elm} = e^2\, Z\, F^4\,\langle \mathcal{Q} U^\dagger \mathcal{Q} U\rangle + e^2\, F^2 \sum_{i=1}^{14} K_i\, O_i^{e^2 p^2} + \mathcal{O}(e^2 p^4)$$

$$\mathcal{L}_{\rm EW}^{\Delta S=1} = e^2\, G_8\, g_{\rm ewk}\, F^6\,\langle \lambda U^\dagger \mathcal{Q} U\rangle + e^2\, G_8\, F^4 \sum_{i=1}^{14} Z_i\, O_i^{EW} + \mathcal{O}(G_F e^2 p^4)$$

LO amplitudes

$e^2 = m_u - m_d = 0$ limit: a $\mathcal{L}^{\Delta S=1}$ insertion needed! $\mathcal{O}(G_F p^2)$

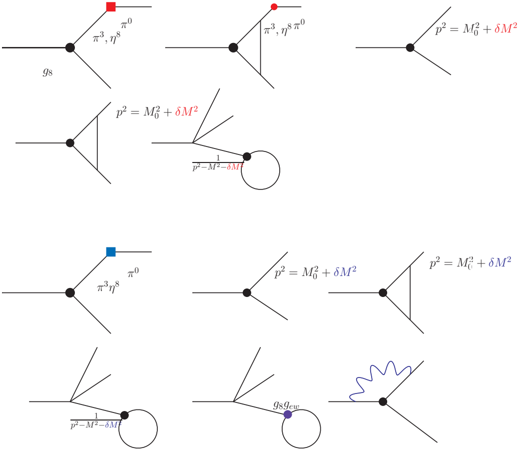
$$\begin{aligned} \mathcal{A}_{1/2} &= -\sqrt{2}\, G_8 F \left[\left(M_K^2 - M_\pi^2\right) \right] - \frac{\sqrt{2}}{9} G_{27} F \left(M_K^2 - M_\pi^2\right) \\ \mathcal{A}_{3/2} &= -\frac{10}{9} G_{27} F \left(M_K^2 - M_\pi^2\right) \\ \mathcal{A}_{5/2} &= 0 \end{aligned}$$

IB pieces

- $\pi^0 - \eta$ mixing. $\mathcal{M}_{12}(\pi_3, \eta_8) \neq 0! \rightarrow$ rotate
- Mass corrections $M_{K^\pm}^2 = M_K^2 - \frac{4\,\varepsilon^{(2)}}{\sqrt{3}} B_0(m_s - \widehat{m}) + 2\, e^2 Z F^2$
- $\mathcal{L}^{\Delta S=1}\ (\mathcal{O}(G_F p^2)) \rightarrow \mathcal{L}_{\mathrm{EW}}^{\Delta S=1}\ (\mathcal{O}(G_F e^2 p^0))$

$G_{27} \ll G_8 \rightarrow$ We neglect IB (Isospin Breaking) from the 27 part

IB at NLO



$$\mathcal{A}_n = -G_{27}\, F_\pi \left(M_K^2 - M_\pi^2\right) \mathcal{A}_n^{(27)} - G_8\, F_\pi \left(M_K^2 - M_\pi^2\right) \left[\mathcal{A}_n^{(8)} + \varepsilon^{(2)}\, \mathcal{A}_n^{(\varepsilon)}\right]$$
$$+ e^2\, G_8\, F_\pi^3 \left[\mathcal{A}_n^{(\gamma)} + Z\, \mathcal{A}_n^{(Z)} + g_{\rm ewk}\, \mathcal{A}_n^{(g)}\right]$$

Low-energy constants

Only remaining piece for a very precise evaluation: The LECs!

- LECs encode the SD information $\rightarrow$ Take $\mathcal{L}_{\text{eff}}^{\Delta S=1}(C_i(\mu)Q_i(\mu))$ and match
- Nonperturbative analytic matching??
- Use large-N_c limit: $\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle (1 + \mathcal{O}(\frac{1}{N_c})) \rightarrow \langle J \rangle_{\text{quarks}} = \langle J \rangle_{\text{chiral}}$

- $\{C_i(\mu_{SD}), m_{s,d}(\mu_{SD}), L_i, K_i\} \rightarrow \{g_8^\infty, g_{27}^\infty, e^2(g_8 g_{ewk})^\infty\}$
- $\{C_i(\mu_{SD}), m_{s,d}(\mu_{SD}), L_i, K_i, X_i\} \rightarrow \{(g_{27}D_i)^\infty, (g_8 N_i)^\infty, (g_8 Z_i)^\infty\}$

Updates:

- $C_i(\mu_{SD})$. Inputs from PDG, FLAG'19, ATLAS
- $m_{s,d}$ from FLAG '19
- L_i from FLAG '19, Bijnens '14
- K_i (mostly) from Ananthanarayan '04
- X_i from Cirigliano '06 (+ reassessment of some resonance parameters)

Fitting the CP even parts of the LO couplings

$$\frac{\mathrm{Re}g_8}{\mathrm{Re}g_{27}} \sim 5\,\frac{\mathrm{Re}g_8^\infty}{\mathrm{Re}g_{27}^\infty}$$

In the CP even sector, $\mathcal{O}\left(\frac{1}{N_c}\right)$ correction cannot be neglected in the LO couplings

They encode key information about operator running (see next talk)

We can fit them to data (together to the phase shift difference)

	LO fit	NLO fit
$\mathrm{Re}g_8$	$5.002 \pm 0.002_{\,\mathrm{exp}} \,{}^{+\,0.013}_{-\,0.006}\,{}_{\mu_{\mathrm{SD}}}$	$3.581 \pm 0.001_{\,\mathrm{exp}} \,{}^{+\,0.144}_{-\,0.141}\,{}_{\nu_\chi} \,{}^{+\,0.024}_{-\,0.008}\,{}_{\mu_{\mathrm{SD}}}$
$\mathrm{Re}g_{27}$	$0.251 \pm 0.001_{\,\mathrm{exp}} \,{}^{+\,0.011}_{-\,0.004}\,{}_{\mu_{\mathrm{SD}}}$	$0.296 \pm 0.001_{\,\mathrm{exp}} \,{}^{+\,0.000}_{-\,0.001}\,{}_{\nu_\chi} \,{}^{+\,0.010}_{-\,0.003}\,{}_{\mu_{\mathrm{SD}}}$
$\chi_0 - \chi_2(^\circ)$	$47.97 \pm 0.92_{\mathrm{exp}}{}^{+0.12}_{-0.22}{}_{\mu_{\mathrm{SD}}}$	$51.395 \pm 0.806_{\mathrm{exp}}{}^{+1.033}_{-1.041}{}_{\nu_\chi}{}^{+0.033}_{-0.007}{}_{\mu_{\mathrm{SD}}}$

Isospin breaking parameters in ϵ'/ϵ

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

$$\epsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2-\chi_0)}\, \omega_+ \left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\left(1-\Omega_{\mathrm{eff}}\right) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right]$$

$$\Omega_{\mathrm{IB}} = \frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_2^{(0)}} \cdot \frac{\mathrm{Im}A_2^{\mathrm{non-emp}}}{\mathrm{Im}A_0^{(0)}} \qquad (A_I^{(0)} = \text{isopin limit})$$

$$\Delta_0 = \frac{\mathrm{Im}A_0}{\mathrm{Im}A_0^{(0)}}\,\frac{\mathrm{Re}A_0^{(0)}}{\mathrm{Re}A_0} - 1$$

$$\omega_+ = \frac{\mathrm{Re}A_2^{+}}{\mathrm{Re}A_0} = \omega\left\{1 + f_{5/2}\right\}$$

$$\Omega_{\mathrm{eff}} = \Omega_{\mathrm{IB}} - \Delta_0 - f_{5/2}$$

Results

$$\Omega_{\text{eff}} = \Omega_{\text{IB}} - \Delta_0 - f_{5/2}$$

10^{-2}	$\alpha = 0$ LO	$\alpha = 0$ NLO	$\alpha \neq 0$ LO	$\alpha \neq 0$ NLO
Ω_{IB}	13.7	$17.1^{+8.4}_{-8.3}$	19.6 ± 4.8	26.0 ± 8.2
Δ_0	-0.002	-0.51 ± 0.12	5.6 ± 1.6	$5.7^{+1.7}_{-1.6}$
$f_{5/2}$	0	0	0	$8.2^{+2.4}_{-2.5}$
Ω_{eff}	13.7	$17.6^{+8.5}_{-8.4}$	14.0 ± 4.0	$12.1^{+9.0}_{-8.8}$

Results

$$\Omega_{\rm eff} = \Omega_{\rm IB} - \Delta_0 - f_{5/2}$$

10^{-2}	$\alpha = 0$		$\alpha \neq 0$	
	LO	NLO	LO	NLO
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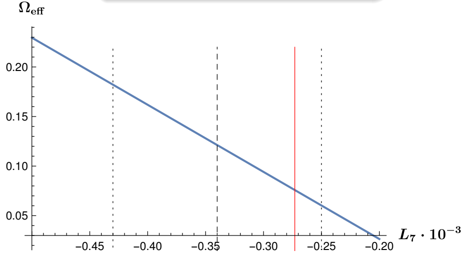
$\Omega_{\rm eff}$	13.7	$17.6^{+8.5}_{-8.4}$	14.0 ± 4.0	$12.1^{+9.0}_{-8.8}$

Set-up	Δ_0	$f_{5/2}$	$\Omega_{\rm IB}$	$\Omega_{\rm eff}$
Central	0.0565	0.0820	0.260	0.121
$\sigma_{\mu_{\rm SD}}$	$^{+0.0066}_{-0.0015}$	$^{+0.0003}_{-0.0011}$	$^{+0.008}_{-0.002}$	$^{+0.001}_{-0.000}$
σ_{ν_χ}	0.0017	$^{+0.0232}_{-0.0244}$	0.034	$^{+0.057}_{-0.055}$
$\sigma_{\rm WC}$	0.0067	0.0009	0.001	0.004
$\sigma_{L_{5,8}}$	0.0136	0.0017	0.040	0.029
σ_{L_7}	0.0011	0.0000	0.060	0.061
σ_{K_i}	0.0019	0.0031	0.018	0.013
σ_{X_i}	0.0021	0.0003	0.003	0.005

Results

$$\Omega_{eff} = (12.1^{+9.0}_{-8.8}) \cdot 10^{-2}$$



Those analyses that include in A_0 the EM penguin contributions should subtract them from Δ_0 :

$$\hat{\Omega}_{\rm eff} \equiv \Omega_{\rm IB} - \Delta_0|_{\alpha=0} - f_{5/2} = (18.3^{+9.6}_{-9.5}) \cdot 10^{-2}$$

Conclusions

$\Delta I = 1/2$ rule makes compulsory assessing isospin breaking corrections in ϵ'/ϵ

Main limitation of analytic methods: Matching of LECs beyond $N_c \to \infty$

Ω_{IB} are $\sim 25\%$ compared to the leading isospin conserving pieces

$\Omega_{eff} = (12.1^{+9.0}_{-8.8}) \cdot 10^{-2}$

Consequence for ϵ'/ϵ? See next talk!