$K \rightarrow \pi \nu \bar{\nu}$ and new physics in the neutrinos

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based on:
X-G He, J. Tandean and G.V. JHEP 1907 (2019) 022
new physics in neutrinos and $K \rightarrow \pi\nu\bar{\nu}$

- SM calculations of $K \rightarrow \pi\nu\bar{\nu}$ are very clean, so precise comparisons to experiment are eventually expected.

- Here we consider somewhat unusual extensions of the SM that may affect these rare modes through the unobserved neutrinos.

- This is in part motivated by the charged B anomalies in $R(D)$, $R(D^*)$.
  - New light sterile neutrino possible explanation.

- Also motivated by a renewed interest in charged lepton flavour violation experiments.
  - SU(2) of SM relates it to neutrino lepton flavour violation.
\[ K \rightarrow \pi\nu\bar{\nu}: \text{SM and ‘usual’ extensions} \]

- adding NP to the SM

\[ \mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W^2} V_{ts}^* V_{td} s \gamma_\mu d \left( (X(x_t) + X_N) \sum_\ell \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell \right) \]

SM (top) ‘usual’ NP

usually related to \( \epsilon, \epsilon' \)
usual predictions/constraints...

- there is interference between SM and NP
- in principle can cover all the allowed window
- green curve: same phase as in SM
- other constraints limit the allowed regions and are model dependent
- this type of NP can’t violate the GN bound

![Graph showing allowed and forbidden regions for $B(K_L \to \pi^0 \nu \bar{\nu})$ and $B(K^+ \to \pi^+ \nu \bar{\nu})$. The green line illustrates the case $X_N$ real, the red line corresponds to $X_N$ having a phase equal to that of the $\lambda_t$ (central value) and the blue line to $X_N$ having a phase equal to minus that of the $\lambda_t$. For comparison the purple marks the SM $1\sigma$ region and the green marks the 90% c.l. from BNL-787 combined with BNL-949. Finally the vertical dashed red line marks a possible future limit for $B_{K^+}$ at 1.3 times the SM.]

**Fig. 1** New physics with lepton flavour conserving left-handed neutrinos. The green line illustrates the case $X_N$ real, the red line corresponds to $X_N$ having a phase equal to that of the $\lambda_t$ (central value) and the blue line to $X_N$ having a phase equal to minus that of the $\lambda_t$. For comparison the purple marks the SM $1\sigma$ region and the green marks the 90% c.l. from BNL-787 combined with BNL-949. Finally the vertical dashed red line marks a possible future limit for $B_{K^+}$ at 1.3 times the SM.

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But the neutrinos are not seen

• can have different neutrinos
  – additional light neutrinos (right-handed - sterile in SM)
  – different flavour neutrinos (lepton flavour violation)
\[ K \rightarrow \pi \nu \bar{\nu} \] additional neutrino

- New light sterile neutrino

\[ \mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W^2} V_{ts}^* V_{td} \bar{s} \gamma_\mu d \left( (X(x_t) + X_N) \sum_\ell \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell + \tilde{X}_N \bar{\nu}_R \gamma^\mu P_R \nu_R \right) \]

- No interference with SM, can only add to SM rates

Not directly related to \( \epsilon, \epsilon' \)

\[ |V_{bd}^R|^2 + |V_{tu}^R| \]

New parameters can be tiny
Additional light (sterile) neutrino

Note 'new' GN bound

same phase as SM top contribution

- SM model to 68.27%
- Arbitrary $|\tilde{X}|$ and $\phi$
- Arbitrary $|\tilde{X}|$, and $\phi = 0$
- Arbitrary $|\tilde{X}|$, and $\phi = \frac{\pi}{2} - \phi_{\lambda_t}$
- Arbitrary $|\tilde{X}|$, and $\phi = -\phi_{\lambda_t}$
- NA62 yesterday 90% c.l
an example of a model

- add a new light neutrino
- needs to be **sterile with respect to SM** to satisfy light neutrino counts. Postulate it is part of a right-handed doublet with the $\tau$
- Interacts with a $W^\prime, Z^\prime$
- needs to **mostly appear with a tau lepton** to satisfy observed patterns of LF universality (and this helps with BBN constraints)
- $W^\prime, Z^\prime$ couple **strongly** (weakly) to the **third** (first two) generations: avoids LHC constraints (low $\sigma_{\text{prod}}$)
- electroweak strength corrections to processes between a third generation fermion pair and one from 1st or 2nd generation
- one such neutrino already appears in models that single out the third generation

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(sd) → νₐνᵣ

neutral currents quark sector

• first 2 generations couple after rotation to mass eigenstates
• three (sets) of parameters come into play
• mixing between Z – Z' (tiny)
• right handed rotation angles (arbitrary but constrained)
• strength of the new interaction (can be large)
Strength of $g_R$ singles out third generation

- Perturbative unitarity: $g_R \lesssim 10 \, g_L$

  - From $Z \rightarrow \tau^+ \tau^-$ at LEP: $\left| \frac{g_R}{g_L} \xi_Z \right| \lesssim 3 \times 10^{-3}$

  - From other fits to LEP data: $g_R M_Z \sim g_L M_{Z'}$

- The $Z'$ ($W'$) can be much lighter than in other models because they evade LHC searches that do not use third generation fermions

- Can be made to satisfy all FCNC constraints, with room to accommodate deviations of EW strength in processes that involve a transition between a third generation fermion and a lighter one, such as $K \rightarrow \pi \nu \bar{\nu}$
**Flavour constraints**

- **Meson mixing**

- Specifically for kaons this implies

\[
\left| V_{Rbs} V_{Rbd}^\star \right| \lesssim 1.5 \times 10^{-4}
\]

- a combination of \( B_s \) mixing and \( B_d \) mixing with a few other assumptions leads to a stronger

\[
\left| V_{Rbs} V_{Rbd}^\star \right| \lesssim 6.7 \times 10^{-7}
\]

- \( B_s \) mixing also limits the new loop contributions, implying for the Inami-Lim function

\[
I(x_t, x_H) \lesssim \mathcal{O}(10)
\]

(this one depends on complicated details of the model, hidden in the \( x_H \))

- FCNC constraints can be summarised by:

\[
V_{Rbi}^D \sim \delta_{bi}
\]
collecting for $K \rightarrow \pi \nu \bar{\nu}$

- the effective coupling is

\[
\tilde{X} = - \left( \frac{M_Z^2}{M_Z^2} \frac{g_R^2}{g_L^2} \right) \left( \frac{s_W^2}{2} I(\lambda_t, \lambda_H) + \frac{\pi s_W^4}{\alpha} \frac{V^d_{Rbs} V^d_{Rbd}}{V^d_{ts} V^d_{td}} \right)
\]

- loop term in second bracket can be $\sim \mathcal{O}(1)$
  - by construction this one has the same phase as the SM top-quark contribution
  - its magnitude is very model dependent (details of the scalar sector and symmetry breaking)

- FCNC tree-level term at most a few percent
  - too small to contribute when made to satisfy B mixing constraints

- first bracket can be order 1 from LEP constraints, has it been improved? (no)
third generation limits on $Z'$

- CMS uses ‘TAT’ (topcolor assisted technicolor) model with
  
  \[ \sigma(pp \to Z') \mathcal{B}(Z' \to \tau\tau)_{TAT} \sim \frac{1}{3} \sigma(pp \to Z') \mathcal{B}(Z' \to \tau\tau)_{SSM} \]

![Graph showing limits on $m(Z')$ vs. $\sigma(pp \to Z') \times \mathcal{B}(Z' \to \tau\tau)$](image)

$2.2 \text{ fb}^{-1}$ (13 TeV)

CMS

- Observed
- Expected $\pm$ 1 s.d.
- Expected $\pm$ 2 s.d.

$\sigma_{NLO}^{SSM}$

$\sigma_{NLO}^{TAT}$

$\mathcal{B}(Z' \to \tau\tau)$

Combined ($\tau_h \tau_h$, $\tau_\mu \tau_h$, $\tau_e \tau_h$, $\tau_e \tau_\mu$)

$m_{Z'} > 1700 \text{ GeV}$

are these limits on $Z'$ mass general? (no)

- can it be 900 GeV? need $(\sigma \mathcal{B}) \sim 3 \times 10^{-2} (\sigma \mathcal{B})_{SSM}$

• at TeV masses the $Z' \rightarrow t\bar{t}$ channel is open

• if third generation dominates then $\mathcal{B}(Z' \rightarrow \tau^+\tau^-) \rightarrow 1/8$

• however production cross section is very suppressed

\[
\frac{g_L}{2 \cos \theta_W} \gamma_\mu (g_V + g_A \gamma_5)
\]

\[
\frac{g_R}{2 \cos \theta_W} \sin \theta_W \ V_{Rtu}^U * V_{Rtu}^U \ \gamma_\mu (1 + \gamma_5)
\]

\[
\sigma \times B(Z' \rightarrow \tau \tau) \sim (\sigma \times B(Z' \rightarrow \tau \tau))_{SSM} \ \frac{12.5 \%}{3.37 \%} \ \frac{V_{Rtu}^U}{V_{Rtu}^U} \ \frac{|V_{Rtu}^U|^2}{g_L^2} \ \text{or} \ \xi_Z^2.
\]

• likely dominated by terms not calculated, but very small
Number of light neutrinos

• there is one light right handed neutrino! why not seen at LEP?
  – LEP standard result \( n = 2.9840 \pm 0.0082 \)
    • assumes Lepton universality and no new particles
  – direct limit \( n = 3.00 \pm 0.08 \)

• basically it has to couple through \( Z - Z' \) mixing

\[
\Gamma(Z \rightarrow \nu_R^3 \bar{\nu}_R^3) = \frac{1}{24} \frac{\alpha}{\cos^2 \theta_W} \frac{g_R^2}{g_L^2} \xi_Z^2 M_Z,
\]

\[
\Gamma(Z \rightarrow \nu_R^3 \bar{\nu}_R^3) < 3 \times 10^{-4} \text{ MeV}
\]

• the limit on new physics from the invisible \( Z \) width
  – there is 13.3 MeV error in this measurement so our right handed neutrino is unobservable by LEP
• it is known that
\[ \Delta N_{\text{eff}} < \begin{cases} 0.28 & \text{for } H_0 = 68.7^{+0.6}_{-0.7} \text{ km/s/Mpc} \\ 0.77 & \text{for } H_0 = 71.3^{+1.9}_{-2.2} \text{ km/s/Mpc} \end{cases} \]

• the new right handed interaction is stronger that weak, at least for third generation

• The \( Z', \ W' \) can scatter with the new light neutrino and bring it to thermal equilibrium with SM particles that could give \( \Delta N_{\text{eff}} \sim 1 \)

• our model only lets them scatter with tau so the decoupling temperature is \( m_\tau \) whereas \( T_{\text{BBN}} \) is of order 1 MeV, so there is a suppression factor

• also known that neutrinos that decouple above the QCD phase transition (200 MeV) only contribute an effective 0.1 per species

number of light neutrinos: cosmology?
this neutrino could explain charged B anomalies

Xiao-Gang He, G. V.: PLB779 (2018), 52
$K \rightarrow \pi \nu \bar{\nu}$ LFV

- Lepton flavour conserving

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W^2} V_{ts}^* V_{td} s_\gamma d \left( (X(x_t) + X_N) \sum_{\ell} \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell + \tilde{X}_N \bar{\nu}_R \gamma^\mu P_R \nu_R \right)$$

SM (top) \quad ‘usual’ NP \quad new RH $\nu$

- Lepton flavour violating (e.g. leptoquark exchange) will in general also produce LFC couplings

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi s_W^2} \bar{s}_\gamma d \left( \sum_{\ell} \left( V_{ts}^* V_{td} X_t + \lambda^5 W_{\ell\ell} \right) \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell + \lambda^5 \sum_{i \neq j} W_{ij} \bar{\nu}_i \gamma^\mu P_L \nu_j \right)$$

SM (top) \quad LFC \quad LFV

- no interference between LFV and SM
$K \rightarrow \pi\nu\bar{\nu}$ constraints on LFV new physics

$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) \times 10^{-11} = \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \times 10^{-11}$$

$$\mathcal{B}_{K_L}(SM) + \frac{\tilde{\kappa}_+}{3} \sum_{i \neq j} \left| W_{ij} \right|^2$$

SM model to 68.27%

$|W_{\mu\ell}| \leq ||W_{\mu\ell}||$, arbitrary $\phi_{\mu\mu}$ and $\phi_{\mu\ell}$

$|W_{\mu\ell}| = ||W_{\mu\ell}||$, $\phi_{\mu\ell} = \pi - \phi_{\mu\mu}$

$|W_{\mu\ell}| = ||W_{\mu\ell}||$, $\phi_{\mu\ell} = -\phi_{\mu\mu}$

--- NA62 yesterday 90% c.l
Charged lepton flavour violation

• New physics at some high scale that respects the symmetries of the SM

• Effective theory at the electroweak symmetry breaking scale is written in terms of left-handed doublets

  • So left handed leptons enter as $l = \left( \begin{array}{c} \nu \\ \ell \end{array} \right)$ into operators such as:

$$\mathcal{L} \supset c_4 \; \bar{s} \gamma^\mu P_R d \; \bar{l} \gamma^\mu P_L l \; + \; \text{h.c.}$$

  • Which can give rise to both types of LFV decays

    - $K_L \rightarrow \mu^\pm e^\mp$ or $\Omega^- \rightarrow \Xi^- \mu^\pm e^\mp$

    - $K^+ \rightarrow \pi^+ \nu_{\mu e} \bar{\nu}_{e\mu}$
Dimension 6 effective Lagrangian

- LFV transitions between down-type quarks

\[ \mathcal{L}_{NP} = \frac{1}{\Lambda_{NP}^2} \left( \sum_{k=1}^{5} \mathcal{C}^{ijxy} \mathcal{Q}^k_{ijxy} + (\mathcal{C}^{ijxy}_6 \mathcal{Q}^k_{ijxy} + \text{H.c.}) \right) \]

- Several structures appear:
  - \( Q^{ijxy}_1 = \bar{q}_i \gamma^n q_j \bar{l}_x \gamma_\eta l_y \)
  - \( Q^{ijxy}_2 = \bar{q}_i \gamma^n \tau_l q_j \bar{l}_x \gamma_\eta \tau_l l_y \)
  - \( Q^{ijxy}_3 = \bar{d}_i \gamma^n d_j \bar{e}_x \gamma_\eta e_y \)
  - \( Q^{ijxy}_4 = \bar{d}_i \gamma^n d_j \bar{l}_x \gamma_\eta l_y \)
  - \( Q^{ijxy}_5 = \bar{q}_i \gamma^n q_j \bar{e}_x \gamma_\eta e_y \)
  - \( Q^{ijxy}_6 = \bar{l}_i \epsilon_j \bar{d}_x q_y \)

- Several structures appear:
  - \( Q_{1,4} \sim (\bar{\nu} \Gamma \nu + \ell \bar{\Gamma} \ell) \), \( Q_2 \sim (\bar{\nu} \Gamma \nu - \ell \bar{\Gamma} \ell) \)

- In terms of previous notation:
  - \( W_{\ell \ell'} \approx 9727 \left( \frac{1 \text{ TeV}}{\Lambda_{NP}} \right)^2 (c_1^{\ell \ell'} - c_2^{\ell \ell'} + c_4^{\ell \ell'}) \)

- \( Q_{3,5,6} \) do not produce neutrino pairs

- Generically different processes are complementary
LFV $K^+ \rightarrow \pi^+$ transitions

\[ c_i^{e\mu} = c_i^{\mu e}, \quad c_4 = 0 \]

\[ \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \leq 2.88 \times 10^{-10} \quad (BNL\ 787 - 949) \]

\[ \mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) \leq 1.85 \times 10^{-10} \quad (NA62\ \text{yesterday}) \]

SM + NP (NP can only add to SM in this case)

\[ \mathcal{B}(K^+ \rightarrow \pi^+e^-\mu^+) < 1.3 \times 10^{-11} \quad (BNL\ 865) \]

\[ \mu^-\text{ Au} \rightarrow e^-\text{ Au} < 7 \times 10^{-13} \quad (SINDRUM\ II) \]
• $K^+ \rightarrow \pi^+ e^- \mu^+$ depends on $(c_1 + c_2 + c_4)^{e\mu,\mu e}$

• whereas $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ depends on $(c_1 - c_2 + c_4)^{e\mu,\mu e}$
  – for these operators the charged LFV mode is more restrictive by a factor of about 4
  – a measurement of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at the SM level with 10% uncertainty will already make this mode more restrictive
  – of course even with current experimental limits the two are complementary
Charged lepton flavour violation

- lepton flavour indices include the third generation
  - in this case left handed leptons enter as $l_3 = \begin{pmatrix} \nu \tau \\ \tau \end{pmatrix}$
- The LFV lagrangian can have terms such as

$$\mathcal{L} \supset c_4 \bar{s} \gamma^\mu P_R d \bar{l}_3 \gamma_\mu P_L l_{1,2} + \text{h.c.}$$

- Which can give rise to both
  - $\tau^- \rightarrow e^- K_S$
  - $K^+ \rightarrow \pi^+ \nu_e \bar{\nu}_\tau$
- golden rare kaon modes can also compete with charged LFV tau-decay modes
$K \rightarrow \pi \nu \bar{\nu}$ vs CLFV $\tau$ decay

\[ c_i^{\tau e} = 0, \quad c_4^{e\tau} = 0 \]

\[ c_i^{\tau \mu} = 0, \quad c_2^{\mu\tau} = 0 \]

\[ \mathcal{B}(\tau^- \rightarrow e^- K_S) < 2.6 \times 10^{-8} \quad \text{(Belle)} \]
\[ \mathcal{B}(\tau^- \rightarrow \mu^- K_S) < 2.3 \times 10^{-8} \quad \text{(Belle)} \]

\[ \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 2.88 \times 10^{-10} \quad \text{(BNL 787 – 949)} \]
Conclusions

• we studied the effects of new physics affecting the neutrinos on the rare decay modes $K \rightarrow \pi \nu \bar{\nu}$

• these modes can provide significant constraints on new light sterile neutrinos that avoid LEP counts and BBN bounds

• these modes also provide constraints on lepton flavour violating new physics that are complementary to bounds from CLFV experiments
  – these extend to CLFV involving tau leptons

• both of these scenarios can only increase the $K \rightarrow \pi \nu \bar{\nu}$ rates as they don’t interfere with the SM